Shadow Banking and Market Discipline on Traditional Banks*

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Abstract

We present a model in which shadow banking arises endogenously and undermines market discipline on traditional banks. Demandable deposits impose market discipline: Without shadow banking, traditional banks optimally pursue a safe portfolio strategy to prevent early withdrawals. Shadow banking constitutes an alternative banking strategy that combines high risk-taking with early liquidation in times of crisis. In equilibrium, shadow banks expand until their liquidation causes a fire-sale and exposes traditional banks to liquidity risk. Higher deposit rates in compensation for liquidity risk deter early withdrawals, undermining market discipline on traditional banks. Constrained-optimal policy interventions deter entry into shadow banking.

Keywords: Shadow banking; Financial crisis; Market discipline; Fire-sales

JEL codes: E44, E58, G01, G21, G23, G28

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1 Introduction

Recent decades have seen rapid growth in financial intermediation by non-bank entities based on a business model that combines highly-leveraged, short-term funding with risky long-term investments such as sub-prime mortgage lending. In the 2008 financial crisis, these "shadow banks" experienced a sudden dry-up in their funding and liquidated their assets. The ensuing turmoil quickly spread to traditional commercial banks which reduced their credit to the private sector. This has contributed to the deepest recession since the Great Depression, raising two important questions.[1] First, what circumstances and mechanisms lead to the emergence and expansion of shadow banking? Second, how does shadow banking affect the portfolio and funding strategies of traditional banks?

In this paper, we propose a framework where depositors may withdraw their deposits early in reaction to crises.[2] These "early withdrawals" constitute an optimal response to adverse changes in banks' solvency prospects and become a source of market discipline: Traditional banks optimally commit to a safe portfolio strategy to prevent early withdrawals. When commitment is costly, shadow banking emerges as an alternative banking strategy that combines a risky portfolio strategy with early withdrawals in times of crisis. To the best of our knowledge, this paper presents the first model where shadow and traditional banks coexist and interact without regulatory arbitrage or direct contractual linkages.[3]

We bring this theoretical model to bear on the 2008 financial crisis, its transmission to the traditional banking sector, and policy debates on shadow banking. In doing so, we account for two key empirical facts: Shadow banks faced a sudden contraction in funding and the liquidation of their assets caused a fire-sale. Traditional banks did not suffer from withdrawals, experienced a sharp rise in their funding costs, and re-allocated their portfolios towards safe and liquid assets.

We develop our analysis by specifying a model with households, outside investors, and a banking sector. Banks collect deposits from households and choose their portfolios of safe, risky, and liquid assets and households lend to banks on terms that depend on their solvency prospects. Following news signals that revise expected asset returns, households decide whether to withdraw their deposits early and banks trade assets in a secondary market with outside

[1] See Brunnermeier (2009) for an in-depth analysis of the financial crisis and its channels of transmission. For detailed descriptions of shadow banking activities, see Adrian and Ashcraft (2012) and Pozsar et al. (2013).

[2] Early withdrawals are distinct from bank-runs à la Diamond and Dybvig (1983). For an early withdrawal to take place, depositors must find it optimal to withdraw their funds even when no other depositor does so.

[3] While regulatory arbitrage through off-balance sheet exposures was a prominent feature of the 2008 financial crisis, many traditional commercial banks without such exposures also became troubled. Notably, the two largest US bank failures during the crisis, that of Washington Mutual Bank and IndyMac, and the failure of Northern Rock in the UK were due to on-balance sheet mortgage losses and a dry-up of wholesale funding (Comlay and Stempel, 2008; Goodhart, 2016; Poirier and Younglai, 2008; Shin, 2009).

investors.[4] As in Stein (2012), fire-sales of assets to outside investors crowd out investment in productive outside projects.

A key element in the model is the equilibrium relationship between fire-sales and bank strategies. During early withdrawals, shadow banks liquidate their assets in the secondary market to repay their depositors. Fire-sale externalities then create strategic substitutabilities in banks' decision to pursue a shadow banking strategy: As the shadow banking sector grows larger, its liquidation causes a deeper fire-sale, reducing the payoff from shadow banking relative to traditional banking, and bringing about an equilibrium where shadow and traditional banks coexist.

Analyzing the effects of fire-sales on market discipline yields important insights for vulnerability to financial crises. A key intuition is that low deposit rates strengthen early withdrawal incentives as depositors stand to lose less in terms of interest foregone. Therefore, market discipline on traditional banks is strong whenever deposit rates are low. This is precisely the case when there are no fire-sales: Interest rates on deposits are low and market discipline drives traditional banks to commit to a safe portfolio strategy. With the prospect of a fire-sale, on the other hand, depositors demand higher rates in compensation for liquidity risk.[5] This weakens early withdrawal incentives and allows traditional banks to risk-shift by pursuing risky portfolios that may leave them in default. In equilibrium, the shadow banking sector expands to a size where its liquidation causes a fire-sale and undermines market discipline through this mechanism.

The model also provides novel and important insights for welfare analysis and policy design. There are two key financial frictions in the model economy: First, banks are funded with demandable deposits, such that their ability to provide financial intermediation hinges on their access to secondary market liquidity.[6] Second, fire-sales are socially costly as they divert outside investor funds from productive projects and have negative externalities on banks.[7] Particularly,

[4] Traditional banks are not protected by deposit insurance guarantees. In this context, the model can be interpreted as an analysis of market discipline arising from short-term, non-depository liabilities or a counterfactual in which shadow banking emerges even without any opportunities for regulatory arbitrage.

[5] What is important for this mechanism is that illiquidity leads to possible losses for depositors that may not be avoided with certainty through an early withdrawal. We introduce this through idiosyncratic liquidity shocks that hit banks simultaneously with the news signal. The restriction on timing arises naturally in an environment with market discipline: When depositors may pre-empt liquidity shocks by withdrawing early, the equilibrium collapses to a solution similar to a Diamond-Dybvig bank-run.

[6] In the interest of brevity, we do not explicitly model the contracting problem that gives rise to demandable deposits, but instead refer to a rich literature that provides several reasons, including liquidity insurance (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005), imposing market discipline (Calomiris and Kahn, 1991; Diamond and Rajan, 2000) and the provision of liquid and transactable claims (Dang et al., 2017; DeAngelo and Stulz, 2015; Gorton and Pennacchi, 1990) among many others. Moreover, our qualitative results are not sensitive to allowing for equity-funding provided it has convex costs.

[7] Fire-sale externalities lead to inefficiencies despite their pecuniary nature because of incomplete markets (Geanakoplos and Polemarchakis, 1986). See also Stein (2012), Davila and Korinek (2018) and Lorenzoni (2008) for other studies where this is the case.

the interplay between liquidity risk and market discipline constitutes a novel source of fire-sale externalities, as it distorts the investment strategies of traditional banks towards assets with lower net present value (NPV).

Entry into shadow banking increases financial intermediation at the expense of exacerbating fire-sales. While the existence of shadow banking is welfare-increasing, the sector grows too large in equilibrium compared to the constrained-efficient due to fire-sale externalities. The optimal policy is a Pigouvian tax on shadow bank activities, or equivalently a subsidy to traditional banks. These interventions deter banks from pursuing a shadow banking strategy and can be used to reduce the size of the shadow banking sector to the constrained-efficient level.

We also show that outcomes from policy interventions differ substantially when adjustments in the size of the shadow banking sector are taken into account. Notably, taxation of secondary market transactions exacerbates the fire-sale at a given sector size, but has no impact on secondary market prices after the resulting contraction in shadow banking. Differentially taxing the sale of risky and safe assets in the secondary market then constitutes a second-best intervention: A tax on risky assets reduces entry into shadow banking, alleviates the fire-sale on safe assets and curtails liquidity risk on traditional banks. This intervention moves the equilibrium closer to the constrained-efficient allocation, but also gives rise to time inconsistency issues. Since entry into shadow banking takes place before the news signal, policymakers face a temptation to eliminate the tax once the fire-sale is underway.

We build upon a growing literature that provides micro-foundations for the existence and growth of shadow banking. The rapid growth of shadow banking in recent decades is well documented (see e.g. Claessens et al., 2012; Pozsar et al., 2013). Gennaioli et al. (2013) emphasize the ability of shadow banks to generate safe assets through securitization. In a similar vein, Moreira and Savov (2017) focus on liquidity transformation whereby shadow banks create money-like assets that become illiquid in times of high uncertainty. Harris et al. (2014), Plantin (2015) and Ordoñez (2017) highlight the role of regulatory arbitrage as a primary cause of shadow banking. In these studies, regulatory constraints restrict intermediation by traditional banks and create regulatory arbitrage opportunities for unregulated shadow banks.

Our contribution to this literature is to show that shadow banking may arise as an equilibrium outcome without any advantages in intermediation technologies or opportunities for regulatory arbitrage. We assume, realistically, that commitment is costly and show that ex-ante identical banks endogenously cluster into traditional and shadow banking strategies, where the former optimally pay a lump-sum cost to commit to a safe portfolio strategy. In this context, commitment costs reflect any costly action undertaken by banks to resolve asymmetric information issues with their depositors, such as providing detailed balance sheet reports and eschewing opaque intermediation processes like securitization.[8]

This paper is also related to a recent strand of literature that analyzes interactions between traditional and shadow banks. Gorton and Souleles (2007), Luck and Schempp (2014) and Gornicka (2016) develop models where traditional banks gain off-balance sheet exposure to shadow banks by extending implicit guarantees. In a framework where shadow and traditional banks have access to a common pool of liquidity, Hanson et al. (2015) show that traditional banks have a comparative advantage in holding illiquid assets with low fundamental risk when they are protected by deposit insurance guarantees. In a similar environment, Luck and Schempp (2018) show that shadow banking may lead to inefficiently low liquidity creation.

We contribute to this literature by deriving an equilibrium where shadow and traditional banks coexist and interact without regulatory arbitrage or (implicit) contractual linkages. By doing this, we provide novel insights about how shadow banking affects market discipline on traditional banks. Particularly, we show that the comparative advantage of traditional banks in holding illiquid safe assets can stem from market discipline instead of deposit insurance. However, market discipline may only be sustained in an equilibrium with free entry into shadow banking when there are policy interventions to offset fire-sale externalities.

This paper also contributes to a rich literature on the disciplining role of demandable debt that dates back to Calomiris and Kahn (1991) and Diamond and Rajan (2000). Cheng and Milbradt (2012) analyze optimal debt maturity when there is a trade-off between inefficient liquidations and market discipline. Eisenbach (2017) finds that market discipline may be insufficiently strong in good aggregate states, while there are excessive liquidations in bad states. Different to these studies, we find that vulnerability to liquidity shocks leads to an erosion of market discipline and a rise in solvency risk. Our findings are similar to Diamond and Rajan (2005), who show that liquidity problems hamper banks' fundamental solvency, with one important distinction: it is the anticipation of liquidity risk that undermines market discipline and creates solvency risk in our model, rather than its realization.

As vulnerability to liquidity shocks arises when there is a fire-sale, our findings on market discipline also contribute to a recent literature on fire-sale externalities. Stein (2012) and Davila and Korinek (2018) show that negative fire-sale externalities may arise from price-dependent financial constraints, while Lorenzoni (2008) focuses on the reallocation of funds to less productive agents during fire-sales. We propose a novel source of fire-sale externalities in the negative impact on market discipline. This externality leads to a relaxation of solvency constraints which allows traditional banks to expand their balance sheets. However, it also skews the composition of their investments towards assets with lower NPV, hence reducing the

[8] Costly commitment differs from regulation in two important ways. First, the ability to commit does not directly constrain traditional banks' leverage or risk-taking choices, both of which are optimally determined and dependent on market discipline. Second, traditional banks are not protected by deposit insurance and may hence be vulnerable to liquidity risk depending on prices in secondary markets.

extent of their financial intermediation.

Our modelling choices draw heavily from observations on the 2008 financial crisis. Acharya et al. (2013) document the collapse in the market for asset-backed commercial papers at the onset of the crisis while Gorton and Metrick (2012) and Krishnamurty et al. (2014) show a similar contraction in repo markets. Together, these two markets accounted for the vast majority of funding for shadow banks. Covitz et al. (2013) find that the dry-up in funding for shadow banks was associated with a rise in macro-financial risks such as uncertainty about sub-prime mortgages values. In our model, the definitive characteristic of the shadow banking strategy is its vulnerability to early withdrawals that closely resemble these events. Consistent with Covitz et al. (2013), early withdrawals are triggered by a negative revision in expected asset payoffs. Krishnamurthy (2010), Merrill et al. (2012), and Mitchell and Pulvino (2012) provide empirical evidence for fire-sales during the financial crisis. We argue that fire-sales caused by the liquidation of shadow banks play a key role in the spread of financial instability to traditional banks.

We proceed as follows: Section 2 presents motivating evidence from the 2008 financial crisis. Section 3 describes the core mechanisms of the model in a simple framework without liquidity risk. Section 4 presents the complete model with liquidity risk. Section 5 provides the numerical results. Section 6 conducts policy analysis. Section 7 concludes.

2 Motivating evidence

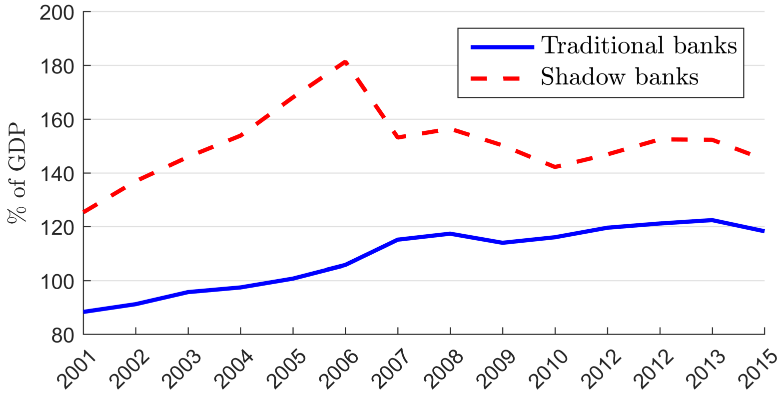
In this section, we present four key stylized facts about shadow banking and the 2008 financial crisis in the United States.

Fact 1. *The shadow banking sector expanded rapidly until its collapse in 2007. The traditional banking sector grew at a slower rate but did not suffer from a collapse.*

Figure 1 shows that shadow bank assets expanded from 125% of GDP in 2002 to over 180% at its peak in 2007. This rapid growth came to an end with a collapse during the financial crisis. The traditional banking sector also expanded but at a relatively modest pace. Although there was no contraction in traditional banking during the crisis, the shadow banking sector remained larger throughout the period 2002-2015.

In our model, the size of the shadow banking sector is endogenously determined through free entry and may rise for two reasons: an increase in the liquidity available in secondary markets and a rise in the cost of commitment. Recent financial innovations such as securitization may have increased both the (opportunity) cost of commitment and the thickness of secondary markets.

Figure 1: **Shadow and traditional bank assets**



Note: Financial assets when available, otherwise total assets. Traditional banks refer to all deposit-taking corporations. Shadow banks refer to all financial corporations that are not classified as central banks, banks, insurance corporations, pension funds, public financial institutions, or financial auxiliaries.

Source: Financial Stability Board (2017) and the World Bank

Fact 2. *At the onset of the financial crisis, shadow banks experienced a sudden dry-up of funding and liquidated their assets.*

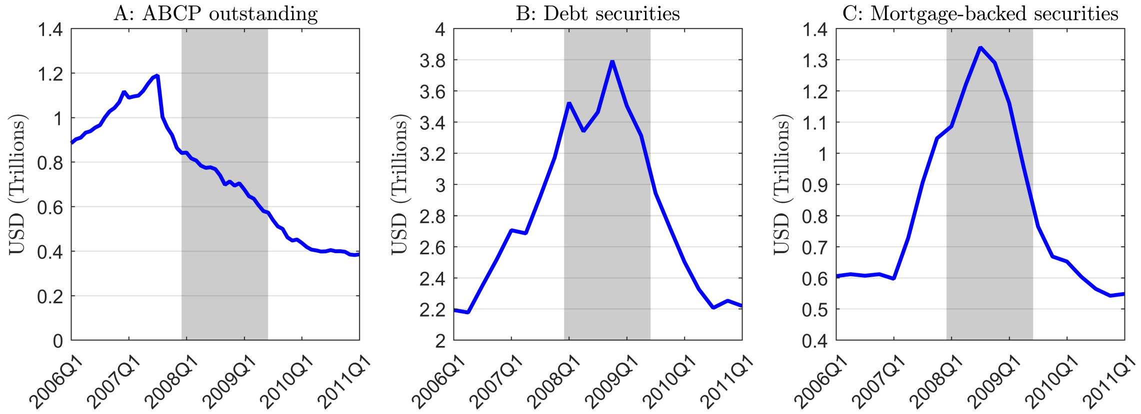
At their peak of $1.2 trillion in July 2007, asset-backed commercial papers (ABCP) were the largest money market instrument in the United States and constituted the main source of funding for shadow banks. Following rising mortgage default rates and the suspension of withdrawals by a number of funds, the market for ABCP contracted by $350 billion in the second half of 2007 and a further $400 billion by the end of 2009 (see Figure 2 Panel A). Faced with this sudden contraction in funding, shadow banks liquidated their asset holdings. Panels B and C show that shadow banks sold $1.5 trillions of debt securities between 2008Q3 and 2010Q1, approximately half of which were mortgage-backed.

In our framework, shadow banks face early withdrawals following adverse news about future macro-financial fundamentals. During a withdrawal, shadow banks are forced to liquidate their assets in the secondary market. Since this generates an adjustment in secondary market prices, there are consequences for traditional banks even in the absence of implicit guarantees or any other form of direct exposure to the shadow banking sector.

Fact 3. *Spreads between private debt securities and Treasury bonds increased sharply during the crisis. Securities with higher perceived risk experienced greater increases in spreads.*

Figure 3 shows the evolution of yield spreads between corporate bonds (grouped according to their credit rating) and Treasury bonds of comparable maturity. It is notable that spreads peaked in the last quarter of 2008, coinciding with the liquidation of shadow bank assets.

Figure 2: **ABCP markets and shadow bank assets**



Note: Shaded areas indicate US recessions. Shadow banks refer to money market mutual funds, security brokers and dealers, and issuers of asset-backed securities. Mortgage-backed securities refer to agency- and GSE-backed securities.

Source: Financial Accounts of the United States

Although there was a sharp rise in spreads for every rating category, the increase was greater for lower-rated corporate bonds. Our model generates changes in secondary market prices that are consistent with these movements in spreads. Because of limited liquidity in secondary markets, the liquidation of shadow bank portfolios causes a fire-sale and (illiquid) assets trade at a significant discount. Portfolio re-allocation by traditional banks from risky to safe and liquid assets also contributes to the fire-sale. We provide evidence for and further discuss portfolio re-allocation under Fact 4.

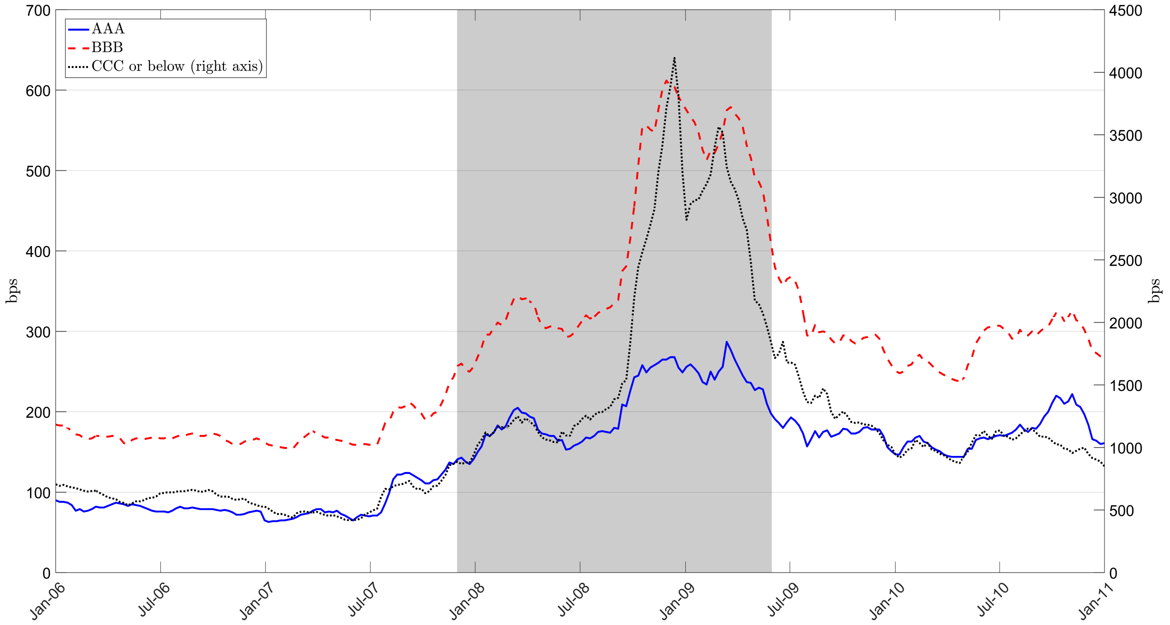
Fact 4. *Traditional banks re-allocated their portfolios toward safe and liquid assets and increased their liabilities. At the same time, they faced a rise in their funding costs.*

Panel A of Figure 4 shows that traditional banks increased their liabilities during the crisis. Although the increase in liabilities was driven by Federal Reserve funding and a rise in deposits protected by deposit insurance guarantees, there was also no decline in large uninsured time deposits. The implication is that traditional banks were perceived to be safe enough to preclude withdrawals even in the absence of government guarantees.

Changes in traditional bank portfolios shown in Panel B lend further support to this interpretation. During the crisis, traditional banks re-allocated their portfolios from bank credit (which is risky and illiquid) to cash assets and government securities (which are liquid and safe). Mortgage-backed securities that were at the epicenter of the financial crisis accounted for only 12% of total assets and this did not change significantly over the crisis.

While traditional banks did not experience a dry-up in funding, there was a significant increase in their funding costs. Panel C shows that the TED spread increased to 200 basis

Figure 3: **Spreads on corporate bonds**



Note: Shaded areas indicate US recessions. AAA (BBB) refers to the spread between Moody's Seasoned Aaa (Baa) Corporate Bonds and 10-Year Treasury bonds (constant maturity). CCC or below refers to the option-adjusted spread between the Bank of America Merrill Lynch US Corporate C Index and a spot Treasury curve.

Source: Moody's Investor Services, Bank of America Merrill Lynch

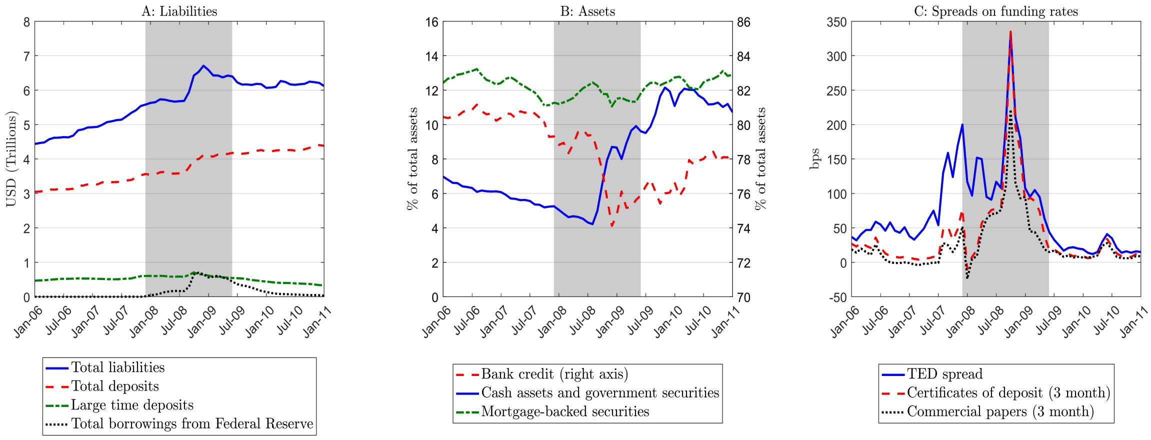
points in December 2007 and 335 basis points in October 2008. The spreads on certificates of deposits and commercial papers also increased significantly.

In the following sections, we show that the interaction between market discipline and liquidity risk may account for these observations. In our framework, market discipline stems from the depositors' ability to withdraw their funds early. To avoid an early withdrawal, traditional banks commit to a minimum recovery rate (in other words, a maximum loss given default) on deposits, which we refer to as a "no-withdrawal constraint", and respond to bad news by re-allocating their portfolios towards safe assets.

The fire-sale discussed in Fact 3 interacts with this mechanism through two distinct transmission channels. First, since risky assets are discounted to a greater extent than safe assets, it reduces traditional banks' capacity for portfolio re-allocation. To satisfy the no-withdrawal constraint, traditional banks are then forced to reduce their risky asset holdings ex-ante. This creates an excess return that increases the expected payoff associated with traditional banking. As the extent of the fire-sale is proportionate to the size of the shadow banking sector, equilibrium is achieved through this mechanism. In the next section, we analyze this mechanism and the properties of the equilibrium in a simple model without liquidity risk.

Second, the fire-sale leaves traditional banks illiquid and vulnerable to liquidity shocks. In Section 4 and 5, we extend the model to allow for liquidity risk and show that this undermines

Figure 4: **Traditional banks during the crisis**



Note: Shaded areas indicate US recessions. Traditional banks refers to "large domestically chartered commercial banks" which are defined as the top 25 domestically chartered commercial banks, ranked by domestic assets. Cash assets and govenment securities include vault cash, cash items in process of collection, balances due from depository institutions and Federal Reserve Banks, and liabilities of the U.S. government, U.S. government agencies and U.S. government-sponsored enterprises. The TED spread is calculated as the spread between LIBOR based on US dollars and Treasury Bills. Certificate of deposit and commercial paper spreads are between the secondary market rates of these securities and the Effective Federal Funds Rate. All spreads refer to 3 month maturities.

Source: Board of Governors of the Federal Reserve System, the Federal Reserve Bank of St. Louis

market discipline on traditional banks: Depositors demand higher interest rates to compensate for liquidity risk, generating a rise in deposit rates similar to Panel C. Higher deposit rates in turn reduce early withdrawal incentives, bringing about greater risk-shifting by traditional banks. In equilibrium, the shadow banking sector reaches a size that causes financial instability through this mechanism.

3 A simple model

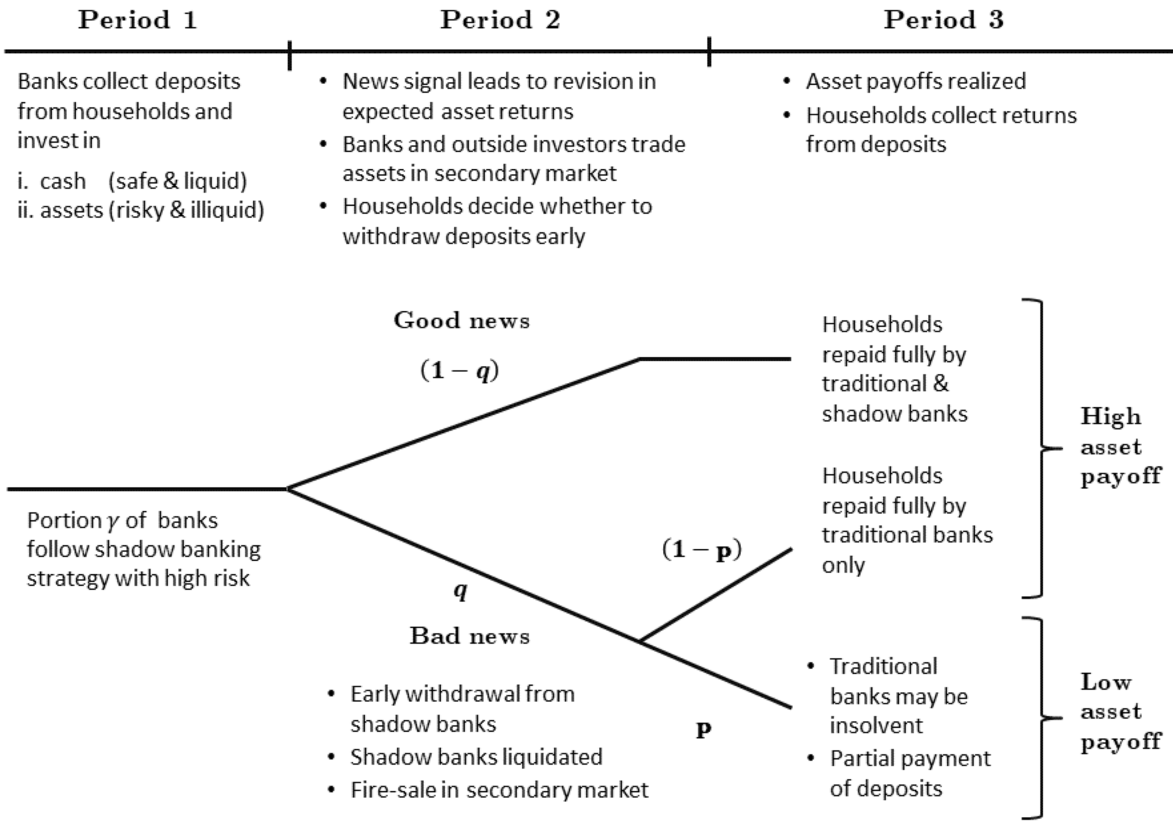
We consider a financial economy populated by households, banks and outside investors. Events unfold over three time periods (see Figure 5 for a graphical timeline). In the first period, banks collect deposits from households and invest in (safe and liquid) cash and (risky and illiquid) assets.[9]

The second period begins with a (public) news signal that leads to a revision of expected asset returns. With probability q, the signal harbors "bad news" leading to a decline in expected asset returns. After observing the signal, households decide whether to withdraw their deposits early and banks trade assets with outside investors in a secondary market.

In the third period, assets yield a high or low payoff contingent on economic fundamen-

[9]In Section 4, we also introduce an illiquid safe asset.

Figure 5: **Timeline**



tals. Following bad news in the second period, fundamentals may turn out to be weak with (conditional) probability p, leading to a low payoff from assets. Depending on their portfolio strategies, banks may then be left with insufficient funds to repay their depositors. In this case, they become insolvent under limited liability and a haircut proportionate to their funding shortfall is imposed on deposits.[10]

Importantly, banks cannot credibly commit to a safe investment strategy unless they pay a commitment cost $\tau > 0$. While we do not explicitly model the underlying information asymmetries between banks and their depositors, the commitment cost may reflect any costly action taken by banks to increase the transparency of their balance sheets and intermediation activities, such as providing independently audited balance sheet reports and foregoing opaque intermediation processes like securitization.

We shall show that, when commitment is sufficiently costly, ex-ante identical banks optimally cluster into two distinct groups according to their investment strategies. An endogenously determined share $\gamma \in [0, 1]$ of banks do not pay the commitment cost and follow a 'shadow

[10] Fundamentals turn out to be strong with certainty after good news (which takes place with probability $(1-q)$) and with (conditional) probability $(1-p)$ after bad news. In this case, assets yield a high payoff and banks are solvent.

banking' strategy that entails high risk-taking, with losses shifted to their depositors under limited liability. Following bad news, households optimally withdraw their deposits from shadow banks due to concerns about their solvency prospects. Shadow bank assets are then liquidated in the secondary market at an endogenous fire-sale discount and depositors receive the liquidation value. The remaining banks pay the cost τ to credibly commit to a portfolio that is safe enough to prevent an early withdrawal. We refer to this as a 'traditional banking' strategy.

It is important to note that early withdrawals are distinct from Diamond-Dybvig bank-runs. An early withdrawal takes place when households find it optimal to withdraw their deposits from a bank in response to a deterioration in its solvency prospects regardless of the extent of withdrawals by other households.

Before explaining these activities in detail, we briefly describe some notational conventions. Table 1 provides a list of variables and parameters. We denote variables that pertain to a shadow (traditional) banking strategy with a superscript 'SB' ('TB'). Deposits, cash and (risky) assets are respectively labelled as (D, M, I). To distinguish between deposits and cash per bank and per household, we denote the latter as (d, m) in lower case. Deposits take the form of two-period time deposits with an early withdrawal option. Upon maturity in the third period, deposits pay a gross interest rate R when the bank is solvent and a recovery rate V otherwise. When they are withdrawn early, deposits pay the principal (i.e. a gross return of 1) if the bank remains solvent and a liquidation value θ otherwise. Asset prices in the first and second period are labelled as (P_1, P_2). In period 3, assets yield a payoff σ_h when fundamentals are strong and σ_l otherwise. We simplify notation by normalizing the risk-free rate to $R^* = 1$ and the unconditional expectation of asset payoffs to unity such that[11]

$$(1 - qp)\,\sigma_h + qp\sigma_l = 1 \tag{1}$$

3.1 Agents and their optimal strategies

3.1.1 Asset origination

In the first period, each bank has access to a separate but identical origination technology with diminishing returns, which gives rise to an (inverted) asset supply schedule[12]

$$P_1 = \frac{1}{\alpha A^{\frac{1}{\alpha}}}\left((1+\omega)\, I_1\right)^{\frac{1-\alpha}{\alpha}} \tag{2}$$

[11] Note that assets may still have a return above unity since (P_1, P_2) are endogenous.

[12] In Appendix A, we microfound this asset supply schedule with the use of relationship lending frictions.

Table 1: **Notation**

Variables

Label	Description
D, d	Deposits
M, m	Cash
I	Assets
R	Interest rate on deposits
K	Capital
P_1, P_2	Asset prices
$\widetilde{I}$	Excess supply in secondary market
$\widetilde{K}$	Outside project
ϕ	Fire-sale discount
θ	Liquidation value
V	Recovery rate
Π	Bank profits
γ	Share of shadow banks
c	Household consumption

Parameters

Label	Description
q	Prob. of bad news
p	Prob. of weak fundamentals
σ_h, σ_l	Asset payoffs
A	Productivity
α	Cobb-Douglas elasticity
R^*	Risk-free rate
E	Household endowment
$\widetilde{E}$	Outside investor endowment
τ	Commitment cost
ω	Origination parameter

where $A > 0$ is a productivity parameter and $\alpha \in (0, 1)$ is the standard Cobb-Douglas elasticity. $\omega > 0$ determines the price elasticity of supply and the markup extracted by banks, which can be written as

$$\mu \equiv \frac{\partial P_1}{\partial I_1} \frac{I_1}{P_1} = \frac{1-\alpha}{\alpha (1+\omega)}$$

To ensure that the mark-up remains below the level where it eliminates insolvency risk in the banking sector, we place the restriction

$$\omega \geq \underline{\omega} \equiv \frac{1 - q\alpha - (1-q)\,\sigma_h}{\alpha (1-q)(\sigma_h - 1)} \tag{3}$$

3.1.2 Secondary market and outside investors

In the second period, banks and outside investors trade assets in a secondary market. First, we consider the state after good news. Since assets yield a high payoff with certainty in this state, secondary markets clear at a price σ_h without any sales to outside investors and any trade between banks is inconsequential to the equilibrium allocation. Therefore, we abuse notation slightly by using the subscript 2 to denote variables after bad news for the remainder of the paper. Following bad news, the excess supply of assets by banks can be written as

$$\widetilde{I}_2 = \gamma\left(I_1^{SB} - I_2^{SB}\right) + (1-\gamma)\left(I_1^{TB} - I_2^{TB}\right) \geq 0 \tag{4}$$

where γ is the share of shadow banks within the banking sector and I_2 denotes asset holdings in the second period. Secondary market prices are given by

$$P_2 = \phi\left[(1-p)\,\sigma_h + p\sigma_l\right]$$

where $\phi \in [0,1]$ is a fire-sale discount and the term in brackets is the expected payoff conditional on bad news. When there is no excess supply ($\widetilde{I}_2 = 0$), assets are priced at their expected return such that $\phi = 1$. For $\widetilde{I}_2 > 0$, outside investors become the marginal buyers and the fire-sale discount adjusts to clear the secondary market.

Our set up for outside investors is based on Stein (2012). Outside investors begin the second period with an endowment $\widetilde{E}$ and allocate their funds between asset purchases in the secondary market and an outside project $\widetilde{K}$ that yields a safe payoff $g\left(\widetilde{K}\right)$ where $g'(.) > 0$, $g''(.) < 0$. Their first order condition equates the expected return between the two investment opportunities and yields an implicit expression for the market-clearing fire-sale discount

$$\phi = \left[g'\left(\widetilde{E} - \phi\left[(1-p)\,\sigma_h + p\sigma_l\right]\widetilde{I}_2\right)\right]^{-1} \tag{5}$$

Without loss of generality, we can write this as $\phi = f\left(\widetilde{I}\right)$ where $f(.)$ is a continuous and decreasing function. We restrict the fire-sale discount to values within the range $\phi \in \left(\underline{\phi}, \bar{\phi}\right]$ by imposing the following restrictions on $f(.)$ (and hence on $\widetilde{E}$ and $g(.)$)

$$\underline{\phi} < f\left((A\alpha^{\alpha})^{\frac{1}{1-\alpha}}\left((1-q)\frac{\sigma_h}{1+\mu} + q\sigma_l\right)^{\frac{\alpha}{1-\alpha}}\right) < \bar{\phi} \tag{6}$$

We relegate the expressions for the bounds $\left(\underline{\phi}, \bar{\phi}\right)$ to Appendix B. The upper bound of this restriction ensures that the fire-sale deepens as the shadow banking sector grows, and is necessary for bringing about an interior equilibrium where shadow and traditional banks coexist. The lower bound restriction only serves to simplify our exposition.[13]

3.1.3 Households

There is a unit continuum of risk neutral households which derive utility only from final period consumption, making their utility maximization problem equivalent to maximizing expected

[13] See Section 3.2 for a formal definition of the equilibrium. For $\phi < \bar{\phi}$, we attain an equilibrium with slightly different properties which are discussed in Appendix C.

consumption[14]

$$E\left[c\right] = \left(1-q\right)c_{gh} + q\left(1-p\right)c_{bh} + qpc_{bl} \tag{7}$$

where (c_{gh}, c_{bh}, c_{bl}) are respectively consumption under good news and bad news with high and low payoff from assets. In the first period, households allocate their endowment E between deposits in traditional and shadow banks $\left(d^{TB}, d^{SB}\right)$ and cash m_1 which transfers funds to the next period at zero net return.[15] The first period budget constraint is

$$d^{SB} + d^{TB} + m_1 = E \tag{8}$$

When the second period begins with good news, there are no withdrawals and households simply retain their cash holdings m_1. Third period consumption is then given by

$$c_{gh} = m_1 + d^{TB}R^{TB} + d^{SB}R^{SB} \tag{9}$$

where $\left(R^{TB}, R^{SB}\right)$ represent the deposit interest rates at traditional and shadow banks.

Following bad news, on the other hand, households observe the decline in expected asset returns and decide whether to withdraw their deposits early. For the moment, we take it as given that shadow banks face an early withdrawal while traditional banks do not.[16] The second period budget constraint under bad news can then be written as

$$m_2 = m_1 + \theta^{SB}d^{SB} \tag{10}$$

where θ^{SB} is the liquidation value of shadow bank portfolios. In the third period, traditional banks may be rendered insolvent by a low payoff from assets. Therefore, consumption is contingent on asset payoffs

$$c_{bh} = m_2 + d^{TB}R^{TB} \tag{11}$$

$$c_{bl} = m_2 + Vd^{TB}R^{TB} \tag{12}$$

with V representing the recovery rate of traditional bank deposits.[17]

The representative household chooses $\left\{d^{TB}, d^{SB}, m_1, m_2\right\}$ to maximize expected consumption (7) subject to (8)-(12). To solve this optimization problem, we work backwards from period

[14] These assumptions only serve to simplify the exposition. Our qualitative findings remain the same under risk aversion and period-by-period discounting.

[15] We assume that E is high enough to ensure that the non-negativity constraints on d^{SB}, d^{TB} and M_1 never bind.

[16] In Section 3.1.4, we show that this arises as an outcome of banks' optimal portfolio strategies.

[17] θ^{SB} and V are endogenous and depend on the optimal investment strategy of traditional and shadow banks. We elaborate further on this in Section 3.1.4.

2 and consider the early withdrawal decision after bad news. It is optimal for households to withdraw their deposits when doing so increases their expected consumption $(1-p)\,c_{bh}+pc_{bl}$. Lemma 1 shows that early withdrawal is optimal unless households expect a recovery rate above a threshold $\bar{V}$.

Lemma 1 *Households withdraw their deposits from traditional banks early unless they anticipate a recovery rate*

$$V \geq \bar{V} \equiv \frac{1}{p}\left(\frac{1}{R^{TB}}-(1-p)\right) \tag{13}$$

Proof. Provided in Appendix H.1 ■

The implication is that traditional banks may avoid an early withdrawal by committing to a recovery rate $V \geq \bar{V}$. Observe that $\bar{V}$ is inversely related to R^{TB}. This is because d^{TB} are time deposits that only pay back the principal amount when withdrawn early. Therefore, higher interest rates R^{TB} increase the interest foregone by withdrawing early and reduce households' incentives to do so.

In the first period, the first order conditions for $\{d^{TB}, d^{SB}, m_1\}$ yield the following expressions for interest rates

$$R^{TB} = 1+\frac{qp\,(1-V)}{1-qp\,(1-V)} \tag{14}$$

$$R^{SB} = 1+\frac{q}{1-q}\left(1-\theta^{SB}\right) \tag{15}$$

Since households anticipate withdrawing their deposits early from shadow banks, R^{SB} is proportional to liquidation value θ^{SB}. Conversely, interest rates on traditional bank deposits depend on the recovery rate V and these banks may borrow at the risk-free rate ($R^{TB}=1$) when they commit to a complete repayment of deposits ($V=1$).

3.1.4 Banks

There is a unit continuum of ex-ante identical, risk neutral banks. In the first period, banks collect deposits D from households and originate assets I_1 at price P_1 as well as holding cash M_1. Their first period budget constraint can then be written as

$$P_1 I_1 + M_1 = D \tag{16}$$

In the second period, banks trade assets in the secondary market. As discussed in Section 3.1.2, following good news, assets are priced at their expected payoff in the secondary market and trade is inconsequential. Bank profits (in the third period) are then given by

$$\Pi_{gh} = \sigma_h I_1 + M_1 - DR$$

Following bad news, banks that are subject to an early withdrawal have a liquidation value

$$\theta = \min\left\{1, \frac{P_2 I_2 + M_2}{D}\right\} \tag{17}$$

and those that are not face the second period budget constraint

$$P_2 I_2 + M_2 = P_2 I_1 + M_1 \tag{18}$$

Observe that a fire-sale reduces the liquidation value as well as the maximum cash M_2 banks may attain through portfolio re-allocation. When assets yield a high payoff σ_h in the third period, banks make a profit

$$\Pi_{bh} = \sigma_h I_2 + M_2 - DR$$

while limited liability binds under a low asset payoff σ_l.[18] Under limited liability, banks make zero profits and the recovery rate on deposits is proportional to the shortfall of funds

$$V = \frac{\sigma_l I_2 + M_2}{DR} \leq 1 \tag{19}$$

The commitment cost $\tau > 0$ creates a discontinuity in the optimization problem of banks such that it can be evaluated as a choice between two distinct investment strategies: a traditional banking strategy (labelled as 'TB') that involves paying a cost τ to gain the ability to commit and a shadow banking strategy (labelled as 'SB') that does not.[19] In the first period, banks adopt the strategy that leads to the highest expected payoff such that shadow banking is preferred when

$$E\left[\Pi^{SB}\right] \geq E\left[\Pi^{TB}\right] - \tau \tag{20}$$

where $E\left[\Pi^{SB}\right]$ and $E\left[\Pi^{TB}\right]$ are the expected payoffs associated with shadow and traditional banking. Below, we solve the optimization problem under each strategy and attain an expression for expected payoffs by combining banks' first order conditions with those of the households from Section 3.1.3. In doing so, we take the fire-sale discount ϕ (and hence the secondary market price P_2) as given.

Shadow banking As shadow banks do not pay the cost τ, they cannot credibly commit to a safe portfolio strategy. Lemma 2 shows that this results in an early withdrawal of shadow bank deposits following bad news.

[18] See Appendix H.2 for the relevant proof.

[19] To simplify the exposition, we treat the commitment cost τ as a utility cost, thereby omitting it from the budget constraints. Since τ is small relative to total bank assets, its inclusion has a negligible impact on the equilibrium outcome.

Lemma 2 *Under the restriction $\omega > \underline{\omega}$, V^{SB} falls short of the minimum recovery rate required by households not to withdraw their deposits early.*

Proof. Provided in Appendix H.2 ■

The early withdrawal leads to the liquidation of shadow bank assets in the secondary market. Since limited liability binds after the liquidation, shadow banks only internalize the payoff after good news. Therefore, the representative shadow bank chooses $\left\{I_1^{SB}, M_1^{SB}, D^{SB}\right\}$ to maximize expected profits

$$E\left[\Pi^{SB}\right] = (1-q)\left(\sigma_h I_1^{SB} + M_1^{SB} - D^{SB} R^{SB}\right) \tag{21}$$

subject to the budget constraint (16). Lemma 3 provides the solution the shadow bank's problem while Figure 6 depicts it graphically.

Lemma 3 *Combining the solution to the shadow bank's problem with the household first order condition (15) yields the following expressions for $\left(\theta^{SB}, R^{SB}, E\left[\Pi^{SB}\right], I_1^{SB}, M_1^{SB}\right)$*

$$\begin{aligned}
\theta^{SB} &= \frac{(1+\mu) P_2}{(1-q)\,\sigma_h + q\,(1+\mu)\,P_2} \\
R^{SB} &= \frac{\sigma_h}{(1-q)\,\sigma_h + q\,(1+\mu)\,P_2} \\
E\left[\Pi^{SB}\right] &= (1-q)\,\frac{\mu}{1+\mu}\sigma_h I_1^{SB} \\
I_1^{SB} &= \frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left((1-q)\,\frac{\sigma_h}{1+\mu} + qP_2\right)^{\frac{\alpha}{1-\alpha}} \\
M_1^{SB} &= 0
\end{aligned}$$

where

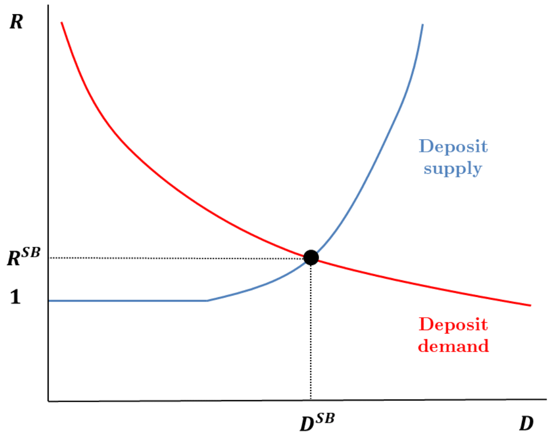
$$\theta^{SB} < 1 \;,\; R^{SB} > 1 \;\forall\; \phi \in [0,1]$$

Proof. Provided in Appendix H.3 ■

The red line depicts banks' demand for deposits, which is inversely related to the asset price P_1. As deposits are used to originate risky assets I_1, its downward slope reflects the positive relationship between I_1 and P_1 given by the asset supply schedule (2). The blue line depicts the supply of deposits by households. It is horizontal at the risk-free rate $R = 1$ when there is no early withdrawal or $\theta = 1$, but becomes upward sloping when a rise in P_1 reduces the liquidation value θ and drives households to require a higher interest rate in compensation as per (15). Under a shadow banking strategy, banks optimally invest up to the intersection of these two curves where the funding costs exceed the risk-free rate.

Traditional banking Having paid the commitment cost τ, it is optimal for traditional banks to commit to a portfolio strategy consistent with a recovery rate $V \geq \bar{V}$ that precludes an

Figure 6: **Shadow banking strategy**



Note: The deposit demand curve is attained by combining (2) and (16). Deposit supply stems from (15) and (39).

early withdrawal.[20] We can use (19) to write this in terms of a no-withdrawal constraint

$$\sigma_l I_2^{TB} + M_2^{TB} \geq \bar{V} D^{TB} R^{TB} \tag{22}$$

which constitutes the key difference between shadow and traditional banking strategies.

The representative traditional banks chooses $\left\{I_1^{TB}, I_2^{TB}, M_1^{TB}, M_2^{TB}, D^{SB}\right\}$ to maximize its expected profits

$$E\left[\Pi^{TB}\right] = (1-q)\left(\sigma_h I_1^{TB} + M_1^{SB}\right) + q(1-p)\left(\sigma_h I_2^{TB} + M_2^{TB}\right) - (1-qp) D^{TB} R^{TB}$$

subject to the budget constraints (16), (18) and the no-withdrawal constraint (22).[21]

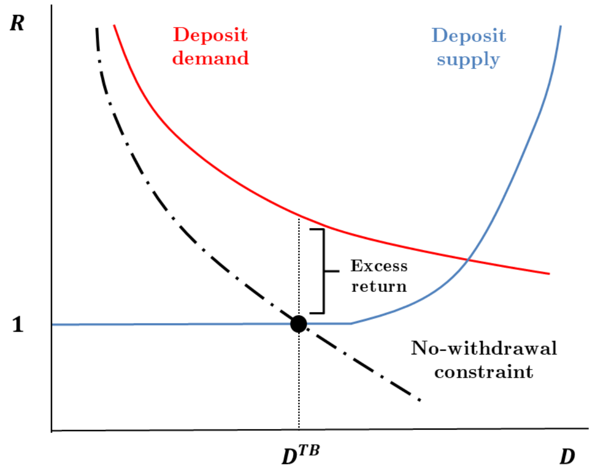
Proposition 1 provides the solution to the traditional bank's problem. It shows that the no-withdrawal constraint generates similar behavior to the observations discussed under Fact 4. To satisfy this constraint, traditional banks respond to bad news by re-allocating their portfolio toward cash.[22] Since the terms of trade between risky assets and cash depend on the secondary market price P_2, the no-withdrawal constraint also reduces banks' risky asset purchases I_1 in period 1 in line with P_2. This in turn creates an excess return that contributes to traditional

[20] This is true by virtue of the segmentation of shadow and traditional banking strategies through the commitment cost. Banks which do not find it optimal to avoid an early withdrawal may further increase their payoff by electing not to pay for the commitment cost. These banks are then classified as shadow banks.

[21] Due to limited liability, traditional banks do not internalize the state with low payoff from assets. It follows from the proof for Lemma 2 that limited liability will bind in this state.

[22] The complete liquidation of risky assets $\left(I_2^{TB} = 0\right)$ and the equivalence between first and second period asset prices $\left(P_1^{TB} = P_2\right)$ are due to the simplifying restrictions we have made in the interest of tractability. In Section 5, we show that the mechanism remains intact in a richer model which generates positive risky asset holdings after the sell-off and $P_1^{TB} > P_2$.

Figure 7: **Traditional banking strategy**



Note: The deposit demand curve is attained by combining (2) and (16). Deposit supply stems from (15) and (39). The no-withdrawal constraint is given by (22), (23), and (24).

bank profits (see Figure 7 for a graphical representation).

Proposition 1 *Combining the solution to the traditional bank's problem with the first order conditions (14) and (13) yields the following expressions for* $\left(I_1^{TB}, I_2^{TB}, M_2^{TB}, E\left[\Pi^{TB}\right], R^{TB}, \bar{V}\right)$

$$\begin{aligned} I_1^{TB} &= \frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega} P_2^{\frac{\alpha}{1-\alpha}} \\ I_2^{TB} &= 0 \qquad (23) \\ M_2^{TB} &= P_2 I_1^{TB} + M_1^{TB} \qquad (24) \\ E\left[\Pi^{TB}\right] &= (1-q)(\sigma_h - P_2) I_1^{TB} \\ R^{TB} &= \bar{V} = 1 \end{aligned}$$

where $M_1^{TB} \geq 0$ *is indeterminate*[23] *and*

$$\frac{\partial E\left[\Pi^{TB}\right]}{\partial P_2} < 0 \; \forall \; P_2 > \alpha\sigma_h$$

Proof. Provided in Appendix H.4. ■

Most importantly, the proposition shows that the no-withdrawal constraint imposes market discipline on traditional banks, eliminating insolvency risk ($\bar{V} = 1$) and reducing their funding

[23] The indeterminacy of M_1^{TB} is due to two reasons. First, it does not contribute towards the no-withdrawal constraint, since each extra unit of cash corresponds to an extra unit of deposit. Second, the no-withdrawal constraint also prevents banks from converting cash to risky assets after bad news such that bank profits are not affected by M_1^{TB}.

costs to the risk-free rate ($R^{TB} = 1$). This is the outcome of a virtuous cycle between deposit rates R^{TB} and the minimum recovery rate $\bar{V}$. As traditional banks guarantee a minimum recovery rate, households demand less compensation for insolvency risk and deposit rates decrease as per (14). Lower deposit rates in turn strengthen the threat of early withdrawal as households stand to lose less in terms of interest foregone. Therefore, the minimum recovery rate $\bar{V}$ increases until all solvency risk is eliminated in equilibrium.

It is important to note that this market discipline result is contingent on the lack of liquidity risk. In Section 4, we show that even a small amount of liquidity risk on traditional banks reverses this virtuous cycle, leading to a rise in insolvency risk and funding costs.

3.2 Equilibrium and welfare

We solve for a rational expectations equilibrium where all optimality conditions and constraints of banks, households and outside investors are satisfied, expectations are confirmed, and deposit and secondary markets clear such that

$$\gamma D^{SB} = d^{SB}$$
$$(1-\gamma) D^{TB} = d^{TB}$$
$$\phi = f\left(\widetilde{I}_2\right)$$

where γ is the share of shadow banks and $\widetilde{I}_2$ reflects the excess supply of assets in the secondary market as per (4)

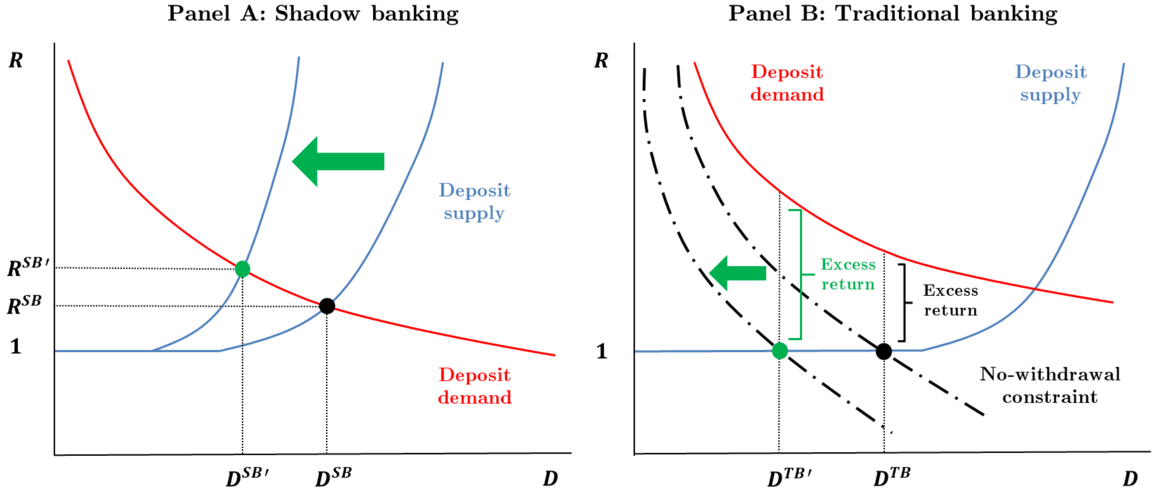
An equilibrium is characterized as 'interior' when traditional and shadow banks coexist with $0 < \gamma < 1$. In an interior equilibrium, banks are indifferent between shadow and traditional banking strategies such that (20) holds with equality

$$E\left[\Pi^{SB}\right] = E\left[\Pi^{TB}\right] - \tau \tag{25}$$

and may be interpreted as a free entry condition that determines γ in equilibrium.

We proceed as follows in our description of the equilibrium solution: Section 3.2.1 builds up on the intuition provided about bank strategies above by discussing their interaction with fire-sales. We focus on fire-sales due to their role in creating *strategic substitutabilities* in banks' decision to enter shadow banking, which is crucial for bringing about an interior equilibrium. Section 3.2.2 then provides the conditions under which an interior equilibrium arises and discusses the implications of a rise in the costs of commitment and a deepening of secondary markets. Finally, Section 3.2.3 analyzes the welfare properties of the interior equilibrium and shows that, while the existence of shadow banking is welfare-raising, the equilibrium size of the

Figure 8: **Fall in the secondary market price**



Note: The deposit demand curve is attained by combining (2) and (16). Deposit supply stems from (15) and (39). The no-withdrawal constraint is given by (22), (23), and (24).

sector is in excess of the constrained-efficient level due to fire-sale externalities.

3.2.1 Fire-sales and bank strategies

There is a two-way interaction between fire-sales and bank strategies: On the one hand, entry into shadow banking increases the excess supply of assets in the secondary market and exacerbates fire-sales.[24] On the other hand, fire-sales reduce the expected payoff from a shadow banking strategy relative to traditional banking and deter entry into the shadow banking sector.

Figure 8 demonstrates the mechanism behind this. As shown in Panel A, a fall in P_2 reduces shadow banks' liquidation value during an early withdrawal. This leads to an upward pivot in the deposit supply curve, raising shadow banks' funding costs and reducing the expected payoff from shadow banking.

In contrast, the expected payoff from traditional banking rises in response to a fall in P_2.[25] This is because it worsens the terms of trade between risky assets and cash after bad news, tightening the no-withdrawal constraint. As shown in Panel B, an inward shift in the constraint forces traditional banks to reduce I_1. Since traditional banks' funding costs remain at the risk-free rate, this increases the excess return and expected payoff from traditional banking.

These interactions constitute fire-sale externalities since banks do not internalize the impact of their entry into shadow banking on the profitability of other banks. Moreover, since

[24] We use the phrases 'adopting a shadow banking strategy' and 'entry into shadow banking' interchangably.

[25] Strictly speaking, Proposition 1 indicates that a rise in profits occurs only in the region $P_2 > \alpha\sigma_h$. This reflects two conflicting effects on traditional bank profits: a rise in the return from risky asset purchases in period 1 versus a fall in the quantity purchased. For $P_2 > \alpha\sigma_h$, the former effect dominates and profits rise.

profitability affects entry incentives, fire-sale externalities create strategic substitutabilities in banks' decisions to adopt a shadow banking strategy. As the shadow banking sector grows, the fire-sale discount on risky assets gets larger (i.e. ϕ falls), reducing the payoff from shadow banking relative to traditional banking until we reach an equilibrium sector size where banks are indifferent between the two strategies.

3.2.2 Interior equilibrium

Proposition 2 provides the conditions for an interior equilibrium. In an interior equilibrium, the free entry condition (25) pins down the size of the shadow banking sector γ and fire-sale discount ϕ through the interactions described in Section 3.2.1. Figure 9 illuminates the underlying mechanism by demonstrating the comparative statics of a rise in commitment costs and a deepening of secondary markets under a numerical example.

Proposition 2 *There is an interior equilibrium under the parameter restrictions $\omega \geq \underline{\omega}$, $\phi \in (\underline{\phi}, \bar{\phi})$, $\alpha < 1/2$, $p < 1 - \alpha$ when commitment costs fall within the range*[26]

$$\underline{\tau} < \tau < \bar{\tau}$$

Proof. Provided in Appendix H.5. ∎

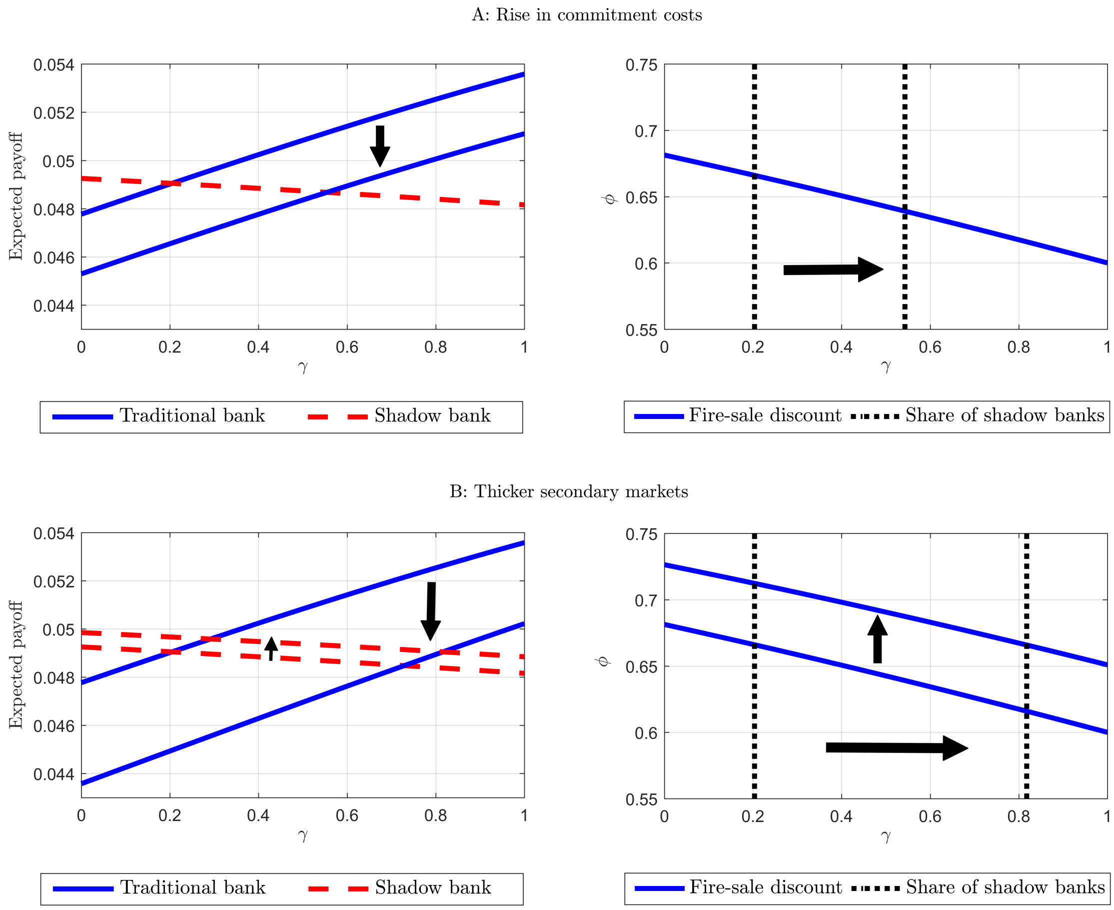
Observe that the expected payoff schedule for traditional (shadow) banking is upward (downward) sloping in γ in line with the intuition from Section 3.2.1. The equilibrium share of shadow banks is at the point where these two schedules intersect as per (25). We also plot this as a vertical bar along with the fire-sale schedule in order to deduce the equilibrium fire-sale discount.

Panel A shows that a rise in commitment costs τ causes a downward shift in the expected payoff schedule for traditional banking. At a given sector size γ, this makes shadow banking relatively profitable and leads to further entry into the sector. This in turn increases the excess supply of assets in the secondary market following bad news and exacerbates the fire-sale. Therefore, in equilibrium, a rise in the commitment cost increases the size of the shadow banking sector and the vulnerability of the economy to fire-sales.

Panel B shows the effects of an increase in the liquidity available in the secondary market due to a rise in the endowment of outside investors. At any given sector size γ, this reduces the fire-sale discount (i.e. a rise in ϕ) bringing about an upward shift in the fire-sale schedule. Consequently, the expected payoff schedule for traditional (shadow) banking shifts down (up) in line with Section 3.2.1, and there is entry into shadow banking until the new schedules intersect at a larger sector size.

[26] We relegate the definitions for $(\underline{\tau}, \bar{\tau})$ to Appendix B.

Figure 9: **Numerical example**



Note: The numerical example correponds to the calibration $A = 1$, $\alpha = 1/3$, $q = p = \sigma_l = 1/2$, $\mu = \bar{\mu}$, and $\tau = 0.0825$. We parameterize the fire-sale function $f(.)$ according to Appendix D and calibrate $\widetilde{E}$ to get $\underline{\phi}$ as a lower bound and set $\kappa = 10\left((1-p)\,\sigma_h + p\sigma_l\right)^{-1}$. Panel A and B respectively display the effects of a small rise in τ and $\widetilde{E}$.

It is important to note that the fire-sale discount returns to its initial value at the new equilibrium. As such, a thicker secondary market for assets increases the size of the shadow banking sector but does not alleviate the fire-sale. This finding stems from an essential property of the interior equilibrium: the fire-sale discount is implicitly determined by the free entry condition (25).

3.2.3 Welfare analysis

We measure social welfare using an equal-weighted sum of agents' utilities and focus on two alternative benchmarks of social optimality: an (unconstrained) efficient allocation (denoted by e) which maximizes welfare without regard to agents' optimality conditions, and a constrained-efficient allocation (denoted by ce) which is subject to all optimality conditions except for the free entry condition.[27] Proposition 3 describes the properties of both allocations.

Proposition 3 *The (unconstrained) efficient allocation is given by*

$$\begin{aligned} P_1^e &= (1-qp)\,\sigma_h + qp\sigma_l \\ \widetilde{K}^e &= \widetilde{E} \end{aligned}$$

The constrained-efficient allocation leads to

$$\begin{aligned} P_1^{TB,ce} &< P_1^{SB,ce} < P_1^e \\ \widetilde{K}^{ce} &< \widetilde{K}^e \end{aligned}$$

and a shadow banking sector size γ^{ce} within the range

$$0 < \gamma^{ce} < \gamma^*$$

where the first inequality is true for sufficiently high $\widetilde{E}$ and γ^* denotes the sector size consistent with the interior equilibrium described in Section 3.2.2.

Proof. Provided in Appendix H.6. ■

Under the (unconstrained) efficient allocation, the extent of financial intermediation is set to equate (risky) asset prices to their expected payoff.[28] Moreover, there is no early liquidation such that outside investors invest all of their funds in productive outside projects $\widetilde{K}$.

Allocations constrained by agents' optimality conditions involve reduced financial intermediation and investment in outside projects, and a trade-off between the two, due to two financial frictions present in the model economy: First, because of their funding by demandable deposits, the extent of financial intermediation provided by (shadow and traditional) banks hinges on their access to secondary market liquidity.[29] Second, liquidation of assets in the secondary market leads to costly fire-sales. Particularly, liquidated assets are purchased (at a fire-sale discount) by outside investors, driving them to divert funds from outside projects.

Entry into shadow banking moves the equilibrium allocation along the trade-off between financial intermediation and outside investments, with two opposing effects on social welfare:

[27] Specifically, the agents are households, bank managers which consume bank profits, outside investors and entrepreneurs which are modeled as described in the microfoundations for asset origination in Appendix A.

[28] Under the efficient allocation, the distinction between shadow and traditional banking strategies ceases to be meaningful as there are no early withdrawals, or necessity to pay commitment costs. Note also that these results are conditional on risk neutrality. With risk aversion, there would be a risk premium such that $P_1^e > (1-qp)\,\sigma_h + qp\sigma_l$.

[29] While banks may also increase their liabilities and/or issue equity, in practice this is likely to be prohibitively costly during an early withdrawal by depositors, which are senior to debt and equity holders. Collateralized liabilities may also have a significant haircut because of the possibility of fire-sales on assets posted as collateral.

First, it raises social welfare by reducing commitment costs and increasing financial intermediation, as shadow banks invest more in risky assets than traditional banks. Second, it reduces social welfare by raising the amount of assets liquidated in the secondary market and crowding out investments in outside projects. Because of diminishing returns to outside projects, the existence of shadow banking is constrained-efficient provided that endowments $\widetilde{E}$ of outside investors are sufficiently high.

Under the interior equilibrium, the shadow banking sector grows larger than the constrained-efficient size. This is because banks do not internalize the negative fire-sale externalities associated with their entry into shadow banking. Specifically, entry into shadow banking deepens the fire-sale, which raises the funding costs of other shadow banks and tightens the no-withdrawal constraint of traditional banks. This results in other banks reducing their financial intermediation (see Figure 8).[30] In the next sections, we show that these fire-sale externalities are exacerbated by the interplay between liquidity risk and market discipline.

4 A model with liquidity risk

In this section, we extend the model to allow for a richer asset space and liquidity risk. We introduce liquidity risk in the form of liquidity shocks which may hit banks in the second period. These shocks are distinct from early withdrawals in that they hit banks idiosyncratically rather than being an optimal response to the deterioration in bank solvency prospects under bad news. The expanded asset space includes three different asset types, liquid, safe, and risky, which we respectively denote with (λ, s, r). The risky asset is identical to the (non-cash) asset in the simple model. Safe and liquid assets both yield a unit payoff with certainty, but they differ in that the safe asset matures in the third period while the liquid asset yields its payoff in the second period

The expanded asset space serves two purposes. First, it allows us to consider the conditions under which safe assets are endogenously liquid due to secondary markets. A priori, it is not clear whether safe assets would be subject to a fire-sale since purchases by traditional banks (which re-allocate their portfolios toward safe assets) may offset sales by shadow banks. Second, it permits us to consider the interplay between fire-sales, market discipline, and liquidity risk. Notably, without a fire-sale on safe assets, portfolio re-allocation required to satisfy the no-withdrawal constraint also increases traditional banks' liquidation values and reduces their vulnerability to liquidity shocks.

In the interest of brevity, we only describe the aspects of the model that differ from Sec-

[30]Note that these fire-sale externalities only partially offset the increase in financial intermediation. We find that financial intermediation always relatives positively to the size of the shadow banking sector.

tion 3.[31] The remainder of the section is organized as follows: Section 4.1 provides further details about liquidity shocks. Section 4.2 extends secondary markets and outside investors to a framework with multiple assets. Finally, Section 4.3 presents analytical results pertaining to the relationship between the fire-sale on safe assets, market discipline, and liquidity risk.

4.1 Liquidity risk

In the second period, banks with a liquidity shortfall $\theta < 1$ may suffer from a liquidity shock leading to their bankruptcy and the liquidation of their asset portfolios in secondary markets.[32] Liquidity shocks take place with probability $\bar{\xi}$ and their realization is idiosyncratic to each bank such that $\bar{\xi}$ may be interpreted as the share of (illiquid) banks hit by shocks.[33] We find it expositionally convenient to let ξ be the effective liquidity shock probability such that

$$\xi = \left\{ \begin{array}{c} \bar{\xi} \text{ if } \theta^{TB} < 1 \\ 0 \text{ otherwise} \end{array} \right\}$$

In terms of timing, liquidity shocks take place simultaneously with the news signal and households' early withdrawal decision. This serves two purposes. First, it allows us to focus solely on liquidity shocks on traditional banks under bad news, as there is no liquidity shortfall under good news and shadow banks face an early withdrawal under bad news. Second, it ensures that depositors may not preempt liquidity shocks through an early withdrawal.[34]

We depict liquidity risk in this simple manner in order to isolate the role of bank portfolio strategies and fire-sales in bringing about a liquidity shortfall in the first place. In Appendix G, we show that our qualitative results remain the same after replacing liquidity shocks with a more sophisticated process that approximates a bank run under the global games framework of Goldstein and Pauzner (2005).

[31] See Appendix E for a complete specification of the model.

[32] Liquidity shocks may be interpreted as an unanticipated need to inject additional cash into a project with no impact on its NPV, or a Diamond-Dybvig bank-run.

[33] This assumption only serves to streamline the exposition. With an aggregate liquidity shock, there would be an additional liquidity shock state where uncertainty is immediately resolved through liquidation of all banks. The states without a liquidity shock would only differ from the equilibrium considered here in that the excess supply of assets in the secondary market would be somewhat lower.

[34] We show in Appendix F that this is a necessary condition for market discipline to be derived from depositors' ability to withdraw early. When depositors attempt to pre-empt liquidity shocks, the early withdrawal decision is no longer guided by solvency concerns and the equilibrium collapses to a solution similar to a Diamond-Dybvig bank-run.

4.2 Secondary market

We extend the secondary market described in Section 3.1.2 to a framework with multiple assets. Since the liquid asset yields its payoff in period 2, only safe and risky assets are traded in the secondary market. The excess supply of each asset is given by the expressions

$$\widetilde{I}(i) = \gamma I_1^{SB}(i) + (1-\gamma)\left(I_1^{TB}(i) - (1-\xi) I_2^{TB}(i)\right) \geq 0 \ \forall \ i \in \{s, r\}$$

and sold to a common set of risk neutral outside investors.[35]

Outside investors allocate their endowment $\widetilde{E}$ between purchases of safe and risky assets and an outside project $\widetilde{K}$ that yields a safe payoff $g\left(\widetilde{K}\right)$ where $g'(.) > 0$, $g''(.) < 0$. Their first order conditions indicate that there is a common fire-sale discount $\phi \in [0,1]$ for the two assets, and it is defined implicitly by the expression

$$\phi = \frac{1}{g'\left(\widetilde{E} - \phi\left[(1-p)\sigma_h + p\sigma_l\right]\widetilde{I}(r) - \phi\widetilde{I}(s)\right)} \tag{26}$$

which suggests that a rise in the total excess supply of safe and risky assets leads to a decrease in ϕ. Crucially, however, an asset type is only subject to the fire-sale discount when it is in excess supply such that its marginal buyer is an outside investor. Therefore, secondary market prices also depend on an asset's own excess supply such that

$$P_2(s) = \begin{cases} 1 & \text{for } \widetilde{I}(s) = 0 \\ \phi & \text{otherwise} \end{cases}$$

$$P_2(r) = \begin{cases} (1-p)\sigma_h + p\sigma_l & \text{for } \widetilde{I}(i) \leq 0 \\ \phi\left[(1-p)\sigma_h + p\sigma_l\right] & \text{otherwise} \end{cases}$$

4.3 Analytical results

Proposition 4 shows that the fire-sale on safe assets plays a definitive role in shaping the relationship between market discipline and liquidity risk.

[35] The commonality of outside investors only serves to simplify the exposition. To attain our key findings, we only need to assume that secondary markets are not completely segmented, that is, there is (at least partial) substitutability between risky and safe assets in these markets such that an increase in the supply of one deepens the fire-sale on the other. Accordingly, our results would also be robust to introducing risk aversion to outside investors. Note also that our assumption of non-segmented secondary markets is consistent with Fact 3 in Section 2.

Proposition 4 *In equilibrium, traditional banks commit to a minimum recovery rate*

$$\bar{V} = 1 - \frac{q}{p} \frac{\xi \left(1 - \theta^{TB}\right)}{1 - q \left(1 - \xi \left(1 - \theta^{TB}\right)\right)} \tag{27}$$

and the interest rate on their deposits is given by

$$R^{TB} = 1 + \frac{q}{1-q} \xi \left(1 - \theta^{TB}\right) \tag{28}$$

With $P_2(s) = 1$, traditional banks have a liquidation value $\theta^{TB} = 1$ such that $\xi = 0$ and (27), (28) yield

$$\bar{V} = R^{TB} = 1$$

With sufficiently low $P_2(s) < 1$, traditional banks have a liquidity shortfall $\theta^{TB} < 1$ which leads to $\xi > 0$. (27), (28) then yield

$$\bar{V} < 1 < R^{TB}$$

Proof. Provided in Appendix H.7 ■

The proposition shows that, when there is no fire-sale on safe assets ($P_2(s) = 1$), satisfying the no-withdrawal constraint is ensures that traditional banks have no liquidity shortfall ($\theta^{TB} = 1$). Since this eliminates liquidity risk ($\xi = 0$), the market discipline results in Proposition 1 retain their validity, that is, traditional banks have no solvency risk and collect deposits at the risk-free rate.

With a sufficiently large fire-sale on safe assets, on the other hand, traditional banks have a liquidity shortfall ($\theta^{TB} < 1$) despite satisfying the no-withdrawal constraint. This leaves them vulnerable to liquidity shocks and creates a vicious cycle between rising deposit interest rates and solvency risk. Households anticipate that they may not be repaid fully in the event of a liquidity shock and demand higher deposit rates as compensation. The rise in deposit rates in turn deters early withdrawals as households stand to lose more in terms of interest foregone. This leads to a decline in the minimum recovery rate $\bar{V}$ required to avoid a withdrawal, allowing traditional banks to take on solvency risk. Finally, the increase in solvency risk brings about a further rise in deposit rates and completes the vicious cycle.

In the next section, we present numerical results that highlight the role of a large shadow banking sector in causing a fire-sale on safe assets, which leaves traditional banks illiquid and undermines market discipline through the mechanism described above.

Table 2: **Calibration**

Parameter	Value	Description	Source
σ_l	0.21	Low payoff from risky assets	Moody's Investors Service (2007)
q	0.41	Prob. of bad news	NBER
p	0.22	Prob. of weak fundamentals	NBER
α	0.33	Cobb-Douglas parameter	-
A	4.98	Productivity	-
ω	10.10	Entrepreneur parameter	World Bank
E	1.00	Household endowment	-
$\widetilde{E}$	3.60	Outside investor endowment	Moody's, Federal Reserve Board
z	2.80	Outside investment parameter	Moody's, Federal Reserve Board
ξ	0.15	Liquidity shock prob.	Federal Reserve Bank of St. Louis
τ	0.11	Commitment cost	Financial Stability Board (2017)

5 Numerical results

This section provides numerical results from the model with liquidity risk. It proceeds as follows: Section 5.1 describes the calibration which targets the United States over the recent financial crisis. Section 5.2 presents and discusses the results from a numerical simulation.

5.1 Calibration

Table 2 reports the calibrated parameters. The payoff of risky asset under weak fundamentals is in line with the average recovery rate from junior debt and σ_h is backed out using the normalization of expected payoffs given in (1). Probabilities (q, p) are calibrated to the frequency of recessions and deep recessions (conditional on a recession) in the United States.[36] Regarding asset origination, we calibrate α to the standard Cobb-Douglas value of $1/3$, while ω is calibrated to attain a mark-up $\mu = 0.18$ consistent with the 5-bank asset concentration in the United States over 2007-2010. A is calibrated to achieve the normalization $A^{\frac{1}{1-\alpha}} / (1+\omega) = 1$.

In the secondary market, we parameterize the payoff function for outside investments to $g\left(\widetilde{K}\right) = z^{-1} \ln\left(\widetilde{K}\right)$ where z is a constant. Our calibration strategy for $\left(\widetilde{E}, z\right)$ targets the rise in the yield spreads between AAA-rated seasoned corporate bonds and the effective Fed-

[36] We use business cycle data from the NBER which covers the period December 1854-June 2017 at a monthly frequency. We label as 'deep recessions' the contractionary episodes of 1873-79 (the Long Depression), 1929-33 (the Great Depression), and 2008-09 (the Great Recession).

eral Funds Rate during the financial crisis.[37] Similarly, the calibration for the liquidity shock probability $\bar{\xi}$ targets TED spreads at the peak of the financial crisis.[38]

We calibrate the commitment cost τ to match the pre-crisis ratio of shadow bank assets to total bank assets, which is approximately 63%. This implies commitment costs amounting to just below 7% of the value of a traditional bank's assets in period 1. Finally, under risk neutrality, household endowment E only serves to shift the level of household consumption. We calibrate it to attain a baseline (risk-free) consumption of 1 when there is no shadow banking.

5.2 Results

Figure 10 plots the numerical solution across a range $\gamma \in [0, 1]$ of shadow banking sector sizes. The equilibrium sector size is denoted by the vertical bar γ^* where traditional and shadow banking strategies yield the same expected payoff. Panel 1 indicates that there is a unique and globally stable interior equilibrium where shadow and traditional banks coexist. This reflects the strategic substitutabilities between banking strategies described in Section 3.2: Entry into shadow banking exacerbates the fire-sale after bad news (Panel 2). This increases shadow bank funding costs (Panel 4) and reduces the expected payoff from shadow banking. In contrast, traditional bank profits rise as the fire-sale gets deeper. Therefore, the shadow banking sector grows until banks are indifferent between the two strategies at γ^*.

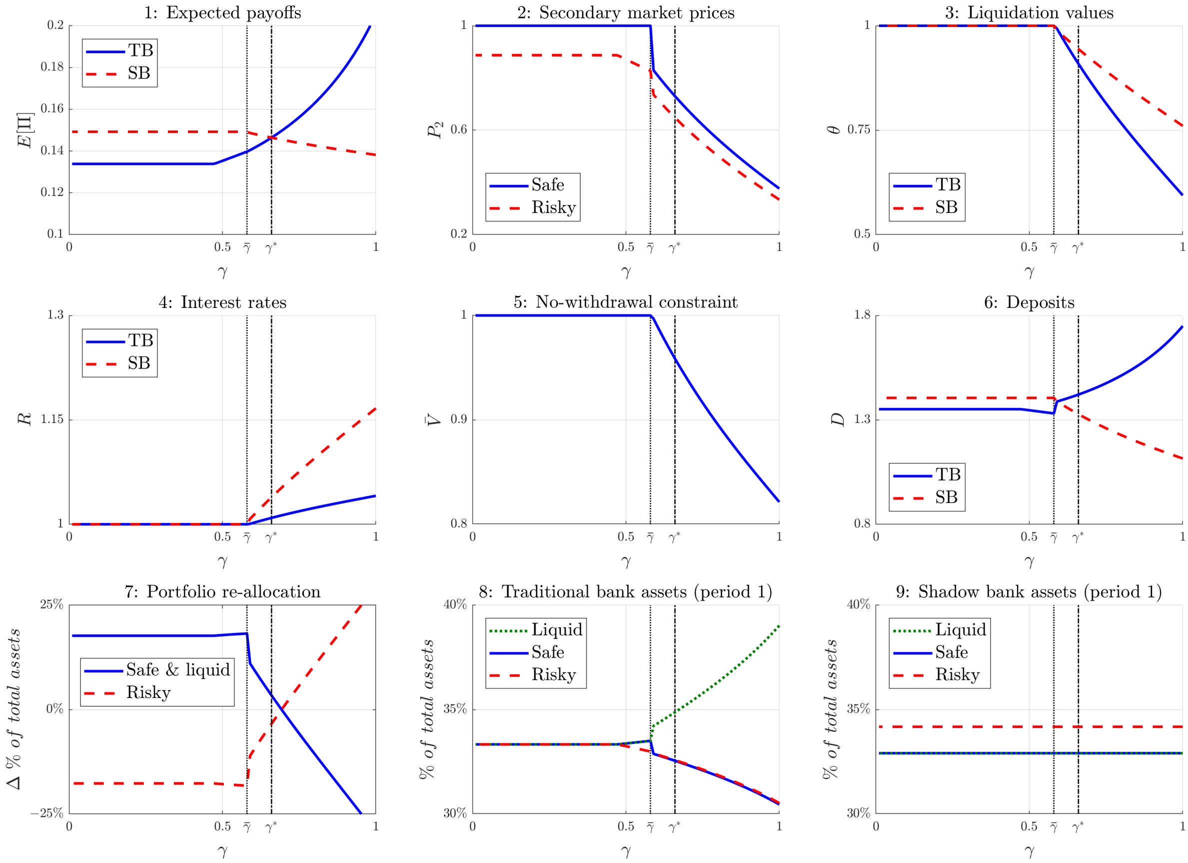
The bar labelled $\bar{\gamma}$ denotes the threshold sector size above which safe assets suffer from a fire-sale. For $\gamma < \bar{\gamma}$, traditional banks have no liquidity shortfall (Panel 3) and the no-withdrawal constraint imposes market discipline such that these banks borrow at the risk-free rate (Panels 4, 5). In equilibrium, however, there is a fire-sale on safe assets $(\gamma^* > \bar{\gamma})$ which reduces the liquidation value of traditional banks, leaving them vulnerable to liquidity shocks (Panel 3). As explained in Section 4.3, liquidity risk increases deposit rates R^{TB} (Panel 4) and undermines market discipline on traditional banks. Accordingly, Panel 5 shows that the minimum recovery rate $\bar{V}$ declines below full repayment for $\gamma > \bar{\gamma}$ such that traditional banks also have fundamental insolvency risk in equilibrium.

Panel 6 shows the evolution of shadow and traditional bank liabilities across γ. As γ rises and the fire-sale deepens, shadow banks respond to the rise in their funding costs by shrinking their balance sheets. Traditional banks, on the other hand, expand their balance sheets due to

[37] Specifically, we back out a percentage decline in bond prices $\Delta\hat{P}/\hat{P}$ from the difference between the peak spread in November 2008 and the average spread for the pre-crisis period between January 2016 and June 2017. We then calibrate $\left(\widetilde{E}, z\right)$ to generate an equilibrium fire-sale discount that matches the decline in the price of safe assets under bad news to $\Delta\hat{P}/\hat{P}$. Since $P_1(s)$ is bank-specific and best interpreted as origination costs in the absence of first period asset trade, our measure for the decline in safe asset prices refers to $P_2(s)$ relative to their expected payoff 1.

[38] See Figure 4 for the evolution of TED spreads. We use TED spreads instead of spreads on deposit rates to exclude the effects of deposit insurance guarantees.

Figure 10: **Numerical results**



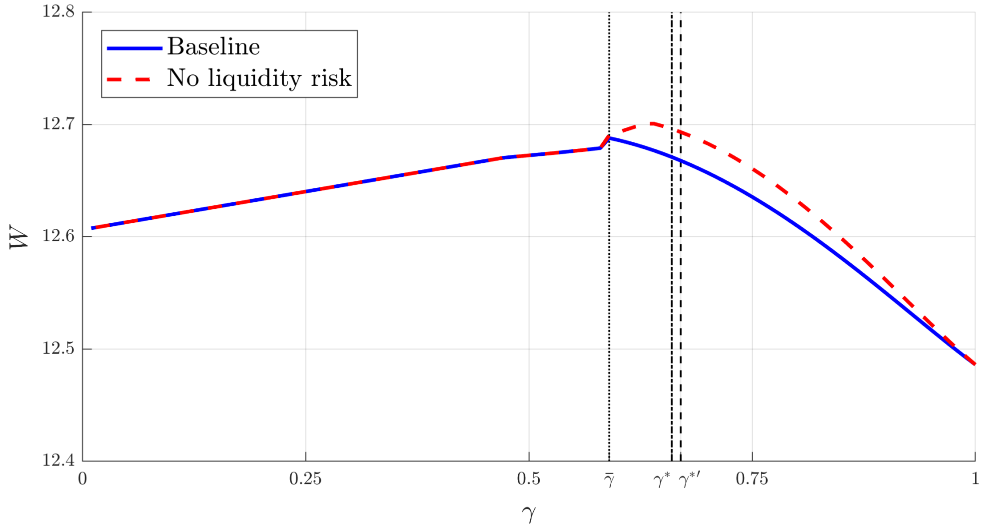
Note: Expected payoffs are inclusive of the commitment cost τ. Total assets in period 1 and 2 are respectively defined as $\bar{I}_1 \equiv \sum_{i\in\{\lambda,s,r\}} I_1(i)$ and $\bar{I}_2 \equiv \sum_{i\in\{s,r\}} P_2(i)\, I_2(i)$. Panel 7 plots $\left(\bar{I}_2^{TB}\right)^{-1} P_2(s)\, I_2^{TB}(s) - \left(\bar{I}_1^{TB}\right)^{-1} \sum_{i\in\{\lambda,s\}} I_1^{TB}(i)$ for safe and liquid assets, and $\left(\bar{I}_2^{TB}\right)^{-1} P_2(r)\, I_2^{TB}(r) - \left(\bar{I}_1^{TB}\right)^{-1} I_1^{TB}(r)$ for risky assets. Panel 8 plots $I_1^{TB}(i)/\bar{I}_1$ respectively for $i = \{\lambda, s, r\}$ and Panel 9 does the same for shadow banks.

the relaxation of the no-withdrawal constraint. This is consistent with observations in Fact 1 of Section 2 which indicate that traditional bank balance sheets expanded at a faster rate in 2002-07 when the shadow banking sector was growing rapidly than in 2008-15 when the shadow banking sector was stagnant.[39]

Panel 7 demonstrates portfolio re-allocation by traditional banks after bad news. When the shadow banking sector is small and market discipline intact, traditional banks re-allocate up to 18% of their portfolio from risky assets to safe and liquid assets in order to prevent an early withdrawal. As the shadow banking sector grows and market discipline is undermined, traditional banks reduce the extent of their re-allocation away from risky assets and eventually the direction is reversed. In equilibrium, there is a re-allocation of approximately 4% away from risky assets. Compared to Fact 4 of Section 2, this predicts the direction of portfolio re-

[39] Although Panel 6 indicates that, in equilibrium, an individual traditional bank has a larger balance sheet than an individual shadow bank, this does not contradict Figure 1, which is at the aggregate level.

Figure 11: **Social welfare**



allocation correctly, but falls somewhat short of the observed amount of 6% to 8% (see Figure 4).

Panel 8 displays the investment strategies of traditional banks in period 1. When there is no fire-sale on safe assets ($\gamma < \bar{\gamma}$), market discipline forces traditional banks to behave as if they internalize asset payoffs under weak fundamentals. Therefore, they devote an equal share of their investment to each asset type. This maximizes the amount of their financial intermediation (i.e. the amount of assets they originate) for a unit of funding, since asset supply schedules are upward sloping in each asset type.[40] When there is a fire-sale on safe assets ($\gamma > \bar{\gamma}$), traditional banks sharply increase their investment in liquid assets to prop up their liquidation value. At the same time, they reduce their holdings of safe assets due to the erosion of market discipline. Despite the expansion in their balance sheets (Panel 6), this distortion in the composition of their portfolios reduces their financial intermediation, since the costs of originating liquid assets rise substantially and exceed those of safe assets (i.e. $(P_1^{TB}(\lambda) > P_1^{TB}(s))$.

Panel 9 shows the investment strategies of shadow banks. Shadow banks skew their investment towards risky assets due to limited liability. Without the ability to commit, they take their funding costs R^{SB} as given and hence do not change their asset composition in response to fire-sales.

Finally, Figure 11 plots social welfare under the baseline simulation described above as well as a version of the model without liquidity risk ($\bar{\xi} = 0$). As with the simple model, entry into shadow banking is welfare-raising when the sector is small but the equilibrium sector

[40] See Appendix E for the expressions for asset supply schedules. Due to decreasing returns to scale in origination and the equivalence of expected payoffs across the three assets (λ, r, s), it is efficient for banks to invest the same amount in the three assets, taking everything else as given. Accordingly, the efficient allocation prescribes $P_1(\lambda) = P_1(s) = P_1(r)$.

size exceeds the constrained-efficient. While this is true regardless of liquidity risk, liquidity risk is far from irrelevant: Recall from Proposition 4 that traditional banks become vulnerable to liquidity risk in the region $\gamma > \bar{\gamma}$ with a fire-sale on safe assets. Liquidity risk reduces social welfare and results in a constrained-efficient shadow bank sector size of $\bar{\gamma}$, which is the largest sector size without a fire-sale on safe assets. In contrast, in the model without liquidity risk, a fire-sale no safe assets is constrained-efficient. Moreover, while the equilibrium sector size is also higher without liquidity risk ($\gamma^{*\prime}$ compared to γ^* in baseline), its distance from constrained-efficiency is smaller in terms of both welfare and sector size.

This is because liquidity risk and its negative impact on market discipline exacerbates fire-sale externalities associated with entry into shadow banking. This is due to two reasons: First, liquidity shocks on traditional banks raise the excess supply of assets in the secondary market. Therefore, when traditional banks are vulnerable to liquidity shocks, a given amount of entry into shadow banking deepens the fire-sale to a greater extent and crowds out more productive outside investments. Second, as explained above, the interplay between market discipline and liquidity risk distorts the composition of traditional bank investments, reducing their contribution to financial intermediation. This partially offsets the gains for financial intermediation from entry into shadow banking.[41]

6 Policy analysis

In this section, we consider two policy interventions aimed at bringing about a constrained-efficient allocation. Section 6.1 considers a tax on shadow bank profits. While it may not be feasible to tax shadow banks in practice, we show that an equivalent outcome may be achieved with a transfer to traditional banks. In Section 6.2, we consider a second-best intervention in the form of a tax on the sale of risky assets in secondary markets, which indirectly reduces the profitability of shadow banks.[42]

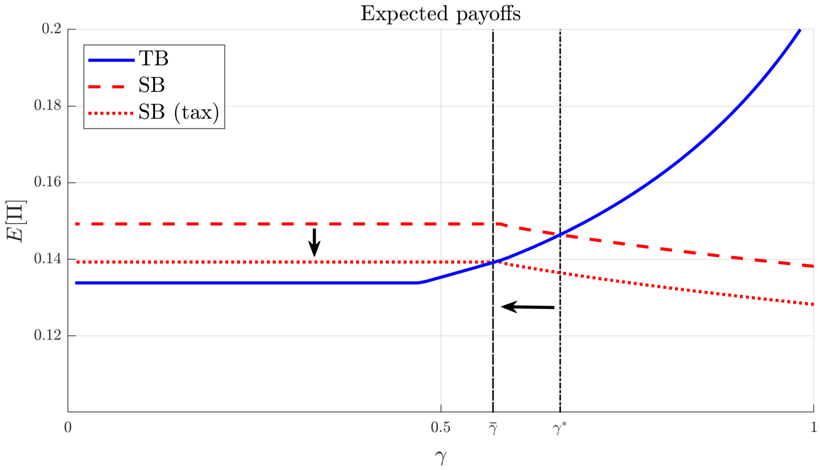
6.1 Tax on shadow bank profits

To begin with, we analyze the taxation of shadow bank profits with the purpose of deterring entry into the sector. This can be considered as a Pigouvian tax because of the negative fire-sale externalities associated with entry into shadow banking. We consider a lump-sum tax T such

[41] Due to decreasing returns to scale in origination and the equivalence of expected payoffs across the three assets (λ, r, s), it is constrained efficient for banks to invest the same amount in the three assets, taking everything else as given. Accordingly, the unconstrained social optimal allocation indicates $P_1(\lambda) = P_1(s) = P_1(r)$.

[42] Note that these interventions remain within the constraints described in Proposition 3. Interventions such as liquidity provision to the banking sector may raise welfare above the constrained-efficient level by using central bank liquidity to relax the trade-off between financial intermedation and investment in outside projects.

Figure 12: **Tax on shadow bank profits**



Note: Expected payoffs are inclusive of the commitment cost τ and tax T.

that the free entry condition becomes

$$E\left[\Pi^{SB}\right] - T = E\left[\Pi^{TB}\right] - \tau$$

and the resulting revenues are redistributed to households with a lump-sum transfer.

Figure 12 shows the outcome under a tax level $0 < T < \tau$ that reduces the equilibrium shadow banking sector size to the constrained-efficient size $\bar{\gamma}$.[43] The tax shifts down the expected payoff schedule for shadow banking and brings about an equilibrium without a fire-sale on safe assets. Since this is a tax on profits and the revenues are rebated to households, the equilibrium allocation at any given sector size γ is identical to the numerical results in Figures 10 and 11. Therefore, reducing the equilibrium sector size to $\bar{\gamma}$ eliminates liquidity risk and restores market discipline in the traditional banking sector.

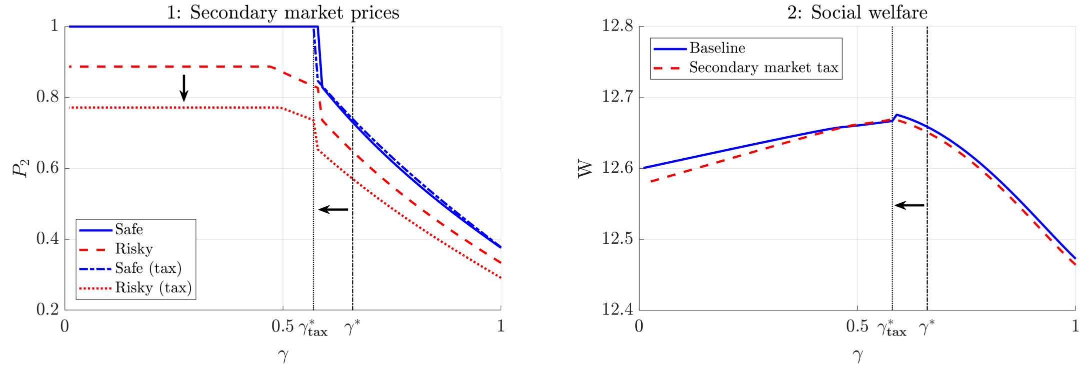
Note that the effect of a tax on shadow banks is equivalent to a decrease in the commitment cost τ. Therefore, when taxing shadow banks is not feasible, the same outcome can also be achieved with a transfer to traditional banks.

6.2 Secondary market tax

When taxation of shadow banks and transfers to traditional banks are not feasible, introducing a tax $T_2(r)$ on the sale of risky assets in secondary markets constitutes a second-best intervention that may move the equilibrium allocation closer to the constrained-efficient. Under this tax,

[43] Quantitatively, our model indicates that the necessary transfer amounts to 9% of commitment costs, which is equivalent to approximately 0.6% of the value of a traditional bank's portfolio in period 1.

Figure 13: **Tax on risky assets in secondary market**



banks face the secondary market price

$$P_{2,tax}(r) = (1 - T_2(r)) P_2(r)$$

while liquidating risky assets. As before, we assume that the revenues are distributed to households in a lump-sum transfer.

Figure 13 shows the outcome of this intervention, which is the opposite of a deepening of secondary markets considered in Section 3.2.2. The intervention shifts down the secondary market price of risky assets at any given sector size γ (Panel 1). This increases the funding costs of shadow banks, reducing the expected payoff from shadow banking and therefore decreasing entry to the sector until we reach a new equilibrium at $\gamma^*_{tax} < \gamma^*$. It is not a coincidence that, at γ^*_{tax}, $P_{2,tax}(r)$ takes the same value as the secondary market price $P_2(r)$ in the pre-intervention equilibrium γ^*. As explained in Section 3.2.1, this is because the fire-sale discount is pinned down by free entry into shadow banking. Therefore, the impact of the tax on $P_{2,tax}(r)$ is offset by adjustments in the sector size and the incidence of taxation falls on outside investors.

Different to Section 3.2.2, however, the tax affects safe and risky assets differentially. Since the sale of safe assets on secondary markets is not taxed, the decline in the shadow bank sector size alleviates the fire-sale on safe assets and reduces liquidity risk on traditional banks. As in the previous section, social welfare is maximized at the largest shadow bank sector size without a fire-sale on safe assets, which we denote with $\bar{\gamma}_{tax}$. Figure 13 depicts a scenario where $T_2(r)$ is set at a level consistent with making this an equilibrium allocation such that $\gamma^*_{tax} = \bar{\gamma}_{tax}$.[44]

Panel 2 shows the impact of the secondary market tax on social welfare across γ. Since the incidence of taxation falls on outside investors, the tax reduces secondary market liquidity both directly and indirectly by driving outside investors to substitute secondary market purchases of risky assets with increased investment in outside projects. This leads to a deeper fire-sale and reduces financial intermediation at any given γ. As a result, the welfare schedule under

[44] Quantatitively, this implies a tax rate of $T_2(r) = 0.13$.

the tax differs from the baseline in two ways: First, it is shifted to the left. Observe from Panel 1 the sector size at which the fire-sale on safe assets takes places is also reduced for the same reason, that is $\bar{\gamma}_{tax} = \gamma^*_{tax} < \bar{\gamma}$. Second, the welfare schedule moves downward compared to the baseline. This downward movement does not constitute a uniform shift, but instead becomes more pronounced as the sector size moves away from the constrained-efficient γ^*_{tax}. When γ is significantly below γ^*_{tax}, the tax leads to a larger fall in welfare because it depresses financial intermediation when it is already far below the constrained-efficient. When γ is above γ^*_{tax}, the tax reduces welfare because it exacerbates liquidity risk on traditional banks. At γ^*_{tax}, however, the downward movement is negligible as there is no liquidity risk and the marginal social benefit from financial intermediation and investment in outside projects are near-equivalent. This ensures that the rise in welfare attained through the reduction in the shadow bank sector size dominates the welfare loss that stems from tax distortions.

However, it is important to note that this intervention might not be time-consistent. This is because entry into shadow banking is determined prior to the news signal such that policymakers may be tempted to eliminate the tax once the sector level is determined and the fire-sale is underway. Notably, Panel 2 indicates that the welfare losses from introducing the tax are low when it is credible and the sector size shifts to γ^*_{tax}, but larger when it is not credible and the sector size remains at γ^*. This highlights the necessity of commitment mechanisms such as binding legislation.

7 Conclusion

We have presented a model of the financial sector in which shadow banking emerges endogenously as an alternative banking strategy. A key aspect of the model is that depositors re-optimize in response to revisions in expectations about asset returns. To prevent early withdrawals by their depositors, traditional banks optimally commit to a safe portfolio strategy while shadow banking strategies combine high risk-taking with the prospect of an early liquidation after bad news.

Two important insights emerge as a consequence. First, costly commitment and fire-sale externalities bring about an equilibrium where ex-ante identical banks optimally cluster into shadow and traditional banking strategies. The size of the shadow banking sector increases in the cost of commitment and the availability of liquidity in the secondary market. Second, because of fire-sale externalities, the shadow banking sector grows inefficiently large in equilibrium with consequences on traditional banks. Particularly, the liquidation of shadow banks leaves traditional banks susceptible to liquidity risk. Higher deposit rates offered by traditional banks in compensation for liquidity risk then weaken market discipline on these banks. This leads to a rise in solvency risk and distorts the composition of traditional bank portfolios.

The model also provides novel insights for policy design. First, we find that the optimal policy is a Pigouvian tax on shadow banks (or equivalently a transfer to traditional banks) that reduces the size of the shadow banking sector to the constrained-efficient level. Second, we show that policy interventions have significantly different implications when the adjustment on the size of the shadow banking sector is taken into account. For example, taxation of secondary market transactions exacerbates the fire-sale at a given sector size, but has no impact on secondary market prices after the resulting reduction in the shadow bank sector size. We find that a tax on risky assets in the secondary market constitutes a second-best intervention: This intervention reduces entry into shadow banking, alleviates the fire-sale on safe assets and curtails liquidity risk on traditional banks. Although the intervention moves the equilibrium closer to the constrained-efficient allocation, it also gives rise to time-inconsistency issues. Since entry into shadow banking takes place before the news signal, policymakers face a temptation to eliminate the tax once the fire-sale is underway.

Finally, it is worth noting that the mechanism considered in this paper is more general than its application to shadow banking and the 2008 financial crisis. In economies without credible deposit insurance guarantees and strict enforcement of banking regulation, financial intermediation strategies that combine high risk-taking with an unstable funding structure may exist within the commercial banking sector. As such, this framework may be relevant for some emerging market economies in contemporary times as well as for the analysis of historical banking panics in the United States prior to the establishment of the Federal Reserve System.

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8 Appendix

A Microfoundations for asset origination

In this section, we provide microfoundations for the asset origination technology described in Section 3.1.1. Let there be a separate but ex-ante identical islands of entrepreneurs which use capital K to produce assets I_1^{ε} with a Cobb-Douglas production technology

$$I_1^{\varepsilon} = AK^{\alpha} \tag{29}$$

Due to relationship lending frictions, banks may only purchase assets from entrepreneurs in their own island, which we denote with I_1.[45] In addition, entrepreneurs may sell assets I_1^l to local (non-bank) creditors such that

$$I_1^{\varepsilon} = I_1 + I_1^l$$

We do not explicitly model local credit markets but posit that asset sales to banks have positive externalities in alleviating information asymmetries in these markets.[46] We depict this by a proportionate relationship

$$I_1^l = \omega I_1$$

where $\omega > 0$ is a parameter. The representative entrepreneur's problem can then be written as

$$\max_{I_1, K} \; P_1 I_1 + P_1^l I_1^l - K$$

subject to the production technology (29) where $\left(P_1, P_1^l\right)$ are the asset prices and the rental rate of capital is normalized to one without loss of generality. The resulting first order conditions take the form of an upward-sloping asset supply schedule

$$P_1 = P_1^l = \frac{1}{\alpha A^{\frac{1}{\alpha}}} \left(I_1 + I_1^l\right)^{\frac{1-\alpha}{\alpha}} \tag{30}$$

where P_1 is specific to each bank. As relationship lending frictions constitute a barrier to entry, banks extract a mark-up

$$\mu \equiv \frac{\partial P_1}{\partial I_1} \frac{I_1}{P_1} = \frac{1-\alpha}{\alpha\left(1+\omega\right)}$$

[45] We implicitly assume that, in the first period, entrepreneurs may also costlessly produce a pseudo-asset that pays zero return and banks may only monitor entrepreneurs in their own island. The same friction also bars households from purchasing assets and banks from trading assets with each other in the first period. Therefore, first period asset prices are best interpreted as the cost of origination rather than the price of a tradable asset.

[46] These externalities could reflect informational spillovers from monitoring by banks or indirect spillovers through external economies of scale in the local monitoring infrastructure.

B Definitions

$$
\begin{aligned}
\underline{\phi} &\equiv \frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} \geq 0 \\
\bar{\phi} &\equiv \min\left[1, \frac{(1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}\right] > \underline{\phi} \\
\underline{\tau} &\equiv \begin{cases} \kappa\left[\frac{\sigma_h - 1}{q}\left(\frac{1-(1-q)\sigma_h}{q}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\sigma_h\left(1 - \frac{\mu}{1+\mu}(1-q)\,\sigma_h\right)^{\frac{\alpha}{1-\alpha}}\right] & \text{for } \mu < \frac{(p-q)(\sigma_h - 1)}{qp(1-q)\sigma_h - (p-q)(\sigma_h - 1)} \\ \kappa\left(\sigma_h - \bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)\left(\bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}} & \\ -\kappa\frac{\mu\sigma_h}{1+\mu}\left((1-q)\,\frac{\sigma_h}{1+\mu} + q\bar{\phi}\left[(1-p)\,\sigma_h + p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}} & \text{otherwise} \end{cases} \\
\bar{\tau} &\equiv \begin{cases} \kappa\,(\sigma_h)^{\frac{1}{1-\alpha}}\left[(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\left(\frac{1-q}{1+\mu} + q\alpha\right)^{\frac{\alpha}{1-\alpha}}\right] & \text{for } \sigma_h \geq \frac{1}{1-qp(1-\alpha)} \\ \kappa\left[\frac{\sigma_h - 1}{qp}\left(\frac{1-(1-qp)\sigma_h}{pq}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\sigma_h\left((1-q)\,\frac{\sigma_h}{1+\mu} + \frac{1-(1-qp)\sigma_h}{p}\right)^{\frac{\alpha}{1-\alpha}}\right] & \text{otherwise} \end{cases}
\end{aligned}
$$

where $\kappa \equiv (1-q)\,(1+\omega)^{-1}\,(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}$.

C Solution under $\phi = \underline{\phi}$

Suppose $\phi < \underline{\phi}$ and hence $P_2 < \sigma_l$ such that traditional banks benefit from buying risky assets both in terms of profits and in terms of the no-withdrawal constraint. Therefore, they find it optimal to hold risky assets until the secondary market price returns to $P_2 = \sigma_l$ (i.e. $\phi = \underline{\phi}$). Let $\tilde{I}_2$ indicate the level of assets that achieve this, implicitly defined by the expression[47]

$$
\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} = f\left(\gamma I_1^{SB} + (1-\gamma)\left(I_1 - \tilde{I}_2\right)\right)
$$

Once the secondary market price reaches $P_2 = \sigma_l$, the no-withdrawal constraint binds and traditional banks behave as described in (1). Therefore, I_1 is set according to $P_1 = P_2 = \sigma_l$ and traditional banks' holdings of safe assets in period 2 is given by

$$
\begin{aligned}
M_2 &= P_2 I_1 - P_2 \tilde{I}_2 \\
&= \sigma_l\left(I_1 - \tilde{I}_2\right)
\end{aligned}
$$

where we have taken advantage of the indeterminacy of $M_1 \geq 0$ to set $M_1 = 0$. Finally, expected

[47]We drop the label 'TB' to simplify the exposition

profits are given by

$$E[\Pi] = (\sigma_h - \sigma_l)\left[(1-q)\frac{A^{\frac{1}{1-\alpha}}(\alpha\sigma_l)^{\frac{\alpha}{1-\alpha}}}{1+\omega} + q(1-p)\tilde{I}_2\right]$$

So far, we have assumed that traditional banks remain net-sellers with $\tilde{I}_2 \leq I_1$. When the excess supply of assets is particularly large, we may have $P_2 < \sigma_l$ even when traditional banks hold on to their risky assets such that $\tilde{I}_2 = I_1$. In this case, they will find it optimal to increase D and M_1 use this to purchase risky assets in period 2 until $P_2 = \sigma_l$. As before, the no-withdrawal constraint will bind at $P_2 = \sigma_l$ and the complete solution is

$$\begin{aligned} P_1 &= P_2 \\ I_2 &= I_1 + \frac{1}{\sigma_l}\tilde{M}_1 \\ M_2 &= 0 \\ D &= P_1 I_1 + \tilde{M}_1 = \sigma_l I_1 + \tilde{M}_1 \end{aligned}$$

where $\tilde{M}_1$ takes on the role of ensuring $P_2 = \sigma_l$ and is implicitly defined by

$$\frac{\sigma_l}{(1-p)\sigma_h + p\sigma_l} = f\left(\gamma I_1^{SB} - (1-\gamma)\frac{1}{\sigma_l}\tilde{M}_1\right)$$

and the expected payoff is

$$E[\Pi] = (\sigma_h - 1)\left(\frac{1-qp}{qp}\frac{A^{\frac{1}{1-\alpha}}}{1+\omega}\left(\alpha\frac{1-(1-qp)\sigma_h}{qp}\right)^{\frac{\alpha}{1-\alpha}} + \frac{q(1-p)}{1-(1-qp)\sigma_h}\tilde{M}_1\right)$$

There are two notable implications. First, the secondary market price cannot go below σ_l. Second, a rise in the shadow banking sector size γ first leads to a rise in $\tilde{I}_2$, and then $\tilde{M}_1$. The above solution shows that $\tilde{M}_1$ and $E[\Pi]$ rises in this case while everything else stays constant. As we move to a limiting case with only shadow banks, safe asset holdings and traditional bank profits both approach infinity

$$\lim_{\gamma\to 1} E\left[\Pi^{TB}\right] = \lim_{\gamma\to 1}\tilde{M}_1 = \infty$$

which guarantees an inferior equilibrium for a sufficiently high commitment cost $\tau > \bar{\tau}$.

D Example fire-sale function

To attain a simple fire-sale function from the outsider investor's problem, we can simply parameterize the payoff function from the outside investment to

$$g\left(\widetilde{K}\right) = \kappa^{-1} \ln\left(\widetilde{K}\right)$$

where $\kappa > 0$. The fire-sale function then becomes

$$f\left(\widetilde{I}\right) = \frac{\kappa \widetilde{E}}{1 + \kappa\left[(1-p)\,\sigma_h + p\sigma_l\right]\widetilde{I}}$$

To satisfy the lower bound condition exactly, we need to set $\widetilde{E}$ at a level that yields $f\left(I_1^{SB}\right) = \underline{\phi}$ at $\gamma = 0$, which is

$$\widetilde{E} = \underline{\phi}\left[\frac{1}{\kappa} + \left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}\left((1-q)\,\frac{\sigma_h}{1+\mu} + \frac{1-(1-qp)\,\sigma_h}{p}\right)^{\frac{\alpha}{1-\alpha}}\right]$$

and the upper bound will approach but never exceed $\bar{\phi}$ as κ rises.

E Description of the model with liquidity risk

We describe banks and entrepreneurs below. Outside investors are described in Section 4.2 while households are described under Appendix H.7.

E.1 Asset origination

Asset origination technologies only differ from those described in Section 3.1.1 in that there is now a separate technology for each asset type $i \in \{\lambda, s, r\}$, which yield the set of asset supply schedules

$$P_1(i) = \frac{1}{\alpha A^{\frac{1}{\alpha}}}\left((1+\omega)\, I_1(i)\right)^{\frac{1-\alpha}{\alpha}} \quad \forall i \in \{\lambda, s, r\} \tag{31}$$

where $P_1(i)$ is specific to each bank and the mark-up extracted by banks is common across asset types

$$\mu \equiv \frac{\partial P_1(i)}{\partial I_1(i)}\frac{I_1(i)}{P_1(i)} = \frac{1-\alpha}{\alpha(1+\omega)} \;\forall i \in \{\lambda, s, r\}$$

E.2 Banks

With a richer asset space, the first period budget constraint becomes

$$\sum_{i\in\{\lambda,s,r\}} P_1(i)\, I_1(i) = D$$

where $I_1(i)$ is the amount purchased of an asset i and $P_1(i)$ is the corresponding asset price.[48] As in the simple model, assets are priced at their expected payoff in the secondary market after good news and trade is inconsequential. Bank profits (in the third period) are then given by

$$\Pi_{gh} = \sigma_h I_1(r) + I_1(s) + I_1(\lambda) - DR \tag{32}$$

Following bad news, banks face the second period budget constraint[49]

$$\sum_{i\in\{s,r\}} \left(P_2(i) - P_1(i)\right) I_1(i) = I_1(\lambda) \tag{33}$$

and have a liquidation value

$$\theta = \min\left\{1, \frac{P_2(r)\, I_2(r) + P_2(s)\, I_2(s)}{D}\right\} \tag{34}$$

When risky assets yield a high payoff σ_h in the third period, banks make a profit

$$\Pi_{gh} = \sigma_h I_2(r) + I_2(s) - DR$$

while limited liability binds under a low asset payoff σ_l. Under limited liability, banks make zero profits and deposits pay a recovery rate

$$V = \min\left\{1, \frac{\sigma_l I_2(r) + I_2(s)}{DR}\right\} \tag{35}$$

which is proportional to the shortfall of funds.

Next, we evaluate the optimal behavior of banks under the two alternative strategies of shadow and traditional banking. The free entry condition is given by (25) as in the simple model.

[48] The absence of cash holdings M_1, M_2 from the bank's choice set is without loss of generality as holding cash is strictly dominated by the liquid asset λ in period 1 and equivalent to holding the safe asset s in period 2.

[49] Observe that the liquid assets λ disappear from the bank's choice set and the economy after yielding their payoff in period 2. This is without loss of generality, since investing in λ is equivalent to or strictly dominated by investing in s in period 2 given that they both yield a certain payoff in period 3.

Shadow banking

Shadow banks choose $\left\{I_1^{SB}(i), D^{SB}, i \in \{\lambda, s, r\}\right\}$ to maximize their expected profits

$$E\left[\Pi^{SB}\right] = (1-q)\left(\sigma_h I_1^{SB}(r) + I_1^{SB}(s) + I_1^{SB}(\lambda) - D^{SB}R^{SB}\right)$$

subject to (32). The optimal portfolio allocation is then determined by the set of first order conditions

$$\begin{aligned} P_1(r) &= \frac{\sigma_h}{(1+\mu)R^{SB}} \\ P_1(s) &= P_1(\lambda) = \frac{1}{(1+\mu)R^{SB}} \end{aligned}$$

which allow us to back out the asset holdings in period 1 using (31).

Traditional banking

Using (35), we can write the no-withdrawal constraint for traditional banks as

$$\sigma_l I_2^{TB}(r) + I_2^{TB}(s) \geq \bar{V} D^{TB} R^{TB} \tag{36}$$

Traditional banks choose $\left\{I_1^{TB}(i), I_2^{TB}(r), I_2^{TB}(s), M_1^{TB}, M_2^{TB}, D^{TB}, i \in \{\lambda, s, r\}\right\}$ to maximize their expected profits

$$\begin{aligned} E\left[\Pi^{TB}\right] &= (1-q)\left(\sigma_h I_1^{TB}(r) + I_1^{TB}(s) + I_1^{TB}(\lambda)\right) \\ &\quad + q(1-p)(1-\xi)\left(\sigma_h I_2^{TB}(r) + I_2^{TB}(s)\right) \\ &\quad - (1-q+q(1-p)(1-\xi))\, D^{TB}R^{TB} \end{aligned}$$

subject to (32), (33) and (36). Due to their ability to commit, traditional banks also internalize the relationship between their liquidation value θ^{TB} given by (34) and the minimum recovery rate $\bar{V}$ and R^{TB} as per the expressions in Proposition 4.

We focus on the case with a slack no-short-sale constraint

$$I_1^{TB}(r) > 0$$

which is the case presented in our results. By combining (33) and (36), we can write the following expressions for second period asset holdings

$$
\begin{aligned}
I_2^{TB}(s) &= \frac{P_2^{TB}(r)\bar{V}R^{TB}D^{TB} - \sigma_l\left(P_2^{TB}(s)I_1^{TB}(s) + P_2^{TB}(r)I_1^{TB}(r) + I_1^{TB}(\lambda)\right)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)} \\
I_2^{TB}(r) &= \frac{P_2^{TB}(s)\left(I_1^{TB}(s) - \bar{V}R^{TB}D^{TB}\right) + P_2^{TB}(r)I_1^{TB}(r) + I_1^{TB}(\lambda)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)}
\end{aligned}
$$

When there is a liquidity shortfall $\theta^{TB} < 1$, we can use the expressions in Proposition 4 and (34) to write the problem as

$$
\begin{aligned}
E\left[\Pi^{TB}\right] &= (1-q)\left(\sigma_h I_1^{TB}(r) + I_1^{TB}(s) + I_1^{TB}(\lambda)\right) \\
&+ \left[P_2^{TB}(s)I_1^{TB}(s) + P_2^{TB}(r)I_1^{TB}(r) + I_1^{TB}(\lambda)\right] * \Bigg[q\xi\frac{1-q+q(1-p)(1-\xi)}{1-q} \\
&+ q\frac{(1-p)(1-\xi)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)}\left((\sigma_h - \sigma_l) - \xi\frac{q(1-p)}{p(1-q)}\left(\sigma_h P_2^{TB}(s) - P_2^{TB}(r)\right)\right)\Bigg] \\
&- (1-q+q(1-p)(1-\xi))\left(1 + \frac{q}{1-q}\xi\right)D^{TB} \\
&- q(1-p)(1-\xi)\left(1 - \xi\frac{q(1-p)}{p(1-q)}\right)\left(\frac{\sigma_h P_2^{TB}(s) - P_2^{TB}(r)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)}\right)D^{TB}
\end{aligned}
$$

s.t.

$$
P_1^{TB}(\lambda)I_1^{TB}(\lambda) + P_1^{TB}(s)I_1^{TB}(s) + P_1^{TB}(r)I_1^{TB}(r) = D^{TB}
$$

which yields the first order conditions

$$
\begin{aligned}
P_1^{TB}(r) &= \frac{1}{1+\mu}\frac{(1-q)\sigma_h + \bar{Z}_1 P_2^{TB}(r)}{\bar{Z}_2} \\
P_1^{TB}(s) &= \frac{1}{1+\mu}\frac{1-q+\bar{Z}_1 P_2^{TB}(s)}{\bar{Z}_2} \\
P_1^{TB}(\lambda) &= \frac{1}{1+\mu}\frac{1-q+\bar{Z}_1}{\bar{Z}_2}
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{Z}_1 &\equiv q\frac{\xi(1-q+q(1-p)(1-\xi))}{1-q} \\
&+ \frac{q(1-p)(1-\xi)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)}\left((\sigma_h - \sigma_l) - \xi\frac{q(1-p)}{p(1-q)}\left(\sigma_h P_2^{TB}(s) - P_2^{TB}(r)\right)\right) \\
\bar{Z}_2 &\equiv (1-q+q(1-p)(1-\xi))\left(1 + \frac{q}{1-q}\xi\right) \\
&+ q(1-p)(1-\xi)\left(1 - \xi\frac{q(1-p)}{p(1-q)}\right)\left(\frac{\sigma_h P_2^{TB}(s) - P_2^{TB}(r)}{P_2^{TB}(r) - \sigma_l P_2^{TB}(s)}\right)
\end{aligned}
$$

When there is no liquidity shortfall $\theta^{TB}=1$, we can use $R^{TB}=\bar{V}=1$ from Proposition 4 to write the problem as

$$\begin{aligned}
E\left[\Pi^{TB}\right] &= (1-q)\left(\sigma_h I_1^{TB}(r)+I_1^{TB}(s)+I_1^{TB}(\lambda)\right) \\
&\quad +q(1-p)(\sigma_h-\sigma_l)\frac{P_2^{TB}(s)\,I_1^{TB}(s)+P_2^{TB}(r)\,I_1^{TB}(r)+I_1^{TB}(\lambda)}{P_2^{TB}(r)-\sigma_l P_2^{TB}(s)} \\
&\quad -\left(q(1-p)\left(\frac{\sigma_h P_2^{TB}(s)-P_2^{TB}(r)}{P_2^{TB}(r)-\sigma_l P_2^{TB}(s)}\right)+(1-qp)\right)D^{TB}
\end{aligned}$$

s.t.

$$P_1^{TB}(\lambda)\,I_1^{TB}(\lambda)+P_1^{TB}(s)\,I_1^{TB}(s)+P_1^{TB}(r)\,I_1^{TB}(r)=D^{TB}$$

The first order conditions FOCs then become

$$\begin{aligned}
P_1^{TB}(r) &= \frac{1}{1+\mu}\frac{(1-q)\,\sigma_h\left(P_2^{TB}(r)-\sigma_l P_2^{TB}(s)\right)+q(1-p)(\sigma_h-\sigma_l)\,P_2^{TB}(r)}{(1-q)\,P_2^{TB}(r)+(q(1-p)\,\sigma_h-(1-qp)\,\sigma_l)\,P_2^{TB}(s)} \\
P_1^{TB}(s) &= \frac{1}{1+\mu}\frac{(1-q)\left(P_2^{TB}(r)-\sigma_l P_2^{TB}(s)\right)+q(1-p)(\sigma_h-\sigma_l)\,P_2^{TB}(s)}{(1-q)\,P_2^{TB}(r)+(q(1-p)\,\sigma_h-(1-qp)\,\sigma_l)\,P_2^{TB}(s)} \\
P_1^{TB}(\lambda) &= \frac{1}{1+\mu}\frac{(1-q)\left(P_2^{TB}(r)-\sigma_l P_2^{TB}(s)\right)+q(1-p)(\sigma_h-\sigma_l)}{(1-q)\,P_2^{TB}(r)+(q(1-p)\,\sigma_h-(1-qp)\,\sigma_l)\,P_2^{TB}(s)}
\end{aligned}$$

Welfare

Social welfare is given by

$$\begin{aligned}
W &= (1+\omega)\sum_{i\in\{\lambda,s,r\}}\left(\left((1-\gamma)\left(1-\alpha P_1^{TB}(i)\right)I_1^{TB}(i)+\gamma\left(1-\alpha P_1^{SB}(i)\right)I_1^{SB}(i)\right)\right) \\
&\quad +E-(1-\gamma)\,\tau+qg\left(\widetilde{K}\right)
\end{aligned}$$

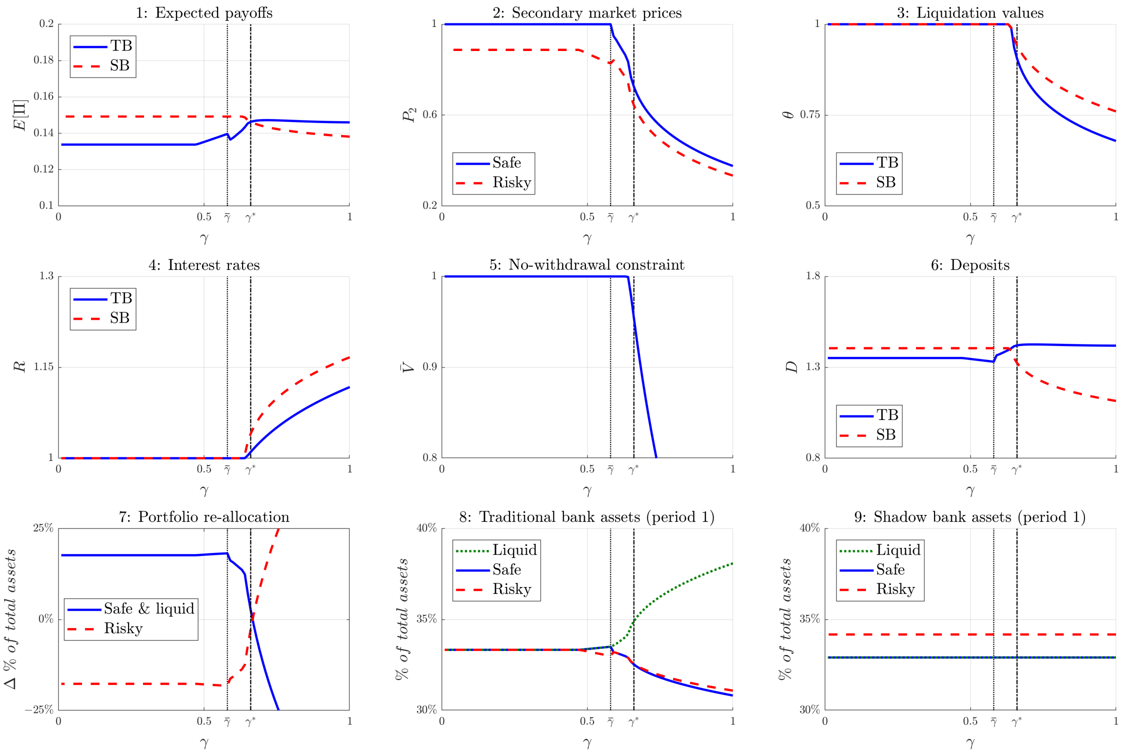
where the only difference from the simple model is the richer asset space.

F Alternative timing for liquidity shocks

In this section, we consider an alternative timing for liquidity shocks where they are preceded by the early withdrawal decision. This allows households to secure themselves from liquidity shocks by withdrawing their deposits early, such that their expected consumption with and without an early withdrawal are respectively given by

$$\begin{aligned}
c_b^w &= m_1+\theta^{SB}d^{SB}+d^{TB} \\
(1-p)\,c_{bh}+pc_{bl} &= m_1+\theta^{SB}d^{SB}+\left[(1-p(1-V))(1-\xi)\,R^{TB}+\xi\theta^{TB}\right]d^{TB}
\end{aligned}$$

Figure 14: **Results under alternative liquidity risk specification**



Note: Expected payoffs are inclusive of the commitment cost τ. Total assets in period 1 and 2 are respectively defined as $\bar{I}_1 \equiv \sum_{i\in\{\lambda,s,r\}} I_1(i)$ and $\bar{I}_2 \equiv \sum_{i\in\{s,r\}} P_2(i)\, I_2(i)$. Panel 7 plots $\left(\bar{I}_2^{TB}\right)^{-1} P_2(s)\, I_2^{TB}(s) - \left(\bar{I}_1^{TB}\right)^{-1} \sum_{i\in\{\lambda,s\}} I_1^{TB}(i)$ for safe and liquid assets, and $\left(\bar{I}_2^{TB}\right)^{-1} P_2(r)\, I_2^{TB}(r) - \left(\bar{I}_1^{TB}\right)^{-1} I_1^{TB}(r)$ for risky assets. Panel 8 plots $I_1^{TB}(i)/\bar{I}_1$ respectively for $i = \{\lambda, s, r\}$ and Panel 9 does the same for shadow banks.

Households will optimally withdraw early when

$$c^w > (1-P)\, c_{bh} + p c_{bl}$$

Using (56), we can write this condition as

$$\xi\left(1-\theta^{TB}\right) + p\,(1-\xi)\,(1-V) > 0$$

which indicates that, when there is liquidity risk (i.e. $\xi > 0$, $\theta^{TB} < 1$), households choose to withdraw even when banks have no fundamental solvency risk ($\bar{V} = 1$). The implication is that the early withdrawal decision becomes completely dependent on liquidity concerns, which may become self-fulfilling in the manner of Diamond and Dybvig (1983).

G Alternative specification for liquidity risk

In this section, we present results for a version of the model where liquidity risk stems from self-fulfilling bank-runs. When a bank has a liquidity shortfall $\theta < 1$, sequential service in withdrawals leads to the emergence of a bank-run equilibrium where households find it optimal to withdraw their deposits given that everyone else is withdrawing.

In order to resolve this multiplicity, we follow the global games solution of Goldstein and Pauzner (2005), and depict the probability ξ that a bank faces a self-fulfilling run as a negative function $\zeta(.)$ of its liquidation value θ such that

$$\xi = \zeta(\theta),$$
$$\zeta'(.) \leq 0, \ \zeta(\theta) \in [0,1] \ \forall \ \theta$$

where $\zeta(1) = 0$ ensures that banks without a liquidity shortfall are not vulnerable to self-fulfilling runs.[50] We parameterize $\zeta(.)$ simply as

$$\zeta(\theta) = \max\left\{0, \min\left\{1, \widetilde{\xi}(1-\theta)\right\}\right\}$$

with $\widetilde{\xi} = 1.64$ calibrated in line with the calibration strategy described in Section 5.1. Figure 14 provides the numerical results under a set up and calibration that are otherwise identical to those presented in Section 5.2. In equilibrium, the two liquidity risk specifications yield exactly the same set outcome. At above equilibrium sizes of shadow banking ($\gamma > \gamma^*$), the bank run specification implies further increases in ξ in line with the decrease in liquidity. This leads to lower traditional bank profits and a sharper decline in minimum recovery rate $\bar{V}$ compared to the baseline model.

H Proofs of Propositions and Lemmata

H.1 Proof of Lemma 1

Expected consumption is given by

$$(1-P)\, c_{bh} + pc_{bl} = m_1 + \theta^{SB} d^{SB} + \left(1 - p\left(1 - \bar{V}\right)\right) d^{TB} R^{TB}$$

when households do not withdraw their deposits early from traditional banks, and

$$c_b^w = m_1 + \theta^{SB} d^{SB} + d^{TB}$$

[50] We also impose $\psi(\theta) \in [0,1] \ \forall \ \theta$ since ξ is a probability.

when they do.[51] Therefore, it is optimal for households not to withdraw their deposits under the condition

$$(1-p)\, c_{bh} + pc_{bl} \geq c_b^w$$
$$\therefore V \geq \bar{V} \equiv \frac{1}{p}\left(\frac{1}{R^{TB}} - (1-p)\right)$$

H.2 Proof of Lemma 2

Consider a bank for which the limited liability constraint does not bind such that it borrows at the risk-free rate R and chooses $\{I_1, I_2, M_1, M_2\}$ to maximize expected profits

$$\max_{I_1, I_2, M_1, M_2} \quad (1-q)\left[\sigma_h I_1 + M_1\right] + q\left(\left[(1-p)\,\sigma_h + p\sigma_l\right] I_2 + M_2\right) - D$$

subject to (16), (18). Since $P_2 \leq (1-p)\,\sigma_h + p\sigma_l$ given $\phi \leq 1$, assets are priced at or below their expected payoff after bad news. Therefore the bank weakly prefers investing in I_2 over M_2 such that we can set

$$\begin{aligned} I_2 &= I_1 + \frac{M_1}{P_2} \\ M_2 &= 0 \end{aligned}$$

This reduces the maximization problem to

$$\max_{I_1, M_1} I_1 + \left(1 - q + \frac{q}{\phi}\right) M_1 - (P_1 I_1 + M_1)$$

and the first order conditions for (I_1, M_1) are respectively

$$I_1 : P_1 = \frac{1}{1+\mu} \tag{37}$$
$$M_1 : \phi = 1 \tag{38}$$

where (38) indicates that the bank will increase its cash holdings until $\phi = 1$. Using these first order conditions, we can attain the following expression for profits in the state with low asset payoffs

$$\begin{aligned} \Pi_{bl} &= \sigma_l I_2 + M_2 - D \\ &= \left(\sigma_l - \frac{1}{1+\mu}\right) I_1 + \left(\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} - 1\right) M_1 \end{aligned}$$

[51] Since households are atomistic, they do not internalize that their decision to withdraw deposits reduces the liquidation value of traditional banks.

which indicates that the limited liability constraint binds in this state under the conditions

$$\sigma_l - \frac{1}{1+\mu} < 0$$
$$\frac{\sigma_l}{(1-p)\,\sigma_h + p\sigma_l} - 1 < 0$$

These conditions are respectively satisfied under (3) and $\sigma_l < \sigma_h$. Therefore, we prove by contradiction that limited liability binds in the state with low asset payoffs. Moreover, it follows from this finding that $V^{SB} < 1$. By combining (13) with (14), we can also show that $\bar{V} = 1$ such that failure to repay deposits fully leads to an early withdrawal.[52]

H.3 Proof of Lemma 3

The first order conditions to this problem indicate that shadow banks do not find it optimal to hold any cash $\left(M_1^{SB} = 0\right)$ when $R^{SB} > 1$.[53] Therefore, their liquidation value can be written as

$$\theta^{SB} = \frac{P_2}{P_1^{SB}} \tag{39}$$

where P_1^{SB} is pinned down by the first order condition for the risky asset

$$P_1^{SB} = \frac{\sigma_h}{1+\mu}\frac{1}{R^{SB}} \tag{40}$$

Combining (40) and (15) yields

$$P_1^{SB} = (1-q)\,\frac{\sigma_h}{1+\mu} + qP_2 \tag{41}$$

and by substituting this into (39) we attain

$$\theta^{SB} = \frac{(1+\mu)\,P_2}{(1-q)\,\sigma_h + q\,(1+\mu)\,P_2} \tag{42}$$

There will be a liquidity shortfall when $\theta^{SB} < 1$. Since θ^{SB} is increasing in $P_2 = \phi\,[(1-p)\,\sigma_h + p\sigma_l]$, setting $\phi = 1$ provides a sufficient condition for this. With some algebra, we can write this condition as

$$(1+\mu)\,[(1-p)\,\sigma_h + p\sigma_l] < \sigma_h$$

[52] Note that, although (13) and (14) are associated with traditional banks in Section 3.1.3, the conditions for a shadow bank to face an early withdrawal are identical under risk neutrality given expectations of no early withdrawal. In Appendix H.3, we also confirm that the optimal strategy followed by shadow banks given expectations of an early withdrawal leads to such a withdrawal.

[53] There is a no-short-sale constraint $(I_1, M_1) \geqq 0$ which is only binding for cash.

A further sufficient condition can be attained by setting the mark-up to its maximum value under $\omega = \underline{\omega}$. The condition then becomes $\sigma_h > 1$ which must be true. To get an expression for interest rates, we combine (42) with (15) such that

$$R^{SB} = \frac{1}{1 - q + q\frac{P_2}{\sigma_h}(1+\mu)}$$

and $R^{SB} > 1$ follows from $\theta^{SB} < 1$. Substituting (16) and (40) into (21) gives an expression for the expected payoff

$$E\left[\Pi^{SB}\right] = (1-q)\frac{\mu}{1+\mu}\sigma_h I_1^{SB}$$

where I_1^{SB} is attained by combining (41) with the asset supply schedule (2) such that

$$I_1^{SB} = \frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left((1-q)\frac{\sigma_h}{1+\mu} + qP_2\right)^{\frac{\alpha}{1-\alpha}}$$

Observe that I_1^{SB}, P_1^{SB} and $E\left[\Pi^{SB}\right]$ are all positive related to P_2.

Finally, we confirm that the early withdrawal from shadow banks is optimal under this solution. It is optimal for households to withdraw their deposits early from shadow banks when

$$\left(1 - p\left(1 - V^{SB}\right)\right)R^{SB} < 1$$

where V^{SB} is defined according to (19). Using (15), (39) and (40), we can write

$$V^{SB} = (1+\mu)\frac{\sigma_l}{\sigma_h}$$

and the condition becomes

$$\frac{(1-p)\sigma_h + p(1+\mu)\sigma_l}{(1-q)\sigma_h + q(1+\mu)P_2} < 1$$

which is true under the restrictions $\phi > \underline{\phi}$, $\omega \geq \underline{\omega}$.

H.4 Proof of Proposition 1

Combining (13) and (14) yields

$$R^{TB} = \bar{V} = 1$$

After substituting for $\left(R^{TB},\bar{V}\right)$ and dropping the label 'TB' to simplify the exposition, the traditional bank's problem can be written in as

$$\Pi=(1-q)\left(\sigma_h I_1+M_1\right)+q\left(1-p\right)\left(\sigma_h I_2+M_2\right)-\left(1-qp\right)D \tag{43}$$
$$\text{s.t.}$$
$$P_1 I_1+M_1=D$$
$$P_2 I_2+M_2=P_2 I_1+M_1 \tag{44}$$
$$\sigma_l I_2+M_2\geq D \tag{45}$$
$$\left(I_1,I_2,M_1,M_2\right)\geqq 0$$

where the last line represents no-short-sale constraints. There are three alternative cases depending on whether the no-withdrawal and no-short-sale constraint on I_2 bind. Below, we describe the case in Proposition 1. In the sections below, we also describe the remaining cases and prove that they may not be valid under the restrictions $\omega\geq\underline{\omega}$, $\phi>\underline{\phi}$.

Proposition 1 describes the case where the no-withdrawal constraint (45) and the no-short-sale constraint on I_2 bind. With $I_2=0$, the second period budget constraint (44) and the no-withdrawal constraint can respectively be written as

$$\begin{aligned} M_2 &= P_2 I_1+M_1 \\ M_2 &= P_1 I_1+M_1 \end{aligned}$$

Therefore, the no-withdrawal constraint may only be satisfied with $I_1>0$ when

$$P_1=P_2$$

which pins down P_1 and also corresponds to

$$I_1=\frac{\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}}{1+\omega}P_2^{\frac{\alpha}{1-\alpha}}$$

as per (2).[54]

Note also that the no-withdrawal constraint prevents the bank from converting M_1 to risky assets in the second period as long as $\phi>\underline{\phi}$. As such, the bank may not profit from holding cash in the first period and M_1 is indeterminate. Therefore, the expected payoff can be written

[54] Any solution with $I_1=0$ is sub-optimal as (2) indicates that P_1 would approach zero.

as

$$
\begin{aligned}
E\left[\Pi^{TB}\right] &= (1-q)\left(\sigma_h - P_2\right) I_1 \\
&= (1-q)\left(\sigma_h - P_2\right) \frac{\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}}{1+\omega} P_2^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

and its derivative with respect to P_2 is

$$
\frac{\partial E\left[\Pi^{TB}\right]}{\partial P_2} = \frac{\left(A\alpha^{\alpha}\right)^{\frac{1}{1-\alpha}}}{1+\omega} \frac{1-q}{1-\alpha}\left(\frac{\alpha\sigma_h}{P_2} - 1\right) P_2^{\frac{\alpha}{1-\alpha}}
$$

such that

$$
\frac{\partial E\left[\Pi^{TB}\right]}{\partial P_2} < 0 \; \forall \; P_2 > \alpha\sigma_h
$$

Alternative cases of the traditional bank's problem

Case 1 In the first alternative case, the no-withdrawal constraint binds but the no-short-sale constraint is slack. By combining (44) and (45), we can write

$$
\begin{aligned}
I_2 &= \frac{P_2 - P_1}{P_2 - \sigma_l} I_1 \\
M_2 &= P_2 \left(\frac{P_1 - \sigma_l}{P_2 - \sigma_l}\right) I_1 + M_1
\end{aligned}
\tag{46}
$$

where $P_2 > \sigma_l$ follows from $\phi > \underline{\phi}$ and M_1 is indeterminate as in Case 1. Substituting these into (43) yields the following first order condition for I_1

$$
P_1 = \frac{1}{1+\mu} \frac{(1-q)\,\sigma_h \left(P_2 - \sigma_l\right) + q\,(1-p)\left(\sigma_h - \sigma_l\right) P_2}{q\,(1-p)\left(\sigma_h - P_2\right) + (1-qp)\left(P_2 - \sigma_l\right)}
\tag{47}
$$

and the expected payoff is

$$
\Pi = \left[(1-q)\,\sigma_h + q\,(1-p)\left(\sigma_h - \sigma_l\right) \frac{P_2}{P_2 - \sigma_l}\right] \frac{\mu}{1+\mu} I_1
\tag{48}
$$

Finally, we derive a condition to eliminate this case by considering the no-short-sale constraint $I_2 \geq 0$. Since $P_2 > \sigma_l$, $I_1 > 0$, (46) indicates that $I_2 \geq 0$ will bind when

$$
P_2 < P_1
$$

Using (47), we can write this as

$$P_2 < \frac{1}{1+\mu} \frac{(1-q)\,\sigma_h\,(P_2-\sigma_l) + q\,(1-p)\,(\sigma_h-\sigma_l)\,P_2}{(1-qp)\,(P_2-\sigma_l) + q\,(1-p)\,(\sigma_h-P_2)} \tag{49}$$

which implicitly establishes a boundary fire-sale discount $\hat{\phi}$ above which the no-short-sale constraint is slack.[55] Case 2 is eliminated for all $\phi \in [0,1]$ when $\hat{\phi} > 1$. The relevant condition can then be attained by combining (49) with $\phi = 1$ such that

$$(1-p)\,\sigma_h + p\sigma_l < \frac{1}{1+\mu}$$
$$\therefore \omega > \underline{\omega}$$

which indicates that the restriction (3) eliminates Case 2. Note also that even in the absence of the restriction (3), the equilibrium fire-sale never occurs under this case since the expected payoff (48) is decreasing in ϕ. Without the restriction (3), the size of the shadow banking sector simply continues to expand until the equilibrium takes the form described in Appendix C with

$$\phi = \underline{\phi} \equiv \frac{(1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}$$

Case 2 In the second alternative case, the no-withdrawal constraint is slack. Due to limited liability, banks strictly prefer to convert their cash to risky assets I_2 following bad news to profit from the decline in P_2. Therefore, we can write

$$\begin{aligned} I_2 &= I_1 + \frac{M_1}{P_2} \\ M_2 &= 0 \end{aligned}$$

and the first order conditions for (M_1, P_1) are respectively written as

$$\begin{aligned} P_1 &= \frac{\sigma_h}{1+\mu} \\ P_2 &< \sigma_h \end{aligned}$$

Since $P_2 < \sigma_h$ even without a fire-sale under bad news, banks optimally hold $M_1 \to \infty$. In other words, with the no-withdrawal constraint is slack, banks find it profitable to hold as much cash as possible in the first period and then convert all of it into risky assets after bad news. Since each unit of M_1 requires a unit of deposits, and risky assets contribute to low state revenues by $\sigma_l < 1$, it is impossible for the no-withdrawal constraint to remain slack under this

[55] With $\sigma_l = 0$, we can get an explicit expression $\hat{\phi} = \frac{1}{1-q}\left(\frac{1}{1+\mu}\frac{1-qp}{1-p} - q\right)$.

investment strategy. Therefore, Case 3 is also eliminated.

H.5 Proof of Proposition 2

For the purposes of the proof, it is convenient to introduce some additional notation. Let ϕ^* denote the equilibrium fire-sale discount and the functions $\left(\pi^{SB}(\phi), \pi^{TB}(\phi)\right)$ map from the fire-sale discount to expected payoffs from shadow and traditional banking such that

$$\begin{aligned}
\pi^{SB}(\phi) &= (1-q)\frac{\mu}{1+\mu}\sigma_h\frac{(A\alpha^\alpha)^{\frac{1}{1-\alpha}}}{1+\omega}\left((1-q)\frac{\sigma_h}{1+\mu}+q\phi\left[(1-p)\sigma_h+p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}} \\
\pi^{TB}(\phi) &= (1-q)\left(\sigma_h-\phi\left[(1-p)\sigma_h+p\sigma_l\right]\right)\frac{(A\alpha^\alpha)^{\frac{1}{1-\alpha}}}{1+\omega}\left(\phi\left[(1-p)\sigma_h+p\sigma_l\right]\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

as per Lemma 3 and Proposition 1. There is an interior equilibrium when the following sufficient conditions are satisfied

$$\begin{aligned}
\pi^{SB}\left(\overline{\phi}\right) &> \pi^{TB}\left(\overline{\phi}\right)-\tau & (50)\\
\pi^{SB}\left(\underline{\phi}\right) &> \pi^{TB}\left(\underline{\phi}\right)-\tau & (51)\\
\frac{\partial\pi^{SB}(\phi)}{\partial\phi} &> \frac{\partial\pi^{TB}(\phi)}{\partial\phi}\ \forall\ \phi\in(\phi^*,1) & (52)\\
\frac{\partial\phi}{\partial\gamma} &< 0\ \forall\ \gamma\in[0,1] & (53)
\end{aligned}$$

where $\tau>0$. In the sections below, we show that these conditions will be satisfied within a range of commitment costs $\tau\in(\underline{\tau},\bar{\tau})$ and also show that this range is non-empty.

Proof for condition (50) The condition depends on the value taken by

$$\bar{\phi}\equiv\min\left[1,\frac{(1-q)\frac{\sigma_h}{1+\mu}+q\sigma_l}{(1-p)\sigma_h+p\sigma_l}\right]$$

When we have

$$\mu<\frac{(p-q)(\sigma_h-1)}{qp(1-q)\sigma_h-(p-q)(\sigma_h-1)} \tag{54}$$

such that $\bar{\phi}=1$, the relevant condition is

$$\pi^{SB}(1)>\pi^{TB}(1)-\tau$$

Using the definitions for $\left(\pi^{SB}(.), \pi^{TB}(.)\right)$, we can write this condition as a minimum commitment cost

$$\tau \geq \underline{\tau} \equiv (1-q)\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}(\sigma_h - 1)\left(\frac{1}{q}\right)^{\frac{1}{1-\alpha}}(1-(1-q)\sigma_h)^{\frac{\alpha}{1-\alpha}}$$
$$-(1-q)\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\frac{\mu}{1+\mu}\sigma_h\left(1-\frac{\mu}{1+\mu}(1-q)\sigma_h\right)^{\frac{\alpha}{1-\alpha}} > 0$$

As an aside, we also show that $\underline{\tau} > 0$ such that a positive commitment cost is necessary for an interior equilibrium. To do this, note that the expected payoff under Case 1 and Case 2 of the traditional bank's problem in Appendix H.4 are equivalent when $\phi = 1$, $\omega = \underline{\omega}$. For any $\omega > \underline{\omega}$, profits under Case 1 are higher. Therefore, we can set $\omega = \underline{\omega}$, $\alpha < 0.5$ to write a sufficient condition

$$(1-q)\sigma_h[(1-q)\sigma_h + q] < 1$$

Note that the RHS is increasing in σ_h. A further sufficient condition is then to set $\sigma_l = 0$ which maximizes σ_h. We can then see that the above condition is true for all $p < 1$. Therefore, we can show that $\tau > 0$ under the two conditions

$$\begin{aligned}\alpha &< \frac{1}{2}\\ \omega &\geq \underline{\omega}\end{aligned}$$

When (54) is not satisfied such that $\bar{\phi} < 1$, the relevant condition for (50) is

$$\pi^{SB}\left(\bar{\phi}\right) > \pi^{TB}\left(\bar{\phi}\right) - \tau$$

which leads to a higher minimum commitment cost

$$\begin{aligned}\underline{\tau} &= (1-q)\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left(\sigma_h - \bar{\phi}[(1-p)\sigma_h + p\sigma_l]\right)\left(\bar{\phi}[(1-p)\sigma_h + p\sigma_l]\right)^{\frac{\alpha}{1-\alpha}} - \\ &\quad (1-q)\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\frac{\mu}{1+\mu}\sigma_h\left((1-q)\frac{\sigma_h}{1+\mu} + q\bar{\phi}[(1-p)\sigma_h + p\sigma_l]\right)^{\frac{\alpha}{1-\alpha}}\end{aligned}$$

where the aside on $\tau > 0$ is still valid.

Proof for condition (51) It follows from Appendix C that (51) will be satisfied when the lower bound restriction on ϕ is violated such that

$$\underline{\phi} > f\left(\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left((1-q)\frac{\sigma_h}{1+\mu} + q\sigma_l\right)^{\frac{\alpha}{1-\alpha}}\right)$$

for any $\tau < \infty$. Therefore, we do not necessarily need an upper bound on the commitment cost for an interior equilibrium. However, the interior equilibrium has different properties in the region $\phi < \underline{\phi}$ (as described in Appendix C) and we impose an upper bound on the commitment cost to prevent this.

Let $\widetilde{\phi} \equiv \frac{\alpha\sigma_h}{(1-p)\sigma_h + p\sigma_l}$ denote the fire-sale discount that maximizes traditional bank profits. The upper bound depends on where $\widetilde{\phi}$ stands relative to $\underline{\phi}$. When the following condition is true

$$\sigma_h \geq \frac{1}{1 - qp(1-\alpha)} \tag{55}$$

such that $\widetilde{\phi} > \underline{\phi}$, the upper bound for commitment costs $\bar{\tau}$ must satisfy

$$\pi^{SB}\left(\widetilde{\phi}\right) < \pi^{TB}\left(\widetilde{\phi}\right) - \tau$$

which yields the upper bound

$$\tau \leq \bar{\tau} = (1-q)\frac{(\sigma_h A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\left(\frac{1-q}{1+\mu} + q\alpha\right)^{\frac{\alpha}{1-\alpha}}\right]$$

When (55) is not satisfied, the fire-sale discount hits $\phi = \underline{\phi}$ before traditional bank profits peak and the upper bound is given by

$$\tau \leq \bar{\tau} = (1-q)\frac{(A\alpha^{\alpha})^{\frac{1}{1-\alpha}}}{1+\omega}\left[(\sigma_h - \sigma_l)\sigma_l^{\frac{\alpha}{1-\alpha}} - \frac{\mu}{1+\mu}\sigma_h\left((1-q)\frac{\sigma_h}{1+\mu} + q\sigma_l\right)^{\frac{\alpha}{1-\alpha}}\right]$$

Note that, regardless of the value taken by $(\bar{\tau}, \underline{\tau})$, it follows from (52) that $\bar{\tau} > \underline{\tau}$.

Proof for condition (52) This follows directly from Lemma 3, which shows that

$$\frac{\partial \pi^{SB}(\phi)}{\partial \phi} > 0 \ \forall \ \phi \in (0,1)$$

and Lemma 1 which shows that

$$\frac{\partial \pi^{TB}(\phi)}{\partial \phi} < 0 \ \forall \ \phi \in \left(\widetilde{\phi}, \bar{\phi}\right)$$

where $\phi^* > \widetilde{\phi}$ since the latter is the peak of traditional bank profits in the range above $\underline{\phi}$.

Proof for condition (53) Recall that the excess supply of assets is given by

$$\widetilde{I} = \gamma I_1^{SB} + (1-\gamma)\left(I_1^{TB} - I_2^{TB}\right) > 0$$

where $\left(I_1^{SB}, I_1^{TB}\right)$ depend on ϕ and $I_2^{TB} = 0$. Given that $f(.)$ is continuous and decreasing, to satisfy (53) we require that

$$\frac{\partial \widetilde{I}}{\partial \gamma} = I_1^{SB} - I_1^{TB} > 0 \; \forall \; \gamma \in [0,1]$$

which is equivalent to

$$I_1^{SB} > I_1^{TB} \; \forall \phi \in (\underline{\phi}, \overline{\phi}]$$

At any given ϕ, we have $I_1^{SB} > I_1^{TB}$ when the following condition is satisfied

$$q\sigma_h > (1+\mu)\,\phi\,[1-(1-q)\,\sigma_h]$$

Since the RHS is increasing in μ and ϕ, a sufficient condition is to set $\phi = 1$, $\mu = \bar{\mu}$, which will be satisfied for $\sigma_h > 1$.

Since $\left(I_1^{SB}, I_1^{TB}\right)$ both decrease in ϕ at different rates, we also need to show that I_1^{SB} at $\underline{\phi}$ is lower than I_1^{TB} at $\bar{\phi}$. This will be true with $\bar{\phi} = 1$ when (54) is satisfied. Otherwise, $\bar{\phi}$ will need to satisfy

$$\bar{\phi} = \frac{(1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}$$

which is precisely how we define the upper bound restriction on the fire-sale discount.

Proof for the non-emptiness of $(\underline{\tau}, \bar{\tau})$ Finally, we prove that $\bar{\tau} > \underline{\tau}$ such that there is a non-empty set of commitment costs that bring about an interior equilibrium. Since there are two alternatives values for both $\bar{\tau}$ and $\underline{\tau}$, we consider each in turn. First suppose that (54) is satisfied so that $\underline{\tau}$ is in line with $\phi = 1$. Then it follows from Proposition 1 that $\bar{\tau} > \underline{\tau}$ regardless of which value $\bar{\tau}$ takes. Second, suppose (54) is not satisfied so that $\underline{\tau}$ is in line with $\bar{\phi} < 1$. When (55) is also not satisfied such that traditional bank profits do not peak until $\underline{\phi}$, $\bar{\tau} > \underline{\tau}$ follows from $\bar{\phi} > \underline{\phi}$.

The only case where we need impose an additional condition corresponds to (55) being satisfied so that $\bar{\tau}$ is in line with the peak $\phi = \frac{\alpha\sigma_h}{(1-p)\sigma_h + p\sigma_l}$ while (54) is not satisfied such that $\bar{\phi} < 1$. The condition for non-emptiness is then equivalent to

$$\widetilde{\phi} < \bar{\phi}$$

$$\therefore \frac{\alpha\sigma_h}{(1-p)\,\sigma_h + p\sigma_l} < \frac{(1-q)\,\frac{\sigma_h}{1+\mu} + q\sigma_l}{(1-p)\,\sigma_h + p\sigma_l}$$

A sufficient condition is

$$\alpha < 1 - p$$

which should be satisfied when $\alpha < 0.5$, $p \leq 0.5$.

H.6 Proof of Proposition 3

To begin with, consider the (unconstrained) efficient allocation. After netting out transfers between agents, the social planner's problem can be written as

$$\max_{I_1(j), I_2(j)} W = (1+\omega) \int_{j\in[0,1]} (1 - \alpha P_1(j)) I_1(j) - \tau(j)\, dj + E + qg\left(\widetilde{K}\right)$$

subject to

$$\begin{aligned}
\widetilde{K} &= \widetilde{E} - \phi\left[(1-p)\sigma_h + p\sigma_l\right] \int_{j\in[0,1]} I_1(j) - I_2(j)\, dj \\
\phi &= \frac{1}{g'\left(\widetilde{K}\right)} \\
P_1(j) &= \frac{1}{\alpha A^{\frac{1}{\alpha}}} \left((1+\omega) I_1(j)\right)^{\frac{1-\alpha}{\alpha}} \ \forall\, j \in [0,1]
\end{aligned}$$

where j indices banks, $\tau(j) = \tau$ when the commitment cost is paid and zero otherwise, and $\widetilde{K}$ is given by

$$\widetilde{K} = \widetilde{E} - \phi\left[(1-p)\sigma_h + p\sigma_l\right] \int_{j\in[0,1]} I_1(j) - I_2(j)\, dj$$

The solution to this problem can then be written as

$$\begin{aligned}
P_1^e(j) &= (1-qp)\sigma_h + qp\sigma_l \ \forall\, j \in [0,1] \\
\tau^e(j) &= 0 \ \forall\, j \in [0,1] \\
I_2^e(j) &= I_1^e(j) \ \forall\, j \in [0,1]
\end{aligned}$$

where the condition for $P_1^e(j)$ also pins down $I_1^e(j)$.

For the constrained-efficient allocation, the social planner chooses $\gamma \in [0,1]$ to maximize the same social welfare function subject to all of the optimality conditions described in Section 3, except for the free entry condition (25). Using the optimality conditions, we can write the social planner's problem as

$$\max_{\gamma\in[0,1]} W = (1-\gamma)\left(\left(1-\alpha P_1^{TB}\right) I_1^{TB} - \tau\right) + \gamma\left(1 - \alpha P_1^{SB}\right) I_1^{SB} + E + qg\left(\widetilde{K}\right)$$

subject to

$$
\begin{aligned}
\widetilde{K} &= \widetilde{E} - \phi\left[(1-p)\,\sigma_h + p\sigma_l\right]\left[\gamma I_1^{SB} + (1-\gamma)\, I_1^{TB}\right] \\
\phi &= \frac{1}{g'\left(\widetilde{K}\right)} \\
I_1^j &= \frac{A^{\frac{1}{1-\alpha}}\left(\alpha P_1^j\right)^{\frac{\alpha}{1-\alpha}}}{1+\omega} \;\forall\; j = \{SB, TB\} \\
P_1^{TB} &= \phi\left[(1-p)\,\sigma_h + p\sigma_l\right] \\
P_1^{SB} &= (1-q)\,\frac{\sigma_h}{1+\mu} + q\phi\left[(1-p)\,\sigma_h + p\sigma_l\right]
\end{aligned}
$$

Note that the last two constraints imply that $(1-qp)\,\sigma_h + qp\sigma_l > P_1^{SB} > P_1^{TB}$ such that financial intermediation falls short of the efficient amount. A marginal rise in γ then has four distinct effects on social welfare: First, since $P_1^{SB} > P_1^{TB}$, the bank that enters shadow banking increases its financial intermediation, which increases social welfare. Second, the bank no longer pays the commitment cost τ, which also increases social welfare as τ constitutes a deadweight loss to the economy. Third, the amount of assets liquidated in secondary markets after bad news rises, leading to crowding out of investments in outside projects $\widetilde{K}$. This reduces social welfare. Fourth, the increase in liquidated assets leads to a deeper fire-sale, reducing the extent of intermediation by all other banks. The constrained-efficient allocation is at the point where the first two positive effects are exactly offset by the latter negative two effects, as indicated by the first order condition

$$
\begin{gathered}
\left(1-\alpha P_1^{SB}\right) I_1^{SB} - \left(1-\alpha P_1^{TB}\right) I_1^{TB} + \tau + g'\left(\widetilde{K}\right)\frac{\partial \widetilde{K}}{\partial \gamma} + \\
\left[(1-\gamma)\left[\left(1-\alpha P_1^{TB}\right)\frac{\partial I_1^{TB}}{\partial \gamma} - \alpha \frac{\partial P_1^{TB}}{\partial \gamma} I_1^{TB}\right] + \gamma\left[\left(1-\alpha P_1^{TB}\right)\frac{\partial I_1^{SB}}{\partial \gamma} - \alpha\frac{\partial P_1^{SB}}{\partial \gamma} I_1^{SB}\right]\right] = 0
\end{gathered}
$$

where

$$
\frac{\partial I_1^{SB}}{\partial \gamma} < 0,\ \frac{\partial I_1^{TB}}{\partial \gamma} < 0,\ \frac{\partial P_1^{SB}}{\partial \gamma} < 0,\ \frac{\partial P_1^{TB}}{\partial \gamma} < 0,\ \frac{\partial \widetilde{K}}{\partial \gamma} < 0,\ g'\left(\widetilde{K}\right) > 0
$$

Since outside projects have diminishing returns with $g''(.) < 0$, for sufficiently high outside investor endowment $\widetilde{E}$, the constrained-efficient allocation will be an interior equilibrium

$$
\gamma^{ce} > 0
$$

where shadow banks exist. However, as the negative effects of entry into shadow banking on intermediation by other banks are externalities, γ^{ce} falls short of the (laissez-faire) equilibrium shadow bank sector size γ^*, that is

$$\gamma^{ce} < \gamma^*$$

H.7 Proof of Proposition 4

We first solve the household's problem to attain the expressions (27), (28). Allowing for liquidity risk, the household's problem can be written as

$$\max_{d^{SB},d^{TB},m_1,m_2} \quad (1-q)\,c_{gh} + q\,(1-p)\,c_{bh} + qpc_{bl}$$

$$\begin{gathered}
\text{s.t.} \\
d^{SB} + d^{TB} + m_1 = E \\
m_2 = m_1 + \theta^{SB} d^{SB} \\
c_{gh} = m_1 + d^{TB} R^{TB} + d^{SB} R^{SB} \\
c_{bh} = m_2 + \left[(1-\xi)\,R^{TB} + \xi\theta^{TB}\right] d^{TB} \\
c_{bl} = m_2 + \left[(1-\xi)\,\bar{V} R^{TB} + \xi\theta^{TB}\right] d^{TB}
\end{gathered}$$

with the first order conditions (15) for deposits in shadow banks and

$$R^{TB} = 1 + \frac{q\xi\left(1-\theta^{TB}\right) + qp\,(1-\xi)\left(1-\bar{V}\right)}{1 - q\left(\xi + (1-\xi)\,p\left(1-\bar{V}\right)\right)} \tag{56}$$

for deposits in traditional banks.

Early withdrawal decision and market discipline

Expected consumption without and with an early withdrawal are respectively given by

$$\begin{aligned}
(1-P)\,c_{bh} + pc_{bl} &= m_1 + \theta^{SB} d^{SB} + \left[\left(1 - p\left(1-\bar{V}\right)\right)(1-\xi)\,R^{TB} + \xi\theta^{TB}\right] d^{TB} \\
c_b^w &= m_1 + \theta^{SB} d^{SB} + \left(1 - \left(1-\theta^{TB}\right)\xi\right) d^{TB}
\end{aligned}$$

where the latter expression indicates that a household that decides to withdraw its deposit early may receive an incomplete repayment due to a liquidity shock. Under this set up, all terms that relate to liquidity risk cancel out and the minimum repayment rate schedule is given by

$$\begin{gathered}
(1-P)\,c_{bh} + pc_{bl} \geq c_b^w \\
\therefore V \geq \bar{V} \equiv \frac{1}{p}\left(\frac{1}{R^{TB}} - (1-p)\right)
\end{gathered} \tag{57}$$

which remains identical to the simple model. Combining (56) and (57) then yields

$$\begin{aligned}\bar{V} &= 1-\frac{q}{p}\frac{\xi\left(1-\theta^{TB}\right)}{1-q\left(1-\xi\left(1-\theta^{TB}\right)\right)}\\ R^{TB} &= 1+\frac{q}{1-q}\xi\left(1-\theta^{TB}\right)\end{aligned}$$

With $\xi=0$, these expressions simplify to $\bar{V}=R^{TB}=1$. Note that $\bar{V}$ is decreasing in ξ and R^{TB} is increasing. Therefore a rise in ξ leads to

$$\bar{V}<1<R^{TB}$$

Fire-sale on the safe asset and liquidity shortfall

Finally, we show that the fire-sale on the safe is a crucial determinant of liquidity risk. Consider the liquidation value given by (34). There will be no liquidity shortfall such that $\theta^{TB}=1$ under the condition

$$P_2(r)\,I_2^{TB}(r)+P_2(s)\,I_2^{TB}(s)\geq D^{TB}$$

First, consider the case without a fire-sale on safe assets such that $P_2(s)=1$. The condition becomes

$$I_2^{TB}(r)+I_2^{TB}(s)\geq D^{TB}$$

and will be true under any investment strategy that satisfies the no-withdrawal constraint (36) as long as

$$P_2(r)\geq\sigma_l$$

To see that $P_2(r)\geq\sigma_l$ must be true, consider what would happen otherwise. Since traditional banks are protected by limited liability, they do not internalize the state with weak fundamentals. Therefore, given $P_2(s)=1\geq P_2(r)$, traditional banks always prefer to purchase risky assets which yield a higher return in the state where they remain solvent. Ordinarily, traditional banks' risky asset purchases are limited by the no-withdrawal constraint. However, with $P_2(r)<\sigma_l$ and $P_2(s)=1$, the strategy of selling a safe asset and purchasing risky assets with the funds increases the recovery rate V. Consequently, traditional banks increase their purchases of risky assets until their price rises to $P_2(r)=\sigma_l$.

Second, consider the case with a fire-sale on safe assets such that $P_2(s)=\phi$. Since shadow banks are liquidated after bad news, and traditional banks re-allocate their portfolio from risky to safe assets, there is an excess supply of risky assets at all times such that $P_2(r)=\phi$. We can then write the condition for $\theta^{TB}=1$ as

$$\phi\left(I_2^{TB}(r)+I_2^{TB}(s)\right)\geq D^{TB} \tag{58}$$

Note that the no-withdrawal constraint is not tightened by a decline in ϕ when both safe and risky assets are in excess supply, since the terms of trade between the two assets do not change, while the value of liquid asset holdings increase. Therefore, for sufficiently low ϕ, (58) fails and there is a liquidity shortfall $\theta^{TB} < 1$.