A Subsidy Inversely Related to the Product Price

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Abstract

This paper proposes a new subsidy scheme for promoting a target good's consumption, where subsidy payment is inversely related to the good's price. Under imperfect competition, this scheme makes the demand faced by producers more elastic, thereby reducing their power to raise prices and increasing subsidy pass-through to consumers. Compared to commonly-used specific or *ad valorem* subsidies, it can lower government expenditure for inducing a given output, and flexibly adjust the incidence on producers. Simulations based on an actual U.S. subsidy programme on electric vehicles indicate up to 50–81% reductions in government spending if it replaces the current specific subsidy.

Keywords: Subsidy efficiency; Subsidy incidence; Imperfect competition; Cournot oligopoly; Electric vehicles

JEL Classification: D43, H21, H22, L13, Q58

1 Introduction

Many government subsidy programmes across countries aim to promote the use of target goods by partially offsetting the purchase cost through financial incentives such as grants, rebates, and tax credits/deductions offered to their consumers (or producers). Such goods are related to, for example, environmental/energy issues (e.g., electric cars and solar PV panels), childcare and education (e.g., nursery), healthcare (e.g., treatment, insurance, and drugs), and housing (e.g., purchase and rental). Typically, subsidy payment per unit of a target good is either independent of or proportional to its price ("specific" or "*ad valorem*").[1] Sometimes it is given by a mixture of the two schemes (e.g., an *ad valorem* subsidy with a cap). This paper presents a new subsidy scheme to be used in such programmes, which embodies a mechanism

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[1]In income tax, a tax credit is in effect a specific subsidy of the amount of the credit, and a tax deduction is an *ad valorem* subsidy at the marginal income tax rate.

to reduce producers' capacity to raise prices and thus significantly saves government expenditure for inducing a target level of market output: simulations based on an actual electric car subsidy in the U.S. indicate a substantial saving of up to 50–81%.

In many cases, the target good of a subsidy programme is produced under imperfect competition, and/or associated with positive externalities or merit-good elements, so its supply without a subsidy falls short of the socially efficient level. Also, it is often desirable from a distributional perspective to help low- and middle-income households acquire such goods. For these reasons, government subsidy programmes intend to lower the effective price that consumers pay out-of-pocket for the good, and thus encourage its usage.

Under imperfect competition, the design of a tax or subsidy policy (for example, specific or *ad valorem*) has welfare implications.[2] A number of previous studies use theoretical models of imperfect competition in a closed-economy context, and analyse the relative efficiency of different policy designs.[3] On taxes, Suites and Musgrave (1953), Delipalla and Keen (1992), Skeath and Trandel (1994), Anderson, de Palma and Kreider (2001), and Hamilton (2009), to name a handful, compare specific with *ad valorem* taxes. Myles (1996), Hamilton (1999), and Carbonnier (2014) examine more general tax designs that contain specific and *ad valorem* taxes as special cases, and their frameworks relate to the subsidy policy proposed here in that the key channel is a policy-induced increase in the elasticity of demand faced by producers. On subsidies, there are a limited number of papers on the issue: Valido et al. (2014) and Liang, Wang and Chou (2017) contrast specific with *ad valorem* subsidies. This paper considers another form of subsidies and compare it with the commonly-used specific and *ad valorem* forms.

This paper has been motivated by two subsidy programmes in Japan (between 2009–2013) and the U.K. (since 2016) that have a distinct feature not observed in the usual specific or *ad valorem* subsidies. In the Japanese subsidy (rebate) programme on the purchase of residential solar photovoltaic (PV) systems, unlike in a specific or *ad valorem* subsidy programme, the rebate amount (per unit quantity) to a buyer is *decreasing* in the pre-rebate, unit price of the system, thereby giving sellers and buyers an incentive to trade at lower (pre-rebate) prices. Specifically, Table 1 shows that as the transaction price of a solar PV system per kW of capacity (inclusive of installation and other related costs) is lowered, the buyer (household) becomes eligible for a higher subsidy per kW. For example, in 2012 a household received no rebate if the (pre-rebate, per-kW) price of the purchased system was above ¥550,000; a rebate of ¥30,000 per kW if it was between ¥475,001–¥550,000; and ¥35,000 if it was equal to or below ¥475,000. The demand for a solar PV system jumps up as its price goes down below a threshold, incentivising sellers to take advantage of this structure. Thus, it was expected that the scheme would work as a mechanism to lower not only consumer prices (i.e., post-rebate,

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[2]In contrast, under perfect competition where the firms are price-takers with no market power, the difference in the design of tax/subsidy policies is insignificant.

[3]There are papers that investigate the difference between the specific and *ad valorem* forms of import tariffs or export subsidies in an open-economy setting where domestic and foreign producers are treated differently (e.g., Brander and Spencer, 1984; Collie, 2006). This paper focuses on a closed-economy setting where producers are treated equally regardless of nationality, and thus export subsidies are out of its scope.

Table 1: Residential Solar PV Installation Subsidy in Japan

Fiscal year	Subsidy (¥/kW)	Condition on pre-rebate price p_{pre} (¥/kW)	
2009	0	if $700{,}000 < p_{\text{pre}}$	
	70,000	if	$p_{\text{pre}} \leq 700{,}000$
2010	0	if $650{,}000 < p_{\text{pre}}$	
	70,000	if	$p_{\text{pre}} \leq 650{,}000$
2011	0	if $600{,}000 < p_{\text{pre}}$	
	48,000	if	$p_{\text{pre}} \leq 600{,}000$
2012	0	if $550{,}000 < p_{\text{pre}}$	
	30,000	if $475{,}000 < p_{\text{pre}} \leq 550{,}000$	
	35,000	if	$p_{\text{pre}} \leq 475{,}000$
2013	0	if $500{,}000 < p_{\text{pre}}$	
	15,000	if $410{,}000 < p_{\text{pre}} \leq 500{,}000$	
	20,000	if	$p_{\text{pre}} \leq 410{,}000$

The table shows rebates (per kW of capacity) offered to households for installing residential solar PV systems. The amount of a rebate is conditional on the (pre-rebate) transaction price (¥/kW) of a system.

out-of-pocket prices for households), but also producer prices (i.e., pre-rebate, transaction prices received by sellers), leading to more than full pass-through of the subsidy to consumers and further accelerating the diffusion of solar PV systems that have positive externalities. Note that introducing a specific or *ad valorem* subsidy on a good normally reduces consumer prices, but increases producer prices, resulting in less than full pass-through of the subsidy to consumers. In the U.K., since 2016 the government has adopted a similar design in its grants for electric and hybrid vehicles: a threshold at the vehicle price of £60,000.

Transaction data suggest that the subsidy design indeed worked well in lowering (pre-rebate) prices. Figure 1 shows the (pre-rebate) price distribution of household solar PV systems that were installed during fiscal year 2012 (April 2012–March 2013). Solar PV system prices bunch in the bins just below the threshold prices (¥475,000 and ¥550,000), indicating that sellers have price-setting power, take account of the subsidy rule, and make transactions at lower prices than they would without such a scheme. Since the threshold prices were lowered significantly year by year, the subsidy design helped rapidly reduce (pre-rebate) prices and further accelerate the diffusion of the technology.

Despite the thought-provoking observation, no previous research exists, to my knowledge, that uses an economics framework to analyse the effect of making subsidy payment conditional on and inversely related to the price of a target product. This paper proposes and evaluates such a new subsidy scheme. More specifically, I consider a subsidy that is offered for the purchase or production of a good on the condition that its price is less than a government-set threshold, and as the price goes down, the subsidy per unit of the good increases in propor-
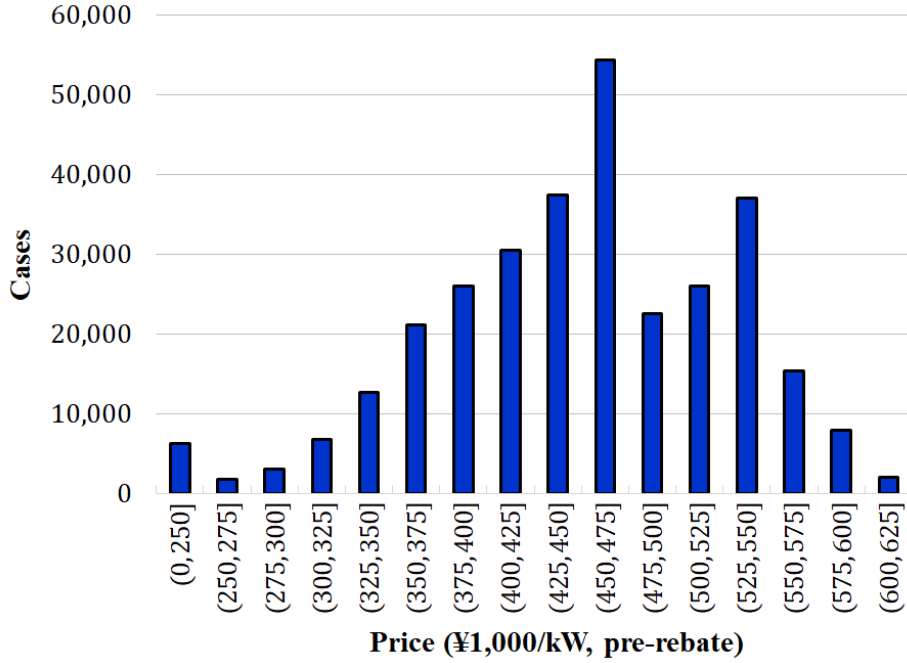
Figure 1: Distribution of Residential Solar PV System Prices (2012)

tion to the difference between the threshold and price. That is, the government sets two policy parameters: the threshold price level, and the rate at which subsidy payment increases with price reduction ($\bar{p}$ and r, respectively, in the model below). With a model of imperfect competition (monopoly or Cournot oligopoly), I contrast this inversely price-related (IPR) subsidy with the benchmark case of no subsidy, and widely-used specific and *ad valorem* subsidies in terms of various characteristics in equilibrium (e.g., output, price, profits, and government expenditure).

From the government's perspective, the advantages of the proposed design consist in efficiency and flexibility. First, the IPR subsidy is more efficient than the specific or *ad valorem* subsidy in the sense that the former requires less government spending than the latter for inducing the same output level through subsidisation. Equivalently, with a given budget, the IPR form can generate a higher output than the specific or *ad valorem* form. The relative efficiency of the IPR scheme follows because it makes the demand curve *faced by producers* more elastic. In effect, the scheme partially compensates producers for cutting prices, so a \$1 reduction in the consumer price can be achieved by a smaller reduction in the producer price. This means that they face more elastic demand than under no subsidy, and under specific or *ad valorem* subsidies. Elastic demand erodes producers' power to raise prices, thus making it easier for the regulator to induce lower prices (higher outputs).

Second, the other side of the coin is that in inducing a given output level, the IPR scheme allows the regulator to pursue an additional goal of adjusting the incidence on producers. Given a target output to be induced by the IPR scheme, the regulator can choose its policy parameters to make a firm's equilibrium profits substantially higher or lower than the case of no government intervention. In other words, besides targeting an output level, the regulator can

in effect choose whether the scheme works as subsidisation or implicit taxation on producers, and to what extent. This flexibility does not hold for specific or *ad valorem* subsidies because they have just one policy parameter, and the regulator's choice of it determines equilibrium output, consumer surplus, and profits at once. On the other hand, the availability of the two parameters allows the IPR form to induce a target output, and at the same time adapt the incidence on producers to suit the government's objectives and market situations. That is, while increasing the supply and use of a subsidised product, the IPR scheme can also serve to, for example, financially support an emerging industry (e.g., electric carmakers), or lower the profits of producers who earn huge economic rents due to imperfect competition.

Simulations with actual market data reveal substantial impacts of the IPR form. As of 2017, buyers of electric vehicles in the U.S. are eligible to claim a subsidy of $7,500 through a tax credit. Using the data on the U.S. electric vehicle (EV) market, I simulate the impact of replacing the current specific subsidy of $7,500 per vehicle with an IPR subsidy in such a way that the market sales remain the same. Simulation results suggest that the IPR form can induce the same market sales with a subsidy payment of just $1,405–$3,737 per vehicle, saving the programme budget by as much as 50–81%. As to the incidence on producers, the regulator can adjust the parameters of the IPR scheme to vary equilibrium producer surplus between $72–150 million higher and $63–169 million lower than under no government intervention.

An issue with the IPR form is the producer's incentive to lower the quality of the product. This is an easy way to reduce the production cost and price of the good, and thus benefit from a higher subsidy. Quality degradation, however, may be undesirable from the perspectives of the government, consumers and society. I show that this incentive can be deterred in a simple way: the regulator can supplement the IPR scheme by making the price threshold for subsidy eligibility increasing in quality, and thus rewarding higher quality with higher subsidy payment. In practice, this result implies that the IPR form works better when information is available about product quality and the price attached to it. For example, it can be applied to subsidy programmes on environment-friendly durable goods, pharmaceuticals, (privately supplied) health insurance, and housing.

The rest of the paper is organised as follows. Section 2 considers the firms' optimisation under imperfect competition, conditional on different subsidy policies, and derives the equilibrium(a) under each policy. Section 3 focuses on the government's choice of policy variables, and compares equilibrium outcomes under different policies. Section 4 evaluates the impact of the IPR scheme through simulations based on an actual U.S. subsidy programme on electric vehicles. Section 5 extends the analysis to discuss how the IPR scheme can be augmented to prevent quality degradation. Section 6 concludes.

2   Subsidy Policies and Corresponding Equilibria

Suppose that a homogeneous good is produced by $n(\geq 1)$ identical firm(s). The cost function is given by $F + cq_i$, where $F(\geq 0)$ is the fixed cost, $c(> 0)$ is the marginal cost, and $q_i(\geq 0)$ is

firm i's output.

The inverse demand function is represented by $p(Q)$, where $Q \equiv \sum_{i=1}^{n} q_i$ is industry output, and holds the following properties:[4]

Assumption 1. $p : \mathbb{R}_{++} \to \mathbb{R}_{+}$ *is continuous and satisfies* $\lim_{Q \to 0} p(Q) > c$ *and* $p(Q) = 0$ *for all sufficiently large* Q. *At all* Q *such that* $p(Q) > 0$, $p(Q)$ *is twice continuously differentiable,* $p'(Q) < 0$, *and* $p''(Q)Q + 2p'(Q) < 0$.[5]

Assumption 1 implies the following inequality to be used throughout the paper: for all Q such that $p(Q) > 0$ and for all $q \in [0, Q]$,

$$p''(Q)q + 2p'(Q) < 0. \tag{2}$$

[Policy A: No Subsidy]

Consider Cournot competition by $n(\geq 2)$ firms (or for $n = 1$, simply the monopolist's profit maximisation).[6] Firm i sets output q_i to maximise its profits

$$\pi_A(q_i, Q_{-i}) = [p(Q_{-i} + q_i) - c]q_i - F, \tag{3}$$

where $Q_{-i} \equiv \sum_{k \neq i} q_k$ if $n \geq 2$ (or $Q_{-i} = 0$ if $n = 1$). With strict concavity of $\pi_A(q_i, Q_{-i})$ with respect to q_i and assuming an interior solution,[7] for each Q_{-i} there is a unique maximiser $q_i = q_A^b(Q_{-i})$ defined by the following first order condition (FOC):

$$p'(Q_{-i} + q_i)q_i + p(Q_{-i} + q_i) - c = 0. \tag{4}$$

With an additional assumption that $p''(Q_{-i} + q_A^b(Q_{-i})) \cdot q_A^b(Q_{-i}) + p'(Q_{-i} + q_A^b(Q_{-i})) < 0$,[8] there exists a unique and symmetric Nash equilibrium under policy A (denoted by E_A), where

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[4]The inverse demand function is derived from the following optimisation problem of a representative consumer with quasi-linear utility $U(x, Q) = x + u(Q)$ (see, for example, Vives, 1999):

$$\max_{x, Q} x + u(Q) \text{ s.t. } x + pQ \leq I, \tag{1}$$

where x is the numéraire good (the composite of all other goods) and I is income. An interior solution is characterised by $p = u'(Q)$, so that the inverse demand function can be written as $p(Q) = u'(Q)$.

[5]The condition that $p''(Q)Q + 2p'(Q) < 0$ limits the convexity of the inverse demand curve: the elasticity of the slope of inverse demand, $-p''(Q)Q/p'(Q)$, is less than 2. More intuitively, this condition means that in case the market is monopolistic, the marginal revenue curve faced by the monopolist is downward sloping.

[6]The paper mostly focuses on an oligopoly ($n \geq 2$) that involves strategic interaction between firms. Similar results also hold for the simpler case of a monopoly ($n = 1$), which I will briefly discuss in passing.

[7]Strict concavity holds because $\partial^2 \pi_A(q_i, Q_{-i})/\partial q_i^2 = p''(Q_{-i} + q_i)q_i + 2p'(Q_{-i} + q_i) < 0$ by (2). It similarly holds for other policies to be discussed below.

[8]This condition ensures that the slope of the best response function is greater than -1: by applying the implicit function theorem to (4),

$$\frac{dq_A^b(Q_{-i})}{dQ_{-i}} = -\frac{p''(Q_{-i} + q_A^b(Q_{-i})) \cdot q_A^b(Q_{-i}) + p'(Q_{-i} + q_A^b(Q_{-i}))}{p''(Q_{-i} + q_A^b(Q_{-i})) \cdot q_A^b(Q_{-i}) + 2p'(Q_{-i} + q_A^b(Q_{-i}))} \in (-1, 0). \tag{5}$$

The slope strictly between -1 and 0 in turn guarantees the existence of a unique and symmetric Nash equilibrium (see, e.g., Vives, 1999). The same comment applies to the analogous functions $q_B^b(Q_{-i})$, $q_C^b(Q_{-i})$, and $q_{D_s}^b(Q_{-i})$ under Policies B, C, and D, respectively, that we consider next.

each firm's output $q_A^* = q_A^b((n-1)q_A^*)$ satisfies

$$p'(nq_A^*)q_A^* + p(nq_A^*) - c = 0. \tag{6}$$

[Policy B: Specific Subsidy]

Suppose that the government offers consumers a specific subsidy of $z(> 0)$ per unit of the good purchased. The subsidy is provided as a rebate or tax credit, for example. Importantly, it makes no difference whether the direct recipients of the subsidy are consumers or producers (physical neutrality; see, e.g., Weyl and Fabinger, 2013). The paper mainly thinks in terms of consumption subsidies (offered directly to consumers), but its results are valid for production subsidies (offered directly to producers) as well.

As demand depends on the price consumers pay out of pocket, the inverse demand function $p(Q)$ now gives the consumer price (i.e., the effective price consumers face after accounting for the subsidy). The producer price p_p (i.e., the price received by a firm) equals $p(Q) + z$. Thus, firm i's profits are given by

$$\pi_B(q_i, Q_{-i}) = [p(Q_{-i} + q_i) + z - c]q_i - F. \tag{7}$$

As in Policy A, for each Q_{-i} there is a unique maximiser $q_i = q_B^b(Q_{-i})$ defined by the following FOC:

$$p'(Q_{-i} + q_i)q_i + p(Q_{-i} + q_i) + z - c = 0. \tag{8}$$

With an additional assumption that $p''(Q_{-i} + q_B^b(Q_{-i})) \cdot q_B^b(Q_{-i}) + p'(Q_{-i} + q_B^b(Q_{-i})) < 0$, there exists a unique and symmetric Nash equilibrium under policy B (denoted by E_B), where each firm's output $q_B^* = q_B^b((n-1)q_B^*)$ satisfies

$$p'(nq_B^*)q_B^* + p(nq_B^*) + z - c = 0. \tag{9}$$

[Policy C: *Ad Valorem* Subsidy]

Suppose that the government offers consumers an *ad valorem* subsidy of vp per unit of the good purchased ($v > 0$). As under Policy B, the consumer price is $p(Q)$, and the producer price is $(1 + v)p(Q)$. Thus, firm i's profits are given by

$$\pi_C(q_i, Q_{-i}) = [(1 + v)p(Q_{-i} + q_i) - c]q_i - F. \tag{10}$$

As in previous policies, for each Q_{-i} there is a unique maximiser $q_i = q_C^b(Q_{-i})$ defined by the following FOC:

$$(1 + v)[p'(Q_{-i} + q_i)q_i + p(Q_{-i} + q_i)] - c = 0. \tag{11}$$

With an additional assumption that $p''(Q_{-i} + q_C^b(Q_{-i})) \cdot q_C^b(Q_{-i}) + p'(Q_{-i} + q_C^b(Q_{-i})) < 0$, there exists a unique and symmetric Nash equilibrium under policy C (denoted by E_C), where

each firm's output $q_C^* = q_C^b((n-1)q_C^*)$ satisfies

$$(1+v)[p'(nq_C^*)q_C^* + p(nq_C^*)] - c = 0. \tag{12}$$

[Policy D: IPR Subsidy]

The government conditionally offers consumers a subsidy that is inversely related to the purchase price. No subsidy is provided if the price of the good is greater than or equal to a government-set threshold $\overline{p}$ (i.e., if $p(Q) \geq \overline{p}$), where we assume $\overline{p} > c$. If the price is below $\overline{p}$, the subsidy per unit of the good increases linearly as the price *decreases*. Specifically, with $p(Q)$ defined as the consumer price, the subsidy per unit of the good when $p(Q) < \overline{p}$ equals $r[\overline{p} - p(Q)]$, where $0 < r < 1$. The producer price p_p is $p(Q) + r[\overline{p} - p(Q)]$.[9] In other words, $p_p = p(Q) + r[\overline{p} - p(Q)]$ is the inverse (market) demand curve faced by the firms. Thus, the assumption $r < 1$ ensures that when $p(Q) < \overline{p}$, the firms face a downward sloping inverse demand curve ($r < 1 \iff \frac{dp_p}{dQ} = (1-r)p'(Q) < 0$). These assumptions on $\overline{p}$ and r are restated for ease of reference:

Assumption 2. $\overline{p} > c$ and $0 < r < 1$.

Define a function $\pi_{D_S} : \mathbb{R}_+^2 \to \mathbb{R}$ as

$$\pi_{D_S}(q_i, Q_{-i}) = \{p(Q_{-i} + q_i) + r[\overline{p} - p(Q_{-i} + q_i)] - c\}q_i - F. \tag{13}$$

Depending on whether or not the price is less than $\overline{p}$ and the good is eligible for the subsidy, $\pi_D(q_i, Q_{-i})$ (firm i's profits under Policy D) equals either $\pi_{D_S}(q_i, Q_{-i})$ or $\pi_A(q_i, Q_{-i})$ (its profits under Policy A):

$$\pi_D(q_i, Q_{-i}) = \begin{cases} \pi_A(q_i, Q_{-i}) & \text{if} \quad p(Q_{-i} + q_i) \geq \overline{p}, \\ \pi_{D_S}(q_i, Q_{-i}) & \text{if} \quad p(Q_{-i} + q_i) < \overline{p}. \end{cases} \tag{14}$$

Two comments follow about the setup of the policy. First, in the paper's framework (Cournot oligopoly), the product price is determined by the aggregate quantity. Note that a Cournot model of quantity competition is considered a reduced form of a two-stage game in which firms commit to quantities in the first stage and set prices in the second (Kreps and Scheinkman, 1983). Under this interpretation, a firm directly sets the price of its product, which is to be compared with the price threshold. (It can be shown that most of the qualitative results of the paper hold under differentiated Bertrand competition as well.)

Second, since $\pi_{D_S}(q_i, Q_{-i}) = [(1-r)p(Q_{-i} + q_i) + r\overline{p} - c]q_i - F$, Policy D is viewed as a combination of an *ad valorem* tax with the rate of r and a specific subsidy of $r\overline{p}$ per unit

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[9]Subsidy payment $r[\overline{p} - p(Q)](= p_p - p(Q))$ is defined in terms of the consumer price $p(Q)$. Alternatively, it can be expressed with the producer price p_p as $r_p[\overline{p} - p_p]$, where the parameter r_p differs from r, but $\overline{p}$ is, by construction of the policy, identical to the one in the consumer price-based definition above. Equating the values from the two definitions gives $p_p - p(Q) = r[\overline{p} - p(Q)] = r_p[\overline{p} - p_p]$. Rearranging this, we obtain $r_p = r/(1-r)$. Since the function $g : (0,1) \to (0,\infty)$ defined as $g(r) = r/(1-r)$ is bijective (one-to-one and onto), it does not matter whether the subsidy is defined in terms of the consumer or producer price.

quantity, with a non-negativity constraint that subsidy payment equals $\max\{r(\overline{p} - p), 0\}$. The constraint reflects the subsidy eligibility condition, and keeps the product from being taxed when it is sold at high prices. With this interpretation, Policy D is related with the models of Myles (1996) and Hamilton (1999) that discuss such a dual scheme with *ad valorem* and specific elements in the context of commodity taxation, i.e., for the case of $r(\overline{p} - p) < 0$ in equilibrium.[10] As taxation is their primary interest, the firms in their models are subject to the linear rule $r(\overline{p} - p)$ regardless of p (without a non-negativity (non-positivity) constraint as above). In contrast, in our model with such a constraint, the firms voluntarily select the subsidy calculation rule for the product ($r(\overline{p} - p)$ or 0) by setting the price lower or higher than $\overline{p}$. In other words, the regulator in our model can only induce the firms to opt in to the scheme by making r and $\overline{p}$ attractive enough for them, rather than enforce the scheme. As shown below, the kinked structure due to the non-negativity constraint results in two potential Nash equilibria in which the firms either opt in to or out of the scheme.

Now, we consider maximising the function $\pi_{D_S}(q_i, Q_{-i})$ in (13) with respect to q_i, conditional on Q_{-i}, with ignoring the eligibility condition $p(Q_{-i} + q_i) < \overline{p}$ in (14) for the moment. With strict concavity of $\pi_{D_S}(q_i, Q_{-i})$ with respect to q_i and assuming an interior solution, for each Q_{-i} there is a unique maximiser $q_i = q_{D_S}^b(Q_{-i})$ defined by the following FOC:

$$(1 - r)\big[p'(Q_{-i} + q_i)q_i + p(Q_{-i} + q_i)\big] + r\overline{p} - c = 0. \tag{15}$$

With an additional assumption that $p''(Q_{-i} + q_{D_S}^b(Q_{-i})) \cdot q_{D_S}^b(Q_{-i}) + p'(Q_{-i} + q_{D_S}^b(Q_{-i})) < 0$, there exists a unique $q_{D_S}^*$ such that $q_{D_S}^* = q_{D_S}^b((n-1)q_{D_S}^*)$. That is, $q_{D_S}^*$ is implicitly and uniquely determined by

$$(1 - r)[p'(nq_{D_S}^*) \cdot q_{D_S}^* + p(nq_{D_S}^*)] + r\overline{p} - c = 0. \tag{16}$$

Lemma 1. $q_A^b(Q_{-i}) < q_{D_S}^b(Q_{-i})$, and $q_A^* < q_{D_S}^*$, where $q_A^b(Q_{-i})$ and q_A^* are defined by (4) and (6), respectively.

Proof. See the Appendix. ■

Next, define $G(Q_{-i})$ as the difference between $\max_{q_i} \pi_A(q_i, Q_{-i}) = \pi_A(q_A^b(Q_{-i}), Q_{-i})$ and $\max_{q_i} \pi_{D_S}(q_i, Q_{-i}) = \pi_{D_S}(q_{D_S}^b(Q_{-i}), Q_{-i})$, where maximisation is unconditional on the (in)eligibility conditions given in (14), and $q_A^b(Q_{-i})$ and $q_{D_S}^b(Q_{-i})$ are defined by (4) and (15), respectively:

$$\begin{aligned} G(Q_{-i}) &\equiv \pi_A(q_A^b(Q_{-i}), Q_{-i}) - \pi_{D_S}(q_{D_S}^b(Q_{-i}), Q_{-i}) \\ &= [p(Q_{-i} + q_A^b(Q_{-i})) - c] \cdot q_A^b(Q_{-i}) - [(1 - r)p(Q_{-i} + q_{D_S}^b(Q_{-i})) + r\overline{p} - c] \cdot q_{D_S}^b(Q_{-i}). \end{aligned} \tag{17}$$

Proposition 1. If $G(0) \geq 0$, there exists a unique $\tilde{Q}_{rest}(\geq 0)$ such that $G(\tilde{Q}_{rest}) = 0$. In this case,

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[10]The product $r\overline{p}$ in this paper is treated in Myles (1996) and Hamilton (1999) as a single parameter representing the specific tax/subsidy per unit quantity.
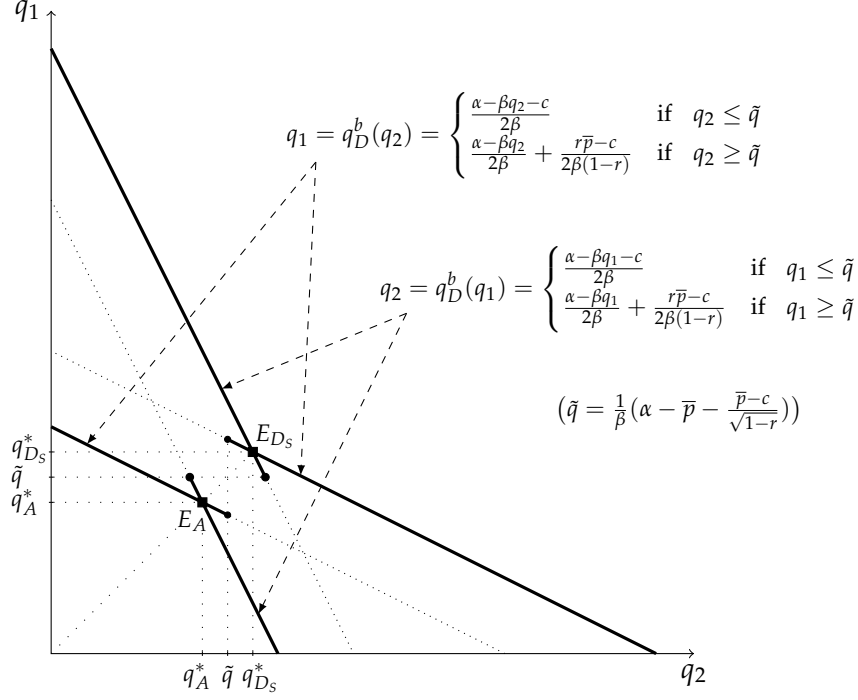
Figure 2: Duopoly under Linear Demand and Policy D

the best response correspondence under Policy D is

$$q_D^b(Q_{-i}) = \begin{cases} q_A^b(Q_{-i}) & \text{if } Q_{-i} \leq \tilde{Q}_{rest}, \\ q_{D_S}^b(Q_{-i}) & \text{if } Q_{-i} \geq \tilde{Q}_{rest}. \end{cases} \tag{18}$$

If $G(0) < 0$, the best response function under Policy D is

$$q_D^b(Q_{-i}) = q_{D_S}^b(Q_{-i}) \ \forall Q_{-i} \geq 0.^{11} \tag{19}$$

Proof. See the Appendix. ∎

In case $G(0) \geq 0$, the best response correspondence $q_i = q_D^b(Q_{-i})$ is single-valued and continuous at any $Q_{-i} \neq \tilde{Q}_{rest}$.[12] Since $q_A^b(Q_{-i}) < q_{D_S}^b(Q_{-i})$ for each Q_{-i} (Lemma 1), $q_i = q_D^b(Q_{-i})$ jumps up once at $Q_{-i} = \tilde{Q}_{rest}$. Intuitively, when the other firms' output Q_{-i} is relatively small ($Q_{-i} \leq \tilde{Q}_{rest}$), firm i needs to supply a large quantity to the market to push down the price below $\overline{p}$ and benefit from the subsidy. If the subsidy scheme is not generous enough to justify this, firm i's optimal choice is to disregard the subsidy and just adhere to the best response function under no subsidy ($q_A^b(Q_{-i})$). The other case $G(0) < 0$ indicates a very generous policy in which a firm will make its product qualify for the subsidy even when the others produce nothing.

As an illustration, let us consider a duopoly case ($n = 2$) with linear demand $p(Q) = \alpha - \beta Q$,

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[11] In the case of a monopoly ($n = 1$), $Q_{-i} = 0$ by definition. Therefore, $q_D^b(0) = q_A^b(0)$ if $G(0) > 0$; $q_D^b(0) = q_{D_S}^b(0)$ if $G(0) < 0$; and $q_D^b(0) = q_A^b(0) = q_{D_S}^b(0)$ if $G(0) = 0$.

[12] $q_A^b(Q_{-i})$ and $q_{D_S}^b(Q_{-i})$ are continuous by the maximum theorem.

where $\alpha > c$ and $\beta > 0$. Figure 2 depicts each firm's best response correspondence, $q_1 = q_{D_S}^b(q_2)$ and $q_2 = q_{D_S}^b(q_1)$ (as well as two potential Nash equilibria E_A and E_{D_S} to be discussed next). In this case, the above-defined threshold value $\tilde{Q}_{rest}$ where the best response switches is given by $\tilde{q} = \frac{1}{\beta}(\alpha - \overline{p} - \frac{\overline{p}-c}{\sqrt{1-r}})$. As the subsidy scheme gets more generous (i.e., r or $\overline{p}$ increases), $\tilde{q}$ goes down, inducing (the product of) each firm to become eligible for the scheme even with smaller supply by the competitor.

The Nash equilibria under Policy D are where the n best response correspondences $q_i = q_D^b(Q_{-i})$, $i = 1, \cdots, n$, intersect. In Figure 2 for a duopoly with linear demand, there exist two equilibria E_A and E_{D_S} because the threshold value $\tilde{q}$ is between q_A^* and $q_{D_S}^*$. If $\tilde{q} < q_A^*$, the two best response correspondences cannot intersect at E_A, so E_{D_S} is the unique equilibrium. Similarly, if $\tilde{q} > q_{D_S}^*$, E_{D_S} cannot be an intersection, so E_A is the unique equilibrium. This observation is generalised in the next proposition.

Proposition 2. *There are at least one, and at most two Nash equilibria under Policy D. All equilibria are symmetric. One possible equilibrium is identical to E_A. The other possible equilibrium (denoted by E_{D_S}) is where all firms follow $q_{D_S}^b(Q_{-i})$. Consider $\tilde{Q}_{rest}(\geq 0)$ such that $G(\tilde{Q}_{rest}) = 0$. Then*

1. *if $G(0) < 0$ ($\iff$ $\tilde{Q}_{rest}$ does not exist), or $\tilde{Q}_{rest} < (n-1)q_A^*$, the only equilibrium under Policy D is E_{D_S};*

2. *if $(n-1)q_A^* \leq \tilde{Q}_{rest} \leq (n-1)q_{D_S}^*$, the two equilibria under Policy D are E_A and E_{D_S};*

3. *if $(n-1)q_{D_S}^* < \tilde{Q}_{rest}$, the only equilibrium under Policy D is E_A.*

Proof. See the Appendix. ∎

3 Comparing Outcomes of Different Policies

Section 2 has considered firm behaviour and market equilibria under exogenous subsidy policies. Suppose now that the government aims to increase social and consumer surplus by setting Policies B, C, or D (z, v, or r and $\overline{p}$) to induce the firms to raise the industry output from the no-subsidy level Q_A^* to a common target level $\hat{Q}$.[13] This section compares equilibria E_B, E_C, and E_{D_S} with one another and the benchmark case of no-subsidy equilibrium E_A. We are especially interested in how the policy variables r and $\overline{p}$ of the IPR form affect the outcomes at E_{D_S}.

[Policies B and C]

Under Policies B and C, substituting $\hat{Q} = Q_B^* = Q_C^*$ into (9) and (12) gives $z = -p'(\hat{Q})\hat{q} - p(\hat{Q}) + c$ and $v = \frac{c}{p'(\hat{Q})\hat{q}+p(\hat{Q})} - 1$, respectively, where $\hat{q} \equiv \hat{Q}/n$. From (9) and (12), $\hat{Q}$ satisfies

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[13]If no externality is associated with the consumption/production of the good, social surplus ($= \int_0^{\hat{Q}} [p(Q) - c]dQ - nF$ = consumer surplus + producer surplus − government expenditure) is maximised with $\hat{Q}$ such that $p(\hat{Q}) = c$. With positive externalities, which are often the very reason for subsidisation but not considered explicitly in our model, socially optimal $\hat{Q}$ will be greater than the level that equates the (consumer) price with c. Note that the following analysis is not about setting $\hat{Q}$ optimally, and thus is not restricted to socially optimal $\hat{Q}$.

$p'(\hat{Q})\hat{q} + p(\hat{Q}) < c$ ($\iff Q_A^* < \hat{Q}$), and additionally, when Policy C is discussed, $p'(\hat{Q})\hat{q} + p(\hat{Q}) > 0$.[14] Government expenditure (GE) and producer surplus (PS) are as follows:[15]

$$\text{GE}_B^*(\hat{Q}) = \left[-p'(\hat{Q})\hat{q} - p(\hat{Q}) + c \right]\hat{Q}, \qquad \text{PS}_B^*(\hat{Q}) = n\left[-p'(\hat{Q})\hat{q}^2 - F \right], \tag{20}$$

$$\text{GE}_C^*(\hat{Q}) = \left[\frac{c}{p'(\hat{Q})\hat{q} + p(\hat{Q})} - 1 \right]p(\hat{Q})\hat{Q}, \quad \text{PS}_C^*(\hat{Q}) = n\left[-\frac{c}{p'(\hat{Q})\hat{q} + p(\hat{Q})}p'(\hat{Q})\hat{q}^2 - F \right]. \tag{21}$$

[Policy D]

Under Policy D, (16) implies pairs of r and $\overline{p}$ that satisfy the following equation can induce $Q_{D_S}^* = \hat{Q}$:

$$(1 - r)[p'(\hat{Q})\hat{q} + p(\hat{Q})] + r\overline{p} - c = 0. \tag{22}$$

Note that (22) and the maintained assumption $c < \overline{p}$ imply $p'(\hat{Q})\hat{q} + p(\hat{Q}) < c$ ($\iff Q_A^* < \hat{Q}$). From (22), $\overline{p}$ is given as a function of r and $\hat{Q}$, and $\frac{\partial \overline{p}}{\partial r} < 0$.[16] Hereafter, $\overline{p}$ is eliminated by using (22), which means that when we consider below the effect of changing r conditional on $\hat{Q}$, $\overline{p}$ is also implicitly changed to satisfy (22), and we observe the total effect of the changes in both r and $\overline{p}$.

First, we investigate the conditions on r (and $\overline{p}$) under which Policy D can induce an equilibrium with $Q_{D_S}^* = \hat{Q}$. Substituting (22) into (15) shows that $q_i = q_{D_S}^b(Q_{-i}; \hat{Q})$ is uniquely determined by the following equation:[17]

$$(1 - r)\left[p'(Q_{-i} + q_i)q_i + p(Q_{-i} + q_i) - p'(\hat{Q})\hat{q} - p(\hat{Q}) \right] = 0. \tag{23}$$

From (13) and (17), π_{D_S} and G clearly depend on r and $\overline{p}$, but this dependence has so far been suppressed as r and $\overline{p}$ have been treated as exogenous parameters for the firms. As this section considers how the regulator sets r (and $\overline{p}$ through (22)) conditional on $\hat{Q}$, the dependence on r (as well as $\hat{Q}$) is now made explicit as $\pi_{D_S}(q_i, Q_{-i}; r, \hat{Q})$ and $G(Q_{-i}; r, \hat{Q})$.

Let r_2 and r_3 be determined by $G(\frac{n-1}{n}Q_A^*; r_2, \hat{Q}) = 0$ and $G(\frac{n-1}{n}\hat{Q}; r_3, \hat{Q}) = 0$. In other words, given the government target $\hat{Q}$ and other firms' output $Q_{-i} = \frac{n-1}{n}Q_A^*$, r_2 makes firm i indifferent between being qualified and disqualified for the subsidy (i.e., between $q_i = q_{D_S}^b(\frac{n-1}{n}Q_A^*; \hat{Q})$ and $q_i = q_A^b(\frac{n-1}{n}Q_A^*)$), and r_3 is described similarly. With (4), (22), and (23) substituted into

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[14]Under Policy B, (9) and $z > 0$ imply $p'(nq_B^*)q_B^* + p(nq_B^*) < c$. Comparing this with (6) and noting that $d[p'(nq)q + p(nq)]/dq = p'(nq) + p''(nq)nq + p'(nq)n = p''(Q)Q + (n+1)p'(Q) < 0$ (by Assumption 1), we find $q_A^* < q_B^* = \hat{q}$. Similarly, under Policy C, (12) and $v > 0$ imply $0 < p'(nq_C^*)q_C^* + p(nq_C^*) < c$, and $q_A^* < q_C^* = \hat{q}$.

[15]These are obtained by substituting $z = -p'(\hat{Q})\hat{q} - p(\hat{Q}) + c$ and $v = c/[p'(\hat{Q})\hat{q} + p(\hat{Q})] - 1$ into the equations below about equilibrium GE and PS under Policies B and C:

$$\text{GE}_B^* = nzq_B^*, \qquad \text{PS}_B^* = n \cdot \pi_B(q_B^*; (n-1)q_B^*) = n[p(nq_B^*) + z - c]q_B^* - nF,$$
$$\text{GE}_C^* = nvp(nq_C^*)q_C^*, \quad \text{PS}_C^* = n \cdot \pi_C(q_C^*; (n-1)q_C^*) = n[(1+v)p(nq_C^*) - c]q_C^* - nF.$$

[16]$\overline{p} = [c - p'(\hat{Q})\hat{q} - p(\hat{Q})]/r + p'(\hat{Q})\hat{q} + p(\hat{Q})$, and $\partial\overline{p}/\partial r = [p'(\hat{Q})\hat{q} + p(\hat{Q}) - c]/r^2 < 0$.

[17]I use the notation $q_i = q_{D_S}^b(Q_{-i}; \hat{Q})$ to make it explicit that $q_{D_S}^b$ is conditional on $\hat{Q}$. Also, (23) means that $q_{D_S}^b$ does not depend on r.

(17),

$$r_2 \equiv 1 - \frac{p'(\frac{n-1}{n}Q_A^* + q_A^b(\frac{n-1}{n}Q_A^*)) \cdot [q_A^b(\frac{n-1}{n}Q_A^*)]^2}{p'(\frac{n-1}{n}Q_A^* + q_{D_S}^b(\frac{n-1}{n}Q_A^*; \hat{Q})) \cdot [q_{D_S}^b(\frac{n-1}{n}Q_A^*; \hat{Q})]^2}$$

$$= 1 - \frac{p'(Q_A^*) \cdot q_A^{*2}}{p'(\frac{n-1}{n}Q_A^* + q_{D_S}^b(\frac{n-1}{n}Q_A^*; \hat{Q})) \cdot [q_{D_S}^b(\frac{n-1}{n}Q_A^*; \hat{Q})]^2}, \tag{24}$$

$$r_3 \equiv 1 - \frac{p'(\frac{n-1}{n}\hat{Q} + q_A^b(\frac{n-1}{n}\hat{Q})) \cdot [q_A^b(\frac{n-1}{n}\hat{Q})]^2}{p'(\frac{n-1}{n}\hat{Q} + q_{D_S}^b(\frac{n-1}{n}\hat{Q}; \hat{Q})) \cdot [q_{D_S}^b(\frac{n-1}{n}\hat{Q}; \hat{Q})]^2}$$

$$= 1 - \frac{p'(\frac{n-1}{n}\hat{Q} + q_A^b(\frac{n-1}{n}\hat{Q})) \cdot [q_A^b(\frac{n-1}{n}\hat{Q})]^2}{p'(\hat{Q}) \cdot \hat{q}^2}. \tag{25}$$

Note that $(0 <) \frac{p'(Q_{-i} + q_A^b(Q_{-i})) \cdot [q_A^b(Q_{-i})]^2}{p'(Q_{-i} + q_{D_S}^b(Q_{-i}; \hat{Q})) \cdot [q_{D_S}^b(Q_{-i}; \hat{Q})]^2} < 1$ because $(0 <) q_A^b(Q_{-i}) < q_{D_S}^b(Q_{-i}; \hat{Q})$ and $\frac{dp'(Q_{-i} + q_i)q_i^2}{dq_i} = p''(Q_{-i} + q_i)q_i^2 + 2p'(Q_{-i} + q_i)q_i < 0$ by (2). Thus, $r_2 \in (0, 1)$ and $r_3 \in (0, 1)$.

The following proposition rephrases Proposition 2 of Section 2 in terms of policy variables (r, $\hat{Q}$, and implicitly $\bar{p}$), and is useful from a policy-making perspective.

Proposition 3. *Given $\hat{Q}$ ($> Q_A^*$), $r_2 < r_3$. Moreover,*

1. *if $r < r_2$, the only equilibrium under Policy D is E_{D_S};*

2. *if $r_2 \leq r \leq r_3$, the two equilibria under Policy D are E_A and E_{D_S};*

3. *if $r_3 < r$, the only equilibrium under Policy D is E_A.*

Proof. See the Appendix. ∎

Given $r \leq r_3$, government expenditure and producer surplus at E_{D_S} are as follows:[18]

$$\text{GE}_{D_S}^*(\hat{Q}, r) = \left[-(1-r)p'(\hat{Q})\hat{q} - p(\hat{Q}) + c \right]\hat{Q}, \quad \text{PS}_{D_S}^*(\hat{Q}, r) = n\left[-(1-r)p'(\hat{Q})\hat{q}^2 - F \right]. \tag{26}$$

It may sound counter-intuitive that (26) implies $\frac{\partial GE_{D_S}^*}{\partial r} = \frac{\partial PS_{D_S}^*}{\partial r} < 0$ because increasing (only) r means a more generous subsidy policy. Note that in this section increasing r simultaneously reduces $\bar{p}$ as discussed after (22), resulting in these negative partial derivatives.

(26) shows that in achieving the policy target $\hat{Q}$, Policy D allows the government to also affect GE and PS by adjusting policy variables r and $\bar{p}$ with (22) satisfied. In contrast, (20) and (21) indicate that with Policy B (C), the government's choice of the only policy variable z (v) determines the equilibrium output, which in turn pins down equilibrium GE and PS. Having one more policy variable provides Policy D the flexibility to impact GE and PS in addition to $\hat{Q}$.

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[18]These are obtained by substituting (22) into the equations below on equilibrium GE and PS at E_{D_S}.

$$GE_{D_S}^* = nr[\bar{p} - p(nq_{D_S}^*)]q_{D_S}^*,$$
$$PS_{D_S}^* = n \cdot \pi_{D_S}(q_{D_S}^*, (n-1)q_{D_S}^*) = n[(1-r)p(nq_{D_S}^*) + r\bar{p} - c]q_{D_S}^* - nF.$$

The next proposition compares Policies B, C, and D in terms of government expenditure (GE) and producer surplus (PS) to attain $\hat{Q}$ in equilibrium.

Proposition 4. *When Policies B, C, and D (with $r \leq r_3$) are all designed to attain $\hat{Q}$ ($> Q_A^*$) in equilibrium, $GE_{D_S}^*(\hat{Q}, r) < GE_B^*(\hat{Q}) < GE_C^*(\hat{Q})$ and $PS_{D_S}^*(\hat{Q}, r) < PS_B^*(\hat{Q}) < PS_C^*(\hat{Q})$. Specifically, the differences are as follows:*

$$GE_B^*(\hat{Q}) - GE_{D_S}^*(\hat{Q}, r) = PS_B^*(\hat{Q}) - PS_{D_S}^*(\hat{Q}, r) = r\frac{-p'(\hat{Q})}{n}\hat{Q}^2 > 0,$$

$$GE_C^*(\hat{Q}) - GE_B^*(\hat{Q}) = PS_C^*(\hat{Q}) - PS_B^*(\hat{Q}) = \left[\frac{c}{p'(\hat{Q})\hat{Q}/n + p(\hat{Q})} - 1\right]\frac{-p'(\hat{Q})}{n}\hat{Q}^2 > 0.$$

(27)

Proof. The results follow from (20), (21), and (26). ∎

Put differently, when $\hat{Q}$ is attained in equilibrium, GE and PS are smaller under Policy D than under Policy B by $-rp'(\hat{Q})\hat{q}$ per unit quantity, or $-rp'(\hat{Q})\hat{q}^2$ per firm, and smaller under Policy B than under Policy C by $-\left[\frac{c}{p'(\hat{Q})\hat{q}+p(\hat{Q})} - 1\right]p'(\hat{Q})\hat{q}$ per unit quantity, or $-\left[\frac{c}{p'(\hat{Q})\hat{q}+p(\hat{Q})} - 1\right]p'(\hat{Q})\hat{q}^2$ per firm.[19] Also, note that the difference in GE equals that in PS because all the three policies lead to the same industry output $\hat{Q}$.[20]

Proposition 4 shows that Policy D is the most efficient among the three subsidy schemes in the sense that it achieves a given target $\hat{Q}$ with the least government spending, and the difference increases with r and $\hat{Q}$, and decreases with n. Policy D can increase output with smaller expenditure because it makes the demand faced by the firms $((1-r)p(Q) + r\bar{p})$ more price-elastic than that under no subsidy ($p(Q)$). A higher price elasticity is a factor to incentivise the firms to increase production in equilibrium, making it easier for the government to induce a higher output. Similarly, as pointed out by Liang, Wang and Chou (2017), Policy B is more efficient than Policy C. In contrast to Policy D, the *ad valorem* subsidy (Policy C) makes the demand faced by the firms less price-elastic, making it harder and more costly for the government to move them to a higher output. Due to the equality of the difference in GE and that in PS, efficiency in terms of government expenditure equivalently means lower producer surplus (the next proposition explores more on producer surplus). Proposition 4 also indicates that as the number of firms (n) increases and a firm's power to control the market price reduces, these differences among the three policies become negligible, a result parallel to the equivalence of specific and *ad valorem* taxation under perfect competition (see, e.g., Keen, 1998).

Next, we compare a firm's profits ($= PS/n$) at the two potential equilibria E_A and E_{D_S} that

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[19]This result is analogous to Proposition 3 of Carbonnier (2014), who in the context of taxation compares different tax schemes that result in the same equilibrium consumer price (and thus output). He finds that in equilibrium an increase in what he terms the elasticity of the tax function ($\epsilon \equiv (dp/p)/(dp_p/p_p)$ in this paper's notation) lowers the producer price (p_p) and raises tax revenue. As for equilibrium values of ϵ under Policies B–D of this paper, it is the case that $\epsilon_C^* < \epsilon_B^* < \epsilon_{D_S}^*$.

[20]Social surplus = consumer surplus + PS − GE, so PS − GE is equal to $\int_0^{\hat{Q}}[p(Q) - c]dQ - nF - \int_0^{\hat{Q}}[p(Q) - p(\hat{Q})]dQ = [p(\hat{Q}) - c]\hat{Q} - nF$ and constant across the three policies.

can be realised under Policy D. With the FOCs (4) and (15),

$$\pi_A(q_A^*, \frac{n-1}{n}Q_A^*) = -p'(Q_A^*)q_A^{*2} - F,$$

$$\pi_{D_S}(\hat{q}, \frac{n-1}{n}\hat{Q}; r) = -(1-r)p'(\hat{Q})\hat{q}^2 - F.$$

Therefore, with $r_1 \equiv 1 - \frac{p'(Q_A^*)q_A^{*2}}{p'(\hat{Q})\hat{q}^2} \in (0,1),$[21]

$$\pi_A(q_A^*, \frac{n-1}{n}Q_A^*) \begin{cases} < \pi_{D_S}(\hat{q}, \frac{n-1}{n}\hat{Q}; r) & \text{if } r < r_1, \\ = \pi_{D_S}(\hat{q}, \frac{n-1}{n}\hat{Q}; r) & \text{if } r = r_1, \\ > \pi_{D_S}(\hat{q}, \frac{n-1}{n}\hat{Q}; r) & \text{if } r > r_1. \end{cases} \tag{28}$$

Proposition 5. *Given $\hat{Q}$ ($> Q_A^*$), $r_1 < r_2(< r_3)$. Therefore, Policy D can make a firm's profits at E_{D_S} higher than, equal to, and lower than at E_A by setting r to satisfy $0 < r < r_1$, $r = r_1$, and $r_1 < r \le r_3$, respectively, and $\overline{p}$ by (22). In particular, if $r_1 < r < r_2$, the unique equilibrium under Policy D (E_{D_S}) observes lower profits than E_A.*

Proof. See the Appendix. ∎

Finally, we examine subsidy pass-through associated with the change from Q_A^* to $\hat{Q}$. Subsidy pass-through to consumers (which is the share of the subsidy benefit passed on to consumers and is denoted by θ) is the ratio of the reduction in the consumer price (i.e., $p(Q_A^*) - p(\hat{Q})$) to the subsidy payment (per unit quantity) needed to induce the price reduction (i.e., $r[\overline{p} - p(\hat{Q})]$ for E_{D_S}). With (6) and (22),

$$\theta_{D_S} = \frac{p(Q_A^*) - p(\hat{Q})}{r[\overline{p} - p(\hat{Q})]} = \frac{-p'(Q_A^*)q_A^* - p(\hat{Q}) + c}{-(1-r)p'(\hat{Q})\hat{q} - p(\hat{Q}) + c}. \tag{29}$$

As shown in Proposition 4, Policy D achieves the same price reduction ($p(Q_A^*) - p(\hat{Q})$) with less subsidy payment than Policies B and C, so that θ is larger at E_{D_S} than at E_B and E_C. Also, (29) implies that, given $\hat{Q}$, θ_{D_S} is increasing in r, and with $r_0 \equiv 1 - \frac{p'(Q_A^*)q_A^*}{p'(\hat{Q})\hat{q}}(< r_1),$[22]

$$\theta_{D_S} \begin{cases} < 1 & \text{if } r < r_0, \\ = 1 & \text{if } r = r_0, \\ > 1 & \text{if } r > r_0. \end{cases} \tag{30}$$

When $r > r_0$, $p(Q_A^*) > p(\hat{Q}) + r[\overline{p} - p(\hat{Q})]$, so the subsidy scheme lowers not only the consumer price, but also the producer price ("over-shifting" or over 100% subsidy pass-through).

In order to graphically summarise the characteristics of Policy D as stated in Propositions 3–5 and (30), Figure 3 plots a firm's equilibrium profits under Policy D against r, conditional on $\hat{Q}$. The domain of the profit function π_{D_S} (π_A) is the interval of r on which E_{D_S} (E_A)

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[21] Note that $0 < r_1 < 1$ because $d[p'(nq)q^2]/dq = [p''(nq)nq + 2p'(nq)]q < 0$ (by Assumption 1), and $0 < q_A^* < \hat{q}$.
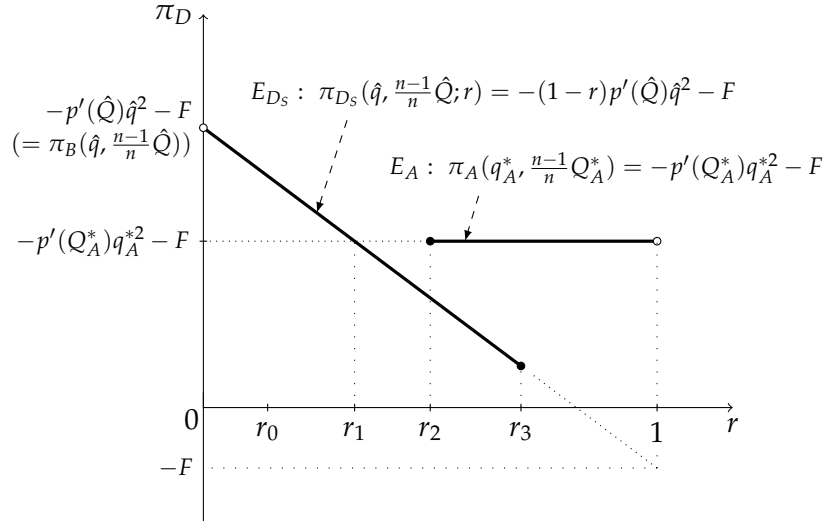[22] Since $q_A^* < \hat{q}$, $r_0 < r_1$.

Figure 3: Equilibrium profits and r under Policy D, conditional on $\hat{Q}$

constitutes a Nash equilibrium under Policy D. The subsidised equilibrium E_{D_S} with $Q_{D_S}^* = \hat{Q}$ can be induced by any pair of policy variables $(r, \overline{p})$ with $r \in (0, r_3]$ and $\overline{p}$ $(> c)$ satisfying (22). If existent, E_{D_S} always offers a firm lower profits (or equivalently, is realised with less government spending) than E_B (where profits equal $-p'(\hat{Q})\hat{q}^2 - F$). If $r \in (0, r_0)$ $(r \in (r_0, r_3])$, the producer price at E_{D_S} increases (decreases) from the no-subsidy equilibrium E_A. For $r \in (0, r_1)$ $(r \in (r_1, r_3])$, profits at E_{D_S} are higher (lower) than at E_A. Furthermore, for $r \in (0, r_2)$, E_{D_S} is the unique equilibrium under Policy D. Thus, with $r \in (r_1, r_2)$, the firms would be better off if they could collude and jointly opt out of the subsidy scheme to earn $-p'(Q_A^*)q_A^{*2} - F$ per firm, but this is not a Nash equilibrium and the prisoner's dilemma ends them up with lower profits at the unique equilibrium E_{D_S}. For $r \in [r_2, r_3]$, both E_{D_S} and E_A are Nash equilibria, but E_A is more plausible since it gives each firm higher profits than, and thus Pareto-dominates (from the firms' perspective), E_{D_S}.[23]

It is clear from Figure 3 that the IPR subsidy can utilise the two policy variables to flexibly control its incidence on the producers, unlike the specific or *ad valorem* subsidy. By changing r in the range of $(0, r_2)$ (and $\overline{p}$ by (22)), the regulator can adjust the level of the programme's benefit or burden distributed to the producers, while attaining a unique equilibrium with the same level of industry output $(\hat{Q})$, and consumer and social surplus. If $r \in (0, r_1)$, Policy D increases both consumer and producer surplus in comparison to the benchmark case E_A of no government intervention. Thus, it is subsidisation for both the consumers and producers. On the other hand, for $r \in (r_1, r_2)$, although Policy D remains to offer the same benefit to the consumers, it works in effect as taxation on the producers because their equilibrium profits decline relative to E_A, and the differential is implicitly transferred to the government as a reduction in government spending to induce $\hat{Q}$. In contrast, with the specific or *ad valorem* form, (20) and (21) mean $\frac{dPS_B^*}{d\hat{Q}} > 0$ and $\frac{dPS_C^*}{d\hat{Q}} > 0$. Therefore, inducing a larger output than

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[23]For the case of a monopoly ($n = 1$), the following changes should be made to Figure 3: the expressions for the profit functions π_{D_S} and π_A remain the same as in Figure 3, but the domain for π_{D_S} is $r \in (0, r_1]$ and that for π_A is $r \in [r_1, 1)$. This is because Policy D cannot make the monopolist worse off than at E_A as the firm always has the option of disqualifying the product from the subsidy and realising the no-subsidy equilibrium E_A.

Q_A^* with a specific or *ad valorem* subsidy necessarily gives out extra profits to the firms who already earn economic rents from imperfect competition, a consequence that may come into conflict with distributional fairness.

With the flexibility provided by the IPR form, the incidence of the policy on producers can be adjusted in line with its objectives and market situations. For example, if a target good requires emerging, innovative technologies (e.g., electric vehicles), producers might have incurred significant fixed costs (e.g., R&D investment). Under these circumstances, the government may want to support the innovative producers by allowing them significantly higher profits than under no subsidy. On the other hand, if prior to government intervention the market for a target good is relatively mature and served by a small number of firms earning large economic rents due to imperfect competition, the government may want to use the IPR subsidy scheme to induce a larger output and at the same time bring down the oligopolists' profits.

4   Application: U.S. Tax Credits for Electric Vehicles

To analyse the impact of the IPR subsidy scheme in an empirical context, we look into tax credits for electric vehicles (EVs) in the United States. As of 2017, buyers of new EVs are eligible to claim a federal income tax credit of $7,500. Effectively, this is a specific subsidy of $7,500 offered to EV buyers. This section provides a simulation on the effect of making this subsidy inversely related to EV prices.

A hypothetical market for mid-sized EVs is constructed as follows. First, based on actual U.S. market data for mid-sized EVs (for 4–5 passengers) in model year 2017, I identify the eight best-selling models, which are produced by eight distinct manufacturers (BMW, Fiat, Ford, General Motors, Kia, Mercedes, Nissan, and Volkswagen), and compute their aggregate sales and (sales-weighted) average price. Then, I conduct a partial equilibrium analysis of this market where the firms engage in Cournot competition as in the previous sections,[24] a specific subsidy of $7,500 is offered to buyers, and the equilibrium price and quantity induced by the subsidy equal the actual average (post-subsidy) price ($27,917 = $35,417 − $7,500) and total sales (50,981 vehicles) (that is, $z = \$7,500$, $p_B^* = \$27,917$, and $Q_B^* = 50,981$). Assuming that the number of (equal-sized) firms, n, can take a non-integer value (as previous studies commonly do), I set $n = 4.64$, which is the reciprocal of the Herfindahl-Hirschman Index calculated from the actual market shares of the eight models (firms).[25]

I consider an inverse demand function of the Bulow-Pfleiderer form (Bulow and Pfleiderer,

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[24]A Cournot model can be regarded as a two stage game where each firm sets the production capacity in the first stage, and the price in the second (Kreps and Scheinkman, 1983). This may be a more realistic description of an EV producer's decision process. On the other hand, the homogeneous product assumption of the paper is certainly a limitation for analysing a vehicle market.

[25]That is, a market with 4.64 equal-sized firms gives the same Herfindahl-Hirschman Index as observed in the data.

1983):

$$p(Q) = \begin{cases} \alpha - \beta Q^{1-\varepsilon} & \text{for } \varepsilon < 1 \text{ and } 0 < Q < \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\varepsilon}}, \\ \alpha - \beta \ln(Q) & \text{for } \varepsilon = 1 \text{ and } 0 < Q < \exp(\frac{\alpha}{\beta}), \end{cases} \tag{31}$$

where $\alpha, \beta > 0$.[26] This function is frequently used in studies of incidence (e.g., Weyl and Fabinger, 2013; Atkin and Donaldson, 2015) due to its property of the constant elasticity of the slope of inverse demand ($-\frac{p''(Q)Q}{p'(Q)} = \varepsilon$). The inverse demand function is linear if $\varepsilon = 0$, and strictly convex (strictly concave) to the origin if $\varepsilon > 0$ (< 0). To see the robustness of the simulation against the shape of the demand curve, I use different values (-0.5, 0, 0.5, and 1) for the curvature parameter ε.

The simulation procedure is summarised as follows. Given a set of assumptions on ε and the price-cost markup rate at E_B ($= \$35,417/c \equiv m$), the inverse demand function is calibrated by using (31) along with (9), which is the FOC to be satisfied at E_B. Based on the computed demand function, we derive the equilibrium with no government intervention, and policy parameters to induce an equilibrium with $\hat{Q} = 50,981$ and $\hat{p} \equiv p(\hat{Q}) = \$27,917$ under the *ad valorem* or IPR subsidy.

Table 2 reports simulation results for different policies under various combinations of ε and m. Q_A^* and p_A^* are the quantity and price at the equilibrium with no subsidy for EVs (Policy A). $Q_A^* < 50,981 = \hat{Q}$ and $p_A^* > \$27,917 = \hat{p}$, so the specific subsidy (Policy B) raises the quantity and lowers the (consumer) price in equilibrium. Note also that for $\varepsilon < 1$, $p_A^* < \$35,417 = \hat{p} + \$7,500$: the producer price rises due to the specific subsidy of $\$7,500$, or equivalently the subsidy pass-through to consumers is less than 100%. For $\varepsilon = 1$, $p_A^* = \$35,417$ by construction, so the producer price remains the same with and without the specific subsidy (100% subsidy pass-through to consumers). Producer surplus under the specific subsidy is $\$72$–$150$ million higher than under no subsidy.

The *ad valorem* subsidy (Policy C) with $v = \frac{z}{c-z}$ induces the same $\hat{Q}$. As the demand curve faced by the firms becomes less elastic, it needs $\$2,011$–$13,010$ extra subsidy payment compared to the specific subsidy, increasing the producer price (by the same amount) and producer surplus (by $\$102$–$663$ million).

Regarding the IPR subsidy (Policy D), the table gives simulated values of r_1 and r_2 (as defined in Section 3). For each of r_1 and r_2, it also reports the price threshold determined by (22) ($\overline{p}_1$ and $\overline{p}_2$), subsidy payment per vehicle ($r_1(\overline{p}_1 - \hat{p})$ and $r_2(\overline{p}_2 - \hat{p})$), producer price ($\hat{p} + r_1(\overline{p}_1 - \hat{p})$ and $\hat{p} + r_2(\overline{p}_2 - \hat{p})$), and producer surplus with the fixed costs disregarded ($[\hat{p} + r_1(\overline{p}_1 - \hat{p}) - c]\hat{Q}$ and $[\hat{p} + r_2(\overline{p}_2 - \hat{p}) - c]\hat{Q}$). I report r_2 rather than r_3 because for $r \in [r_2, r_3]$, equilibrium E_{D_S} is unlikely as it is Pareto-dominated by the other equilibrium E_A (from the firms' perspective), whereas for $r < r_2$, E_{D_S} is a unique equilibrium (see Figure 3).

As for r_1 and r_2, the value of 0.17 (for the case of $\varepsilon = -0.5$ and $m = 2.0$), for example,

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[26]I consider $\varepsilon \leq 1$, which implies that the inverse demand function satisfies Assumption 1, and that given $Q_{-i} > 0$, $p''(Q_{-i} + q_i)q_i + p'(Q_{-i} + q_i) < 0 \; \forall q_i \geq 0$. As discussed in Section 2, this means that the slopes of $q_A^b(Q_{-i})$, $q_B^b(Q_{-i})$, $q_C^b(Q_{-i})$, and $q_{D_S}^b(Q_{-i})$ are strictly between 0 and 1, ensuring the analysis of previous sections to hold.

Table 2: Simulation Results

$\varepsilon =$	−0.5			0.0		
$m \equiv \$35,417/c =$	2.0	1.6	1.2	2.0	1.6	1.2
Policy A (no subsidy)						
Q_A^*	47,401	46,177	39,793	46,983	45,650	38,986
p_A^*	33,585	33,585	33,585	34,028	34,028	34,028
Producer surplus (million \$)	753	529	162	767	543	176
Policy B (specific)						
Producer surplus (million \$)	903	677	301	903	677	301
Policy C (*ad valorem*)						
v	0.73	0.51	0.34	0.73	0.51	0.34
$v\hat{p}$ (= subsidy/car (\$))	20,510	14,306	9,511	20,510	14,306	9,511
Producer price (\$)	48,427	42,223	37,428	48,427	42,223	37,428
Producer surplus (million \$)	1,566	1,024	403	1,566	1,024	403
Policy D (IPR) with $r = r_1$						
r_1	0.17	0.22	0.46	0.15	0.20	0.42
$\overline{p}_1$ (\$)	55,273	48,854	38,257	59,973	52,474	40,077
$r_1(\overline{p}_1 - \hat{p})$ (= subsidy/car (\$))	4,553	4,589	4,774	4,831	4,867	5,049
Producer price (\$)	32,470	32,506	32,691	32,748	32,784	32,966
Producer surplus (million \$)	753	529	162	767	543	176
Policy D (IPR) with $r = r_2$						
r_2	0.34	0.43	0.72	0.34	0.42	0.70
$\overline{p}_2$ (\$)	32,000	32,064	32,380	32,405	32,467	32,757
$r_2(\overline{p}_2 - \hat{p})$ (= subsidy/car (\$))	1,405	1,785	3,229	1,516	1,914	3,379
Producer price (\$)	29,322	29,702	31,146	29,433	29,831	31,296
Producer surplus (million \$)	592	386	83	598	392	91

$\varepsilon =$	0.5			1.0		
$m \equiv \$35,417/c =$	2.0	1.6	1.2	2.0	1.6	1.2
Policy A (no subsidy)						
Q_A^*	46,867	45,534	39,158	46,534	45,139	38,769
p_A^*	34,687	34,687	34,687	35,417	35,417	35,417
Producer surplus (million \$)	796	572	203	824	600	229
Policy B (specific)						
Producer surplus (million \$)	903	677	301	903	677	301
Policy C (*ad valorem*)						
v	0.73	0.51	0.34	0.73	0.51	0.34
$v\hat{p}$ (= subsidy/car (\$))	20,510	14,306	9,511	20,510	14,306	9,511
Producer price (\$)	48,427	42,223	37,428	48,427	42,223	37,428
Producer surplus (million \$)	1,566	1,024	403	1,566	1,024	403
Policy D (IPR) with $r = r_1$						
r_1	0.12	0.16	0.33	0.09	0.11	0.24
$\overline{p}_1$ (\$)	73,460	62,741	44,960	96,183	80,087	53,324
$r_1(\overline{p}_1 - \hat{p})$ (= subsidy/car (\$))	5,400	5,429	5,571	5,955	5,978	6,086
Producer price (\$)	33,317	33,346	33,488	33,872	33,895	34,003
Producer surplus (million \$)	796	572	203	824	600	229
Policy D (IPR) with $r = r_2$						
r_2	0.33	0.41	0.67	0.32	0.40	0.64
$\overline{p}_2$ (\$)	32,894	32,952	33,214	33,496	33,550	33,779
$r_2(\overline{p}_2 - \hat{p})$ (= subsidy/car (\$))	1,645	2,062	3,547	1,797	2,234	3,737
Producer price (\$)	29,562	29,979	31,464	29,714	30,151	31,654
Producer surplus (million \$)	604	400	99	612	409	109

The table reports simulation results about Policies A–D for different combinations of demand curvature ε and the price-cost markup rate m.

indicates that if the subsidy eligibility is met ($p < \overline{p}$), the policy (implicitly) compensates the producer with \$0.17 for every \$1 reduction in the consumer price p. Equivalently, for every \$1 reduction in the producer price $p_p = p + r(\overline{p} - p)$, which in this subsidy programme equals the transaction price paid by the consumer to the producer, the consumer receives a subsidy of \$0.20 = \$0.17/(\$1 - \$0.17) (as discussed in footnote 9).

The results associated with r_1 are interpreted as follows. In order for $(r, \overline{p})$ to induce $(\hat{Q}, \hat{p})$ as a unique equilibrium while guaranteeing equilibrium profits at least as large as under no government intervention (Policy A), it must be the case that $r \leq r_1$ and subsidy payment per vehicle $\geq r_1(\overline{p}_1 - \hat{p})$.[27] In Table 2, r_1 ranges from 0.09 to 0.46, and $r_1(\overline{p}_1 - \hat{p})$ from \$4,553 to \$6,086 for various combinations of ε and m. To put it another way, given the constraint that each firm cannot be worse off than under the no-subsidy equilibrium E_A, switching from Policy B to Policy D can reduce the government's expenditure (equivalently, the producer price and profits) per vehicle by up to \$1,414–\$2,947 ($= 7,500 - r_1(\overline{p}_1 - \hat{p})$) or 19–39%. As (30) implies, the producer price (transaction price) at $r = r_1$ is lower than p_A^* in spite of the subsidy ("over-shifting" or over 100% subsidy pass-through). By construction, $r = r_1$ results in the same producer surplus as Policy A.

The results associated with r_2 can be interpreted similarly. In order for $(r, \overline{p})$ to induce $(\hat{Q}, \hat{p})$ as a unique equilibrium, it must be the case that $r < r_2$ and subsidy payment per vehicle $> r_2(\overline{p}_2 - \hat{p})$. In Table 2, r_2 ranges from 0.32 to 0.72, and $r_2(\overline{p}_2 - \hat{p})$ from \$1,405 to \$3,737 for various combinations of ε and m. In other words, by switching from Policy B to Policy D in such a way that keeps the firms from realising the opt-out equilibrium E_A, the government can reduce its subsidy expenditure (equivalently, the producer price and profits) per vehicle by up to \$3,763–\$6,095 ($= 7,500 - r_2(\overline{p}_2 - \hat{p})$) or 50–81%. As expected, over-shifting occurs, and producer surplus is substantially (\$63–169 million) lower than under Policy A.

Overall, the simulation suggests the usefulness of the IPR scheme (Policy D). Consistently across different assumptions on ε and m, it can significantly (up to 50–81%) save the government budget for achieving a given target $\hat{Q}$, compared to the specific subsidy. At the same time, the government can significantly affect the policy's incidence on the firms through its choice of policy variables. By letting $r \approx 0$, the IPR subsidy is (almost) equivalent to the specific subsidy (as shown in Figure 3) and can make producer surplus up to \$72–150 million higher than under no government intervention. On the other hand, as r approaches r_2, it works as implicit taxation on the firms by making producer surplus up to \$63–169 million lower than under no government intervention. The government can flexibly set producer surplus within this range depending on whether and how much it wants to subsidise or tax the producers.

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[27]For $r \leq r_1$, subsidy payment per vehicle $= r[\overline{p} - p(\hat{Q})] = c - p'(\hat{Q})\hat{q} - p(\hat{Q}) + rp'(\hat{Q})\hat{q} \geq c - p'(\hat{Q})\hat{q} - p(\hat{Q}) + r_1 p'(\hat{Q})\hat{q} = r_1[\overline{p}_1 - p(\hat{Q})]$. Note that $\hat{p} \equiv p(\hat{Q})$ and that (22) is used for the second and third equalities.

5 Extension: Product Quality

When Policy D is in place, a firm may have an incentive to lower the quality of the product because that is a simple way to reduce its production cost and price, thus making it eligible for a higher subsidy. Lower quality, however, is perhaps undesirable for the government, consumers, and society. This section considers how the regulator can augment Policy D to prevent quality degradation. In short, this is possible by making $\bar{p}$ increasing in quality.

I supplement the cost and demand functions with quality as follows, while maintaining the basic structure of the model discussed so far. The marginal cost c_i of firm i producing a unit of the good depends on its quality $w_i \in \mathbb{R}_+$: $c_i = c(w_i)$ with $c'(w_i) > 0$ and $c''(w_i) > 0$. The inverse demand function is derived by adding on w_i to the setup of Section 2 (footnote 4). Suppose that a unit of the good with quality w_i increases a representative consumer's utility by $f(w_i)$, where $f'(w_i) > 0$ and $f''(w_i) < 0$, so that total utility due to product quality equals $\sum_{i=1}^{n} f(w_i)q_i$. By adding this term to $U(x, Q) = x + u(Q)$ of footnote 4, her utility is now redefined as $U(x, \{q_i, w_i\}_{i=1,\cdots,n}) = x + u(Q) + \sum_{i=1}^{n} f(w_i)q_i$. Given $\{p_i, w_i\}_{i=1,\cdots,n}$, she solves the following utility maximisation problem:

$$\max_{x, q_1, \cdots, q_n} x + u(Q) + \sum_{i=1}^{n} f(w_i)q_i \text{ s.t. } x + \sum_i p_i q_i \leq I. \tag{32}$$

An interior solution is characterised by $p_i = u'(Q) + f(w_i) \ \forall i$. In other words, the inverse demand function for a good with quality w_i is given by $p(Q, w_i) = u'(Q) + f(w_i)$, as compared to $p(Q) = u'(Q)$ in Section 2 (see footnote 4). So $f(w_i)$ is considered a premium attributable to product i's quality, as in hedonic models. In Section 2, we have made some assumptions on $p'(Q)(= u''(Q))$ and $p''(Q)(= u'''(Q))$, and this section maintains the same assumptions on $u''(Q)$ and $u'''(Q)$.

[Policy A]

Before looking into Policy D, let us consider the determination of Nash equilibrium quantity and quality in the baseline case of no subsidy. Conditional on Q_{-i}, firm i simultaneously sets q_i and w_i to maximise its profits $\pi_A(q_i, w_i, Q_{-i}) = [u'(Q_{-i} + q_i) + f(w_i) - c(w_i)]q_i - F$. With an interior solution assumed, the FOCs are

$$u''(Q_{-i} + q_i)q_i + u'(Q_{-i} + q_i) + f(w_i) - c(w_i) = 0, \tag{33}$$

$$[f'(w_i) - c'(w_i)]q_i = 0. \tag{34}$$

As $f(w_i) - c(w_i)$ is strictly concave, there exists a unique w_A^* that satisfies (34) for all i:

$$f'(w_A^*) - c'(w_A^*) = 0. \tag{35}$$

The marginal price (= marginal utility) and marginal cost of quality improvement are equalised at w_A^*. Then, given Q_{-i}, (33) with $w_i = w_A^*$ defines a unique optimal output $q_A^b(w_A^*, Q_{-i})$, as

in (4) in Section 2.[28] It follows that, as in (6), the output per firm at the unique and symmetric Nash equilibrium, q_A^*, is implicitly determined by

$$u''(nq_A^*)q_A^* + u'(nq_A^*) + f(w_A^*) - c(w_A^*) = 0. \tag{36}$$

[Policy D]

Suppose that the regulator sets $\overline{p}$ as a function of w_i, whereas r remains independent of w_i. In particular, let $\overline{p}$ be linear in w_i: $\overline{p}(w_i) = aw_i + b$, where $a(>0)$ and b are constant. Conditional on Q_{-i}, firm i simultaneously sets q_i and w_i to maximise its profits $\pi_{D_S}(q_i, w_i, Q_{-i}) = \{(1 - r)[u'(Q_{-i} + q_i) + f(w_i)] + r\overline{p}(w_i) - c(w_i)\}q_i - F$. With an interior solution assumed, the FOCs are

$$(1 - r)[u''(Q_{-i} + q_i)q_i + u'(Q_{-i} + q_i) + f(w_i)] + r[aw_i + b] - c(w_i) = 0, \tag{37}$$

$$[(1 - r)f'(w_i) + ra - c'(w_i)]q_i = 0. \tag{38}$$

As $(1 - r)f(w_i) + r[aw_i + b] - c(w_i)$ is strictly concave, there exists a unique $w_{D_S}^*$ that satisfies (38) for all i:

$$(1 - r)f'(w_{D_S}^*) + ra - c'(w_{D_S}^*) = 0. \tag{39}$$

To be consistent with Assumption 2, Policy D is set to meet $\overline{p}(w_{D_S}^*) = aw_{D_S}^* + b > c$. Then, given Q_{-i}, (37) with $w_i = w_{D_S}^*$ defines a unique optimal output $q_{D_S}^b(w_{D_S}^*, Q_{-i})$, as in (15) in Section 2. It follows that, as in (16), the output per firm in equilibrium, $q_{D_S}^*$, is implicitly determined by

$$(1 - r)[u''(nq_{D_S}^*)q_{D_S}^* + u'(nq_{D_S}^*) + f(w_{D_S}^*)] + r[aw_{D_S}^* + b] - c(w_{D_S}^*) = 0. \tag{40}$$

(39) means that the government can affect the equilibrium quality $w_{D_S}^*$ by adjusting $\overline{p}'(w_i) = a$. Importantly, b does not appear in (39), but only in (40) that determines the equilibrium quantity $q_{D_S}^*$. As a result, (39) does not restrict the regulator's ability to choose the level of $\overline{p}(w_{D_S}^*) = aw_{D_S}^* + b$ and influence $q_{D_S}^*$. Hence, augmenting Policy D by making $\overline{p}$ dependent on w_i does not change the analysis of the previous sections.

Proposition 6. *The equilibrium product quality under the IPR subsidy ($w_{D_S}^*$) is compared with that under no subsidy (w_A^*) as follows:*

$$w_{D_S}^* \begin{cases} < w_A^* & \text{if } a < f'(w_A^*), \\ = w_A^* & \text{if } a = f'(w_A^*), \\ > w_A^* & \text{if } a > f'(w_A^*). \end{cases} \tag{41}$$

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[28]As for the second order condition (SOC), the Hessian of $\pi_A(q_i, w_i, Q_{-i})$, where q_i and w_i are the variables and Q_{-i} is given, is negative definite at the unique critical point $(q_A^b(w_A^*, Q_{-i}), w_A^*)$ of $\pi_A(q_i, w_i, Q_{-i})$, making it a unique global maximiser. Analogous reasoning confirms the SOC for the profit maximisation problem under Policy D to be discussed next.

Proof. See the Appendix. ∎

Proposition 6 implies that by letting $\bar{p}$ increase with w_i at the rate equal to the marginal utility (= marginal price) of quality at the no-subsidy equilibrium E_A (i.e., $a = f'(w_A^*)$), Policy D can induce the equilibrium quantity $Q_{D_S}^*(> Q_A^*)$ while maintaining the quality level realised at E_A (i.e., $w_{D_S}^* = w_A^*$). This result can extend to the case of multiple (K) attributes $\mathbf{w}_i = (w_{1i}, \cdots, w_{Ki})$.

With regard to actual implementation of the IPR scheme, Proposition 6 indicates the importance of the measurability of the product's attributes. If they are relatively easily measured (e.g., various attributes of electric cars), policymakers can be guided by estimated monetary values of these attributes and set the rule for linking $\bar{p}$ with quality. On the other hand, the IPR scheme is unlikely to be suited for those goods whose attributes are hard to be defined or measured (e.g., services such as childcare and education), for it is then difficult to change $\bar{p}$ to prevent quality degradation.

Following the above argument, let us consider several cases in which monetary values of product attributes can be estimated relatively easily, allowing the IPR form to perform better. First, as illustrated by the solar PV panel and electric car subsidies discussed so far, various subsidy programmes are in place today across countries to encourage the faster diffusion of environment-friendly durable goods such as renewable energy systems, low-emission vehicles, energy-efficient home appliances, and energy-saving building renovations. Information on energy costs/benefits (as well as other attributes) is usually available for these goods. Indeed, the rate of a (specific) subsidy is often dependent on product quality: for example, U.S. federal tax credits for plug-in hybrid vehicles increase with battery capacity.

Second, many countries regulate pharmaceutical drug prices to ensure quality and affordability for consumers: the out-of-pocket price paid by consumers for a drug is set below its unregulated level through a negotiation between the regulator and a pharmaceutical firm. In return, a (specific or *ad valorem*) subsidy is paid to the firm. The regulated price and subsidy rate depend on drug quality, which is determined by clinical evidence and other assessments (see, e.g., OECD, 2008; Paris and Belloni, 2013). With theoretical models based on the negotiation process in Australia, Johnston and Zeckhauser (1991) and Wright (2004) suggest the possibility that the government can take advantage of its bargaining with firms and inter-firm strategic interaction to increase social surplus and flexibly adjust producer surplus at the same time, two functions featured by the IPR scheme as well.[29]

Other examples include subsidies on health insurance and housing. In the U.S., low- and middle-income households are subsidised for their purchase of health insurance from private firms. The attributes (i.e., benefits and costs) of a health insurance plan are clearly defined (though complicated), and the marginal price of an attribute can be actuarially calculated. Likewise, low- and middle-income households in many countries are subsidised for the purchase or rental cost of housing, and hedonic regression models provide good estimates of the

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[29]Wright (2004) points out that in practice the government does not well exploit the second function in the negotiations, as it simply benchmarks the subsidy level (and thus firm profitability) against foreign markets.

prices of housing attributes.

6 Conclusion

This paper has been motivated by a unique structure and experience of a government subsidy programme in Japan or the U.K. in which a higher subsidy is paid as the price of a target product (residential solar PV system for Japan, or low-emission vehicle for the U.K.) is cut, inducing lower (post- and) pre-subsidy prices and larger sales. To my knowledge, this type of policy design has not previously been studied in the literature, making this paper the first to analyse the effects of making subsidy payment inversely related to the price. Specifically, I consider a particular scheme (termed the IPR scheme) in which subsidy payment is conditional on the product's price being less than a government-set threshold, and is proportional to the differential between the threshold and price.

With a model of imperfect competition (Cournot oligopoly, or monopoly), I find that the IPR subsidy has two advantages over the widely-used specific or *ad valorem* subsidy.[30] First, it can induce a given level of market output (the government's target) with less government expenditure than the other forms. Second, while achieving this target, the government can also seek a second goal of controlling the incidence of the policy on producers: profits can be made larger or smaller than under no government intervention.

Simulations on an existing electric car subsidy programme in the U.S. indicate the substantial magnitude of these advantages. The actual market sales induced by the current specific subsidy would be realised with 50–81% less government outlay if the IPR structure is used instead. Additionally, depending on the regulator's choice of policy variables, producer surplus induced by the IPR scheme can range from \$72–150 million higher to \$63–169 million lower than under the case of no government intervention.

The IPR form can be used in various subsidy programmes, especially if the pecuniary value of the target good's quality is estimable. Such goods would include environment-friendly durables (as in the actual cases in Japan and the U.K.), pharmaceuticals, (privately supplied) health insurance, and housing (purchase or rental).

A Proofs

A.1 Proof of Lemma 1

Proof. Equation (15) and the assumption $\overline{p} > c$ give

$$p'(Q_{-i} + q_i) \cdot q_i + p(Q_{-i} + q_i) = \frac{c - r\overline{p}}{1 - r} < c, \tag{42}$$

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[30]Most of the qualitative results of the paper carry over to the differentiated Bertrand competition framework.

Since $\frac{\partial[p'(Q_{-i}+q_i)q_i+p(Q_{-i}+q_i)]}{\partial q_i} = p''(Q_{-i}+q_i)q_i + 2p'(Q_{-i}+q_i) < 0$ (i.e., marginal revenue is decreasing in q_i) by (2), comparing (42) with (4) proves $q_A^b(Q_{-i}) < q_{D_S}^b(Q_{-i})$.

Substituting $q_i = q_{D_S}^*$ $\forall i$ into (42) gives $p'(nq_{D_S}^*) \cdot q_{D_S}^* + p(nq_{D_S}^*) < c$. Comparing this with (6) and noting that $\frac{dp'(nq)q+p(nq)}{dq} = p'(nq) + p''(nq)nq + p'(nq)n = p''(Q)Q + (n+1)p'(Q) < 0$ (by Assumption 1), we find $q_A^* < q_{D_S}^*$. ∎

A.2 Proof of Proposition 1

Proof. The definition of $G(Q_{-i})$ means that conditional on Q_{-i},

$$G(Q_{-i}) \geq 0 \iff \pi_A(q_A^b(Q_{-i}), Q_{-i}) = \max_{q_i}\{\max\{\pi_A(q_i, Q_{-i}), \pi_{D_S}(q_i, Q_{-i})\}\} \qquad (43)$$

We now refer to (14) and take the (in)eligibility conditions there into account. Given Q_{-i}, if $G(Q_{-i}) \geq 0$ and additionally if the eligibility requirement is not satisfied at $q_i = q_A^b(Q_{-i})$ (that is, $p(Q_{-i} + q_A^b(Q_{-i})) \geq \bar{p}$), then (14) and (43) imply that $q_A^b(Q_{-i})$ is firm i's best response. Similarly, conditional on Q_{-i},

$$G(Q_{-i}) \leq 0 \iff \pi_{D_S}(q_{D_S}^b(Q_{-i}), Q_{-i}) = \max_{q_i}\{\max\{\pi_A(q_i, Q_{-i}), \pi_{D_S}(q_i, Q_{-i})\}\}. \qquad (44)$$

Thus, given Q_{-i}, if $G(Q_{-i}) \leq 0$ and additionally if the eligibility requirement is satisfied at $q_i = q_{D_S}^b(Q_{-i})$ (that is, $p(Q_{-i} + q_{D_S}^b(Q_{-i})) < \bar{p}$), then (14) and (44) imply that $q_{D_S}^b(Q_{-i})$ is firm i's best response. The following lemma states the relationship between $G(Q_{-i})$ and the (in)eligibility conditions.

Lemma 2. $G(Q_{-i}) \geq 0$ *implies* $p(Q_{-i} + q_A^b(Q_{-i})) > \bar{p}$, *and* $G(Q_{-i}) \leq 0$ *implies* $p(Q_{-i} + q_{D_S}^b(Q_{-i})) < \bar{p}$. *Equivalently,* $p(Q_{-i} + q_A^b(Q_{-i})) \leq \bar{p}$ *implies* $G(Q_{-i}) < 0$, *and* $p(Q_{-i} + q_{D_S}^b(Q_{-i})) \geq \bar{p}$ *implies* $G(Q_{-i}) > 0$.

Proof. Suppose $p(Q_{-i} + q_A^b(Q_{-i})) \leq \bar{p}$. Then,

$$
\begin{aligned}
\pi_{D_S}(q_{D_S}^b(Q_{-i}), Q_{-i}) &> \pi_{D_S}(q_A^b(Q_{-i}), Q_{-i}) \\
&= [(1-r)p(Q_{-i} + q_A^b(Q_{-i})) + r\bar{p} - c] \cdot q_A^b(Q_{-i}) - F \\
&= [p(Q_{-i} + q_A^b(Q_{-i})) - c] \cdot q_A^b(Q_{-i}) + r[\bar{p} - p(Q_{-i} + q_A^b(Q_{-i}))] \cdot q_A^b(Q_{-i}) - F \\
&= \pi_A(q_A^b(Q_{-i}), Q_{-i}) + r[\bar{p} - p(Q_{-i} + q_A^b(Q_{-i}))] \cdot q_A^b(Q_{-i}) \\
&\geq \pi_A(q_A^b(Q_{-i}), Q_{-i}).
\end{aligned}
$$

That is, $G(Q_{-i}) < 0$. In other words, if $G(Q_{-i}) \geq 0$, then $p(Q_{-i} + q_A^b(Q_{-i})) > \bar{p}$.

Suppose $p(Q_{-i} + q^b_{D_S}(Q_{-i})) \geq \overline{p}$. Then,

$$
\begin{aligned}
\pi_A(q^b_A(Q_{-i}), Q_{-i}) &> \pi_A(q^b_{D_S}(Q_{-i}), Q_{-i}) \\
&= [p(Q_{-i} + q^b_{D_S}(Q_{-i})) - c] \cdot q^b_{D_S}(Q_{-i}) - F \\
&\geq [p(Q_{-i} + q^b_{D_S}(Q_{-i})) - c] \cdot q^b_{D_S}(Q_{-i}) + r[\overline{p} - p(Q_{-i} + q^b_{D_S}(Q_{-i}))] \cdot q^b_{D_S}(Q_{-i}) - F \\
&= [(1 - r)p(Q_{-i} + q^b_{D_S}(Q_{-i})) + r\overline{p} - c] \cdot q^b_{D_S}(Q_{-i}) - F \\
&= \pi_{D_S}(q^b_{D_S}(Q_{-i}), Q_{-i}).
\end{aligned}
$$

That is, $G(Q_{-i}) > 0$. In other words, if $G(Q_{-i}) \leq 0$, then $p(Q_{-i} + q^b_{D_S}(Q_{-i})) < \overline{p}$. ∎

Lemma 2 and the argument preceding it suggest that the best response correspondence under Policy D conditional on Q_{-i}, denoted by $q^b_D(Q_{-i})$, is as follows:

$$
q^b_D(Q_{-i}) = \begin{cases} q^b_A(Q_{-i}) & \text{if} \quad G(Q_{-i}) \geq 0, \\ q^b_{D_S}(Q_{-i}) & \text{if} \quad G(Q_{-i}) \leq 0 \end{cases} \tag{45}
$$

The next lemma helps simplify the conditions in (45).

Lemma 3. *1. If there exists $\check{Q}_{-i}$ such that $G(\check{Q}_{-i}) \geq 0$, then $G(Q_{-i}) > 0$ for all $Q_{-i} < \check{Q}_{-i}$.*

2. If there exists $\check{Q}_{-i}$ such that $G(\check{Q}_{-i}) \leq 0$, then $G(Q_{-i}) < 0$ for all $Q_{-i} > \check{Q}_{-i}$.

3. There is at most one $\tilde{Q}_{rest}$ such that $G(\tilde{Q}_{rest}) = 0$.[31]

Proof. By the envelope theorem, $\frac{d\pi_A(q^b_A(Q_{-i}), Q_{-i})}{dQ_{-i}} = p'(Q_{-i} + q^b_A(Q_{-i})) \cdot q^b_A(Q_{-i})$, and $\frac{d\pi_{D_S}(q^b_{D_S}(Q_{-i}), Q_{-i})}{dQ_{-i}} = (1 - r)p'(Q_{-i} + q^b_{D_S}(Q_{-i})) \cdot q^b_{D_S}(Q_{-i})$. Thus,

$$
\begin{aligned}
G'(Q_{-i}) &= p'(Q_{-i} + q^b_A(Q_{-i})) \cdot q^b_A(Q_{-i}) - (1 - r)p'(Q_{-i} + q^b_{D_S}(Q_{-i})) \cdot q^b_{D_S}(Q_{-i}) \\
&< \{p'(Q_{-i} + q^b_A(Q_{-i})) \cdot [q^b_A(Q_{-i})]^2 - (1 - r)p'(Q_{-i} + q^b_{D_S}(Q_{-i})) \cdot [q^b_{D_S}(Q_{-i})]^2\} \frac{1}{q^b_{D_S}(Q_{-i})} \\
&= -G(Q_{-i})\frac{1}{q^b_{D_S}(Q_{-i})},
\end{aligned} \tag{46}
$$

where the inequality follows because $q^b_A(Q_{-i}) < q^b_{D_S}(Q_{-i})$ by Lemma 1, and the last equality results from substituting the FOCs (4) and (15) into (17). Thus, $G(Q_{-i}) = 0$ implies $G'(Q_{-i}) < 0$. Therefore, the curve $G = G(Q_{-i})$ on the Q_{-i}-G plane can cross the horizontal axis $G = 0$ only from above and at most once, so the three statements of the lemma follow. ∎

Note that $p(Q_{-i}) \leq c$ for Q_{-i} sufficiently large (by Assumption 1), and $c < \overline{p}$ (by Assumption 2). Then, for Q_{-i} sufficiently large, $p(Q_{-i} + q^b_A(Q_{-i})) < p(Q_{-i}) < \overline{p}$, and thus by Lemma 2, $G(Q_{-i}) < 0$. Hence, Lemma 3 and the intermediate value theorem imply that there exists a unique $\tilde{Q}_{rest}(\geq 0)$ such that $G(\tilde{Q}_{rest}) = 0$ if and only if $G(0) \geq 0$.

---

[31]Since the functions $\pi_A(q^b_A(Q_{-i}), Q_{-i})$ and $\pi_{D_S}(q^b_{D_S}(Q_{-i}), Q_{-i})$ are identical across the firms, $\tilde{Q}_{rest}$ is common to all i (so $\tilde{Q}_{rest}$ is used rather than $\tilde{Q}_{-i}$).

With (45) and Lemma 3, the best response correspondence is given by (18) if $G(0) \geq 0$, and by (19) if $G(0) < 0$.

∎

A.3 Proof of Proposition 2

Proof. First, it is proved by contradiction that no asymmetric equilibrium exists. If there is one, there are (at least) two firms (j and k) such that $q_j \neq q_k$ in equilibrium. Without loss of generality, let $q_j < q_k$. Then, $Q_{-j} - Q_{-k} = q_k - q_j > 0$, implying that $slope \equiv \frac{q_j - q_k}{Q_{-j} - Q_{-k}} = -1$. But this leads to a contradiction because $slope > -1$ as shown below. If both firms respond with q_A^b, then $slope = \frac{q_A^b(Q_{-j}) - q_A^b(Q_{-k})}{Q_{-j} - Q_{-k}} > -1$ because $\frac{dq_A^b(Q_{-i})}{dQ_{-i}} > -1$ everywhere, as shown in (5) of footnote 8. Similarly, if both firms respond with $q_{D_S}^b$, then $slope = \frac{q_{D_S}^b(Q_{-j}) - q_{D_S}^b(Q_{-k})}{Q_{-j} - Q_{-k}} > -1$ because $\frac{dq_{D_S}^b(Q_{-i})}{dQ_{-i}} > -1$ everywhere. Next, suppose that firm j responds with $q_{D_S}^b$, and firm k with q_A^b. Then $slope = \frac{q_{D_S}^b(Q_{-j}) - q_A^b(Q_{-k})}{Q_{-j} - Q_{-k}} > \frac{q_A^b(Q_{-j}) - q_A^b(Q_{-k})}{Q_{-j} - Q_{-k}} > -1$ because $q_{D_S}^b(Q_{-j}) > q_A^b(Q_{-j})$. Lastly, it never happens that firm j responds with q_A^b, and firm k with $q_{D_S}^b$, because in that case (18) implies $Q_{-j} \leq \tilde{Q}_{rest} \leq Q_{-k}$, but we have assumed above that $Q_{-j} - Q_{-k} = q_k - q_j > 0$.

Second, we look into symmetric equilibria. The best response correspondence (function) (18) and (19) imply that there are two potential cases to occur at a symmetric equilibrium: [1] all firms follow q_A^b, so $q_i = q_A^b((n-1)q_A^*) = q_A^*$ for all i (as determined by (6)); or [2] all firms follow $q_{D_S}^b$, so $q_i = q_{D_S}^b((n-1)q_{D_S}^*) = q_{D_S}^*$ for all i (as determined by (16)). In case [1], the potential equilibrium is nothing other than E_A, the unique and symmetric equilibrium under Policy A. According to (18), $q_D^b(Q_{-i}) = q_A^b(Q_{-i})$ if and only if $Q_{-i} \leq \tilde{Q}_{rest}$. Therefore, the n best response correspondences $q_i = q_D^b(Q_{-i})$, $i = 1, \cdots, n$, intersect at E_A (that is, E_A is an equilibrium under Policy D) if and only if $(n-1)q_A^* \leq \tilde{Q}_{rest}$.

In case [2], the potential equilibrium (denoted by E_{D_S}) is nothing but the unique intersection of n functions $q_i = q_{D_S}^b(Q_{-i})$, $i = 1, \cdots, n$. When $G(0) \geq 0$, (18) shows $q_D^b(Q_{-i}) = q_{D_S}^b(Q_{-i})$ if and only if $Q_{-i} \geq \tilde{Q}_{rest}$. Thus, provided $G(0) \geq 0$, E_{D_S} is an intersection of the n best response correspondences $q_i = q_D^b(Q_{-i})$, $i = 1, \cdots, n$, if and only if $(n-1)q_{D_S}^* \geq \tilde{Q}_{rest}$. When $G(0) < 0$ ($\iff$ $\tilde{Q}_{rest}$ does not exist, by Lemma 3 in the proof of Proposition 1), (19) shows $q_D^b(Q_{-i}) = q_{D_S}^b(Q_{-i})$ for all $Q_{-i} \geq 0$. Thus, provided $G(0) < 0$, the n best response correspondences $q_i = q_D^b(Q_{-i})$, $i = 1, \cdots, n$, intersect at E_{D_S}. Therefore, in case [2], E_{D_S} is an equilibrium under Policy D if and only if $(n-1)q_{D_S}^* \geq \tilde{Q}_{rest}$ or $\tilde{Q}_{rest}$ does not exist.

Since $(n-1)q_A^* < (n-1)q_{D_S}^*$ by Lemma 1, statements 1–3 of the proposition follow.

∎

A.4 Proof of Proposition 3

Proof. With $\hat{Q}$ and r set by the government, substituting the FOCs (4) and (15) into (17) and letting $Q_{-i} = 0$ yields $G(0; r, \hat{Q}) = -p'(0 + q_A^b(0)) \cdot [q_A^b(0)]^2 + (1 - r)p'(0 + q_{D_S}^b(0; \hat{Q})) \cdot [q_{D_S}^b(0; \hat{Q})]^2$. So

$$\text{sgn}(G(0; r, \hat{Q})) = \text{sgn}\left(r - \left\{1 - \frac{p'(0 + q_A^b(0)) \cdot [q_A^b(0)]^2}{p'(0 + q_{D_S}^b(0; \hat{Q})) \cdot [q_{D_S}^b(0; \hat{Q})]^2}\right\}\right). \tag{47}$$

[*Case 1*]

If $G(0; r, \hat{Q}) \leq 0$, or equivalently if $r \leq 1 - \frac{p'(0 + q_A^b(0)) \cdot [q_A^b(0)]^2}{p'(0 + q_{D_S}^b(0; \hat{Q})) \cdot [q_{D_S}^b(0; \hat{Q})]^2}$, Lemma 3 (in the proof of Proposition 1) implies that the condition given in statement 1 of Proposition 2 holds (because $G(0; r, \hat{Q}) < 0$, or in case $G(0; r, \hat{Q}) = 0$, $\tilde{Q}_{rest} = 0$, where $\tilde{Q}_{rest}$ is as defined in Proposition 2). Therefore, the only equilibrium under Policy D is E_{D_S}.

[*Case 2*]

If $G(0; r, \hat{Q}) > 0$, or equivalently if $r > 1 - \frac{p'(0 + q_A^b(0)) \cdot [q_A^b(0)]^2}{p'(0 + q_{D_S}^b(0; \hat{Q})) \cdot [q_{D_S}^b(0; \hat{Q})]^2}$, then conditional on $\hat{Q}$ and r, the intermediate value theorem and Lemma 3 imply that there exists a unique $\tilde{Q}_{rest}(> 0)$ such that $G(\tilde{Q}_{rest}; r, \hat{Q}) = 0$.[32] Plugging (22) into $G(\tilde{Q}_{rest}; r, \hat{Q}) = 0$ yields

$$\begin{aligned}
0 &= G(\tilde{Q}_{rest}; r, \hat{Q}) \\
&= [p(\tilde{Q}_{rest} + q_A^b(\tilde{Q}_{rest})) - c] \cdot q_A^b(\tilde{Q}_{rest}) \\
&\quad - \left\{(1 - r)p(\tilde{Q}_{rest} + q_{D_S}^b(\tilde{Q}_{rest}; \hat{Q})) - (1 - r)[p'(\hat{Q})\hat{q} + p(\hat{Q})]\right\} \cdot q_{D_S}^b(\tilde{Q}_{rest}; \hat{Q}).
\end{aligned} \tag{48}$$

(46) and $G(\tilde{Q}_{rest}; r, \hat{Q}) = 0$ imply $\frac{\partial G(\tilde{Q}_{rest}; r, \hat{Q})}{\partial \tilde{Q}_{rest}} = p'(\tilde{Q}_{rest} + q_A^b(\tilde{Q}_{rest})) \cdot q_A^b(\tilde{Q}_{rest}) - (1 - r)p'(\tilde{Q}_{rest} + q_{D_S}^b(\tilde{Q}_{rest}; \hat{Q})) \cdot q_{D_S}^b(\tilde{Q}_{rest}; \hat{Q}) < 0$. On the other hand,

$$\begin{aligned}
\frac{\partial G(\tilde{Q}_{rest}; r, \hat{Q})}{\partial r} &= [p(\tilde{Q}_{rest} + q_{D_S}^b(\tilde{Q}_{rest}; \hat{Q})) - p'(\hat{Q})\hat{q} - p(\hat{Q})] \cdot q_{D_S}^b(\tilde{Q}_{rest}; \hat{Q}) \\
&= -p'(\tilde{Q}_{rest} + q_{D_S}^b(\tilde{Q}_{rest}; \hat{Q}))[q_{D_S}^b(\tilde{Q}_{rest}; \hat{Q})]^2 \\
&> 0,
\end{aligned} \tag{49}$$

where the second equality is due to (23).[33] Thus, by the implicit function theorem, $\frac{d\tilde{Q}_{rest}}{dr} = -\frac{\partial G(\tilde{Q}_{rest}; r, \hat{Q})/\partial r}{\partial G(\tilde{Q}_{rest}; r, \hat{Q})/\partial \tilde{Q}_{rest}} > 0$. Hence, noting that r_2 and r_3 are respectively determined by $G(\frac{n-1}{n}Q_A^*; r_2, \hat{Q}) = 0$ and $G(\frac{n-1}{n}\hat{Q}; r_3, \hat{Q}) = 0$, where $\frac{n-1}{n}Q_A^* < \frac{n-1}{n}\hat{Q}$, we find $r_2 < r_3$. Moreover, $0 < \tilde{Q}_{rest} < \frac{n-1}{n}Q_A^*$ if and only if $1 - \frac{p'(0 + q_A^b(0)) \cdot [q_A^b(0)]^2}{p'(0 + q_{D_S}^b(0; \hat{Q})) \cdot [q_{D_S}^b(0; \hat{Q})]^2} < r < r_2$; $\frac{n-1}{n}Q_A^* \leq \tilde{Q}_{rest} \leq \frac{n-1}{n}\hat{Q}$ if and only if $r_2 \leq r \leq r_3$; and $\frac{n-1}{n}\hat{Q} < \tilde{Q}_{rest}$ if and only if $r_3 < r$.

---

[32] As discussed after Lemma 3, $G(Q_{-i}; r, \hat{Q}) < 0$ for Q_{-i} sufficiently large.

[33] In this section, r and $\overline{p}$ co-move (more precisely, move in opposite directions) to satisfy (22). Without this condition, r and $\overline{p}$ are independent parameters as in Section 2, and $\frac{\partial G(\tilde{Q}_{rest}; r, \overline{p})}{\partial r} = [p(\tilde{Q}_{rest} + q_{D_S}^b(\tilde{Q}_{rest}; r, \overline{p})) - \overline{p}] \cdot q_{D_S}^b(\tilde{Q}_{rest}; r, \overline{p}) < 0$, where with some abuse of notation $\overline{p}$ replaces $\hat{Q}$ in the functions G and $q_{D_S}^b$. The last inequality follows from Lemma 2 and $G(\tilde{Q}_{rest}; r, \overline{p}) = 0$.

Therefore, [*Case 1*] and [*Case 2*] above show that statements 1–3 of Proposition 2 are equivalent to those of this proposition.

$\blacksquare$

A.5 Proof of Proposition 5

Proof. By rewriting the denominator of (24) with (23),

$$r_2 = 1 - \frac{-p'(Q_A^*)q_A^{*2}}{[p(\frac{n-1}{n}Q_A^* + q_{D_S}^b(\frac{n-1}{n}Q_A^*; \hat{Q})) - p'(\hat{Q})\hat{q} - p(\hat{Q})] \cdot q_{D_S}^b(\frac{n-1}{n}Q_A^*; \hat{Q})}. \tag{50}$$

Similarly,

$$r_1 = 1 - \frac{-p'(Q_A^*)q_A^{*2}}{[p(\frac{n-1}{n}\hat{Q} + q_{D_S}^b(\frac{n-1}{n}\hat{Q}; \hat{Q})) - p'(\hat{Q})\hat{q} - p(\hat{Q})] \cdot q_{D_S}^b(\frac{n-1}{n}\hat{Q}; \hat{Q})}. \tag{51}$$

Note that (23) implies that the denominators of (50) and (51) are positive. By the envelope theorem,[34]

$$\frac{d[p(Q_{-i} + q_{D_S}^b(Q_{-i}; \hat{Q})) - p'(\hat{Q})\hat{q} - p(\hat{Q})] \cdot q_{D_S}^b(Q_{-i}; \hat{Q})}{dQ_{-i}} = p'(Q_{-i} + q_{D_S}^b(Q_{-i}; \hat{Q})) \cdot q_{D_S}^b(Q_{-i}; \hat{Q})$$

$$< 0. \tag{52}$$

(50), (51), and (52) imply that $r_1 < r_2$ because $\frac{n-1}{n}Q_A^* < \frac{n-1}{n}\hat{Q}$. The remaining statements follow from Proposition 3 and (28).

$\blacksquare$

A.6 Proof of Proposition 6

Proof. If $\alpha < f'(w_A^*)$, $(1-r)f'(w_A^*) + r\alpha - c'(w_A^*) = f'(w_A^*) - c'(w_A^*) + r[\alpha - f'(w_A^*)] < 0$ by (35). This means $w_{D_S}^* < w_A^*$ because of (39) and $\frac{d[(1-r)f'(w_i)+r\alpha-c'(w_i)]}{dw_i} = (1-r)f''(w_i) - c''(w_i) < 0$. The two other cases can be proven similarly. $\blacksquare$

---

[34]Substituting (22) into (13) shows that $q_{D_S}^b(Q_{-i}; \hat{Q})$ is a maximiser of $[p(Q_{-i} + q_i) - p'(\hat{Q})\hat{q} - p(\hat{Q})]q_i$, conditional on $\hat{Q}$ and Q_{-i}.

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