Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation

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Abstract

I build a quantitative model of economic growth that can be used to evaluate the impact of environmental policy interventions on final-use energy consumption, an important driver of carbon emissions. In the model, energy demand is driven by endogenous and directed technical change (DTC). Unlike existing DTC models, I consider the case where multiple technological characteristics are embodied in each capital good, rather than in different sectors. Energy supply is subject to increasing extraction costs. The model is consistent with aggregate evidence on energy use, efficiency, and prices in the United States. In my primary analysis, I examine the impact of new energy taxes and compare the results to the standard Cobb-Douglas approach used in the environmental macroeconomics literature, which is not consistent with data. When examining a realistic and identical path of energy taxes in both models, the directed technical change model predicts 24% greater cumulative energy use over the next century. I also use the model to study the macroeconomic consequences of energy efficiency mandates. I find large rebound effects that undermine the environmental effectiveness of such policies.

Keywords Energy, Climate Change, Directed Technical Change, Growth

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1 Introduction

Discussions of climate change mitigation often focus on substitution between clean and dirty sources of energy. Carbon accounting, however, shows that improvements in energy efficiency have been the primary driver of long-run reductions in the carbon intensity of output (e.g., Raupach et al., 2007; Nordhaus, 2013). In this paper, I further demonstrate that the increases in energy efficiency are driven by improvements in *final-use energy efficiency*. In other words, reductions in the carbon-intensity of output tend to occur when capital goods and consumer durables require less energy to run, not when an economy uses more renewable energy sources or gets better at turning primary energy (e.g., coal) in to final-use energy (e.g., electricity).

Despite the importance of final-use energy efficiency in determining long-run trends in carbon emissions, this margin has received relatively little attention in the existing environmental macroeconomics literature. Existing studies with directed technical change and climate change focus on clean versus dirty sources of energy and do not consider energy efficiency as a separate source of technological improvement (e.g., Acemoglu et al., 2012, 2016). Existing studies with exogenous technology, other the other hand, frequently assume that final-use energy is combined with capital and labor in a Cobb-Douglas fashion (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014), even though this is at odds with well known data patterns (Atkeson and Kehoe, 1999; Hassler et al., 2012, 2016b). Given the importance of final-use energy in determining long-run trends in carbon emissions, our understanding of climate change mitigation policies will be incomplete until we have a model that can be used to understand responses along this important margin.

This paper constructs the first quantitative macroeconomic model focusing on final-use energy. I show that a simple and tractable growth model is consistent with existing evidence on aggregate energy use, efficiency, and prices. I then calibrate the model to macroeconomic data from the United States and investigate the impacts of environmental policy on final-use energy consumption.

The demand side of the model highlights the role of endogenous and directed technical change.[1] Existing evidence suggests that there is a near-zero short-run elasticity of substitution between energy and non-energy inputs, but a unit-elastic long-run elasticity. The model captures this fact by differentiating between *ex post* substitution – which captures substitution when technology is fixed – and *ex ante* substitution, which occurs through the choice of technology (e.g., Jones, 2005; Caselli et al., 2006; Leon-Ledesma and Satchi, 2016). For a given set of technologies, energy and non-energy inputs must be combined in fixed proportions. Capital good producers, however, respond to increases in the relative price of energy by lowering the energy input ratio through directed research and development activity. Thus, the long-run elasticity of substitution is higher than the short-run elasticity.

I develop a new underlying model of directed technical change. The standard Acemoglu (1998,

[1]As discussed below section, this modelling strategy builds on the insight of Hassler et al. (2012, 2016b) who note that directed technical change models can explain observed data on energy use and prices, but do not examine the implications for climate policy.

2002) approach focuses on the role of innovation in different sectors.[2] To focus on final-use energy efficiency – rather than the efficiency of the energy sector – I consider the case where two types of technology are embodied in each capital good. One type captures the ability of the capital good to produce output. The other captures the energy efficiency of final good production.

Since the model focuses on the role of final-use energy, I consider a simple representation of primary energy supply (e.g., coal, oil). There is a single aggregate energy composite that is available in infinite supply, but is subject to increasing extraction costs. Most directed technical change models of energy use consider the case where the finiteness of energy resources drives long-run trends in prices (Hassler et al., 2012, 2016b; André and Smulders, 2014).[3] I show, however, that this Hotelling (1931) approach is at odds with aggregate data, which demonstrate that energy use is increasing on the balanced growth path. The increasing extraction cost model is consistent with data and suggests that energy consumption growth will continue at current rates in the absence of policy intervention or an environmental disaster.

Aside from energy considerations, the model is made up of neoclassical elements. As a result, it yields a standard balanced growth path and can be calibrated to aggregate data from the United States. I use the calibrated model to investigate the impacts of climate change mitigation policies on final-use energy consumption.[4]

My primary analysis considers the case of energy taxes and compares the new model to the standard Cobb-Douglas approach with exogenous technology (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014). Existing studies argue that the Cobb-Douglas model is an appropriate stand-in for a model with endogenous and directed technical changes, because both approaches feature unit-elastic long-run substitution between energy and non-energy inputs (Golosov et al., 2014; Barrage, forthcoming; Hassler et al., 2016c, 2017). When explicitly comparing the predictions of the two models, however, I find that they yield significantly different predictions about the impact of environmental policy.

The rationale for the conventional wisdom is straightforward: the study of climate change is inherently concerned with long-run outcomes. Moreover, the Cobb-Douglas and directed technical change models have identical long-run predictions in the absence of policy. Thus, they are likely yield similar long-run predictions in a world with policy. I show, however, that this reasoning does not hold up to quantitative scrutiny. Climate change is a function of the stock of carbon in the atmosphere, rather than the flow of emissions. Since the directed technical change model accurately captures the low short-run elasticity of substitution observed in the data, it predicts that energy efficiency will react slowly to new energy taxes. By contrast, the Cobb-Douglas approach

[2]As a result, it has been used effectively to consider the role of clean versus dirty energy sectors (e.g., Acemoglu et al., 2012, 2016; Fried, 2018).

[3]This literature is focused on the economic consequences of exhaustible resources, rather than climate change and environmental policy.

[4]In this sense, the paper complements the existing directed technical change literature, which focuses on substitution between energy sources and abstracts from investments in energy efficiency (Acemoglu et al., 2012, 2016; Fried, 2018). Combining the two approaches to gain a complete quantitative understanding of the impact of climate policy is a particularly important step for future work. I do not undertake such an analysis here in order to focus on the important role of final-use energy efficiency.

predicts that reactions will occur immediately, because it is relatively easy to substitute between energy and non-energy inputs. In other words, the directed technical change model features slower transitions. As a result, the two models yield different predictions for medium-term and cumulative energy use, even in cases where they predict the same flow of energy use at some point in the future.

The easiest way to compare the quantitative predictions of the two models is to examine their predicted reactions to the same path of future energy taxes. I simulate taxes in the Cobb-Douglas model that are needed to reduce energy use by 40% between 2005 and 2055.[5] With the same path of taxes, the directed technical change model misses the energy use target by 15% percentage points. Thus, the slow transition path implies that policy designed with the Cobb-Douglas model is unlikely to achieve intended targets for medium-run flows of emissions. At the same time, the consumption cost of the energy taxes is twice as large in the directed technical change model. The directed technical change model also predicts 24% greater *cumulative* energy use over the next century. Given that the new model better explains past patterns of energy use, these results provide evidence that existing analyses are too optimistic about the impacts of climate change mitigation policy, at least when considering the important margin of final-use energy efficiency.

In a subsequent analysis, I investigate the macroeconomic consequences of research subsidies and mandates for energy efficiency technologies, which are commonly used in attempts mitigate climate change and achieve energy security (Gillingham et al., 2009; Allcott and Greenstone, 2012). Despite their popularity, these policies may be ineffective due to rebound in energy use. Rebound occurs when economic behavior lessens the reduction in energy use following efficiency improvements. A long existing literature attempts to indirectly evaluate the effectiveness of such policies by estimating the size of rebound effects, usually in partial equilibrium or static settings (Gillingham et al., 2016). The directed technical change model, however, makes it possible to directly analyze the broader motivating question: can policies that improve energy efficiency achieve long-term reductions in energy use, even if they do not increase energy prices? I start by considering the standard rebound exercise of a one-off improvement in energy efficiency. Consistent with existing evidence, such shocks lead to short-run reductions in energy use (e.g., Davis, 2017), but they also lower the incentive for future investment in energy efficient technology. As a result, the interventions lead to temporary increases in medium-term energy use relative to world without policy, an extreme form of rebound known as 'backfire.' Eventually, the short-term reductions and medium-term backfire offset each other, leaving cumulative energy use unchanged. Permanent policy interventions can overcome rebound effects to achieve long-run reductions in energy use relative to laissez faire, but cannot achieve absolute decreases in energy use.

[5]This is consistent with goals laid out in the Paris Agreement, which suggests that the United States adopt policies consistent with a 80% reduction in carbon emissions by 2050 (Heal, 2017). Thus, I examine a case where half of the required reduction in carbon emissions comes from reductions in energy use. This is consistent with existing predictions (Williams et al., 2014). The goals are outlined in the Intended Nationally Determined Contribution (INDC) submitted by the United States to the United Nations Framework Convention on Climate Change (UNFCC), which is available at: `http://www4.unfccc.int/submissions/INDC/Published%20Documents/United%20States%20of%20America/1/U.S.%20Cover%20Note%20INDC%20and%20Accompanying%20Information.pdf`.

Related Literature. — This paper contributes to several existing literatures. First, it extends our understanding of carbon accounting. An existing literature focusing on the Kaya identity demonstrates that energy efficiency, rather than the carbon intensity of energy, has driven reductions in carbon emissions in the United States and around the world (Raupach et al., 2007; Nordhaus, 2013). I take this finding one step further and show that these reductions in energy use are driven by *final-use energy efficiency*.

Second, this paper contributes to the quantitative macroeconomic literature on climate change by constructing a model focusing on final-use energy. Existing studies on directed technical change (DTC) and climate change focus on clean versus dirty sources of energy and do not consider energy efficiency as a separate source of technological improvement (e.g., Acemoglu et al., 2012, 2016; Fried, 2018). Meanwhile, the literature on endogenous, but not directed, energy efficiency improvements focuses on the efficiency of the energy sector (e.g., Popp, 2004; Bosetti et al., 2006). While both of these margins are important, the data strongly suggest that the overlooked margin of final-use energy efficiency is an important long-run driver of carbon emissions. Existing studies with exogenous technology frequently assume that final-use energy is combined with capital and labor in a Cobb-Douglas fashion (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014; Barrage, forthcoming), but this is at odds with existing data (Atkeson and Kehoe, 1999; Hassler et al., 2012, 2016b). As noted above, I show that this difference in modelling has important consequences for understanding the impacts of climate change mitigation policy.

Third, this paper is related to the literature on directed technical change and energy use, which focuses on questions of long-run sustainability (e.g., Di Maria and Valente, 2008; André and Smulders, 2014). The most closely related paper is that of Hassler et al. (2012, 2016b), who show that a model of directed technical changes in consistent with data on energy demand from the United States. They use this observation to examine how a social planner should manage a finite resource and generate predictions for future consumption growth. I build on their findings in several ways. First, I develop a decentralized model that can be used to investigate the impacts of environmental policy. While Hassler et al. (2012, 2016b) do not analyze the impacts of policy, their findings have been used to support the use of the Cobb-Douglas assumption in climate change economics (e.g., Golosov et al., 2014; Barrage, forthcoming; Hassler et al., 2016c, 2017). By explicitly analyzing the impact of policy in both models, I show the opposite result: constraining the model to match short-run data – as in the DTC model – leads to very different long-run reactions to policy. I also build on the work of Hassler et al. (2012, 2016b) by considering an alternative model of primary energy supply. In particular, I consider the case of increasing extraction costs, rather than finite energy supplies. I show that (i) of the two models, only the increasing cost formulation is consistent with aggregate data and (ii) the increasing cost formulation leads to different predictions about growth in consumption and energy use in the absence of climate change mitigation policy.

Finally, this paper contributes to the broader literature on the modelling of directed technical change. Employing the directed technical change model of Acemoglu (1998, 2002) in the context of energy efficiency requires focusing on technological improvements in the energy sector (e.g., Smulders and De Nooij, 2003; André and Smulders, 2014). In order to focus on final-use energy efficiency, therefore, I construct a new model where both types of technology are embodied in the capital good.[6]

Section 2 discusses the empirical motivation underlying the theory. The model is presented in Section 3 and the calibration in Section 4. Section 5 reports the results of the quantitative analyses, and Section 6 concludes.

2 Empirical Motivation

2.1 The Importance of Final-Use Energy

In this section, I demonstrate that final-use energy efficiency has played a crucial role in reducing the carbon-intensity of output in the United States and around the world. To analyze the determinants of the carbon intensity of output, I consider the following decomposition:

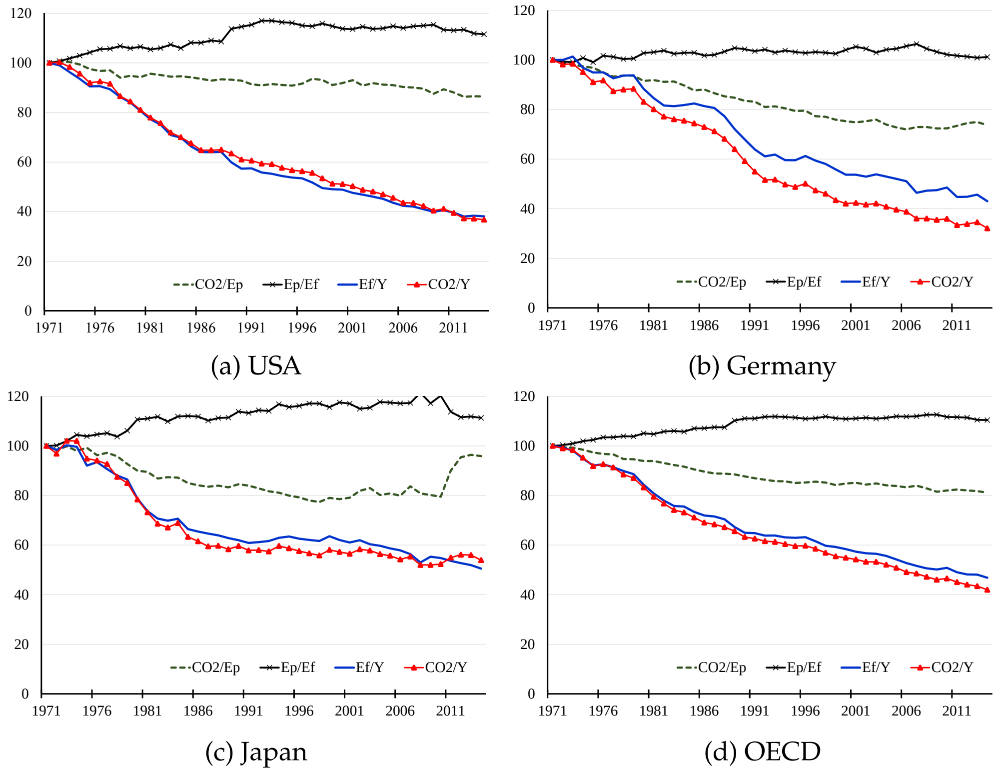
$$\frac{CO_2}{Y} = \frac{CO_2}{E_p} \cdot \frac{E_p}{E_f} \cdot \frac{E_f}{Y}, \tag{1}$$

where CO_2 is yearly carbon emissions, Y is gross domestic product, E_p is primary energy use (e.g., coal, oil), and E_f is final-use energy consumption (e.g., electricity, gasoline). The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, captures substitution between clean and dirty sources of energy (e.g., coal versus solar). The efficiency of the energy sector, which transforms primary energy into final-use energy, is captured by $\frac{E_p}{E_f}$. For example, the ratio decreases when power plants become more efficient at transforming coal into electricity. The final-use energy intensity of output, $\frac{E_f}{Y}$, measures the quantity of final-use energy used per unit of output. For example, the ratio decreases when manufacturing firms use less electricity to produce the same quantity of goods.

The results of this decomposition are presented in Figure 1, which plots each component from equation (1) from 1971-2014. Data are normalized to 1971 values.[7] Panel (a) shows the results for the United States, which will be the focus of the quantitative policy analysis. The carbon intensity of output fell over 60% during this time period, and this decline is matched almost exactly by the decline in the final-use energy intensity of output. The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, declined approximately 15% over this period. While this is a significant improvement for environmental outcomes, it is relatively small compared to the overall improvements in the carbon intensity of output. Finally, the efficiency of the energy transformation sector, as measured by the

[6] An extension of the model also incorporates aspects of 'second-wave' endogenous growth theory (e.g., Peretto, 1998; Howitt, 1999) to eliminate the scale effects present in most directed technical change models (e.g., Acemoglu, 2002; Acemoglu et al., 2012; Hassler et al., 2012, 2016b). This allows for labor to move between production and research even as population grows, which is important for the current context because (i) growing population is an important driver of energy use and (ii) environmental policy may increase the incentive for investment in energy efficient technologies.

[7] Appendix Section A describes the data and provides links to the original sources.



(a) USA (b) Germany

(c) Japan (d) OECD

Figure 1: This figure decomposes the decline in the carbon intensity of output using the identity: $\frac{CO_2}{Y} = \frac{CO_2}{E_p} \cdot \frac{E_p}{E_f} \cdot \frac{E_f}{Y}$, where CO_2 is yearly carbon emissions, Y is GDP, E_p is primary energy, and E_f is final-use energy. This figure demonstrates that the fall in the carbon intensity of output, $\frac{CO_2}{Y}$, has been driven by decreases in final-use energy intensity of output, $\frac{E_f}{Y}$, rather than the use of cleaner energy sources, $\frac{CO_2}{E_p}$, or a more efficient energy transformation sector, $\frac{E_p}{E_f}$. Data are from the International Energy Agency (IEA) and the World Development Indicators (WDI). All values are normalized to 1971 levels.

inverse of $\frac{E_p}{E_f}$, actually declined roughly 15% over this period.[8]

Similar results hold for other developed countries. Panel (b) shows that this general patterns holds for Germany, which has had significantly greater reductions in the carbon intensity of output. Panel (c) shows that similar results hold for Japan, and panel (d) presents aggregate results for the OECD.

To the best of my knowledge, this is the first paper to perform a carbon accounting exercise using equation (1). Existing studies often focus on the Kaya Identity, which only considers the role of primary energy. These studies show that energy efficiency is the main driver of long-run trends in carbon emissions (e.g., Raupach et al., 2007; Peters et al., 2017). Thus, this is the first study to demonstrate the relative importance of final-use energy efficiency, as compared to the efficiency of the energy sector. Motivated by these findings, this paper focuses on final-use energy efficiency and its role in climate change mitigation.

[8] This result is driven by differences in the efficiency of transformation across different sources of primary energy, rather than technological regress.

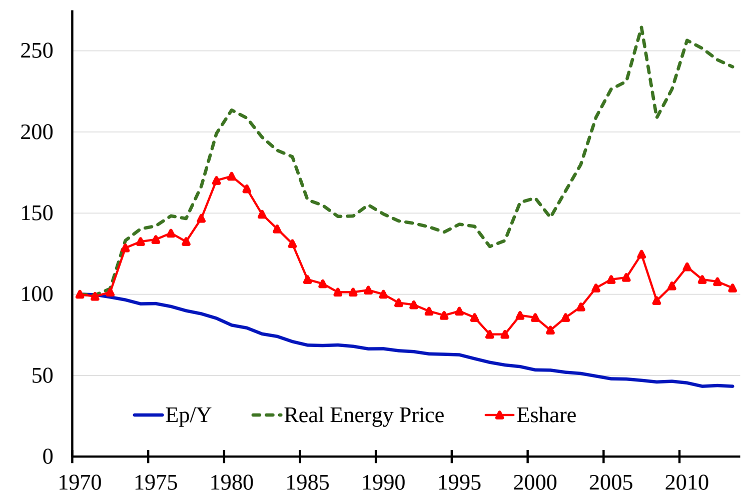


Figure 2: This figure shows the energy expenditure share (E_{share}), the primary energy intensity of output ($\frac{E_p}{Y}$), and the average real energy price (p_E) in the United States from 1971-2014. These objects are related through the following identity: $E_{share} = p_E \cdot \frac{Ep}{Y}$. This figure demonstrates that short-run movements in energy prices affect the energy expenditure share of output in the short-run, but not the energy intensity of output, suggesting that it is difficult to substitute between energy and non-energy inputs in the short-run. At the same time, there is no long-run trend in the energy expenditure share of output, despite increasing prices. Data are taken from the Energy Information Administration (EIA) and the Bureau of Economic Analysis (BEA). All values are normalized to 1971 levels.

2.2 Energy Demand

The model presented in this paper can recreate key stylized facts about energy use, prices, efficiency and expenditure observed in U.S. data, suggesting that it is a useful framework to think about the impacts of policy on final-use energy. Figure 2 summarizes important evidence on the demand for energy. In particular, it shows the expenditure share of energy (E_{share}), the primary energy intensity of output ($\frac{E_p}{Y}$), and the average real energy price (p_E) in the United States from 1971-2014.[9] These objects are related through the following identity:

$$E_{share} = p_E \cdot \frac{Ep}{Y}. \tag{2}$$

The data indicate that the expenditure share, but not the energy intensity of output, reacts to short-term price fluctuations, suggesting that it is difficult to substitute between energy and non-energy inputs in the short-run. Indeed, Hassler et al. (2012, 2016b) provide a formal maximum likelihood estimate of the short-run elasticity of substitution between energy and non-energy inputs and find an elasticity of substitution very close to zero. At the same time, there is no long-run trend in the energy expenditure share of output, despite increasing prices.

Hassler et al. (2012, 2016b) show that a DTC model can recreate these facts. With fixed technology, the elasticity of substitution between energy and non-energy inputs is essentially zero.

[9]This figure focuses on primary, rather than final-use, energy due to limitations on expenditure data.

Over longer time horizons, agents in the economy respond to higher energy prices by investing in energy efficiency, driving down energy use. As a result, the expenditure share is constant on the balanced growth path, despite increasing prices and a low short-run elasticity of substitution.[10] [11]

I build on their work by constructing a decentralized model that can be used to examine the impacts of policy. Motivated by the evidence presented in Figure 1, the model focuses on the demand for final-use energy coming from final good production, rather than the demand for primary energy that coming from the energy sector. To capture this important margin, the new model departs from the seminal DTC approach of Acemoglu (1998, 2002) and considers the case where multiple types of productivity – including energy efficiency – are embodied in each capital good. When combined with Leontief production in the short-run, this yields a tractable model that can be used to study the impact of environmental policy on final-use energy consumption.

In macroeconomic studies of climate change, it is common to assume that energy and non-energy inputs are combined in a Cobb-Douglas fashion, even though this is at odds with the short-run data provided in Figure 2 (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014; Barrage, forthcoming). While Hassler et al. (2012, 2016b) do not investigate the impacts of policy, there work has been used to motivate the Cobb-Douglas assumption, because the two models have the same long-run elasticity of substitution (e.g., Golosov et al., 2014; Barrage, forthcoming; Hassler et al., 2016c, 2017).

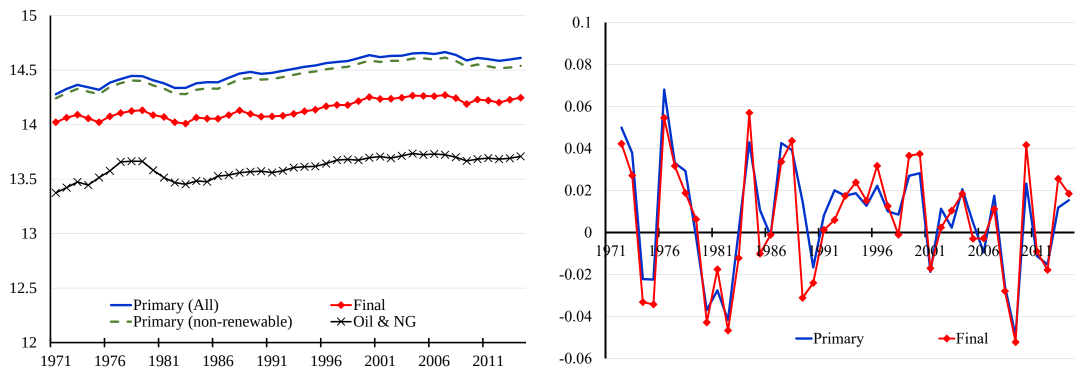
Using the decentralized model developed in this paper, I can compare the the two approaches and show that they lead to significantly different quantitative predictions about the impacts of policy. The difference can be seen through equation (2). New taxes effectively increase the price of energy. The Cobb-Douglas model assumes that energy intensity must immediately fall by enough to leave the expenditure share unchanged. Both the data and the new model, however, suggest that energy intensity will be unchanged in the very short-run and the expenditure share will spike. Then, efficiency will improve over time and the expenditure share will converge back to its long-run level. The difference in transition paths implies that the two models will have different predictions for both cumulative and medium-term energy use, even in cases where they have identical predictions for energy use at some point in the future.

2.3 Energy Supply

Since this paper is focused on the role of final-use energy, I consider a simple representation of the supply of primary energy. As discussed above, the trendless energy expenditure share, rising

[10]See also, Hart (2013) and André and Smulders (2014). For related results focusing on the elasticity of substitution between capital and labor, see Jones (2005), Caselli et al. (2006), and Leon-Ledesma and Satchi (2016), amongst others.

[11]Of course, not all improvements in energy efficiency need to driven by technical change. In particular, sectoral reallocation could explain aggregate changes in energy use. Decomposition exercises suggest that improvements in intra-sectoral efficiency, rather than reallocation, have been the key driver of falling energy intensity over this period (Sue Wing, 2008; Metcalf, 2008). They also suggest that, prior to 1970, sectoral reallocation was the primary driver of falling energy intensity. The calibration will focus on the post-1970 period. Existing work suggests that there was a significant regime shift in both energy prices and energy efficiency improvements after this period (e.g., Hassler et al., 2012, 2016b; Baumeister and Kilian, 2016; Fried, 2018). See Hart (2018) for a model focusing on earlier periods where energy efficiency was driven sectoral reallocation.



(a) Energy Use (log of ktoe) (b) Growth rate

Figure 3: This figures demonstrates that aggregate energy use has been increasing in the United States over the period 1971-2014, even as the energy expenditure share was constant (see figure 2). Thus, the data are inconsistent with a standard Hotelling (1931) model. Data are from the International Energy Agency (IEA). *ktoe* is kilotons of oil equivalent, a measure of energy content.

prices, and improving energy efficiency are all consistent with the balanced growth path of a directed technical change model. To differentiate between possible causes of the rising prices, I now turn to considering trends in energy use.[12]

Studies with aggregate energy use almost always use one of two underlying models of energy supply to explain long-run trends in prices: optimal depletion of finite resources (e.g., Hotelling, 1931; Dasgupta and Heal, 1974) or increasing extraction costs (e.g., Pindyck, 1978; Slade, 1982). Existing work on directed technical change and the environment focuses on the former (Di Maria and Valente, 2008; Hassler et al., 2012, 2016b; André and Smulders, 2014). Of the two approaches, however, only the increasing extraction cost model is consistent with aggregate evidence from the United States. In particular, if rising prices are driven by forward looking behavior and finite supplies, then energy use must decrease on the balanced growth path, which is when the energy expenditure share is constant. Figure 3, however, shows that energy use has been increasing over the period of study.

Thus, in this paper, I consider the case of increasing extraction costs, which allows for increasing energy use on the balanced growth path. This has several important implications. First, the underlying model of energy costs will help determine the equilibrium impacts of environmental policy. When policy decreases energy use, extraction prices decrease as well, partially offsetting the environmental benefits of the original intervention. The strength of this feedback depends on the nature of supply. Relatedly, the fact that energy use will increase in the absence of policy implies

[12]As demonstrated in Figure 2, the price of energy in the United States had an upward trend from 1971-2014. Once again, this is a good match for post-1970 data, but not for U.S. data in the preceding two decades, where energy prices actually declined. Consistent with the predictions of the model, decomposition exercises suggest that intra-sectoral energy efficiency declined during this period of falling prices (Sue Wing, 2008). In this paper, I focus on the case where prices increase in the long run, though this is not central to any of the policy analysis. The EIA predicts the energy prices will increase across a wide range of sources and end-uses over the next several decades. See 'Table 3. Energy Prices by Sector and Source' at https://www.eia.gov/outlooks/aeo/.

that larger interventions are needed to hit specific environmental policy goals, when compared to to a world where energy use would decreases even without targeted policy.

The nature of energy supply also has important implications for long-run sustainability in the absence of policy. The fact that prices are driven by increasing extraction costs, rather than optimal response to finite supplies, does not necessarily imply that energy supplies are actually infinite. Thus, the increasing extraction cost model is more likely to lead to an 'environmental disaster' in the absence of policy (Acemoglu et al., 2012; Lemoine, 2017). Such a disaster could occur when energy inputs are exhausted, or when the climate consequences of fossil energy use hit a 'tipping point' (Stern, 2008; Lemoine and Traeger, 2014). Existing evidence suggests that the latter is a greater concern and that it is virtually impossible that all available sources of fossil fuels will eventually be used (Rogner, 1997; Rogner et al., 2012; Covert et al., 2016).

It is also worth noting that the increasing extraction cost formulation leads to different predictions about the sustainability of long-run growth in the absence negative environmental consequences of energy use. In particular, Hassler et al. (2012, 2016b) use the observation of a trendless energy expenditure share to motivate their model, but find that the energy expenditure share cannot be constant at its current level (see also, André and Smulders, 2014). By considering a model where prices are dictated by extraction costs, I demonstrate how to overcome this discrepancy. In this way, the new model generates that novel prediction that, the United States can maintain its current energy expenditure share and consumption growth rate indefinitely, barring negative environmental consequences of energy use.

3 Model

3.1 Structure

3.1.1 Final Good Production

Final good production is perfectly competitive. The model extends the standard endogenous growth production function to account for energy use. To match the extremely low short-run elasticity of substitution between energy and non-energy inputs, I will consider a Leontief structure

$$Q_t = \int_0^1 \min[\left(A_{N,t}(i)X_t(i)\right)^{\alpha} L_t^{1-\alpha}, A_{E,t}(i)E_t(i)]\, di, \tag{3}$$

$$s.t. \quad A_{E,t}(i)E_t(i) \leq A_{N,t}(i)X_t(i)^{\alpha} L_t^{1-\alpha} \ \forall i, \tag{4}$$

where Q_t is gross output at time t, $A_{N,t}(i)$ is the the quality of capital good i, $X_t(i)$ is the quantity of capital good i, L_t is the aggregate (and inelastic) labor supply, $A_{E,t}(i)$ is the energy efficiency of capital good i, and $E_t(i)$ is the amount of energy devoted to operating capital good i. Several components of the production function warrant further discussion. As in the standard endogenous growth production function, output is generated by a Cobb-Douglas combination of aggregate labor, L_t, and a series of production process, each of which uses a different capital good, indexed by i. Unlike the endogenous growth literature, each production process also requires energy to run. Thus, the usual capital-labor composite measures the potential output that can be created using each production process, and the actual level of output depends on the amount of energy devoted to each process, $E_t(i)$. The notion of potential output is captured by constraint (4). Each capital good i has two distinct technological characteristics. The quality of the capital good, $A_{N,t}(i)$, improves its ability to produce output. The energy efficiency of the capital good, $A_{E,t}(i)$, lowers the amount of energy needed to operate the production process at full potential.[13]

3.1.2 Energy Sector

Energy is available in infinite supply, but is subject to increasing extraction costs (see, e.g., Heal, 1976; Pindyck, 1978; Lin and Wagner, 2007). Extraction costs are paid in final goods, and energy is provided by a perfectly competitive sector with open access. The increasing extraction cost incorporates two main forces that govern long-run energy availability. First, it captures the increase in cost needed to extract conventional energy resources from harder-to-access areas.[14] Second, it captures the increase in cost that may occur when a particular energy source is exhausted, necessitating a switch to a type of energy that is more difficult to extract.

The fact that production is open-access implies that Hotelling (1931) rents play no role in determining prices. In other words, agents do not behave as if energy resources are finite. As discussed in Section 2.3, this is necessary to match data on long-run energy use. When examining the implications of the model, I also assume that the underlying energy supply limits are never reached. This is consistent with existing models and geological evidence. In particular, the infinite supply of energy and increasing extraction costs capture the existence of 'unconventional' energy sources, which have high extraction costs but are available in vast quantities (Rogner, 1997; Rogner et al., 2012).[15] As in Golosov et al. (2014), the treatment of energy sources as infinite in potential supply also incorporates the abundance of coal, which is predicted to be the major driver of climate change (van der Ploeg and Withagen, 2012; Hassler et al., 2016a).[16] Together, the vast quantities

[13]Consistent with the econometric literature on energy use, energy requirements depend both on the amount of capital and the amount of labor being used in the production process (Van der Werf, 2008; Hassler et al., 2012, 2016b). Second, consistent with both the econometric and DTC literatures, improvements in non-energy technology, $A_N(i)$, raise energy requirements (e.g., Smulders and De Nooij, 2003; Van der Werf, 2008; Hassler et al., 2012, 2016b; Fried, 2018). This framework is isomorphic to one in which $A_N(i)$ is the relative price of investment.

[14]For example, recent research suggests that most new oil production comes from the exploitation of new geographic areas, rather than improved technology applied to existing sources of energy (Hamilton, 2012).

[15]For example, Rogner et al. (2012) estimate a resource base of 4,900 – 13,700 exajoules (EJ) for conventional oil, compared with annual production of 416 EJ across all energy sources. Thus, constraints on availability of conventional oil sources may be binding. The ability to exhaust fossil fuel energy sources, however, appears much less likely when considering other options. The resource base for unconventional sources of oil is estimated to be an additional 3,750 – 20,400 EJ. Meanwhile, the resource base for coal and natural gas (conventional and unconventional) are 17,300–435,000 EJ and 25,100 – 130,800 EJ, respectively. These estimates rely on projections regarding which resources will be profitable to extract from the environment. When considering the full range of energy sources that could become profitable to extract as resource prices tend towards infinity, the numbers grow even larger. In particular, such 'additional occurrences' are estimated to be larger than 1 million EJ for natural gas and 2.6 million EJ for uranium.

[16]Technically, Golosov et al. (2014) specify a finite amount of coal, but assume it is not fully depleted. Thus, it has no

of coal and 'unconventional' energy sources imply that using too much fossil energy, rather than exhausting supply, is the relevant environmental concern (Covert et al., 2016).

The marginal cost of extraction, which will also be equal to the price, is given by

$$p_{E,t} \quad = \quad \xi \bar{E}_{t-1}^{\psi}, \tag{5}$$

where $\bar{E}_{t-1}$ is total energy ever extracted at the start of the period. The law of motion for the stock of extracted energy is given by

$$\bar{E}_t = E_{t-1} + \bar{E}_{t-1}. \tag{6}$$

The fact that extraction costs are constant within each period is a useful simplification. As motivation, it is intuitive to consider the case where energy producers exploit new sources of energy in each period and the difficulty of extraction is constant within each source.[17,18]

3.1.3 Final Output

Final output is given by gross production less total energy extraction costs, which are equal to energy expenditures by the final good producer. As long as equation (4) holds with equality,[19] final output is given by

$$Y_t \quad = \quad L_t^{1-\alpha} \int_0^1 \left[1 - \frac{p_{E,t}}{A_{E,t}(i)}\right] \left(A_{N,t}(i) X_t(i)\right)^{\alpha} \, di. \tag{7}$$

This formulation further illuminates the continuity between the production function used here and the standard approach in endogenous growth models. Output has the classic Cobb-Douglas form with aggregate labor interacting with a continuum of capital goods. The model developed in

scarcity rent, although it does have an extraction cost. Oil, by contrast, is assumed to have no extraction cost, but does have a positive scarcity rent. Hart and Spiro (2011) survey the empirical literature and find little evidence that scarcity rents are a significant component of energy costs. They suggest that policy exercises focusing on scarcity rents will give misleading results.

[17]This is consistent, for example, with recent evidence from the oil industry, where drilling, but not within-well production, responds to changes in prices (Anderson et al., 2014).

[18]A primary goal of this paper is to compare the results of the new DTC model to the standard Cobb-Douglas approach used in integrated assessment models (IAMs) (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014). Since IAMs examine worldwide outcomes, it is crucial to consider the equilibrium effect of policy on energy prices. Hence, the comparison between models is most accurate when considering endogenous prices. At the same time, I also use the model to investigate the effect of policies pursued in the United States. In this case, endogenous energy prices can be motivated in two ways. First, it is possible to think of the United States as a closed economy, which is a good match for some, but not all, sources of primary energy. Alternatively, one can imagine the policies being applied on a worldwide level with the United States making up a constant fraction of total energy. To ensure that the key qualitative results of the paper are not driven by this assumption, I also consider the opposite extreme of exogenous energy prices, which implicitly treats the United States as a small open economy taking unilateral policy actions. In this case, energy prices will increase at a constant exogenous rate.

[19]To ensure that equation (4) holds with equality, it is sufficient, but not necessary, to assume that capital fully depreciates after each period. If capital fully depreciates, then forward looking consumers will never 'over-invest' in capital and drive its return to zero. This assumption will be maintained in the empirical analysis and is also employed in Golosov et al. (2014).

this paper extends the standard endogenous growth set-up model by considering a broader notion of aggregate productivity, $\left[1 - \frac{p_{E,t}}{A_{E,t}(i)}\right] \cdot \left(A_{N,t}(i)\right)^{\alpha}$. In words, productivity is determined by two different types of embodied technology, as well as energy extraction costs. The functional form is driven by the fact that underlying production function is Leontief. I show that this updated formulation leads to a tractable growth model. Moreover, in the long-run, the updated technology index grows at a constant rate and the model can explain all of the usual growth facts.

Final output can either be consumed or saved for next period. In the empirical application, each period will be ten years. Following existing literature, I assume complete depreciation during production (Golosov et al., 2014). Thus, market clearing in final goods implies

$$Y_t = C_t + K_{t+1} = L_t w_t + r_t K_t + \Pi_t + p_t^R + T_t, \tag{8}$$

where K_t is aggregate capital, Π_t is total profits, T_t is total tax revenue, and p_t^R is total payments to R&D inputs (discussed in the next section). When examining the effects of environmental policy, I assume that the government balances the budget using lump-sum taxes or transfers.

3.1.4 Capital Goods and Research

Each type of capital good is produced by a single profit-maximizing monopolist in each period. This monopolist also undertakes in-house R&D activities to improve the embodied technological characteristics, $A_{N,t}(i)$ and $A_{E,t}(i)$. The R&D production function is given by

$$A_{J,t}(i) \quad = \quad \left[1 + \eta_J R_{J,t}(i) R_{J,t}^{-\lambda}\right] A_{J,t-1}, \quad J = N, E, \tag{9}$$

where $R_{J,t}(i)$ is R&D inputs assigned to characteristic J by firm i in period t, $R_{J,t} \equiv \int_0^1 R_{J,t}(i)di$, and $A_{J,t-1} \equiv \max\{A_{J,t-1}(i)\}$. In words, R&D builds on aggregate knowledge, $A_{J,t-1}$, and current period within-firm research allocations, $R_{J,t}(i)$, but is also subject to a congestion externality $R_{J,t}^{-\lambda}$ caused by duplicated research effort. When the period ends, patents expire and the best technology becomes available to all firms. Monopolists make decisions to maximize single period profits.[20]

There are a unit mass of R&D inputs, yielding[21]

$$R_{N,t} + R_{E,t} = 1 \; \forall t. \tag{10}$$

[20] This can be motivated in several ways. Most directly, the identity of the firm producing capital good i could change after each period. Alternatively, it could be the case that firms are infinitely lived but myopic, which seems reasonable considering the ten year period length. The set-up presented here is isomorphic to one where firms are infinitely lived and the aggregate technology, $A_{J,t-1}$, is given by the average of the previous period's technology as in Fried (2018). This would open up the possibility of technological regress, though it would not occur in equilibrium.

[21] This is consistent with both existing literature on DTC and the environment (Acemoglu et al., 2012; Fried, 2018) and the social planner model provided by (Hassler et al., 2012, 2016b). Often, models of directed technical change refer to the fixed set of research inputs as scientists (e.g., Acemoglu et al., 2012; Fried, 2018). This would be applicable here, though generating the standard Euler equation would require the representative household to ignore scientist welfare (in the environmental literature, directed technical change and capital accumulation are generally not included simultaneously). This would be a close approximation to a more inclusive utility function as long as scientists made up a small portion of the overall population. For simplicity, I refer to research inputs, which could be scientists, research labs, etc.

I assume that the investment price is fixed at unity. Thus, market clearing implies that

$$\int_0^1 X_t(i)di = K_t, \tag{11}$$

where K_t is aggregate capital.

3.1.5 Consumer Problem

The consumer side of the problem is standard. In particular, the representative household chooses a path of consumption such that

$$\{C_t\}_{t=0}^{\infty} = \text{argmax} \sum_{t=0}^{\infty} \beta^t L_t \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma}, \tag{12}$$

where $\tilde{c}_t = C_t/L_t$. Population growth is given exogenously by

$$L_{t+1} = (1+n)L_t. \tag{13}$$

I am interested in the decentralized equilibrium. Thus, I consider the case where the representative household takes prices and technology as given. In other words, the household's budget constraint is given by the second equality in (8).

3.2 Analysis

As demonstrated in Appendix Section B.1, the first order conditions for the final good producer yield the following inverse demand functions:

$$p_{X,t}(i) = \alpha A_{N,t}(i)^{\alpha}\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big] L_t^{1-\alpha} X_t(i)^{\alpha-1}, \tag{14}$$

$$w_t = (1-\alpha)\int_0^1 \big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big] L_t^{-\alpha}\big(A_{N,t}(i)X_t(i)\big)^{\alpha} di, \tag{15}$$

where $\tau_t \geq 1$ is a proportional tax on energy. The intuition for the result is straightforward. The final good producer demands capital goods until marginal revenue is equal to marginal cost. Unlike the usual endogenous growth model, marginal revenue is equal to marginal product minus the cost of energy needed to operate capital goods. Consider the case where the final good producer is already operating at a point where $\big(A_{N,t}(i)X_t(i)\big)^{\alpha} L_t^{1-\alpha} = A_{E,t}(i)E_t(i)$. If the final good producer purchases more capital, it receives no increase in output unless there is a corresponding increase in energy purchased. The final good producer realizes this when making optimal decisions and adjusts demand for capital accordingly. This iso-elastic form for inverse demand maintains the tractability of the model.

Monopolist providers of capital goods must decide on optimal production levels and optimal research allocations. See Appendix Section B.2 for a formal derivation of the monopolists' behavior. Given the iso-elastic inverse demand function, monopolists set price equal to a constant markup over unit costs. Since capital goods must be rented from consumers, the unit cost is given by the rental rate, r_t. Thus, monopolist optimization yields

$$p_{X,t}(i) = \frac{1}{\alpha} r_t, \tag{16}$$

$$X_t(i) = \alpha^{\frac{2}{1-\alpha}} r_t^{\frac{-1}{1-\alpha}} A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}, \tag{17}$$

$$\bar{\pi}_{X,t}(i) = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}, \tag{18}$$

where $\bar{\pi}_{X,t}(i)$ is production profits (i.e., profits excluding research costs) of the monopolist.

To understand research dynamics, it is helpful to look at the relative prices for research inputs,

$$\frac{(1-\eta_t^S) p_{E,t}^R(i)}{p_{N,t}^R(i)} = \frac{\tau_t p_{E,t} A_{N,t}(i)}{\alpha A_{E,t}(i)^2 \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]} \frac{\eta_E R_{E,t}^{-\lambda} A_{E,t-1}}{\eta_N R_{N,t}^{-\lambda} A_{N,t-1}}, \tag{19}$$

where $p_{J,t}^R(i)$ is the rent paid to research inputs used by firm i to improve technological characteristic J at time t and $\eta_t^S \in [0, 1)$ is a subsidy for energy efficient research. There are several forces affecting the returns to R&D investment. First, increases in the tax-inclusive price of energy increase the relative return to investing in energy efficiency. Second, the return to investing in a particular type of R&D is increasing in its efficiency. Research efficiency, in turn, depends on inherent productivity, η_J, accumulated knowledge, $A_{J,t-1}$, and the amount of congestion, $R_{J,t}^{-\lambda}$. Third, since energy and non-energy inputs are complements in production, increases in $A_{N,t}(i)$ raise the return to investing in $A_{E,t}(i)$ and vice versa. These effects, however, are asymmetric. To maximize profits, monopolists balance two forces that drive demand for their products: 'output-increasing' technological progress, $A_{N,t}(i)$, and 'cost-saving' technological progress, $A_{E,t}(i)$. The asymmetry occurs because energy efficiency, $A_{E,t}(i)$, has a negative and convex effect on the cost of energy per unit of final output, $\frac{\tau_t p_{E,t}}{A_{E,t}(i)}$. Finally, the return to investing in the quality of capital goods is increasing in the share of final output paid to capital good producers, α.

In the usual DTC model, this analysis would demonstrate the role of *market size effects* and *price effects* in research incentives (Acemoglu, 1998, 2002). As demonstrated in equation (19), however, aggregate inputs do not affect R&D decisions in this model. In other words, market size effects play no role in this model. This is due to the short-run complementarity between energy and non-energy inputs. Moreover, the price effects in this model differ from those in the usual DTC model. Since the price of the final good is the numeraire, $\frac{\tau_t p_{E,t}}{A_{E,t}(i)}$ is the cost of energy per unit of final good production, and $1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}$ is the cost of non-energy inputs in final good production. Thus, the relative input prices do affect research allocations, but the relative price is completely determined by the cost of energy extraction. Moreover, as explained above, the relative price of energy – along with lagged technology levels – enter asymmetrically, unlike in the seminal model. These theoretical differences highlight the importance of considering the case where improvements in energy efficiency are driven by final-use energy, rather than using the more common approach where innovation occurs in different sectors.

To understand the intuition of the model, it is helpful to consider the laissez-faire case ($\tau_t = 1,\ \eta^S_{t=0}$). Then, noting that the price of research inputs must be the same for each technology, (19) can be re-written as:

$$\frac{1+g_{E,t}(i)}{1+g_{N,t}(i)} = \frac{E_{share}(i)}{\alpha}\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N R_{N,t}^{-\lambda}},$$

where $E_{share}(i) = \frac{\frac{p_{E,t}}{A_{E,t}(i)}}{1-\frac{p_{E,t}}{A_{E,t}(i)}}$ is the fraction of gross output produced with capital good i that ends up paying energy suppliers, relative to final output produced with capital good i. When the expenditure share of energy is high, the final good producers want to save on energy inputs, motivating the capital good producers to invest in energy efficiency. Conversely, if the energy expenditure share is low, there is little investment in energy efficiency. Thus, in the long-run the model moves toward a balanced growth path with a constant energy expenditure share.

Given that all firms use common technology at the start of the period, they make identical R&D decisions and, as a result, they end the period with identical technology. Moreover, there is a unit mass of monopolists. Thus, $R_{J,t}(i) = R_{J,t}\ \forall i, J, t$. The optimal research allocations are given by the implicit solution to (20) and (21),

$$R_{E,t} = \frac{\sqrt{\frac{\tau_t p_{E,t}}{A_{E,t-1}}}\sqrt{\frac{1}{\alpha(1-\eta^S_t)}\left[\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}} + \eta_E R_{E,t}^{-\lambda} - \eta_E R_{E,t}^{1-\lambda}\right] + (1+\eta_E R_E^{1-\lambda})} - 1}{\eta_E R_{E,t}^{-\lambda}}, \tag{20}$$

$$R_{N,t} = 1 - R_{E,t}. \tag{21}$$

This formulation highlights the simple closed form solution in the special case where $\lambda = 0$ and $\eta^S_t = 0$.

To analyze the determinants of research activity, it is instructive to consider multiplying both sides of (20) by $\eta_E R_{E,t}^{-\lambda}$ so that the growth rate of energy efficiency technology is given as a function of the other parameters. Since $\eta^S_t \in [0,1)$, the left-hand side is strictly increasing in $R_{E,t}$ in this formulation and the right-hand side is strictly decreasing in $R_{E,t}$. Thus, $R_{E,t} = \Gamma(\frac{\tau_t p_{E,t}}{A_{E,t-1}})$, for some well defined function $\Gamma(\cdot)$.

Two implications can be immediately read from equations (20) and (21). First, on a balanced growth path, $\frac{p_{E,t}}{A_{E,t-1}}$ must be constant. As discussed in Section 3.1.3, this implies that the relevant technology index in the economy will grow at a constant rate and that the model will have a balanced growth path that the resembles the standard neoclassical growth model. Second, the model is relatively easy to investigate computationally, because conditional on the price of energy, it is possible to solve for the full sequence of technology parameters independently of the consumer saving decisions.

Utility maximization yields

$$(\frac{\tilde{c}_t}{\tilde{c}_{t+1}})^{-\sigma} = \beta r_{t+1}. \tag{22}$$

Noting that all monopolists make the same decisions and that there is a unit mass of monopolists, the real interest rate is given by

$$r_t \quad = \quad \alpha^2 A_{N,t}^{\alpha}\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right] L_t^{1-\alpha} K_t^{\alpha-1}, \tag{23}$$

where the market clearing condition from equation (11) has been applied.

3.3 Equilibrium

Definition 1. *A* competitive equilibrium *is a sequence of prices, $\{w_t, p_{X,t}, r_t, p_t^R, p_{E,t}\}_{t=0}^{\infty}$, allocations, $\{C_t, K_t, L_t, E_t, R_{N,t}, R_{E,t}\}_{t=0}^{\infty}$, technology levels, $\{A_{N,t}, A_{E,t}\}_{t=0}^{\infty}$, and environmental policies, $\{\tau_t, \eta_t^S\}_{t=0}^{\infty}$, such that each of the following conditions holds $\forall t$:*

- *The economy obeys market clearing conditions for final goods, (8), and capital goods, (11).*
- *Optimal research allocations solve (20) and (21).*
- *The dynamics for technology follow (19), noting that all monopolists make identical decisions.*
- *Consumer behavior follows the Euler equation, (22).*
- *Factor prices are given by (5), (15), (16), and (23), noting that all monopolists make identical decisions and that the market for capital goods clears.*
- *The economy obeys laws of motion for total extracted energy, (6), and population, (13).*
- *Initial Conditions $A_{J,-1}$ for $J \in \{E, N\}$, K_0, L_0, and $\bar{E}_{-1}$ are given.*

3.4 Balanced Growth under Laissez Faire

In this section, I examine long-run outcomes in the absence of environmental policy. To focus on empirically relevant cases, I maintain the following assumption for the remainder of the paper:

$$\eta_E > n, \tag{A.1}$$

which rules out extreme cases where all research activity is devoted to improving energy efficiency even in the absence of environmental policy. Section 4 shows that this assumption is satisfied by an order of magnitude in the data.

Definition 2. *A* laissez-faire equilibrium *is a* competitive equilibrium *without environmental policy. Formally, $\tau_t = 1$ and $\eta_t^S = 0$ $\forall t$.*

Definition 3. *A* balanced growth path *(BGP) occurs when final output, technology, and consumption grow at constant rates.*

On a balanced growth path (BGP), research allocations must remain fixed. Consider the laissez-faire case where there is no energy policy. From equations (20) and (21), it is immediate that $\frac{p_{E,t}}{A_{E,t-1}}$ is constant. Intuitively, this occurs because of the non-linear relationship between energy efficiency, $A_{E,t}$, and the cost of energy per unit of output, $\frac{p_{E,t}}{A_{E,t}}$. When energy prices increase, monopolists have greater incentive to invest in energy efficient technology, but this incentive dissipates as technology improves and the expenditure share falls. As a result, energy prices and energy efficient technology grow at the same constant rate, g_E^*, on the BGP.[22] Thus, the increasing price of energy is exactly offset by improvements in energy efficiency.

Definition 4. *The energy share of expenditure, denoted by θ_E, is the sum of resources paid to energy producers and energy taxes as a fraction of final output. Formally, $\theta_{E,t} \equiv \frac{\tau_t p_{E,t} E_t}{Y_t}$.*

Given that energy prices and energy efficient technology grow at the same rate on the BGP, it is straightforward to show that the energy share of expenditure is constant in a laissez-faire equilibrium. In particular,

$$\theta_{E,t} = \frac{p_{E,t}/A_{E,t}}{1 - p_{E,t}/A_{E,t}}, \tag{24}$$

which must be constant given that $\frac{p_{E,t}}{A_{E,t-1}}$ is fixed and the growth rate of energy efficient technology is constant.[23] Thus, despite the Leontief nature of production, the model still delivers a constant long-run energy expenditure share. As demonstrated in Section 2, this is consistent with aggregate data on U.S. energy use. Importantly, the expenditure share is only constant on the BGP.

The fact that energy efficient technology and the price of energy grow at the same rate yields the first of two key BGP relationships. In particular, noting the relationship between energy use and the price of energy, as given by (5) and (6), yields

$$(1 + g_M^*)^\psi = (1 + g_E^*), \tag{BGP-RD}$$

where g_M^* is the growth rate cumulative energy use. On the BGP, this must also be the growth rate of per period energy use. This equation summarizes the conditions for a BGP on the research side of the economy.

I now move to considering the remainder of the economy. Consider the growth rate of TFP in this model.

Definition 5. Total factor productivity *is defined as in the standard neoclassical growth model. Formally, $TFP \equiv \frac{Y_t}{K_t^\alpha L_t^{1-\alpha}}$.*

[22] For the price of energy to grow at a constant rate, energy use must also grow at a constant rate, which will occur on the BGP.

[23] See Hart (2013) for a general discussion of the relationship between factor shares and directed technical change.

It is immediate that

$$TFP_t = A_{N,t}^{\alpha}\big[1 - \frac{p_{E,t}}{A_{E,t}}\big]. \tag{25}$$

Since $\frac{p_{E,t}}{A_{E,t}}$ is constant on the BGP in the absence of policy, TFP grows at rate, $(1+g_N^*)^{\alpha}-1$, which is also constant. Since the consumer problem is standard, the model now reduces to the neoclassical growth model with monopolistic competition, implying that the directed technical change will have the usual BGP properties. In particular, both final and gross output will grow at rate $g_Y^* = (1+g_N^*)^{\frac{\alpha}{1-\alpha}}(1+n)-1$. Given equation (3), the growth rate of energy use (both cumulative and per period) is given by

$$1 + g_M^* = \frac{(1+g_N^*)^{\frac{\alpha}{1-\alpha}}}{1+g_E^*}(1+n). \tag{BGP-QE}$$

Together, equations (BGP-RD) and (BGP-QE) determine the relative growth rates of technology on the unique BGP. Adding in market clearing for R&D inputs, (10), yields the optimal research allocations and applying the law of motion for technology, (9), gives the technology and energy use growth rates. The technology growth rates are then sufficient to characterize the output-side of the BGP, which behaves as in the standard model.

Remark 1. *In a* laissez-faire equilibrium, *energy use is strictly increasing on the* BGP, *i.e.,* $g_M^* > 0$.

Proof. The remark follows from equation (BGP-RD) and the proof to Proposition 1, which demonstrates that research allocations are interior on the BGP. □

Contrary to a world with only exhaustible energy sources, the current model predicts that energy use will be increasing in the long-run in the absence of environmental policy. Thus, it matches the data presented in Section 2, which suggest that energy use is increasing on a balanced growth path. Intuitively, this result holds because there is only incentive for energy efficient research when cumulative energy use (and, therefore, the price of energy) is increasing.

Proposition 1 summarizes and extends the results from this section. In particular, it uses the relationship between equations (BGP-RD) and (BGP-QE) to explicitly characterize the balanced growth path.

Proposition 1. *In a* laissez-faire equilibrium, *there exists an* unique BGP *on which each of the following holds true:*

1. *The research allocations are implicitly given by*
$R_E^* = \left\{\frac{\left[(1+\eta_N(1-R_E^*)^{1-\lambda})^{\frac{\alpha}{1-\alpha}}(1+n)\right]^{\frac{1}{1+1/\psi}}-1}{\eta_E}\right\}^{\frac{1}{1-\lambda}}.$
2. *Technological growth rates are given by* $g_E^* = \eta_E(R_E^*)^{1-\lambda}$ *and* $g_N^* = \eta_N(1-R_E^*)^{1-\lambda}$. *The relationship between growth rates can be expressed as:*
$(1+g_E^*)^{\frac{\psi+1}{\psi}} = (1+g_N^*)^{\frac{\alpha}{1-\alpha}}(1+n).$

3. *Output per worker and consumption per worker grow at a constant rate,* $g_R^* = (1+g_N^*)^{\frac{\alpha}{1-\alpha}} - 1$.

4. *Total output and the capital stock grow at a constant rate,* $g_Y^* = (1+g_R^*)(1+n) - 1$, *which implies that the capital-output ratio is fixed.*

5. *The real interest rate,* r_t, *is constant.*

6. *Energy use grows at rate* $g_M^* = \frac{1+g_R^*}{1+g_E^*}(1+n) - 1 > 0$.

7. *The expenditure shares of energy, capital, labor, R&D inputs, and profits are all constant. In particular, the expenditure share of energy is implicitly given by*
$\frac{\theta_E^*}{1+\theta_E^*} = \frac{1+\eta_E(R_E^*)^{1-\lambda}}{\frac{1}{\alpha}\left[\frac{\eta_E(R_E^*)^{-\lambda}}{\eta_N(1-R_E^*)^{-\lambda}} + \eta_E(R_E^*)^{-\lambda} - \eta_E(R_E^*)^{1-\lambda}\right] + 1 + \eta_E(R_E^*)^{1-\lambda}}$.

Proof. The intuition is provided in the text, and a formal proof is provided in Appendix Section B.4. □

3.5 Balanced Growth with Environmental Policy

In this section, I consider long-run outcomes in the presence of environmental policy.

Definition 6. *An* equilibrium with environmental policy *is a* competitive equilibrium *where* $\tau_t = \tau_0(1+g_\tau)^t$, $g_\tau, \tau_0 > 0$ *and* $\eta_t^S = \eta^S \geq 0 \; \forall t$.[24]

In a world with increasing energy taxes, equations (20) and (21) now imply that the growth rate of energy efficiency is equal to the product of growth in the energy price and the growth of the taxes. Thus, balanced growth on the research side of the economy requires

$$(1+g_M^*)^{\psi}(1+g_\tau) = (1+g_E^*), \tag{BGP-RD$'$}$$

which is equivalent to the laissez-faire condition if $g_\tau = 0$. This also implies that, on a BGP, $\lim_{t\to\infty} \frac{p_{E,t}}{A_{E,t}} = 0$. Thus, $\lim_{t\to\infty}[Q_t - Y_t] = 0$ and $\lim_{t\to\infty} \theta_{E,t} = \frac{\tau_t p_{E,t}}{A_{E,t}}$, which is constant. In the limit, the model again reduces to that of the standard neoclassical growth model with monopolistic competition. As a result, the BGP condition for the output side of the economy is unchanged:

$$1+g_M^* = \frac{(1+g_N^*)^{\frac{\alpha}{1-\alpha}}}{1+g_E^*}(1+n). \tag{BGP-QE$'$}$$

The economy will not reach a BGP in finite time. Using the same steps as in Section 3.4, it is now possible to characterize the BGP. Noting the similarity between (BGP-RD′) and (BGP-QE′) on one hand and (BGP-RD) and (BGP-QE) on the other, it is immediate that the growth rate of technological progress is unaffected by the level of taxes or the research subsidy.

[24]I restrict the formal analysis to the case of exponentially increasing taxes and a fixed research subsidy for analytic convenience. In particular, this restriction allows for the simple characterization of a balanced growth path, but does not drive any of the underlying intuition.

Remark 2. *In an* equilibrium with environmental policy*, changes in energy research subsidies and the level of energy taxes have no effect on the BGP growth rate of energy. Formally,* $\frac{dg_M^*}{d\tau_0} = \frac{dg_M^*}{d\eta^S} = 0$*.*

Proof. The intuition follows from the preceding discussion. Formally, the remark follows from Proposition 2. □

Noting that changes in the level of subsidies do not affect the long-run allocation of research inputs, examination of (20) indicates that research subsidies do affect the energy expenditure share and, therefore, the level of energy use. This creates another significant difference with the Cobb-Douglas model, where the energy expenditure share is virtually fixed in response to environmental policy.[25] This result is summarized in the following remark.

Remark 3. *In an* equilibrium with environmental policy*, increases in the research subsidy decrease the energy expenditure share on the BGP. Formally,* $\frac{d\theta_E^*}{d\eta^S} < 0$*.*

Proof. The remark follows from Proposition 2. The intuition is given in the preceding discussion. □

As demonstrated in equation (BGP-RD′), the existence of increasing energy taxes weakens the link between the cost of energy extraction, $p_{E,t}$, and energy efficient research. In particular, there can be incentives for energy efficient research even when the price of energy is decreasing, as long as the tax on energy is increasing quickly enough. Thus, it is possible to have an equilibrium with a constant energy price.

Remark 4. *In an* equilibrium with environmental policy*, energy use is weakly increasing on the* BGP*. Formally,* $g_M^* \geq 0$*. Moreover,* $\frac{dg_M^*}{dg_\tau} < 0$*.*

Proof. The remark follows from the proof to Proposition 2. □

All of the results presented thus far are summarized and extended in Proposition 2. In particular, it uses the relationship between equations (BGP-RD′) and (BGP-QE′) to explicitly characterize the BGP in the presence of environmental policy.

Proposition 2. *In an* equilibrium with environmental policy*, there exists an* unique BGP *on which each of the following holds true:*

1. *The research allocations are implicitly given by*
$$R_E^* = \left\{ \frac{\left[(1+\eta_N(1-R_E^*)^{1-\lambda})^{\frac{\alpha}{1-\alpha}}(1+n)(1+g_\tau)^{1/\psi} \right]^{\frac{1}{1+1/\psi}} - 1}{\eta_E} \right\}^{\frac{1}{1-\lambda}}.$$
2. *Technological growth rates are given by* $g_E^* = \eta_E (R_E^*)^{1-\lambda}$ *and* $g_N^* = \eta_N (1-R_E^*)^{1-\lambda}$*. The relationship between growth rates can be expressed as*
$$(1+g_E^*)^{\frac{\psi+1}{\psi}} = (1+g_N^*)^{\frac{\alpha}{1-\alpha}}(1+n)(1+g_\tau).$$

[25] Tax-inclusive energy expenditure is a constant share of gross output, but the rebate of taxes implies that the share in total output decreases slightly in response to an increase in taxes.

3. *Output per worker and consumption per worker grow at a constant rate,* $g_R^* = (1+g_N^*)^{\frac{\alpha}{1-\alpha}} - 1$.

4. *Total output and the capital stock grow at a constant rate,* $g_Y^* = (1+g_R^*)(1+n) - 1$, *which implies that the capital-output ratio is fixed.*

5. *The real interest rate,* r_t, *is constant.*

6. *Energy use grows at rate* $g_M^* = \frac{1+g_R^*}{1+g_E^*}(1+n) - 1 \geq 0$.

7. *The expenditure shares of energy, capital, labor, R&D inputs, and profits are all constant. In particular, the expenditure share of energy is implicitly given by*
$\theta_E^* = \frac{1+\eta_E(R_E^*)^{1-\lambda}}{\frac{1}{\alpha(1-\eta^S)}\left[\frac{\eta_E(R_E^*)^{-\lambda}}{\eta_N(1-R_E^*)^{-\lambda}}+\eta_E(R_E^*)^{-\lambda}-\eta_E(R_E^*)^{1-\lambda}\right]+1+\eta_E(R_E^*)^{1-\lambda}}$.

Proof. The intuition is provided in the text, and a formal proof is provided in Appendix Section B.4. □

3.6 Comparison to Cobb-Douglas

As mentioned in the introduction, the standard approach in climate change economics is to treat energy as a Cobb-Douglas component of the aggregate production function (Nordhaus and Boyer, 2000; Golosov et al., 2014). The standard Cobb-Douglas production function is given by

$$Q_t^{CD} = A_t^{CD} K_t^{\gamma} E_t^{\nu} L_t^{1-\alpha-\nu},$$

where A_t^{CD} grows at an exogenous rate, g_{CD}. Since energy extraction costs $p_{E,t}$ units of the final good, final output is given by

$$Y_t^{CD} = (1 - \frac{\nu}{\tau}) A_t^{CD} K_t^{\gamma} E_t^{\nu} L_t^{1-\alpha-\nu}.$$

As a result, the energy expenditure share under Cobb-Douglas is given by

$$\theta_{E,t}^{CD} = \frac{\nu}{1 - \frac{\nu}{\tau_t}}.$$

In the absence of policy, the energy expenditure share is constant, matching the long-run elasticity of substitution between energy and non-energy inputs, but not the near-zero short-run elasticity of substitution. This has important implications for climate policy. In the Cobb-Douglas model, a tax on energy use – no matter how large – immediately generates declines in energy use that are sufficient to leave the expenditure share essentially unchanged.[26]

Since addressing climate change inherently involves long-run outcomes, the existing literature argues that the Cobb-Douglas approach may provide accurate predictions about the reaction of

[26] In response to new energy taxes, there is actually a slight *decrease* in the energy expenditure share, which is due purely to the tax rebate. This effect is quantitatively unimportant.

energy use to policy interventions over the relevant time frame, even though it cannot match short-run responses (e.g., Golosov et al., 2014; Barrage, forthcoming; Hassler et al., 2016c, 2017). The analytically results from Section 3.5, however, cast doubt on this assertion. The directed technical change model matches both the short- and long-run elasticities, suggesting that it will more accurately predict the effect of environmental taxes on energy use. This new model suggests that, in response to policy, energy use will not fall by enough to leave the expenditure share unchanged. In particular, the energy expenditure share will not be constant on the transition path, and the balanced growth level of the energy expenditure share may even increase permanently in response to policy. Thus, there is good reason to expect that the Cobb-Douglas approach overestimates the decline in energy use following an environmental policy intervention. Section 5.1 quantifies the difference in predictions between the models.[27]

4 Calibration

4.1 External Parameters

I solve the model in 10 year periods. As discussed above, the consumer and non-energy production portions of the model are standard. Thus, I take several parameters from the existing literature. In particular, I follow Golosov et al. (2014) and set $\alpha = .35$, $\delta = 1$, $\sigma = 1$, and $\beta = .860$.[28] I assume that the economy starts without environmental policy. Thus, all taxes and subsidies can be thought of as relative to 'business as usual' case, which serves as the baseline.

In addition to standard neoclassical elements, the DTC model includes R&D and energy extraction. Thus, the parameters from these segments of the model cannot be taken from the existing literature. I calibrate them to aggregate U.S. data. Data sources and details can be found in Appendix A. Due to limitations on energy expenditure data, I restrict attention to the period 1971-2014. For energy use, I use the consumption of primary energy across all sources.[29]

Following the structure of the model, I calculate gross output, Q_t, as final output, Y_t, plus energy expenditure. I measure $A_{E,t} = Q_t/E_t$, yielding $g_E^* = 0.21$ on the BGP (2.0% annual growth). On the BGP, the growth rate of income per capita is given by $g_R^* = (1 + g_N^*)^{\frac{1}{1-\alpha}} - 1$. In the data, $g_R^* = 0.19$ (1.8% annual growth), which yields $g_N^* = 0.39$. The average energy expenditure share in the data is 8.5%, which I take to be the balanced growth level. In the data, $n = 0.10$.

[27] In Appendix Section B.5, I explain the calibration procedure for Cobb-Douglas and describe the balanced growth path. I calibrate both models so that they have identical predictions for output and energy use in the absence of environmental taxes. Due to other differences between the models, especially the difference in market structure – monopolistic competition in the DTC model and perfect competition in the Cobb-Douglas model – predictions for interest rates and levels (though not growth rates) of consumption and capital differ between the models. Given that incentives for innovation are an important part of the difference between the two models, I maintain these differences in the quantitative analysis.

[28] I normalize $TFP_0 = E_0 = L_0 = 10$. This normalization simply sets the units of the analysis and has no effect on the quantitative results of the model. I also assume that the economy is on the BGP at time $t = 0$. Given the other parameters in the model, this yields $Y_0 = 93.50$, $K_0 = 8.25$, $p_{E,0} = 0.80$, $A_{E,0} = 10.15$, and $A_{N,0} = 909.03$.

[29] The model abstracts from energy transformation, implying that primary and final-use energy use are the same. Due to limitations on the price data for final-use energy, the calibration focuses on primary energy.

Below, I calibrate the R&D sector of the model to match key BGP moments. The BGP is uninformative about research congestion, λ, which measures the trade-off between advances in overall productivity and energy efficiency. As a base value, I take $\lambda = 0.21$ from Fried (2018), who also captures the congestion of moving research inputs from energy-related research to general purpose research, making it a natural starting point for the quantitative exercises presented here. I will also consider cases where $\lambda \in \{0, 0.105, 0.31\}$ for robustness.

4.2 R&D Calibration

The key R&D parameters remaining to be calibrated are the inherent efficiencies of each sector, η_N and η_E.[30] To calibrate them, it is also necessary to solve for R_E^*. To start, I re-write the research arbitrage equation in terms of observables,

$$\frac{1+g_E^*}{1+g_N^*} = \frac{\theta_E^*}{\alpha}\frac{\eta_E}{\eta_N}\left(\frac{R_E^*}{1-R_E^*}\right)^{-\lambda}. \tag{26}$$

This equation has a natural interpretation. Monopolists must trade off the relative benefits and costs of investing in the two types of technology. The ratio $\frac{\theta_E^*}{\alpha}$ is a summary measure of the relative return to investment in energy efficiency. The energy expenditure share, θ_E^*, captures the benefit to energy efficiency improvements. Meanwhile, α gives the fraction of increased final output that will be paid to capital good producers. The remaining terms on the right-hand side capture the inverse of relative costs – i.e. research efficiencies – of investing in the two types of technology, which are determined by inherent productivity and the degree of congestion.[31] To complete the R&D calibration, I add the following two equations,

$$g_E^* = \eta_E (R_E^*)^{1-\lambda}, \tag{27}$$

$$g_N^* = \eta_N (1-R_E^*)^{1-\lambda}, \tag{28}$$

which ensure that rates of technological progress match their values in the data.

Taking the ratio of (27) and (28) and substituting into (26) yields

$$\frac{R_E^*}{1-R_E^*} = \frac{g_E^*}{1+g_E^*}\frac{1+g_N^*}{g_N^*}\frac{\theta_E^*}{\alpha}, \tag{29}$$

[30] An alternate approach would be to assume that $\eta_E = \eta_N$ and calibrate λ. Such an approach leads to a significantly higher value of λ (0.69). As shown below, this would greatly magnify the difference between the DTC and Cobb-Douglas approaches.

[31] Hassler et al. (2016b) identify a similar relationship between equilibrium growth rates and the expenditure share of energy when considering a social planner solution with a general CES production function and a finite set of energy resources that can be extracted from the environmental without cost. In their framework, the long-run equilibrium must also conform to the social planner's optimal depletion condition for the energy resource. This pins down the long-run expenditure share and technology growth rates. Since energy use is currently rising, the data suggests that the BGP conditions are not met in the Hassler et al. (2016b) world, leading to the prediction that the energy expenditure share will increase and consumption growth will decrease in the long run.

which captures the equilibrium relationship between research allocation and growth rates on the BGP. As expected, there is a positive relationship between R_E^* and both g_E^* and θ_E^*. All of the variables on the right-hand side are observable and imply that 13.3% of research expenditure is spent improving the energy efficiency of capital goods. This result is independent of the level of research congestion, λ, and inherent efficiencies, $\{\eta_E, \eta_N\}$.[32] Intuitively, the structure of the model implies that investment in energy efficiency must be relatively low because both the incentive for R&D in energy efficiency – captured by the ratio of expenditure shares – and the relative growth rate of energy efficient technology – captured by the remaining two terms – are low.

To solve for the research efficiencies, I first consider the ratio of (27) and (28), which yields

$$\frac{\eta_E}{\eta_N} = \frac{g_E^*}{g_N^*}\left(\frac{1 - R_E^*}{R_E^*}\right)^{1-\lambda}. \tag{30}$$

Since the growth rate of energy efficient technology is large relative to the research allocation, it must be the case that the inherent efficiency of this type of research is high. In particular, applying the results found above yields $\frac{\eta_E}{\eta_N} = 2.42$.[33] To complete the calibration, I plug R_E^* into equation (28) to find $\eta_N = 0.44$ and then use (30) to find $\eta_E = 1.05$.

4.3 Energy Sector Calibration

To calibrate the energy sector parameters, I start by noting that, on the BGP, both cumulative and per period energy use grow at a constant rate, g_M^*. The most important parameter for the energy sector is ψ, which captures the rate at which growth in energy use translates into growth in energy prices,

$$\psi = \frac{ln(1 + g_E^*)}{ln(1 + g_M^*)}. \tag{31}$$

In the model, environmental policy will decrease energy use, which in turn lowers the price of energy and the incentive for energy efficient research. The size of this effect depends directly on ψ. In the data, energy efficiency grows significantly faster than energy use, which leads to an estimate of $\psi = 2.31$.

Next, to ensure that the economy starts in a steady state, it must be the case that total extracted energy grows at a constant rate. Thus, I calculate the initial level of extracted energy as

$$\bar{E}_{-1} = g_M^*/E_0, \tag{32}$$

where $\bar{E}_{-1}$ is the cumulative energy used prior to the first period. Conditional on ψ, the ratio between the initial stock and the per period flow of energy use determines the degree to which

[32] Unfortunately, existing data sources do not separate expenditure by different characteristics of the same good, making it difficult to compare this result to existing evidence.

[33] Research efficiency could be greater in energy research for a number of reasons. Appendix Section **??** provides a simple example where there is a greater diversity of research tasks necessary to improve non-energy technology.

Table 1: Parameters

Parameter	Value	Description	Source
α	.35	Capital share of income	Golosov et al. (2014)
δ	1	Depreciation	Golosov et al. (2014)
β	.860	Discount factor	Golosov et al. (2014)
σ	1	Inter-temporal substitution	Golosov et al. (2014)
n	0.10	Population growth	EIA
λ	0.21	Research congestion	Fried (2018)
η_E	1.05	Research efficiency	Calibrated
η_N	0.44	Research efficiency	Calibrated
ψ	2.31	Energy cost growth	Calibrated
ξ	$1.40 \cdot 10^{-5}$	Energy cost scale	Calibrated
$\bar{E}_{-1}$	114	Initial extracted energy	Calibrated

energy prices fluctuate in response to policy-induced changes in energy use. If the stock of consumed energy is large, then per period energy use fluctuations will only have a small effect on extraction costs. The calibration yields $\bar{E}_{-1} = 114$ with per period energy use normalized to 10. As discussed in Section 5.1, the calibration implies that the endogenous price of energy plays a significant role in the quantitative outcomes, but not in the qualitative conclusions from comparing the DTC and Cobb-Douglas models.

Finally, ξ is a scale parameter calibrated to the starting price,

$$\xi \quad = \quad \frac{p_{E,0}}{\bar{E}_{-1}^{\psi}}. \tag{33}$$

Conditional on the other parameters, ξ simply reflects the normalization decisions. Values for all parameters are provided in Table 1.

4.4 Solving the Model

Conditional on the price of energy, the model can separated into three pieces: the R&D allocations, the standard neoclassical growth model with monopolistic competition, and the energy sector. The fact that innovation occurs in different characteristics of capital goods, rather than in different sectors, facilitates the solution of the model. In particular, equations (20) and (21) demonstrate that, conditional on the price of energy, the R&D allocations and technology growth rates can be solved independently of the consumer problem. To find the competitive equilibrium, I employ the following steps:[34]

1. Guess a vector of energy prices.

[34] In all quantitative applications, this procedure is sufficient to find a competitive equilibrium. I have not shown that such a procedure must converge to an equilibrium. In all cases, I use the BGP in the absence of energy taxes to generate the initial guess of energy prices.

2. Solve for productivity paths and R&D allocations using equations (9), (20), and (21), noting that all monopolists make identical research decisions.
3. Solve the neoclassical growth model conditional on the path of productivities using equations (B.29) – (B.35) in Appendix Section B.4.1.
4. Back out implied energy use and energy prices using equations (3), (5), and (6). This takes advantage of the fact that (4) holds with equality in all periods.
5. Check if the initial guess and resulting prices are the same. If they are, then consumers have made optimal decisions taking all future prices as given and the economy is in equilibrium.
6. If the economy is not in equilibrium, start from step 1 with a convex combination of initial guess and resulting prices.

5 Results

5.1 Energy Taxes

In this section, I examine the effect of energy taxes in the new DTC and compare the results to those in the standard Cobb-Douglas model. The time period in the model is ten years. All future policies are announced in the initial period, which I take as 2005 to match the stated objectives of international climate agreements. All policies take effect in 2015. The gap between the announcement and implementation of the policy allows one round of endogenous and directed technical change to occur before comparing the outcomes across the two models. If the policy were unexpected, the final good producer in the Cobb-Douglas model could react, whereas there would be no adjustment in the DTC model due to the Leontief structure.

To best understand the quantitative impacts of the new model of energy use developed in this paper, it is helpful to consider a realistic path of future energy taxes. Under the Paris Agreement on climate change, the United States aims to adopt policies consistent with an 80% reduction in carbon emissions by the year 2050, when compared to 2005 levels. I apply taxes such that half of this gain, a 40% reduction, comes from lower energy use.[35] The evidence in Figure 1 suggests that energy efficiency has been responsible for well more than half of past decreases in the carbon intensity of output.

As in Section 3.4, I consider a path of proportional energy taxes that grow at a constant rate,

$$\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2005}{10}}. \tag{34}$$

To achieve the environmental goals given above, the DTC model requires $g_\tau = .36$.[36] When taking into account the general equilibrium effect of energy use on extraction costs, this yields a tax-inclusive energy price that is 273% higher than the laissez-faire level in 2055.

[35]Since the model is solved in ten year periods, I choose taxes such that the 40% reduction occurs by 2055.

[36]To find the minimum tax necessary to achieve the policy goal, I search with a 1% step size.

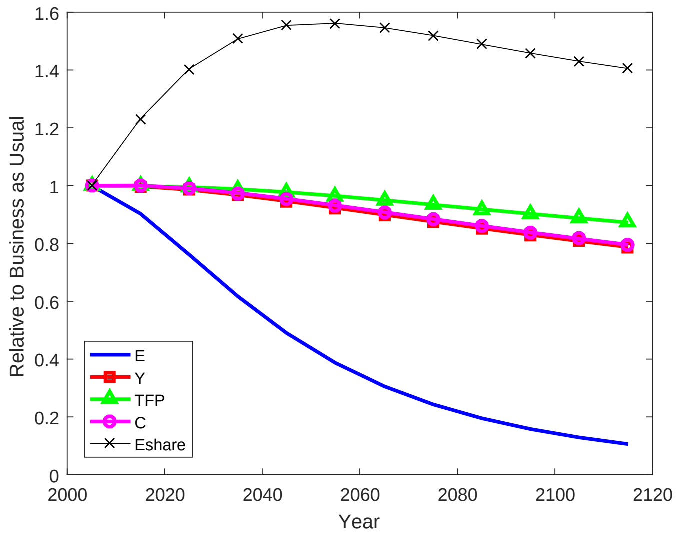


Figure 4: This figure demonstrates the effect of energy taxes in the DTC model. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2005}{10}}$, with $g_\tau = .36$. This level of taxation achieves a 40% reduction in energy in by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. All outcomes in the figure are given as a fraction of the outcomes in the baseline scenario, which has no energy taxation.

Figure 4 presents the results. In particular, it presents the paths of energy use, output, TFP, consumption, and the energy expenditure share from 2005 to 2115.[37] All outcomes are given as a fraction of the business as usual scenario. As expected, energy taxes simultaneously increase the energy expenditure share and decrease energy use. In other words, capital good producers have increased incentive to invest in energy efficiency, but the resulting improvement is insufficient to fully offset the increase in the price of energy. In this way, it is already apparent that the results will differ from those in the Cobb-Douglas model. By 2055, the economy experiences a 6.8% decrease in consumption and 3.5% decrease in TFP relative to the baseline. Energy use plummets to 11% of baseline by 2115, one century after the policy is initially implemented. At the same time, consumption decreases by 20%, and TFP is 12.7% lower than in the business as usual scenario.

Figure 5 repeats the analysis for the standard Cobb-Douglas model with exogenous technological progress. The effect of policy in the Cobb-Douglas approach differs considerably from the DTC model. In this case, $g_\tau = 0.26$ is sufficient to achieve a 40% reduction in energy use by 2055. This leads to a tax-inclusive energy price that is 136% greater than baseline. To achieve the environmental policy priorities, consumption decreases by 2.1% in 2055 and 6.4% by 2115, relative to baseline. By 2115, energy use is 18.4% of baseline levels.

As expected, the energy share of expenditure is essentially unchanged in the Cobb-Douglas model.[38] Thus, energy use decreases by enough to fully offset the increase in energy prices. This

[37] In empirical applications, taxes grow for 500 years and then remain constant.

[38] The slight decrease in the energy expenditure share is due to the lump sum tax rebates. The expenditure share of energy in gross output is constant, but after taxes are implemented, a proportion of energy expenditure is rebated to consumers.

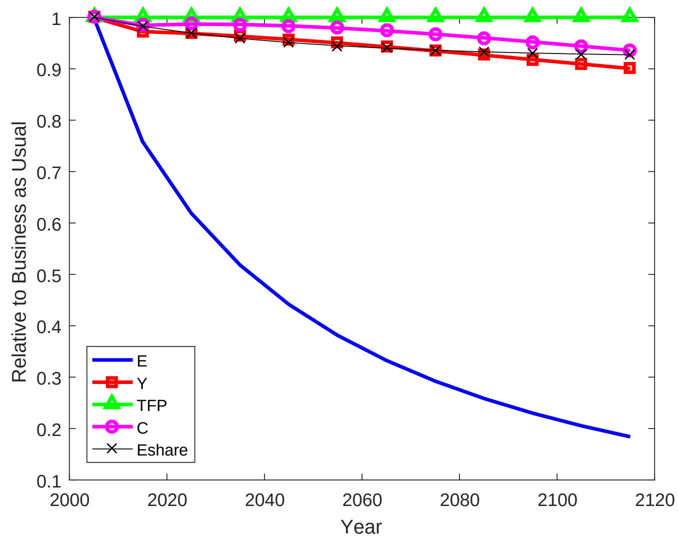


Figure 5: This figure demonstrates the effect of energy taxes in the standard Cobb-Douglas model with exogenous technological progress. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2005}{10}}$, with $g_\tau = 0.26$. This level of taxation achieves a 40% reduction in energy in by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. All outcomes in the figure are given as a fraction of the outcomes in the baseline scenario, which has no energy taxation.

can be seen in how quickly the Cobb-Douglas model responds to new taxes. In 2015, energy use decreases by almost 25% relative to the baseline, in comparison to a 10% decrease in the DTC model. This occurs even though the tax rate is lower in the Cobb-Douglas model.

Figure 6 provides a direct comparison of energy use and consumption in the two models when applying the same path of energy taxes, specifically those necessary to achieve environmental policy goals in the Cobb-Douglas model. Thus, the analysis quantifies the error that would occur if policy was designed with the Cobb-Douglas model, but the true economy was given by the DTC model. Energy use is measured as a fraction of the 2005 level, and consumption is measured relative to the baseline.[39]

When applying the requisite taxes from the Cobb-Douglas model to the DTC model, energy use in 2055 declines by 25% when compared to 2005 levels, missing the environmental target by 15 percentage points. Forgone consumption is roughly twice as large in the DTC model. Despite the stated goals of policy, cumulative energy use is most important for long-run environmental outcomes. The difference in cumulative energy use between the two models is given by the area between the two energy use curves. Over the course of the century, cumulative energy use is 24% higher in the DTC model. These results further illuminate the important differences between the two models and demonstrate that policy designed for the Cobb-Douglas model would yield drastically different outcomes in a world more closely resembling the DTC model.

Figure C.1 in Appendix Section C presents the results from several robustness exercises. As

[39]Given the difference in market structure, the baseline level of consumption, but not the growth rate of consumption, differs in the two models.

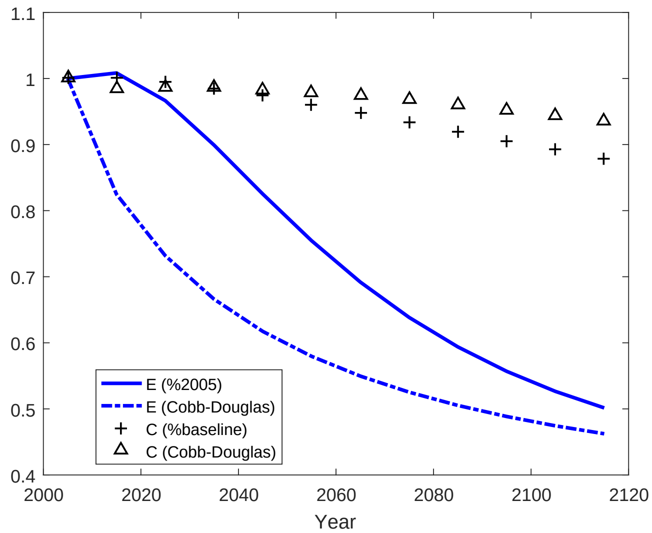


Figure 6: This figure demonstrates the difference between the DTC model and the standard Cobb-Douglas model with exogenous technological progress. Energy taxes are proportional to the price of energy and grow at a constant rate: $\tau_t = 1 \cdot (1 + g_\tau)^{\frac{t-2015}{10}}$, with $g_\tau = 0.26$. In the Cobb-Douglas model with exogenous technical change, this level of taxation achieves a 40% reduction in energy use by 2055, compared to 2005 levels. All taxes are rebated to consumers in a lump sum fashion. Energy use is measured as a fraction of 2005 levels. Consumption is measured relative to the baseline, which does not include energy taxes. The baseline level of consumption differs in the two models.

discussed in Section 4, the research congestion parameter, λ, was set exogenously. So, I consider several alternate values. Most importantly, in panel (a), I consider the limiting case without congestion, i.e., $\lambda = 0$. This minimizes the difference between the two models by making research input reallocation as effective as possible. The quantitative results still differ substantially between the two models. In particular, cumulative energy use with the DTC model is 11% greater by 2115, and the DTC model misses the policy goal by 5 percentage points. Panel (b) considers the case of $\lambda = .105$, which splits the difference between the baseline and most conservative estimates. Cumulative energy use is 17% greater by 2115 with the DTC model, and applying the Cobb-Douglas tax rates causes the model to miss the policy target by 9.6 percentage points in 2055. Naturally, the differences are magnified with considering greater values of λ. In particular, cumulative energy use is 32% higher by 2115 and the policy target is missed by 22 percentage points in the DTC model when $\lambda = 0.31$, as demonstrated in panel (c).

The model was calibrated to the United States. As noted in Section 3.1.2, the fact that energy prices are fully endogenous can be motivated in two ways. First, we can think of the U.S. as a closed economy. Second, we can think of policy being applied to the whole world, with the US making up a constant fraction of total energy use. To ensure that the assumption of fully endogenous energy prices is not driving the results, I consider the case where the price of energy is exogenous and grows at the steady state rate. This captures the scenario where the U.S. is a small open economy taking unilateral action to lessen energy use. The results are presented in panel (d). In this case, cumulative energy use is 38% higher in the DTC model by 2115 and the policy target is missed by 26 percentage points. Thus, taking energy prices as fully endogenous is a conservative approach that lessens the difference between the DTC and Cobb-Douglas models. Moreover, the Cobb-Douglas model only requires energy taxes to grow at $g_\tau = 0.18$ to meet the policy target, implying that the general equilibrium reaction of energy prices is quantitatively important.

5.2 Research Subsidies

Many policy makers are in favor of policy approaches, such as research subsidies or energy efficiency mandates, that try to reduce energy use without raising prices (Gillingham et al., 2009; Allcott and Greenstone, 2012).[40] A large academic literature, however, suggests that rebound effects will undermine the effectiveness of these approaches (Gillingham et al., 2016). Rebound occurs when economic behavior following improvements in energy efficiency leads to increases in energy use, at least partially undoing the initial reduction. Existing work attempts to indirectly gauge the effectiveness of such policies by measuring the degree of rebound. Using the DTC model, however, I can address the broader motivating question and directly analyze the impact of such policies on long-run energy use at the macroeconomic level, the scale which is relevant for climate change mitigation policy.

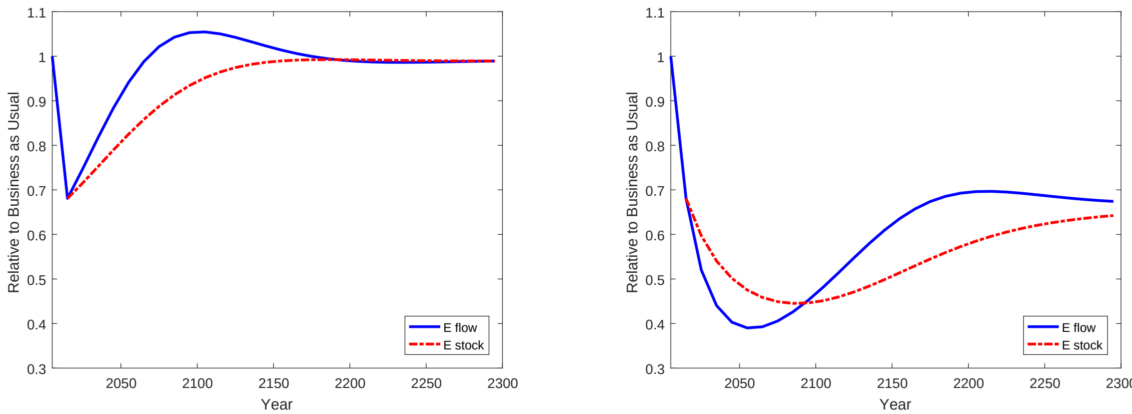
Figure 7 presents the results. Panel (a) considers a single period research subsidy of 73% in 2015. This is analogous to the setting in most of the existing literature, which examines one-off efficiency improvements. In the short-run, energy use decreases considerably, which is unsurprising given the low short-run elasticity of substitution between energy and non-energy inputs. Over time, however, energy use catches back up with the baseline. By the end of the century, energy use is actually higher than in the business as usual case. This is known as 'backfire' in the literature. Long-run energy use is identical to the laissez-faire case. In the literature, this is known as 'full rebound' (Wei, 2010). Intuitively, full rebound occurs because one-off policy interventions do not change the long-run incentives of capital good producers. Thus, when energy efficiency increases in the short-run, the incentive for further investment in energy-saving technology decreases, and the economy converges back to the original BGP.

In terms of environmental policy goals, this result is also more pessimistic than the existing macroeconomic literature, which suggests less than full rebound (Gillingham et al., 2016), but does not consider the potential for contemporary efficiency improvements to alter research incentives.[41] At the same time, the transition path is long. As a result, energy efficiency policies may serve as useful complements to other policy interventions by delaying fossil energy use.

While the existing literature generally focuses on one-off shocks in order to estimate the degree of rebound, there is no particular reason why attempts to reduce long-run energy use would be constrained to temporary interventions. In panel (b), I consider a permanent subsidy of 73% to

[40] In the DTC model, all innovation occurs in different characteristics of capital goods. Thus, research subsidies and efficiency mandates are equivalent. In particular, for any given subsidy, there is an equivalent energy efficiency mandate that yields the same research allocation.

[41] Recent work by Lemoine (2016) also suggests a higher potential for backfire by considering the general equilibrium response of energy prices to efficiency improvements.



(a) Single Period Subsidy

(b) Permanent Subsidy

Figure 7: The effects of research subsidies on energy use. *Panel A* demonstrates the effects of a single period research subsidy of 73%. *Panel B* demonstrates the effects of a permanent subsidy of 73%. This policy achieves a 40% reduction in energy use by 2055, compared to 2005 levels. $E\ flow$ refers to per period energy use. $E\ stock$ refers to cumulative energy use since 2015, the first year the policy takes effect.

energy efficiency research. This subsidy is sufficient to achieve the 40% reduction in energy use discussed in the previous section.

Unlike the case of a single period research subsidy, permanent interventions reduce long-run energy use relative to a business as usual scenario. As demonstrated theoretically in Section 3.5, however, R&D subsidies are not sufficient to generate absolute long-run declines in energy use. Intuitively, the initial reduction in energy use again lowers the incentive for future investments in energy efficient technology. Since the subsidy is permanent, however, the return to investing in energy-saving technology is greater than the in the laissez-faire case for any given energy expenditure share. Thus, the economy converges to a BGP with a lower energy expenditure share, which translates to a lower level of energy use. Still, the logic of the model implies that, for a fixed level of taxes and subsidies, the growth rate of energy use is positive and constant in the long run. Given the need to decrease total carbon emissions in order to avoid dangerous levels of warming, it appears that taxes, or other policies that increase the effective price of energy, are a necessary component of mitigation policies.

6 Conclusion

Economic analysis of climate change has benefited substantially from the study of growth models (e.g., Nordhaus, 1993, 2014). This paper contributes to this ongoing effort by focusing on the demand for energy coming from final good production, a crucial margin for climate change mitigation policy. In particular, I develop a DTC model that can explain both short- and long-run patterns of energy use in the U.S. By contrast, much of the existing literature either abstracts from energy use (e.g., Nordhaus, 1993, 2014) or uses a Cobb-Douglas approach that cannot replicate the same facts (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014). At the same time, the existing literature on directed technical change and the environmental focuses on substitution between energy sources (e.g., Acemoglu et al., 2012) or on the efficiency of the energy sector (e.g., André and Smulders, 2014), rather than the energy efficiency of final good production.

I use the new DTC model to conduct two policy analyses. In my primary exercise, I find that policy conclusions based on the standard Cobb-Douglas model likely overestimate policy-induced reductions in energy use. In a second analysis, I find that innovation-driven rebound effects will prevent policies like R&D subsidies from generating long-run declines in energy use, highlighting the need for policies that increase effective prices.

This paper has focused on the importance of final-use energy and abstracted from other important elements of climate change economics. The model, however, is designed such that it can be easily incorporated into broader integrated assessment models (IAMs) (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014). In particular, the dynamics of the DTC model closely follow those of the neoclassical growth model, with tractable extensions to account for the supply and demand on energy.

Such an extension could be used to answer several interesting questions. Including the third margin of technological investment in clean versus dirty energy sources would make it possible to gain a more complete understanding of the effect of carbon taxes on emissions. Combined with a model of the carbon cycle, such an analysis could yield updates to existing estimates of optimal carbon taxes and the social cost of carbon. It would also allow for the comparison of second-best policies. For example, it would be interesting to compare subsidies for renewable energy, which would limit the incentive to improve energy efficiency, and energy taxes, which provide no incentive to invest in clean energy sources.

It would also be interesting to examine the model presented here is a broader geographic scope. In particular, existing work with exogenous technological progress suggest that unilateral policy actions among rich countries will have small impacts on overall carbon emissions (Nordhaus, 2010). In a world with endogenous technological progress and diffusion or trade, however, unilateral policies would improve worldwide energy efficiency, leading to greater environmental benefit (Di Maria and Van der Werf, 2008; Hémous, 2016). This magnifies the difference with the standard Cobb-Douglas approach, where substitution of capital for energy in one country would have no direct impact on other countries. The positive implications of these international spillovers could potentially outweigh the more pessimistic conclusions that result from considering the DTC model in a closed economy.

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A Data Appendix

A.1 Figure 1

Primary Energy (E_p). Total energy extracted from the environment (i.e., production) plus net imports. For renewables used in electricity generation, production is equal to electricity generated. Measured in kilotonnes of oil equivalent (ktoe). Data available from 1971-2014. Source: 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/energybalances/.

Final-Use Energy (E_f). Total energy consumption: total primary energy minus losses occurring during transformation and energy industry own use. Measured in ktoe. Data available from 1971-2014. Source: 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/energybalances/.

Carbon Dioxide Emissions (CO_2). Carbon dioxide emissions from fuel combustion. Measured in megatonnes (Mt). Data available from 1971-2014. Source: 'IEA Headline Energy Data' at http://www.iea.org/statistics/topics/energybalances/.

Real GDP (Y). Real gross domestic product in 2009 chained dollars. Data available from 1929-2015. Source: NIPA Table Section 1. Accessed via 'Table D1: Population, U.S. gross domestic product, and implicit price deflator, 1949– 2011' at https://www.eia.gov/totalenergy/data/annual/.

A.2 Figure 2

Energy Expenditure Share ($Eshare$). Energy expenditure as a share of GDP (%). Data available from 1970–2014. Source: 'Table 1.5: Energy consumption, expenditures, and emissions indicators estimates, 1949–2011' at https://www.eia.gov/totalenergy/data/annual/.

Energy Intensity of Output (E/Y). Total primary energy consumption per real dollar of GDP. Measured in thousand Btu per chained (2009) dollar. Data available from 1949–2016. Source: 'Table 1.5: Energy consumption, expenditures, and emissions indicators estimates, 1949–2011' at https://www.eia.gov/totalenergy/data/annual/.

Nominal Energy Expenditure. Energy expenditure in millions of nominal dollars. Data available from 1970–2014. Source: 'Table 1.5: Energy consumption, expenditures, and emissions indicators estimates, 1949–2011' at https://www.eia.gov/totalenergy/data/annual/.

Primary Energy Consumption. Total Primary Energy Consumption. Measured in Quadrillion Btu. Data available from 1949–2016. Source: 'Table 1.5: Energy consumption, expenditures, and emissions indicators estimates, 1949–2011' at https://www.eia.gov/totalenergy/data/annual/.

GDP Price Deflator. U.S. GDP implicit price deflator with base year 2009. Data available from 1929-2015. Source: NIPA Table Section 1. Accessed via 'Table D1: Population, U.S. gross domestic product, and implicit price deflator, 1949– 2011' at https://www.eia.gov/totalenergy/data/annual/.

Real Energy Price. Average real price of primary energy in 2009 chained dollars. Author's calculations: Nominal Energy Expenditure divided by Primary Energy Consumption divided by GDP Price Deflator.

A.3 Calibration

See above for details regarding **Real GDP**, **Primary Energy Consumption** (from figure 2), and the **Energy Expenditure Share**.

Population. Total resident population of the United States. Accessed via 'Table D1: Population, U.S. gross domestic product, and implicit price deflator, 1949– 2011' at https://www.eia.gov/totalenergy/data/annual/.

Gross Output. Author's calculations. Using the structure of the model, gross output is calculated as: $Y/\left(1-\frac{E_{share}}{1+E_{share}}\right)$.

Energy Efficiency. Author's calculations: Gross Output / Primary Energy Consumption.

B Online Appendix

B.1 Final Good Producer Problem

In this section, I derive the inverse demand functions (14) and (15). Consider the maximization of (3) subject to (4) with $v_t(i)$ as the Lagrange multiplier attached to capital good i,

$$\mathcal{L} = \int_0^1 A_{E,t}(i)E_t(i)di - w_t L_t - \int_0^1 p_{X,t}(i)X_t(i)di - \tau_t p_{E,t}\int_0^1 E_t(i)di \\ - \int_0^1 v_t(i)\big[A_{E,t}(i)E_t(i) - \big(A_{N,t}(i)X_t(i)\big)^{\alpha} L_t^{1-\alpha}\big]di. \tag{B.1}$$

Complementary slackness implies

$$v_t(i)\big[A_{E,t}(i)E_t(i) - \big(A_{N,t}(i)X_t(i)\big)^{\alpha} L_t^{1-\alpha}\big] = 0\ \forall i. \tag{B.2}$$

I focus on the case where the constraint is always binding. This will necessarily be true in the empirical exercise, because $\delta = 1$ is a sufficient, but not necessary, condition for the constraint to bind. The first order conditions with respect to $E_t(i)$, $X_t(i)$, and L_t are given by:

$$v_t(i) = 1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}, \tag{B.3}$$

$$v_t(i) = \frac{p_{X,t}(i)}{\alpha A_{N,t}^{\alpha}(i) L_t^{1-\alpha} X_t(i)^{\alpha-1}}, \tag{B.4}$$

$$w_t = \int_0^1 v_t(i)(1-\alpha)A_{N,t}^{\alpha}(i)L_t^{-\alpha}X_t(i)^{\alpha}di. \tag{B.5}$$

Substituting (B.4) and (B.5) into (B.3), respectively, and multiplying through yields

$$p_{X,t}(i) = \alpha A_{N,t}(i)^{\alpha}\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]L_t^{1-\alpha}X_t(i)^{\alpha-1}, \tag{B.6}$$

$$w_t = (1-\alpha)\int_0^1 \big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]L_t^{-\alpha}\big(A_{N,t}(i)X_t(i)\big)^{\alpha}di. \tag{B.7}$$

Thus, we have arrived at equations (14) and (15) from the text. A key result is that inverse demand is iso-elastic, which allows for simple closed form solutions. This is shown in the next section.

B.2 Monopolist Problem

The monopolist maximizes profits subject to demand and research productivity constraints:

$$max\ \pi_{X,t}(i) = p_{X,t}(i)X_t(i) - r_t X(i) - (1-\eta_t^S)p_{E,t}^R R_E(i) - p_{N,t}^R R_N(i) \tag{B.8}$$

$$\tag{B.9}$$

subject to

$$
\begin{aligned}
p_{X,t}(i) &= \alpha A_{N,t}(i)^{\alpha}\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]L_t^{1-\alpha}X_t(i)^{\alpha-1}, && \text{(B.10)}\\
A_{J,t}(i) &= \big[1+\eta_J R_{J,t}(i)R_{J,t}^{-\lambda}\big]A_{J,t-1}, \ \ J\in\{N,E\}, && \text{(B.11)}\\
R_{J,t}(i) &\in [0,1], \ \ J\in\{N,E\}. && \text{(B.12)}
\end{aligned}
$$

In equilibrium, the research allocation must be interior due to the congestion effects. Thus, I ignore the last constraint for the remainder of this section. First, substitute (B.10) into (B.8) and take the first order condition with respect to $X(i)$. Constraint (B.11) is independent of the production level, $X_t(i)$. Hence, the model yields the standard first order conditions and results, adjusted for the effective cost of energy:

$$
\begin{aligned}
r_t &= \alpha^2 A_{N,t}(i)^{\alpha}\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]L_t^{1-\alpha}X_t(i)^{\alpha-1}, && \text{(B.13)}\\
X_t(i) &= \alpha^{\frac{2}{1-\alpha}}r^{\frac{-1}{1-\alpha}}A_{N,t}(i)^{\frac{\alpha}{1-\alpha}}L_t\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, && \text{(B.14)}\\
p_{X,t}(i) &= \frac{1}{\alpha}r_t. && \text{(B.15)}
\end{aligned}
$$

Next, to find optimal profits, we can re-write the monopolist problem after substituting in results we have found so far:

$$
max \ \ \pi_{X,t}(i) = \tilde{\alpha}r_t^{\frac{-\alpha}{1-\alpha}}A_{N,t}(i)^{\frac{\alpha}{1-\alpha}}L_t\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}} - (1-\eta_t^S)p_{E,t}^R R_{E,t}(i) - p_{N,t}^R R_{N,t}(i) \quad \text{(B.16)}
$$

subject to

$$
A_{J,t}(i) = \big[1+\eta_J R_{J,t}(i)R_{J,t}^{-\lambda}\big]A_{J,t-1}, \ \ J\in\{N,E\}, \quad \text{(B.17)}
$$

where $\tilde{\alpha} = (\frac{1}{\alpha}-1)\alpha^{\frac{2}{1-\alpha}}$. Let κ_J be the Lagrange multiplier for constraint (B.17). The first order conditions for technology levels and research scientist allocations yield

$$
\begin{aligned}
p_{N,t}^R &= \kappa_N A_{N,t-1}R_{N,t}^{-\lambda}, && \text{(B.18)}\\
(1-\eta_t^S)p_{E,t}^R &= \kappa_E A_{E,t-1}R_{E,t}^{-\lambda}, && \text{(B.19)}\\
\kappa_N &= \frac{\alpha}{1-\alpha}\tilde{\alpha}r_t^{\frac{-\alpha}{1-\alpha}}A_{N,t}(i)^{\frac{\alpha}{1-\alpha}-1}L_t\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}}, && \text{(B.20)}\\
\kappa_E &= \frac{1}{1-\alpha}\tilde{\alpha}r_t^{\frac{-\alpha}{1-\alpha}}A_{N,t}(i)^{\frac{\alpha}{1-\alpha}}L_t\big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\big]^{\frac{1}{1-\alpha}-1}\tau_t p_{E,t}A_{E,t}^{-2}. && \text{(B.21)}
\end{aligned}
$$

Putting these together, we have

$$
\begin{aligned}
p^R_{N,t} &= \alpha\psi_t A_{N,t}^{\frac{\alpha}{1-\alpha}-1}\Big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\Big]^{\frac{1}{1-\alpha}}\eta_N R_{N,t}^{-\lambda}A_{N,t-1}, && \text{(B.22)}\\
(1-\eta^S_t)p^R_{E,t} &= \psi_t A_{N,t}^{\frac{\alpha}{1-\alpha}}\tau_t p_{E,t}A_{E,t}(i)^{-2}\Big[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\Big]^{\frac{1}{1-\alpha}-1}\eta_E R_{E,t}^{-\lambda}A_{E,t-1}, && \text{(B.23)}
\end{aligned}
$$

where $\psi_t = \frac{\tilde{\alpha}}{1-\alpha}r_t^{\frac{-\alpha}{1-\alpha}}L_t$ is common to both terms. In the next section, I shown the optimal research allocations resulting from these first order conditions. Taking ratios of these first order conditions yields (19) in the main text.

B.3 R&D Allocations

In this section, I derive the optimal research allocations given in equations (20) and (21). First, note that $R_{J,t}(i) = R_{J,t}\ \forall i,t,J$. This occurs because all monopolists make identical decisions, and there are a unit mass of monopolists. This also implies that $A_{J,t}(i) = A_{J,t}\ \forall i,t,J$. Also, factor mobility ensures that $p^R_{E,t} = p^R_{N,t}\ \forall t$. Thus, equation (19) can be re-written as

$$
(1-\eta^S_t)\frac{A_{E,t}}{A_{E,t-1}}\Big[\frac{A_{E,t}}{\tau_t p_{E,t}}-1\Big] = \frac{A_{N,t}}{A_{N,t-1}}\frac{\eta_E R_E^{-\lambda}}{\alpha\eta_N R_N^{-\lambda}}. \tag{B.24}
$$

Replacing growth rates and technology levels with the values given by (9) and applying the resource constraint (10) yields

$$
(1-\eta^S_t)(1+\eta_E R_{E,t}^{1-\lambda})\Big[\frac{(1+\eta_E R_{E,t}^{1-\lambda})A_{E,t-1}}{\tau_t p_{E,t}}-1\Big] = (1+\eta_N(1-R_{E,t})^{1-\lambda})\frac{\eta_E R_{E,t}^{-\lambda}}{\alpha\eta_N(1-R_{E,t})^{-\lambda}} \tag{B.25}
$$

Dividing by $(1-\eta^S_t)$, then multiplying through on the left-hand side and isolating the term with energy prices yields

$$
(1+\eta_E R_E^{1-\lambda})^2\frac{A_{E,t-1}}{\tau_t p_{E,t}} = \frac{1}{1-\eta^S_t}\Big[\frac{\eta_E R_E^{-\lambda}}{\alpha\eta_N(1-R_E)^{-\lambda}}(1+\eta_N(1-R_E)^{1-\lambda})\Big] + (1+\eta_E R_E^{1-\lambda}). \tag{B.26}
$$

Distributing terms on the right-hand side leaves

$$
(1+\eta_E R_E^{1-\lambda})^2\frac{A_{E,t-1}}{\tau_t p_{E,t}} = \frac{1}{\alpha(1-\eta^S_t)}\Big[\frac{\eta_E R_E^{-\lambda}}{\eta_N(1-R_E)^{-\lambda}} + \eta_E R_E^{-\lambda} - \eta_E R_{E,t}^{1-\lambda}\Big] + (1+\eta_E R_E^{1-\lambda}). \tag{B.27}
$$

Now, (20) can be derived by multiplying through by $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$, taking the square root of both sides, subtracting one, and dividing by $\eta_E R_E^{-\lambda}$.

B.4 Solving the Model

B.4.1 Intensive Form

In this section, I show how to solve the model in intensive form. This is helpful both for the quantitative exercise (see Section 4.4) and in proving the propositions in Sections 3.4 and 3.5. For any variable Z_t, I define

$$z_t \equiv \frac{Z_t}{L_t A_{R,t}}, \tag{B.28}$$

where $A_{R,t} = TFP_t^{\frac{1}{1-\alpha}}$ and $TFP_t = A_{N,t}^{\alpha}\big[1 - \frac{p_{E,t}}{A_{E,t}}\big]$. Applying (7), (8), and (11), this yields

$$y_t = k_t^{\alpha}, \tag{B.29}$$

$$k_{t+1} = \frac{y_t - c_t}{(1+g_{R,t+1})(1+n)}, \tag{B.30}$$

where $1 + g_{R,t} = \frac{A_{R,t}}{A_{R,t-1}} = (1 + g_{TFP,t})^{\frac{1}{1-\alpha}}$. Moreover, the Euler equation yields

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{\beta r_{t+1}}{(1+g_{R,t+1})^{\sigma}}. \tag{B.31}$$

Finally, when considering the interest rate, it is also important to keep track of the energy tax rate, τ_t. Let $\tilde{A}_{R,t} = A_{N,t}^{\alpha}\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\big]$ be TFP adjusted for energy taxes. Then, from equation (17),

$$r_t = \alpha^2 A_{N,t}^{\alpha}\big[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\big] K_t^{\alpha-1} L_t^{1-\alpha} \tag{B.32}$$

$$= \alpha^2 \left(\frac{K_t}{\tilde{A}_{R,t} L_t}\right)^{\alpha-1} \tag{B.33}$$

$$= \alpha^2 \left(\frac{A_{R,t}}{\tilde{A}_{R,t}}\right)^{\alpha-1} \left(\frac{K_t}{A_{R,t} L_t}\right)^{\alpha-1} \tag{B.34}$$

$$= \tilde{\tau}_t \alpha^2 k_t^{\alpha-1}, \tag{B.35}$$

where $\tilde{\tau}_t \equiv \left(\frac{A_{R,t}}{\tilde{A}_{R,t}}\right)^{\alpha-1} = \frac{1 - \frac{\tau_t p_{E,t}}{A_{E,t}}}{1 - \frac{p_{E,t}}{A_{E,t}}}$ is the interest rate wedge caused by the introduction of energy taxes.

When solving the model, I guess on a path of energy prices and then solve for the research allocations and growth rates. Then, the solution to the remainder of the model is given by (B.29), (B.30), (B.31), and (B.35). As described above, this is just the standard neoclassical growth model with a few additions. The α^2 term in (B.35) is the standard adjustment for monopolistic competition, $\tilde{\tau}_t$ is the wedge in the interest rate caused by energy taxes, and $g_{R,t}$ may not be constant due to endogenous research allocations and energy prices.

B.4.2 Proof to Propositions ??, 1, and 2.

Proof of items 3 – 5 of Propositions 1 and 2. To find the BGP, first note that $\tilde{\tau}_t = \bar{\tau}$, a constant. In the laissez-faire case, $\bar{\tau} = 1$. In the case of environmental policy (EP), $\bar{\tau} = \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right]$, which is also constant. In the EP case, the economy does not converge to the BGP in finite time. As discussed in the main text, $g_{TFP} = (1 + g_N^*)^\alpha - 1$ on the BGP because $\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]$ is fixed (at 1 in the case of EP). Thus, the growth rate of output per person is given by $g_R^* = (1 + g_N^*)^{\frac{\alpha}{1-\alpha}} - 1$. With constant growth rates of technology, the BGP is given by:

$$\bar{r} = \frac{(1 + g_R^*)^\sigma}{\beta}, \tag{B.36}$$

$$\bar{k} = \left(\frac{\bar{\tau}\alpha^2}{\bar{r}}\right)^{\frac{1}{1-\alpha}}, \tag{B.37}$$

$$\bar{y} = \bar{k}^\alpha, \tag{B.38}$$

$$\bar{c} = \bar{y} - (1 + g_R^*)(1 + n)\bar{k}, \tag{B.39}$$

where $\bar{z}$ denotes the steady state value of z. Thus, r_t is constant, Y_t/L_t and C_t/L_t grow at rate g_R^*, and Y_t and K_t grow at rate $g_Y^* = (1+g_R^*)(1+n)-1$. This proves parts (3) – (5) of Propositions 1 and 2.

Proposition 1 and Item 6 of Propositions 1 and 2. On the BGP, both per period and cumulative energy use grow at the same, weakly positive rate. At any point in time, energy use is given by

$$E_t = \frac{A_{N,t}^\alpha}{A_{E,t}} K_t^\alpha L_t^{1-\alpha}. \tag{B.40}$$

On the BGP, therefore, the growth rate of energy is given by

$$g_M^* = \frac{(1 + g_N^*)^{\frac{\alpha}{1-\alpha}}}{(1 + g_E^*)}(1 + n) - 1. \tag{B.41}$$

This proves item (6) of the propositions, except for the sign restrictions. Research allocations must be interior due to the congestion effects. This implies that $\tau_t p_{E,t}$ is growing on the BGP. In a *laissez-faire equilibrium*, this implies that $p_{E,t}$ is growing and, as a result, that the growth rate of energy use is positive. In the EP case, if g_τ is sufficiently high, then a BGP requires $1 + g_E^* > (1 + g_N^*)^{\frac{\alpha}{1-\alpha}}(1 + n)$ (see item (1)). If the per period growth rate of energy use is negative, then the cumulative growth rate and the growth rate of the price of energy both converge to zero in the limit. In this case, energy efficiency grows at rate g_τ on the BGP.

Items 1 and 2 of Propositions 1 and 2. On a BGP, energy efficiency grows at the rate of the energy price times the growth in energy taxes. Writing the price of energy is terms of energy use,

$$(1 + g_M^*)^\psi (1 + g_\tau) = (1 + g_E^*). \tag{B.42}$$

Combining these last two equations yields

$$(1+g_E^*)^{1+1/\psi}(1+g_\tau)^{-1/\psi} = (1+g_N^*)^{\frac{\alpha}{1-\alpha}}(1+n). \tag{B.43}$$

The existence of an interior solution is guaranteed by assumption (A.1). Applying (9) and (10) to this equation yields item (2) of Propositions 1 and 2. Rearranging yields item (1).

Item 7 of Propositions 1 and 2. All that remains to show for these two propositions is that expenditure shares are constant. To start, from equation (15) note that

$$w_t L_t = (1-\alpha)A_{N,t}^{\alpha}[1-\frac{\tau_t p_{E,t}}{A_{E,t}}]K^{\alpha}L^{1-\alpha} = \tilde{\tau}_t(1-\alpha)Y_t, \tag{B.44}$$

which implies that the share is constant on the BGP. Next, from (23) and (17),

$$r_t K_t = \alpha^2 A_{N,t}^{\alpha}[1-\frac{\tau_t p_{E,t}}{A_{E,t}}]K^{\alpha}L^{1-\alpha} = \tilde{\tau}_t \alpha^2 Y_t, \tag{B.45}$$

which again implies that the share is constant on the BGP The remaining share, $(1-\alpha-\alpha^2)$, is the production profits of the monopolists. This can be further divided into pure profits and payments to research inputs. All research inputs are hired at the same rate. By equation (B.22), total payments to research inputs is given by

$$\begin{aligned} p_t^R &= \frac{\alpha}{1-\alpha}(\frac{1}{\alpha}-1)\frac{r_t X_t}{A_{N,t}}\eta_N R_N^{-\lambda} A_{N,t-1} && \text{(B.46)} \\ &= \frac{\alpha}{1-\alpha}(\frac{1}{\alpha}-1)\cdot\frac{\eta_N (R_N)^{-\lambda}}{1+g_N}\cdot\tau_t\alpha^2 Y_t, && \text{(B.47)} \end{aligned}$$

noting that there is a unit mass of research inputs. The remaining share of final output is paid to monopolists as pure profits.

To get the energy expenditure share in either case, rearrange equation (20) to isolate $\frac{\tau_t p_{E,t}}{A_{E,t-1}} = \frac{\tau_t p_{E,t}(1+g_E^*)}{A_{E,t}}$, which is constant. In the laissez-faire case, $\tau_t = 1$ and $\frac{p_{E,t}}{A_{E,t}} = \frac{\theta_E^*}{1+\theta_E^*}$. In the EP case, $\lim_{t\to\infty}\frac{p_{E,t}}{A_{E,t}} = 0 \Rightarrow \lim_{t\to\infty}\theta_{E,t} - \frac{\tau_t p_{E,t}}{A_{E,t}} = 0$. Thus, item (7) of the propositions is proven.

B.5 The Cobb-Douglas Model

In this section, I derive the dynamics, BGP, and calibration procedure for the Cobb-Douglas model. To start, I note that, due to perfect competition, aggregate energy use is given by

$$E_t = (\frac{\nu}{\tau_t p_{E,t}})^{\frac{1}{1-\nu}}(A_t^{CD})^{\frac{1}{1-\nu}}K_t^{\frac{\gamma}{1-\nu}}L_t^{\frac{1-\gamma-\nu}{1-\nu}}. \tag{B.48}$$

This, in turn, yields

$$
\begin{aligned}
Q_t &= \Big(\frac{\nu}{p_{E,t}\cdot\tau_t}\Big)^{\frac{\nu}{1-\nu}}(A_t^{CD})^{\frac{1}{1-\nu}}K_t^{\frac{\gamma}{1-\nu}}L_t^{\frac{(1-\gamma-\nu)}{1-\nu}}, && \text{(B.49)}\\
Y_t &= \Big(1-\frac{\nu}{\tau}\Big)Q_t. && \text{(B.50)}
\end{aligned}
$$

To analyze the model in intensive form, I define

$$
z_t = \frac{Z_t}{L_t(A_t^{CD})^{\frac{1}{1-\gamma-\nu}}(\tau_t\cdot p_{E,t})^{\frac{-\nu}{1-\gamma-\nu}}}, \tag{B.51}
$$

for any variable Z_t. This notation is specific to Appendix Section B.5.

The Euler equation is the same as in the putty-clay case. In intensive form,

$$
\frac{c_{t+1}}{c_t} = \frac{\beta r_{t+1}}{(1+g_{CD})^{\frac{1}{1-\gamma-\nu}}(1+\tilde{g}_{P,t+1})^{\frac{-\nu}{1-\gamma-\nu}}}, \tag{B.52}
$$

where $1+\tilde{g}_{P,t+1} = (1+g_{\tau,t+1})(1+g_{P,t+1})$, $1+g_{\tau,t} = \frac{\tau_t}{\tau_{t-1}}$, and I have imposed $\sigma = 1$. The rest of the dynamics are given by

$$
\begin{aligned}
k_{t+1} &= \frac{y_t - c_t}{(1+g_{CD,t+1})^{\frac{1}{1-\gamma-\nu}}(1+\tilde{g}_{P,t+1})^{\frac{-\nu}{1-\gamma-\nu}}(1+n)}, && \text{(B.53)}\\
y_t &= \Big(1-\frac{\nu}{\tau}\Big)\nu^{\frac{\nu}{1-\nu}}k_t^{\frac{\gamma}{1-\nu}}, && \text{(B.54)}\\
r_t &= \gamma k_t^{\frac{\gamma-(1-\nu)}{1-\nu}}. && \text{(B.55)}
\end{aligned}
$$

As in the case of the putty-clay model, I solve the model by first guessing a path of energy taxes and then solving the growth model with equations (B.52) – (B.55).

I consider the BGP in a *laissez-faire equilibrium*. This gives

$$
\begin{aligned}
\bar{r} &= \frac{(1+g_{CD}^*)^{\frac{1}{1-\gamma-\nu}}(1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}}}{\beta}, && \text{(B.56)}\\
\bar{k} &= (\bar{r}/\gamma)^{\frac{1-\nu}{\gamma-(1-\nu)}}, && \text{(B.57)}\\
\bar{y} &= (1-\nu)\nu^{\frac{\nu}{1-\nu}}\bar{k}^{\frac{\gamma}{1-\nu}}, && \text{(B.58)}\\
\bar{c} &= \bar{y}-(1+g_{CD}^*)^{\frac{1}{1-\gamma-\nu}}(1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}}(1+n)\bar{k}. && \text{(B.59)}
\end{aligned}
$$

As a result, r_t is constant, Y_t/L_t and C_t/L_t grow at rate $(g_R^*)^{CD} = (1+g_{CD}^*)^{\frac{1}{1-\gamma-\nu}}(1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}} - 1$, and Y_t and K_t grow at rate $g_Y^{CD} = (1+g_R^*)^{CD}(1+n) - 1$.

I calibrate the model to the BGP using the same data as employed for the putty-clay model, leading to observationally equivalent paths for output and energy use. To match the energy expenditure share, I set

$$\frac{\nu}{1-\nu} = \theta_E^* \tag{B.60}$$

and

$$\gamma = \alpha - \nu. \tag{B.61}$$

All that remains is to ensure that total output grows at the same rate in the two models, which implies that energy use will also grow at the same rate. Since the energy sector is equivalent in the two models, this further implies that the price of energy will grow at the same rate. Thus, I set $(g_R^*)^{CD} = g_R^*$, where the latter comes from the putty-clay model in Section B.4.1. This implies that

$$g_R^* = (1+g_{CD}^*)^{\frac{1}{1-\gamma-\nu}}(1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}} - 1 \Rightarrow \tag{B.62}$$

$$g_{CD}^* = (1+g_R^*)^{1-\gamma-\nu}(1+g_E^*)^{\nu} - 1. \tag{B.63}$$

The calibration yields $g_{CD}^* = .42$, which corresponds to an annual growth rate of 3.5%. The growth rate of TFP is higher in the Cobb-Douglas case because it needs to overcome the drag of rising energy prices to achieve the same BGP rates of growth in consumption and output.

B.6 Extension of Base Model

In this section, I consider an extension of the putty-clay model that includes two additional margins of adjustment: (i) entry of new capital good producers and (ii) labor allocation between production and research. This extended model incorporates insights from 'second wave' endogenous growth theory (e.g., Peretto, 1998; Young, 1998; Howitt, 1999). As a result, it eliminates scale effects that are present in existing models of directed technical change (e.g., Acemoglu, 1998, 2002; Hassler et al., 2012, 2016b). I show that the extended model continues to explain the key patterns observed in U.S. data. In the future, I will update the quantitative exercise to include these two margins. For simplicity, this appendix focuses on the laissez-faire equilibrium.

Consider the following extension of the aggregate production function:

$$Q_t = \int_0^{M_t} \min\left[(L_t/M_t)^{1-\alpha}\,(A_{N,t}(i)X_t(i))^{\alpha}\,,\; A_{E,t}(i)E_t(i)\right] di, \tag{B.64}$$

where M_t gives the mass of (atomistic) capital good producers in operation at time t. This particular functional form is standard in the existing literature and eliminates the 'love of variety' in production. I will refer to L_t as production workers. To operate in period t, a capital good producer must hire φ^{-1} workers to cover fixed costs. This yields

$$M_t = \varphi F_t, \tag{B.65}$$

where F_t is the total number of fixed cost workers. There is free entry into capital good production.

For the creation of new technologies, I assume

$$A_{J,t}(i) = \left[1 + \eta_J R_{J,t}(i) \bar{R}_{J,t}^{-\lambda}\right] A_{J,t-1}, \quad J \in \{N, E\}, \tag{B.66}$$

where $\bar{R}_{J,t} = \frac{1}{M_t} \int_0^{M_t} R_{J,t}(i)\, di$ and $A_{J,t} = \frac{1}{M_t} \int_0^{M_t} A_{J,t}(i)\, di$.[42] This is the functional form from the main text updated to accommodate changes in the mass of capital good producers. I now assume that research is conducted by scientists. Thus, the labor market clearing condition is given by:

$$N_t = L_t + F_t + R_t \; \forall t. \tag{B.67}$$

The remainder of the model is unchanged.

Using the same steps outlined in earlier appendix sections, it is straightforward to derive the following key expressions:

$$Y_t = \left[1 - \frac{p_{E,t}}{A_{E,t}}\right] \left(A_{N_t} K_t\right)^{\alpha} L_t^{1-\alpha}, \tag{B.68}$$

$$w_t = (1-\alpha) L^{-\alpha} \left(A_{N_t} K_t\right)^{\alpha}, \tag{B.69}$$

$$\bar{\pi}_{X,t} \propto r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}^{\frac{\alpha}{1-\alpha}} \left(L_t / M_t\right) \left[1 - \frac{p_{E,t}}{A_{E,t}}\right]^{\frac{1}{1-\alpha}}, \tag{B.70}$$

$$p_t^R \propto r_t^{\frac{-\alpha}{1-\alpha}} \left(L_t / M_t\right) A_{N,t}(i)^{\frac{\alpha}{1-\alpha}} \left[1 - \frac{p_{E,t}}{A_{E,t}}\right]^{\frac{1}{1-\alpha}} \eta_N \bar{R}_{N,t}^{-\lambda} \frac{1}{1 + g_{N,t}}, \tag{B.71}$$

where I have applied the relevant market clearing conditions and the fact that all capital good producers face an identical problem and make identical decisions.

I will show that there exists a balanced growth path with constant expenditure shares, matching the data. This BGP will have constant labor allocations (in shares). Output per capita and wages will grow at rate $(1 + g_N^*)^{\frac{\alpha}{1-\alpha}} - 1$, as in the baseline model developed in the main text.

There are a few key relationships to note. First, $\frac{p_{E,t}}{A_{E,t}}$ will be constant, which is necessary for constant factor shares. Second, to have a constant growth rate of technological progress, the number of researchers hired by each capital good producer, $\bar{R}^* = \bar{R_N}^* + \bar{R_E}^*$, will be constant. This is consistent with increases in the total number of researchers, $R_t(i)$, because of expanding varieties. This is the essence of 'second wave' endogenous growth theory.

To have a constant fraction of workers dedicated to overcoming fixed costs, F_t – and therefore M_t – must grow at rate n, the rate of population growth. If the fraction of production workers is constant, then L_t also grows at rate n and it is immediate that L_t / M_t is constant on the BGP. For the number of scientists hired by each firm to be constant, the total number of scientists grows at the same rate as the number of capital good producers, which is also n.

So, to find a BGP, I need only show that there is such an allocation that is consistent with free

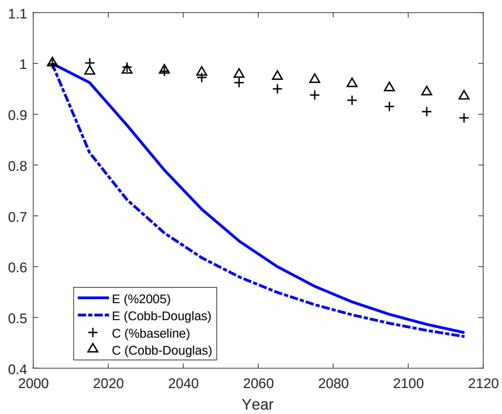
[42] It is equivalent to assume that there is diminishing returns to research within each firm.

mobility of individuals across occupations. In particular, I will show that, on a BGP, the wages of all three occupations grow at rate $(1+g_N^*)^{\frac{\alpha}{1-\alpha}}-1$. This is immediate for production worker – equation (B.69) – and scientists – equation (B.71). Payments to fixed cost workers are determined by the free entry condition.

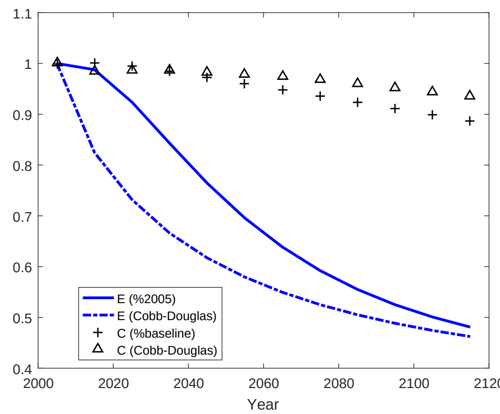
$$\begin{aligned} w_t^F &= \frac{1}{\varphi}\left(\bar{\pi}-p_t^R\right) && \text{(B.72)} \\ &\propto A_{N,t}(i)^{\frac{\alpha}{1-\alpha}}. && \text{(B.73)} \end{aligned}$$

So, the model has a BGP with free mobility between occupations. The labor allocation shares are determined by the market clearing condition (B.67) and the free mobility condition, $w_t = w_t^F = p_t^R$.

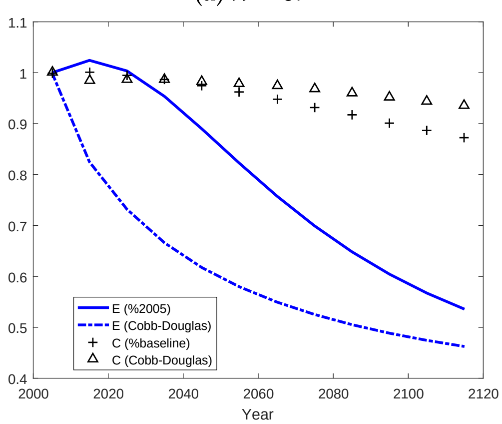
C Robustness Exercises



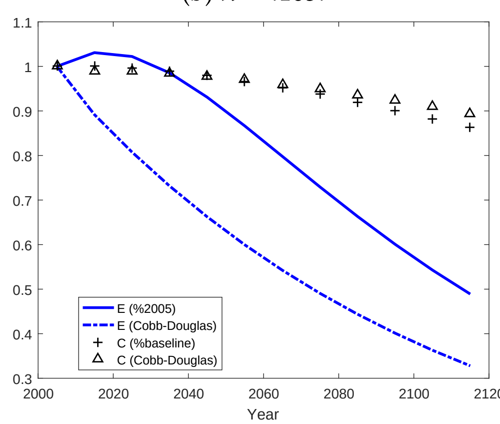
(a) $\lambda = 0$.



(b) $\lambda = .105$.



(c) $\lambda = .31$.



(d) Exogenous $p_{E,t}$.

Figure C.1: Robustness exercises. Panels (a) – (c) consider alternate values of research congestion, λ. In each case, $g_\tau = 0.26$, which is the tax rate requires to achieve policy goals with the Cobb-Douglas model. Panel (d) presents the results when energy prices grow exogenously at rate, $g_P = 0.21$. This matches the growth rate of energy prices on the BGP in the baseline scenarios. The tax rate for panel (d) is given by $g_\tau = 0.18$, which is the tax rate requires to achieve policy goals with the Cobb-Douglas model.