Optimal Taxation, Household Production and Intra-Household Exchange

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Abstract

This paper presents an analysis of the optimal taxation of couples based on a model of the household as a two-person economy engaged in intra-household production and exchange (Apps 1982). The aim is to gain further insight into the impact of joint and individual taxation systems by comparing the efficiency and distributional effects, particularly within the household, of differential rates on primary and second earnings.

1 Introduction

Discussion of the "gender wage gap" as an indicator of the degree of discrimination against women in the labour market is typically framed in terms of the difference in gross wage rates between women and men. This overlooks the point that ultimately what determines the realised levels of people's well being, as well as important dimensions of household behaviour such as consumption and time use, saving and fertility, are *after-tax* wage levels and therefore the system of taxation households face.[1] In the light of the fact that in recent decades many developed countries have achieved a significant reduction in the gender pay gap, due largely to legislative reforms in the labour market, the extent to which the tax system reinforces these gains or reverses them is an important issue. The starting point for this paper therefore is the belief that both normative and positive analyses of taxation polices have an important role to play in any discussion of gender and the labour market.

The central concern of optimal tax theory is the rigorous analysis of the effects of taxation on the (dis)incentives it creates to supplying labour to the

[1] Bick et al. (2018) estimate that on average over a group of eight major countries, including the U.S., Germany, the U.K. and France, labour income taxes are the key driving force in explaining the time series behaviour of hours worked per employed married woman over the period 1983 - 2016.

market - the *efficiency effects* - and its effects on the distribution of wellbeing across the individuals bearing the taxes - the *equity effects*. Given the fact that the majority of taxpayers live in couple-households,[2] the question of whether the base of an income tax system should be joint incomes, as in the USA and Germany, or individual incomes, as in most other countries, is a central issue.[3] The former system equalises the marginal tax rates of male and female partners in a couple. The latter, as long as it has a progressive rate scale, will generally result in second earners facing lower rates, to an extent determined by the progressivity of the tax system and the width of the tax brackets across which tax rates are defined. From the point of view of the effects of taxation on the well being of women, individual income-based taxation is likely to be preferred.

A seminal analysis of the optimal taxation of the household addressing this question is that of Boskin and Sheshinski (1983) who, using a "unitary model"[4] of the household find that individual taxation is superior for reasons of efficiency: it reduces disincentive effects on labour supply in line with the classic Ramsey principle of taxation. There is now an extensive empirical literature that finds both equity and efficiency effects.[5] An important contribution is that of Alesina et al. (2010), who examine equity effects under alternative tax systems with a focus on the endogenous sources of higher labour supply elasticities for women. Given their emphasis on gender equity, the authors base their analysis of within-household allocations on a Nash bargaining model of the household.[6] They find that a system with a lower tax on the earnings of the female partner achieves not only a higher degree of efficiency but also gains in both overall equity and, importantly, in gender equity.[7] More recently, Apps and Rees (2018) find that in an economy with a high degree of inequality, joint taxation is not only costly in terms of efficiency losses but also severely limits the redistributional capacity of a progressive rate scale. This finding is attributed to the opportunity joint taxation provides for tax avoidance by primary earners in the top percentiles of

[2] Throughout this paper when we refer to "households" we mean households formed by couples, rather than the single-person household of most of economic theory.

[3] While the US and Germany are recognised as countries with full income splitting, others, for example the UK and Australia, have partial income splitting or "quasi-joint taxation" due to the withdrawal of family payments on joint income. For a comparative analysis of the Australian, German, UK and US systems, see Apps and Rees (2009), Ch. 6, and for a recent analysis of the impact of the Australian system on the net-of-tax wage gap, see Apps (2017).

[4] This model, which has been much criticised in the family economics literature, in essence assumes that the household can be modelled as if it were a single individual with two types of labour supply or, equivalently, two types of non-market time allocation labelled "leisure'. The model has been very useful in optimal tax analysis and has the advantage of avoiding the counterfactual assumption, made in a number of models, that non-market time is used entirely for own consumption. For a relatively concise survey of the literature on household models up to the late 2000's see Apps and Rees (2009), chs 2-5.

[5] See, for example, LaLumia (2008) and Steiner and Wrohlich (2004, 2008).

[6] Further studies with a focus on the economics of the household include Basu (2006), Konrad and Lommerud (2000) and Gugl (2009).

[7] Alesina et al. termed this "gender-based taxation" (GBT), though if the focus is to be on role rather than gender, we might refer to the household as consisting of a primary and a second earner, where the latter has a higher labour supply elasticity because of the strong elasticity of substitution between domestic and market work.

the wage distribution, by having the secondary earner switch from market to home production. The paper highlights the role of joint taxation in widening the net-of-tax gender pay gap, thus counteracting the policy measures that have reduced the gross wage gap. The aim of the present paper is to investigate this issue in more depth, by replacing the unitary model used in Apps and Rees (2018) with a model that can explore the issue of the within-household distribution of utility.

A first step in the analysis is the choice of a model with which to explore household decision-taking behaviour. Since we wish to combine the analysis of the within-household distribution of utility with that of the overall distribution across all households, we draw on the model of the household as a small economy engaged in intra-household production and exchange in Apps (1982). The original formulation of this is a general equilibrium model in which market wage rates are endogenous. The gender wage gap is driven by the "crowding" of women into "female" occupations, which leads to a higher male wage and lower implicit price for the household good.[8] Here we simplify by assuming that the terms of this exchange are determined exogenously. A lower outside net of tax wage sets the within household terms of trade in the form of an implicit price for the household good. We do not model labour market discrimination explicitly. We assume that each individual in the household maximises utility subject to a budget constraint determined by her/his own income, equal to the value of their production of both market and household goods, or equivalently, the value of their time endowments at their own net of tax market wage rates.[9]

This contrasts with the standard approach in the household taxation literature in two essential respects. First, it emphasises the importance of the output of household production as a tradeable good, as opposed to own consumption of "leisure" time, when analysing the within-household utility distribution.[10] Secondly, it relaxes the assumption, implicit in the formulation of the household pooled budget constraint, that within-household lump sum transfers of any magnitude can be made.

The significance of these aspects of our model can be better appreciated if we contrast the approach that has been followed in some recent papers which analyse the within-household aspects of optimal taxation. For example, Cremer et al.(2016)[11] have argued that models that specifically take account of within household decision processes may overturn the conclusion that the tax rate on

[8] The analysis draws on the Bergmann (1971) crowding model of racial discrimination.

[9] See Grossbard-Shechtman (1984) for an alternative model of the couple household in which one partner works in the market and the other "works in the household", or WiHo, where a perfectly competitive market for WiHo establishes an equilibrium price, or wage for household work, by the usual process of supply and demand. For a more recent exposition see Grossbard (2015).

[10] While this emphasis on the importance of household production is of course not new (see Apps and Rees (2009) chs 2,6 for a literature survey), it does stand in contrast to recent optimal tax analyses of couple-households. See for example Brett (2007), Cremer et al. (2016) and Meier and Rainer (2015).

[11] Meier and Rainer (2015) come to a similar conclusion using a somewhat different analytical approach.

women should be lower than that on men. It is straightforward to challenge this result by pointing out the full implications of their assumptions.

Consider the line of argument presented in their paper. Their analysis is based on the "collective model" of the household, in which the household places weights, often referred to as "bargaining weights", on the utilities of the two earners, and maximises the weighted sum of their utilities[12] subject to a pooled income constraint. If the weight placed on the utility of the male partner is higher than that on the female,[13] then she will receive a worse allocation than he does. In the model proposed by Cremer et al., this results in the conclusion that women should optimally be taxed at higher marginal rates than men.

This perhaps surprising result follows from the assumption that the two goods household members consume are a market good and leisure time. In this framework, a woman's lower bargaining weight means that she receives less of both these goods, which then implies that since her time is spent either at leisure or doing market work, she does more of the latter than if she had a higher weight in the household. Because of their lower bargaining power, women are spending too much time at work and are not getting enough leisure.

Enter a social planner, who places a higher weight on women's utility than does the household. Because individual consumptions cannot be objectively measured or observed, the planner cannot ensure directly that a woman has more consumption[14] but can ensure, by taxing her market labour supply more heavily than that of her spouse, that the household reduces her labour supply and increases his, thus shifting the pattern of leisure consumption in her favour. This argument can lead to the conclusion that men's wage rates should be subsidised, though this result in only unambiguous when the issue of across-household equity is ignored.

There are two problems with this story. The first is the implicit assumption that the bargaining weights are exogenously given and fixed and are not affected by the tax system. In fact, in both the theoretical and empirical literature on the collective model, the weights depend on outside opportunities in general and wage rates in particular. We might expect that if the tax system discriminates against a woman's market labour supply and earning power, it is likely actually to reduce her ability to influence household decisions.[15] This then works against the effect identified in the paper.

Secondly, as the empirical data on time use show convincingly, a major proportion of the time use of women as second earners is taken up by household production, in particular by child care at a crucial phase in the household's life cycle. The data show that women do work more hours overall than men,

[12] In the language of Welfare Economics, the household utility function is a weighted utilitarian social welfare function.

[13] This literature does identify role with gender..

[14] The seemingly obvious device of making a lump sum transfer to the woman, as suggested by the studies of Lundberg et al. (1997) and Lundberg et al. (2016), is not considered in the paper, presumably because in this model framework this simply increases consumption of both partners with the woman gaining less than the man. This is again because in Cremer et al. the weights are exogenously fixed and all income is pooled.

[15] See for example Basu (2006), who analyses this idea in a game theoretic context.

especially when children are present in the household, but, because of the importance of household production, it does not follow that they are working more in the market than they would if they had more say in household decisions.

This paper is set out as follows. In the next section we present the theoretical model. Following that we derive the results for the optimal tax analysis. Then in section 4 we illustrate the analysis with a numerical example. Section 5 concludes.

2 The Household Exchange Model

Both individuals divide their working time between market labour supply and production of a household good that they both consume. On the production side, the household chooses its time allocation efficiently by solving the problem

$$\min_{L_{ih}} \sum_{i=1}^{2}(1-t_i)w_{ih}L_{ih} \ \text{ s.t. } z_h \geq f(k_{1h}L_{1h}, k_{2h}L_{2h}) \ \ h=1,2,...,H \tag{1}$$

where $h = 1, 2, ..., H$ denotes the household, L_{ih} are time inputs into household production, k_{ih} are exogenously given productivity parameters, and the production function $f(.)$ is linear homogenous and strictly quasiconcave. The tax system appears in the form of $a_i \geq 0$, a lump sum transfer, and t_i the marginal tax rate.

Each has the time constraint

$$l_{ih} = T - L_{ih} \tag{2}$$

where l_{ih} is market labour supply and T is total (working) time available. The solution to this problem yields household labour demand functions $L_{ih}((1-t_1)w_{1h}, (1-t_2)w_{2h}, z_h)$, labour supply functions $l_{ih}((1-t_1)w_{1h}, (1-t_2)w_{2h}, z_h)$, and a unit cost function $c((1-t_1)w_{1h}, (1-t_2)w_{2h})$ independent of the level of output. This defines an implicit price of the domestic good, denoted by $p_h = c(.)$. These prices of course also depend on the productivity parameters k_{ih}. Note also, by Shepard's Lemma:

$$\frac{\partial p_h}{\partial(1-t_i)w_{ih}} = L_{ih}^{(1)} \tag{3}$$

where $L_{ih}^{(1)}$ is the optimal input of L_{ih} required to produce one unit of z_h and $p_h = \sum_i (1-t_i)w_{ih}L_{ih}^{(1)}$.

The constant returns to scale assumption implies that there is a separation between production and consumption, which greatly simplifies the tax problem.

Turning to the consumption side, the utility functions are:

$$u_{ih} = u(x_{ih}, z_{ih}) \ \ i = 1,2 \tag{4}$$

where x_{ih} denotes consumption of the market good and z_{ih} the consumption of the household good.

A key departure the model makes from standard household models is to assume that the individuals maximise their utility subject to separate budget constraints, and do not pool their incomes into a single household budget constraint. In other words we relax the assumption that lump sum transfers can be made between the partners in the household. Instead, given the implicit price for the household good, p_h, the full income budget constraints of the individuals in household h are

$$x_{ih} + p_h z_{ih} \leq a_i + (1 - t_i) w_{ih} T \equiv y_{ih} \tag{5}$$

Maximising utility subject to this constraint yields demand functions $x_{ih}(y_{ih}, p_h)$, $z_{ih}(y_{ih}, p_h)$ and indirect utility functions $v_{ih}(y_{ih}, p_h)$. These are of course of great importance for the tax analysis. Letting μ_{ih} denote an individual's marginal utility of full income, we have the usual results:

$$\frac{\partial v_{ih}}{\partial y_{ih}} = \mu_{ih}; \ \frac{\partial v_{ih}}{\partial p_h} = -\mu_{ih} z_{ih}; \frac{\partial v_{jh}}{\partial y_{ih}} = 0 \quad i, j = 1, 2, i \neq j \tag{6}$$

However, since both y_{ih} and p_h depend on the tax rates, we will in the next section express these indirect utility functions as $v_{ih}(a_i, t_1, t_2)$ and focus on their properties.

Finally we have the fundamental equilibrium condition for this household economy:

$$\sum_{i=1}^{2} z_{ih}(y_{ih}, p_h) = z_h = f(k_{1h} L_{1h}, k_{2h} L_{2h}) \tag{7}$$

We call this a model of an exchange economy because each individual achieves their desired consumption pair by trading one good for the other at a rate of exchange given by the price p_h. For example if 2 has a lower market labour supply than 1 but produces more z, she can relax her constraint $x_{2h} \leq w_{2h} l_{2h}$ by exchanging z for x. In effect she "sells" z to 1 at the price p_h and uses the proceeds to "buy" more x on the goods market. This is simply a classic two-person economy with "exports" to the rest of the economy in the form of labour supplies, and internal trade in a "non-externally traded" good. This is seen most sharply of course in the "traditional household", in which $L_{2h} = l_{1h} = T$. An overly casual observer noting 2's positive consumption of the market good may interpret this as resulting from 1 making a lump sum transfer to 2, an error arising from ignoring the importance of household production.[16]

3 Optimal Tax Analysis

We consider two optimal tax problems. In the first, tax systems are individual in the sense that, consistently with the notation we have used so far, individual

[16] For example in the version of the collective model in Chiappori (1988), since it omits household production altogether, we must interpret any excess of 2's consumption over her labour earnings as a lump sum transfer from 1. This contrasts with the version of the collective model in Apps and Rees (1988), which incorporates household production.

i receives the lump sum $a_i \geq 0$ and pays the marginal tax rate t_i, $i = 1, 2$. In the second, we have joint taxation with $a_1 = a_2 = a$, and $t_1 = t_2 = t$. Note that in principle both lump sum transfers could be positive, but we rule out negative transfers because that allows lump sum taxation and so distortionary taxation need not exist. Whether both lump sum transfers *should* be positive is to be determined by the optimal tax model.

It greatly simplifies the presentation of the results of the tax analysis, while not changing anything of real significance to this paper, if we assume that the utility functions given in (4) are in fact quasilinear, so can be written as

$$u_{ih} = x_{ih} + u(z_{ih}) \tag{8}$$

It follows from the demand analysis set out above that we can derive indirect utility functions $v_{ih}(a_i, t_1, t_2)$ with, derivatives, for $i, j = 1, 2$, $i \neq j$, $h = 1, ..., H$:

$$\frac{\partial v_{ih}}{\partial a_i} = 1; \frac{\partial v_{jh}}{\partial a_i} = 0 \tag{9}$$

$$\frac{\partial v_{ih}}{\partial t_i} = \frac{\partial v_{ih}}{\partial y_i}\frac{\partial y_i}{\partial t_i} + \frac{\partial v_{ih}}{\partial p_h}\frac{\partial p_h}{\partial t_i} = w_{ih}(z_{ih}L_{ih}^{(1)} - T) \tag{10}$$

$$\frac{\partial v_{jh}}{\partial t_i} = \frac{\partial v_{jh}}{\partial p_h}\frac{\partial p_h}{\partial t_i} = w_{ih}z_{jh}L_{ih}^{(1)} \tag{11}$$

Note finally that since $z_h = z_{1h} + z_{2h}$ we have

$$\frac{\partial v_{ih}}{\partial t_i} + \frac{\partial v_{jh}}{\partial t_i} = w_{ih}(z_h L_{ih}^{(1)} - T) = -w_{ih}l_{ih} \tag{12}$$

We make use of these results in the optimal tax analysis to follow.

3.1 Individual taxation

The Lagrange function is[17]:

$$\mathcal{L} = S[v_{11}(a_1, t_1, t_2), v_{21}(a_2, t_1, t_2), ..., v_{1H}(a_1, t_1, t_2), v_{2H}(a_2, t_1, t_2)]$$
$$+\lambda[\sum_{h=1}^{H}\sum_{i=1}^{2} t_i w_{ih} l_{ih} - H(a_1 + a_2)] \tag{13}$$

The general form of the FOCs, given the constraints $a_i \geq 0$, are

$$\frac{\partial \mathcal{L}}{\partial a_i} = \sum_{h=1}^{H} S_{ih}\frac{\partial v_{ih}}{\partial a_i} - \lambda H \leq 0;\ a_i \geq 0;\ a_i\frac{\partial \mathcal{L}}{\partial a_i} = 0 \quad i = 1, 2, \tag{14}$$

[17] Note that social welfare is a function of individual utilities. In a unitary model this would be a function of "household utility", in a collective model with fixed weights as in Cremer et al, it would be a function of a given weighted sum of individual utilities. The tax problem arises when the "planner" attaches different weights to the individual utility functions than does the household.

where $S_{ih} \equiv \partial S / \partial v_{ih}$;

$$\frac{\partial \mathcal{L}}{\partial t_i} = \sum_{h=1}^{H} (S_{ih} \frac{\partial v_{ih}}{\partial t_i} + S_{jh} \frac{\partial v_{jh}}{\partial t_i}) + \lambda \sum_{h=1}^{H} (w_{ih} l_{ih} + t_i w_{ih} \frac{\partial l_{ih}}{\partial t_i} + t_j w_{jh} \frac{\partial l_{jh}}{\partial t_i}) = 0 \quad i, j = 1, 2, \; i \neq j \tag{15}$$

together with the constraint:

$$\sum_{h=1}^{H} \sum_{i=1}^{2} t_i w_{ih} l_{ih} - H(a_1 + a_2) = 0 \tag{16}$$

3.2 Joint taxation

The Lagrange function is:

$$\begin{aligned} \mathcal{L} = S[v_{11}(a,t), v_{21}(a,t), ..., v_{1H}(a,t), v_{2H}(a,t)] \\ + \lambda [t \sum_{h=1}^{H} \sum_{i=1}^{2} w_{ih} l_{ih} - 2Ha] \end{aligned} \tag{17}$$

The general form of the FOCs are

$$\frac{\partial \mathcal{L}}{\partial a} = \sum_{h=1}^{H} \sum_{i=1}^{2} S_{ih} \frac{\partial v_{ih}}{\partial a} - 2\lambda H] = 0 \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial t} = \sum_{h=1}^{H} \sum_{i=1}^{2} S_{ih} \frac{\partial v_{ih}}{\partial t} + \lambda \sum_{h=1}^{H} \sum_{i=1}^{2} (w_{ih} l_{ih} + t w_{ih} \frac{\partial l_{ih}}{\partial t}) = 0 \tag{19}$$

together with the constraint:

$$t \sum_{h=1}^{H} \sum_{i=1}^{2} w_{ih} l_{ih} - 2Ha = 0 \tag{20}$$

In principle, there are four possible cases for the optimal lump sums, corresponding to both positive, both negative, or one positive and the other negative. The marginal tax rates could be positive or negative in general. Of course certain constellations can be ruled out *a priori*, for example if both lump sums are zero then there is no optimal tax problem, for at least one positive marginal tax rate we need at least one positive lump sum. The precise solutions depend on the parameters of the model and, in particular, the wage distributions of males and females. To investigate this further, we turn to a numerical analysis in which the model parameters are calibrated on empirical data on the distributions of wage rates and hours worked.

4 The Numerical Model

In this section we specify functional forms for the relationships discussed in the theoretical section, then go on to calibrate them on empirical data and derive the implications of the model for optimal taxation

4.1 Empirical specification

We choose a CES functional form for production of the household good:

$$z_h = \gamma[(k_{1h}L_{1h})^\alpha + (k_{2h}L_{2h})^\alpha]^{1/\alpha} \tag{21}$$

The household then derives its optimal time allocation by solving the problem:

$$\min_{L_i} \sum_i (1-t_i)w_{ih}L_{ih} \quad \text{s.t. } z_h \geq \gamma[(k_{1h}L_{1h})^\alpha + (k_{2h}L_{2h})^\alpha]^{1/\alpha} \tag{22}$$

Defining $r \equiv \alpha/(\alpha-1)$, we obtain from the FOCs of the problem the input demand functions:

$$L_{1h} = \frac{z_h}{\gamma}\{k_{1h}^\alpha((1-t_i)w_{1h}k_{2h}^\alpha)^r + k_2^\alpha((1-t_i)w_{2h}k_{1h}^\alpha)^r\}^{(1/r)-1}((1-t_i)w_{1h}k_2^\alpha)^{1-r} \tag{23}$$

$$L_{2h} = \frac{z_h}{\gamma}\{k_{1h}^\alpha((1-t_i)w_{1h}k_{2h}^\alpha)^r + k_2^\alpha((1-t_i)w_{2h}k_{1h}^\alpha)^r\}^{(1/r)-1}((1-t_i)w_{2h}k_1^\alpha)^{1-r} \tag{24}$$

The implicit price of the household good is found by setting $z_h = 1$ and substituting into the minimand of the problem to obtain

$$p_h = \frac{(w_1k_{2h})^r + (w_2k_{1h})^r}{\gamma}\{k_{1h}^\alpha(w_1k_{2h}^\alpha)^r + k_{2h}^\alpha(w_2k_{1h}^\alpha)^r\}^{(1/r)-1} \tag{25}$$

which, given the linear homogeneity of the production function, is independent of the scale of output z.

On the consumption side, the individual utility functions are also CES and so the individual's problem is

$$\max_{x_{ih},z_{ih}} x_{ih}^\beta + z_{ih}^\beta \quad \text{s.t. } x_{ih} + p_h z_{ih} \leq a_i + (1-t_i)w_{ih}T = y_{ih}, \quad i = 1,2 \tag{26}$$

where p_h is derived as above and w.l.o.g. $\beta \in (0,1)$.

The solutions of the individual optimisation problems yield the demand functions

$$x_{ih}(p_h, y_{ih}) = (1+p_h^s)^{-1}y_{ih}; \quad z_{ih}(p_h, y_{ih}) = p_h^{s-1}(1+p_h^s)^{-1}y_{ih} \quad i = 1,2 \tag{27}$$

where $s \equiv \beta/(\beta-1)$, and the indirect utility functions are:

$$v_{ih}(p_h, y_{ih}) = [(1+p_h^s)^{-1}y_{ih}]^\beta + [p_h^{s-1}(1+p_h^s)^{-1}y_{ih}]^\beta \tag{28}$$

$$v_i(p_h, y_{ih}) = p_h^s y_{ih}^\beta(1+p_h^s)^{-\beta} \quad i = 1,2 \tag{29}$$

where μ_{ih} is the individual's marginal utility of full income

For solution of the household model we have the equilibrium condition:

$$\sum_i p_h^{s-1}(1+p_h^s)^{-1}y_{ih} = z_h = \gamma[(k_{1h}L_{1h})^\alpha + (k_{2h}L_{2h})^\alpha]^{1/\alpha} \tag{30}$$

where the left hand side is the sum of the demand functions and the right hand side is the supply derived from the production side.

Note also that we need the constraints:

$$a_i \geq 0 \quad i = 1, 2 \tag{31}$$

Otherwise we might simply get lump sum taxation of the primary earners to pay a lump sum transfer to the second earners.

4.2 Numerical analysis

We draw on household survey data to solve for the optimal tax parameters of the two linear tax systems by maximising a social welfare function of the form $[\sum_{h=1}^{H} \sum_{i=1}^{2} v_{ih}^{\pi}]^{1/\pi}$, where π is the measure of inequality aversion. We first construct empirically realistic percentile distributions of primary and second earnings and go on to select parameters values which can generate labour supplies that are broadly consistent with the available data.

It is important to recognise the limitations of available household survey data in providing information crucially important for the estimation of the parameters of labour supply models and for the analysis of tax policy. While the surveys report hours of work and earnings from which we can derive wage rates, they do not contain information on individual productivities in market work. To deal with this problem we follow the convention of the labour supply literature. We set market productivities equal to wage rates and define market consumption, x, as a Hicksian composite good which we choose as numeraire with price set to 1.

Data on the productivity of time allocated to household production, k_1 and k_2, are also missing. The convention in empirical work on labour supply is to assume that an hour of time allocated to home production, labelled "leisure", produces one unit of output priced at the wage, thereby implicitly setting $k_2 = 1$. Here we assume that the secondary earner has the same productivity in home and market production and therefore set $k_2 = w_2$. We set $k_1 \leq w_2$, given that his higher wage is derived from a market job with restricted entry and select values that can give plausible labour supplies for a given set of parameters.

In addition to home productivities, data on individual consumptions, z_1 and z_2, are missing. As noted previously, studies that follow the Chiappori (1992) "collective" model, such as Cremer et al. (2016), assume that non-market time is spent on own consumption, and is therefore an assigned good. This may represent a harmless simplification for the primary earner, but it is completely counterfactual for a secondary earner specialising in home production, with or without dependent children present.[18]

[18] It is important to note that the Samuelson "unitary" model of the household and the optimal tax models cited earlier based on this model do not make this counterfactual assumption. The demand functions in the Samuelson two-person model are estimated as aggregate household demand functions.

4.3 Data

The data set for couple households is drawn from the Australian Bureau of Statistics (ABS) Survey of Income and Housing for 2015-16, the most recent that is currently available. The sample is selected on the criteria that the primary earner is aged from 25 to 59 years and works at least 25 hours per week for a wage of at least $20.00 (approx. the minimum wage in 2015). The sample contains 4260 records. We first construct a percentile primary wage distribution from the data for primary earnings and hours of market work. The wage in each percentile is calculated as average gross hourly earnings, with hours smoothed across the distribution.[19] A second profile representing the average second earner wage at each primary wage percentile is constructed from the data on second earnings and hours.[20] The two profiles are plotted in Figure 1.

Figure 1 Percentile wage distributions, 2015-16

Primary earner hours rise only slightly across the primary wage distribution, with an overall average of around 8.5 hours per day for a five day working week. This is consistent with data from previous surveys. Second earner hours, while relatively flat, tend to rise across the middle percentiles and then decline towards the top percentiles, with an overall mean close to 4.5 hours per day. However, in contrast to primary hours, there is a high degree of heterogeneity. Over a third of second earners work part time with widely varying hours, around a third work full time and the remainder are not in the workforce. Very little of the variation in second hours can be explained by wage rates or demographics at a given primary wage. We suggest that it can, however, be explained by variation in child care prices, with an impact not only during the child rearing years but across the entire life cycle.[21] In the analysis to follow we do not attempt to capture the full degree of the observed variation.

4.4 Numerical results

The most striking finding from the numerical analysis relates to the role of the shapes of the wage distributions in determining the optimal tax parameters. The wage profiles in Figure 1 show two important characteristics. First, up to around the 90th percentile, wage rates increase virtually linearly for both earner types and the gender wage gap is relatively small when compared with the wage gap between the top percentile and the lower or middle percentiles. Second, in the top decile there is a very sharp increase in primary earner wages leading to both a high degree of inequality in the primary earner distribution and a large gender wage gap.

These two characteristics of the wage distributions have very different effects on the optimal tax system derived on conventional assumptions about the social welfare function. To gain an insight into the extent to which the optimal

[19] We apply the Lowess method to obtain a smoothed profile.

[20] We correct for selectivity bias based on an analysis of predicted wage rates for participant and non-participant sub-samples within quintiles of the primary wage distribution.

[21] In Apps and Rees, 2018, labour supply heterogeneity is driven by child care price variation.

parameters are driven by a gender wage gap alone, as opposed to rising top incomes, we first derive results separately for a subset of four couples for whom the gender wage gap is the sole determinant of inequality. Table 1, Panels (a) to (d), reports the optimal tax parameters and labour supplies of each couple selected from across the primary wage distribution as follows:

- Panel (a): 25th percentile: $w_1 = 28$ and $w_2 = 24$ (wage gap = 16.7%)
- Panel (b): 50th percentile: $w_1 = 36$ and $w_2 = 30.5$ (wage gap = 18.0%)
- Panel (c): 75th percentile: $w_1 = 48$ and $w_2 = 39$ (wage gap = 18.8%)
- Panel (d): 100th percentile: $w_1 = 210$ and $w_2 = 89$ (wage gap = 135.5%)

The time constraint, T, is set to 10 hours per day. Given that the wage gap is driven by "crowding" in "feminised" occupations to the point where working at home becomes a close substitute, the second earner is assumed to be equally productive in both sectors of the economy. We therefore set $k_2 = w_2$. To obtain labour supplies broadly consistent with data means we specify the following parameter values: $\alpha = 0.6$, $\beta = 0.6$ and $\gamma = 0.5$. Table 1 presents results for $k_1 = 1$, $k_1 = 0.5w_2$ and $k_2 = w_2$, with the inequality aversion parameter, π, set to 0.5.

Table 1, Panels (a) to (d): Results for varying gender wage gaps

Not surprisingly, given that inequality is due to a wage gap alone, only the primary earner pays tax while only the second earner receives a non-zero lump sum. An entirely "gender based" tax (GBT) system is optimal. Note, however, that as the primary earner's home productivity rises from $k_1 = 1$ to $k_1 = w_2$ within each panel, his labour supply falls because, as his home productivity rises, work at home becomes a closer substitute for market work. Consequently, his tax rate falls and, in turn, the second earner's lump sum falls, reflecting a rising efficiency cost as the primary earner's labour supply becomes more responsive to the tax rate on his earnings. At the same time as he becomes more productive at home, her labour supply rises.

In contrast, as the wage gap rises across the panels, her labour supply falls. In the top percentile, with a wage gap of 135.5%, she becomes close to a non-participant if, as implied by $k_1 = 1$, he is only minimally productive at home. Even if he is equally productive, her labour supply rises to only 2.27 hours per day. These results suggest that there may be considerable gains from switching to a two-bracket "GBT" rate scale under which a high rate can be applied to the top primary income earners alone.

We now turn to the optimisation problem when all records in the wage distribution are included. Given that the preceding results indicate that setting $k_1 = 0.5w_2$ generates labour supplies that most closely reflect the data across Panels (a) to (c), we solve for the optimal tax rates and lump sums for this productivity gap, and we specify the same parameter values. Table 2(a) presents the optimal tax rates and lump sums, together with the median labour supplies, at the optimum. To provide more information on what is driving the results, Table 2(b) reports labour supplies, net incomes and individual utilities at the 25th, 50th, 75th and 100th percentiles of the primary wage distribution.

Table 2(a) Optimal tax parameters and median labour supplies

Table 2(b) Labour supplies, net incomes and utilities at the optimum

The results for the optimal tax parameters differ dramatically from those reported in Table 1. Both a_1 and a_2 are now positive. However, the most significant changes are in the optimal solutions for t_1 and a_1. The optimal t_1 rises to 0.30, a rate well above rates shown Table 1. Even in the case of the top percentile with a wage gap of 135.5% the new optimal tax rate is 5 percentage points above the rate reported in Panel (d) of Table 1. In the case of the lump sums, a_1 is not only positive but greater than a_2, at 76.27 and 66.75, respectively. The explanation lies in the gap between wages in the top percentiles and across the remainder of the distribution. The gender wage gap is relatively small in comparison.

Given the objective of reducing the overall degree of inequality, and with the policy instrument limited to a linear income tax, it is necessary to impose a high tax rate across the entire distribution of primary earners. The effect of an optimal rate of $t_1 = 0.30$ is to reverse the pre-tax gender wage gap across much of the primary wage distribution. The original gap according to gender, or to primary/secondary earning status, remains only in the top percentiles. This is evident from the reported full incomes and utilities in Table 2(b) which show $y_1 < y_2$ and $v_1 < v_2$ up to and including the 75th percentile. To reduce the new negative gender wage gap created by the higher t_1 across these percentiles, the optimal solution requires $a_1 > a_2$.

As we might expect, primary earner labour supplies fall from around 8.5 hours to around 7.5 hours until towards the top percentiles. In the top percentile, primary earner labour supply rises marginally, from 9.28 hours to 9.33 hours. In the case of the second earner, she no longer has a zero tax rate and, since market and home production are close substitutes, her labour supply falls to below 4 hours. These result indicate the potential for significant gains from switching from an individual based linear GBT system to a highly progressive, individual based, piecewise linear system.

5 Conclusions

In this paper we have extended the approach of Apps and Rees (2018) to focus on the effects of incorporating concerns about within-household - essentially gender - equity on the structure of optimal taxation. We have placed a considerable emphasis on the importance of household production and individual productivities in determining the outcomes of the analysis. There are two dimensions of inequality that are of concern: the first is represented by the gender gap in gross wage rates; and the second is that which characterises the male wage distribution in the higher percentiles. We have isolated the form of the optimal tax system in the former case and shown that it essentially involves a positive wage tax on men with a zero tax on the earned income of, and a lump sum payment to, women. However, when we also incorporate the inequality in the male wage distribution there is a dramatic change, which is driven by the across- rather than within-household inequality. In this case, tax rates on men rise dramatically, as does the lump sum payment they receive. Such is the degree of inequality at the top of the male wage distribution that this redistributional effect dominates the impact of the effect of the gender wage gap, which, however, is still anything but trivial.

A further insight suggested by the analysis in this paper is the potential for significant equity gains from switching from an individual-based *linear* to an individual-based *piecewise linear* gender-based tax system. The latter would allow a selectively higher tax rate on top incomes, as required for the redistribution of income from top to middle and lower wage earners in an economy with the high degree of inequality indicated in Figure 1. This result is supported by the solution for the optimal piecewise linear individual based tax system based on the "unitary" model of the household in Apps and Rees (2018), as well as by the results in Andrienko et al. (2016), (which however considered only single-earner households). We show that moving from simple linear taxation to multi-bracket piecewise linear taxation has important effects on the progressivity and bracket structure of the tax system and we would conjecture that very much the same would happen here. A tax system with 3 or 4 brackets would be likely to show positive tax rates on high wage earners of both types, though lower for women, but much lower rates further down the distribution, with lower wage females quite probably paying zero taxes.

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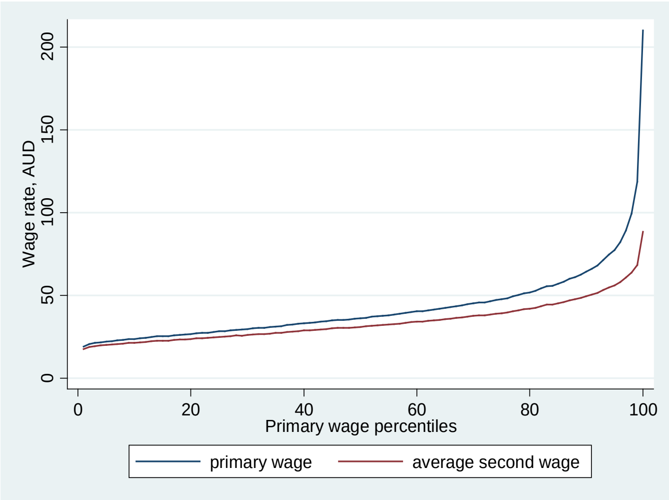

Figure 1 Percentile wage distributions, 2015-16

Panel (a): 25th percentile: w_1=28 and w_2=24 (wage gap = 16.7%)

	t_1	a_1	t_2	a_2	l_1	l_2
k_1=1	0.07	0.00	0.00	19.78	9.96	4.36
k_1= 0.5w_2	0.06	0.00	0.00	13.33	8.55	4.78
k_1=w_2	0.04	0.00	0.00	7.79	6.50	5.40

Panel (b): 50th percentile: w_1=36 and w_2=30.5 (wage gap = 18.0%)

	t_1	a_1	t_2	a_2	l_1	l_2
k_1=1	0.07	0.00	0.00	24.87	9.97	4.35
k_1=0.5w_2	0.06	0.00	0.00	16.69	8.56	4.79
k_1=w_2	0.04	0.00	0.00	8.52	6.51	5.42

Panel (c): 75th percentile: w_1=48 and w_2=39 (wage gap = 18.8%)

	t_1	a_1	t_2	a_2	l_1	l_2
k_1=1	0.09	0.00	0.00	44.79	9.98	4.18
k_1=0.5w_2	0.07	0.00	0.00	30.62	8.63	4.61
k_1=w_2	0.06	0.00	0.00	18.60	4.77	6.71

Panel (d): 100th percentile: w_1=210 and w_2=89 (wage gap = 135.5%)

	t_1	a_1	t_2	a_2	l_1	l_2
k_1=1	0.29	0.00	0.00	604.48	9.99	0.01
k_1=0.5w_2	0.25	0.00	0.00	494.75	9.28	1.66
k_1=w_2	0.22	0.00	0.00	385.95	8.31	2.27

Table 1, Panels (a) to (d): Results for varying gender wage gaps

	t_1	a_1	t_2	a_2	l_1	l_2
k_1=0.5w_2	0.30	76.27	0.06	66.75	7.40	3.70

Table 2(a) Optimal tax parameters and median labour supplies

Percentile	l_1	l_2	y_1	y_2	v_1	v_2
25^{th}	7.21	3.51	276	283	369	378
50^{th}	7.27	3.70	315	321	420	428
75^{th}	7.54	4.55	463	518	626	701
100^{th}	9.33	2.15	1553	905	1985	1157

Table 2(b) Labour supplies, net incomes and utilities at the optimum