A Portfolio-Based Measure of Economic Uncertainty

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Abstract

Financial uncertainty and macroeconomic uncertainty are commonly proxied separately by the volatility of stock returns or key macroeconomic variables, respectively. We propose a portfolio-based measure (PBMEU) that aims to capture aggregate uncertainty in both financial markets and the macroeconomy. Our measure focuses on the volatility of a broad market portfolio including stocks, bonds, and commodities, where correlations between these individual markets have significant implications for the consequences of shocks to the economy. When there are significant and persistent economy-wide shocks, the PBMEU produces higher level of uncertainty than the sum of financial and macroeconomic uncertainties, and in turn yields more significantly negative effects on macroeconomy. This asymmetric effect cannot be ascertained by the commonly used proxies such as VIX, aggregate uncertainty of Jurado et al. (2015) and economic policy uncertainty of Baker et al. (2015).

Keywords: Economic Uncertainty, Broad Market Portfolio, Correlation between Individual Markets, Market Specific and Economy-wide Shocks.

JEL Classification: C58, D81, G12, G17

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1 Introduction

Financial uncertainty and macroeconomic uncertainty are often proxied separately. For instance, financial uncertainty is commonly estimated as the stock market volatility or VIX (e.g., Bloom (2009)), while macroeconomic uncertainty is measured as the volatility of key macroeconomic variables such as Inflation and output growth (e.g., Berger et al. (2017) and total factor productivity (e.g. Bloom et al. (2012)). Given that shocks in financial markets may affect uncertainty in the real economy and vice versa, it is unclear how the interaction between financial markets and macroeconomy affects uncertainty in its own as well as their aggregation.

In this paper, we propose a portfolio-based measure of uncertainty (PBMEU) that aims to capture the aggregate uncertainty of both financial markets and macroeconomy. The PBMEU is defined as the conditional volatility of a broad market portfolio including stocks, bonds, oil, and other commodities. In our setting correlations between these markets helps incorporate the interactions between shocks in financial markets and the real economy, which has significant implications for the measure of aggregate uncertainty. 

Jurado et al. (2015) argue that economic uncertainty is not the same as uncertainty in any single series. Specifically, stock market volatility is tightly linked to financial markets and may not be well-suited to measuring aggregate uncertainty. The authors introduce a measure of economic uncertainty based on a large number of macroeconomic and financial variables and demonstrate that their aggregate measure displays “significant independent variations from popular uncertainty proxies.” The economic uncertainty measure in Jurado et al. (2015) is the first to take into account the interaction between financial markets and macroeconomy. Consistent with their aim, the authors use the historical data to extract common factors that capture comovement between financial and macroeconomic variables. Given that macroeconomic data are released at low frequencies and often subject to revision, the use of macro variables to proxy uncertainty may not be suitable for investors who need to make real-time decisions.

1 A number of fundamentals are found to be correlated with stock market returns, which suggests that shocks to stock market have implications for the real economy and vice versa. These variables include the investment-capital ratio (e.g. Cochrane (1991)), the dividend-earnings ratio (e.g. Lamont (1998)), and the ratio of consumption to wealth (e.g. Lettau and Ludvigson (2001)). In addition, Constantinides and Ghosh (2017) find that two broad categories of macroeconomic variables are highly correlated with the market price-dividend ratio. The first category consists of price levels, including the Consumer Price Index for all urban consumers as well as the Producer Price Index. The second category includes labor-related variables, namely the average hourly earnings, average hours of production, and numbers of employees in private non-farm payrolls in different sectors.

2 They first extract common factors from these variables using principal components analysis (PCA) and then aggregate conditional volatility of individual disturbances as the aggregate uncertainty measure.

3 See the abstract of Jurado et al. (2015).
Our goal in this paper is to use market price data, in particular, from stock, bond and commodity markets to estimate the aggregate uncertainty of financial markets and macroeconomy. This gives us an opportunity to sample at higher frequency, i.e. daily, weekly and monthly observations that allow market participants to monitor uncertainty dynamics and respond more quickly, rather than quarterly or annually as in the case for fundamental data. The market price is also immediately verified by market participants, in contrast to macroeconomic data that can be revised ex-post.

While correlations between stock and other markets are important in our aggregate uncertainty measure, the stock-oil comovement plays a critical role in identifying different implications of oil-specific or economy-wide shocks on our uncertainty measure—PBMEU-dynamics. We use the sign and magnitude of the stock-oil correlation to identify and distinguish oil-specific shocks from economy-wide events (Kilian (2009), Hitzemann (2016), Rapaport (2016), and Ready (2016)). As a result, supply or oil-specific shocks yield negative correlations, but economy-wide and demand-driven (from an energy market perspective) shocks are associated with strong positive comovements between these markets. While neither stock nor oil volatility themselves capture the broad macroeconomic effects, the stock-oil correlation allows us to forge links between financial markets and macroeconomic uncertainty.

We compare the PBMEU to alternative uncertainty measures: VIX, the economic uncertainty of Jurado et al. (2015) (JLN), and the text-based Economic Policy Uncertainty (EPU) index of Baker et al. (2015), and document significant differences between these uncertainty measures over some periods of time. For example, the JLN is higher than PBMEU during the April 2004-November 2007 period or the economic expansion phase classified by the National Bureau of Economic Research (NBER), but it reverses after global financial crisis (GFC) from early 2010 to mid-2012. More specifically, JLN is consistently above its sample mean prior to the GFC, while PBMEU fluctuates below its average level. Right after the GFC, JLN quickly decayed to its sample mean, but PBMEU consistently stayed above the pre-GFC level until the period end. In short, the JLN estimates suggest uncertainty remains more or less the same for several years before and after the GFC. While VIX and PBMEU comove over early 1996 to 1999 and

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4 According to NBER, this period was classified as economic expansion stage, and both the stock and oil markets performed well as indicated by their low volatilities.

5 This symmetry may result from the JLN measure's inability to separate the nature of shocks. Jurado et al. (2015) adopt a historical perspective and estimate their measure across the entire sample to remove "predictable" components. They discuss reasons for using historically revised data rather than higher frequency data to estimate the aggregate uncertainty, see page 1190-1191, in that it improves accuracy of their uncertainty proxy. This feature of macroeconomic variables however presents considerable challenges to decision makers who need up-to-date uncertainty measures.
2004-2007 expansion phase, the former strongly fluctuates around JLN in the post GFC period. In general, EPU is higher and more volatile than the other measures after GFC.

The main driver of this dynamic difference is that our PBMEU implicitly incorporates the two types of shocks, which allows us to know when the aggregate uncertainty is higher or lower than the sum of individual market uncertainties through the sign of stock-oil correlation. We first document the persistence of negative (positive) correlations prior to (following) the GFC. Using changes in global oil production and global economic activity (GEA), see Kilian (2009), as proxies for oil supply shocks and economy-wide shocks, respectively, we find that the positive stock-oil correlation is significantly associated with economy-wide shocks, which pushes the PBMEU higher during the post-GFC recovery period. Meanwhile, the negative correlation between these markets is weakly related to oil production shocks, which yields a lower level in PBMEU during the 2004-2007 expansion period. We find that changes in PBMEU due to economic shocks via the positive stock-oil correlation result in an unique negative impact on Industrial Production and Employment while the PBMEU dynamics associated with oil-specific shocks or the negative stock-oil correlation weakly generate positive impacts on these macroeconomic variables. This asymmetric effect of uncertainty lead by different shock types can not be ascertained by VIX, EPU and JLN, but has significant implications on the dynamics and persistence of economic regimes.

Our paper is closely related to Jurado et al. (2015) in capturing uncertainty at the aggregate level including financial markets and macroeconomy. While both emphasize the importance of their interaction in measuring economic uncertainty, our PBMEU is different from JLN measure in several ways. Given a large number of individual financial and macroeconomic series in JLN construction, the linkage between financial markets and macroeconomy in the JLN measure is captured by dynamics of common factors extracted from these series using the PCA method. The disturbances of these series are uncorrelated cross sectionally, so the JLN uncertainty is equivalent to the sum of conditional volatilities of the individual disturbances. Although this approach is statistically powerful and capable of dealing with a huge dataset, it provides limited insights on the economic intuition of comovement between financial markets and macroeconomy. To overcome this limitation, our PBMEU captures the correlation dynamics and relate them to shocks to a specific market or macroeconomy. With this focus, our analysis starts at the aggregate level, rather than individual markets of stock, bond, and commodity, which gives us an advantage of low dimensionality and timely available market data, and more importantly, directly capturing the interaction between these markets. To increase the ability in reflecting real economic activities, we further separate oil from
commodity markets\textsuperscript{6} and find that oil plays an important role in many aspects. Not only does it explain the difference between PBMEU and JLN but also helps us better understand the uncertainty dynamics associated with different economic regimes.

The stock-oil comovements embedded in the PBMEU link our paper to recent studies on the interactions between such markets, see Kilian and Park (2009), Rapaport (2016), Hitzemann (2016) and Ready (2016) who document a strong relationship between the stock-oil correlation and shocks to the oil market or global economy. Given the recent observations of strong comovements between stock and oil markets, our portfolio approach is motivated by the asset allocation literature, for example, see Erb and Harvey (2006, 2016), and stresses the spillover effects at times of economy shocks, i.e. the association between economy-wide shocks and positive stock-oil correlations. While our findings of negative effects of uncertainty shocks on macroeconomy align with the literature of uncertainty proxy, see Bloom (2009), Jurado et al. (2015), Baker et al. (2015), and Rossi and Sekhpoyan (2015), we also document the Granger causality of uncertainty on macro volatilities whose source comes from commodity sector, not equity nor bond markets. Finally, since PBMEU uses market price data, it does not rely on any subjective forecasts as in dispersion measure, see Bachmann et al. (2013) and D’Amico and Orphanides (2008), nor keywords in newspaper as in Baker et al. (2015).

The remainder of this paper proceeds as follows. Section 2 describes proxies of uncertainty. Section 3 discusses the role of oil in measuring economic uncertainty. Section 4 introduces our measure PBMEU through the model specification of stochastic volatility and correlation between different asset classes in a broad market portfolio. Section 5 describes data and links our empirical measure to historical economic events. Section 6 examines the impact of uncertainty measure on macroeconomic activity, and Section 7 concludes.

2 Measuring economic uncertainty

Research on uncertainty measure has mainly focused on metrics of volatility and cross-sectional dispersion in both objective measures, e.g. firm-level earnings and sales, and subjective measures such as analysts forecasts. Bloom (2009) uses the stock market volatility, the Chicago Board of Options Exchange Volatility index (VXO)\textsuperscript{7} to gauge uncertainty and finds a strong countercyclical relation between real activities, i.e. pro-

\textsuperscript{6}This is because it is highly liquid (e.g. Gibson and Schwartz (1990) and Trolle and Schwartz (2010)), and has a strong link to the macroeconomy (e.g., Hamilton (2013), Kilian (2009), and Dew-Becker et al. (2017)).

\textsuperscript{7}VXO is the implied volatility of S&P 100 index options over the next 30-day period.
duction, working hours, and employment, and the uncertainty measure. Individual stock volatility has also been previously used as a proxy for uncertainty at the firm level, see Leahy and Whited (1996) and Bloom et al. (2007). To measure macroeconomic uncertainty, Berger et al. (2017) study the inflation and output growths in the stochastic volatility framework, while Bloom et al. (2012) use the conditional heteroskedasticity of total factor productivity from the GARCH(1,1) model. Alternatively, Beber and Brandt (2009) measure macro uncertainty by implied volatility extracted from economic derivatives prices, although such products were only available in 2002-2007 period.

Uncertainty proxy with cross-sectional dispersion variables focuses on the analysts’ expectations, see Bachmann et al. (2013) and D’Amico and Orphanides (2008), or firm- and industry-level earnings, see Bloom et al. (2012). The proxies however have drawbacks related to forecasters’ heterogeneous beliefs in the subjective measure or the loadings of common risk factors in the objective metric, which also explain the variables’ dynamics. In other words these measures may capture differences in opinion or the cyclicality of firms’ business activity, not economic uncertainty; see Diether et al. (2002) and Mankiw et al. (2004). For example, the literature on investors’ disagreement has documented the distorting biases in analysts’ forecasts, see Hong et al. (2000) and So (2013). Bachmann et al. (2013) and Scotti (2013) acknowledge and address these issues in their measures.

A different approach to assessing uncertainty relies on newspaper coverage. Baker et al. (2015) introduce the Economic Policy Uncertainty (EPU) index based on uncertainty-related keywords in daily news. The EPU examines articles in 10 leading U.S. newspapers to see whether they contain the following three-word combinations: “economic” or “economy”; “uncertain” or “uncertainty”; and one or more of the policy terms “Congress”, “Deficit”, “Federal Reserve”, “legislation”, “regulation” or “White House”. The authors document that an increase in EPU raises stock price volatility, reduces investment activities in policy-sensitive sectors at the firm level, and forewarns declines in real activity at the macro level. Similarly, Gulen and Ion (2015) find a negative response of corporate investment to EPU at the firm level, and Stock and Watson (2012) conclude that the policy uncertainty or EPU plays a strong role behind the 2007-2009 recession and slow recovery afterwards.

Jurado et al. (2015) argue that the common measures based on volatility or dispersion incorrectly convey information on uncertainty. They note that economic uncertainty is defined as the conditional volatility of disturbances from a comprehensive set of objective variables that represents the actual information set of economic agents. This contrasts the common use of unconditional volatility, e.g. stock market volatility, and helps avoid

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8Scotti (2013) uses surprises in Bloomberg forecasts to construct uncertainty measures.
subjective bias inherited in the forecast dispersion measure. To do this, the authors first form the common factors using principle component analysis (PCA) from a large variable set as predictable components of individual variables in the set, estimate the forecast error volatilities, and aggregate them as a measure of economic uncertainty.

Under this approach, Jurado et al. (2015) show that the first and second components (PCA) load heavily on stock market portfolio returns and measures of real economic activity, respectively. In addition, periods of notably high uncertainty occur far more infrequently than those from simpler volatility-based measures such as VIX, but the effects on the macroeconomy are more pronounced and persistently correlated with real economic activity. Meanwhile, other common uncertainty proxies exhibit frequent spikes in periods of non-recession or relative macroeconomic dormancy. In addition, a large positive uncertainty shock is associated with, but does not necessarily cause, a sizable and protracted decline in real activity, opposite to the overshooting pattern noted by Bloom (2009). The PCA method is capable of dealing with a large number of financial and macroeconomic series; however, the extracted common factors provide limited economic intuition how the interaction between financial markets and macroeconomy affects economic uncertainty. In addition, the issues in macro data such as low frequency and ex-post revision can hinder investors from measuring uncertainty on a real-time basis for better decision making, especially at times of high uncertainty. It consequently makes market prices more appealing given that it is immediately verified by market participants.

The PBMEU introduced in this paper employs price data of different asset classes, namely equity, bond and commodities. Not only does PBMEU allow shocks to second moment in individual asset classes as in JLN uncertainty, the portfolio approach also integrates comovements between different markets into the measure, where the asset correlations are reported to have straightforward economic implications. Within the commodity sector, oil is further separated from the rest given its importance to the economy. As either input or output, it affects almost all economic activities. High oil prices reduce consumption of goods and services and increase inflation, and low oil prices supports consumers but deteriorate the profits of oil producers. More importantly, the sign of stock-oil correlation matters with positive sign associated with economy-wide shocks, which is both supported by recent literature and empirical evidence in this paper. Berger et al. (2017) illustrate that oil prices among others act as a driver of the global macroeconomic uncertainty, stressing the need to treat oil as a separate sector in PBMEU construction.
3 Oil and economic uncertainty

Oil is essential to real economy. Elder and Serletis (2012) show that increases in oil prices heighten the likelihood that the economy transitions from expansion to recession. Stock and Watson (2012) suggest that oil price shocks were one of the factors driving the recession of 2007-2009 as well as slowing the subsequent recovery. Engemann et al. (2011) document that an increase in oil price volatility has a significantly negative effect on investment, durable consumption, and aggregate output, and Jo (2014) report similar findings with the world industrial production.

Oil shocks not only have profound impacts on the real economy but also induce wealth transfers and portfolio rebalancing, which in turn affects financial markets, see Driesprong et al. (2008); or they can command nontrivial prices of risk as in Chiang et al. (2015). Gao et al. (2016) demonstrate that oil price volatility has a distinguished impact on economic growth and asset prices that is not captured by aggregate macroeconomic and financial volatilities or other business cycle variables. Barrero et al. (2016) find that oil price volatility plays an important role in shaping short-run uncertainty. Taken together, the effects of oil shocks on the real economy and financial markets suggest an unique role of oil market in driving the dynamics of financial and macroeconomic variables, which is essential for measuring aggregate uncertainty. This motivates us to separate oil from other commodities.

3.1 Links between oil and financial markets: the stock-oil correlation

The low correlation between stock and oil markets was established in early studies, e.g. Jones and Kaul (1996), but recent papers of Kilian and Park (2009) and Ready (2016) show that these markets can strongly covary with direction dependent on the underlying forces driving oil price changes. In particular, when oil supply shocks play as the main driver of oil price movements, stock market responds to these shocks negatively, which induces a negative contemporaneous correlation between the two markets. On the other hand, Rapaport (2016) suggests that both oil supply and demand shocks contribute to the negative stock-oil correlation as long as these shocks are specific to the oil market, but economy-wide shocks such as macroeconomic growth lead to a positive contemporaneous correlation. This is consistent with the implications from a general equilibrium model developed in Hitzemann (2016) and recent observations in the economy.

Motivated by

There has been a growing interest in the comovement between oil and stock markets. An article on the Wall Street Journal on January 25, 2016 reported that the 20-day rolling correlation between Brent oil prices and the S&P 500 index reached 0.97, higher than any calendar month since 1990. It was also
this stream of literature, particularly Rapaport (2016) and Hitzemann (2016), we use the sign and magnitude of the stock-oil correlation as a means to investigate the effect of two types of shocks, oil-specific shocks and economy-wide shocks, on aggregate uncertainty. This channel is naturally built into our uncertainty measure and explains how PBMEU is different from alternative volatility-based measures such as the stock market volatility or JLN economic uncertainty.

3.2 Stock-oil correlation and economic uncertainty

The idea of aggregate uncertainty was first introduced by Jurado et al. (2015) that neither stock market nor macroeconomic variable volatilities adequately captures economic uncertainty. The interaction between financial markets and real economy plays a crucial role in shaping uncertainty. To capture this important linkage, Jurado et al. (2015) extracts the common factors from a large dataset of financial and macroeconomic variables. These factors however depend on the variable selection and sample periods since they are designed to convert a huge sample of correlated variables into a smaller set of linearly uncorrelated components. The time-varying nature of correlation between financial and macro series suggests that the factors under PCA analysis are not necessarily identical under different economic regimes, in particular before and after financial crises. In addition, the factors are extracted from historical data in which macro variables are often revised after their initial releases. The revised observations in turn contain forward-looking information that are unknown to investors when they are first announced.

We propose a portfolio-based measure of economic uncertainty to directly capture the linkage between financial markets and real economy via asset correlations. Our portfolio is based on three key asset classes, namely equity, bond, and commodity, and the conditional volatility of portfolio returns proxies for economic uncertainty. More importantly, we treat oil market separately from other commodities given its importance documented in literature. This version of volatility-based uncertainty incorporates information from the bond, oil and commodity markets, which are tightly linked to macroeconomy through interest rates, inflation, and GDP growth among others. In our framework, the correlations between individual markets can play a key role in as-

reported that low oil prices reduced aggregate earnings for companies in the S&P 500 by more than 6% in 2015, see http://www.marketwatch.com/story/sovereign-wealth-funds-could-pull-another-404-billion-out-of-stock-market-2016-02-22 and http://www.etf.com/sections/features-and-news/why-stocks-oil-are-correlated. The tendency for stock values to fall with oil prices, or a positive stock-oil correlation, implies that oil and stock markets react to a common factor, which reduces both corporate profits and the demand for oil, e.g. Bernanke (2016).
sessing economic uncertainty, which may not be achieved in a study of single equity or macroeconomic variable volatility. In addition, our portfolio-based measure provides an uncertainty proxy from a perspective of decision makers who need to make real-time decisions. The high frequency of market prices helps provide more timely information than macro data subject to lower frequency and revision problem, so this implies that PBMEU can be updated at daily and weekly frequencies if necessary.

4 Our uncertainty measure: PBMEU

In this section, we formally introduce the PBMEU and discuss the role of stock-oil correlation in its construction. Let $m$ denote a market portfolio including equities, bonds, oil and other commodities, the broad market portfolio return, $R_{m,t}$, is then defined as:

$$R_{m,t} = \omega_e R_{e,t} + \omega_b R_{b,t} + \omega_o \gamma_o R_{o,t} + \omega_c (1 - \gamma_o) R_{c,t},$$  

where $R_{e,t}$, $R_{b,t}$, $R_{o,t}$, and $R_{c,t}$ are the aggregate return of stock, bond, oil, and commodities excluding oil markets, respectively; and $\omega_e, \omega_b, \omega_c$ are the weights of equity, bond, and commodity in the portfolio and $\gamma_o$ is the weight of oil in commodity class.

The uncertainty is measured by the conditional volatility of unpredictable component in the broad market portfolio return, so PBMEU is constructed as follows. Let $\epsilon_{i,t} = R_{i,t} - E(R_{i,t})$ for $i = e$ (equity), $b$ (bond), $o$ (oil), and $c$ (commodity), $\Omega_t$ is the information set available at time $t$, and $E(.)$ is the expectation operator. Hence:

$$PBMEU_t \equiv \left( \text{var} \left[ R_{m,t} - E(R_{m,t}) \mid \Omega_{t-1} \right] \right)^{\frac{1}{2}} = \left( \text{var} \left[ \omega_e \epsilon_{e,t} + \omega_b \epsilon_{b,t} + \omega_c (1 - \gamma_o) \epsilon_{c,t} + \omega_o \gamma_o \epsilon_{o,t} \mid \Omega_{t-1} \right] \right)^{\frac{1}{2}},$$

where $\epsilon_{i,t} = [R_{i,t} - E(R_{i,t})] \mid \Omega_{t-1}$ is the residual at time $t$ of asset class $i$. By definition, uncertainty captures the extent to which economic activity is unpredictable, so it requires that the expected component, $E(R_{m,t})$, be properly removed from portfolio returns. The PCA method is applied in JLN uncertainty from a sample of 279 financial and macro variables to form the expected component, which leaves the individual series’ residuals cross-sectionally uncorrelated. Alternatively, several economic factors can shape the expected components, but the imperfectly identified predictor set that omits some other factors may drive the residuals to be correlated in the cross section. In our framework, we
consider a simple set of predictors, including the own- and cross-lagged asset returns and the index of global economic activity (GEA) in industrial commodity market. Kilian (2009) constructs GEA as a proxy for the aggregate demand in all industrial commodities of the economy and finds that an increase in GEA leads to higher real oil price. Kilian and Park (2009) show that the U.S. real stock returns react differently to changes in oil price, which depends on whether the supply or demand shocks drive the oil market. Kilian and Murphy (2014) recently attribute the 2003-2008 oil price surge to global demand shocks. Taken together, the evidence supports GEA as one of predictors in the portfolio returns.

To see clearly the separate effects of each market volatility from that of stock-oil correlation on the PBMEU, we expand Eq.(2) as:

\[(PBMEU_t)^2 = \omega^2 \sigma_{e,t}^2 + \omega^2 \sigma_{b,t}^2 + \omega^2 \gamma_o \sigma_{o,t}^2 + \omega^2 (1 - \gamma_o) \sigma_{c,t}^2 + 2 \rho_{eb,t} \omega_e \sigma_{e,t} \sigma_{b,t} + 2 \rho_{eo,t} \omega_e \omega_o \gamma_o \sigma_{e,t} \sigma_{o,t} + 2 \rho_{ec,t} \omega_e \omega_c (1 - \gamma_o) \sigma_{e,t} \sigma_{c,t} + 2 \rho_{bo,t} \omega_b \sigma_{b,t} \sigma_{o,t} + 2 \rho_{bc,t} \omega_b \omega_c (1 - \gamma_o) \sigma_{b,t} \sigma_{c,t} + 2 \rho_{co,t} \omega_c \omega_o (1 - \gamma_o) \sigma_{c,t} \sigma_{o,t},\]

where \(\sigma_{i,t} \equiv \text{var}(\varepsilon_{i,t} | \Omega_{t-1})\) is the corresponding residual volatility of individual markets, respectively, and \(\rho_{ij,t}\), where \(i, j = e, b, o, \) and \(c\), is the correlation for each pair of assets. By construction, the sign of correlation between any two markets clearly affects the PBMEU. To focus on the stock-oil correlation, let \(\sigma_{e+b+c,t}^2\) be the residual variance of a portfolio excluding oil and rewrite Eq.(3) as:

\[(PBMEU_t)^2 = \sigma_{e+b+c,t}^2 + \omega_c^2 \gamma_o \sigma_{o,t}^2 + 2 \rho_{co,t} \omega_c \gamma_o \sigma_{e,t} \sigma_{o,t} + 2 \rho_{bc,t} \omega_b \omega_c (1 - \gamma_o) \sigma_{o,t} \sigma_{c,t} \]

Given that oil and other commodities belong to the same asset class, they are closely linked to each other. While there is limited empirical evidence on the oil-bond correlation, a general consensus is that a rise of oil price would increase inflation and leads to higher bond yield. As a result, \(PBMEU_t\) depends largely on the sign of stock-oil correlation \(\rho_{eo,t}\) and exceeds the oil-excluding portfolio volatility when these two markets move in the same direction. This is because all asset correlations \(\rho_{eo,t}, \rho_{co,t}\) and \(\rho_{ob,t}\) together magnify the contribution of individual market volatilities in \(PBMEU_t\), while a negative \(\rho_{eo,t}\) partially offsets the effects of other positive asset correlations. If the JLN

\(^{10}\text{Anderson et al. (2012), Hong et al. (2012) and Narayan and Gupta (2015) find supportive evidence on return autocorrelation and cross-asset return predictability.}\)
uncertainty was assumed to be close to $\sigma_{e+b+c,t}$, the positive (negative) stock-oil correlation would push PBMEU higher (lower) relative to economic uncertainty suggested in JLN. More recently, the movement direction in stock and oil markets is reportedly associated with the nature of shocks to economy, e.g. [Rapaport (2016)]. This indicates that the uncertainty implied in PBMEU rises at times of economy-wide shocks or positive stock-oil correlation, and the opposite holds for oil-specific shocks or negative correlation between these markets.

When PBMEU is compared with stock market volatility $\sigma_{s}^2$, their difference is subject to the sign of stock-oil correlation as well as the stock volatility magnitude relative to bond, oil and other commodities. When stock market volatility dominates those in other asset classes, a strong stock-oil comovement leads to a smaller gap between the PBMEU and $\sigma_{s}^2$. On one hand, the inclusion of other assets in a broad portfolio reduces the stock weight and hence the impact of $\sigma_{s}^2$ on PBMEU. On the other hand, such reduced contribution is partly alleviated by the positive stock-oil correlation, narrowing the difference between PBMEU and $\sigma_{s}^2$.

The PBMEU construction however is associated with two possible drawbacks. The portfolio itself is misspecified because the true market portfolio $R_{m,t}$ is unobservable to market participants. Alternatively, the predictable component of portfolio return is measured inappropriately, which introduces biases in the volatility of estimated disturbances. Nevertheless, the PBMEU is based on the broad market volatility to incorporate the correlations of different markets, and this helps better capture the interaction between financial markets and real economy. In addition, the availability of timely market data makes PBMEU attractive to decision makers who need to respond quickly to changes in the economic uncertainty.

4.1 Estimating PBMEU: multivariate stochastic volatility models

The multivariate GARCH framework, in particular the dynamic conditional correlation (DCC)-GARCH models introduced by [Engle (2002) and Tse and Tsui (2002)], have been widely used to capture dynamics of correlations between individual time series. Under the DCC-GARCH model, both the conditional correlations and variances are specified explicitly as a deterministic function of past returns, and model parameters are estimated by maximum likelihood. Instead, the multivariate stochastic volatility (MSV) framework assumes the stochastic behavior of conditional covariance matrix and relies on Markov Chain Monte Carlo (MCMC) methods to make statistical inference on the model parameters and stochastic components, see [Chib et al. (2009)] for a summary.
review. The MSV modeling approach is more consistent with the theoretical framework of measuring uncertainty in that it allows for independent shocks to higher moments and separate stochastic processes of asset correlations and variances. This motivates us to focus on the MSV approach in PBMEU estimation.

We next describe the MSV model specification proposed by Asai and McAleer (2009) for our broad market portfolio returns. Let $X_{i,t}$ be the set of predictors for each asset class. The forecasting equation is expressed as:

$$RRR_t = BXX_{t-1} + \epsilon_t,$$

where $RRR_t = (R_{e,t}, R_{b,t}, R_{o,t}, R_{c,t})'$ and $X_{i,t-1} = (R_{e,t-1}, R_{b,t-1}, R_{o,t-1}, R_{c,t-1}, GEA_{t-1})$, $\epsilon_t \sim N(0, \Omega_t)$, $\Omega_t = D_t P_t D_t$, $D_t = \text{diag}\{\exp(0.5h_{i,t})\}$, $D_t$ and $P_t$ are the diagonal stochastic volatility matrix and stochastic correlation matrix, respectively. The individual log stochastic volatility process is specified as:

$$h_{i,t} = \gamma_{i1} + \gamma_{i2}(h_{i,t-1} - \gamma_{i1}) + \sigma_i \eta_{i,t}, \eta_{i,t} \sim N(0, 1),$$

and the stochastic correlation matrix has the following form:

$$P_t = \begin{bmatrix} 1 & \rho_{eb,t} & \rho_{eo,t} & \rho_{ec,t} \\ \rho_{eb,t} & 1 & \rho_{bo,t} & \rho_{bc,t} \\ \rho_{eo,t} & \rho_{bo,t} & 1 & \rho_{oc,t} \\ \rho_{ec,t} & \rho_{bc,t} & \rho_{oc,t} & 1 \end{bmatrix}.$$

To construct the stochastic correlation matrix $P_t$, we start with a positive definite matrix $Q_t$ and normalize it by its diagonal elements, e.g.

$$P_t = (\text{diag}\{\text{vecd}(Q_t)\})^{-1/2}Q_t(\text{diag}\{\text{vecd}(Q_t)\})^{-1/2},$$

where the operator $\text{diag}(\cdot)$ diagonalizes a vector into a square matrix, and the operator $\text{vecd}(\cdot)$ creates a vector from the diagonal elements of a matrix. The inverse of $Q_t$ then follows a 4-variate Wishart distribution

$$Q_t^{-1}|k, S_{t-1} \sim W(k, S_{t-1}), S_t = \frac{1}{k}Q_t^{-d/2}AQ_t^{-d/2},$$

where $k$ and $S_t$ are the degrees of freedom and scale parameter of the Wishart distribution, $A$ is a positive definite symmetric parameter matrix, $d$ is a scalar parameter, and $Q_t^{-d/2}$ is computed from spectral decomposition. By construction, the scale matrix $S_t$
contains information on $Q_t$ to generate the future $Q_{t+1}$, see Asai and McAleer (2009) and reference therein for the scale matrix specification.

Once volatilities and correlations at time $t$ are obtained, the $PBMEU_t$ are computed using Eq.(3), where 50%, 30%, 10% and 10% are the portfolio weights allocated to stock, bond, oil, and other commodity markets, respectively. We note that Gao and Nardari (2017) consider passive investment strategies with predetermined weights of 60% stocks, 30% bonds, and 10% commodities; and similar asset allocations are found in Erb and Harvey (2006) and Erb and Harvey (2016). For robustness check, we also study the time-varying portfolio weights and show similar results on the uncertainty effects in Appendix.

By construction, both the individual asset classes’ stochastic volatilities and correlations contribute to the uncertainty measure, and this helps identify the economic consequences on uncertainty of different shock types via the stock-oil correlation. We estimate PBMEU from the entire sample using monthly data as in Jurado et al. (2015) for the comparison purpose, but this would not be a challenge with higher frequency data and using only observations up to date. In the following, we rather focus on the monthly PBMEU estimates to study the difference between PBMEU and other uncertainty measures documented in the literature.

### 5 Data and a historical view on PBMEU

#### 5.1 Data

Our sample period starts January 1984 to December 2015 due to data limitation on oil prices before 1984. Datastream is the primary data source in our study, where the MSCI USA (mnemonic: MSUAML) proxies the aggregate stock market index.\(^{11}\) The US Broad Investment-Grade Bond Index (mnemonic: SBBIGBI) represents the aggregate bond market index, which covers a wide range of bonds including US Treasury and corporate bonds with a minimum maturity of one year.\(^{12}\) The WTI Spot Cushing oil price (mnemonic: CRUDOIL) and Thomson Reuters’ TR/CC CRB ex-energy TR index (mnemonic: TRJCENT) are proxies for oil and non-oil commodity prices, respectively.\(^{13}\)

\(^{11}\)The MSCI USA index represents the investable stock market since it is designed to measure the performance of U.S. large and mid cap segments and covers approximately 85% of the free float-adjusted market capitalization in the U.S. Over our sample period, the MSCI USA and the S&P500 returns are highly correlated with a coefficient of 0.999.

\(^{12}\)The detail can be found at [https://www.yieldbook.com/m/indices/single.shtml?ticker=USBIG](https://www.yieldbook.com/m/indices/single.shtml?ticker=USBIG).

\(^{13}\)This index is based on exchange traded futures on 15 commodities (excluding Energy) with high exposure to agricultural commodities and metals, see [http://financial.thomsonreuters.com/content/](http://financial.thomsonreuters.com/content/)
The ex-energy TR index only starts from January 1994, so we extend this series back to 1984 by following the index construction and using futures data of individual components. In addition, the monthly global economic activity data is collected from Kilian (2009).

5.2 A historical view on PBMEU

Our discussion begins with the volatility and correlation estimates for the four markets: stock, bond, oil, and other commodities. The top (bottom) panel of Figure 1 presents the stochastic volatility estimates of stock and bond markets (oil and other commodities) by solid lines, and the realized volatilities, defined as the square root of the sum of squared daily returns in each month, by dots. Daily bond market return data is only available from January 1994, limiting their estimates of monthly realized volatility.

For our sample period, the overall stochastic volatilities are greatest for oil, followed by stock, other commodity, and bond markets, where the corresponding average volatilities are 8.71%, 4.09%, 3.34% and 1.21%. The oil volatility is more than double stock volatility over the sample period and consistent with Gao et al. (2016). The stochastic volatility patterns follow those from realized volatilities in that the volatility is generally higher in Black Monday October 1987, the 1998 Long-Term Capital Management (LTCM) default and early 2000’s IT bubble for equity; the rising Treasury yield in late 2003 for bond; the oil production increase by Saudi Arabia in March 1986, Gulf War I in August 1990, and oil price crisis in early 1998 for oil; and the pre-1989 and post-GFC 2010-early 2011 period for other commodities. Notably, all markets reached their highest volatility during the 2008-2009 GFC.

Figure 2 presents the stochastic and realized correlation estimates in solid lines and dots, respectively, where the latter is computed using daily returns in each month. The stock-oil, stock-other commodities, and oil-other commodities correlations exhibit similar pattern in that they fluctuate over a wide range from -0.5 to 0.5 until several months


In particular, we collect futures prices for corn, soybeans, live cattle, gold, copper, sugar, cotton, cocoa, coffee, wheat, lean hogs, orange juice and silver traded on the U.S. exchanges from Morningstar. For aluminum (nickel) traded on LME, we obtain from Datastream the official price of cash/3-month constant futures contracts, namely LAHCASH/LAH3MTH from October 1988 (LNICASH/LNI3MTH from January 1984), and the futures contracts maturing at January 1994, namely LAH0194 (LNI0194) from December 1993. We follow Gorton et al. (2012) to linearly interpolate the futures prices using the cash and fixed-term maturity of 3-month futures contracts for both commodities over the period prior to December 1993. To extend the aluminum futures prices back to January 1984, we use the futures contracts traded on COMEX. The correlation between daily values of the original index and our constructed series in January 1994 is 0.996.

See \[ \text{http://www-personal.umich.edu/~lkilian/reupdate.txt} \]
before the GFC, then rise and remain positive through the remainder of the sample. In contrast, the stock-bond correlations are positive through 1999 and become negative thereafter except for a short period from 2004 to mid-2006. The oil-bond correlation does not exhibit a clear pattern until the end of 2006, after which it is mainly negative. Overall, the stochastic correlation patterns are comparable with those from realized correlations.

To support the linkage between different shock types, namely oil-specific and economy-wide, and stock-oil correlation sign, we present the correlation estimates, major economic and oil-related events, and shocks to oil market and economic growth in Panels A, B, and C of Figure 3, respectively. In particular, the change in GEA and log global oil production are proxies of economy-wide and oil supply shocks (see Kilian (2009), respectively. Over the pre-GFC period, several large oil supply shocks (and associated events) occur around December 1996-February 1997 (overproduction, see Hamilton (2011)), November 1999 (declining oil production in Iraq and Iran), September 2001 (11/9 attack), January-March 2003 (Venezuela unrest and Gulf War II), October 2004, and August 2005 (Katrina hurricane), which are caused by political or natural events, and the oil supply-demand imbalance gradually built on economic growth and technology development. Both the Gulf War II and Katrina hurricane cause a decline in global oil production and economic activity, but their effects are not the same with the stronger impact of Gulf War II (Katrina hurricane) on oil supply (overall economy). Accordingly, the stock-oil correlation is reportedly negative (positive) in March 2003 (August 2005) and consistent with the dominance of oil production shocks over economy-wide shocks (vice versa) at these times. Similarly, the unanticipated 11/9 attack has no adverse impact on oil production but results in the U.S. economy falling into recession, and this is further supported by positive stock-oil correlation around the event.

With respect to the economy-wide (GEA) shocks, there are a few large shocks in October 1986, December 1996, and December 2003, which are associated with the close-to-zero stock-oil correlations. The Lehman Brothers collapse leads to the deepest negative GEA shock, which induces a stronger comovement between stock and oil markets. After the GFC, economy-wide shocks become much larger in magnitude and more persistent,

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15 Hamilton (2011) notes that a general strike in Venezuela eliminated 2.1 million barrels per day (mb/d) of oil production in January 2003, and the U.S. attack on Iraq removed an additional 2.2 mb/d in March 2003. Kilian (2009) points out that the oil supply shocks caused by Gulf War II differs from those in earlier periods in that the war was anticipated and without a well-defined end, which promoted a preemptive increase in oil production from other oil producing countries such as Saudi Arabia and Kuwait. This in confirmed by a deep reduction in global oil production in March 2003 followed by quick rebound in Panel C of Figure 3 and Hamilton (2011), and the temporary shortfall has no significant effects on GEA.
e.g. the Eurozone crisis, Libya War, and Debt Ceiling Debate; in addition, the stock-oil correlation reaches and remains at its highest positive level until the sample period end. Overall, the dynamics in stock-oil correlations, oil-specific and economy-wide shocks support the tight link between the nature of shocks and stock-oil correlation sign.

5.3 The dynamics of PBMEU and other measures

In this section we study the difference between dynamics in PBMEU and other uncertainty proxies documented in literature. We first normalize individual measures\(^\text{17}\) and present them in Figure 4. We also overlay key economic events, as well as NBER recessions by shaded portions in the figure. The realized stock-oil correlations are illustrated with dots, whose statistically significant values (at the 95% confidence level) are represented by filled dots. Since JLN and PBMEU are designed to capture aggregate uncertainty, we document four time periods in which their difference are well defined, namely (1) mid-1986 to end-1988, (2) early-1996 to end-1999, (3) early-2004 to end-2007, and (4) begin-2010 to late-2012. VIX and PBMEU are also useful for better understanding on the time-varying nature of uncertainty. Overall, VIX moves with PBMEU during the second and third periods, and strongly fluctuates around the JLN over the last period; while EPU is higher and more volatile than the others in the post-GFC time window. To highlight the role of individual asset volatilities, e.g. stock and oil markets due to their large magnitude relative to the others, and the stock-oil correlation, Figure 5 illustrates these variables, JLN and PBMEU in separate time windows, where the left (right) vertical axis indicates the magnitude of normalized market volatilities and uncertainty proxies (correlation).

The first period (mid-1986 to end-1988) witnesses the soaring oil volatility over May-November 1986 followed by that in the stock market afterwards. The high oil volatility is associated with a substantial decline in oil prices and caused by oil supply shocks, when Saudi Arabia increases production to gain more market shares in September 1985. The shrinking revenue in oil producing firms makes them more vulnerable to a credit crunch and leads to a negative reaction of stock market. When the oil market recovers from oil supply shocks, stock market volatility reaches its highest level in Black Monday or October 1987. Overall, the individual market volatilities push PBMEU higher than average, while JLN remains unchanged and close to sample mean. However, both the stock and oil markets are not tightly correlated to each other, which attenuates the

\(^{17}\)We normalize them by subtracting the sample mean and then dividing by the sample standard deviation.
effects of their volatility shifts on PBMEU. This leads to PBMEU levels lower than oil volatility at times of oil supply shocks, and than VIX during the stock market crash.

Over the time window from mid-1996 to end-1999, a number of Asian countries including Thailand, Indonesia and South Korea experienced a severe currency depreciation in summer 1997, which leads to a drop of 56% in the MSCI Emerging Markets Index a year later. The U.S. market seemingly avoids the turmoil as indicated by close-to-average levels in VIX, JLN, and PBMEU. However, the Asian monetary crisis and following decline in demand for crude oil and nonferrous metals severely affect Russian foreign exchange reserves and make its government bonds default on 17 August 1998. The financial meltdown in Russia leads to heavy losses in U.S. investment banks, e.g. LTCM hedge fund, and global financial markets, and consequently raises the U.S stock market volatility. Finally, the oil sector is dragged down and associated with a sharp increase in volatility over April-December 1998. Overall, both JLN and PBMEU exhibit a slight increasing trend, and the former (latter) is consistently lower than (close to) sample mean. In addition, a spike in oil volatility following the oil cut agreement among OPEC countries is coupled with stock-oil comovement in April-May 1999, further pushing PBMEU away from its average level.

Over the pre-GFC period from early-2004 to end-2007, JLN consistently fluctuates above its sample mean while PBMEU consistently fluctuates below its sample mean. The low PBMEU level results from the calm stock and oil volatilities and a loose linkage between these markets. NBER classifies this time period as economic expansion, where both the stock and oil markets also performed well. If the latent uncertainty was indeed high, EPU would move proportionally to JLN, but it is reported to be lower than average. Taken together, PBMEU and other measures indicate that concerns on future economic uncertainty are not as high as suggested by JLN.

In the GFC aftermath from 2010 to 2012, the persistence in positive stock-oil correlation is distinguishable from the patterns observed in the previous periods. Such phenomenon is associated with many geopolitical events such as the Eurozone Crisis, Libya War, and Debt Ceiling Debate, which also helps explain the wide fluctuation in EPU. The strong comovements between stock and overall commodity markets play the dominant role in explaining the higher PBMEU level relative to others. Consequently, PBMEU does decrease to its mean level at a much lower rate, contrasting the quickly declining pattern in asset volatilities, JLN and VIX. Overall, the oil volatility and stock-oil correlation are the two main factors explaining the PBMEU dynamics distinct from others. Surprisingly, JLN remains more or less the same or decreases at times of high oil volatility.
The correlations play an important role in varying market volatility effects on the PBMEU, where the following examples illustrate different patterns between PBMEU and individual asset volatilities. Although the stock volatility estimates and VIX uncertainty reach their highest levels in October stock market crash 1987, the PBMEU is much lower due to the contemporaneous negative correlations between different markets. The oil volatility and VIX display a sharp increase in July-August 1990 given concerns on oil supply shocks by Gulf War I and early U.S. recession, respectively; however, the PBMEU remains modest as these markets are negatively correlated. An increase in Treasury yield in 2003 causes bond volatility to reach its historically high due to worries about a pickup in economic growth and accompanying expected rise in inflation, but the PBMEU again remains at its sample average and consistent with the NBER economic expansion stage because of the low correlations between bond and other markets. Overall the evidence suggests that by incorporating the interaction between financial markets and macroeconomy via the asset correlations, the PBMEU displays strikingly different patterns from individual market volatilities. This further supports the argument made by Jurado et al. (2015) that the volatility of either stock market returns or key macroeconomic variables is not a good proxy for aggregate economic uncertainty.

6 The uncertainty effects on macroeconomy

6.1 The impulse response analysis

We follow the literature to conduct the impulse response analysis by a VAR model that characterizes the dynamic relation between uncertainty and macro variables. Similar to Baker et al. (2015), we focus on Employment and Industrial Production in measuring the effects of uncertainty shocks on macroeconomy in the VAR models separately for different uncertainty proxies. The baseline VAR model consists of three lags of all variables with the following causal ordering:

\[
\begin{bmatrix}
\text{uncertainty proxy} \\
\log(S&P500) \\
\text{Federal Funds Rate} \\
\log(\text{Employment}) \\
\log(IP)
\end{bmatrix}
\] (baseline). \hspace{1cm} (10)

The baseline VAR is further extended to include more variables as in Bloom (2009).

and presented as below:

\[
\begin{bmatrix}
\text{uncertainty proxy} \\
\log(\text{S&P500}) \\
\text{Federal Funds Rate} \\
\log(\text{Wage}) \\
\log(\text{CPI}) \\
\log(\text{Hours}) \\
\log(\text{Employment}) \\
\log(\text{IP})
\end{bmatrix}
\]

\[(\text{Expanded}), \quad (11)\]

where we collect Industrial Production (IP) in manufacturing (FRB G17, series code B00004) from Federal Reserve, the Federal Funds Rate (FEDFUNDS), Employment (MANEMP), Wage (CES30000000008), CPI (CPIAUCSL), and Hours (AWHMAN) from FRED Economic Data, and the S&P500 from WRDS at monthly frequency. We rely on Cholesky decomposition to identify orthogonal shocks for the impulse response analysis, and the variable ordering in the VAR model has been discussed in Baker et al. (2015) and Bloom (2009), respectively. We refer readers to their papers for more detailed discussion.

Figure 6 presents the mean effect (middle line) and its 90% confidence band to show different patterns of the impulse response of Industrial Production and Employment to a one standard deviation of uncertainty shock. The horizontal axis represents the 3-year period following an uncertainty shock at month 0, and the vertical axis illustrates the percentage change in the response variable. The EPU shocks weakly foreshadow macroeconomic performance relative to those in VIX, PBMEU and JLN shocks. The maximum drops in impulse response of Industrial Production are -0.44% for EPU (at the 11th month), followed by -0.54% for VIX (at the 22nd month), -0.8% for PBMEU (at the 27th month), and -0.94% for JLN (at the 19th month). Similarly, Employment negatively responds to uncertainty shocks whose largest magnitude is -0.35% for EPU (at the 15th month), followed by -0.47% for VIX (at the 19th month), -0.61% for PBMEU (at the 27th month), and -0.74% for JLN (at the 19th month). While JLN and PBMEU generate comparable macro effects, an uncertainty shock in the latter has a more resilient impact than the other with barely diminishing effects up to 36 months.

We verify the empirical findings with a number of modifications of the variable set and causal ordering in the baseline VAR, see in Baker et al. (2015). We study the expanded VAR (expanded), increase the number of lags to 6 (lags6), and reverse the variable ordering (adding the prefix rev). Baker et al. (2015) alternatively include the
Michigan Consumer Sentiment index which surveys U.S. households to determine their views on their own financial situation and short- and long-term economy in the analysis, so we also include it in front of or following PBMEU (mich first and mich second, respectively). Figure 11 in Appendix depicts the similar mean effects of PBMEU across different model specifications with the maximum drops around -0.81% for Industrial Production and -0.65% for Employment, and it confirms that both the sentiment index and PBMEU do not overlap much information in the sample period.

6.2 The asymmetric effects on macroeconomy

We have argued that the stock-oil correlation helps identify nature of different shocks, that is, economy-wide (oil-specific) shocks are associated with positive (negative) correlations. We also document the long-lasting negative effects of PBMEU on Industrial Production and Employment and argue that this originates from economy-wide shocks which induce a strong comovement between stock and oil markets. Hence, the positive and negative stock-oil correlations are expected to have distinct effects on the macroeconomy. We recall that the stock-oil correlation in Figure 3 fluctuates around zero with some negative spikes prior to GFC, but remains significantly positive over the post-GFC period. Consequently, PBMEU slowly decays and stays between the highly fluctuating EPU or VIX and JLN. We report in this section that global economic activity shocks are more frequently negative with large magnitude at times of the positive stock-oil correlation, supporting the ability of PBMEU to incorporate such nature of shocks, especially after GFC.

To determine how different shock types affect the stock-oil correlation, the global economic activity and oil production shocks are defined as the first difference in GEA index and log global oil production adjusted for seasonality by US Census’ X-12 procedure respectively. We divide the sample into two groups based on the stock-oil correlation estimates and shocks sign separately to form four intersections. Panel A of Table 1 shows that the global economic shocks are on average negative and left-skewed at times of positive stock-oil correlation, implying the frequent occurrence of negative economy-wide shocks with sizable magnitude. Meanwhile, these shocks are on average

19See http://www.sca.isr.umich.edu/
20We also include a time trend, study the bivariate case with uncertainty measure and Industrial Production, remove S&P500, or add VIX following the uncertainty measure. The results are both quantitatively and qualitatively similar and available upon request.
21We exclude the monthly consumption of 8 OECD countries in regression as proxy for global oil consumption since its pattern does not reconcile with the increasing trend in annual global oil demand after the GFC, see Figure 20 and 21 for monthly OECD and annual global oil consumptions, respectively.
positive and right-skewed in the domain of negative stock-oil correlation. This indicates that poor economic growth is more likely to occur when the financial markets and fundamentals closely link to each other, while both move in their own way at times of high economic performance. In addition, Panel B of Table 1 documents a significantly negative correlation between negative GEA shocks and positive stock-oil correlation, so it suggests that stock and oil markets comove to a great extent at times of negative shocks on global economy.

The results above are also supported by truncated regression of stock-oil correlation on the economy-wide and global oil production shocks in Panel C of Table 1. The lagged stock-oil correlation is included in regression to control for potential autocorrelation in the series, and the regression results are presented in Panel C of Table 1. The coefficient of GEA shocks is only significantly negative at 1% level with magnitude of -0.176 in the domain of positive correlation, indicating that a negative global economic shock corresponds to a rise in stock-oil comovement and hence higher PBMEU uncertainty. In contrast, the global oil production shocks have positive coefficient but insignificant. In addition, the coefficients on lagged variable are statistically significant at 1% level with values between 0.85 and 0.92, and this supports the correction for serial correlation. Taken together, the regression results suggest that the economy-wide shocks are associated with the positive stock-oil correlation, where negative shocks coincide with a tight comovement between these markets. However, no strong evidence on the significant relation between oil production shocks and negative correlations is documented.

Given that the stock-oil correlation sign can identify the shock type of either economy-wide or oil-specific, we introduce a variable $CDF_{\rho_t} = \sum_{i=1}^{T}(\rho_i \leq \rho_t)/T [1 - \sum_{i=1}^{T}(\rho_i \leq \rho_t)/T]$ to proxy the likelihood of economy-wide [oil specific] shocks at time $t$, where $T$ is the sample size, $\rho_i$ or $\rho_t$ are the correlation estimates at time $i$ or $t$, $i = 1, ..., T$. PBMEU is then decomposed into two components, upside PBMEU$_t \cdot CDF_{\rho_t}$ and downside PBMEU$_t \cdot (1 - CDF_{\rho_t})$, respectively.\textsuperscript{22} Intuitively, when the uncertainty is more likely to originate from global economic shocks at times of positive stock-oil correlation, the interaction between PBMEU$_t$ and CDF$_{\rho_t}$ can capture this by giving more weight to upside uncertainty, i.e. higher CDF$_{\rho_t}$, and vice versa during periods of negative stock-oil correlation.

We report the impulse response analysis of upside and downside uncertainties on

\textsuperscript{22}We note that the use of indicator function $I(CDF_{\rho_t} > 0.5)$ to specify nature of shocks does not distinguish similar magnitudes of CDF$_{\rho_t}$, e.g. 0.49 vs 0.51 or 0.6 vs 0.7. One can alternatively change the domain of correlation to unit space by monotonic transformation, e.g. $(1 + \rho_t)/2$, and use the resulting value as the likelihood of economy-wide shocks. This interpretation however ignores the variable magnitude, e.g. if $\rho_t = 0.8$ is the maximum value, the likelihood of global economic shock is 1 under original ranking but 0.9 under such transformation, though the variable ordering is preserved.
Industrial Production and Employment from the baseline VAR model in Figures 7 and 8, respectively. For the comparison purpose, we also present the impulse response analysis of upside and downside uncertainties decomposed using the estimated correlation between oil and bond markets, although economic meanings of this correlation are unknown. From the baseline model, Panels A and B in Figures 7 and 8 depict that the upside uncertainty yields a negative impact on Industrial Production with a maximum drop of -0.7% for the oil-stock correlation compared to -0.48% for the oil-bond correlation, and on Employment with a maximum drop of -0.47% for the oil-stock compared to -0.44% for the oil-bond correlation. The negative effect on Industrial Production from the upside uncertainty using the oil-stock correlation is much larger than that using the oil-bond correlation. Meanwhile, Panels C and D in Figures 7 and 8 display the weakly positive impact of the downside uncertainty, with the greatest effect on Industrial Production of 0.25% for the oil-stock compared to 0.02% for the oil-bond correlation, and on Employment of 0.02% for oil-stock compared to 0.05% for the oil-bond correlation. The results are robust to different VAR model specifications documented in Figures 12 to 15 in Appendix. In general, the upside (downside) uncertainty generated by oil-stock correlations uniquely yields the negative (positive) impact on macroeconomy, especially the Industrial Production, compared to those from oil-bond correlations. This asymmetric effect cannot be ascertained by VIX, aggregate uncertainty of Jurado et al. (2015) and economic policy uncertainty of Baker et al. (2015).

7 Conclusion

We propose a new measure of aggregate economic uncertainty, PBMEU, that embeds the interaction between economic fundamentals and financial markets. We find that PBMEU uncertainty can be higher or lower than the sum of individual uncertainties in different markets, e.g. the stock market or macroeconomic volatilities. Several advantages of the PBMEU are also discussed. First, it captures uncertainty as viewed by market participants since it can be estimated at any point in time over various horizons using market price data. Second, it provides a channel to incorporate the nature of shocks, oil-specific or economy-wide. This in turn enables us to show asymmetric effects of uncertainty due to different shock types on the macroeconomy. Alternative uncertainty measures such as those based on news coverage EPU, stock market volatility VIX, and aggregate uncertainty JLN do not capture such asymmetric effects of different shock types.
This figure displays the stochastic and monthly realized volatility estimates for stock (MSCI USA), bond (US Broad Investment-Grade Bond), oil (WTI Spot Cushing) and other commodity (Thomson Reuters’ TR/CC CRB ex-energy TR index) over the sample period January 1984 through December 2015. The stochastic volatility estimate at time $t$ for asset class $i$ is based on monthly data and computed as the mean of 5,000 iterations of $\exp(0.5h_{i,t})$ in MCMC method, where $h_{i,t}$ is the individual log stochastic volatility. The monthly realized volatility in month $t$ is computed as the square root of sum of daily squared returns in that month. For bond market, the monthly realized volatility started in January 1994 due to daily data unavailability from January 1984 to December 1993.
This figure displays the stochastic and monthly correlation estimates for stock (MSCI USA), bond (US Broad Investment-Grade Bond), oil (WTI Spot Cushing) and other commodity (Thomson Reuters’ TR/CC CRB ex-energy TR index) over the sample period January 1984 through December 2015. The stochastic correlation estimate at time $t$ between asset classes $i$ and $j$ is based on monthly data and computed as the mean of 5,000 iterations of $\rho_{\text{stochastic}}$ in MCMC method. The monthly correlation in month $t$ between any two assets is computed based on their daily returns in that month. For the monthly correlations related to bond market, they are not computed over the period before January 1994 due to the daily bond return data unavailability from January 1984 to December 1993.
This figure displays the stochastic and monthly correlation estimates for stock (MSCI USA) and oil (WTI Spot Cushing) markets, and the first-order difference or shocks on Global Economic Activity index and log global oil production and over the period January 1994 to December 2015. The stochastic correlation estimate at time $t$ between asset classes $i$ and $j$ is based on monthly data and computed as the mean of 5,000 iterations of $\rho_{ij,t}$ in MCMC method. The monthly correlation in month $t$ between any two assets is computed based on their daily returns in that month. The Global Economic Activity index is available from Kilian [2009], and the global oil production is downloaded from [https://www.eia.gov](https://www.eia.gov). The global oil production series are seasonally adjusted following the X-12 procedure of the U.S. Census. The shocks are defined as first difference of each series. The key political economic events are represented by shaded areas in each panel.
This figure displays different (normalized) uncertainty measures and stock-oil correlations over the sample period January 1984 through December 2015, namely the one-month uncertainty index from Jurado et al. (2015) JLN1, VIX, Economic Policy Uncertainty EPU in Baker et al. (2015), and portfolio-based measure of economic uncertainty PBMEU. Since uncertainty measures have different scales, they are normalized by first deducting its sample mean and then dividing by its sample standard deviation. The monthly correlation between stock (MSCI USA) and oil (WTI Spot Cushing) markets is computed from daily returns in that month, and the solid red dots represent the monthly correlations that are significantly different from zero at 5% level. The key political economic events (NBER recessions) over the sample period are represented by dark (light) shaded areas.
This figure displays different (normalized) asset volatilities, uncertainty measures and stock-oil correlations over the sample period January 1984 through December 2015, namely the oil and stock stochastic volatility, one-month uncertainty index from Jurado et al. (2015) JLN1 and portfolio-based measure of economic uncertainty PBMEU over different periods. Each normalized series is calculated by first deducting its sample mean and then dividing by its sample standard deviation.
This figure displays the impulse responses of Industrial Production (IP) and Employment in the VAR model framework with different uncertainty measures over the sample period January 1984 through December 2015, namely the one-month uncertainty index from Jurado et al. (2015) JLN1, Economic Policy Uncertainty EPU in Baker et al. (2015), stock market volatility VIX, and portfolio-based measure of economic uncertainty PBMEU. Panel A and B present the IP and employment mean response (middle line) following a unit of standard deviation of different uncertainty shocks in the “base” VAR model and the 90% confidence band.
This figure displays the impulse responses of Industrial Production and Employment in the VAR model framework with upside and downside correlation-based PBMEU over the sample period January 1984 through December 2015. The upside (downside) correlation-based PBMEU at time $t$ defined as the interaction between PBMEU and a ranking variable $CDF_{\rho_{t}} = \frac{\sum_{i=1}^{T} \rho_{i} \leq \rho_{t}}{T} (1 - \frac{\sum_{i=1}^{T} \rho_{i} \leq \rho_{t}}{T})$, where $T$ is the sample size and $\rho$ is the stochastic stock-oil correlation estimate at time $t$. Panel A and B (C and D) present the Industrial Production and Employment mean response (middle line) following a unit of standard deviation of upside (downside) correlation-based PBMEU in VAR model and the 90% confidence band.
This figure displays the impulse responses of Industrial Production and Employment in the VAR model framework with upside and downside correlation-based PBMEU over the sample period January 1984 through December 2015. The upside (downside) correlation-based PBMEU at time $t$ defined as the interaction between PBMEU, and a ranking variable $CDF_{\rho_t} = \sum_{i=1}^{T} \rho_i \leq \rho_t \{ 1 - \sum_{i=1}^{T} \rho_i \leq \rho_t \}$, where $T$ is the sample size and $\rho_t$ is the stochastic oil-bond correlation estimate at time $t$. Panel A and B (C and D) present the Industrial Production and Employment mean response (middle line) following a unit of standard deviation of upside (downside) correlation-based PBMEU in VAR model and the 90% confidence band.
Figure 9: The impact of upside and downside realized stock-oil correlation on Industrial Production for different uncertainty measures.

This figure displays the impulse responses of Industrial Production in the VAR model framework with upside and downside realized correlation-based uncertainty over the sample period January 1984 through December 2015 for different uncertainty measures. The upside (downside) correlation-based uncertainty at time $t$ is defined as the interaction between uncertainty measure at $t$ and a ranking variable $CDF_{t} = \sum_{1}^{T} \rho_{i} \leq \rho_{t} (1 - \sum_{1}^{T} \rho_{i} \leq \rho_{t})$, where $T$ is the sample size, and $\rho_{t}$ is the realized stock-oil correlation at $t$ estimated from 3-, 6-, and 12-month rolling window of daily returns. The panels present the Industrial Production mean response (middle line) following a unit of standard deviation of upside (downside) stock-oil correlation-based uncertainty in VAR model and the 90% confidence band.
Table 1: The relation between different types of shocks and stock-oil correlation

<table>
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This table presents the relation between different types of shocks and stock-oil correlation over the sample January 1984 to December 2015. Summary statistics on GEA shocks under positive and negative domains of the stock-oil correlation is shown in Panel A, and Panel B presents the correlation coefficients between stock-oil correlation and GEA or global oil production shocks, where each variable is divided into two groups of positive and negative values. Panel C presents the results of truncated regression of stochastic stock-oil correlation on different types of shocks. The Global Economic Activity index is available from Kilian (2009), and the global oil production is downloaded from https://www.eia.gov. The global oil production series are seasonally adjusted following the X-12 procedure of the U.S. Census and available from January 1994 to December 2015. The shocks are defined as first difference of each series. The truncated regression is run for the positive and negative domains of stock-oil correlation, and the robust t-statistics are reported in parentheses. ***, **, and * denote statistical significance at 1%, 5% and 10% level.
Appendix: Estimation procedure

7.1 Estimation procedure

We follow Asai and McAleer (2009) to implement the two-stage procedure in which feasibility, speed, numerical accuracy and speed are well-balanced within the MCMC framework. The algorithm in the paper is presented as follows:

- for the \(i^{th}\) return series, estimate the parameters \((\beta_i, \gamma_{i1}, \gamma_{i2}, \sigma_i)\) and log volatilities \(\{h_i\}_{t=1}^{T}\) via the MCMC method. Obtain the standardized series \(z_{it} = (r_{it} - \beta_i X_{i,t-1})w_{it}\), where \(X_{i,t-1}\) is the set of predictor values at time \(t - 1\), \(\beta_i\) and \(w_{it}\) are the MCMC estimates of beta loadings on the predictors and inverse of volatility at time \(t\) obtained by averaging MCMC draws as \(\beta_i = (1/M)\Sigma_{\tau=1}^{M} \beta_i^{(\tau)}\) and \(w_{it} = (1/M)\Sigma_{\tau=1}^{M} e^{(-0.5h^{(\tau)}_{it})}\), respectively, where \(M\) is the number of iterations.

- based on the standardized vector \(z_{it}^{T} = \{z_1, z_2, z_3, z_4\}_{t=1}^{T}\), estimate the parameters \(A, d, k\) and \(\{Q\}_{t=1}^{T}\) via the MCMC method.

The first stage allows to estimate the individual equations 4 times in multivariate volatility models faster than estimating a system of 4 equations because each process \(r_{it}, i = 1, ..., 4\) follows a univariate stochastic volatility model, so the estimates of volatilities \(h_{it}\) and \(\beta_i\) in the first stage is valid. Within the first stage, for the \(i^{th}\) series, we sample as follows

- sample \(\beta_i|\gamma_{i1}, \gamma_{i2}, \sigma_i, \{h_i\}_{t=1}^{T}\).

- sample \(\gamma_{i1}, \gamma_{i2}, \sigma_i, \{h_i\}_{t=1}^{T}|\beta_i\).

Given the prior \(\beta_i \sim N(\mu_0, \sigma_0)\), where \(\mu_0, \sigma_0 = (E(r_{it}), 0_{1 \times J})', V_{0,\beta_i} = diag([2.5^2\sigma^2_{it}, \{2.5^2\sigma^2_{it}/\sigma^2_{X_{i,t-1}}\}_{j=1}^{J}]),\) where \(J\) is the number of predictors, the posterior distribution \(p(\beta_i|.) \sim N(\mu', \sigma')\), where \(\mu'_{\beta_i} = (\mu_{0,\beta_i}V_{0,\beta_i}^{-1} + \Sigma_{t=1}^{T}r_{it}e^{-h_{it}}X_{i,t-1})V_{\beta_i}^{-1}\) and \(V_{\beta_i} = (V_{0,\beta_i}^{-1} + \Sigma_{t=1}^{T}e^{-h_{it}}X_{i,t-1}X_{i,t-1}'),\) respectively.

To sample \(\gamma_{i1}, \gamma_{i2}, \sigma_i, \{h_i\}_{t=1}^{T}|\beta_i\), we first compute the residual \(e_{i,t} = r_{i,t} - \beta_i X_{i,t-1}\) and follow Kim et al. (1998) to implement the estimation procedure with the following priors for the parameters governing \(\{h_i\}_{t=1}^{T}\): diffuse prior for \(\gamma_{i1}, (\gamma_{i2} + 1)/2 \sim Beta(20, 1.5)\), and \(\sigma_i \sim IG(5, 0.05)\).

In the second stage, the MCMC sampler is as follows:

- initialize \(A^{(1)}, d^{(1)}, k^{(1)}\) and \(\{Q^{(1)}\}_{t=1}^{T}\).

- at the \(\tau^{th}\) iteration, sample \(Q_{it}^{(\tau)}|Q_{it}^{(\tau-1)}, A^{(\tau-1)}, d^{(\tau-1)}, k^{(\tau-1)}, z_{it}^{T}\).
• sample $A^{(\tau)}|\{Q^{(\tau)}\}_{t=1}^T, d^{(\tau-1)}, k^{(\tau-1)}, z_t^{(\tau)}$.

• sample $d^{(\tau)}|\{Q^{(\tau)}\}_{t=1}^T, A^{(\tau)}, k^{(\tau-1)}, z_t^{(\tau)}$.

Given $P_t = (\text{diag}\{\text{vecd}(Q_t)\})^{-1/2}Q_t(\text{diag}\{\text{vecd}(Q_t)\})^{-1/2}$, the standardized residuals $z_t \sim N(0, P_t)$ yield the following likelihood function where,

$$L(z_t^T|A, d, k, \{Q\}_{t=1}^T) \propto \prod_{1=1}^T P_t^{-1}|P_t^{-1}z_t^T \times |S_{-1}^{-1}| \frac{1}{2} Q_t^{-1}z_T^{-1} \exp(-\frac{1}{2} \text{tr}[S_{-1}^{-1}Q_t^{-1}])$$

where $S_t = (1/k)Q_t^{-d/2}AQ_t^{-d/2}$, $\Gamma_4(k) = \Gamma(k)\ldots\Gamma(k-3)$, $Q_0^{-1} = I_A$, $A^{-1} \sim W_A(4, 0.25I_A)$, $d \sim U(-1, 1)$, $k \sim Exp(5)I(k > 4)$, where $W_A$ is the 4-variate Wishart distribution. The conditional posterior distribution of $Q_t^{-1}$ is as follows

$$p(Q_t^{-1} | .) \propto W_A(Q_t^{-1}|k, S_{t-1})N(0, P_t)W_A(Q_{t+1}^{-1}|k, S_t)$$

$$\propto |Q_t^{-1}|^{(k-d)-4} \exp(-\frac{1}{2} \text{tr}[S_{t-1}^{-1}Q_t^{-1}]) - \frac{1}{2} z_t^P P_t^{-1}z_t - \frac{1}{2} \text{tr}[S_{t-1}^{-1}Q_t^{-1}])$$

$$\propto W_A(Q_t^{-1}|k + 1, (S_{t-1}^{-1} + z_t z_t^{-1})^{-1})f(Q_t^{-1})$$

where

$$f(Q_t^{-1}) = |P_t^{-1}|^{\frac{1}{2}} |Q_t^{-1}|^{-\frac{1}{2} - d} e^{\frac{1}{2} \text{tr}[S_{t-1}^{-1}Q_t^{-1}]} - \frac{1}{2} \text{tr}[(P_t^{-1} - Q_t^{-1})z_t z_t^{-1}], t < T$$

$$f(Q_T^{-1}) = \exp(-\frac{1}{2} \text{tr}[(P_T^{-1} - Q_T^{-1})z_T z_T^{-1}]), t = T.$$ 

The Metropolis-Hastings algorithm is used to sample $Q_t^{-1}$ from the posterior distribution with the proposal $W_A(Q_t^{-1}|k + 1, (S_{t-1}^{-1} + z_t z_t^{-1})^{-1})$ and acceptance ratio $1 \wedge f(Q_t^{-1})/f(Q_t^{-1})$, where $Q_t^{-1}$ is the proposal and current values, respectively.

The conditional posterior distribution of $A^{-1}$ is derived as

$$p(A^{-1}, \cdot) \propto W_A(4, 0.25I_A)W_A(\gamma, C) \propto |A^{-1}|^{-\frac{1}{2}} \exp(-\frac{1}{2} \text{tr}[A^{-1}])$$

$$\times |A^{-1}|^{\frac{T}{2}} \exp(-\frac{1}{2} \text{tr}[C A^{-1}]) \propto W_A(A^{-1}|\hat{\gamma}, \hat{C})$$

where $\hat{C}^{-1} = 4I_A + C^{-1}$, $C^{-1} = k \sum_{t=1}^T Q_t^{-1}Q_t^{-d/2}$, $\hat{\gamma} = \gamma - 1, \gamma = kT + 5$. $A^{-1}$ is sampled from Gibbs sampler.
The conditional posterior distribution of \( d \) is derived as

\[
p(d | \cdot) \propto p(d) \prod_{t=1}^{T} |S_{t-1}^{-1}Q_{t}^{-1}|^{-\frac{k}{2}} \exp \left( -\frac{1}{2} tr[S_{t-1}^{-1}Q_{t}^{-1}] \right)
\]

\[
\propto |Q_{t-1}^{-1}|^{-\frac{d}{2}} \exp \left( -\frac{1}{2} tr[A^{-1}C^{-1}] \right) I_{(-1,1)}(d)
\]

\[
\propto \exp \left( -(kd/2)T \ln |Q_{t-1}^{-1}| - \frac{1}{2} tr[A^{-1}C^{-1}] \right) I_{(-1,1)}(d).
\]

The conditional posterior distribution of \( k \) is written as

\[
p(k | \cdot) \propto \exp \left( -5k + 2Tk \ln(k/2) - \frac{T}{2} k \ln|A| - T \sum_{i=1}^{4} \ln \Gamma \left( \frac{k + 1 - i}{2} \right) \right)
\]

\[
\times \exp \left( \frac{k}{2} T \ln|Q_{t-1}^{d/2}Q_{t-1}^{-1}Q_{t}^{d/2}| - \frac{1}{2} tr[A^{-1}C^{-1}] \right)
\]

We apply the Adaptive Rejection Metropolis Sampling of [Gilks et al., 1995] for \( p(d | \cdot) \) and Adaptive Random Walk Metropolis Sampling of [Haario et al., 2001] for \( p(k | \cdot) \) with acceptance ratio \( 1 \wedge p(k^p | \cdot)/p(k^c | \cdot) \), where \( k^p \) and \( k^c \) are the proposal and current values, respectively. The MCMC simulation is conducted with 10,000 iterations, where the first 5,000 draws are discarded, and the next 5,000 iterations are used to calculate the posterior means.

**Appendix: time-varying asset weight**

In this section we illustrate the calculation of time-varying portfolio weights in equity, bond, oil, and other commoditess. We obtain the global asset weights in equity and (government + non-government) from [Doeswijk et al., 2014] at annual frequency over 1959-2012. We assume that the 2012 values are held constant through the period 2013-2015 and that the monthly asset weights of bond and equity carry the annual value of the same year.

We calculate the percentage of equity investment relative to commodity (equity-to-commodity ratio) by the total notional amount of equity and commodity outstanding in global OTC derivative markets. The data is available from Bank of International Settlements [23] and is collected by BIS every 6 months since 1998. We assume that the equity-to-commodity ratio value in the first half of 1998 is assumed to hold constant prior to 1998 and that the monthly equity-to-commodity ratios carry the values of the same half-year.

[23]See https://www.bis.org/statistics/full_data_sets.htm
Given the global equity and bond asset weights and equity-to-commodity ratio, we are able to derive the weights in these asset classes at monthly frequency. To calculate the contribution of oil or energy market in commodity asset class, we collect the data on futures contracts of constituents in Thomson Reuters/Core Commodity CRB (Non-Energy) Index from Datastream. For each commodity, we compute the month-end dollar values of open interest from the nearest contract month, and aggregate all dollar values of constituents in non-energy and all (energy + non-energy) sectors. The monthly dollar value ratio of non-energy (energy) over all sectors is multiplied by commodity weight to be the weight in other commodities (oil). The correlation between PBMEU computed from constant and time-varying weights is 0.8623.

This figure displays different (normalized) portfolio-based measure of economic uncertainty PBMEU with constant and varying asset weights over the sample period January 1984 through December 2015 and time-varying asset weights. In Panel A, each normalized series is calculated by first deducting its sample mean and then dividing by its sample standard deviation. In Panel B, the time-varying asset weights are presented.
This figure displays the mean impulse responses of Industrial Production (IP) and Employment in the VAR model framework with portfolio-based measure of economic uncertainty PBMEU over the sample period January 1984 through December 2015. Panel A and B present the mean impulse response of IP and employment response following a unit of standard deviation of PBMEU uncertainty shock under different model specifications: “base” or VAR model (1), “base full” or VAR model (2), “lags6” by increasing the number of lags in model (1), “mich first” by including the Michigan Consumer Sentiment index (MCSI) before PBMEU in model (1), and “mich second” by including MCSI after PBMEU in model (1). As a robustness check, the variable ordering is reversed for all model specifications. These are noted by adding a “rev” prefix to the model name.
Figure 12: The robust impact of upside correlation on Industrial Production

This figure displays the mean impulse responses of Industrial Production in the VAR model framework with upside correlation-based PBMEU over the sample period January 1984 through December 2015. The upside correlation-based PBMEU at time $t$ defined as the interaction between PBMEU$_t$ and a ranking variable $CDF_{p_t} = \sum_{\rho_t \leq \rho}^{\frac{T}{1+\rho}}$, where $T$ is the sample size and $\rho_t$ is the stochastic correlation estimate at time $t$. Panel A and B present the Industrial Production response following a unit of standard deviation of upside correlation-based PBMEU in the different VAR model specifications for equity-oil and oil-bond correlations. The considered model specifications include: “base” or VAR model (1), “base full” or VAR model (2), “lags6” by increasing the number of lags in model (1), “mich first” by including the Michigan Consumer Sentiment index (MCSI) before PBMEU in model (1), and “mich second” by including MCSI after PBMEU in model (1). As a robustness check, the variable ordering is reversed for all model specifications. These are noted by adding a “rev” prefix to the model name.
Figure 13: The robust impact of upside correlation on Employment

This figure displays the mean impulse responses of Employment in the VAR model framework with upside correlation-based PBMEU over the sample period January 1984 through December 2015. The upside correlation-based PBMEU at time $t$ is defined as the interaction between PBMEU$_t$ and a ranking variable $CDF_{\rho_t} = \sum_{i=1}^{T} I(\rho_i \leq \rho_t)$, where $T$ is the sample size and $\rho_t$ is the stochastic correlation estimate at time $t$. Panel A and B present the Industrial Production response following a unit of standard deviation of upside correlation-based PBMEU in the different VAR model specifications for equity-oil and oil-bond correlations. The considered model specifications include: “base” or VAR model (1), “base full” or VAR model (2), “lags6” by increasing the number of lags in model (1), “mich first” by including the Michigan Consumer Sentiment index (MCSI) before PBMEU in model (1), and “mich second” by including MCSI after PBMEU in model (1). As a robustness check, the variable ordering is reversed for all model specifications. These are noted by adding a “rev” prefix to the model name.
This figure displays the mean impulse responses of Industrial Production in the VAR model framework with downside correlation-based PBMEU over the sample period January 1984 through December 2015. The downside correlation-based PBMEU at time $t$ defined as the interaction between PBMEU at time $t$ and a ranking variable $1 - \text{CDF}_p = 1 - \sum_{\rho_i \leq \rho_t} \frac{T}{T}$, where $T$ is the sample size and $\rho_t$ is the stochastic correlation estimate at time $t$. Panel A and B present the Industrial Production response following a unit of standard deviation of downside correlation-based PBMEU in the different VAR model specifications for equity-oil and oil-bond correlations. The considered model specifications include: “base” or VAR model (1), “base full” or VAR model (2), “lags6” by increasing the number of lags in model (1), “mich first” by including the Michigan Consumer Sentiment index (MCSI) before PBMEU in model (1), and “mich second” by including MCSI after PBMEU in model (1). As a robustness check, the variable ordering is reversed for all model specifications. These are noted by adding a “rev” prefix to the model name.
Figure 15: The robust impact of downside correlation on Employment

This figure displays the mean impulse responses of Employment in the VAR model framework with downside correlation-based PBMEU over the sample period January 1984 through December 2015. The downside correlation-based PBMEU at time $t$ defined as the interaction between PBMEU and a ranking variable $1 - CDF_{\rho_t} = 1 - \sum_{\rho_i \leq \rho_t} \frac{i}{T}$, where $T$ is the sample size and $\rho_t$ is the stochastic correlation estimate at time $t$. Panel A and B present the Industrial Production response following a unit of standard deviation of downside correlation-based PBMEU in the different VAR model specifications for equity-oil and oil-bond correlations. The considered model specifications include: “base” or VAR model (1), “base full” or VAR model (2), “lags6” by increasing the number of lags in model (1), “mich first” by including the Michigan Consumer Sentiment index (MCSI) before PBMEU in model (1), and “mich second” by including MCSI after PBMEU in model (1). As a robustness check, the variable ordering is reversed for all model specifications. These are noted by adding a “rev” prefix to the model name.
This figure displays the impulse responses of Industrial Production (IP) and Employment in the VAR model framework with portfolio-based measure of economic uncertainty PBMEU over the sample period January 1984 through December 2015. PBMEU is constructed from different weight sets, namely constant and time-varying. Panel A and B present the IP and employment mean response (middle line) following a unit of standard deviation of different uncertainty shocks in the “base” VAR model and the 90% confidence band. Panel C and D present the mean impulse response of IP and employment response following a unit of standard deviation of PBMEU uncertainty shock (with time-varying weight) under different model specifications: “base” or VAR model (1), “base full” or VAR model (2), “lags6” by increasing the number of lags in model (1), “mich first” by including the Michigan Consumer Sentiment index (MCSI) before PBMEU in model (1), and “mich second” by including MCSI after PBMEU in model (1). As a robustness check, the variable ordering is reversed for all model specifications. These are noted by adding a “rev” prefix to the model name.
This figure displays the impulse responses of Industrial Production and Employment in the VAR model framework with upside and downside correlation-based PBMEU over the sample period January 1984 through December 2015. PBMEU is constructed from different weight sets, namely constant and time-varying. The upside (downside) correlation-based PBMEU at time $t$ defined as the interaction between PBMEU and a ranking variable $CDF_{\rho_t} = \frac{\sum_{i=1}^{T} \rho_i \leq \rho_t}{T} (1 - \frac{\sum_{i=1}^{T} \rho_i \leq \rho_t}{T})$, where $T$ is the sample size and $\rho_t$ is the stochastic stock-oil correlation estimate at time $t$. Panel A and B (C and D) present the Industrial Production and Employment mean response (middle line) following a unit of standard deviation of upside (downside) correlation-based PBMEU in VAR model and the 90% confidence band.
This figure displays the impulse responses of Industrial Production and Employment in the VAR model framework with upside and downside stock-oil correlation-based PBMEU over the sample period January 1984 through December 2015. PBMEU is constructed from different weight sets, namely constant and time-varying. The upside (downside) correlation-based PBMEU at time $t$ defined as the interaction between PBMEU$_t$ and a ranking variable $CDF_{\rho_t} = \frac{\sum_{i=1}^{T} \mathbb{1}_{\rho_i \leq \rho_t}}{T}$, where $T$ is the sample size and $\rho_t$ is the stochastic oil-bond correlation estimate at time $t$. Panel A and B (C and D) present the Industrial Production and Employment mean response (middle line) following a unit of standard deviation of upside (downside) correlation-based PBMEU in VAR model and the 90% confidence band.
Figure 19: The impact of upside and downside realized stock-oil correlation on Industrial Production for time-varying weight

This figure displays the impulse responses of Industrial Production in the VAR model framework with upside and downside realized stock-oil correlation-based uncertainty over the sample period January 1984 through December 2015 for PBMEU with different asset weights. The upside (downside) correlation-based uncertainty at time $t$ is defined as the interaction between uncertainty measure at $t$ and a ranking variable $CDF_{\rho_t} = \frac{1}{T} \left( \frac{\sum_{i=1}^{T} \rho_i \leq \rho_t}{T} \right)$, where $T$ is the sample size, and $\rho_t$ is the realized stock-oil correlation at $t$ estimated from 3-, 6-, and 12-month rolling window of daily returns. The panels present the Industrial Production mean response (middle line) following a unit of standard deviation of upside (downside) correlation-based uncertainty in VAR model and the 90% confidence band.
This figure presents the log monthly oil consumption, production and inventory of 8 OECD countries and US over the period January 1994 to December 2015. The seasonally adjusted series are also computed following the X-12 procedure of the U.S. Census. The key political economic events are represented by shaded areas in each panel.
Figure 21: Annual log consumption and production

This figure presents the log annual oil consumption and production at the global level over 1980-2014.
References


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