

The yield curve and the stock market: mind the long run*

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Abstract

We extract cycles from the term spread and study their role for predicting the equity premium using linear models. When properly extracted, the trend of the term spread is a strong and robust out-of-sample equity premium predictor, both from a statistical and an economic point of view. It outperforms several variables recently proposed as good equity premium predictors. Our results support recent findings in the asset pricing literature that the low-frequency components of macroeconomic variables play a crucial role in shaping the dynamics of equity markets. Hence, for policymakers and financial market participants interested in gauging equity market developments, the trend of the term spread is a promising variable to look at.

Keywords: equity premium, term spread, predictability, frequency domain

JEL classification: C58, G11, G12, G17

1 Introduction

Central banks' monetary policy actions affect a broad spectrum of interest rates, which in turn impact on stock markets and, ultimately, on stockholders (*i.e.* households) wealth. It is not surprising then that central banks closely monitor stock market developments (Greenspan, 1996, Bernanke, 2005 and Yellen, 2017) as they want to better understand both the effects of interest rates movements on the stock market and whether they should react or not to stock market movements. Furthermore, as stock/asset prices usually lead the business cycle, they are often used by macroprudential authorities as early warning indicators of possible threat to macroeconomic and financial stability (see Tölö et al. 2018, table 1).

A variable of particular interest to policymakers and financial markets participants alike is the slope of the yield curve, for which the term spread of interest rates is a common proxy. The term spread (TMS) is straightforward to compute from publicly available data -- it is simply the difference between the long- and the short-term interest rate. Throughout the years, several papers have analyzed the relationship between the TMS and the equity market. In their seminal studies, Chen et al. (1986), Campbell (1987) and Fama and French (1989) find that the term structure of interest rates predicts the equity premium. In particular, Fama and French (1989) propose a (rough) frequency-domain analysis of the equity premium predictability, in the sense that they analyze how different financial variables track different frequency components of the equity premium. They show that the default spread and the dividend yield track long-term business conditions, while the TMS tracks variation in expected returns in response to shorter-term business-cycle fluctuations. That is, the default spread and the dividend yield help predicting the low-frequency fluctuations of stock return, while the TMS its business-cycle fluctuations.¹

¹ A similar reasoning underlies the Ferreira and Santa-Clara (2011) sum-of-parts method for forecasting stock returns, where different parts of stock market returns (the dividend-price ratio, earnings growth, and price-earnings ratio growth) capture different frequencies of stock returns.

In the spirit of Fama and French (1989) analysis, in this paper we consider three economically-motivated frequency components of the TMS as potential equity premium predictors: the high-frequency component, the business-cycle-frequency component, and the low-frequency component. Our main focus is on the out-of-sample forecasting performance of these frequency components, as it is well known since Goyal and Welch (2008) that the predictability power of the TMS (and several other variables) is rather poor when the forecasting exercise is done out-of-sample.

The main result of this paper is that the low-frequency component of the TMS, extracted using wavelet filtering methods, is found to be a strong and robust out-of-sample predictor of the equity premium (both from a statistical and an economic point of view) for forecasting horizons ranging from one month to two years. Its outperformance versus the historical mean benchmark is remarkably good for the one-month horizon, increases with the forecasting horizon, and is consistently stable throughout an out-of-sample period comprising 28 years of monthly data. It also outperforms several variables that have recently been proposed as good equity premium predictors. Differently, the remaining frequency components of the TMS are poor equity premium out-of-sample predictors.

The main reason behind this good predictive power is that the forecast made with the low-frequency component of the TMS captures remarkably well the low-frequency fluctuations of the equity premium (and to a less extent its business-cycle frequency fluctuations). This finding further suggests that the level and price of aggregate risk in equity markets are strongly linked to low-frequency economic fluctuations, supporting time-varying expected returns and return predictability (see *e.g.* Dew-Becker and Giglio, 2016). From a theoretical point of view, return predictability is also compatible with market efficiency. In fact, asset returns depend on the state of the real economy, which is characterized by significant business-cycle fluctuations. So, if the quantity and price of aggregate risk are linked to economic fluctuations, then one should expect time-varying returns and return predictability, even if

markets are efficient.

The rest of the paper is organized as follows. In section 2 we review related literature. Section 3 presents the data and the method used to construct the predictors. Section 4 presents the in-sample (IS) predictability analysis (sub-section 4.1), the out-of-sample (OOS) forecasting results (sub-section 4.2), the results of the asset allocation exercise (sub-section 4.3), and a discussion about possible interpretations of the results (sub-section 4.4). Robustness analyses are done in section 5. Finally, section 6 concludes.

2 Related literature and our contribution

This research is mainly related with two streams of literature – an older one on forecasting the equity premium using the TMS, and a more recent one on using frequency domain methods in finance. In this section we give a brief overview of these streams of literature, as well as of the specific frequency-domain technique we use.

2.1 The term spread as equity premium predictor

Within the literature on forecasting the equity premium,² this paper is primarily related with the papers that analyze the equity premium forecasting properties of the TMS. Fama and French (1989) find that the equity premium on US stocks is positively related to the slope of the yield curve of US Treasury securities. Aspren (1989) studies the relationship between the US TMS and the returns on stocks of ten European countries. Boudoukh et al. (1993) and Ostdiek (1998) show how risk premia on US stocks and the world stock portfolio are negative in periods preceded by inverted yield curves. McCown (2001) finds empirical evidence about the relationships between the yield curves of larger economies (US, Germany,

² See *e.g.* the literature reviews of Rapach and Zhou (2013) and Harvey et al. (2016).

and Japan) and risk premia of stocks for eight industrialized countries. Nyberg (2013) finds that the US TMS is a powerful predictive variable for bear equity markets in the US. While this literature mainly analyzes the IS predictability of the TMS, our main focus is on its OOS forecasting power.

Hence, this paper also contributes to the literature that focuses on finding good OOS predictors of the equity premium. As pointed out by Goyal and Welch (2008), most equity premium predictors perform poorly OOS on US data up to 2008. Since Goyal and Welch (2008), several new predictors have been developed and tested, most notably macro, financial market- and behavioral-related variables.³ Here we use three frequency components of the TMS as OOS equity premium predictors.

2.2 Frequency domain asset pricing

Our paper is also related to the literature that focuses on the spectral properties of financial asset returns. Frequency domain tools have long been used in economics (*e.g.* Granger and Hatanaka, 1964 and Engle, 1974). In finance, the interest in using frequency domain tools has been growing more recently. Harris and Yilmaz (2009) decompose the spot exchange rate into its regular and irregular components (using high-pass filters) and use the short-term momentum in the low frequency trend component to generate forecasts of the spot exchange rate. Dew-Becker and Giglio (2016) develop a frequency domain decomposition of innovations

³ With regards to macro variables, Cooper and Priestley (2009, 2013) use the output gap and the world business cycle, respectively, Favero et al. (2011) consider a demographic variable (the proportion of middle-aged to young population), Li et al. (2013) study the aggregate implied cost of capital, Chava et al. (2015) study the predictive power of bank lending standards, and Moller and Rangvid (2015, 2018) study different US-based macroeconomic variables and global economic growth, respectively, by focusing on their fourth-quarter growth rate. Financial market variables include the variance risk premium (Bollerslev et al., 2009), lagged US market returns for the OOS predictability of stock returns of other industrialized countries (Rapach et al., 2013), the stock-bond yield gap (Maio, 2013), technical indicators (Neely et al., 2014), option-implied state prices (Metaxoglou and Smith, 2017), and risk neutral variance of the equity market return measured from index option prices (Martin, 2017). Behavioral-related variables include the investment sentiment indexes (Huang et al., 2015) and information on short-interest positions (Rapach et al., 2016).

to the pricing kernel. They derive frequency-specific risk prices that capture the price of risk of fluctuations in consumption growth at different frequencies. This allows to measure the relative importance of economic fluctuations at different frequencies and to assess whether they are priced in risky asset markets. Chaudhuri and Lo (2016) apply spectral analysis techniques to quantify stock-return dynamics across multiple time horizons and propose a spectral portfolio theory. Finally, Bandi et al. (2018) and Faria and Verona (2018) use models where returns and predictors are linear aggregates of components operating over different frequencies, and where predictability is frequency-specific. In this paper we investigate if and how different frequencies of the TMS individually capture relevant information regarding the future dynamics of equity markets.

2.3 Wavelet filtering methods

Wavelets have long been popular in many fields such as geophysics, engineering, medicine, and biomedical engineering. Notably, Yves Meyer, a French mathematician, received the 2017 Abel Prize “for his pivotal role in the development of the mathematical theory of wavelets.”⁴

This paper also contributes to the literature that uses wavelets methods to forecast (out-of-sample) economic and financial time series. Examples include Rua (2011, 2017), who propose a factor-augmented wavelets approach to forecast GDP growth and inflation; Kilponen and Verona (2016), who forecast aggregate investment using the Tobin’s Q theory of investment; and Zhang et al. (2017) and Faria and Verona (2018), both focused on stock market returns predictability.

⁴ The Abel Prize is, with the Fields Medal, considered to be the highest honor a mathematician can receive. These awards have often been described as the mathematician’s “Nobel Prize”.

2.3.1 Time-frequency decomposition of time series using wavelet filtering methods

Wavelet multiresolution analysis (MRA) allows to decompose any variable – regardless of its time series properties – into a trend, a cycle, and a noise component in a way which is similar to the traditional time series trend-cycle decomposition approach (Watson, 1986) or other filtering methods like the Hodrick and Prescott (1997) or the Baxter and King (1999) band-pass filter. In particular, using a wavelet filter, any time series y_t can be decomposed as

$$y_t = \sum_{j=1}^J y_t^{D_j} + y_t^{S_J} \quad , \quad (1)$$

where $y_t^{D_j}$, $j = 1, 2, \dots, J$, are the J wavelet detail components and $y_t^{S_J}$ is the wavelet smooth component. Equation (1) shows that the original series y_t , exclusively defined in the time domain, can be decomposed in different time series components, each defined in the time domain and representing the fluctuation of the original time series in a specific frequency band. In particular, for small j , the j wavelet detail components represent the higher frequency characteristics of the time series (*i.e.* its short-term dynamics). As j increases, the j wavelet detail components represent lower frequencies movements of the series. Finally, the wavelet's smooth component captures the lowest frequency dynamics (*i.e.* its long-term behavior).

In this paper, we perform wavelet decomposition analysis by applying the maximal overlap discrete wavelet transform (MODWT) MRA. This methodology i) is not restricted to a particular sample size; ii) is translation-invariant, so that it is not sensitive to the choice of starting point for the examined time series; and iii) does not introduce phase shifts in the wavelet coefficients, *i.e.* peaks or troughs of the original time series are perfectly aligned with similar events in the MODWT MRA. This last feature is especially relevant in a OOS forecasting exercise.⁵

⁵ Our presentation here is limited to basic facts that are directly relevant to our empirical analysis. A

2.3.2 The advantages of wavelets over standard econometric techniques

Traditional econometric techniques (time series and spectral/frequency analysis) impose strong assumptions about the data generating process. In particular, they presuppose a variable to be stationary. However, several economic and financial time series are hardly stationary as they exhibit trends and patterns such as structural breaks, volatility clustering and long memory.

Unlike Fourier analysis, wavelets are defined over a finite window in the time domain, which is automatically and optimally resized according to the frequency of interest. That is, using a short time window allows to isolate the high frequency features of a time series, while looking at the same signal with a large time window reveals its low frequency features. Hence, by varying the size of the time window, it is possible to capture simultaneously both time-varying and frequency-varying features of the time series. Wavelets are thus extremely useful when dealing with non-stationary time series, regardless of whether the non-stationarity comes from the level of the time series (*i.e.* from long-term trend or jumps) and/or from higher order moments (*i.e.* from changes in volatility).

Wavelet filtering methods allow a decomposition of a time series into different frequency bands. To obtain the decomposition, an appropriate cascade of wavelet filters is applied. This is essentially equivalent to filtering by a set of band-pass filters so as to capture the fluctuations of the time series in different frequency bands.

One may ask the question of why not just use the more popular Baxter and King (1999) or Christiano and Fitzgerald (2003) band-pass filters, which also permit the isolation of fluctuations in different frequency bands. The band-pass filter is a combination of a Fourier

more detailed analysis of wavelets methods can be found in Appendix A and in Percival and Walden (2000). Papers using the MODWT MRA decomposition include Galagedera and Maharaj (2008), Gallegati et al. (2011), Xue et al. (2013), Bekiros and Marcellino (2013), Gallegati and Ramsey (2013), Barunik and Vacha (2015), Caraianni (2015), Crowley and Hughes Hallett (2015), Bekiros et al. (2016), Berger (2016), Zhang et al. (2017) and Faria and Verona (2018).

decomposition in the frequency domain with a moving average in the time domain. It is optimized by minimizing the distance between the Fourier transform and an ideal filter. Like a short-time Fourier transform, it applies an “optimal” Fourier filtering on a sliding window in the time domain with constant length regardless of the frequency being isolated. Wavelet filtering, in contrast, provides better resolution in the time domain as the wavelet basis functions are both time-localized and frequency-localized. Guay and St.-Amant (2005) observe that the band-pass filter is not an ideal filter, as it is a finite representation of an infinite moving-average filter, and that it performs well at business-cycle frequencies but not at low and high frequencies. Moreover, Murray (2003) points out that the band-pass filter may introduce spurious dynamic properties.⁶

3 Data and predictors

We use monthly data from January 1973 to December 2017 and focus on the predictability of the S&P500 index excess returns, measured as the log return on the S&P500 index (including dividends) minus the log return on a one-month Treasury bill. Data for the S&P500 index total return is from CRSP and for the one-month Treasury bill is from the FRED2 database. The TMS (TMS_{TS}) is computed as the difference between the US 10-year government bond yield and the 3-month T-bill time series, and the time series are obtained from the New York Federal Reserve Bank website.⁷

Although the relationship between stock market returns, economic growth, and the TMS_{TS}

⁶ For a description of other disadvantages of band-pass filters, see Gallegati et al. (2017) and Gallegati and delli Gatti (2018).

⁷ We start our sample period in 1973 for two main (and related) reasons. First, the beginning of the sample coincides with the collapse of the Bretton Woods system, which led to a different way of conducting monetary policy. The second reason concerns the issue of model instability, which typically becomes more apparent in longer samples and can make finding a good forecasting model more difficult. In fact, unless structural breaks (like different monetary policy regimes) are properly modelled, past data can be of limited use in constructing useful forecasting models to be used at the end of the sample.

has been extensively studied in previous research (see references in section 2), we are particularly motivated by the conjecture of Fama and French (1989) that the TMS tracks variation in expected returns in response to business cycles. So, besides the TMS_{TS} , we evaluate three frequency components of the TMS_{TS} as individual equity premium predictors. The first, denoted TMS_{HF} , captures the high-frequency fluctuations of the series (HF stands for high frequency). The second, denoted TMS_{BCF} , broadly corresponds to business cycle fluctuations. The third, denoted TMS_{LF} , captures the low-frequency fluctuations of the series (LF stands for low frequency).

To compute those frequency components, we start by running a $J=6$ level MODWT MRA to the TMS_{TS} using the Haar wavelet filter with reflecting boundary conditions (as done by *e.g.* Manchaldore et al., 2010 and Malagon et al., 2015).⁸ As we use monthly data, the first component ($TMS_t^{D_1}$) captures oscillations of the TMS_{TS} between 2 and 4 months, while components $TMS_t^{D_2}$, $TMS_t^{D_3}$, $TMS_t^{D_4}$, $TMS_t^{D_5}$ and $TMS_t^{D_6}$ capture oscillations of the TMS_{TS} with a period of 4-8, 8-16, 16-32, 32-64 and 64-128 months, respectively. Finally, the smooth component $TMS_t^{S_6}$ captures oscillations of the TMS_{TS} with a period exceeding 128 months (10.6 years).⁹

Subsequently, the high-frequency component (TMS_{HF}) is computed as $TMS_{HF,t} = \sum_{i=1}^3 TMS_t^{D_i}$, the business-cycle-frequency component ($TMS_{BCF,t}$) is computed as $TMS_{BCF,t} = \sum_{i=4}^6 TMS_t^{D_i}$, whereas the low-frequency component (TMS_{LF}) corresponds to $TMS_t^{S_6}$.

To illustrate the rich set of different dynamics aggregated (and thus hidden) in the original time series, Figure 1 plots the time series of the TMS_{TS} and its three frequency components under analysis. As expected, the lower the frequency, the smoother the resulting filtered

⁸ Results are robust using different wavelet filters (like *e.g.* Daubechies). As regards the choice of J , the number of observations dictates the maximum number of frequency bands that can be used. In our case, $N = 204$ is the number of observations in the in-sample period, so J is such that $J \leq \log_2 N \simeq 7.7$.

⁹ In the MODWT, each wavelet filter at frequency j approximates an ideal high-pass filter with passband $f \in [1/2^{j+1}, 1/2^j]$, while the smooth component is associated with frequencies $f \in [0, 1/2^{j+1}]$. The level j wavelet components are thus associated to fluctuations with periodicity $[2^j, 2^{j+1}]$ (months, in our case).

time series. We note that, by summing these three frequency components, we get the exact original TMS_{TS} .

The summary statistics of the predictors (and of the equity premium) and their correlations are reported in Panel A and B of Table 1, respectively. The monthly excess market return has a mean of 0.43% and a standard deviation of 4.44%, which implies a monthly Sharpe ratio of 0.10. The excess market return has little autocorrelation, while the predictors are quite persistent (with the exception of the TMS_{HF}). The frequency components of the TMS_{TS} are low correlated.

4 Empirical results

4.1 In-sample predictability

Let r_t be the equity premium for month t and h the forecasting horizon. For each predictor x_t , the predictive regression is

$$r_{t:t+h} = \alpha + \beta x_t + \varepsilon_{t:t+h} \quad \forall t = 1, \dots, T - h, \quad (2)$$

where $r_{t:t+h} = (1/h)(r_{t+1} + \dots + r_{t+h})$. The objective of the IS analysis is to estimate equation (2) by OLS in order to test the significance of estimated β coefficients. As there are some concerns about the statistical inferences from equation (2) (related with Stambaugh, 1999 and Campbell and Yogo, 2006 bias), to make reliable inferences we follow Rapach et al. (2016) and use a heteroskedasticity- and autocorrelation-robust t -statistic and compute a wild bootstrapped p -value to test $H_0 : \beta = 0$ against $H_A : \beta > 0$ in equation (2). To enhance comparisons across predictors, we also standardize each predictor to have a unitary standard deviation before estimating equation (2). After accounting for lags and overlapping

observations, we thus have 540, 538, 535, 529 and 517 to estimate equation (2) for one-month-ahead ($h=1$), one-quarter-ahead ($h=3$), one-semester-ahead ($h=6$), one-year-ahead ($h=12$), and two-years-ahead ($h=24$) forecasting horizons.

Panel A of Table 2 reports, for each predictor and forecasting horizon, the OLS estimate of β in equation (2), its t -statistic, and the R^2 of the regression.

Starting with the monthly horizon ($h=1$), the high and business-cycle frequencies of the TMS_{TS} (TMS_{HF} and TMS_{BCF}) are not statistically significant, whereas the TMS_{TS} and the TMS_{LF} are significant at the 10% and 5% level, respectively. Overall, the R^2 s are rather small, which is expectable due to the large unpredictable component in monthly data. Campbell and Thompson (2008) argue, however, that a monthly R^2 of about 0.5% represents an economically relevant degree of return predictability. The monthly R^2 s of the statistically significant predictors are indeed slightly above that threshold.

Looking at longer forecasting horizons ($h \geq 3$), the estimated β s for the TMS_{TS} and the TMS_{LF} are similar to the ones obtained for $h=1$. Those two predictors continue to be statistically significant (at least at the 10% level) at all forecasting horizons. As it is common in this literature, the fit of the regression increases as the forecasting horizon increases. The high-frequency component is never significant, while the business-cycle-frequency component becomes significant at longer horizons ($h \geq 12$).

Thus, the TMS_{TS} and its low-frequency component are statistically significant IS predictors of the equity premium for all forecasting horizons.

4.2 Out-of-sample forecasting

Besides being more relevant from the perspective of an investor, an OOS exercise allows to avoid some econometric problems like in-sample over-fitting, small-sample size distortions and look-ahead bias (see *e.g.* Goyal and Welch, 2008 and Huang et al., 2015).

The OOS forecasts are produced using a sequence of expanding windows. We use an initial sample (1973:01 to 1989:12) to make the first OOS forecast. The sample is then increased by one observation and a new OOS forecast is produced. This is the procedure until the end of the sample. The full OOS period runs from 1990:01 to 2017:12 (336 observations).

The h -step-ahead OOS forecast of the excess returns, $\hat{r}_{t:t+h}$, is computed as

$$\hat{r}_{t:t+h} = \hat{\alpha}_t + \hat{\beta}_t x_t \quad , \quad (3)$$

where $\hat{\alpha}_t$ and $\hat{\beta}_t$ are the OLS estimates of α and β in equation (2), respectively, using data from the beginning of the sample until month t . Importantly, as the MODWT MRA at a given point in time uses information of neighboring data points (both past and future), we recompute the time-frequency series components at each iteration of the OOS forecasting process. This ensures that our method does not suffer from any look-ahead bias as the forecasts are made with current and past information only.

The OOS forecasting performance of each predictor is evaluated using the Campbell and Thompson (2008) R_{OS}^2 statistic. As is standard in the literature, the benchmark model is the historical mean (HM) forecast \bar{r}_t , which is the average excess return up to time t . The R_{OS}^2 statistic measures the proportional reduction in the mean squared forecast error for the predictive model ($MSFE_{PRED}$) relative to the historical mean ($MSFE_{HM}$) and is given by

$$R_{OS}^2 = 100 \left(1 - \frac{MSFE_{PRED}}{MSFE_{HM}} \right) = 100 \left[1 - \frac{\sum_{t=t_0}^{T-h} (r_{t:t+h} - \hat{r}_{t:t+h})^2}{\sum_{t=t_0}^{T-h} (r_{t:t+h} - \bar{r}_t)^2} \right] \quad ,$$

where $\hat{r}_{t:t+h}$ is the excess return forecast from the model using each of the alternative predictors. A positive (negative) R_{OS}^2 indicates that the predictive model outperforms (underperforms) the HM in terms of MSFE.

As in Rapach et al. (2016), the statistical significance of the results is evaluated using the

Clark and West (2007) statistic. This statistic tests the null hypothesis that the MSFE of the HM model is less than or equal to the MSFE of the predictive model against the alternative hypothesis that the MSFE of the HM model is greater than the predictive model ($H_0 : R_{OS}^2 \leq 0$ against $H_A : R_{OS}^2 > 0$).

Columns two to six of Panel A in Table 3 show the R_{OS}^2 of each predictor for the entire OOS period. As in the IS analysis, five forecasting horizons h are considered.

For forecasting horizons up to six months, the TMS_{TS} is a poor OOS predictor of the equity premium (negative R_{OS}^2). However, it outperforms the HM benchmark (positive and statistically significant R_{OS}^2) at the one-year- and two-years-ahead forecasting horizons.

The results for the different frequency components of the TMS_{TS} allow us to uncover some interesting features about the OOS predictive power of the TMS_{TS} . Its high- and business-cycle frequencies (TMS_{HF} and TMS_{BCF}) perform rather poorly as OOS equity premium predictors. This result is hardly surprising given their poor IS performance. In contrast, the TMS_{LF} has a remarkable OOS forecasting power for all forecasting horizons under analysis. Its R_{OS}^2 ranges between 2.09% for $h=1$ and 31.9% for $h=24$.

To evaluate the consistency over time of the OOS performance of the predictors, we look at the dynamics of the difference between the cumulative square forecasting error for the HM forecasting model and the cumulative square forecasting error when the TMS_{TS} and the TMS_{LF} are used as equity premium predictors. The results are plotted in Figure 2 and should be read as follows. When the line rises (falls), the predictive regression using the TMS_{TS} (black line) and the TMS_{LF} (blue line) outperforms (underperforms) the HM. A forecasting model/variable that consistently outperforms the HM over time would then feature an upward-sloping curve. Furthermore, the R_{OS}^2 is positive when the end point is above the zero line. For all forecasting horizons, the TMS_{LF} consistently outperforms the HM benchmark during the entire OOS period (excluding the first five years for $h=1, 6$ and

12).

Overall, from a statistical point of view, these results show that the low-frequency component of the TMS_{TS} is a remarkably good predictor of the equity premium for forecasting horizons from one month to two years.¹⁰

4.3 Asset allocation analysis

To analyze the economic value of the different predictive models from an asset allocation perspective, we consider a mean-variance investor allocating his wealth between equities and risk-free bills. At the end of month t , the investor optimally allocates

$$w_t = \frac{1}{\gamma} \frac{\hat{R}_{t+h}}{\hat{\sigma}_{t+h}^2} \quad (4)$$

of the portfolio to equity for the period from t to $t+h$. In equation (4), γ is the investor's relative risk aversion coefficient, \hat{R}_{t+h} is the model prediction of stock return at time t for the period $t+h$, and $\hat{\sigma}_{t+h}^2$ is the forecast of the variance of the stock return. As in Rapach et al. (2016), we assume a relative risk aversion coefficient of three, use a ten-year moving window of past excess returns to estimate the variance of the excess return, and constrain the weights w_t to a range between -0.5 and 1.5. These constraints introduce limits to the possibilities of short-selling and leveraging the portfolio.

We assume that the rebalancing frequency of the portfolio is equal to the forecasting horizon h . Taking the semester horizon ($h=6$) as an example, the procedure is as follows. The investor uses the model prediction of excess returns over the next six months and the rule (4) to define the equity weight for the next six months. Then, at the end of that semester,

¹⁰ This is an improvement with respect to previous results using wavelet filtering methods in out-of-sample forecasting exercises (see *e.g.* Rua, 2011 and Kilponen and Verona, 2016), which find improved predictability only at very short horizons.

the investor updates the model prediction of excess returns and determines the new weight using the non-overlapping return forecasts.

The average utility (or certainty equivalent return, CER) of an investor that uses the portfolio rule (4) is given by $CER = \overline{RP} - 0.5\gamma\sigma_{RP}^2$, where \overline{RP} and σ_{RP}^2 are the sample mean and variance of the portfolio return, respectively. We report the annualized utility gain from using the predictive models associated with different predictors. The utility gain is computed as the difference between the CER for an investor that uses the predictive model to forecast excess returns and the CER for an investor who uses the HM benchmark for forecasting. The difference can be interpreted as the annual portfolio management fee that an investor is willing to pay for access to the alternative forecasting model versus the historical average forecast. The analysis of different forecasting/rebalancing horizons (from one month to two years) allows us to take into account the perspective of agents with different profiles, as they include those with short- and medium-term approaches (*e.g.* some mutual funds) and those with longer-term horizons (*e.g.* central banks, pension and sovereign wealth funds).

Reported results in columns seven to eleven in Panel A of Table 3 show that the performance of the TMS_{LF} is strong also from an economic point of view. The CER gains obtained are remarkable and range from 453 basis points ($h=24$) to 659 basis points ($h=3$). From a practical standpoint, this means that the information contained in the TMS_{LF} may be useful to investors with different profiles regarding their forecasting and rebalancing horizons.

To complement this analysis, Figure 3 provides a dynamic perspective of the portfolio and cumulative wealth for an investor that uses a trading strategy (for $h=1$) based on the equity premium forecast using the HM model (black dashed line), the TMS_{TS} (black solid line), and the TMS_{LF} (blue line). Panel A presents the dynamic equity weights (resulting from equation 4). Three results stand out. First, changes in the equity exposure of the TMS_{LF} portfolio are much smoother than those using the TMS_{TS} . Second, a trading strategy based on the TMS_{LF} is unaffected by the lower bound on the equity weight (-0.5), but is quite often

constrained by the upper bound (1.5). Third, the strategy based on the TMS_{LF} displays excellent market timing in the three business cycle recessions. The exposure of the TMS_{LF} based portfolio to the equity market smoothly decreases before the occurrence of a recession (leading the portfolio to enter the recession with a rather low exposure to the risky asset), starts to increase in the late stage of the recession period, and continues to increase, smoothly, at the beginning of the subsequent expansionary period.

Panel B in Figure 3 shows the log cumulative wealth for an investor that begins with \$1 and reinvests all proceeds. Consistent with the results reported in Table 3 and Figure 2, the strategy based on the TMS_{LF} clearly outperforms those based on the HM and on the TMS_{TS} . In particular, an investor who invests \$1 at the end of December 1989 would have accumulated approximately \$43 (\$8.6 / \$6.9) by the end of December 2017 using a strategy based on the TMS_{LF} (TMS_{TS} / HM). In other words, it pays off to be focused on the long-term dynamics of the TMS_{TS} and to ignore its higher-frequency fluctuations.

4.4 Interpretation of the results

We first analyze the economic source of equity premium predictability of the TMS_{TS} and of its frequency components, and then discuss some possible interpretations of the predictive power of the TMS_{LF} .

4.4.1 Channels of predictability

The value of a stock is the discounted value of its expected cash flows. Stock return can thus reflect changes in the discount rate, changes in the expectations of cash flows, or both. That is, a variable that predicts lower (higher) stock market return should either predict an increase (decrease) in the discount rate or a decrease (increase) in cash flow expectations, or both (Baker and Wurgler, 2006). Following Cochrane (2008), we use the log dividend-price

ratio (DP) as the proxy for the discount rate channel (supported by the evidence that changes in aggregate DP ratio comes primarily from changes in discount rates) and the log dividend growth (DG) as proxy for the cash flow channel.

A common way of disentangling variations in expected discount rates versus variations in future cash flows is by means of the Campbell and Shiller (1988) log linearization of stock returns (Cochrane, 2008):

$$R_{t+1} = \kappa + DG_{t+1} - \rho DP_{t+1} + DP_t , \quad (5)$$

where R_{t+1} is the one-month-ahead stock market return, and κ and ρ are positive log-linearization constants.

Equation (5) implies that, if a variable has forecasting power of the next-period market return (beyond that of DP_t), then it must predict DP_{t+1} , DG_{t+1} , or both. As DP_{t+1} and DG_{t+1} are proxies for the discount rate and cash flow channels, respectively, evaluating their predictability from a variable provides insight into the economic source of the market return predictability power of that variable.

We use the approach in Huang et al. (2015) and estimate two bivariate predictive regressions for the TMS_{TS} and for each of its frequency components:

$$Y_{t+1} = \varrho + \delta X_t + \psi DP_t + \vartheta_{t+1} , \quad (6)$$

where $X = TMS_{TS}, TMS_{HF}, TMS_{BCF}, TMS_{LF}$ and $Y = DP, DG$. To make reliable inferences, we use a heteroskedasticity- and autocorrelation-robust t -statistic and compute a wild bootstrapped p -value to test $H_0 : \delta = 0$ against $H_A : \delta < 0$ and $H_0 : \psi = 0$ against $H_A : \psi > 0$ in (6).¹¹

¹¹ Data is from Goyal and Welch (2008) updated database and the sample period is 1973-2016.

Table 4 reports the results. The lagged DP ratio has a strong predictive power for the one-month-ahead DP ratio, with very high persistence as given by the auto-regressive coefficient of 0.99, and no forecasting power for the one-month-ahead DG . This supports the claim of Cochrane (2008) that the dividend-price ratio captures the time change in discount rates. The TMS_{TS} has predictive power for both the discount rate and cash-flow proxies, as both slope estimates are statistically significant. This suggests that the predictability power of the TMS_{TS} comes from both channels. Interestingly, this analysis also unveils that the predictability power of the TMS_{TS} operating through the cash-flow channel is concentrated at the business cycle frequency, while the one operating through the discount rate channel is concentrated at the low frequency.

4.4.2 Good market timing of the TMS_{LF}

The previous analysis shows that the equity premium predictability power of the TMS_{LF} is concentrated in the discount rate channel. Accordingly, high (low) TMS_{LF} predicts high (low) future returns, because it predicts low (high) discount rates. This implies an increased (decreased) appetite for risk-taking, triggering an increased (decreased) future equity exposure.

Figure 4 shows the good market timing of the TMS_{LF} as equity premium predictor. Let us start from the most left point A in Figure 4, where both the TMS_{LF} and the optimal equity exposure are at their maximum. After point A, the TMS_{LF} starts to decrease while the optimal equity exposure still stays at its maximum for approximately four more years (until point B), after which the equity exposure starts to decrease. At point C the TMS_{LF} reaches its relative minimum, while the equity exposure continues to decrease and reaches its relative minimum immediately before the beginning of the recession. It stays around that level until the very end of the recession (point D), after which it starts to increase again. After point D, approximately the same lead-lag pattern restarts. Interestingly, this pattern does not seem

to be altered by the quantitative easing policy of the Federal Reserve in the latter part of the sample.

4.4.3 Low-frequency fluctuations of the equity premium

In Figure 5 we plot the dynamics of the one-month ahead equity premium forecast using the TMS_{LF} and the high-, business-cycle- and low-frequency components of the equity premium (top, middle and bottom graph, respectively). It is clear that the equity premium predictability power of the TMS_{LF} comes essentially from its ability to capture the low-frequency dynamics of the equity premium. In fact, the correlation between the forecast with the TMS_{LF} and the low-frequency component of the equity premium is 0.62, while the correlations between the forecast with the TMS_{LF} and the other frequency components are much lower. That is, the forecast made with the low-frequency component of the TMS_{TS} captures remarkably well the low-frequency fluctuations of the equity premium and, similarly to Fama and French (1989) findings but to a less extent, its business-cycle frequency fluctuations.

This is consistent with empirical evidence that there are low-frequency, decades-long shifts in asset values relative to measures of macroeconomic fundamentals in the U.S. (see *e.g.* Bianchi et al., 2017). Additionally, Dew-Becker and Giglio (2016) find that low-frequency shocks in the consumption growth are significantly priced in the U.S. equity markets, which affects the one-period innovation in the stochastic discount factor, whereas business-cycle and higher frequencies are not priced. This supports the existence of aversion to low-frequency fluctuations by investors in the equity market, and is related to our findings in the sense that it is the dynamics of the low-frequency component of macroeconomic variables – rather than their business cycle or higher frequencies components – that is really relevant for the equity markets evolution. As illustrated in Figure 3, it indeed pays off to be focused on the long-term dynamics of the TMS_{TS} and of the equity premium and to ignore their higher-frequency

fluctuations.

5 Robustness analyses

5.1 Different sample periods

We first test the robustness of the results by evaluating the forecasting performance of the low-frequency component of the TMS_{TS} in different sample periods.

First, we divide the OOS period into two sub-periods: from January 1990 to December 2006, which broadly corresponds to the great moderation period, and from January 2007 onwards, which includes the great financial crisis and its aftermath. Panel A of Table 5 presents the R_{OS}^2 and CER gains for the TMS_{TS} and the TMS_{LF} . For both sub-sample periods and all forecasting horizons considered, the TMS_{LF} has strong statistical and economic performances.

Second, we evaluate the one-month-ahead return forecasts based on the TMS_{TS} and TMS_{LF} during periods of bad, normal, and good economic growth. These regimes are defined as the bottom, middle, and top-third of sorted growth rates of industrial production in the US, respectively.¹² This analysis is motivated by the fact that, while there is common agreement in the empirical literature that return predictability is usually concentrated in recessions, there is an ongoing debate about OOS returns predictability during expansions and good times. Henkel et al. (2011) and Neely et al. (2014) find no return predictability during expansions, whereas Dangl and Halling (2012) and Huang et al. (2017) find statistically significant levels of OOS predictability during expansions and good times when using models with time-varying or state-dependent coefficients.

¹² Data on US industrial production was downloaded from Federal Reserve Economic Data at <http://research.stlouisfed.org/fred2/>.

We report the R_{OS}^2 and the CER gains for each regime in Panel A of Table 6. Overall, the TMS_{TS} is never significant while the TMS_{LF} is statistically significant in all sub-samples – even in good growth periods. In bad growth periods, the TMS_{LF} delivers R_{OS}^2 of 2.87% and CER gains of 752 basis points. These values are higher than in the full sample case, thus confirming that return predictability and utility gains are higher in bad times.

5.2 Comparison with different filtering methods

We evaluate the importance of the filtering method used to extract the low-frequency component of the TMS_{TS} by using two alternative filters. The first one is the Christiano and Fitzgerald (2003) asymmetric band-pass filter, assuming a unit root with drift. The frequency bands of the filter are chosen so as to extract exactly the same frequency components as in our analysis with wavelets: the high-frequency (TMS_{BP-HF}), the business-cycle-frequency (TMS_{BP-BCF}) and low-frequency (TMS_{BP-LF}) components. The second filter is the one-sided Hodrick and Prescott (1997) filter, which is used to isolate the business-cycle component (TMS_{HP-CY}) from its low-frequency component (TMS_{HP-TR}).¹³

Panels B of Tables 2, 3, 5 and 6 report the R^2 , R_{OS}^2 and CER gains for TMS_{BP-HF} , TMS_{BP-BCF} , TMS_{BP-LF} , TMS_{HP-CY} and TMS_{HP-TR} for the full sample and for the different sub-sample periods. We only discuss the results for the TMS_{BP-LF} and TMS_{HP-TR} , as our main interest is to compare the performance of the TMS_{LF} with that of predictors with similar characteristics.

As regards the IS analysis (Table 2), the low-frequency components of the TMS_{TS} obtained using the alternative filtering methods are statistically significant (at least at the 10% level) for all forecasting horizons. Looking at the OOS results (Tables 3 to 6), the TMS_{BP-LF} is a poor equity premium predictor as it is never statistically significant. Regarding the

¹³ See Mehra (2004). As we use monthly data, we set the smoothness parameter of the HP filter to 129600 as suggested by Ravn and Uhlig (2002) and de Jong and Sakarya (2016).

TMS_{HP-TR} , it features positive and statistically significant R_{OS}^2 s for all forecasting horizons when looking at the entire sample period. However, its forecasting performance across different sub-sample periods is mixed. It performs poorly during the great moderation period for the one-month horizon and in periods of normal and good growth, but performs reasonably well in the other periods/horizons. Despite being a good equity premium predictor, the TMS_{HP-TR} lacks robustness across sample periods.

These results show that the methodology used to extract the low-frequency component of the TMS is crucial for the quality of the equity premium forecasting exercise. Wavelet filtering methods enable the extraction of a low-frequency component with a forecasting performance clearly superior to that obtained using alternative filters.

5.3 Comparison with other predictors

We evaluate the OOS equity premium predictability performance of some variables which have been recently proposed as good equity premium predictors. We consider two financial market variables – the excess bond premium (EBP, Gilchrist and Zakrajsek, 2012) and the yield gap (Maio, 2013), a macro variable – the output gap (Cooper and Priestley, 2009), a technical indicator based on financial market variable (TI-MA(2,12), Neely et al., 2014), and a behavioral-related variable – the short-interest positions (SII, Rapach et al., 2016).

Due to data availability, the sample period starts in January 1973 and ends in December 2014. The OOS period spans from January 1990 to December 2014. Panel C of Table 3 reports the results. For comparison, we also report the results for the TMS_{LF} for this sample period. Overall, none of these alternative predictors outperforms the TMS_{LF} . Except for the SII, these variables are not good predictors of the equity premium during the OOS under consideration. As shown by Rapach et al. (2016), the SII is a good predictor up to a one-year horizon. Its forecasting power, however, deteriorates significantly at the two-year

horizon. Panels C of Tables 5 and 6 report the results for the OOS periods as in sub-section 8.1. As shown in Table 5, the SII exhibits an unstable performance as its success as equity premium predictor is a fairly recent phenomenon. In fact, it strongly underperforms the HM benchmark during the first sub-sample period (up to 2006), while it features a remarkable performance in the second sub-sample period (from 2007 onwards).

6 Concluding remarks

The term spread of interest rates, commonly used as a proxy for the slope of the yield curve, is a variable of particular interest to policymakers and financial markets participants. We show that the term spread is a good and robust out-of-sample predictor of stock market return, once its time series is *properly* purged from its short-term noise and medium-term fluctuations. *Properly* means that it is crucial the way the higher-frequency fluctuations are eliminated, as certain filtering methods (namely wavelet filters) enable the extraction of a low-frequency component with forecasting performance clearly superior to that other filters (like band-pass filters).

The out-of-sample forecasting performance of the low-frequency component of the term spread is strong (both from a statistical and an economic point of view) for forecasting horizons ranging from one month to two years. It is also consistently stable throughout an out-of-sample period comprising 28 years of monthly data. Importantly, it performs well also in expansions and outperforms several variables that have recently been proposed as good equity premium predictors. Hence, for policymakers and financial market participants interested in gauging equity market developments, the proper trend of the term spread is a promising variable to look at. This reinforces recent findings in asset pricing literature that it is the low-frequency components of macroeconomic variables – rather than their business cycle or higher frequencies components – that shape the dynamics of equity markets.

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Table 1: Summary statistics and correlations

Panel A reports the summary statistics for the equity premium and the predictors. Panel B reports the correlation coefficients for the predictors. Predictors are the original time series of the term spread TMS_{TS} and the three frequency components TMS_{HF} , TMS_{BCF} and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS_{TS} of less than 16 months, between 16 and 128 months and greater than 128 months, respectively. The database contains 540 monthly observations from 1973:01 to 2017:12.

Panel A: Summary statistics						
Variable	Mean	Median	1 st percentile	99 th percentile	Std. dev.	AR(1)
Equity premium (%)	0.43	0.85	-11.7	10.5	4.40	0.05
TMS_{TS} (% , ann.)	1.58	1.75	-2.38	3.67	1.35	0.95
TMS_{HF} (% , ann.)	0.00	-0.01	-1.03	1.67	0.42	0.60
TMS_{BCF} (% , ann.)	0.00	0.09	-2.00	1.53	0.95	0.99
TMS_{LF} (% , ann.)	1.58	1.76	0.56	2.37	0.52	1.00

Panel B: Correlations				
Variable	TMS_{TS}	TMS_{HF}	TMS_{BCF}	TMS_{LF}
TMS_{TS}	1			
TMS_{HF}	0.47	1		
TMS_{BCF}	0.89	0.22	1	
TMS_{LF}	0.61	0.02	0.31	1

Table 2: In-sample predictive regression results

This table reports the β estimation by OLS of the predictive model (2) and the corresponding R^2 statistic (in percentage), for the various forecasting horizons ($h = 1, 3, 6, 12, 24$) and different predictors. The predictors in Panel A are the original time series of the term spread TMS_{TS} , and the three frequency components TMS_{HF} , TMS_{BCF} , and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS_{TS} less than 16 months, between 16 and 128 months, and greater than 128 months, respectively. The predictors in Panel B are the high-, business-cycle- and low-frequency components (TMS_{BP-HF} , TMS_{BP-BCF} and TMS_{BP-LF}) of the TMS_{TS} obtained using the BP filter and the cycle (TMS_{HP-CY}) and the low-frequency component (TMS_{HP-TR}) of the TMS_{TS} obtained using the one-sided HP filter. Each predictor variable is standardized to have a standard deviation of one. Brackets below the β estimates contain the heteroskedasticity- and autocorrelation-robust t -statistics for $H_0 : \beta = 0$ versus $H_A : \beta > 0$. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively, accordingly to wild bootstrapped p -values. The sample period runs from 1973:01 to 2017:12, monthly frequency.

Predictor	$h=1$		$h=3$		$h=6$		$h=12$		$h=24$	
	$\hat{\beta}$	R^2								
PANEL A: Predictors										
TMS_{TS}	0.33	0.55	0.30	1.33	0.28	2.22	0.36	6.95	0.32	12.9
	[1.64]*		[1.71]*		[1.63]*		[2.35]**		[2.99]**	
TMS_{HF}	0.17	0.16	0.03	0.02	-0.09	0.23	0.04	0.10	0.01	0.01
	[0.91]		[0.22]		[-0.76]		[0.56]		[0.15]	
TMS_{BCF}	0.21	0.22	0.23	0.79	0.26	1.90	0.30	5.04	0.29	10.0
	[1.12]		[1.37]		[1.53]		[1.96]*		[2.15]*	
TMS_{LF}	0.33	0.56	0.33	1.61	0.33	3.00	0.34	6.26	0.32	12.1
	[1.68]**		[1.93]**		[1.99]*		[2.25]**		[3.72]***	
PANEL B: Alternative filtering methods										
TMS_{BP-HF}	0.13	0.09	-0.03	0.01	-0.16	0.74	-0.02	0.02	-0.02	0.07
	[0.71]		[-0.19]		[-1.40]		[-0.27]		[-0.83]	
TMS_{BP-BCF}	0.20	0.21	0.23	0.78	0.26	1.95	0.30	4.78	0.28	9.68
	[1.11]		[1.40]		[1.56]		[1.95]*		[2.19]**	
TMS_{BP-LF}	0.28	0.40	0.28	1.14	0.27	2.09	0.28	4.13	0.23	6.47
	[1.38]*		[1.55]*		[1.60]*		[1.77]*		[2.63]**	
TMS_{HP-CY}	-0.22	0.25	-0.26	1.00	-0.28	2.15	-0.15	1.22	-0.06	0.39
	[-1.04]		[-1.22]		[-1.36]		[-0.93]		[-0.74]	
TMS_{HP-TR}	0.51	1.35	0.51	3.93	0.51	7.30	0.49	12.9	0.38	17.4
	[2.53]***		[2.81]***		[2.74]**		[2.74]**		[2.82]**	

Table 3: Out-of-sample R-squares (R_{OS}^2) and annualized CER gains

Columns two to six report the OOS R-squares R_{OS}^2 (in percentage) for the excess returns forecasts at h -month horizon from the model as given by equation (3). The R_{OS}^2 measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean HM. The h -month-ahead OOS forecast of excess return is generated using a sequence of expanding windows. Panel A reports the results for the original time series of the term spread TMS_{TS} , and the three frequency components TMS_{HF} , TMS_{BCF} , and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS_{TS} less than 16 months, between 16 and 128 months and greater than 128 months, respectively. Panel B reports the results for the high-, business-cycle- and low-frequency components (TMS_{BP-HF} , TMS_{BP-BCF} and TMS_{BP-LF}) of the TMS_{TS} obtained using the BP filter, and the cycle (TMS_{HP-CY}) and the low-frequency component (TMS_{HP-TR}) of the TMS_{TS} obtained using the one-sided HP filter. Columns seven to eleven present the annualized certainty equivalent return (CER) gains (in percent) for an investor who allocates his or her wealth between equities and risk-free bills according to the rule (4), using stock return forecasts from model in equation (3) with alternative predictors under analysis instead of the forecasts based on the HM. Panel C reports the R_{OS}^2 and the CER gains obtained using alternative predictors from the literature (excess bond premium, yield gap, output gap, technical indicator based on moving averages and the short interest index). The sample period is from 1973:01 to 2017:12. The OOS period is from 1990:01 to 2017:12, monthly frequency. Asterisks denote significance of the OOS $MSFE$ -adjusted statistic of Clark and West (2007). ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Predictor	R_{OS}^2					CER gains				
	$h=1$	$h=3$	$h=6$	$h=12$	$h=24$	$h=1$	$h=3$	$h=6$	$h=12$	$h=24$
PANEL A: Predictors										
TMS_{TS}	-0.72	-1.99	-1.28	3.47**	15.0***	0.10	0.52	0.33	1.38	1.84
TMS_{HF}	-0.87	-1.70	0.58*	-1.63	2.37	-1.16	-0.78	0.05	-0.81	-0.03
TMS_{BCF}	-1.52	-5.01	-8.16	-7.78	5.19*	-2.25	-1.96	-2.19	-0.58	0.81
TMS_{LF}	2.09***	6.36***	12.0***	22.9***	31.9***	5.91	6.59	6.34	5.46	4.53
PANEL B: Alternative filtering methods										
TMS_{BP-HF}	-0.13	-0.28	-5.25	-0.43	-0.50	1.28	-0.14	-1.05	-0.01	-0.30
TMS_{BP-BCF}	-0.68	-2.30	-3.81	-2.58	6.46**	-1.29	-1.33	-1.24	-0.21	0.78
TMS_{BP-LF}	-0.01	0.06	0.59	1.55	8.26	0.90	1.89	1.63	1.83	2.73
TMS_{HP-CY}	0.21	0.79	1.91	-0.98	1.41	0.38	0.39	0.52	-0.82	0.64
TMS_{HP-TR}	1.24**	3.89***	8.10***	15.5***	20.5***	3.83	4.03	3.98	3.40	2.21
PANEL C: Alternative predictors (OOS period: 1990-2014)										
EBP	0.97	0.66	-7.23	-10.1	-11.6	3.90	5.09	0.14	-1.30	1.44
Yield gap	-1.13	-4.22	-8.79	-15.7	-16.2	-0.21	-0.25	-1.01	-1.29	-0.87
Output gap	-3.24	-7.40	-8.52	-5.25	-8.02	-3.73	-2.86	-1.65	-0.75	0.70
TI-MA(2,12)	1.20*	0.76	2.55	0.86	-0.34	4.78	1.76	2.53	0.67	0.21
SII	1.94***	6.52***	11.6***	13.1**	4.85	4.18	4.67	5.44	3.40	-0.02
TMS_{LF}	2.17***	6.49***	12.1***	23.1***	31.0***	6.29	6.99	6.68	5.63	4.24

Table 4: Economic channel analysis

This table reports the estimation results of equation (6) considering four predictors (X): the original time series of the term spread TMS_{TS} , and the three frequency components TMS_{HF} , TMS_{BCF} , and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS_{TS} less than 16 months, between 16 and 128 months and greater than 128 months, respectively. DP stands for the dividend-price ratio and represents the discount rate channel. DG is the dividend growth and represents the cash flow channel. The regression slopes and the R^2 (in percentage) are reported. ***, ** and * denote significance at the 1%, 5% and 10% levels, respectively, accordingly to wild bootstrapped p -values. The sample period runs from 1973:01 to 2016:12, monthly frequency.

X_t	Y_{t+1}	δ	ψ	R^2
TMS_{TS}	DP	-0.24*	0.99***	98.9
	DG	-0.08***	0.04	3.8
TMS_{HF}	DP	-0.30	0.99***	98.9
	DG	-0.02	0.07	0.3
TMS_{BCF}	DP	-0.24	0.995***	98.9
	DG	-0.15***	0.09*	6.1
TMS_{LF}	DP	-0.68**	0.99***	98.9
	DG	-0.04	0.05	0.4

Table 5: Out-of-sample R-squares (R_{OS}^2) and annualized CER gains

Columns three to six present the OOS R-squares R_{OS}^2 (in percentage) for the excess returns forecasts at h -month horizon from the model as given by equation (3). Panel A reports the results for the original time series of the term spread TMS_{TS} , and the low-frequency component of the term spread TMS_{LF} , obtained through wavelets decomposition capturing oscillations of the TMS_{TS} greater than 128 months. Panel B reports the results for the low-frequency component (TMS_{BP-LF}) of the TMS_{TS} obtained using the BP filter, and the low-frequency component (TMS_{HP-TR}) of the TMS_{TS} obtained using the one-sided HP filter. Panel C reports the results obtained using alternative predictors from the literature (excess bond premium, yield gap, output gap, technical indicator based on moving averages and the short interest index). The R_{OS}^2 measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean HM. The h -month-ahead OOS forecast of excess returns is generated using a sequence of expanding windows. Columns seven to ten present the annualized certainty equivalent return (CER) gains (in percent) for an investor who allocates his wealth between equities and risk-free bills according to the rule (4), using stock return forecasts from model in equation (3) with alternative predictors under analysis instead of the forecasts based on the HM. The sample period runs from 1973:01 to 2017:12. Two OOS forecasting periods are considered. The first runs from 1990:01 to 2006:12 and the second from 2007:01 to 2017:12, monthly frequency. Asterisks denote significance of the OOS $MSFE$ -adjusted statistic of Clark and West (2007). ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Sample period	Predictor	R_{OS}^2				CER gains			
		$h=1$	$h=3$	$h=6$	$h=12$	$h=1$	$h=3$	$h=6$	$h=12$
PANEL A: Predictors									
1990-2006	TMS_{TS}	-1.12	-3.13	-3.44	-2.37	0.72	1.67	0.71	0.13
	TMS_{LF}	1.66***	6.13***	13.5***	25.1***	5.51	6.57	5.62	4.72
2007-2017	TMS_{TS}	-0.16	-0.74	0.57	9.38***	-0.87	-1.28	-0.37	3.08
	TMS_{LF}	2.67***	6.62***	10.8***	20.8***	6.50	6.59	7.39	6.44
PANEL B: Alternative filtering methods									
1990-2006	TMS_{BP-LF}	0.05	0.79	2.57	4.17	1.34	2.95	2.19	3.22
	TMS_{HP-TR}	0.35	1.54*	4.29**	8.64***	2.57	3.20	2.28	1.47
2007-2017	TMS_{BP-LF}	-0.10	-0.74	-1.09	-1.09	0.24	0.30	0.90	0.04
	TMS_{HP-TR}	2.48**	6.48***	11.3***	22.4***	5.77	5.26	6.48	6.20
PANEL C: Alternative predictors (OOS period: 1990-2014)									
1990-2006	EBP	0.64	3.41**	-6.47	-10.7	2.59	4.31	-0.98	-1.23
	Yield gap	-0.99	-3.33	-8.43	-21.5	-0.89	-0.40	-1.35	-1.35
	Output gap	-4.05	-9.31	-13.0	-8.74	-5.12	-3.96	-2.60	-1.13
	TI-MA(2,12)	1.05	1.67	6.25**	0.24	4.17	2.15	2.72	-0.10
	SII	-0.15	-0.51	-0.95	-9.49	0.88	0.74	0.59	-0.38
2007-2014	EBP	1.48	-2.62	-7.90	-9.56	6.68	6.69	2.42	-1.47
	Yield gap	-1.34	-5.27	-9.11	-9.48	1.24	0.06	-0.28	-1.10
	Output gap	-1.96	-5.14	-4.50	-1.48	-0.80	-0.64	-0.11	-0.99
	TI-MA(2,12)	1.44	-0.33	-0.76	1.53	6.06	0.95	2.33	2.33
	SII	5.24***	14.9***	22.9***	37.4***	11.2	13.1	15.9	11.2

Table 6: Out-of-sample R-squares (R_{OS}^2) and annualized CER and SR gains

The sample period runs from 1973:01 to 2017:12. We divide the OOS in periods of bad growth, normal growth, and good growth. These regimes are defined as the bottom, middle, and top-third of sorted growth rates of industrial production in the US, respectively. This table reports, for the three regimes, the OOS R-squares R_{OS}^2 (in percentage) for the excess returns forecasts at the one-month horizon ($h = 1$) from the model as given by equation (3). Panel A reports the results for the original time series of the term spread TMS_{TS} , and the low-frequency component of the term spread TMS_{LF} , obtained through wavelets decomposition capturing oscillations of the TMS_{TS} greater than 128 months. Panel B reports the results for the low-frequency component (TMS_{BP-LF}) of the TMS_{TS} obtained using the BP filter, and the low-frequency component (TMS_{HP-TR}) of the TMS_{TS} obtained using the one-sided HP filter. Panel C reports the results obtained using alternative predictors from the literature (excess bond premium, yield gap, output gap, technical indicator based on moving averages and the short interest index). The R_{OS}^2 measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean HM. The one-month-ahead OOS forecast of excess return is generated using a sequence of expanding windows. The table also reports the annualized certainty equivalent return (CER) gains (in percent) for an investor who allocates his wealth between equities and risk-free bills according to the rule (4), using stock return forecasts from model in equation (3) with alternative predictors under analysis instead of the forecasts based on the HM. Asterisks denote significance of the OOS $MSFE$ -adjusted statistic of Clark and West (2007). ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Predictor	Bad growth		Normal growth		Good growth	
	R_{OS}^2	CER gains	R_{OS}^2	CER gains	R_{OS}^2	CER gains
PANEL A: Predictors						
TMS_{TS}	0.57	0.92	-2.61	-1.70	-0.82	1.03
TMS_{LF}	2.87***	7.52	2.17**	4.95	1.16**	5.24
PANEL B: Alternative filtering methods						
TMS_{BP-LF}	-0.10	1.24	-0.44	-0.81	0.37	2.30
TMS_{HP-TR}	2.51***	6.29	0.12	1.33	0.62	3.84
PANEL C: Alternative predictors (OOS period: 1990-2014)						
EBP	-0.30	5.97	3.26**	4.39	0.77	1.37
Yield gap	-1.87	-0.39	0.20	0.46	-1.22	-0.67
Output gap	-2.23	-4.18	-5.23	-4.37	-2.97	-2.69
TI-MA(2,12)	3.47*	10.3	2.49*	5.58	-2.23	-1.46
SII	2.88**	4.98	0.13	1.73	2.16*	5.79
TMS_{LF}	3.00***	7.88	2.13**	5.55	1.29**	5.45

Figure 1: Time series of the term spread and of its frequency components

This figure reports the original time series of the term spread TMS_{TS} (black line) and of its three frequency components TMS_{HF} , TMS_{BCF} and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS less than 16 months (green line), between 16 and 128 months (red line), and greater than 128 months (blue line), respectively. Gray bars denote NBER-dated recessions. Sample period runs from 1973:01 to 2017:12, monthly frequency.

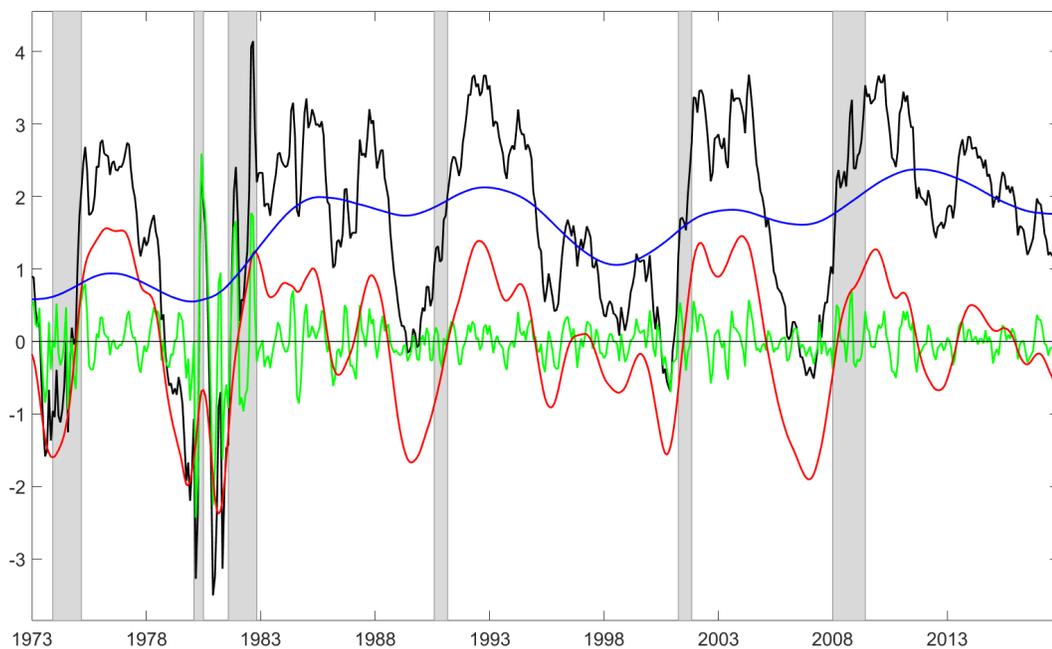


Figure 2: Cumulative sum of squared forecast errors

This figure reports the difference between the cumulative square forecasting error for the HM forecasting model and the cumulative square forecasting error for the predictive regression based on the model (3) for the original time series of the term spread TMS_{TS} (black line) and the low-frequency component of the term spread TMS_{LF} (blue line). Gray bars denote NBER-dated recessions. The sample period runs from 1973:01 to 2017:12. The OOS forecasting period runs from 1990:01 to 2017:12, monthly frequency.

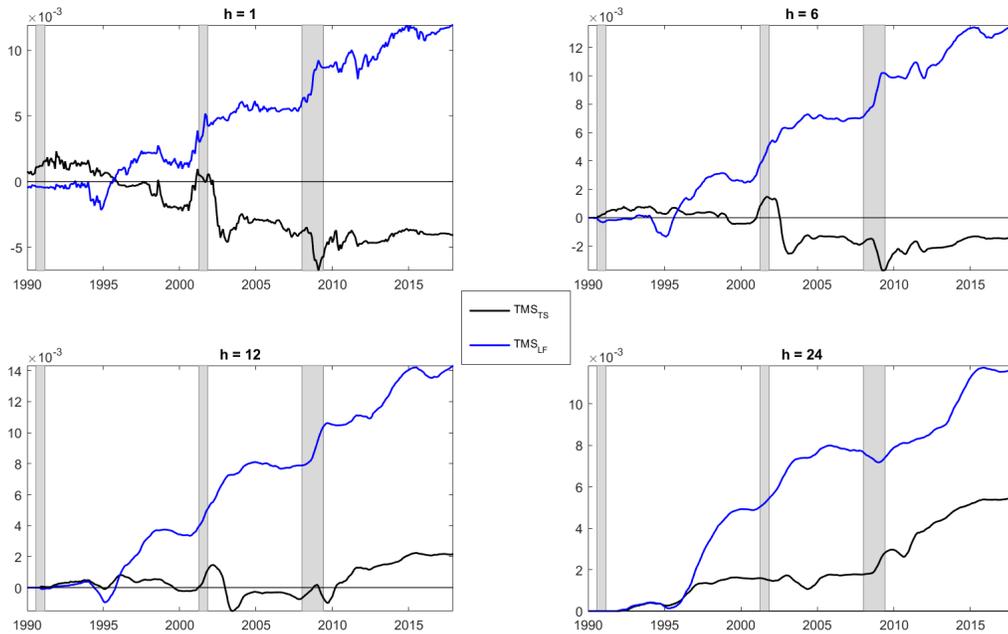
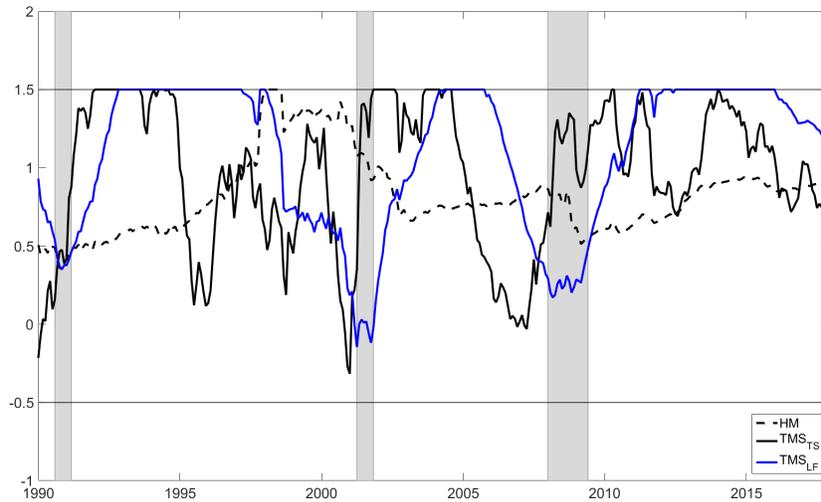


Figure 3: Equity weights and log cumulative wealth

Panel A plots the dynamics of the equity weight for a mean-variance investor who allocates monthly his wealth between equities and risk-free bills according to the rule (4), using stock return forecasts based on the HM benchmark (dashed black line), the original time series of the term spread TMS_{TS} (solid black line), and the low-frequency component of the term spread TMS_{LF} (blue line). The equity weight is constrained to a range between -0.5 and 1.5. Panel B delineates the corresponding log cumulative wealth for the investor that begins with \$1 and reinvests all proceeds. The investor is assumed to have a relative risk aversion coefficient of three. Gray bars denote NBER-dated recessions. Sample period runs from 1990:01 to 2017:12, monthly frequency.

A. Equity weights



B. Log cumulative wealth

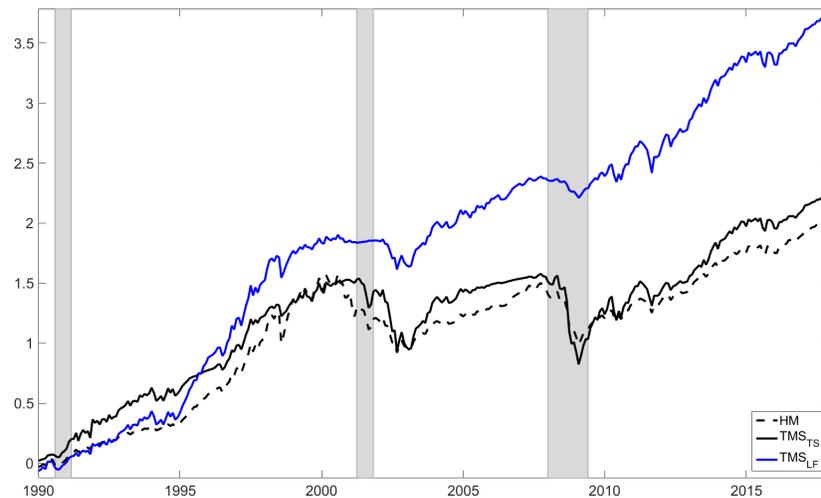


Figure 4: Equity weights and low-frequency component of the term spread

This figure plots the dynamics of the low-frequency component of the term spread (TMS_{LF} , black line) and the equity weight (blue line) for a mean-variance investor who allocates monthly his wealth between equities and risk-free bills according to the rule (4) using stock return forecasts based on the TMS_{LF} . For readability, the TMS_{LF} has been demeaned and centered around 1. The investor is assumed to have a relative risk aversion coefficient of three. Gray bars denote NBER-dated recessions. Sample period runs from 1990:01 to 2017:12, monthly frequency.

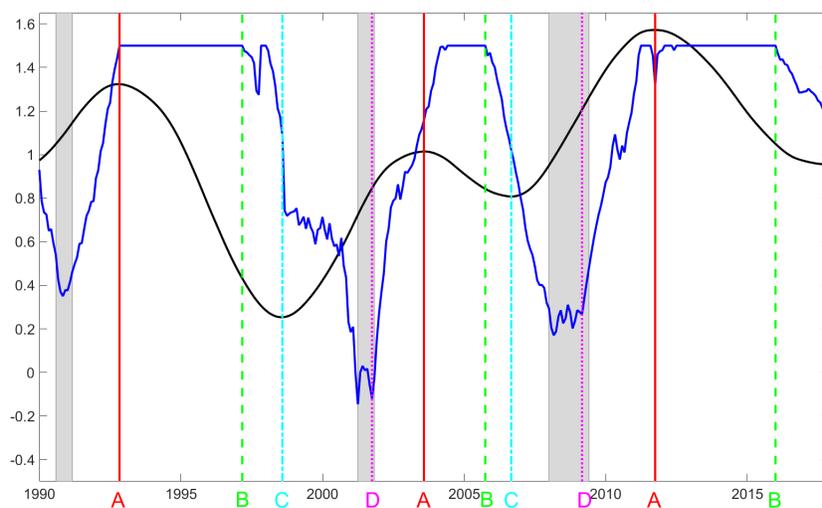
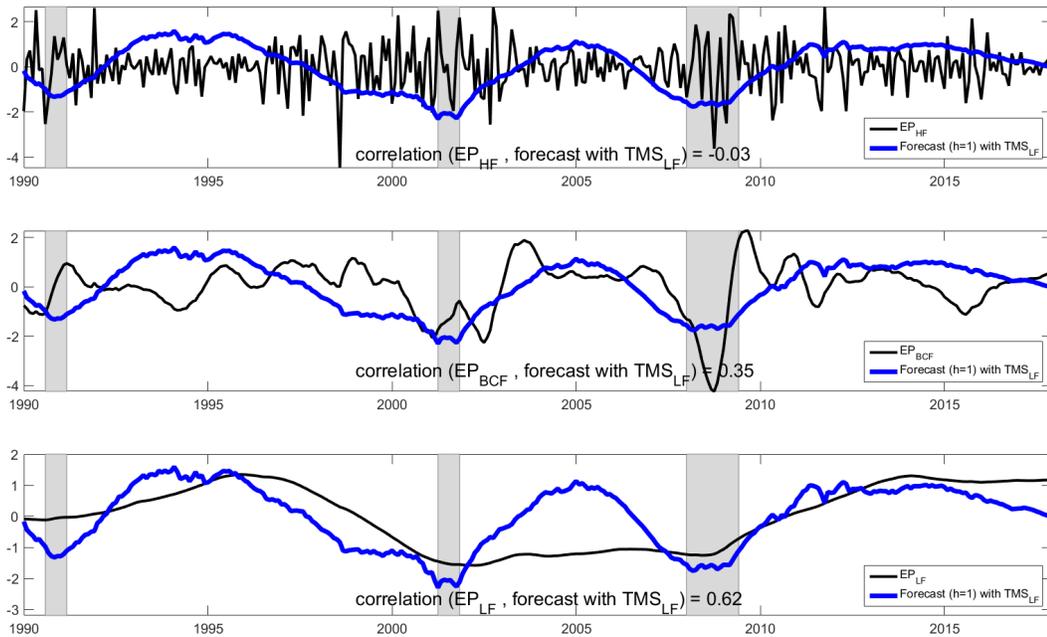


Figure 5: Equity premium frequency components and equity premium forecast ($h = 1$) based on the TMS_{LF}

This figure plots the dynamics of the one-month ahead equity premium forecast based on the low-frequency component of the term spread (TMS_{LF} , blue line) and the high-, business-cycle- and low-frequency components of the equity premium (top, middle and bottom graphs, respectively, black lines). The series are centered to have zero mean and scaled to have standard deviation 1. Gray bars denote NBER-dated recessions. Sample period runs from 1990:01 to 2017:12, monthly frequency.



Appendix A

The discrete wavelet transform (DWT) multiresolution analysis (MRA) allows the decomposition of a time series into its constituent multiresolution (frequency) components. There are two types of wavelets: father wavelets (ϕ), which capture the smooth and low-frequency part of the series, and mother wavelets (ψ), which capture the high-frequency components of the series, where $\int \phi(t) dt = 1$ and $\int \psi(t) dt = 0$.

Given a time series y_t with a certain number of observations N , its wavelet multiresolution representation is given by

$$y_t = \sum_k s_{J,k} \phi_{J,k}(t) + \sum_k d_{J,k} \psi_{J,k}(t) + \sum_k d_{J-1,k} \psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k} \psi_{1,k}(t) \quad , \quad (7)$$

where J represents the number of multiresolution levels (or frequencies), k defines the length of the filter, $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ are the wavelet functions and $s_{J,k}$, $d_{J,k}$, $d_{J-1,k}$, \dots , $d_{1,k}$ are the wavelet coefficients.

The wavelet functions are generated from the father and mother wavelets through scaling and translation as follows

$$\begin{aligned} \phi_{J,k}(t) &= 2^{-J/2} \phi(2^{-J}t - k) \\ \psi_{j,k}(t) &= 2^{-j/2} \psi(2^{-j}t - k) \quad , \end{aligned}$$

while the wavelet coefficients are given by

$$\begin{aligned} s_{J,k} &= \int y_t \phi_{J,k}(t) dt \\ d_{j,k} &= \int y_t \psi_{j,k}(t) dt , \end{aligned}$$

where $j = 1, 2, \dots, J$.

Due to the practical limitations of DWT in empirical applications, we perform wavelet decomposition analysis here by applying the maximal overlap discrete wavelet transform (MODWT). The MODWT is not restricted to a particular sample size, is translation-invariant so that it is not sensitive to the choice of the starting point of the examined time series, and does not introduce phase shifts in the wavelet coefficients (so peaks or troughs in the original time series are correctly aligned with similar events in the MODWT MRA). This last property is especially relevant in the forecasting exercise.