When the Tail Wags the Dog: The Predictive Power of Tail Risk Premia on Individual Stock Returns

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The impact of tail events on returns is well documented at the aggregate market level, but not so much is known about its impact at the individual stock level. This paper introduces a novel, option-free, methodology to directly calculate the tail risk premium for individual stocks, and then examines the characteristics of this premium in the cross section of stock returns. The existence of a premium for bearing either negative or positive tail risk is significantly associated with negative returns up to one month in the future, although only weakly so in the case of positive tail risk. Also, the larger is the tail risk premium, the greater and longer lasting is its impact on expected future returns. The paper discusses several theories, such as the lottery effect, differential probability weightings, crash risk, and momentum, among others, in an attempt to explain the relationship between tail risk and future returns. It also introduces to the finance literature the concept of Unrealistic Optimism; whereby human beings have an optimistic bias about their personal risk. People believe that negative (positive) future events are less (more) likely to happen to themselves than to the average person. A consequence of Unrealistic Optimism is that investors – irrespective of whether the tail risk is positive or negative – may pay too much when tail risk is present, which will yield a negative relationship between the existence of a tail risk premium and future returns. Of the competing theories, Unrealistic Optimism seems to be the most consistent with our results.

Keywords: tail risk, asymmetry, cross-section of stock returns, return prediction, behavioral finance, investor psychology, empirical asset pricing. JEL Classification: G11, G12, G17

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I. Introduction

Compensation for extreme, tail event risk is formally referred to in the academic finance literature as a “tail risk premium.” Bollerslev, Todorov and Xu (2015) shows that a majority of the predictability in the variance risk premium is attributed to this premium for bearing jump tail risk, and that, specifically, it is negative tail risk and not positive tail risk that seems to be priced\(^1\). The impact of tail events on returns is well documented at the aggregate market level, but not so much is known about its impact at the individual stock level. One reason why is that out of the money call and put options are required to determine the tail risk in the risk neutral probability space. Although out of the money options are prevalent for an index such as the S&P500, they either do not exist or are illiquid for most stocks. For this reason, to date no paper has directly examined both the impact of tail risk and its premium on the cross section of individual stock returns. Given that the return distribution for individual stocks will likely exhibit a greater proclivity for extreme events than the return distribution for a diversified market portfolio (where extreme negative events in some securities might be tempered by extreme positive events in others) one would expect that tail risk should play a more prominent role in the returns for individual stocks than it would for a market portfolio. Consequently, a careful study of tail risk premia for individual stocks may yield new and heretofore unseen insights into their predictive power for

\(^1\) Variance risk premium of market index possesses return predictive power was first documented in Bollerslev, Tauchen and Zhou (2009).
future returns. Moreover, it is of interest to examine whether there a difference in the way in which the existence of positive and negative tail risk premia affect future returns.

Kelly and Jiang (2014) is the first to examine tail risk in the cross section of individual stock returns. This important paper employs an aggregate measure of time-varying tail risk that relies on panel estimation from the cross section of stock returns. It then measures a stock’s sensitivity to this measure of tail risk by sorting portfolios into quintiles based on tail beta-exposure, and documents that the lowest tail beta quintile is associated with the lowest future returns, while the highest tail beta quintile is associated with the highest future returns. Although Kelly and Jiang provides strong evidence that tail risk is priced in individual stocks, it does not directly calculate the tail risk premium nor does it examine any asymmetry in the way positive and negative tail risk premia affect future returns.

The current paper differentiates itself from Kelly and Jiang (2014) in three critical ways. First, rather than using an aggregate measure of tail risk and indirectly examining the sensitivity of a stock’s return to this aggregate measure, the current paper directly calculates the tail risk premium for individual stocks and examines how this premium varies across the cross-section of stock returns. Specifically, the paper introduces a novel, non-parametric approach to directly determine the tail risk premium. The approach avoids the need for the use of liquid out of the money stock options (which don’t exist for most stocks). The second contribution is that this new approach allows stocks to be sorted by their exposure to tail risk, so that the impact of positive and negative tail risk premia on future returns can be examined separately. Stocks with exposure to negative tail risk require a tail risk premium that is positive (investors demand a higher return today than otherwise expected for bearing negative tail risk) while those with positive tail risk require a tail risk premium that is negative (investors are willing to accept a lower return today
when there is a chance for extreme positive events). Third, this paper documents that almost all of the extreme jumps are concentrated in the first and tenth deciles; consequently, an analysis of deciles, and even percentiles, rather than the quintiles examined in prior studies, is necessary if researchers are to better understand how extreme jump tail risk is priced.

The results in this paper provide an interesting new perspective on the differential pricing of information related to negative and positive tail risk. Bollerslev, Todorov and Xu (2015) and Kelly and Jiang (2014) find evidence of pricing for negative tail risk, but neither fully examines the extent to which positive tail risk is priced. The current paper finds that both positive and negative tail risk are priced in the cross section of individual stock returns. The evidence further reveals that information about positive tail risk and negative tail risk seem to be processed by the market differentially. The existence of a premium for bearing positive tail risk today has a negative effect on future monthly returns, but the predictive power is weak. In contrast, existence of a premium for bearing negative tail risk today is associated with significantly lower future monthly returns. In addition, this paper presents evidence that it is not only the sign of the tail risk premium that matters in predicting future returns, but also its magnitude. The larger and more positive the current tail risk premium (that is, the greater the concerns about a big negative jump), the more negative and persistent will be the association with future returns.

The paper’s empirical methodology controls for several explanations previously offered in the literature for the existence and pricing of tail risk including momentum (Lehman, 1990 and Jegadeesh, 1993), lottery effects (Barberis and Huang, 2008 and Bali, Cakci and Whitelaw 2011), idiosyncratic volatility (Ang, Hoderick, Xing, and Zhang, 2006), illiquidity (Amihud, 2002), market beta (Sholes and Williams, 1977 and Dimson 1979), maximum and minimum monthly return (Bali, Cakici and Whitelaw, 2011). The predictive power of the premium for bearing
negative or positive tail risk on future returns survives the inclusion of these control variables. Consequently, the search for another potential explanation is warranted. To this end, the paper introduces a heretofore unexplored in-the-finance-literature explanation for tail risk pricing behavior: Unrealistic Optimism. Unrealistic Optimism is the well documented psychological phenomenon whereby human beings have an optimistic bias about their personal risk. People believe that negative (positive) future events are less (more) likely to happen to themselves than to the average person. A consequence of Unrealistic Optimism is that investors - irrespective of whether the tail risk is positive or negative – may pay too much when tail risk is present, which will yield a negative relationship between the existence of a tail risk premium and future returns.

This paper is organized as follows: Section II contains the literature review; Section III demonstrates individual stock level tail risk premium estimation and the data; Section IV contains the tail risk premium cross-sectional pricing characteristics and cross-sectional return tests; Section V includes robustness checks; Section VI introduces the concept of Unrealistic Optimism and discusses several possible other theories to explain the empirical findings and Section VII concludes the paper.

II. Literature Review

In addition to the papers mentioned in the introduction, there are several recent papers that examine the pricing of downside risk which are related to the current paper. Ang, Chen and Xing (2006) finds that stocks that covary strongly with the market during periods of market decline tend to have higher average returns than other stocks. Investors are downside risk averse and therefore require a premium to hold these assets. Bali, Cakici and Whitelaw (2014) introduces a hybrid tail

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covariance risk measure that measures stock return tail covariance risk. The measure is based on
the basic form of lower partial moments. The paper documents a significant positive premium for
bearing negative tail risk captured in the cross section.

This paper is also related to the literature on crash risk. Kelly and Jiang (2014) is among
the first papers that examine extreme crash risk on stock returns. The paper find that stocks with
high loadings on market tail risk earn higher abnormal returns. Chabi-Yo, Ruenzi and Weigert
(2015) finds that investors are crash-averse; that is, they receive positive compensation for holding
crash-sensitive stocks through the measure of “lower tail dependence” from individual stock price
distributions. The findings in these papers are consistent with the downside risk literature (Ang,
Chen and Xing (2006) and Bali, Cakici and Whitelaw (2014)) that investors are downside risk
averse and require a positive premium for holding the crash risk sensitive stocks.

Bali, Cakici and Whitelaw (2014) constructs a firm-specific tail risk measure based on
lower partial moments of stock returns and finds that it negatively predicts future stock returns.
Almeida et al. (2017) adopts a risk-neutral excess expected shortfall approach to construct a
nonparametric tail risk measure. The paper finds that the risk-neutral tail risk measure possesses
negative predictive power for intermediate horizon stock returns. Lu and Murray (2017) constructs
a proxy for bear-market risk and finds it to be negatively priced; that is, stocks with a high
sensitivity to bear-market risk are found to underperform their low-sensitivity counterparts.

Our paper is also related to the asset pricing literature on higher moments. Traditional
finance theory assumes a normal distribution of asset returns, for which mean and variance
together are sufficient to characterize the entire return distribution. The capital asset pricing model
(Sharpe (1964), Lintner (1965) and Mossin (1966)) predicts that market volatility is a determinant
of the market equity premium. Contrary to this notion, Ang, et al. (2006) examines whether
aggregate volatility innovation is priced in cross-section of stock returns, and concludes that high sensitivity stocks have subsequent lower average returns. Given this controversy, it is natural to ask is whether other return distributional characteristics are also priced in the cross section. Chang, Christofferson and Jacobs (2013) shows that the cross-section of stock returns has substantial exposure to higher moments. Cremers, Halling and Weinbaum (2015) finds that although both jumps and volatility are priced in cross section, jumps seem to have larger impact on returns than does volatility. Bali, Cakici and Whitelaw (2011) finds that stocks with maximum returns have a significant negatively return in the following month. These pricing findings are consistent with the erroneous probability weighting of investors as modeled in Barberis and Huang (2008) and optimal belief framework of economic agents modeled in Brunnermeier, Gollier and Parker (2007).

III. Calculation of the Tail Risk Premium in the Cross Section of Individual Stock Returns

A. Methodology

This section discusses the construction of the tail risk premium associated with jumps in returns for individual stocks. The methodology is an innovation on the well-established notion - Bollerslev, Todorov and Xu (2015), Carr and Wu (2009) and others - that the jump tail risk premium can be calculated as the difference between the expectation of the tail variation in the physical probability space (P-space) and its counterpart in the risk-neutral probability space (Q-space).

To this end, we define the infinite-order polynomial variation of log returns, which captures not only the second-order (quadratic) variation (see Carr and Wu, 2009), but also the higher-order (third-order and up) variations, which Jiang and Oomen (2008) has shown to be associated with
jumps in stock returns. We denote the simple return \( R_{t+1} = \frac{S_{t+1} - S_t}{S_t} \) and logarithmic return \( r_{t+1} = \ln \left( \frac{S_{t+1}}{S_t} \right) \) over a period from \( t \) to \( t + 1 \). Formally, based on Merton (1976)'s jump diffusion process, the realized infinite-order polynomial variation (\( \mathbb{P}V \)) for individual asset \( i \) at time \( t + 1 \) can be expressed as follows:

\[
\mathbb{P}V_{i,[t,t+1]} = 2\left(R_{i,t+1} - r_{i,t+1}\right) = \int_t^{t+1} \sigma_{i,t}^2 dt + \sum_{n=2}^{\infty} \int_t^{t+1} \int_{\mathbb{R}} x_i^n \mu(dx_i, dt) = \mathbb{C}V_{i,[t,t+1]} + \mathbb{J}P\mathbb{V}_{i,[t,t+1]}.
\]

(1)

where \( \sigma \) is the volatility. \( \mu(dx_i, dt) \) is the Poisson random measure for the compound Poisson process with compensator equal to \( \lambda \frac{1}{\sqrt{2\pi}\sigma_J^2} e^{-\frac{1}{2}(x-a)^2} \), with \( \lambda \) as the jump intensity. \( \mathbb{C}V \) is the integral of the continuously instantaneous variance (often referred to as the integrated volatility), and \( \mathbb{J}P\mathbb{V} \) represents the realized jump component of the infinite-order polynomial variation. Analogously, the second-order polynomial variation (the realized quadratic variance, denotes \( \mathbb{Q}V \)) can be written by the following equation:

\[
\mathbb{Q}V_{i,[t,t+1]} = r_{i,t+1}^2 = \int_t^{t+1} \sigma_{i,t}^2 dt + \int_t^{t+1} \int_{\mathbb{R}} x_i^2 \mu(dx_i, dt) = \mathbb{C}V_{i,[t,t+1]} + \mathbb{JQV}_{i,[t,t+1]}.
\]

(2)

By subtracting Equation 2 from Equation 1, we then have the realized tail-jump variation at time \( t \) such that

\[
\mathbb{TV}_{i,[t,t+1]} = 2\left(R_{i,t+1} - r_{i,t+1}\right) - r_{i,t+1}^2 = \mathbb{P}V_{i,[t,t+1]} - \mathbb{Q}V_{i,[t,t+1]}
\]
\[
\sum_{n=3}^{\infty} \frac{2}{n!} \int_{t}^{t+1} \int_{\mathbb{R}^n} x^n \mu(dx_i, dt).
\] (3)

Now that we have the unconditional realized tail-jump variation, we next present the conditional ex-ante estimation of the tail-jump variation and then will develop a proxy for the tail-risk premium.

Following Bollerslev, Tauchen and Zhou (2009), under the assumption that \( \mathbb{TV} \) is a martingale\(^3\), the \( \mathbb{P} \)-space expected tail-variation of returns at time \( t \) can be expressed as follows:

\[
E_t^\mathbb{P}(\mathbb{TV}_{i,[t,t+1]}) = 2(R_{i,t} - r_{i,t}) - r_{i,t}^2.
\] (4a)

Then, the difference between expected \( \mathbb{TV} \) in the \( \mathbb{P} \)-space and that in the risk-neutral \( \mathbb{Q} \)-space, \( E_t^\mathbb{P}(\mathbb{TV}_{i,[t,t+1]}) - E_t^\mathbb{Q}(\mathbb{TV}_{i,[t,t+1]}) \), serves as a proxy for the tail-risk premium. The advantages of using \( E_t^\mathbb{P}(\mathbb{TV}_{i,[t,t+1]}) \) as a tail risk measure in the physical space are threefold. First, it is non-parametric, i.e., it does not require the estimation of a cutoff value as in Kelly and Jiang (2014) or Bollerslev and Todorov (2011b). Second, it does not require the estimation of a jump compensator in order for the instantaneous arithmetic stock return to be a semi-martingale process as in Bollerslev and Todorov (2011a), Bollerslev and Todorov (2015), or Bollerslev, Todorov and Xu (2015). Third, \( E_t^\mathbb{P}(\mathbb{TV}_{i,[t,t+1]}) \) only relies on stock price information and can be easily calculated using databases such as the WRDS CRSP database and the TAQ database. Thus, relative to the measures used in the aforementioned papers, our measure not only lessens estimation error, but

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\(^3\) Under Merton (1976) jump diffusion model assumption, the compensated compound Poisson process \( \mu(dx_i, dt) \), with compensator \( \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}(x-a)^2} \), is a martingale process; consequently, \( \mathbb{P}V_{i,[t,t+1]} \) is also a martingale. Todorov and Tauchen (2011) provides empirical evidence that the VIX index is a pure-jump process without a continuous component, which supports the notion that \( \mathbb{P}V_{i,[t,t+1]} \) is a martingale process. (Du and Kapadia (2012) demonstrates that \( \mathbb{P}V_{i,[t,t+1]} \) is the underlying process of the VIX index). Furthermore, it is widely accepted in both theory and practice that \( \mathbb{Q}V_{i,[t,t+1]} \) is a martingale process, consequently, the claim that \( \mathbb{TV}_{i,[t,t+1]} \) is a martingale process has both theoretical and practical support.
also shortens the calculation time, and because it only relies on prices, it is broadly applicable to other asset classes. Once the $\mathbb{P}$-space tail risk measure is calculated, then one can examine the innovations in this measure

$$\Delta E_t^\mathbb{P}(T^\mathbb{P}V_{l,[t,t+1]}) = \Delta[2(R_{l,t} - r_{l,t}) - r_{l,t}^2]$$ (4b)

In contrast to the $\mathbb{P}$-space tail variation measure which is easy to calculate, as will be shown below, the corresponding calculation of Equation 4a in the $\mathbb{Q}$-space for individual stocks is much more problematical since the necessary data is not readily available. If, instead of examining individual stocks, one were interested in calculating the $\mathbb{Q}$-space tail variation of the market as a whole, then the methodology would be relatively easy to implement. For example, Carr and Wu (2009) shows that the CBOE VIX index is a measure of moment combinations, and therefore a polynomial variation in the risk neutral probability space. Specifically, that paper argues that the VIX index measures the risk-neutral expectation of the polynomial variation process for the S&P 500 market index,

$$E_t^\mathbb{Q}(\mathbb{P}V_{M,[t,t+1]}) = VIX_t^2$$ (5)

Once Equation 5 has been calculated, Du and Kapadia (2012) and Chow, Jiang and Li (2014) show that the $\mathbb{Q}$-space measure of tail variation for the market can be calculated as the difference between the square of the VIX and the centralized Bakshi, Kapadia, and Madan’s (2003) volatility measure, $V_{BKM}^C$ such that

$$V_{BKM}^C = V_{BKM} - \mu_{BKM}^2,$$

where $\mu_{BKM} = \ln \left( \frac{K_0}{S_0}\right) + \left( \frac{F_0}{K_0} - 1 \right) - e^{rT} \left[ \int_{K_0}^{K} \frac{1}{K} C_T(K)dK + \int_{0}^{K_0} \frac{1}{K^2} P_T(K)dK \right]$ and $V_{BKM} = \ln^2 \left( \frac{K_0}{S_0}\right) + 2 \ln \left( \frac{K_0}{S_0}\right) \left( \frac{F_0}{K_0} - 1 \right) + 2e^{rT} \left[ \int_{K_0}^{K} \frac{1 - \ln \left( \frac{K}{S_0}\right)}{K^2} C_T(K)dK + \int_{0}^{K_0} \frac{1 + \ln \left( \frac{K}{S_0}\right)}{K^2} P_T(K)dK \right]$ as in Bakshi, Kapadia, and Madan (2003).
\[ E_t^Q \left( \mathbb{T} \mathbb{V}_{M,[t,t+1]} \right) = VIX_t^2 - V_{BKM,t}^C \] (6)

It is important to note that the calculation of both the VIX and \( V_{BKM}^C \) rely on highly liquid out-of-the-money put and call options\(^6\), which fortunately are prevalent on the S&P 500 index. Recent papers by Gao, Gao and Song (2018) and Gao, Lu and Song (2018) estimate tail risk based on the \( \mathbb{Q} \)-space tail variation measure (in Equation 6) for the market index and portfolio of assets where there exists liquid option trading. Unfortunately, these options often either do not even exist for individual stocks or, if they do exist, are not frequently traded, and thus an analogue of the aforementioned methodology to calculate the \( \mathbb{Q} \)-space tail variation for individual stocks is impossible to implement. Consequently, an alternative is required.

To this end, we propose a new methodology for the estimation of the \( \mathbb{Q} \)-space tail variation. The measure is based on the groundbreaking work on tracking portfolios established by Breeden, Gibbons and Litzenberger (1989), Lamont (2001), and Ang, Hodrick, Xing and Zhang (2006). Specifically, we create a tracking portfolio in the \( \mathbb{Q} \)-space that mimics the actual innovations in tail variation that occur in the \( \mathbb{P} \)-space. According to Lamont (2001), “A tracking portfolio for any variable \( y \) can be obtained as the fitted value of a regression of \( y \) on a set of base asset returns. The portfolio weights for the economic tracking portfolio for \( y \) are identical to the coefficients of an OLS regression.” Note that in our analysis we follow Ang, Hodrick, Xing and Zhang (2006) and use the first-order difference in the \( \mathbb{P} \)-space and \( \mathbb{Q} \)-space tail variation measures, rather than the raw measures themselves, in order to capture innovations in tail risk which will ultimately be

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\(^6\) Demeterfi, Derman, Michael and Zou (1999) present a methodology for estimating the \( \mathbb{Q} \)-space measure of implied volatility for individual securities that is based on the variance swap concept, which requires highly liquid out of the money put and call options.
used to estimate tail risk premia. Accordingly, we estimate the following ordinary least squares regression for each stock in each month to obtain our portfolio weights, $\beta_i$\(^7\)

\[
\Delta[2(R_{it} - r_{it}) - r_{it}^2] = \alpha_i + \beta_i \cdot \Delta(VIX_{t-22}^2 - V_{BK,M,t-22}^C) + \epsilon_{i,t}
\]

(7)

Where $VIX_{t-22}^2 - V_{BK,M,t-22}^C$ represents the tail variation in the $\mathbb{Q}$-space, as delineated in Equation 6, and $2(R_{it} - r_{it}) - r_{it}^2$ represents the tail variation in the $\mathbb{P}$-space, as shown in Equation 4a. Note that we follow the precedent set by Bekaert and Hoerova (2014) which estimates $\mathbb{P}$-spaced conditional realized variation utilizing a 22-day lag. Their approach is based on the notion that options-based $\mathbb{Q}$-spaced measures, such as the VIX, are forward-looking, and thus there is a time lag error of one month (22 trading days) that must be corrected.

Once the $\beta_i$ coefficients have been obtained, then $\beta_i \cdot \Delta(VIX_{t-22}^2 - V_{BK,M,t-22}^C)$ represents the $\mathbb{Q}$-space tracking portfolio that mimics innovations in tail variation that occur in the $\mathbb{P}$-space. Formally,

\[
\Delta E_t^Q(\mathbb{T}V_{i,[t,t+1]}) = \beta_i \cdot \Delta(VIX_{t-22}^2 - V_{BK,M,t-22}^C)
\]

(8)

Now that the daily innovations in the $\mathbb{P}$-space and $\mathbb{Q}$-space tail variation measures (Equations 4b and 8, respectively) have been obtained, then the daily tail risk premium for any asset can be estimated by taking their difference

\[
TRP_{i,[t,t+1]}^{daily} = \Delta E_t^P(\mathbb{T}V_{i,[t,t+1]}) - \Delta E_t^Q(\mathbb{T}V_{i,[t,t+1]})
\]

(9)

Since there are 22 trading days in a month, the corresponding monthly tail risk premium for each individual stock can be estimated as

\[
TRP_{i}^{Monthly} = 22 \cdot TRP_{i,[t,t+1]}^{daily}
\]

(10)

\footnote{Stocks must have at least 17 observations in any given month to be included in that month’s regression.}
B. Data

We run the baseline regression model in Equation 7 for all common stocks on AMEX, NASDAQ, and NYSE, with more than 17 daily observations in any given month. Daily stock returns come from the WRDS CRSP dataset, over the sample period from January 1990 to September 2014. S&P index option data are obtained from IVolatility.com, which provides end-of-day and high frequency option data on major stock market indices across countries.

Table 1 reports the summary statistics for portfolios sorted into deciles by the tail risk premium. Definitions for all the variables can be found in the Appendix. Panel A presents the decile portfolio firm-specific characteristics sorted by the tail risk premium. Firms with a higher tail risk premium tend to have lower lagged 1-month returns (short-term return reversal effect). Firms that fall into the extreme first and 10th decile also tend to be smaller firms that have higher market betas, higher idiosyncratic volatility, more illiquidity, higher maximum monthly returns, lower minimum monthly returns, lower trading volumes and lower prices.

To examine the correlation structure among the explanatory variables, we report in percentage form Pearson correlation coefficients of the variables in Table 2.

Idiosyncratic volatility (Ang, et al. (2006)) is negatively correlated with size (correlation coefficient of -49.70%), which is consistent with the findings in Fu (2009). Moreover, idiosyncratic volatility is also correlated with maximum and minimum monthly returns (Bali, Cakici and Whitelaw (2011)), with correlation coefficients of 89.72% and 80.77%, respectively). Maximum and minimum monthly returns (Bali, Cakici and Whitelaw (2011)) are correlated with size, but to a much lesser extent (-37.24% and 39.00%, respectively).
IV. Predictive Power of the Tail Risk Premium on Future Returns

A. Portfolios Sorted by the Tail Risk Premium

To investigate the predictive power of the tail risk premium in the cross section, we first calculate the tail risk premium for each of the stocks in our sample, and then sort the stocks into decile portfolios by the magnitude of their monthly tail risk premium. We next calculate the one month forward buy and hold returns for each decile portfolio. We term these returns as the 1/0/1 (sort in one month, examine the one-month return for the following month) return. The results are reported in Panel A of Table 3.

[Insert Table 3 Here]

The lowest (decile 1) tail risk premium portfolio earns the highest return of 2.02% in the following month, while the highest (decile 10) portfolio earns the lowest return of 0.27%. The difference between the lowest and highest quintile portfolio is 1.75% monthly, and has a t-statistic of -11.26. After a Newey-West (1987) adjustment for heteroskedasticity and autocorrelation, the t-statistic is still strongly significant with a value of -7.30.

We next examine the length of time it takes for the market to correct this pricing error, by comparing the results for the to 1/0/1 (sort in one month, examine the one-month return for the following month) portfolio strategy discussed in the previous paragraph with 1/1/1 and 1/2/1 (sort in one month, examine the one-month return starting two months from now, and three months from now, respectively) portfolio strategies.

These results are reported in Panels B and C of Table 3. The existence of a tail risk premium at time $t$ possesses virtually no impact on future returns moving from the second-next month into the future. This suggests that the adjustment period for market perception of tail risk seems to be
somewhere between one month and two months, after which the market fully incorporates information about tail risk into the price.

B. Cross-Sectional Return Test for the Predictive Power of the Tail Risk Premium

The above evidence suggests that tail risk is priced at the individual stock level. Consistent with prior studies, we perform a more thorough firm-level cross-sectional returns and examine whether the predictive power of the tail risk premium remains. Specifically, we estimate the following monthly regression:

\[ R_{i,t+1} = \gamma_0_{t+1} + \gamma_{1,t+1} \times TRP_{i,t}^{Monthly} + \phi_{t+1} \times Z_{i,t} + \varepsilon_{i,t+1} \]  \hspace{1cm} (11)

where \( R_{i,t+1} \) is the monthly stock return for stock \( i \) in month \( t + 1 \), \( TRP_{i,t}^{Monthly} \) is the individual stock tail risk premium, delineated by Equation 10. \( Z_{i,t} \) represents a vector of characteristics and controls for firm \( i \) at the end of month \( t \) such as size, book-to-market ratio and market beta. Controls are also provided for illiquidity following Amihud (2002), idiosyncratic volatility following Ang, et al. (2006), lagged 1-month return for short-term return reversal effect following Jegadeesh (1990) and Lehmann (1990), lagged 12-month return accounting for the momentum effect, and maximum and minimum monthly return following Bali, Cakici and Whitelaw (2011).

Table 4 reports the time-series average of \( \gamma \) and \( \phi \) coefficients for the cross-sectional regressions.

[Insert Table 4 Here]

Column 1 provides univariate results and Column 2 adds firm-specific control variables. The coefficient for the tail risk premium is negative and is statistically significant in both the univariate and multivariate regressions, with coefficients of -0.801 and -1.155 and Newey-West (1987) \( t\)-

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8 See Appendix for variable definitions.
statistic equal to -5.60 and -5.50, respectively. Specifically, stocks with tail risk require a premium in the current month, and this premium is associated with lower returns the following month.

The results for the impact of overall tail risk on one-month future returns are interesting, but tail risk involves concerns about both extreme positive events and extreme negative ones. Consequently, it may be of interest to examine, separately, the impact of positive and negative tail risk on future returns.

1. The Monthly Predictive Power of Positive versus Negative Tail Risk Premia

To investigate the extent to which positive and negative tail risk may be priced differentially in the cross-section of returns, we again perform firm-level cross-sectional monthly regressions, but this time we include dummy variables to identify those stocks in the top and bottom deciles when sorted by their tail risk premia. The regression is specified in Equation 12.

\[
R_{i,t+n} = \gamma_{0,t+n} + \gamma_{1,t+n} \times TRP_{i,t}^{\text{Monthly}} \times I_{[\text{Decile 1 TRP}_{i,t}^{\text{Monthly}}]} + \gamma_{2,t+n} \times TRP_{i,t}^{\text{Monthly}} \times I_{[\text{Decile 10 TRP}_{i,t}^{\text{Monthly}}]} + \phi_{t+n} \times Z_{i,t} + \epsilon_{i,t+n}
\]  

(12)

Where \( I_{[\text{Decile 1 TRP}_{i,t}^{\text{Monthly}}]} \) is a dummy variable that equals 1 if \( TRP_{i,t}^{\text{Monthly}} \) is in decile 1 and equals 0 otherwise, and \( I_{[\text{Decile 10 TRP}_{i,t}^{\text{Monthly}}]} \) is the corresponding dummy variable for decile 10. \( R_{i,t+n} \) is monthly stock return for stock \( i \) in month \( t + n \), where \( N = 1,2 \). \( TRP_{i,t}^{\text{Monthly}} \) is the individual stock tail risk premium calculated in Equation 10. \( Z_{i,t} \) represents a vector of characteristics and controls for firm \( i \) at the end of month \( t \) such as size, B/M ratio, market beta, illiquidity, etc.

[Insert Table 5 Here]
Table 5 presents the results. In the t+1 regression, the coefficient on the interacted variable $\text{TRP}_{t,t}^{Monthly} \times I_{[Decile \ 10 \ TRP_{t,t}^{Monthly}]}$ is statistically significant with $\gamma$ coefficient of -1.971 and Newey-West (1987) $t$-statistic of -5.09. The negative coefficient implies that the greater the tail risk premium today (i.e., the greater the negative tail risk) the more negative will be the next month’s return. The coefficient on the decile 1 interacted variable is also negative with a $\gamma$ coefficient of -0.830, but is statistically significant at only the 10% level, thus we refrain from making any strong claims about its impact on future returns. In the t+2 regression, neither the Decile 1 nor the Decile 10 interacted dummies are significant. The 10% significance level for the Decile 1 interacted dummy variable in the t+1 regression begs the question of whether the interacted variable would have any stronger predictive power at shorter investment horizons.

2. The Daily Predictive Power of Positive versus Negative Tail Risk Premia

In order to more fully examine the relationship between positive and negative tail risk premia and future returns, we replicate the study conducted in the previous section using daily returns. Specifically, we conduct a firm-level cross-sectional predictive regression as in Equation 13. We add the caveat that, at the daily level, there is likely to be some noise in our estimates, thus any conclusions should be tempered somewhat.

$$R_{t,t+n}^{Daily} = \gamma_{0,t+n} + \gamma_{1,t+n} \times TRP_{t,t}^{Daily} \times I_{[Decile \ 1 \ TRP_{t,t}^{Daily}]} + \gamma_{2,t+n} \times TRP_{t,t}^{Daily}$$
$$\times I_{[Decile \ 10 \ TRP_{t,t}^{Daily}]} + \phi_{t+n} \times Z_{t,t} + \varepsilon_{t,t+n}$$

([Insert Table 6 Here])

Panel 1 of Table 6 presents the results of the regression of the relationship in Equation 13, day into the future. Interestingly, unlike the monthly findings (where the coefficient on the
interacted variable for decile 1 was negative and significant at only the 10% level and the coefficient for the interacted variable for decile 10 was negative and significant at the 1% level), in the daily regression, the coefficients on the interacted variables for Deciles 1 and Decile 10 are now both negative and significant at the 1% level, with NW t-statistic of -6.78 and -5.17, respectively. Thus, the existence of a premium for bearing either positive or negative tail risk is associated with lower returns one day in the future. Panels 2-5 show that the predictive power of the premium associated with bearing positive tail risk is gone by day 2. In contrast, the existence of a premium for bearing negative tail risk continues to have predictive power for about 10 days. Even on day 10, the coefficient for the interacted variable for decile 10 is -5.128 with NW t-stat -2.43.

The daily results corroborate the findings of the previous section and offer additional evidence on the way that concerns about extreme positive and negative tail events impact future returns. The predictive power of the premium for bearing positive tail risk dissipates quickly, while the predictive power of the premium for negative tail risk seems to persist.

C. Do Larger Tail Risk Premia Have More Predictive Power?

The results in the previous section suggest that the larger the tail risk premium, the greater its impact on future returns. This suggests that using a finer grid to sort stocks, for example, sorting the stock by tail risk premia into percentiles rather than deciles, and then redoing the analysis conducted in Section IV.A. may yield interesting results. To this end, we sort and then split the stocks contained in deciles 1 and 10 into deciles once again; that is, we effectively create ten extreme high and low percentile portfolios, with percentiles 1-10 belonging to decile Portfolio 1 and percentiles 91-100 belonging to decile Portfolio 10. We then apply the 1/0/1 (sort in one month,
examine the one-month return for the following month) portfolio strategy for percentile 1 and 100 portfolios, 2 and 99 portfolios and 3 and 98 portfolios and report the results in Table 7.

[Insert Table 7 Here]

The results are consistent with the notion that there is a monotonic relationship between the magnitude of the tail risk premium and the impact on future returns. The t+1 return difference is most negative when comparing the two most extreme (1 and 100) portfolios, and decreases monotonically thereafter. The return difference (in percentage) for the 100-1 portfolio is -3.91, for the 99-2 portfolio is -2.50, and the 98-3 portfolio is -2.44, respectively. All are significant at the 1% level.

As a robustness check, we combine percentiles 98-100 into one portfolio and percentiles 1-3 into another portfolio, and then do the same for percentiles 97-99 and 2-4. The t+1 return difference for the (98-100)-(1-3) portfolio is -2.95, for the (97-99)-(2-4) portfolio is -2.35, with both being significant at the 1% level, once again lending support to the notion that predictive power of the current premium for bearing negative tail risk should be directly related to its magnitude.

V. Robustness Checks

We perform a variety of robustness checks in order to ensure that our results are not being driven by other factors.

A. Monte Carlo Analysis of Regression Beta

The first robustness check is on the beta of the baseline regression model in Equation 7. There may be a concern that the beta may not be statistically different from zero both cross-sectionally and in the time series. The standard unidimensional t-test cannot capture this
possibility. Instead, to capture both the time series and cross-sectional properties of the regression beta, we use Monte Carlo simulation to test whether beta is statistically different from zero. Monte Carlo simulation has two advantages. First, it is a distributional-free approach. Second, it allows us to make statistical inferences on both the cross sectional and time-series dynamics of the regression beta in Equation 7.

In our sample, the number of firms that have more than 17 trading days in a given month ranges from 3626 to 7471. We denote sample size as $S$, number of random draws as $N$. For a given month in a given year, we perform the following simulation,

1) Random draw (with placement) $\beta_1, \beta_2, \beta_3, \ldots, \beta_S$ and compute the mean of $\beta_1, \beta_2, \beta_3, \ldots, \beta_S$, denote $\bar{\beta}_n$.

2) Repeat 1) $N$ times and get $\bar{\beta}_1, \bar{\beta}_2, \bar{\beta}_3, \ldots, \bar{\beta}_N$.

3) Compute $t$-statistic for $\bar{\beta}_1, \bar{\beta}_2, \bar{\beta}_3, \ldots, \bar{\beta}_N$.

We then compute the average of the (time series) year-month $t$-statistic to get the simulated $t$-statistics.

[Insert Figure 1 Here]

From panels A through B in Figure 1, we observe that the bootstrapped $t$-statistic is statistically and significantly different from zero even if we limit to sample size to only 500 firms in each independent random draw\(^9\). This indicates that the beta of the baseline regression model in Equation 7 is both statistically and economically important that carries important pricing information. In other words, our tail risk premium estimation methodology indeed captures the difference between the $\mathbb{P}$- and $\mathbb{Q}$- spaced expectations of tail risk variation.

---

\(^9\) In figure 1 we limit the number of random draws to 10000. We also perform the Monte Carlo simulation by varying number of random draws from 1000 to 10000 and sample size 500 to 3000, results are similar.
B. Sensitivity to Market Aggregate Tail Risk Premium

The second robustness check is to ensure that our results are not being driven by the sensitivity the individual stock’s loadings to the market tail risk premium. To this end, we follow Ang, et al. (2006) which adopts a “beta approach.” They obtain an individual stock’s sensitivity (beta) to innovation in market aggregate volatility (specifically $\Delta VIX$), and then determine whether this beta has predictive power for the next-month’s stock returns. We run the following regression model,

$$ r_t^i = \beta_0 + \beta_{MKT}^i MKT_t + \beta_{\Delta TRP}^i \Delta TRP_{Market}^t + \epsilon_t^i $$

(14)

where $MKT$ is the market excess return and $\Delta TRP_{Market}^t$ is estimated tail risk premium for the S&P 500, which is our proxy for innovations in the market aggregate tail risk compensation, that is, factor loading, $\beta_{\Delta TRP}^i Market$, captures the sensitivity of individual stock monthly returns to the change in market aggregate tail risk premium. The results are reported in Table 8.

[Insert Table 8 Here]

The value weighted mean $t+1$ return for deciles 1 and 10 are 1.16 and 1.31, respectively. This difference is not statistically significant, which implies that sensitivity to the market aggregate tail risk premium has no predictive power for these stocks. Moreover, it implies that our tail risk premium estimation methodology captures an individual stock’s idiosyncratic tail risk premium, which provides pricing information beyond the individual stock’s loadings to the market tail risk premium.

C. Contemporaneous Regression

Our paper uses a 22-day lag adjustment between the risk neutral and the physical probability space measures. However, in the literature on the variance risk premium normally does not require a lag adjustment for the $Q$- spaced variables calculation in the baseline regression
model in Equation 7. For example, Bollerslev, Tauchen and Zhou (2009) is among the first to document the variance risk premium’s return predictability at the quarterly horizon. They compute the variance risk premium using a relatively “conventional” approach, where the risk premium of return variation is defined as the difference between the time series conditional expected future return variation in the (options based) risk-neutral (Q-spaced) framework and that in the physical probability (P-) space in a contemporaneous manner; however, this approach is inherently biased in that it assumes the risk neutral measures are “backward-looking”.

As a robustness check, we employ the non-lagged methodology of Bollerslev, Tauchen and Zhou (2009) and redo the analysis presented in Secion V.A. We sort the stocks into ten equal groups (decile portfolios) by $TRP_{i,Monthly,notag}$ calculated on the following regression model,

$$\Delta[2(R_{i,t} - r_{i,t}) - r_{i,t}^2] = \alpha_i + \beta_i \cdot \Delta(VIX_t^2 - V_{BK,t}^C) + \epsilon_{i,t}$$

(15)

The results are reported in Table 9.

[Insert Table 9 Here]

As can be seen by comparing the results of Table 9 to those presented in Table 3, using contemporaneous rather than 22-day lagged Q-spaced measures makes little qualitative difference.

VI. Potential explanations for the asymmetric way positive and negative tail risk premia impact returns.

Tversky and Kahneman (1992) proposes the concept of prospect theory, whereby individuals perceive the utility of gains and losses differentially. Barberis and Huang (2008) applies this theory to investor behavior and argue for the existence of a lottery effect, where biases in the probability weighting of investors cause them to overvalue stocks that have a small probability of a large positive return. The lottery effect predicts that positively skewed securities
will be overvalued, and thus the existence of a negative premium for bearing positive tail risk today would imply lower returns in the future. Bali, Cakici and Whitelaw (2011) documents that when stocks are sorted by their monthly returns, those stocks with the maximum (minimum) monthly return tend to have a lower (higher) return in the following month. They interpret these results as support for the lottery effect. We control for the MAX and MIN effect in our regression analysis, and find that predictive power of both the positive and negative tail survive the inclusion of these variables. Moreover, in fact, our results for negative tail risk not only survive the inclusion of the MIN control variable, but are in stark contrast to the predictions of the lottery effect. If biases in investors probability weightings cause them to overvalue stocks with positive skewness, then it stands to reason that these same biases should also be causing them to undervalue stocks with negative skeweness. In which case, the existence of a premium for bearing negative tail risk today, should predict a higher return in the future. However, we find just the opposite: the existence of a premium for bearing negative tail risk predicts lower future returns, not higher ones. Thus, the lottery effect, although consistent with our findings for positively skewed stocks, cannot explain our results regarding negatively skewed ones.

An alternative explanation for the relationship between positive skewness and lower returns is presented in Brunnermeier, Gollier, and Parker (2007). The paper presents a general equilibrium model where individuals optimally balance a bias towards optimism with the real costs of making bad decisions. The result is that investors prefer heterogeneous, underdiversified portfolios that overweight assets with positive skewness, so they can obtain skewed portfolio returns. This preferential weighing scheme, in turn, raises the prices of and lowers the returns of positively skewed assets. However, it is reasonable to expect that, if investors are overweighting positively skewed stocks in their portfolios, they would also be underweighting negatively skewed
ones. If this were the case, then – similar to the predictions of the lottery effect - stocks with negative skewness should be undervalued, and thus be associated with higher future returns. Again, the opposite is observed in the data.

Although Barberis and Huang (2008) and Brunnermeier, Gollier, and Parker (2007) offer well accepted explanations for the association between positive tail risk and negative future returns, neither can explain our findings regarding stocks with negative tail risk. A potential explanation, however, may come from the psychology literature: Unrealistic Optimism.

Harris and Guten (1979) and Weinstein (1980, 1982, 1984, 1987, and 1989) document the existence of Unrealistic Optimism, a phenomenon whereby human beings have an optimistic bias about their personal risk; specifically, they perceive their own future as more optimistic compared to others.\(^{10}\) People believe that extreme negative future events are less likely to happen to themselves than to the average person, and extreme positive future events as more likely to happen to themselves than to others. In other words, humans believe that negative (positive) tail events have a lower (higher) probability of occurring to themselves than occurring to others. Consequently, when people determine the expected impact of extreme tail events on value, the results are exaggerated and optimistically biased. In addition, the more extreme the event, the greater is the exaggeration of reality.

Sharot, Guitart-Masip, Korn, Chowdhury and Dolan (2012), Sharot, Korn and Dolan (2011) and Sharot, Kanai, Marston, Korn, Rees and Dolan (2012) provide evidence that the human memory process actually reinforces the distortions associated with Unrealistic Optimism. People update their beliefs more frequently in response to information that is better than expected

compared to information that is worse than expected. In addition, Moutsiana, Garett, Clarke, Lotto, Blakemore and Sharot (2013) shows that humans possess a natural tendency to discount bad news while incorporating good news into beliefs.

Taken as a whole, the literature on Unrealistic Optimism yields interesting predictions for asset pricing. If Unrealistic Optimism causes investors to overestimate the likelihood of positive tail events and to simultaneously underestimate the likelihood of negative tail events, then investors will tend to pay too much for securities with exposure to either kind of tail risk. Consequently, the existence of a premium for bearing tail risk, irrespective of whether it is negative or positive tail risk, will be associated with lower future returns as the overpricing is eventually corrected. Moreover, the more extreme the tail events the greater will be the exaggeration/overpricing, and the longer lasting should be the predictive power of the associated tail risk premium on future returns. We are not claiming that Unrealistic Optimism is the only explanation consistent with our empirical results, we merely show that are results are consistent with its predictions.

VII. Conclusion

This paper introduces a novel methodology to directly determine the tail risk premium for individual stocks, and then employs this measure to examine the impact of equity tail risk in the cross section of stock returns. Bollerslev, Todorov and Xu (2015) and Kelly and Jiang (2014) find evidence of pricing for negative tail risk, however, neither examines the pricing of positive tail risk, nor the impact on the magnitude of the tail risk on the predictive power of its associated premium. The current paper finds that both positive and negative tail risk are priced in the cross section of individual stock returns.
At the monthly level, the existence of a premium for bearing positive tail risk today holds weak predictive power for lower future returns, while its counterpart for bearing negative tail risk has significant predictive power for lower future returns.

The paper also examines the pricing phenomenon at the daily level and reveals interesting insights. To our knowledge, this paper is the first to document any kind of statistically significant predictive power associated with the premium for bearing positive tail risk. The premium associated with bearing positive tail risk is associated with a negative return one-day in the future, but thereafter the predictive power quickly disappears. In contrast, the predictive power associated with a premium for bearing negative tail risk lasts for 10 trading days. Moreover, the size of the current premium for bearing negative tail risk matters. The larger the premium associated with exposure to negative tail risk, the more negative and longer lasting is its impact on expected future returns.

The paper discusses several potential explanations for our results including the lottery effect, selective probabilty weighting, crash risk, and momentum, among others. The paper is the first in the finance literature to introduce the concept of Unrealistic Optimism, and to discuss its consequences for asset pricing. Unrealistic Optimism is the well documented psychological phenomenon whereby people believe that extreme negative future events are less likely to happen to themselves than to the average person, and extreme positive future events are more likely to happen to themselves than to the average person; in other words, they are overly optimistic about their prospects. Further, the more extreme and remote the likelihood the event, the greater is the optimistic bias. The result of this optimistic bias is that investors will tend to pay too much for a security that has extreme positive or negative tail risk, because they will overestimate the likelihood of the extreme positive payoffs and underestimate the likelihood of the extreme negative
payoffs. Thus, one would expect the existence of a tail risk premium, regardless of whether it is for bearing positive or negative tail risk, would be associated with lower future returns. Our empirical results are consistent with this finding.

The methodology in this paper can be easily extended to other asset classes and to investor behavior in different countries, for example, bond markets, foreign exchange markets, commodity markets in both US and foreign markets. As future research, it would be interesting to investigate how tail risk is priced in these other asset classes; especially in the presence of liquidity risk.
References


Appendix  Variable Definitions

Tail risk premium: We compute tail risk premium as in Equation 10 in Section III A.

Log (Size): Following Fama and French (1993), size is computed each June as stock price times number of shares outstanding (in hundreds). Size is measured in hundred thousand. We control for size effect by taking natural logarithm of Size.

Log (B/M): Following Fama and French (1993), book-to-market is computed as the ratio of book common equity over market capitalization (size). Book common equity is calculated using Compustat’s book value of stockholders’ equity plus balance-sheet deferred taxes and investment tax credit minus the book value of preferred stock. The ratio is computed as the book common equity at the end of fiscal year over size as the December end of fiscal year end.\(^{11}\)

Market beta: We follow Sholes and Williams (1977) and Dimson (1979) to address nonsynchronous trading in beta estimation. We run regression including lag, current and lead market risk premium as independent variables as in Equation 16,

\[ R_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i} (R_{market,t-1} - r_{f,t-1}) + \beta_{2,i} (R_{market,t} - r_{f,t}) + \beta_{3,i} (R_{market,t+1} - r_{f,t+1}) + \epsilon_{i,t} \]  

(16)

where \( R_{i,t} \) is return for stock \( i \) on day \( t \). \( R_{market,t} \) is market return on day \( t \) and \( r_{f,t} \) is risk free rate on day \( t \). We estimation the above equation for each stock using daily returns within each month. For each month, the market beta is estimated as follows in Equation 17 for each stock \( i \),

\[ \bar{\beta}_i = \bar{\beta}_{1,i} + \bar{\beta}_{2,i} + \bar{\beta}_{3,i} \]  

(17)

\(^{11}\) To avoid issues with extreme values, the book-to-market ratios are winsorized at the 1% and 99% levels.

35
**Idiosyncratic Volatility**: Following Ang, Hodrick, Xing and Zhang (2006), idiosyncratic volatility is calculated as

\[ Idiosyncratic \ volatility = \sqrt{\text{var}(\varepsilon_{t,t})} \]            \hspace{1cm} (18)

where \( \varepsilon_{t,t} \) is the error term from the three-factor Fama and French (1993) regression. The regression is estimated monthly with more than 17 daily observations in a month.

**Lagged 1-month return**: Following Jegadeesh (1990) and Lehmann (1990), we use lagged 1-month return to account for short-term return reversal effect, the reversal variable for each stock in month \( m \) is defined as the return on the stock over the previous month, i.e., the return in month \( m - 1 \).

**Lagged 12-month return**: Jegadeesh and Titman (1993) documented intermediate-term momentum effect, we use lagged 12-month return to account for momentum effect, it is defined as return \( m - 12 \) for each stock in month \( m \).

**Illiquidity**: Following Amihud (2002), we compute stock illiquidity for each stock \( i \) in each month \( m \) as the ratio of the absolute monthly stock return to its dollar trading volume:

\[ \text{Illiquidity}_{i,m} = \frac{|R_{i,m}|}{|\text{Volume}_{m} \times \text{Price}_{m}|} \]            \hspace{1cm} (19)

**Maximum (Minimum) monthly return**: Following Bali, Cakici and Whitelaw (2011), we control for maximum (minimum) monthly return for each stock \( i \) in month \( m \) as the maximum (minimum) daily return within month \( m \).

\[ \text{Maximum monthly return}_{m} = \max\{R_{i,t}\}, t = 1, \ldots, T \]            \hspace{1cm} (20)

\[ \text{Minimum monthly return}_{m} = \min\{R_{i,t}\}, t = 1, \ldots, T \]            \hspace{1cm} (21)
where $T$ is the maximum number of daily observations in month $m$. These are estimated monthly with more than 17 daily observations in a month.

**Log (trading volume):** trading volume is the sum of the trading volumes during that month. We control for size effect by taking natural logarithm of Size.

**Price:** the price on the last trading date of the month.
Table 1. Characteristics of Portfolios Sorted by Tail Risk Premium
Each week, stocks in CRSP database are ranked by their respective tail risk premium. The equal-weighted characteristics of each quintile are computed over the same week. The procedure is repeated for every month from January 1990 to September 2014. Tail risk premium and illiquidity are in $10^6$. Variance risk premium and polynomial variation risk premium are in basis points. Lagged 1-month return, Lagged 12-month return, Maximum monthly return, Minimum monthly return are in percentages. Log (Size), Log (B/M), Market beta, Idiosyncratic Volatility, Log (trading volume), Price are in absolute values. See Appendix for variable definitions.

<table>
<thead>
<tr>
<th>Characteristics of Portfolios Sorted by Tail Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quintiles</td>
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<tr>
<td>Tail risk premium</td>
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<tr>
<td>Log (Size)</td>
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<td>Log (B/M)</td>
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<tr>
<td>Market beta</td>
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<tr>
<td>Idiosyncratic volatility</td>
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<tr>
<td>Lagged 1-month return</td>
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<td>Lagged 12-month return</td>
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<tr>
<td>Illiquidity</td>
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<tr>
<td>Maximum monthly return</td>
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<tr>
<td>Price</td>
</tr>
<tr>
<td>Number of stocks</td>
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Table 2. Pearson Correlation Matrix
Pearson correlation coefficients reported as percent for firm characteristics of all CRSP stocks from January 1990 to September 2014. See Appendix for variable definitions.

<table>
<thead>
<tr>
<th></th>
<th>Tail risk premium</th>
<th>Log (Size)</th>
<th>Log (B/M)</th>
<th>Market beta</th>
<th>Idiosyncratic volatility</th>
<th>Lagged 1-month return</th>
<th>Lagged 12-month return</th>
<th>Illiquidity</th>
<th>Maximum monthly return</th>
<th>Minimum monthly return</th>
<th>Log (trading volume)</th>
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</thead>
<tbody>
<tr>
<td>Tail risk premium</td>
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<td>-0.09</td>
<td>-0.33</td>
<td>0.28</td>
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<td>Maximum monthly return</td>
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Table 3. Portfolios Returns Sorted into Deciles by Tail Risk Premium
We form value-weighted decile portfolios every month by sorting stocks based on tail risk premium in Equation 10. Portfolios are formed every month, based on tail risk premium in Equation 10 computed using daily data over the previous month. Panel A displays the 1/0/1 portfolio strategy (sort in one month, examine the one-month return for the following month), Panel B for 1/1/1 portfolio strategy (sort in one month, examine the one-month return starting two month from now) and Panel C for 1/2/1 portfolio strategy (sort in one month, examine the one-month return starting three month from now). Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) previous month tail risk premium. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to the total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "10-1" refers to the difference in monthly returns between portfolio 10 and portfolio 1. NW t-stat refers to robust Newey-West (1987) t-stat. Pre-formulation TRP is reported in basis points. The sample period is January 1990 to September 2014.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>% Mkt Share</th>
<th>Size</th>
<th>B/M</th>
<th>Pre-Formation TRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.02</td>
<td>7.19</td>
<td>1.76%</td>
<td>4.03</td>
<td>0.94</td>
<td>-1.023</td>
</tr>
<tr>
<td>2</td>
<td>1.70</td>
<td>6.25</td>
<td>4.45%</td>
<td>4.92</td>
<td>0.79</td>
<td>-0.042</td>
</tr>
<tr>
<td>3</td>
<td>1.61</td>
<td>5.31</td>
<td>8.72%</td>
<td>5.53</td>
<td>0.73</td>
<td>-0.013</td>
</tr>
<tr>
<td>4</td>
<td>1.49</td>
<td>4.52</td>
<td>14.48%</td>
<td>6.03</td>
<td>0.70</td>
<td>-0.004</td>
</tr>
<tr>
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<td>1.43</td>
<td>3.98</td>
<td>19.92%</td>
<td>6.40</td>
<td>0.70</td>
<td>-0.001</td>
</tr>
<tr>
<td>6</td>
<td>1.31</td>
<td>4.10</td>
<td>19.73%</td>
<td>6.39</td>
<td>0.70</td>
<td>0.002</td>
</tr>
<tr>
<td>7</td>
<td>1.22</td>
<td>4.71</td>
<td>15.20%</td>
<td>6.07</td>
<td>0.70</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>1.09</td>
<td>5.48</td>
<td>9.44%</td>
<td>5.56</td>
<td>0.73</td>
<td>0.014</td>
</tr>
<tr>
<td>9</td>
<td>1.02</td>
<td>6.47</td>
<td>4.40%</td>
<td>4.93</td>
<td>0.78</td>
<td>0.044</td>
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<tr>
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<td>0.27</td>
<td>7.39</td>
<td>1.88%</td>
<td>4.03</td>
<td>0.93</td>
<td>1.063</td>
</tr>
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<td>10-1</td>
<td>-1.75</td>
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<td></td>
</tr>
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</table>

\( t \)-stat \((-11.26)\)

NW \( t \)-stat \((-7.30)\)
### Panel B: Portfolios Sorted by Tail Risk Premium, 1/1/1 Portfolio Strategy

<table>
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<tr>
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<th>Mean</th>
<th>Std. Dev.</th>
<th>%Mkt Share</th>
<th>Size</th>
<th>B/M</th>
<th>Pre-Formation TRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.76</td>
<td>7.51</td>
<td>1.76%</td>
<td>4.05</td>
<td>0.94</td>
<td>-0.982</td>
</tr>
<tr>
<td>2</td>
<td>0.97</td>
<td>6.36</td>
<td>4.45%</td>
<td>4.93</td>
<td>0.79</td>
<td>-0.042</td>
</tr>
<tr>
<td>3</td>
<td>1.11</td>
<td>5.54</td>
<td>8.75%</td>
<td>5.53</td>
<td>0.73</td>
<td>-0.013</td>
</tr>
<tr>
<td>4</td>
<td>1.12</td>
<td>4.62</td>
<td>14.46%</td>
<td>6.04</td>
<td>0.70</td>
<td>-0.004</td>
</tr>
<tr>
<td>5</td>
<td>1.06</td>
<td>4.13</td>
<td>19.97%</td>
<td>6.40</td>
<td>0.70</td>
<td>-0.001</td>
</tr>
<tr>
<td>6</td>
<td>1.06</td>
<td>4.19</td>
<td>19.75%</td>
<td>6.40</td>
<td>0.70</td>
<td>0.002</td>
</tr>
<tr>
<td>7</td>
<td>1.06</td>
<td>4.66</td>
<td>15.22%</td>
<td>6.07</td>
<td>0.70</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>1.06</td>
<td>5.48</td>
<td>9.41%</td>
<td>5.56</td>
<td>0.72</td>
<td>0.014</td>
</tr>
<tr>
<td>9</td>
<td>0.97</td>
<td>6.52</td>
<td>4.37%</td>
<td>4.93</td>
<td>0.78</td>
<td>0.044</td>
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<tr>
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<td>1.85%</td>
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<td>0.997</td>
</tr>
</tbody>
</table>

10-1  | 0.08 |           |             |      |     |                  |

\( t\)-stat (0.73)

NW \( t\)-stat (0.76)
### Panel C: Portfolios Sorted by Tail Risk Premium, 1/2/1 Portfolio Strategy

<table>
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<tr>
<th>Rank</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>%Mkt Share</th>
<th>Size</th>
<th>B/M</th>
<th>Pre-Formation TRP</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.72%</td>
<td>4.06</td>
<td>0.95</td>
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<td>2</td>
<td>1.24</td>
<td>6.60</td>
<td>4.44%</td>
<td>4.93</td>
<td>0.79</td>
<td>-0.042</td>
</tr>
<tr>
<td>3</td>
<td>1.16</td>
<td>5.60</td>
<td>8.78%</td>
<td>5.53</td>
<td>0.73</td>
<td>-0.013</td>
</tr>
<tr>
<td>4</td>
<td>1.24</td>
<td>4.70</td>
<td>14.48%</td>
<td>6.04</td>
<td>0.70</td>
<td>-0.004</td>
</tr>
<tr>
<td>5</td>
<td>1.16</td>
<td>4.18</td>
<td>19.96%</td>
<td>6.41</td>
<td>0.70</td>
<td>-0.001</td>
</tr>
<tr>
<td>6</td>
<td>1.18</td>
<td>4.11</td>
<td>19.79%</td>
<td>6.40</td>
<td>0.70</td>
<td>0.002</td>
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<tr>
<td>7</td>
<td>1.18</td>
<td>4.67</td>
<td>15.21%</td>
<td>6.08</td>
<td>0.70</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>1.21</td>
<td>5.38</td>
<td>9.43%</td>
<td>5.57</td>
<td>0.73</td>
<td>0.014</td>
</tr>
<tr>
<td>9</td>
<td>1.14</td>
<td>6.41</td>
<td>4.37%</td>
<td>4.94</td>
<td>0.78</td>
<td>0.044</td>
</tr>
<tr>
<td>10</td>
<td>0.99</td>
<td>7.33</td>
<td>1.82%</td>
<td>4.06</td>
<td>0.94</td>
<td>0.969</td>
</tr>
<tr>
<td>10-1</td>
<td>0.03</td>
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<td></td>
</tr>
</tbody>
</table>

$t$-stat (0.18)

NW $t$-stat (0.18)
Table 4. Fama-MacBeth Cross-Sectional Regression

Results of a Fama-MacBeth cross-sectional regression of stock returns for the following:

\[
R_{i,t+1} = \gamma_{0,t+1} + \gamma_{1,t+1} \times TRP_{i,t}^{Monthly} + \phi_{t+1} \times Z_{i,t} + \epsilon_{i,t+1}
\]  

(11)

\(R_{i,t+1}\) is monthly stock return for stock \(i\) in month \(t + 1\). \(TRP_{i,t}^{Monthly}\) is individual stock tail risk premium for stock \(i\) in month \(t\), calculated in Equation 10. \(Z_{i,t}\) represents a vector of characteristics and controls for firm \(i\) at the end of month \(t\) such as size, B/M ratio, market beta, illiquidity, etc. The control variables are described in the Appendix. The Newey-West (1973) HAC robust \(t\)-statistic is reported in parentheses. Specification (1) is univariate regression, Specification (2) is multiple regression adding control variables. The sample period is from January 1990 to September 2014.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.009</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(2.42)</td>
<td>(6.16)</td>
</tr>
<tr>
<td>Tail risk premium</td>
<td>-0.801</td>
<td>-1.155</td>
</tr>
<tr>
<td></td>
<td>(-5.60)</td>
<td>(-5.50)</td>
</tr>
<tr>
<td>Log (Size)</td>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.19)</td>
</tr>
<tr>
<td>Log (B/M)</td>
<td></td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.52)</td>
</tr>
<tr>
<td>Market beta</td>
<td></td>
<td>0.000</td>
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<tr>
<td></td>
<td></td>
<td>(0.74)</td>
</tr>
<tr>
<td>Illiquidity</td>
<td>19.715</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.11)</td>
<td></td>
</tr>
<tr>
<td>Idiosyncratic volatility</td>
<td>-0.001</td>
<td>(-1.90)</td>
</tr>
<tr>
<td>Lagged 1-month return</td>
<td>0.003</td>
<td>(0.85)</td>
</tr>
<tr>
<td>Lagged 12-month return</td>
<td>-0.001</td>
<td>(-0.40)</td>
</tr>
<tr>
<td>Maximum monthly return</td>
<td>-0.032</td>
<td>(-2.02)</td>
</tr>
<tr>
<td>Minimum monthly return</td>
<td>-0.019</td>
<td>(-1.06)</td>
</tr>
<tr>
<td>Log (trading volume)</td>
<td>-0.001</td>
<td>(-0.80)</td>
</tr>
</tbody>
</table>

Adjusted \(R^2\) (%) | 0.133 | 4.765 |
Table 5. Fama-MacBeth Regression including dummy variables for Decile 1 and Decile 10 Tail Risk Premia

Results of a Fama-MacBeth cross-sectional regression of stock returns for the following:

\[ R_{i,t+n} = \gamma_{0,t+n} + \gamma_{1,t+n} \times TRP_{i,t}^{Monthly} \times I_{[Decile\ 1\ TRP_{i,t}^{Monthly}]} + \gamma_{2,t+n} \times TRP_{i,t}^{Monthly} \times I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]} + \phi_{t+n} \times Z_{i,t} + \varepsilon_{i,t+n} \]  (12)

Where \( I_{[Decile\ 1\ TRP_{i,t}^{Monthly}]} \) is a dummy variable that equals 1 if \( TRP_{i,t}^{Monthly} \) is in decile 1 and equals 0 otherwise, and \( I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]} \) is the corresponding dummy variable for decile 10. \( R_{i,t+n} \) is monthly stock return for stock \( i \) in month \( t + n \), where \( N = 1,2 \). \( TRP_{i,t}^{Monthly} \) is individual stock tail risk premium for stock \( i \) in month \( t \), calculated in Equation 10. \( Z_{i,t} \) represents a vector of characteristics and controls for firm \( i \) at the end of month \( t \) such as size, B/M ratio, market beta, illiquidity, etc. The control variables are described in the Appendix. The Newey-West (1973) HAC robust \( t \)-statistic is reported in parentheses. Specification (1) is the multiple regression adding control variables for monthly return \( R_{i,t+1} \), specification (2) is the multiple regression adding control variables for monthly return \( R_{i,t+2} \). The sample period is from January 1990 to September 2014.
<table>
<thead>
<tr>
<th></th>
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<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Return</strong></td>
<td>t+1</td>
<td>t+2</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.020</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(5.92)</td>
<td>(6.51)</td>
</tr>
<tr>
<td>( TRP_{lt}^{Monthly} \times I_{\text{Decile 1}} )</td>
<td>-1.971</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(-5.09)</td>
<td>(-0.05)</td>
</tr>
<tr>
<td>( TRP_{lt}^{Monthly} \times I_{\text{Decile 10}} )</td>
<td>-0.803</td>
<td>-0.668</td>
</tr>
<tr>
<td></td>
<td>(-1.86)</td>
<td>(-1.39)</td>
</tr>
<tr>
<td>Log (Size)</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.63)</td>
</tr>
<tr>
<td>Log (B/M)</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(3.46)</td>
<td>(3.19)</td>
</tr>
<tr>
<td>Market beta</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.55)</td>
<td>(0.23)</td>
</tr>
<tr>
<td>Illiquidity</td>
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<td>14.994</td>
</tr>
<tr>
<td></td>
<td>(1.73)</td>
<td>(1.33)</td>
</tr>
<tr>
<td>Idiosyncratic volatility</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(-2.73)</td>
<td>(-2.73)</td>
</tr>
<tr>
<td>Lagged 1-month return</td>
<td>0.003</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>(0.91)</td>
<td>(4.47)</td>
</tr>
<tr>
<td>Lagged 12-month return</td>
<td>-0.002</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(-0.97)</td>
<td>(-1.72)</td>
</tr>
<tr>
<td>Maximum monthly return</td>
<td>-0.026</td>
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<tr>
<td></td>
<td>(-1.98)</td>
<td>(0.40)</td>
</tr>
<tr>
<td>Minimum monthly return</td>
<td>-0.025</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(-1.62)</td>
<td>(1.58)</td>
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<tr>
<td>Log (trading volume)</td>
<td>-0.000</td>
<td>-0.001</td>
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<tr>
<td></td>
<td>(-0.52)</td>
<td>(-1.41)</td>
</tr>
<tr>
<td><strong>Adjusted R^2 (%)</strong></td>
<td>4.063</td>
<td>3.947</td>
</tr>
</tbody>
</table>
Table 6. Fama-MacBeth Regression of Positive and Negative Tails using Daily Returns

Results of daily Fama-MacBeth cross-sectional regression of stock returns for the following:

\[ R_{i,t+n}^{Daily} = \gamma_{0,t+n} + \gamma_{1,t+n} \times TRP_{i,t}^{Daily} \times I_{[Decile \ 1 \ TRP_{i,t}^{Daily}]} + \gamma_{2,t+n} \times TRP_{i,t}^{Daily} \times I_{[Decile \ 10 \ TRP_{i,t}^{Daily}]} + \phi_{t+n} \times Z_{i,t} + \varepsilon_{i,t+n} \]  

(13)

Where \( I_{[Decile \ 1 \ TRP_{i,t}^{Daily}]} \) is a dummy variable that equals 1 if \( TRP_{i,t}^{Daily} \) is in decile 1 and equals 0 otherwise, and \( I_{[Decile \ 10 \ TRP_{i,t}^{Daily}]} \) is the corresponding dummy variable for decile 10. \( R_{i,t+n}^{Daily} \) is one-day holding period return for stock \( i \) from day \( t + n - 1 \) to day \( t + n \), where \( n = 1, 2 \ldots, 18 \). A maximum of 18 days is used since we require stocks to have a minimum of 18 days to be included in the sample. \( TRP_{i,t}^{Daily} \) is individual stock tail risk premium for stock \( i \) in month \( t \), calculated in Equation 10. \( Z_{i,t} \) represents a vector of characteristics and controls for firm \( i \) at the end of month \( t \) such as size, B/M ratio, market beta, illiquidity, etc. The control variables are described in the Appendix. The table reports predictive regression using next-month daily return (day 1, 7, 10, 14, 18) as dependent variable. The Newey-West (1973) HAC robust \( t \)-statistic is reported in parentheses. Decile portfolio sort section, similar to table 3, reports decile 10 return minus decile 1 return difference in basis points, as well as \( t \)-statistic and Newey-West (1973) HAC robust \( t \)-statistic associated with it (see table 3 for detailed testing methodology). For ease of reading, significant coefficients are highlighted in bold.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>Day 1</td>
<td>Day 7</td>
<td>Day 10</td>
<td>Day 14</td>
<td>Day 18</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.004</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(-4.75)</td>
<td>(0.96)</td>
<td>(0.25)</td>
<td>(2.23)</td>
<td>(1.25)</td>
</tr>
<tr>
<td>( TRP_{t,t}^{Monthly} \times I_{\text{Decile 10} TRP_{t,t}^{Monthly}} )</td>
<td>-20.741</td>
<td>-5.553</td>
<td>-5.128</td>
<td>1.091</td>
<td>4.642</td>
</tr>
<tr>
<td></td>
<td>(-5.17)</td>
<td>(-2.30)</td>
<td>(-2.43)</td>
<td>(0.33)</td>
<td>(1.59)</td>
</tr>
<tr>
<td>( TRP_{t,t}^{Monthly} \times I_{\text{Decile 1 TRP}_{t,t}^{Monthly}} )</td>
<td>-30.483</td>
<td>2.082</td>
<td>-2.008</td>
<td>-3.090</td>
<td>-1.127</td>
</tr>
<tr>
<td></td>
<td>(-6.78)</td>
<td>(0.77)</td>
<td>(-0.51)</td>
<td>(-1.20)</td>
<td>(-0.47)</td>
</tr>
<tr>
<td>Log (Size)</td>
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<td>-0.000</td>
<td>0.000</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(1.98)</td>
<td>(-1.19)</td>
<td>(1.38)</td>
<td>(-1.98)</td>
<td>(-1.12)</td>
</tr>
<tr>
<td>Log (B/M)</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(4.28)</td>
<td>(1.54)</td>
<td>(2.09)</td>
<td>(2.60)</td>
<td>(1.05)</td>
</tr>
<tr>
<td>Market beta</td>
<td>0.000</td>
<td>-0.000</td>
<td>-0.000</td>
<td>-0.000</td>
<td>0.000</td>
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<tr>
<td></td>
<td>(0.50)</td>
<td>(-0.64)</td>
<td>(-0.38)</td>
<td>(-2.18)</td>
<td>(0.94)</td>
</tr>
<tr>
<td>Illiquidity</td>
<td>11.861</td>
<td>5.409</td>
<td>12.591</td>
<td>4.639</td>
<td>-1.809</td>
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<tr>
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<td>(2.87)</td>
<td>(1.73)</td>
<td>(2.67)</td>
<td>(1.75)</td>
<td>(-0.57)</td>
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<tr>
<td>Idiosyncratic volatility</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(1.15)</td>
<td>(2.01)</td>
<td>(1.81)</td>
<td>(1.95)</td>
<td>(1.81)</td>
</tr>
<tr>
<td>Lagged 1-month return</td>
<td>-0.001</td>
<td>-0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(-1.28)</td>
<td>(-0.17)</td>
<td>(0.68)</td>
<td>(1.98)</td>
<td>(0.61)</td>
</tr>
<tr>
<td>Lagged 12-month return</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(-1.26)</td>
<td>(-1.68)</td>
<td>(-1.19)</td>
<td>(0.05)</td>
<td>(-0.12)</td>
</tr>
<tr>
<td>Maximum monthly return</td>
<td>-0.011</td>
<td>-0.004</td>
<td>-0.007</td>
<td>-0.009</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(-1.71)</td>
<td>(-0.81)</td>
<td>(-1.83)</td>
<td>(-2.20)</td>
<td>(-0.62)</td>
</tr>
<tr>
<td>Minimum monthly return</td>
<td>-0.011</td>
<td>0.002</td>
<td>0.000</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(-1.66)</td>
<td>(0.38)</td>
<td>(0.03)</td>
<td>(0.37)</td>
<td>(0.60)</td>
</tr>
<tr>
<td>Log (trading volume)</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.000</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(1.43)</td>
<td>(0.05)</td>
<td>(-0.87)</td>
<td>(-0.45)</td>
<td>(-0.16)</td>
</tr>
<tr>
<td>Adjusted R² (%)</td>
<td>5.074</td>
<td>2.587</td>
<td>2.527</td>
<td>2.313</td>
<td>2.013</td>
</tr>
</tbody>
</table>
Table 7. Extreme Tail: Percentile Portfolio Returns Sorted by Tail Risk Premium
We form value-weighted percentile portfolios every month by sorting stocks based on tail risk premium in Equation 10. Portfolios are formed every month, based on tail risk premium in Equation 10 computed using daily data over the previous month. The table displays the 1/0/1 portfolio strategy (sort in one month, examine the one-month return for the following month). Portfolio 1 (100) is the portfolio of stocks with the lowest (highest) previous month tail risk premium. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to the total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "100-1" refers to the difference in monthly returns between portfolio 100 and portfolio 1, the row "99-2" refers to the difference in monthly returns between portfolio 99 and portfolio 2, and the row "98-3" refers to the difference in monthly returns between portfolio 98 and portfolio 3. The row “98-100 minus 1-3” stands for the difference between mean monthly returns of portfolio 98 through 100 and mean monthly returns of portfolio 1 through 3. The row “97-99 minus 2-4” represents for the difference between mean monthly returns of portfolio 97 through 99 and mean monthly returns of portfolio 2 through 4. NW t-stat refers to robust Newey-West (1987) t-stat. Pre-formulation TRP is reported in basis points. The sample period is January 1990 to September 2014.
Percentile Portfolio Returns Sorted by Tail Risk Premium

<table>
<thead>
<tr>
<th>Rank</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>%Mkt Share</th>
<th>Size</th>
<th>B/M</th>
<th>Pre-Formation TRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.74</td>
<td>9.19</td>
<td>6.18%</td>
<td>3.28</td>
<td>1.10</td>
<td>-8.330</td>
</tr>
<tr>
<td>2</td>
<td>2.20</td>
<td>8.54</td>
<td>6.68%</td>
<td>5.53</td>
<td>1.06</td>
<td>-1.072</td>
</tr>
<tr>
<td>3</td>
<td>2.14</td>
<td>8.21</td>
<td>6.70%</td>
<td>3.74</td>
<td>1.01</td>
<td>-0.570</td>
</tr>
<tr>
<td>4</td>
<td>2.32</td>
<td>7.97</td>
<td>8.25%</td>
<td>3.91</td>
<td>0.97</td>
<td>-0.368</td>
</tr>
<tr>
<td>5</td>
<td>1.83</td>
<td>7.81</td>
<td>8.63%</td>
<td>4.04</td>
<td>0.93</td>
<td>-0.264</td>
</tr>
<tr>
<td>96</td>
<td>0.44</td>
<td>8.39</td>
<td>8.23%</td>
<td>4.02</td>
<td>0.93</td>
<td>0.268</td>
</tr>
<tr>
<td>97</td>
<td>0.20</td>
<td>7.91</td>
<td>8.03%</td>
<td>3.89</td>
<td>0.96</td>
<td>0.373</td>
</tr>
<tr>
<td>98</td>
<td>-0.30</td>
<td>7.84</td>
<td>7.05%</td>
<td>3.74</td>
<td>1.00</td>
<td>0.572</td>
</tr>
<tr>
<td>99</td>
<td>-0.30</td>
<td>8.56</td>
<td>6.58%</td>
<td>3.54</td>
<td>1.04</td>
<td>1.074</td>
</tr>
<tr>
<td>100</td>
<td>-1.17</td>
<td>8.81</td>
<td>6.29%</td>
<td>3.26</td>
<td>1.12</td>
<td>8.504</td>
</tr>
</tbody>
</table>

| 100-1 | -3.91 | (-8.34) |
| NW t-stat | (-7.21) |
| 99-2  | -2.50 | (-7.14) |
| NW t-stat | (-7.24) |
| 98-3  | -2.44 | (-7.01) |
| NW t-stat | (-6.35) |
| 98-100 minus 1-3 | -2.95 | (-13.02) |
| NW t-stat | (-9.14) |
| 97-99 minus 2-4 | -2.35 | (-12.11) |
| NW t-stat | (-9.58) |
Table 8. Portfolios Returns Sorted into Deciles by Sensitivity to Market Aggregate Tail Risk Premium

We form value-weighted decile portfolios every month by sorting stocks based on sensitivity to market tail risk premium, $\beta_{MKT}^t$, in Equation 14.

\[ r^t_i = \beta_0 + \beta_{MKT}^t MKT_t + \beta_{\Delta TRP}^t Market \Delta TRP_{Market}^t + \epsilon^t_i \] (14)

Portfolios are formed every month, based on sensitivity to market tail risk premium, $\beta_{MKT}^t$, in Equation 14 computed using daily data over the previous month. The table displays the 1/0/1 portfolio strategy (sort in one month, examine the one-month return for the following month), Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) previous month tail risk premium. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to the total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "10-1" refers to the difference in monthly returns between portfolio 10 and portfolio 1. NW t-stat refers to robust Newey-West (1987) t-stat. Pre-formulation TRP is reported in basis points. The sample period is January 1990 to September 2014.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>%Mkt Share</th>
<th>Size</th>
<th>B/M</th>
<th>Pre-Formation TRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.16</td>
<td>7.05</td>
<td>2.31%</td>
<td>4.20</td>
<td>0.90</td>
<td>-24.86</td>
</tr>
<tr>
<td>2</td>
<td>1.34</td>
<td>5.71</td>
<td>6.73%</td>
<td>5.17</td>
<td>0.76</td>
<td>-10.00</td>
</tr>
<tr>
<td>3</td>
<td>1.26</td>
<td>5.01</td>
<td>11.29%</td>
<td>5.65</td>
<td>0.73</td>
<td>-5.73</td>
</tr>
<tr>
<td>4</td>
<td>1.35</td>
<td>4.66</td>
<td>14.23%</td>
<td>5.92</td>
<td>0.72</td>
<td>-3.08</td>
</tr>
<tr>
<td>5</td>
<td>1.31</td>
<td>4.42</td>
<td>14.98%</td>
<td>6.03</td>
<td>0.72</td>
<td>-1.03</td>
</tr>
<tr>
<td>6</td>
<td>1.33</td>
<td>4.37</td>
<td>15.28%</td>
<td>6.00</td>
<td>0.73</td>
<td>0.83</td>
</tr>
<tr>
<td>7</td>
<td>1.33</td>
<td>4.58</td>
<td>14.54%</td>
<td>5.95</td>
<td>0.72</td>
<td>2.86</td>
</tr>
<tr>
<td>8</td>
<td>1.38</td>
<td>5.06</td>
<td>11.41%</td>
<td>5.68</td>
<td>0.74</td>
<td>5.47</td>
</tr>
<tr>
<td>9</td>
<td>1.45</td>
<td>5.83</td>
<td>6.89%</td>
<td>5.20</td>
<td>0.77</td>
<td>9.71</td>
</tr>
<tr>
<td>10</td>
<td>1.31</td>
<td>6.84</td>
<td>2.34%</td>
<td>4.20</td>
<td>0.92</td>
<td>24.64</td>
</tr>
<tr>
<td>10-1</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$-stat</td>
<td>(1.08)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>NW $t$-stat</td>
<td>(0.95)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 9. Portfolios Returns Sorted into Deciles by Contemporaneous Tail Risk Premium
We form value-weighted decile portfolios every month by sorting stocks by tail risk premium estimated based on Equation 15. Portfolios are formed every month, based on tail risk premium in Equation 15 computed using daily data over the previous month. Panel A displays the 1/0/1 portfolio strategy (sort in one month, examine the one-month return for the following month), Panel B for 1/1/1 portfolio strategy strategy (sort in one month, examine the one-month return starting two month from now) and Panel C for 1/2/1 portfolio strategy (sort in one month, examine the one-month return starting three month from now). Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) previous month tail risk premium. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to the total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "10-1" refers to the difference in monthly returns between portfolio 10 and portfolio 1. NW t-stat refers to robust Newey-West (1987) t-stat. Pre-formation TRP is reported in basis points. The sample period is January 1990 to September 2014.

Panel A: Portfolios Sorted by Tail Risk Premium, 1/0/1 Portfolio Strategy

<table>
<thead>
<tr>
<th>Rank</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>%Mkt Share</th>
<th>Size</th>
<th>B/M</th>
<th>Pre-Formation TRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.00</td>
<td>7.23</td>
<td>1.83%</td>
<td>4.03</td>
<td>0.94</td>
<td>-1.006</td>
</tr>
<tr>
<td>2</td>
<td>1.69</td>
<td>6.31</td>
<td>4.46%</td>
<td>4.92</td>
<td>0.79</td>
<td>-0.043</td>
</tr>
<tr>
<td>3</td>
<td>1.59</td>
<td>5.23</td>
<td>8.89%</td>
<td>5.54</td>
<td>0.73</td>
<td>-0.013</td>
</tr>
<tr>
<td>4</td>
<td>1.52</td>
<td>4.38</td>
<td>14.92%</td>
<td>6.05</td>
<td>0.70</td>
<td>-0.005</td>
</tr>
<tr>
<td>5</td>
<td>1.40</td>
<td>4.02</td>
<td>19.56%</td>
<td>6.39</td>
<td>0.70</td>
<td>-0.001</td>
</tr>
<tr>
<td>6</td>
<td>1.30</td>
<td>4.14</td>
<td>19.84%</td>
<td>6.39</td>
<td>0.70</td>
<td>0.001</td>
</tr>
<tr>
<td>7</td>
<td>1.26</td>
<td>4.75</td>
<td>15.16%</td>
<td>6.07</td>
<td>0.70</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>1.19</td>
<td>5.44</td>
<td>9.05%</td>
<td>5.55</td>
<td>0.72</td>
<td>0.014</td>
</tr>
<tr>
<td>9</td>
<td>0.91</td>
<td>6.46</td>
<td>4.41%</td>
<td>4.92</td>
<td>0.78</td>
<td>0.045</td>
</tr>
<tr>
<td>10</td>
<td>0.34</td>
<td>7.45</td>
<td>1.89%</td>
<td>4.03</td>
<td>0.94</td>
<td>1.108</td>
</tr>
<tr>
<td>10-1</td>
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<td></td>
<td></td>
<td></td>
<td>(-9.92)</td>
</tr>
<tr>
<td>t-stat</td>
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<td></td>
<td></td>
<td></td>
<td>(-6.96)</td>
</tr>
</tbody>
</table>

NW t-stat
### Panel B: Portfolios Sorted by Tail Risk Premium, 1/1/1 Portfolio Strategy

<table>
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<th>Rank</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>%Mkt Share</th>
<th>Size</th>
<th>B/M</th>
<th>Pre-Formation TRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.72</td>
<td>7.60</td>
<td>1.82%</td>
<td>4.04</td>
<td>0.94</td>
<td>-0.973</td>
</tr>
<tr>
<td>2</td>
<td>0.89</td>
<td>6.48</td>
<td>4.48%</td>
<td>4.93</td>
<td>0.79</td>
<td>-0.043</td>
</tr>
<tr>
<td>3</td>
<td>1.13</td>
<td>5.54</td>
<td>8.91%</td>
<td>5.55</td>
<td>0.73</td>
<td>-0.013</td>
</tr>
<tr>
<td>4</td>
<td>1.08</td>
<td>4.72</td>
<td>14.93%</td>
<td>6.06</td>
<td>0.70</td>
<td>-0.004</td>
</tr>
<tr>
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<td>19.62%</td>
<td>6.39</td>
<td>0.70</td>
<td>-0.001</td>
</tr>
<tr>
<td>6</td>
<td>1.08</td>
<td>4.12</td>
<td>19.85%</td>
<td>6.39</td>
<td>0.70</td>
<td>0.001</td>
</tr>
<tr>
<td>7</td>
<td>1.14</td>
<td>4.55</td>
<td>15.14%</td>
<td>6.07</td>
<td>0.70</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>1.12</td>
<td>5.45</td>
<td>9.01%</td>
<td>5.55</td>
<td>0.72</td>
<td>0.014</td>
</tr>
<tr>
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<td>0.99</td>
<td>6.45</td>
<td>4.36%</td>
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<td>0.045</td>
</tr>
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<td>4.05</td>
<td>0.94</td>
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<td></td>
<td></td>
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<td>NW t-stat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.73)</td>
</tr>
</tbody>
</table>
Panel C: Portfolios Sorted by Tail Risk Premium, 1/2/1 Portfolio Strategy

<table>
<thead>
<tr>
<th>Rank</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>%Mkt Share</th>
<th>Size</th>
<th>B/M</th>
<th>Pre-Formation TRP</th>
</tr>
</thead>
<tbody>
<tr>
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<td>7.55</td>
<td>1.78%</td>
<td>4.06</td>
<td>0.94</td>
<td>-0.922</td>
</tr>
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<td>2</td>
<td>1.27</td>
<td>6.70</td>
<td>4.46%</td>
<td>4.93</td>
<td>0.79</td>
<td>-0.043</td>
</tr>
<tr>
<td>3</td>
<td>1.19</td>
<td>5.62</td>
<td>8.91%</td>
<td>5.55</td>
<td>0.73</td>
<td>-0.013</td>
</tr>
<tr>
<td>4</td>
<td>1.23</td>
<td>4.78</td>
<td>14.96%</td>
<td>6.06</td>
<td>0.70</td>
<td>-0.005</td>
</tr>
<tr>
<td>5</td>
<td>1.21</td>
<td>4.18</td>
<td>19.64%</td>
<td>6.40</td>
<td>0.70</td>
<td>-0.001</td>
</tr>
<tr>
<td>6</td>
<td>1.20</td>
<td>4.07</td>
<td>19.85%</td>
<td>6.40</td>
<td>0.70</td>
<td>0.001</td>
</tr>
<tr>
<td>7</td>
<td>1.18</td>
<td>4.62</td>
<td>15.15%</td>
<td>6.08</td>
<td>0.70</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>1.11</td>
<td>5.30</td>
<td>9.03%</td>
<td>5.56</td>
<td>0.72</td>
<td>0.014</td>
</tr>
<tr>
<td>9</td>
<td>1.11</td>
<td>6.37</td>
<td>4.37%</td>
<td>4.92</td>
<td>0.79</td>
<td>0.044</td>
</tr>
<tr>
<td>10</td>
<td>0.99</td>
<td>7.31</td>
<td>1.86%</td>
<td>4.06</td>
<td>0.95</td>
<td>1.010</td>
</tr>
</tbody>
</table>

10-1  | -0.04|

\(t\)-stat (-0.33)
NW \(t\)-stat (-0.37)
Figure 1. Monte Carlo Analysis of Regression Beta

The Monte Carlo analysis is performed to test whether the baseline regression (7) beta equal to zero. Denote sample size as $S$, number of random draws as $N$. For a given month in a given year, we perform the following,

1) Random draw (with placement) $\beta_1, \beta_2, \beta_3, \ldots, \beta_S$ and compute the mean of $\beta_1, \beta_2, \beta_3, \ldots, \beta_S$, denote $\overline{\beta_n}$.
2) Repeat 1) $N$ times and get $\overline{\beta_1}, \overline{\beta_2}, \overline{\beta_3}, \ldots, \overline{\beta_N}$.
3) Compute $t$-statistic for $\overline{\beta_1}, \overline{\beta_2}, \overline{\beta_3}, \ldots, \overline{\beta_N}$.

We then compute the average of the (time series) year-month $t$-statistic to get the simulated $t$-statistic. The sample period is January 1990 to September 2014. The following graphs, panels A through C plots the Monte Carlo simulated regression beta values for different combinations sample size ($S=500, 1000$) and number of random draw values ($N=10000$).

Panel A: Sample Size ($S$) = 500, Number of Random Draw ($N$) = 10000. Average $t$-statistic=3.684.
Panel B: Sample Size ($S$) = 1000, Number of Random Draw ($N$) = 10000. Average $t$-statistic=5.152.