# The Long Run Evolution of Absolute Intergenerational Mobility

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#### Abstract

This paper combines cross-sectional and longitudinal income data to present the evolution of absolute intergenerational income mobility in several developed economies in the 20th century. We show that detailed panel data are unnecessary for estimating absolute mobility in the long run. We find that in all countries absolute mobility decreased during the second half of the 20th century. Increasing income inequality and decreasing growth rates have contributed to the decrease. Yet, growth is the dominant contributor to this decrease in most countries. We derive a model for the relationship between absolute mobility, growth, inequality and relative mobility. Ceteris paribus, absolute and relative mobility are inversely related.

**Keywords:** Mobility, inequality, copula modeling **JEL Codes:** D3, E2, H0, J6

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# 1 Introduction

The question whether next generations will be better off than previous ones has been central in the recent public economic and political debate. Chetty et al.  $(2014\underline{b})$ discuss a "growing public perception that intergenerational income mobility [...] is declining in the United States." Others argue that "people's frustrations [... are] rooted in the fear that their kids won't be better off than they were." (Obama, 2013) This has led scholars to quantify absolute intergenerational mobility – "the likelihood a child will be financially better off than their parent at around the same age." (Halikias and Reeves, 2016) This paper estimates the long run evolution of absolute intergenerational mobility in income in several major countries. In all, we identify a significant downward trend for post-war era birth cohorts.

Absolute mobility, which we denote by A, is defined as the fraction of children with higher real incomes than their parents at the same age (Chetty et al., 2014a). This captures the chances of children to have a higher standard of living than their parents. Chetty et al. (2017) studied the historical evolution of absolute mobility in the United States. They found that it has fallen from around 90% for children born in 1940 to 50% for children born in the 1980s (see Fig. 1). This result is still inline with the increase of living standards among young adults compared to previous generations. According to the US Current Population Survey (United States Bureau of Labor Statistics, 2017), real wages of American 30-year-olds grew annually on average by 1.0% during 1965–1995 and by 0.6% during 1985–2015. The decrease of these growth rates is one of the drivers of the documented decline in mobility. Chetty et al. (2017) also found that the increase of income inequality explains most of the documented decrease in absolute mobility in the United States.

Estimating absolute mobility for longer time periods and more countries poses challenges. First, matching incomes of parents and children requires historical panel data. These usually do not cover the whole income distribution and are available for a very limited range of birth cohorts. In many countries, such data are rare. Second, income data are many times disjoint from microdata such as age and gender. This further complexifies the identification of parents and children. These issues are of particular importance when considering early 20th century, or earlier, cohorts. In such cases, the existing data sources on income are very limited.

This paper presents the long run evolution of absolute mobility in a group of de-

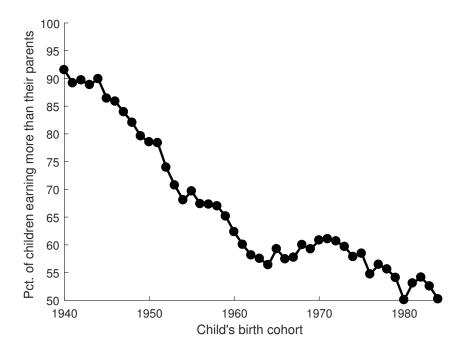


Figure 1: Trends in absolute mobility in the United States. Source: Chetty et al. (2017).

veloped economies. In France, Sweden and the United States it was possible to estimate absolute mobility for birth cohorts of early 20th century. This goes far beyond the existing literature on absolute income mobility. Our approach combines the marginal income distributions for parents and children and their copula – the joint distribution of parent and child income ranks. It overcomes the described difficulties. We find that long panel data series are unnecessary for estimating absolute mobility. We show that the estimates of absolute mobility depend mainly on the marginal income distributions. Their copula plays only a minor role in determining absolute mobility, within plausible limits. We also find that plausible changes in the copula cannot explain the long run evolution of absolute mobility, and only changes in the marginal distributions can. In short, we are able to provide robust evidence that the concept of absolute mobility is driven primarily by economic growth, secondarily by changes in inequality, and thirdly (and weakly) by changes in relative mobility.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Intergenerational mobility is typically divided into two classes: relative and absolute. Relative measures gauge children's propensity to occupy a different position in the income distribution than their parents. Absolute measures gauge their propensity to have higher incomes than their parents in real terms.

These observations make the estimation of absolute mobility possible even for countries in which panel data are scarce. We then combine available data on intergenerational copulas and historical income distributions to provide absolute mobility estimates for several developed countries over the coarse of a few decades.

We find a significant and nearly monotonic decrease in the chances of children to have a higher income than their parents in all countries over the post-war period. We also find that this decrease followed a rapid increase in absolute mobility from the early 1900s until World War II. This increase reflects the economic boom and the decreasing income inequality in developed countries during the three decades that followed the end of the war.

The decrease in absolute mobility in the United States was mainly due to increasing income inequality. We find that in France, Denmark, Sweden and Norway, the slow economic growth of the past several decades was the key contributor to a similar decrease. Despite higher growth rates and regardless of "the American Dream" ethos, the absolute intergenerational mobility in the United States for late 1970s and early 1980s birth cohorts is among the lowest within the group of countries. The increasing income inequality did play a dominant role in the decrease of absolute mobility in Germany and the United Kingdom, yet still not as major as in the United States.

Using new data on France, we also find evidence that the rise in female labor force participation rates did not lead to an increase in absolute intergenerational mobility. This is inline with the findings of Chetty et al. (2014b) for the US. They found that "comparing children's family incomes to their parents' family incomes [...] we find similar declines in absolute mobility for sons and daughters".

We study a simplified model, based on a standard regression model for intergenerational income dynamics. We derive closed-form expressions for absolute mobility as a function of income growth, income inequality and relative intergenerational mobility. This allows quantifying the sensitivity of absolute mobility to potential changes in income inequality and growth. The model also shows that absolute intergenerational mobility of 50%, to which several of the countries are very close, is a plausible lower bound. When income growth is close to zero, or when the increase in inequality is dramatic, absolute mobility approaches this bound. The model describes well the long run evolution of absolute mobility. We also find that one should not expect the co-movement of absolute and relative mobility. Ceteris paribus, these two types of mobility are mechanically inversely related. This seemingly counter-intuitive relationship stems from a conceptual difference between the two categories of mobility. When relative mobility is low, children are likely to stay at the same income rank as their parents'. In this case, even the slightest income growth would result in very high absolute mobility. When relative mobility is high, children's income ranks cannot be predicted by their parents'. Thus, absolute mobility is likely to be close to random, or to 50%. As described, the effects of growth and inequality on absolute mobility are more important than that of relative mobility. Thus, despite the inverse relationship between the two mobility indicators, countries characterized by high relative mobility, such as the Nordic countries, are also characterized by high absolute mobility, in practice.

Our findings are also consistent with those of Katz and Krueger (2017). They quantify absolute mobility by the share of children in a given birth cohort that have higher income than the median father of these children. In the case of the United States this measure is nearly identical to absolute intergenerational mobility. This hints that indeed, the marginal distributions may allow estimating absolute mobility without the need for longitudinal data. Yet, we find that in some cases, especially in countries with high relative mobility, the Katz-Krueger mobility (KKM) could lead to estimates that are very different from those of absolute intergenerational mobility. Also, while the findings of Chetty et al. (2017) "fit well with what has previously been established regarding rising U.S. income inequality and stagnating real median earnings," (Katz and Krueger, 2017) this is not the case in other countries.

Relative mobility has been studied for decades.<sup>2</sup> Yet, investigations of absolute mobility in income remain "scarce, mainly because of the lack of large, high-quality panel data sets linking children to their parents." (Chetty et al., 2017) Absolute mobility has been studied in the context of class and occupation. Class mobility, occupational mobility and mobility in educational attainment are central in the sociological mobility literature, both using relative and absolute measures.<sup>3</sup> The differences in outcome variables and the different nature of measures used make

<sup>&</sup>lt;sup>2</sup>See, for example, Becker and Tomes (1979); Borjas (1992); Piketty (2000); Mazumder (2005); Aaronson and Mazumder (2008); Lee and Solon (2009); Hauser (2010); Corak (2013); Chetty et al. (2014b); Berman (2018); Kraay and Van der Weide (2017); Vosters and Nybom (2017).

<sup>&</sup>lt;sup>3</sup>See, for example, Lipset and Rogoff (1954); Lipset and Zetterberg (1959); Erikson, Goldthorpe and Portocarero (1979); Goldthorpe (1987); Erikson and Goldthorpe (1992); Breen and Jonsson (2005); Breen and Rottman (2014).

the results of this paper difficult to compare to this literature. Yet, we note that already Lipset and Rogoff (1954) identified that "the overall pattern of social mobility appears to be much the same in the industrial societies of various Western countries".  $^4$ 

Our contribution is threefold. First, from an empirical perspective, the primary contribution of this paper is to provide new series on absolute intergenerational mobility. In particular, we are able to provide evidence on absolute intergenerational mobility for early 20th century birth cohorts. We find that in all countries absolute mobility decreased during the second half of the 20th century. We also describe the effect of income growth and income inequality on absolute mobility and decompose the changes in absolute mobility into the contribution of each. We show that the decrease of income growth rates is the main determinant of changes in absolute mobility in most countries. In the United States, and to a lesser extent in the United Kingdom, increasing inequality was more important for the absolute mobility long run trend than growth.

Second, from a methodological perspective, we describe in detail the low sensitivity of absolute mobility to plausible copulas. This allows estimating absolute mobility in the long run, without the necessity for detailed panel data. Notably, we show that copulas of different countries share a similar form. Together with the low sensitivity of absolute mobility to changes in the copula, this enables using only a single parameter, such as the rank correlation, for estimating absolute mobility. The same methodology could be applied to other countries in the years ahead. Also, since household surveys are a common practice in most countries today, it will be possible to continue tracking absolute mobility even without detailed panel data, which are still much less common.

Third, from a theoretical perspective, this paper describes a mathematical inverse relationship between relative and absolute mobility. This does not mean that a country characterized by high absolute mobility will be characterized by low relative mobility and vice versa. Yet, for similar changes in inequality and for similar growth, absolute mobility is a decreasing function of relative mobility. This exposes problems that can arise if we treat both as capturing a similar phenomenon.

 $<sup>^{4}</sup>$ This observation was based on absolute mobility between three occupation classes – farm, manual and non-manual. The limited data at the time did not allow more nuanced conclusions.

The paper is organized as follows. Section 2 lays out our methodology, addressing the necessity of panel data for producing reliable estimates of absolute mobility. In Section 3 we specify our data sources. Section 4 presents the main results, describing the evolution of absolute mobility in several major countries. Section 5 discusses a simplified model for the relationship between absolute mobility, income growth, income inequality and relative intergenerational mobility. We conclude in Section 6.

# 2 Methodology

In an ideal setting, measuring the rate of absolute intergenerational mobility – the fraction of children with greater real-terms income than their parents – is trivial. For every birth cohort of children we could trace back their parents and compare their incomes at a certain age. However, such data are usually available for small samples and do not cover the whole income distribution or available for a very limited range of birth cohorts. Notably, in many countries, such data are rare. Mobility in education or occupation are easier to measure (see, for example, the recently compiled Global Database on Intergenerational Mobility (Narayan et al., 2018)), since it is possible to survey children and ask about their parents' education or occupation at a certain age. However, for income, which captures a more accurate picture of well-being than occupation or level of education, this would not be possible. As a result, estimating typical measures of relative income mobility over a long period of time is challenging and very unreliable in some countries.

Yet, in the case of absolute intergenerational mobility, one is able to provide reliable estimates with narrow confidence intervals, even in the absence of historical detailed panel data. The reason is double. First, as we demonstrate below, the structure of realistic copulas, the joint distributions of parent and child income ranks, is roughly similar. When two realistic copulas differ in a measure of relative mobility, they are very likely to differ proportionally in other relative mobility measures, which are theoretically independent from one another. The practical implication of this observation is that describing the copula using a single measure of relative mobility is empirically justified.

Second, as we shall also demonstrate below, the sensitivity of the absolute mobility estimates to plausible changes in the copula is low. In particular, plausible changes in relative mobility measures cannot explain long term changes in absolute mobility. Therefore, assuming a fixed copula in time will provide meaningful and reliable estimates of absolute mobility.

Our methodology builds on the approach of Chetty et al. (2017). We also use repeated cross-sections and combine them using a copula. Yet, using real intergenerational copulas, we are able to justify empirically the two points made above. Using these copulas, covering different countries, birth cohorts, pre- and post-tax incomes and considering the intergenerational links between fathers and daughters and fathers and sons, we are also able to provide realistic bounds for the absolute mobility estimates.

## 2.1 Empirical copulas and measures of relative mobility

We first use copulas measured for different birth cohorts, different countries and for both pre-tax and post-tax incomes and compare them in terms of different measures of relative mobility. Our aim is to demonstrate that although relative mobility is measured by theoretically distinct measures, in practice, differences in one measure translate into proportional changes in other measures. These measures of relative mobility are effectively interchangeable.

We consider copulas as transition (doubly stochastic) matrices  $P \in \mathscr{P}(N)$ , where  $p_{ij}$  represents the probability of transferring to quantile j (child) for those starting in quantile i (parent) and N is the number of income quantiles. We consider four standard measures of relative mobility:

• Spearman's rank correlation (Spearman, 1904) (or rank-rank slope, *RRS*), defined as

$$\rho_S(P) = \frac{12\sum_{i=1}^N \sum_{j=1}^N ijp_{ij} - 3N(N+1)^2}{N(N^2 - 1)}$$
(2.1)

• Bartholomew's index (Bartholomew, 1967) (average absolute jump), defined as

$$B(P) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} |i - j| p_{ij}$$
(2.2)

• Average absolute non-zero jump, defined as the average absolute jump while excluding the trace of P, or

$$NZ(P) = \frac{N \cdot B(P)}{N - tr(P)}$$
(2.3)

• Shorrocks' trace index (Shorrocks, 1978), defined as

$$S(P) = \frac{N - tr(P)}{N - 1} \tag{2.4}$$

The different measures are mathematically related, however they are not linearly dependent. Specifically, it is possible to construct matrices which have the same trace index, but very different rank correlation, average absolute non-zero jump measure or Bartholomew's index and vice versa. Bartholomew (1967); Shorrocks (1978); Atkinson and Bourguignon (1982); Atkinson (1983) provide several constructive examples demonstrating the differences between such measures. They describe mathematical constructions of copulas such that one measure is preserved while others may change.

For our analysis of relative intergenerational mobility measures we use 28 transition matrices estimated using survey and tax data covering pre-tax incomes in Denmark, Finland, Norway, Sweden, UK and US (Jäntti et al., 2006) for different birth cohorts in each, estimated for fathers and daughters and for fathers and sons; post-tax incomes in Germany, UK and US (Eberharter, 2014); de-identified federal income tax returns for pre-tax incomes in US (Chetty et al., 2014a).

Figure 2 depicts the relationship between the relative mobility measures calculated for the various transition matrices. It demonstrates that despite the a priori independence of the relative mobility measures, they are, in fact, almost linearly related across time and countries, both for pre- and post-tax incomes. We conclude, therefore, that the shape of the copulas is similar and they can be practically summarized by a single parameter. We use the rank correlation since in many countries the estimated relative mobility is simply reported using the rank correlation or the intergenerational elasticity (from which the rank correlation can be deduced), rather than providing the entire copula. Also, the rank correlation proves to be more empirically robust compared to other measures (Chetty et al., 2014a).

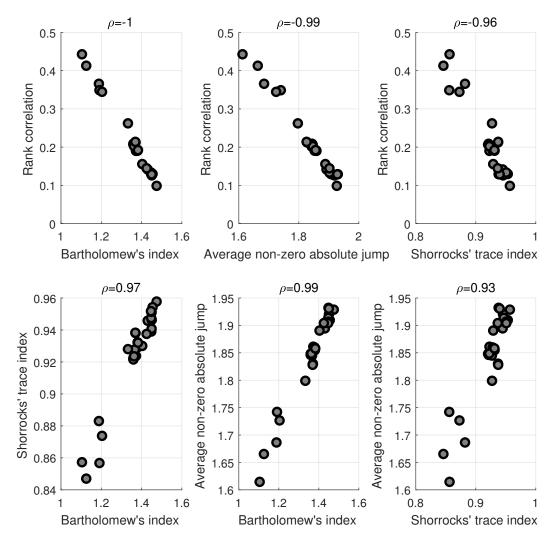


Figure 2: The relationship between relative mobility measures in empirical copulas.

### 2.2 Empirical copulas and absolute mobility

Figure 2 demonstrates that a single parameter characterizing relative intergenerational mobility can practically describe the estimated intergenerational copulas. We will now demonstrate, in addition, that the sensitivity of absolute mobility to plausible changes in the copula is low. Together, these observations would allow us to argue that the marginal income distributions and a single observation of a relative mobility measure, such as the rank correlation, can provide reliable estimates of absolute mobility for various countries. We note that conceptually, the same rank correlation could deliver very different absolute mobility estimates. Yet, the similar shape of empirical copulas described above, makes this practically implausible (see Appendix B). We also test the sensitivity of absolute mobility to different standard copula models. This test is presented in Appendix C, showing that for the same rank correlation, this sensitivity is very small.

Chetty et al. (2017) used linear programming for constructing plausible copulas, used for checking the robustness of their estimates. We use, instead, the same 28 copulas described above, which cover various countries and birth cohorts, as well as pre- and post-tax incomes. Using the same marginal distributions used for the United States absolute mobility estimates as in Chetty et al. (2017) (based on the United States Census, the CPS data and tax data), we estimated the United States absolute intergenerational mobility, each time using a different empirical copula as fixed in time, producing 28 different estimates of the absolute mobility evolution in time.

The results are presented in Fig. 3. They demonstrate that estimating the absolute mobility in the United States with different copulas, which may be very different from the one characterizing the United States, results in a very similar evolution in time. The estimates obtained using the various copulas differ from the benchmark estimates of Chetty et al. (2017) by 0.77 percentage points on average. We conclude that letting the copula change in time within the boundaries defined by the copulas used, cannot explain more than a change of several percentage points in absolute mobility over a long period of time. Thus, plausible changes in the copula cannot explain large long run trends like we find in all countries. Only changes in the marginal distributions can explain such trends.

The shaded area in Fig. 3 covers an area that is generally above the baseline absolute mobility estimate. This is due to the use of post-tax copulas in addition to pre-tax copulas. The post-tax copulas reflect usually lower relative mobility (Eberharter, 2014). This leads, as we will see in Section 5, to higher estimates of absolute mobility. If we only used pre-tax copulas, the variation of the different absolute mobility estimates would have been even smaller. Their average would have been closer to the baseline estimate.

In addition, in many countries the income distribution is not very well documented

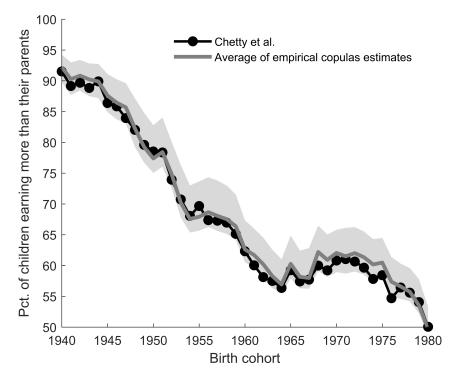


Figure 3: Effect of changes in copula on absolute intergenerational mobility in the United States. Mobility was estimated using the marginal distributions used in Chetty et al. (2017) and 28 empirical copulas for Denmark, Finland, Norway, Sweden, UK, US and Germany. The shaded gray area is the area covered by the various absolute mobility estimates. The gray curve is the arithmetic mean of all 28 estimates in each year. The black circles are the estimates reported in Chetty et al. (2017).

due to lack of high quality distributional national accounts and administrative data. Yet, for estimating absolute mobility, it may be sufficiently accurate to use survey data. Such surveys usually fail to cover well the top 1% of incomes, mainly due to income under-reporting and non-response (Moore and Welniak, 2000; Korinek, Mistiaen and Ravallion, 2006; Cowell, 2011), but the rest of the distribution is generally well-covered (Yonzan et al., 2018). Therefore the error in the estimation of absolute mobility due to this data limitation would be at most 1 percentage point.

## 3 Data

The above results demonstrate that absolute intergenerational mobility mainly depends on the income distributions of parents and children, rather than on their copula. Following the steps of Chetty et al. (2017) requires such distributions for 30-year-olds only, in years that are 30 years apart. Namely, in order to estimate absolute mobility for children born in 1980, we wish to possess the marginal income distribution of 30-year-olds in 1980 and in 2010. In the United States this became possible combining census, tax data and the CPS data for 1940–1984 birth cohorts. In France this has also became possible for 1970–2014 birth cohorts, using the tax data studied in Garbinti, Goupille-Lebret and Piketty (2018). However, we have good knowledge of the income distribution of the entire adult population even at much earlier years, based on tax data, surveys and national accounts.

Such data, in different levels of accuracy and details, are available from The World Inequality Database (2018). We consider only years in which data was sufficiently detailed to use the generalized Pareto curve interpolation method (Blanchet, Fournier and Piketty, 2017). When available, we use pre-tax total income data in the "equalsplit" assumption: individuals in tax units that are composed of more than one income-contributing individuals are assumed to contribute each an equal part to the total income. The equal-split assumption is compatible with the income specification in Chetty et al. (2017), using total family income of children and parents rather than individual incomes (see Alvaredo et al. (2016) for further details on this assumption). In the cases in which the incomes of equal-split adults are not available we use tax units as the unit of observation and when these are not available we use individual incomes. In Appendix D the potential bias this difference may have on the results is discussed. Using data from Garbinti, Goupille-Lebret and Piketty (2018), it is also possible to explicitly compare the results for individual adults and equal-split adults in France. This has a very small effect on the results (see Fig. 15).

Our data are available for the following countries and time periods:

Country	Time period	
Canada	1950 - 2010	
Denmark	1950 - 2010	
France	1915 - 2014	
Germany	1965 - 2013	
Norway	1950 - 2011	
Sweden	1903 - 2013	
United Kingdom	1959 - 2014	
United States	1917-2014	

Table 1: The availability of the income distribution for developed countries in The World Inequality Database (2018)

Data for the rank correlation in each of the above countries was taken from different studies. The data and their sources are presented in Tab. 2.<sup>5</sup> For each country we consider a nominal value; a lower bound -0.1, smaller than the smallest value estimated in any of the countries; and an upper limit -0.5, larger than the largest value estimated in any of the countries. These bounds not only account for possible measurement error, but also for the constant rank correlation we use for each country. In practice, the correlation may have changed over time. Using these bounds we are able to account for the fixed correlation possible effect on the results.

Country	Rank correlation	Source
Canada	0.27	Corak, Lindquist and Mazumder (2014)
Denmark	0.19	Jäntti et al. (2006)
France	0.3	Lefranc and Trannoy (2005)
Germany	0.3	Checchi (1997)
Norway	0.21	Bratberg, Anti Nilsen and Vaage (2005)
Sweden	0.2	Jäntti et al. (2006)
United Kingdom	0.3	Jäntti et al. (2006)
United States	0.3	Chetty et al. $(2014\underline{a})$

Table 2: Rank correlation values used in the absolute mobility analysis

<sup>5</sup>For the countries in which the interegenerational elasticity was reported, rather than the rank correlation, we use the relationship  $\rho = \frac{\sigma_p}{\sigma_c} \beta$ , where  $\sigma_p$  and  $\sigma_c$  are the standard deviations of the parents and children marginal income distributions and  $\beta$  is the estimated intergenerational income elasticity. The rank correlation is approximated by  $\rho_S \approx \frac{6 \operatorname{arcsin}(\rho/2)}{\pi}$  (see Trivedi and Zimmer (2007) and Section 5).

## 4 Results

#### 4.1 Estimating absolute mobility in developed economies

Following the discussion on the sensitivity of absolute mobility to changes in the copula and assuming that empirical copulas can be represented using a single parameter such as the rank correlation, we present estimates of trends in absolute mobility in several developed countries as described in Section 3 – Canada, Denmark, France, Germany, Norway, Sweden, United Kingdom and United States. We assume that the rank correlation in each country is fixed in time and in order to take into account plausible changes in time or the lack of accurate data, we consider a range of values – a nominal value (see Tab. 2), a lower bound (0.1) and an upper bound (0.5).

In order to get the marginal income distributions we use data from The World Inequality Database (2018) and the generalized Pareto curve interpolation method (Blanchet, Fournier and Piketty, 2017) to generate simulated samples of the pre-tax income distribution (the sample size was  $N = 5 \cdot 10^5$ , large enough to reduce the statistical uncertainty of the estimates to zero). For some years the data we use cover only the top 10% of the distribution. The generalized Pareto curve interpolation method might not be able to describe well the bottom 90% in such cases. In Appendix E we use the detailed data for the US and France to show this does not create a sizable bias in the estimates of absolute mobility.

Since the available historical data include all adult population and it is not possible to restrict the data to 30-year-olds only (or specific marital status, family size, etc.), these results differ from those obtained above (Fig. 3) conceptually. However, as long as the income growth and inequality among 30-year-olds is similar to that of the entire adult population, absolute mobility estimates will be also similar if the entire adult population is considered. In most countries, this specification difference would only have a small effect. For example, in the United States, the difference between the absolute mobility estimates using the marginal distributions used by Chetty et al. (2017) and using The World Inequality Database (2018) marginal distributions is lower than 2 percentage points, excluding several birth cohorts in the mid 1940s, in which the difference is 6–8 percentage points. The small difference is driven by two effects – income growth is slightly lower among 30-year-olds than among the entire adult population, while inequality among 30-year-olds is also lower. The effects of these differences on absolute mobility almost cancel out (see also Section 5). Similarly, in France, "[...inequality] is almost as large within each age group as for the population taken as a whole." (Garbinti, Goupille-Lebret and Piketty, 2018) This cancels out with the slightly lower growth among 30-year-olds than among the whole population.

Despite these observations, the difference may be potentially large in some countries. For this reason we currently restrict our analysis to developed countries only, in which the age-income structure was well-studied and for which the dependence of growth and inequality on age groups changes only little the picture for the entire adult population. Therefore, in order to obtain a good understanding of the long run evolution of absolute mobility, rather than a very accurate picture of the absolute mobility for a specific birth cohort, it is possible to use the historical data of the entire adult population. We also note that this data limitation does not invalidate the methodology – the low sensitivity of absolute mobility to the copula is valid in any case, as described above.

In France, detailed tax data from 1970s onward (Garbinti, Goupille-Lebret and Piketty, 2018), allow testing the robustness of the estimates to this assumption for three birth cohorts. Such a robustness check is presented in Appendix F. In addition, comparing our results to the absolute mobility levels found in the US (Chetty et al., 2017) and in Canada (Ostrovsky, 2017), we find that the simplifying assumptions made here have a very small effect and match the limited existing empirical evidence closely.

The main results are presented in Fig. 4. They indicate that the documented decrease in absolute mobility rates found in the United States occurred in all the countries we consider. Using the historical data on the income distribution in France, Sweden and the US, it is also possible to show that the absolute mobility in those countries increased rapidly for the children born in the 1910s–1940s, the direct beneficiaries of the *Trente Glorieuses* in France and the *Rekordåren* in Sweden. The decrease of absolute mobility from the 1940s birth cohorts onward was mainly due to decreasing income growth rates rather than rising inequality (see Section 4.2). The extension of the time series to the birth cohorts of the 1920s and 1930s in the US shows a similar inverted-U shape to that obtained for France and Sweden.

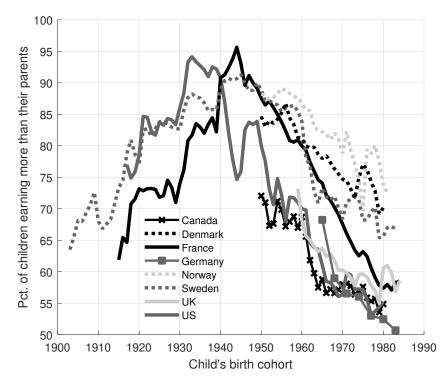


Figure 4: The evolution of absolute intergenerational mobility in developed economies.

The Nordic countries, which are characterized by low income inequality and high relative mobility (Corak, 2013), also stand out in terms of absolute mobility. As we will see in the next section, this is a direct implication of the low inequality in those countries – the same income growth rate may lead to substantially different absolute mobility, depending mainly on the level of income inequality.

The estimates for Germany are expected to be downward biased. They are based on samples that exclude East Germany before 1990, but include East Germany after 1990 (Bartels, 2018). Before the 1990 reunification, wages in the east were half of wages in the west. By 2015 they were roughly 75% of wages in the west (Berlin-Institut für Bevölkerung und Entwicklung, 2015). This, alongside massive immigration from the east to the west, would lead to high absolute mobility for children in East-German families who became adults after the reunification. For these reasons, our data for Germany underestimate growth rates between generations and hence also absolute mobility. The real trend would be similar to our estimates, shifted 5–10 percentage points upward.

As described above, the assumptions made while estimating absolute intergenerational mobility introduce uncertainties. In order to consider the effect of the fixed rank correlation assumption, we estimated the absolute intergenerational mobility in each of the countries assuming that the rank correlation was within the range [0.1, 0.5], which covers a wider range of values than the combined range of estimated values in the countries we consider. These estimates have a very little effect on the results (see Appendix G). We therefore conclude that the simplifying assumptions discussed above and in detail in Appendices D, E, F and G may potentially lead to a sizable bias of several percentage points in the absolute intergenerational mobility estimates. However, as explained, they cannot explain decreases as significant as identified in Fig. 4.

Using the detailed data from Garbinti, Goupille-Lebret and Piketty (2018) and Piketty, Saez and Zucman (2018) it is possible to test the robustness of the results for France and US to changes in income definition and to different units of observation. In particular, the data from Garbinti, Goupille-Lebret and Piketty (2018) allow comparing absolute mobility for individual adults and equal-split adults in France after 1970. This makes it possible to assess the effect of changes in women's labor force participation and gender inequality on the evolution of absolute mobility. We find that the individualized-based estimates do not differ significantly from the baseline estimates. This indicates that the rise in female labor force participation rates did not lead to a substantial increase in absolute intergenerational mobility, despite its mitigating effect on the increase in inequality (Piketty, Saez and Zucman, 2018). We also use these data to show that the absolute mobility estimates are robust to changes in income concept (*i.e.* labor income and total income). Appendix H presents these results.

We also note that absolute mobility estimates based on disposable income are expected to be somewhat higher than what we find for pre-tax income. Income inequality changes are milder for disposable income and relative mobility is lower (see Section 2). Together, as will further be discussed in Section 5, these would lead to higher absolute mobility.

## 4.2 Absolute mobility decomposition

Figure 4 shows absolute intergenerational mobility decreasing in several developed countries. It is possible to decompose the dynamics of the absolute mobility estimates in order to understand the sources of the long run trend.

As noted, the rank correlation, hence relative mobility, is assumed to be fixed in time and it was demonstrated that plausible changes in relative mobility cannot be the source of long run changes in absolute mobility. Therefore, the observed trend can be attributed to two factors – the generally decreasing income growth rates and the generally increasing income inequality. For each country analyzed we wish to understand what is the contribution of those factors to the absolute intergenerational mobility trend. For that purpose, we produce, in addition to the baseline estimate of each country, two counterfactual calculations: one in which the shape of the income distribution is kept constant in time and similar to the earliest distribution in the data, but with the average income changing according to its real historical values; and another, in which the distribution shape changes according to historical data, but the annual income growth rate is fixed in time and equal to the average real income growth rate over the entire period considered.

Such calculations are presented in Fig. 5. It can be generally divided into two groups – countries in which the fixed inequality counterfactual scenario follows closely the baseline estimate (most notably Denmark and France) and countries in which it is not the case, in which the baseline estimate is very similar to the fixed income growth scenario. In the former case it is clear that the decrease in absolute mobility is due to the decrease in income growth rates, since fixing inequality has only a small effect. In the latter case, the effect of inequality is more dominant. It is particularly visible for the US and the UK.

We formalize the different contributions by calculating the fraction of the overall change in absolute mobility produced by each counterfactual. Those contributions are presented in Tab. 3.

In most of the countries considered, the decreasing income growth rates are the main explanation for the decreasing absolute mobility. Particularly in France, Canada and the Nordic countries. In the UK, Germany and the US the role of increasing income inequality is much more important, especially in the US, as was already identified

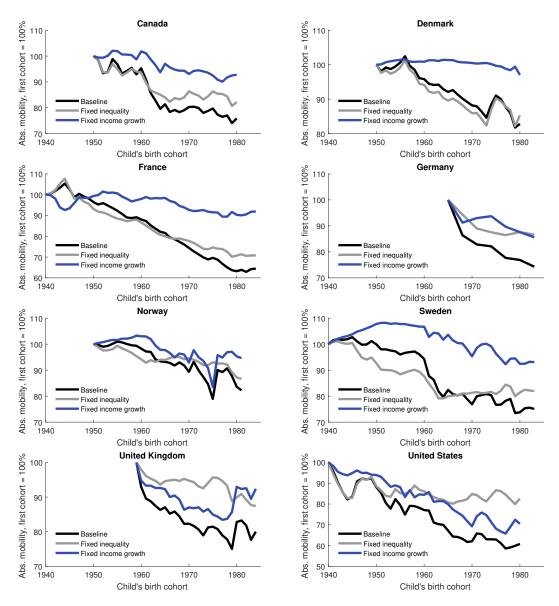


Figure 5: Counterfactual calculations of absolute mobility in a group of developed economies. For comparability, we set the absolute mobility to 100% for the earliest cohort.

by Chetty et al. (2017). It follows that despite the similarity in the evolution of absolute mobility in the various countries, the US stands out as being the only one in which the increasing income inequality played such a dominant role in the decreasing mobility.

Country	Decrease in mobility $(pp)$	Growth contrib. $(pp)$	Inequality contrib. $(pp)$
Canada	24.1	17.1	7
Denmark	17.2	14.3	2.9
France	35.5	27.9	7.7
Germany	25.7	12.4	13.3
Norway	17.7	12.7	5.1
Sweden	24.9	18	6.9
UK	20	12.5	7.5
US	39.2	14.5	24.6

Table 3: Growth and inequality changes contribution to the evolution of absolute mobility from 1940 (or later) onward

# 5 Growth, inequality, relative mobility and absolute mobility

The absolute intergenerational mobility estimates presented in Fig. 4 raise the need for understanding the trends detected in different countries. Since the absolute mobility depends on marginal distributions and their copula, it is possible to reframe this dependence using

- Growth quantifying the change in the average income between the marginal distributions;
- Inequality describing the intergenerational change in the shape of the marginal distributions;
- Relative mobility quantifying the likelihood of changing ranks in the income distribution across two generations.

This is specifically important for understanding the drivers of the observed absolute mobility trends in different countries, as captured in Fig. 5 and in Tab. 3. The basic intuition for the dependence of absolute intergenerational mobility on income growth, income inequality and relative mobility is illustrated in Fig. 6. This figure presents three hypothetical scenarios and schematic income distributions of parents and children:

- In the scenario on the left, there is no change in inequality between the two generations and the growth is high enough so there is no overlap between the distributions, *i.e.* the poorest child is still richer than the richest parent. In this case absolute mobility will be 100% regardless of relative mobility.
- The middle panel presents a more realistic scenario inequality is still unchanged between the generations, however, growth is not as high as in the previous scenario. In this case the level of absolute mobility depends on relative mobility. In the extreme case of no relative mobility, absolute mobility will still be 100%, since the richest parent is mapped into the richest child and so do the second richest parent and child and so on. Realistically, because relative mobility exists, absolute mobility will be lower than 100%, and as relative mobility increases, absolute mobility would decrease.
- In the right panel growth is as high as in the left panel but inequality is higher among the children. This case is similar to the middle one – absolute mobility will be lower than 100% and as relative mobility increases, absolute mobility would decrease.

These scenarios summarize the basic intuition on the determinants of absolute mobility – absolute mobility increases with income growth, but decreases with increasing inequality and relative mobility.

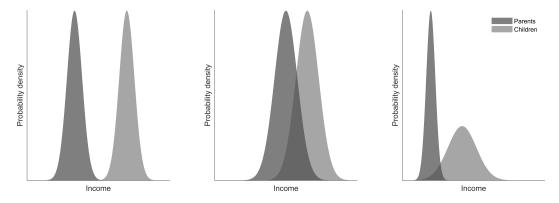


Figure 6: Descriptive scenarios of intergenerational changes in the income distribution.

Figure 6 highlights an important and seemingly counter-intuitive finding. Given two marginal distributions, for parents and children, absolute mobility will be a decreasing function of relative mobility. This observation is general and do not depend on any assumptions. In practice, however, one cannot conclude that in a cross-country comparison, countries characterized by high relative mobility will be necessarily characterized by low absolute mobility. Absolute mobility is more sensitive to growth and to changes in inequality, which differ significantly between countries. It is only weakly dependent on relative mobility, as demonstrated above. Thus, in practice, countries characterized by high relative mobility, such as the Nordic countries, are also characterized by high absolute mobility.

In order to mathematically characterize the dependence of absolute mobility on growth, inequality and relative mobility we present a simplified model for intergenerational mobility. For this purpose we also introduce the intergenerational earnings elasticity (IGE), a canonical measure of relative intergenerational mobility. It is defined as the elasticity of the logarithm of child income with respect to the logarithm of parent income and we denote it by  $\beta$  (Mulligan, 1997; Lee and Solon, 2009; Chetty et al., 2014a). IGE, like the rank correlation, is a measure of immobility rather than of mobility: the larger it is, the stronger the relationship between parent and child incomes. Therefore,  $R_1 \equiv 1 - \beta$  is used as a measure of relative mobility. Similarly, we denote  $R_2 \equiv 1 - \rho_S$  as the measure of relative mobility corresponding to the rank correlation  $\rho_S$ .

### 5.1 Model

Our starting point is a population of N parent-child pairs – which may represent individuals or families. We denote by  $Y_p^i$  and  $Y_c^i$  the respective real incomes of the parent and the child (at the same age) in family  $i = 1 \dots N$ . We assume the incomes are all positive and define the log-incomes  $X_p^i = \log Y_p^i$  and  $X_c^i = \log Y_c^i$ .

The intergenerational earnings elasticity is defined as the slope  $(\beta)$  of the linear regression

$$X_c = \alpha + \beta X_p + \epsilon \,, \tag{5.1}$$

where  $\alpha$  is the regression intercept and  $\epsilon$  is the error term.

The rate of absolute mobility, A, is the fraction of children earning more than their parents in real terms, equal to the probability  $P(X_c - X_p > 0)$ . Assuming that  $\epsilon$ and  $X_p$  are normally distributed, so does  $X_c$ . In such case the marginal income distributions are log-normal and the joint parent-child log-income distribution is a bivariate normal distribution. These simplifying assumptions are standard, since "the lognormal is a good approximation of empirical income distributions, leads to tractable results, and allows for an unambiguous definition of inequality," (Bénabou, 2000) as also described in detail by Pinkovskiy and Sala-i-Martin (2009). It has a mechanistic basis as the long run attractor distribution for quantities undergoing random multiplicative growth (Aitchison and Brown, 1957; Guvenen et al., 2015; Adamou and Peters, 2016). Although Chetty et al. (2014a) argued that "the income distribution is not well approximated by a bivariate log-normal distribution" and while such a simplified model may not explain adequately some aspects of mobility and inequality, we find that for the purpose of estimating long run absolute mobility trends, this model is satisfactory.

In the bivariate log-normal model for the joint parent-child income distribution, the marginal income distributions of both parents and children are log-normal and the correlation between their log-incomes is defined by a single parameter  $\rho$ . We denote the marginal log-income distributions of the parents and the children as  $\mathcal{N}(\mu_p, \sigma_p^2)$  and  $\mathcal{N}(\mu_c, \sigma_c^2)$ , respectively. The joint distribution is therefore fully characterized by five parameters:  $\mu_p$ ,  $\sigma_p$ ,  $\mu_c$ ,  $\sigma_c$  and  $\rho$ .

The choice of model may substantially affect the analysis of the absolute and relative mobility measures. Other possible models for the joint income distribution of parents and children can include marginal distributions which are not log-normal, as well as other types of copula. In the bivariate log-normal model the copula is Gaussian. Other copula types, such as the Clayton, the Gumbel and the Plackett copula families (Trivedi and Zimmer, 2007; Bonhomme and Robin, 2009), may prove to be a better description of the relationship between the marginal income distributions. In their study of relative intragenerational mobility in France, Bonhomme and Robin (2009) argue that the Gaussian copula "tends to underestimate the dependence in the middle of the distribution, that is, the probabilities of remaining in the second, third, and fourth quintiles" and show that the empirical copula is best estimated by the Plackett copula. In Appendix C we show that the choice of copula model, assuming a similar rank correlation, has practically no effect on absolute mobility. This observation is consistent with our observations on empirical copulas (see Section 2), showing that absolute mobility estimates are insensitive to plausible changes in the copula.

### 5.2 Theoretical results

We first address the properties of the bivariate log-normal approximation. We derive closed-form expressions for the measures of mobility – A,  $R_1$ , and  $R_2$  – in terms of the model parameters:

**Proposition 1** For a bivariate normal distribution with parameters  $\mu_p$ ,  $\sigma_p$  (for the parents marginal distribution),  $\mu_c$ ,  $\sigma_c$  (for the children marginal distribution) and correlation  $\rho$ , the rate of absolute mobility is

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (2R_1 - 1) + \sigma_c^2}}\right) \,, \tag{5.2}$$

where  $\Phi$  is the cumulative distribution function of the standard normal distribution.

Equation (5.2) enables the calculation of A assuming the knowledge of the marginal income distributions and  $R_1$  (or  $\beta$ ). It is therefore also possible to calculate A in terms of  $R_2$ , by replacing  $R_1$  and  $R_2$  according to the known relationship between them in a bivariate log-normal model (Trivedi and Zimmer, 2007).<sup>6</sup>

Our next step is to demonstrate that the bivariate log-normal model for the joint income distribution is empirically sound. For that purpose we compare the model prediction for the historical rate of absolute mobility in France with the results presented in Section 4.1. We use pre-tax national income per adult data and the income share data of 14 percentiles (The World Inequality Database, 2018)<sup>7</sup> to obtain  $\mu_p$ ,  $\sigma_p$ ,  $\mu_c$  and  $\sigma_c$  every year.

The  $\sigma$  parameters are obtained by OLS estimation of the log-normal Lorenz curve. The Lorenz curve of the log-normal distribution log  $\mathcal{N}(\mu, \sigma^2)$  is (Cowell, 2011)

$$L(z) = \Phi\left(\Phi^{-1}(z) - \sigma\right).$$
(5.3)

After obtaining  $\sigma$ , we estimate  $\mu$ . Denoting the per-adult pre-tax income as m, it follows that the parameter  $\mu$  is

 $<sup>{}^{6}1 -</sup> R_2 = \frac{6 \arcsin \frac{\rho}{2}}{\pi}$ 

$$\mu = \log\left(m\right) - \frac{\sigma^2}{2}.$$
(5.4)

The absolute mobility estimates for France, presented in Section 4.1, are compared to those resulting from the log-normal approximation in Fig. 7. Assuming similar rank correlations to those used in Section 4.1, we find that the difference between the estimated absolute mobility values is 2.2 percentage points on average. The log-normal approximation somewhat overstates the effect of increasing income inequality, which produces a downward bias in the mobility estimates. However, this bias affects very little the long run evolution of the absolute mobility.

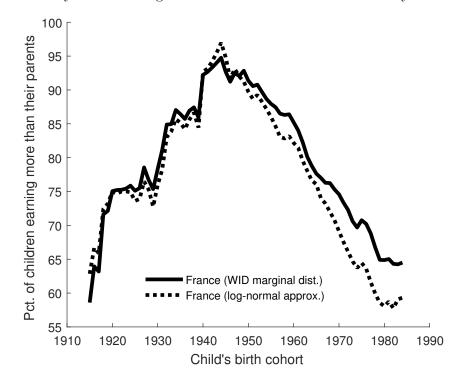


Figure 7: Estimated absolute mobility in France under the bivariate log-normal approximation. The marginal distributions are based on the generalized Pareto curve interpolation method and on The World Inequality Database (2018) data (solid line) and assuming the log-normal approximation (dotted line). We assumed a rank correlation of 0.3 in France as done in Section 4.1.

Figure 7 demonstrates that despite its comparative methodological naïvety, the bivariate log-normal model can be used to describe the long run evolution of absolute mobility. In order to further illustrate the relationship between absolute mobility, income growth and income inequality, we can rewrite Eq. (5.2) and obtain

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 - 2\rho\sigma_p\sigma_c + \sigma_c^2}}\right).$$
(5.5)

For simplicity we assume that inequality does not change between the two generations, so  $\sigma_p = \sigma_c$  and we denote the variance of logarithms (which is the same for both generations) as VL. We then obtain

$$A \approx \Phi\left(\frac{g}{\sqrt{2\left(1-\rho\right)VL}}\right),\tag{5.6}$$

where g is the income growth rate between the two generations.

Equation (5.6) quantifies the dependence of absolute mobility on its three determinants – income growth, income inequality and relative mobility. It illustrates that as growth increases, absolute mobility gets closer to 100%. It also shows that when inequality increases, absolute mobility becomes close to 50%. The 50% limit serves as a lower bound, as long as growth is positive. Since realistically, real-term growth over a period of 30 years was positive in all developed countries since World War II, 50% is a realistic lower bound of absolute mobility. The results in Fig. 4 show that Canada, United Kingdom and the United States have recently approached this lower bound.

We note that if inequality is not assumed as similar for both generations, it is possible to obtain absolute mobility that is lower than 50% with positive growth, if the increase in inequality is drastic enough, as described Prop. 2. However, such extreme conditions have not been realized in any of the countries we discussed, even in periods of rapid increase in income inequality. We also note that this result is independent of relative mobility.

Proposition 2 Under Prop. 1 notations

$$A > 50\% \iff g > e^{\frac{\sigma_c^2 - \sigma_p^2}{2}} - 1 \tag{5.7}$$

## 5.3 The relationship between absolute and relative mobility

It is now also possible to use the model to further study the relationship between absolute and relative mobility. This serves as the primary purpose of the model.

Equation (5.2) demonstrates that the rate of absolute mobility can be explicitly described as a function relative mobility. Fig. 8 shows A as a function of  $1 - \rho_S$  for different birth cohorts in the United States. It shows that the bivariate normal model – with positive income growth and inequality changes consistent with data, but absent other effects – predicts an inverse relationship between absolute and relative mobility.

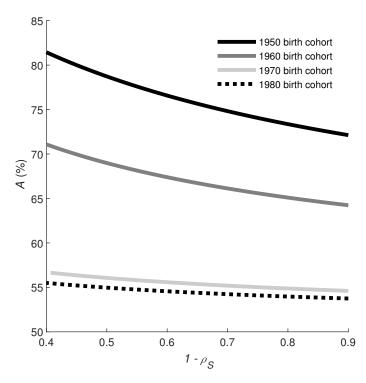


Figure 8: The theoretical relationship between absolute and relative mobility.

Proposition 1 illustrates that the rate of absolute mobility increases with increasing income growth and decreases with increasing income inequality, as described by Chetty et al. (2017). It also demonstrates that an additional mechanism can be at play, since absolute mobility decreases with increasing relative mobility. This seemingly counter-intuitive inverse relationship repeats the basic intuition illustrated in Fig. 6. Figure 8 seemingly stands in contrast to the findings about absolute intergenerational mobility in Fig. 4, since the absolute and relative mobility in those countries were found to be ordered similarly. Yet, in practice, the relationship between relative mobility and absolute mobility cannot be isolated as we do above. Changes in relative mobility can be associated with changes in inequality and growth, which, in turn, affect absolute mobility. Therefore, our results should be considered as an idealized case in which growth and inequality are held constant.

#### 5.4 Absolute mobility and median incomes

Chetty et al. (2017) also find that the share of children earning more than the median parent declined from 92% in the 1940 birth cohort to 45% in the 1984 cohort (Katz and Krueger, 2017). This alternative measure of absolute mobility (KKM) moves almost identically to A across cohorts in the United States (Katz and Krueger, 2017). Denoting KKM as  $\tilde{A}$ , it follows, in the bivariate log-normal model, that  $\tilde{A}$  is defined as

$$\tilde{A} \equiv \Phi\left(\frac{\mu_c - \mu_p}{\sigma_c}\right) \,, \tag{5.8}$$

where  $\Phi$  is the cumulative distribution function of the standard normal distribution.

Using  $\tilde{A}$  has obvious advantages over A. In particular, they can be "directly computed from standard public-use cross-sectional household survey data and do not require data that longitudinally link children to parents." (Katz and Krueger, 2017) However,  $\tilde{A}$  would be close to A only if the IGE is close to 1/2:

**Proposition 3** For a bivariate normal distribution with parameters  $\mu_p$ ,  $\sigma_p$  (for the parents marginal distribution),  $\mu_c$ ,  $\sigma_c$  (for the children marginal distribution) and assuming IGE of  $\beta$ , then

$$A = \tilde{A} \iff \beta = \frac{1}{2}.$$
(5.9)

It is therefore no surprise that for the United States A and  $\tilde{A}$  are relatively similar – Aaronson and Mazumder (2008) estimate the IGE for the 1950–1970 birth cohorts at 0.46–0.58. In countries such as Canada or Denmark, in which the IGE is

substantially lower than 0.5 (Corak, 2013), using  $\tilde{A}$  neglects the part high relative mobility plays in determining A. This will lead to overestimation of absolute intergenerational mobility, if measured as KKM. For example, in Denmark, the average estimated A for 1950–1980 birth cohorts is 78.5%. If  $\tilde{A}$  is considered, the average value is 87.2%. For the United States this difference would be considerably smaller: 62.2% and 63.7%, respectively. Setting aside the normative question of which measure of absolute intergenerational mobility is of most interest, we emphasize that  $\tilde{A}$ cannot be used as a proxy for A, unless the IGE is close to 0.5.

# 6 Conclusion

Our findings highlight a decreasing absolute mobility trend in several developed countries for post-World War II birth cohorts. The sources of this trend, however, differ from country to country. In the United States and to a lesser extent, in the United Kingdom, the rising income inequality is the main contributor for decreasing absolute mobility. In other countries, a similar historic evolution of absolute mobility is predominantly explained by the decrease of income growth rates. We were also able to detect an increase in absolute intergenerational mobility in France, Sweden and United States for pre-World War II birth cohorts.

Our findings imply that it is possible to produce estimates of absolute intergenerational mobility without the need for high-quality panel data sets. We find the structure of realistic copulas to be roughly similar and that different measures of relative mobility are effectively interchangeable. This means that collapsing the copula into a single representative measure of relative mobility is empirically justified for estimating absolute mobility rates. Also, the sensitivity of the absolute mobility to relative mobility is low. Thus, realistic changes in relative mobility cannot explain the evolution of absolute mobility. Assuming a fixed copula in time will provide meaningful and reliable estimates of absolute mobility. We also find that a simplified model as simple as a bivariate log-normal distribution is satisfactory for describing the long run dynamics of absolute mobility.

Our findings join recent work on global inequality and the effects of globalization (Bourguignon, 2015; Milanovic, 2016; Rodrik, 2017; Alvaredo et al., 2017). Some scholars link the recent global "populism wave" to globalization and inequality and their relationship (Milanovic, 2016; Rodrik, 2017). Absolute intergenerational mobility captures this relationship. This can explain, for example, similarities in the recent political trends in France, the United Kingdom and the United States (see also Piketty (2018)). These countries have experienced the so-called "populism wave" during the recent presidential elections and the EU referendum in the United Kingdom. Inequality in the United States, and to a lesser extent in the United Kingdom, has been sharply increasing during the past few decades. Yet, in France, inequality remained largely stable and at much lower levels. Gross domestic product growth, however, was almost consistently higher in the United States and in the United Kingdom than in France at the same period. Absolute intergenerational mobility captures an aspect of the chances of young adults to achieve a higher standard of living than their parents. It is low when growth is slow or when income inequality is very high (or both). Thus, the observed downward trend in absolute mobility demonstrates a possible partial explanation for the common political phenomena of the recent years.

The seemingly counter-intuitive inverse relationship between absolute and relative mobility stems from a conceptual difference between the two categories of mobility. It exposes the problems that can arise if both measure the same phenomenon. In particular, absolute mobility is very sensitive to across-the-board economic growth. For example, during the Middle Ages relative mobility rates were low because social class and profession were predominantly inherited (Goldthorpe, 1987; Clark, 2014). In this case, even the slightest positive or negative income growth would result in very high or very low absolute mobility. A misleading picture of intergenerational mobility may arise if these basic properties are overlooked. Thus, addressing intergenerational mobility requires careful delineation of the phenomena of interest and the manner in which quoted measures reflect them.

The methodology applied here can be used to estimate not only absolute intergenerational mobility, but also intragenerational mobility. For example, developed countries have gone through a recovery from a major crisis during the past decade. It has benefited most of the population, but not the entire population. In the United States, national income per adult increased in 13% between 2009 and 2016. Yet, using data from The World Inequality Database (2018); Panel Study of Income Dynamics (2017) we estimate that only 55% of the adults enjoyed a higher standard of living in 2016 than in 2009. A thorough analysis of intragenerational mobility is left for future work.

Our findings are also related to the large body of work dedicated to welfare measurement issues (Deaton and Zaidi, 2002; Chen and Ravallion, 2010; Aghion et al., 2016). In this paper and in previous analyses standard price index deflators account for changes in prices. Yet, it is well known that such deflators are imperfect for welfare measurement (Aghion et al., 2016). Such measures do not take into account in a sufficient manner major technological changes. Accounting for such changes would not change the main findings in this paper. Yet, it may lead to less dramatic interpretations of the decreasing absolute mobility trends. Such trends would be partially compensated by changes that were not taken into account in the standard deflators. This is also related to the reference levels to which one would compare her income. These levels would, in turn, affect one's welfare. Specifically, it questions whether when discussing mobility, it is valuable to use parents' incomes as these reference levels (Atkinson and Bourguignon, 1982). The welfare literature emphasizes the importance of absolute levels of income or their average growth over time. Yet, this ignores that people do consider their parents' incomes as reference levels. Still, this is only a partial picture – one would also compare her income to her peers' and her expectations of oneself. These, in turn, are determined by numerous factors. A related question is how parents' welfare depends on their children's well-being, in comparison to theirs. The importance of this question is yet another reason to continue studying absolute mobility in the future.

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#### A Proofs

#### A.1 Proof of proposition 1

First, by definition, the correlation  $\rho$ , between  $X_p$  and  $X_c$  equals to their covariance, divided by  $\sigma_p \sigma_c$ 

$$\rho = \frac{\operatorname{Cov}\left[X_p, X_c\right]}{\sigma_p \sigma_c} \,. \tag{A.1}$$

 $\beta$  can be directly calculated as follows, by the linear regression slope definition:

$$\beta = \frac{\sum_{i=1}^{N} (X_{p}^{i} - \bar{X}_{p}) (X_{c}^{i} - \bar{X}_{c})}{\sum_{i=1}^{N} (X_{p}^{i} - \bar{X}_{p})}, \qquad (A.2)$$

where  $\bar{X}_p$  and  $\bar{X}_c$  are the average parents and children log-incomes, respectively. It follows that  $\operatorname{Cov}[X_p, X_c]$ 

$$\beta = \frac{\operatorname{Cov}\left[X_p, X_c\right]}{\sigma_p^2} \,. \tag{A.3}$$

We immediately obtain

$$\beta = \frac{\sigma_c}{\sigma_p} \rho \tag{A.4}$$

and therefore

$$1 - \beta = R_1 = 1 - \frac{\sigma_c}{\sigma_p}\rho.$$
(A.5)

Now we define a new random variable  $Z = X_c - X_p$ . It follows that calculating A is equivalent to calculating the probability P(Z > 0).

Subtracting two dependent normal distributions yields

$$Z \sim \mathcal{N} \left( \mu_c - \mu_p, \sigma_p^2 + \sigma_c^2 - 2 \text{Cov} \left[ X_p, X_c \right] \right) , \qquad (A.6)$$

and it follows, due to Eq. (A.4), that

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2\right) \,. \tag{A.7}$$

If follows that

$$\frac{Z - (\mu_c - \mu_p)}{\sqrt{\sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2}} \sim \mathcal{N}\left(0, 1\right) , \qquad (A.8)$$

so we can now write

$$P(Z > 0) = P\left(\frac{Z - (\mu_c - \mu_p)}{\sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2}} > -\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2}}\right) = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2}}\right) = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (2R_1 - 1) + \sigma_c^2}}\right),$$
(A.9)

where  $\Phi$  is the cumulative distribution function of the standard normal distribution.

#### A.2 Proof of proposition 2

Following Eq. (5.6)

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 - 2\rho\sigma_c\sigma_p + \sigma_p^2}}\right).$$
(A.10)

Defining the rate of growth as

$$g \equiv \frac{e^{\mu_c + \frac{\sigma_c^2}{2}}}{e^{\mu_p + \frac{\sigma_p^2}{2}}} - 1$$
 (A.11)

It follows that

$$A > 50\% \iff \mu_c > \mu_p \iff e^{\mu_c} > e^{\mu_p} \iff e^{\mu_c + \frac{\sigma_c^2}{2}} > e^{\mu_p + \frac{\sigma_c^2}{2}}.$$
(A.12)

We can rewrite

$$e^{\mu_c + \frac{\sigma_c^2}{2}} = (1+g) e^{\mu_p + \frac{\sigma_p^2}{2}}$$
 (A.13)

and substitute in the above to get

$$A > 50\% \iff (1+g) e^{\mu_p + \frac{\sigma_p^2}{2}} > e^{\mu_p + \frac{\sigma_c^2}{2}} \iff g > e^{\frac{\sigma_c^2 - \sigma_p^2}{2}} - 1.$$
(A.14)

### A.3 Proof of proposition 3

Following Eq. (5.2)

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2}}\right) \,. \tag{A.15}$$

Following Eq. (5.8)

$$\tilde{A} = \Phi\left(\frac{\mu_c - \mu_p}{\sigma_c}\right) \,, \tag{A.16}$$

and therefore

$$\tilde{A} = A \iff \frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 \left(1 - 2\beta\right) + \sigma_c^2}} = \pm \frac{\mu_c - \mu_p}{\sigma_c} \,. \tag{A.17}$$

We then obtain

$$\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2}} = \pm \frac{\mu_c - \mu_p}{\sigma_c} \iff \sigma_c = \pm \sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2} \iff \beta = \frac{1}{2}.$$
(A.18)

### B Sensitivity of absolute mobility for fixed rank correlation

In our estimates we use Spearman's rank correlation (or rank-rank slope) to describe the copula between the income distributions of parents and children. We then continue to estimate the absolute intergenerational mobility. We assume that given the marginal distributions, the rank correlation determines absolute mobility. Conceptually, the same rank correlation could deliver very different absolute mobility estimates. Yet, as we describe below, this requires the copulas to be unrealistic.

We consider copulas as transition (doubly stochastic) matrices  $P \in \mathscr{P}(N)$ , where  $p_{ij}$  represents the probability of transferring to quantile j (child) for those starting in quantile i (parent) and N is the number of income quantiles. Evidence shows that the diagonal elements are generally higher and the transition probabilities decrease with the transition distance. The probability to move between two ranks i and j within two generations is a decreasing function of |i - j| (see, for example Jäntti et al. (2006); Chetty et al. (2017)). Preserving the rank correlation, while creating a large effect on absolute mobility requires breaking this regularity.

The rank correlation of a transition matrix is

$$\rho_S(P) = \frac{12\sum_{i=1}^N \sum_{j=1}^N ijp_{ij} - 3N(N+1)^2}{N(N^2 - 1)},$$
(B.1)

thus, only the sum  $\sum_{i=1}^{N} \sum_{j=1}^{N} i j p_{ij}$  depends on the matrix elements.

We now define a  $\Delta$ -local rank-correlation preserving move as a change to 8 elements in the matrix  $-p_{i_1,j_1}, p_{i_1+1,j_1}, p_{i_1+1,j_1}, p_{i_1+1,j_1+1}$  and  $p_{i_2,j_2}, p_{i_2+1,j_2}, p_{i_2+1,j_2}$ ,

 $p_{i_2+1,j_2+1}$  in the following way:

$$p_{i_{1},j_{1}} \rightarrow p_{i_{1},j_{1}} + \Delta$$

$$p_{i_{1}+1,j_{1}+1} \rightarrow p_{i_{1}+1,j_{1}+1} + \Delta$$

$$p_{i_{1}+1,j_{1}} \rightarrow p_{i_{1}+1,j_{1}} - \Delta$$

$$p_{i_{1},j_{1}+1} \rightarrow p_{i_{1},j_{1}+1} - \Delta$$

$$p_{i_{2},j_{2}} \rightarrow p_{i_{2},j_{2}} - \Delta$$

$$p_{i_{2}+1,j_{2}+1} \rightarrow p_{i_{2}+1,j_{2}+1} - \Delta$$

$$p_{i_{2}+1,j_{2}} \rightarrow p_{i_{2}+1,j_{2}} + \Delta$$

$$p_{i_{2},j_{2}+1} \rightarrow p_{i_{2},j_{2}+1} + \Delta$$
(B.2)

where  $\Delta$  can be either positive or negative (as long as all the elements remain non-negative) and  $i_1$ ,  $j_1$ ,  $i_2$  and  $j_2$  can be any quantiles between 1 and N-1.

Such a change trivially preserves the sum  $\sum_{i=1}^{N} \sum_{j=1}^{N} ijp_{ij}$  and therefore the rank correlation. By composing several  $\Delta$ -local rank-correlation preserving moves it is possible to change a given copula while preserving the rank correlation.

In general, rank-correlation preserving moves have the effect of increasing the trace while also increasing the extreme ends of the transition matrix, or vice versa. This is demonstrated in the three copulas in Fig. 9. They all share the same rank correlation (0.3), but are very different from one another. The copulas were constructed by composing several  $\Delta$ -local rank-correlation preserving moves on copula A, which is the copula used for producing the baseline estimates of absolute mobility in France. Only copula A is realistic and has the typical form of the empirical copulas (compare with Jäntti et al. (2006); Eberharter (2014)). Copula C is far from being plausible, attaching very high probabilities to the diagonal, zeros to some off diagonal elements, but non-zero probability to make the largest possible moves within generations.

Nevertheless, copulas A and B produce almost the same absolute mobility. This was tested for France, using the same marginal distributions used for the baseline estimates. For each of the three copulas we produced a series of estimated absolute mobility values. The results are also presented in Fig. 9. As expected, copula C leads to results that are different from the baseline estimates (copula A). Yet, the trend remains very similar to the baseline estimates and the results are 3.3 percentage points higher than the baseline estimate on average. Copula B leads to results that

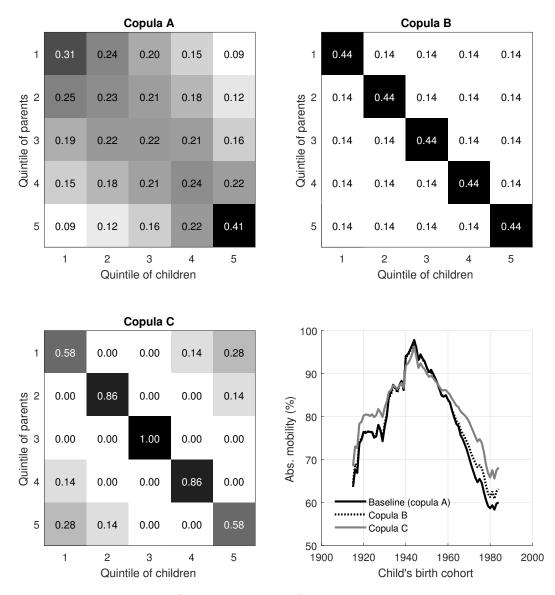


Figure 9: Three copulas (transition matrices) constructed by composing several  $\Delta$ local rank-correlation preserving moves. Copula A is the copula used for producing the baseline estimates of absolute mobility in France. The bottom right panel shows the absolute mobility estimates for France when using the different copulas.

are almost identical to the baseline. This is regardless of it being unrealistic.

Together with the robustness of absolute mobility to different copula models (see Appendix C), our results indicate that if empirical copulas or standard copula models such as the Gaussian, Clayton, Gumbel or Plackett models are used, as we do in

our estimates and in our model, using the rank correlation as a single parameter describing the copula does not introduce any significant uncertainty to the absolute mobility estimates.

#### C Log-normal model sensitivity to the copula model

In general, within the methodology presented, the copula model choice may affect the estimated absolute mobility. We demonstrate that as long as the rank correlation is the same, the copula model effect on estimated absolute mobility is, in practice, insignificant. We compare four copula models – Gaussian, which is the copula in the bivariate log-normal model, as well as the Clayton, the Gumbel and the Plackett copula families (Trivedi and Zimmer, 2007; Bonhomme and Robin, 2009). In their study of relative mobility in France, Bonhomme and Robin (2009) argue that the Gaussian copula "tends to underestimate the dependence in the middle of the distribution, that is, the probabilities of remaining in the second, third, and fourth quintiles" and show that the empirical copula is best estimated by the Plackett copula.

Figure 10 demonstrates that the differences between the absolute mobility estimates when using different copula models, while assuming the same rank correlations, are negligible. The average difference between each of the time series was less than 1 percentage point, *i.e.* an effect of less than 2%.

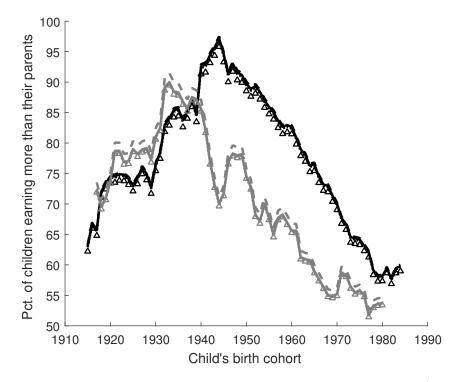


Figure 10: The copula model effect on the absolute mobility in France (black) and the United States (gray). The copula models used were Gaussian (solid lines), Clayton (dashed), Gumbel (dotted) and Plackett (triangles).

## D Potential effect of individual and family incomes differences on absolute mobility

In order to make our analysis results comparable to those in Chetty et al. (2017) we use equal-split adults as the unit of observation of our income data when possible. In several countries, the income data are based, however, on individual or tax unit incomes. Tax units may be either individuals or families, depending on the country and the year. In some countries taxes are declared on an individual basis today, but not in the past.

In order to keep the method as simple as possible, and since this as only a mild effect on the results, the baseline estimates do not take into account those differences. In Fig. 11 we show that under conservative assumptions, ignoring the differences between individual and family income may lead to a downward bias of 5–6.5 percentage points in absolute intergenerational mobility, but would not change at all the trend.

The calculation assumes that the samples of individual incomes are randomly divided into two sub-samples of equal size -  $\{A_i\}$  and  $\{B_i\}$ . These sub-samples are then matched assuming a Gaussian copula with rank correlation of zero (meaning perfectly random matching) so that for a specific index j,  $A_j$  corresponds to  $B_j$ . These represent spouses in a family and we assume that each family is composed of two spouses exactly. The matched incomes are then summed to create a new sample  $\{C_i\}$ , for  $C_i = A_i + B_i$ . This is done for every year and then absolute mobility is estimated the same way as the baseline estimate but assuming the  $\{C_i\}$  samples rather than the original samples, based on individual incomes.

This is a conservative estimate since it ignores assortative mating. Assortative mating effectively increases the rank correlation between spouses' incomes, which we assume are 0. For a rank correlation of 1, the absolute mobility estimates will stay the same. The effect increases with decreasing rank correlation. Based on the CPS data (United States Bureau of Labor Statistics, 2017), the income rank correlation between spouses in the US is 0.3 and was very stable from 1964 onward. Also, we assume that all families have two spouses and we ignore single-person families, for which the individual data reflects the family data. In Norway, Sweden and Denmark, for example, 40%–50% of households are single-person families (Eurostat, 2018). We

also consider a more realistic estimate for these countries, in which we assume that for each of the individual income samples half remains unchanged and the other half is divided and matched in way explained assuming a rank correlation of 0.3 with Gaussian copula. Fig. 11 also presents these estimates, which are very close to the baseline estimates.

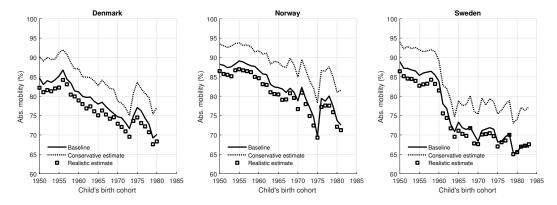


Figure 11: The absolute intergenerational mobility in Denmark, Norway and Sweden implementing assortative mating on individual income data.

#### E Sensitivity to lack of data outside the top 10%

For some of the countries we consider and in some years, the data on which we rely include distributional information only at the top of the population, usually the top 10% (please refer to the World Inequality Database website for more details (The World Inequality Database, 2018)). Since absolute mobility is estimated as a probability attached to the entire population, this limitation might have a large effect on our estimates. Using the detailed data on the shape of the income distribution in France from 1970 onward and in the US from 1962 onward, it is possible to test the sensitivity of our estimates to the lack of data outside the top 10% of the income distribution.

For this purpose we estimated absolute mobility in France and US using only income data for the top 10%, while ignoring the rest of the available data. We then compared the results to the baseline estimates, which include information on the entire distribution. The results are presented in Fig. 12. They show that the absolute mobility estimated using only top 10% are slightly underestimated, and the difference is consistent over time. At the same time, the obtained difference is rather small – less than one percentage point on average. In addition, not including information on the entire distribution, but on the top of the distribution only does not seem to have any effect on the observed trend.

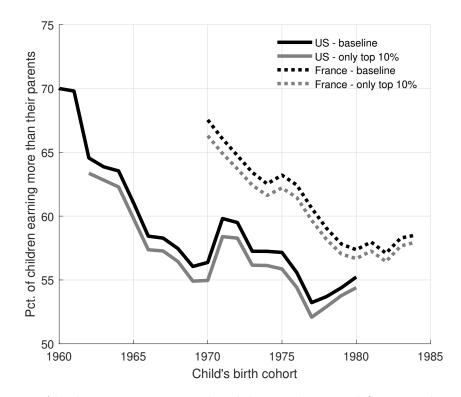


Figure 12: Absolute intergenerational mobility in the United States and in France – baseline estimates (black) and estimates based on top 10% data only (gray).

# F Absolute intergenerational mobility for the entire population and for age groups

Garbinti, Goupille-Lebret and Piketty (2018) "combine national accounts, tax and survey data in a comprehensive and consistent manner to build homogeneous annual series on the distribution of national income by percentiles over the 1900–2014 period, with detailed breakdown by age, gender and income categories over the 1970–2014 period." Using the tabulated age-grouped data, it is possible to estimate the absolute intergenerational mobility in France for 1970, 1975 and 1979 "birth cohorts", by considering the income distribution of adults aged 20–39 in 1970, 1975 and 1979 as parents and in 2000, 2005 and 2009 as children. Assuming rank correlation of 0.3, the same as in the baseline estimates, we find that the estimates assuming 20–39 year-old adults, are lower than the baseline estimates by 2–5 percentage points. This difference is not statistically significant, due to the statistical error associated with the tabulated data. The results are presented in Fig. 13.

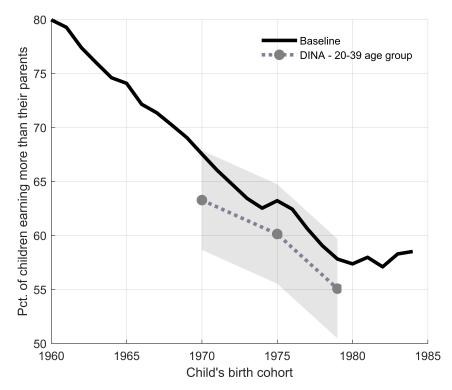


Figure 13: The absolute intergenerational mobility in France. The gray circles are based on tabulated age-grouped data (Garbinti, Goupille-Lebret and Piketty, 2018) for adults aged 20–39 in 1970, 1975 and 1979 considered as parents and in 2000, 2005 and 2009 as children. The shaded gray areas represents 95% confidence intervals for the estimates produced by bootstrapping.

# G Estimating absolute intergenerational mobility assuming rank correlation bounds

Using the lower and upper bounds of the rank correlation for each country, we can estimate the uncertainty in our baseline estimates which are due to the assumption that the rank correlation is fixed in time. The results are presented in Fig. 14, assuming the rank correlation is bounded between 0.1 and 0.5. As these results demonstrate, in none of the countries this uncertainty can explain the observed long run trends.

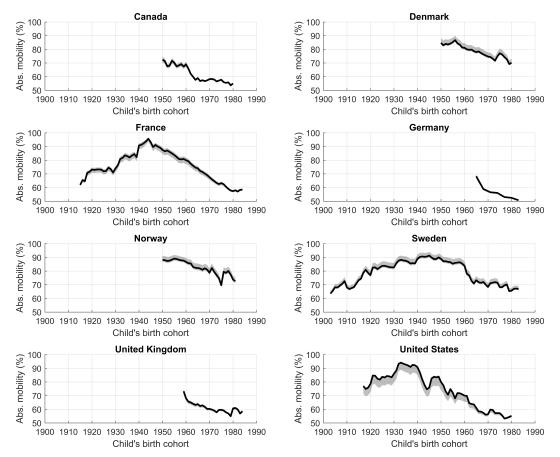


Figure 14: The absolute intergenerational mobility in various countries based on nominal rank correlations (black). The shaded gray areas are the areas that cover the lower and upper bound of the estimates assuming the rank correlation is within the range [0.1, 0.5] for each country. In some of the countries the shaded areas are too narrow to be visible.

## H Robustness of absolute mobility to changes in unit of observation and income concept

Using the detailed data from Garbinti, Goupille-Lebret and Piketty (2018) and Piketty, Saez and Zucman (2018) it is possible to test the robustness of the results for France and US to changes in income definition and to different units of observation. In particular, the data from Garbinti, Goupille-Lebret and Piketty (2018) allow comparing absolute mobility for individual adults and equal-split adults in France after 1970. The individualized series assign zero labor income to nonworking spouses. Thus, comparing absolute mobility estimates based on the two different income specifications makes it possible to assess the effect of changes in women's labor force participation and gender inequality on the evolution of absolute mobility. The individualized series is more unequal than the equal-split series by design. Income growth is similar in both. Therefore, the individualized-based estimates should be lower than the baseline estimates. Fig. 15 presents the results, showing that the individualized-based estimates are indeed lower. Yet, the differences between the estimates are small and the average difference is 1 percentage point. This indicates that the rise in female labor force participation rates did not lead to a substantial increase in absolute intergenerational mobility.

The data in Garbinti, Goupille-Lebret and Piketty (2018) also allow considering absolute mobility in France after 1970 for labor income only. The changes in labor income inequality are milder than for total income. This is due to the rising share of capital income in national income after the 1970s (Piketty and Zucman, 2014; Garbinti, Goupille-Lebret and Piketty, 2018). Yet, labor income growth is also slower than for total income. Thus, the differences between the absolute mobility estimates for the two income concepts are expected to be small. This was also found for the US by Chetty et al. (2017). Detailed labor income data and its distribution are available for the US from 1962 onward (Piketty, Saez and Zucman, 2018). Fig. 16 presents a comparison between the baseline absolute mobility estimates and labor income-based estimates for the US and France. We find that absolute mobility for labor income is lower than for total income. However, the differences between the estimates are small and the average difference is less than 1.5 percentage points in both countries. These results, together with those presented in Fig. 15, indicate that the absolute mobility estimates are robust to changes in income definition and

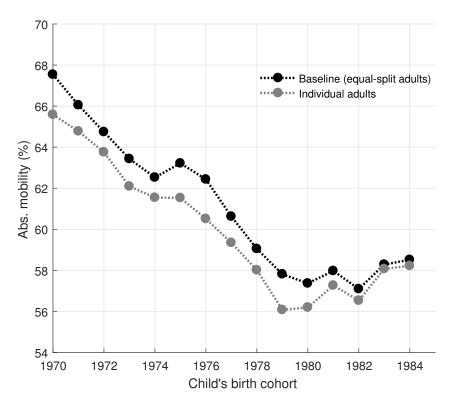


Figure 15: Absolute intergenerational mobility in France using equal-split and individualized income data.

to different units of observation (see also Appendix D). This further implies that the detected decrease in absolute mobility is indeed robust.

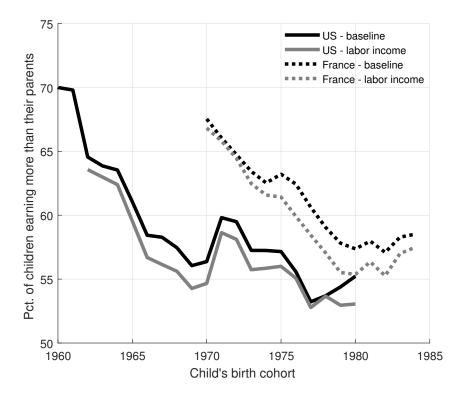


Figure 16: Absolute intergenerational mobility in the United States and in France for total income (black) and labor income (gray).