Banking Industry Dynamics and Size-Dependent Capital Regulation

Tirupam Goel*
Bank for International Settlements

December 2018

Abstract

This paper develops a model of a dynamic and heterogeneous banking sector, embedded in a general equilibrium framework. The objective is two-fold. The first is to offer a model of the banking industry where leverage is a driver of the dynamics, and where the implied distribution of bank capital closely matches the one that observed in data. The second objective is to characterize the optimal size-dependent capital regulation. The rationale for regulation arises because deposit insurance induces banks to assume more leverage – defined as the ratio of assets to capital – than what is socially optimal. As such, a limit on leverage improves welfare. Crucially, the optimal regulation is size-specific – it tighter for larger banks. Intuitively, this is for three reasons. First, allowing small banks to assume more leverage enables them to grow faster, leading to a *growth effect*. Second, although more leverage by small banks results in a higher failure rate among them, the failing banks are the less productive ones, leading to a *cleansing effect*. Third, relative to small banks, large banks entail more correlated shocks per unit asset, and are therefore, riskier. As such, tighter regulation for large banks lowers the overall riskiness of banks' assets, leading to a *stabilization effect*. The model provides support for policies – such as the GSIB framework – that impose tighter regulation on large and systemically important banks.

JEL Classification: G21, G28, E50, C60
Keywords: Size distribution, Default, Heterogeneous agent models, Size-dependent policy

---

*Email: tirupam.goel@bis.org, Address: Bank for International Settlements, Centralbahnplatz 2, 4051, Basel, Switzerland. Acknowledgments: I am extremely grateful to Assaf Razin, Karel Mertens, Eswar S. Prasad, and Maxim Troshkin for invaluable guidance and encouragement. I am very thankful to John M. Abowd, Julieta Caunedo, and Christopher Huckfeldt for numerous useful discussions. I am also thankful to Viral V. Acharya, Javier Bianchi, Frederic Boissay, Markus Brunnermeier, Dean Corbae, Ingo Fender, Benjamin Moll, Guillermo L. Ordonez, Edouard Schaal, Alp Simsek, Julia Thomas, and various conference and seminar participants for useful comments. I thank Cornell Institute for Social and Economic Research for providing state-of-the-art computing resources, and I gratefully acknowledge funding from the Labor Dynamics Institute and from the Department of Economics at Cornell University. Any errors in this paper are the author's responsibility.

1 Introduction

A bank's leverage decision,[1] where leverage is defined as the ratio of assets to net worth, or capital, has implications for growth and solvency. On the one hand, higher leverage increases the potential for higher return on bank capital.[2] On the other hand, leverage increases the variance of returns as well as the probability of failure, that is, an event where the bank's capital falls below zero. Thus, leverage has implications not only for individual banks, but also for industry dynamics such as entry, exit, and size-distribution, and for such macroeconomic aggregates as output and investment. The first objective of this paper is to develop a model of a heterogeneous banking industry where leverage is the key driver of the dynamics. A key design consideration is to have the model be able to generate the observed distribution of bank capital.

In the presence of deposit insurance, bank leverage decisions can be socially inefficient. This inefficiency rationalizes capital regulation, defined here as a cap on bank leverage.[3] Moreover, heterogeneity in banks' sizes – as measured by capital – implies that, in general, the inefficiency depends on the size distribution.[4] The intuition is as follows. Consider a large bank on the one hand, and two small banks obtained by breaking up the large bank on the other hand. Greater leverage by the large bank can lead to greater variance in its return on capital compared to the case of smaller banks. This is because in the latter case, there is some diversification of shock across the two banks', whereas in the former case, the bank-wide shock applies to all the assets. As such, optimal regulation is likely size-dependent, characterizing which is the second objective of this paper.

The question of bank capital regulation gained considerable importance in the aftermath of the great financial crisis, precisely because high leverage played a critical role in scaling up the crisis. In response, international standards *a'la* Basel III (BCBS [2011]) envisioned not only more stringent regulation relative to Basel II, but also tighter regulation for global systemically important banks (GSIBs) relative to the smaller banks. In designing these standards, regulators faced two trade-offs, both of which I study in this paper. The first relates to the extent by which regulation should be tightened, as also discussed in, for example Van den Heuvel [2008] and Corbae and D'erasmo [2014]. The second relates to size-dependence of regulation. If regulators impose relatively tighter regulation on smaller banks, this might constrain their growth and end up benefiting larger banks, which could be undesirable from a financial stability as well as competition perspective. Moreover, if larger banks are allowed to assume higher leverage, they may become more prone to failures, which can be systemically more costly. Figure 1 documents that failures of larger banks incur bigger costs which are often borne by tax-payers, via the deposit insurance framework for instance. Yet, constraining large banks might reduce overall financial intermediation, not least because of the scale economies banks might exhibit (see for e.g.

[1] In this paper, the term *bank* stands for any financial intermediary.

[2] Higher leverage may help gain from a tax shield on debt, but this aspect is not pursued here.

[3] This formulation is in the spirit of the Basel III leverage requirement, as opposed to the risk-weight capital-ratio requirement.

[4] I measure bank size via capital. This measurement is of consequence only in the case of size-dependent policy, and I follow this approach for mathematical convenience. An alternative approach where size is measured via assets would lead to an equivalent result in terms of optimal policy.

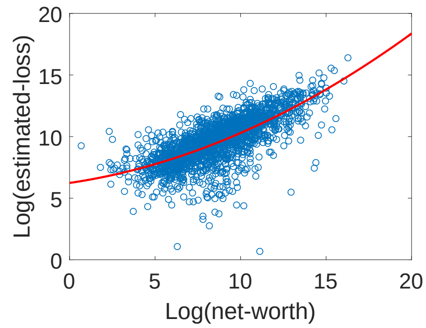

Figure 1: US savings and commercial banks: Log of net-worth versus log of estimated losses during 1985 - 2014. Source: FDIC Historical Statistics on Banking, and Call and Thrift Financial Reports. The net worth is as of the last regulatory filing by the bank before its failure. The estimated loss is the amount of FDIC funds disbursed to meet the shortfall in liabilities. In this sense, the estimated loss captures only the idiosyncratic as opposed to systemic cost of bank failure.

Hughes and Mester [2013]). As a result of these trade-offs, the nature of optimal capital regulation is not obvious *ex-ante* .

Model overview To this end, I develop a model of a dynamic and heterogeneous banking sector embedded in a general equilibrium framework. Industry dynamics are modeled in the spirit of Hopenhayn [1992], while leverage is introduced as a key driver. Banks are financial intermediaries that accept deposits from households and purchase shares in firms that operate a standard production technology to produce consumption goods. There is no aggregate uncertainty, although banks receive independent shocks to the value of their assets. A bank fails when its capital falls below zero, which can happen because of a large shock, or because of high leverage ex-ante. New banks are free to enter the industry upon paying a fixed cost, and receive a randomly drawn seed-capital. The representative household supplies labor to the firms in return for wages, It also owns the banks, and receives dividend income in return. The household saves via bank deposits that are fully covered by a deposit insurance scheme. Deposit insurance rationalizes capital regulation in the model by lowering the cost of funding for banks and inducing them to take more leverage than what is optimal.

Calibration I focus on three aspects of dynamics in the US banking industry to discipline my model. First is the annual failure rate of US savings and commercial banks (see left-hand panel of figure 2). I target the mean failure rate during 1995-2007 as this period exhibits fewer structural or regulatory changes in the industry.[5] Second, I consider the mean leverage of these banks, precisely because leverage is central to their business models

[5]For instance, the Riegle-Neal Interstate Banking and Branching Efficiency Act, which was implemented in June 1997, but US states were already active in removing geographic limits to banking prior to 1995 (Jayaratne and Strahan [1997])

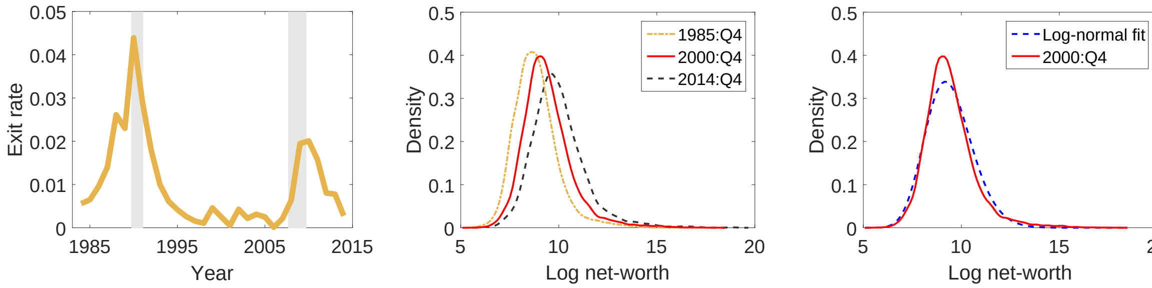

Figure 2: Left-hand panel: Annual failure rate of US commercial and savings banks; The shaded regions mark the two business cycle contractions that also featured financial crises: (i) the savings and loan crisis of the 1990s and (ii) the great financial crisis of 2008; Source: FDIC Historical Statistics on Banking. Center panel: Kernel density estimate of the distribution of log capital. Right-hand panel: Log-normal fit to 2000Q4 data; Source: FDIC Call and Thrift Financial Reports.

and dynamics. Third, I target the distribution of bank capital (see figure 2, center panel). Even though the distribution is shifting to the right over time, distributions that are a few years apart are not statistically significantly different based on the two-sample Kolmogorov-Smirnov (KS) test. It is useful to note that the distributions are fairly well approximated, based on the Kolmogorov-Smirnov Goodness of fit test, by a log-normal distribution (right-hand panel).

Key Results The calibrated model generates a distribution of bank capital that is very close to what is observed in the US data. In addition, the model provides useful insights into how industry dynamics respond to a tightening in capital regulation. For one, the bank failure rate is lower, which increases the mean age of banks. This produces a larger mass of banks in the equilibrium. Relatedly, the distribution of bank capital shifts to the right.

The analysis of counter-factual regulation using the calibrated model reveals that optimal size-independent regulation is tighter relative to the benchmark, and results in a welfare gain of 4.05% in consumption equivalent terms. When size-dependent, regulation is more stringent for larger banks, and the welfare gain is equal to 4.12%. Intuitively, an overall tightening of regulation lowers the costs associated with industry turnover, which more than compensates for the lower overall financial intermediation and final output. In addition, when regulation is size-dependent, it leads to three welfare producing effects. First, allowing small banks to assume more leverage enables them to grow faster, leading to a *growth effect*. Second, although more leverage by small banks results in a higher failure rate among them, the failing banks are the less productive ones, leading to a *cleansing effect*. Third, relative to small banks, large banks entail more correlated shocks per unit asset, and are therefore, riskier. As such, tighter regulation lowers the overall riskiness of banks' assets, leading to a *stabilization effect*.

Related Literature The first strand of literature to which this paper is related, aims to understand how inefficiencies in the banking sector affect the real economy, and to identify the tools regulators could use to address these inefficiencies. The key contribution of this paper is to shed light on industry dynamics as a channel through which leverage can affect aggregate output, consumption, and welfare. Few other papers have this feature. For one, Corbae and D'erasmo [2014] develop a quantitative model of an imperfectly competitive banking industry, taking aspects of the distribution as given. Dynamics in their paper is driven by shocks to borrowers' production technology, and the authors show how competition between banks affects interest rates and thus borrowers' lending and risk-taking decisions. In contrast, I develop a stylized model of a competitive banking industry. This delivers tractability, and also the ability to generate an empirically consistent distribution of bank capital. Moreover, because entry, exit and capital distribution respond to regulation, I am able to study size-dependent policy.

With regards to size-independent policy, Covas and Driscoll [2014] consider capital and liquidity regulation in a dynamic model of banking, to study their effect on loan supply, output, and interest rates. In a related paper, Boissay and Collard [2016] show that capital and liquidity requirement reinforce each other, and must be set relatively high. This leads to welfare gains from more productive lending, despite lower aggregate supply of credit. Nguyen [2014] studies the welfare implications of capital requirements in the presence of moral hazard due to government bailouts. Christiano and Ikeda [2013] study the welfare implications of leverage restrictions in a macroeconomic model with unobservable banker effort. Begenau [2016] shows that higher capital requirements might actually increase bank lending when households value safe and liquid assets, providing another rationale for tighter regulation. Other papers on this topic, some of which also study time-varying regulation, include Benes and Kumhof [2011], Zhu [2007], De Nicolo et al. [2014] and Christensen et al. [2011].

Another strand of the literature of which this paper relates is one that focuses on embedding meaningful financial sectors in macroeconomic models. For example, Gertler and Kiyotaki [2010] and Gertler and Karadi [2011] develop a model in which financial frictions arise due to a limited commitment issue, and use this model to study policy questions, including unconventional monetary policy. Adrian and Boyarchenko [2012] use a value-at-risk constraint to generate endogenous borrowing constraints that amplify financial shocks. In contrast, the focus in this paper is on heterogeneity, and in exploring its implications for macroeconomic aggregates and optimal policy.

This paper is naturally related to studies in industry dynamics. For example, Cooley and Quadrini [2001], Clementi and Hopenhayn [2006], Albuquerque and Hopenhayn [2004] and Miao [2005] study the theoretical implications of various forms of borrowing constraints on firm dynamics, while Lang et al. [1996] for instance, looks at this issue from an empirical side. But these studies focus on non-financial firms that exhibit lower balance sheet leverage relative to banks (Kalemli-Ozcan et al. [2012]), making leverage a less important driver of firm dynamics. In fact, as DeAngelo and Stulz [2013] show and Hanson et al. [2010] argue, high leverage is at the core of banks' business models, wherein obtaining cheap funding is the optimal strategy to surviving the competition. In this spirit, I incorporate leverage in this paper. Relatedly, focusing on the effect of industry dynamics on the real economy, Clementi and Palazzo [2015] show that after a positive aggregate productivity shock, the number of entrants increases. The surviving entrants generate a wider and longer expansion – thus amplifying the shock. Relatedly, in my paper, by allowing smaller banks to take on more leverage, optimal regulation enables surviving entrants to grow faster.

The remainder of the paper is organized as follows. I present the model in Section 2, define the stationary equilibrium in Section 3, discuss the calibration and numerical solution of the model in Section 4, and discuss the issue of optimal regulation in Section 5. Section 6 concludes. Proofs are in the appendix.

2 Model

Overview Time is discrete and the horizon is infinite. The model economy is populated with a representative household, firms, capital producers, banks, and a government. The household supplies labor and owns the banks. The production side of the economy has an island setting – there is a continuum of islands, each with an atomistic firm and an atomistic bank. Firms produce output using physical capital and labor. While firms own the physical capital located on their respective islands, they rent labor in the *national* market. Shares in the firm on a given island are owned by the bank located on that island. Banks are financial intermediaries who fund themselves using own equity and household deposits in order to purchase shares in the firm located on its island. There are no contracting frictions between banks and firms – firms pledge all their profits to their shareholders.

There is no aggregate uncertainty, but physical capital is subject to island-specific *quality* shocks. These shocks are uncorrelated across islands, and lead to an increase or decrease in physical capital stock on each island. As such, these shocks affect firm output and profitability, by extension, the value of its shares, and thus, the value of banks' assets. Banks grow, shrink, or fail and exit the industry depending on these shocks. New bank are free to enter the industry upon payment of a fixed dead-weight cost. Upon entry, they receive a random seed funding from the government. The government also runs a deposit insurance scheme that it funds via lump-sum taxation on the household. And, it sets capital regulation for banks. The sequence of events within a given time period is as follows:

1. Quality shocks are realized on each island.
2. Firms produce output, and pay wages to the household and dividends to the banks.
3. The value of banks' assets is determined. Banks that can pay their liabilities continue to operate, while those that cannot, are considered *failed*. Their liabilities are covered by the government.
4. New banks enter and the distribution of bank capital is determined.
5. All banks choose how much dividends to pay, and decide how much assets and liabilities to assume for the next period. Households consume.

Representative Household The representative household supplies one inelastic unit of labor to the firms, and owns the banking sector. It maximizes utility subject to a budget constraint:

$$\max_{C_t, D_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

$$C_t + D_t = W_t + E_t + R_{t-1} D_{t-1} - O_t$$

where C_t is consumption, D_t is bank deposits, R_t is deposit interest rate, W_t is wages, E_t is dividends received from banks, and O_t is a lump-sum tax.

Firms The firm on a given island produces goods using labor that costs W_t, and using the physical capital that it owns. I assume that firms across islands operate identical and constant-returns-to-scale (CRS) production technologies. This assumption implies that the distribution of capital across islands does not matter, so that aggregate output Y_t can be expressed as a function of aggregate capital K_t. This assumption, combined with the fact that labor is hired nationally, also implies that the dividend rate Z_t that firms pay to their shareholders banks is the same across islands. Thus, the firms' problems admit a representative form:

$$\max_{L_t} \quad K_t^{\alpha} L_t^{1-\alpha} - W_t L_t$$

Shares of the firm are held by the bank on the same island, and each share represents claim to one unit of capital. Consequently, the price of a firm's shares is equal to the price of physical capital. The firm issues new shares to the bank on its own island to purchase additional physical capital. The dividend rate is:

$$Z_t = \frac{Y_t - W_t L_t}{K_t}, \quad \text{where } Y_t = K_t^{\alpha} L_t^{1-\alpha}.$$

Capital producers Firms purchase physical capital from capital producers. These entities are owned by the household. Moreover, they are identical and are perfectly competitive. As such, they admit a representative form, and make zero profit in equilibrium. Capital producers use one unit of numeraire good to produce one unit of physical capital, and therefore, the price of physical capital in terms of the numeraire good is unity.

Banks The objective of an atomistic bank on island, say j, is to maximize the present discounted value of the dividends e_t^j it pays to the household (In what follows, I omit island specific indexing):

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_t e_t$$

where Λ_t – which equals the discounted ratio of inter-temporal marginal utilities of the household – is the stochastic discount factor. The date-t decisions of a bank that wakes up with capital n_t include how much dividends to pay, how much assets s_t to hold, and how much deposits d_t to raise, subject to cash-flow and capital regulation constraints:

$$n_t + d_t = e_t + s_t; \tag{2.0.1}$$

$$x_t = \frac{s_t}{n_t - e_t} \leq \chi(n_t). \tag{2.0.2}$$

Here $\chi(n_t)$ is the maximum post-dividend leverage x_t the bank is allowed to assume. Note that $\chi(n_t)$ reflects the possibility that regulation is size-dependent. Bank capital evolves as a result of retained earnings and losses that result from cash-inflows due to the bank's assets and cash-outflows due to the bank's liabilities. While outflows in the next period are pre-determined and equal $R_t d_t$, inflows depend on the performance of the firm whose shares the bank holds. Formally, the bank's assets and firm's end-of-period physical capital are related as follows:

$$s_t = \Big(i_t + (1-\delta)k_t\Big) \tag{2.0.3}$$

where i_t is investment and $(1-\delta)$ is the fraction of un-depreciated capital. Physical capital evolves based on randomly distributed quality shocks ψ_{t+1}:

$$k_{t+1} = \Big(i_t + (1-\delta)k_t\Big)\psi_{t+1} \tag{2.0.4}$$

and so does the value of bank assets. As such, the bank's net cash-flow in the subsequent period, which is also its new capital, is given as follows:

$$n_{t+1} = \mathcal{H}\Bigg(max\Big(0, \underbrace{(Z_{t+1} + 1 - \delta)\psi_{t+1}s_t}_{\text{inflow}} - \underbrace{R_t d_t}_{\text{outflow}}\Big)\Bigg) \tag{2.0.5}$$

The $\mathcal{H}$ operator represents the bank's technology, which I assume exhibits decreasing returns to scale.[6] The max operator denotes the fact that when net cash flow is negative, the bank fails and exits the market. For brevity, let $(Z_{t+1} + 1 - \delta) = R^k_{t+1}$ denote the expected return on bank assets. Then the shock cutoff for bank failure equals:

$$\psi^*_{t+1} = \frac{R_t d_t}{R^k_{t+1} s_t}$$

A failed bank's assets are liquidated by the government, which covers any shortfall in its liabilities. Liquidated assets are purchased by other banks, including entrants.

Recursive formulation Given aggregate certainty, it is feasible to assert that all aggregates stay constant despite idiosyncratic uncertainty, and verify this assertion later. Accordingly, the recursive problem of a bank with capital n can be written as follows:

[6] The bank's technology $\mathcal{H}$ can be motivated based on the 'span of control' theory discussed in Lucas Jr [1978]. This theory points to the increasingly limited ability of a manager to operate the bank at larger scales. For banks, this motivation is particularly relevant because larger banks operate in more diversified and complicated financial markets, making the CEO's job harder, and therefore, potentially resulting in decreasing returns to scale.

$$
\begin{aligned}
V(n) = \max_{s,d,e} \quad & \left(e + \Lambda \int_{\psi^*} V(n') f(\psi') d\psi' \right) \\
n' &= \mathcal{H}\Big(R^k \psi' s - Rd\Big) \\
n + d &= s + e \\
0 \leq s &\leq \chi(n)(n - e) \\
0 \leq e &\leq n \\
0 &\leq d
\end{aligned}
$$

Here, V is the value function of the bank, and $f(.)$ is the density function of the quality shocks (with the corresponding cumulative distribution function $F(.)$). Note that the integral captures the fact that if ψ' is smaller than ψ^*, then the bank fails. We assume that bank owners have limited liability, so that they are allowed to simply walk away from the failed bank with zero continuation value. We also assume that bank cannot issue debt to pay dividends, which implies that dividends cannot exceed the bank's capital. In Appendix 7.1, I prove the following proposition.

Proposition 1. *If $\mathcal{H}$ is concave and bounded, then there exists a unique value function V that solves the bank's problem.*

The decision trade-offs the bank faces are as follows. A *ceteris paribus* increase in dividends e increases the current payoff for share holders, but reduces n' and thus decreases future payoff. A similar increase in leverage increases the expected future payoff, but also increases its variance. Moreover, the following proposition holds, which I prove in Appendix 7.2.

Proposition 2. *Bank failure probability is increasing in leverage.*

Given these trade-offs, we find that larger banks assume more leverage: $x'(n) \geq 0$. This is because for larger banks, variance in future cash-flows is less *costly* due to diminishing marginal returns. As such, the benefit of higher return that more leverage brings, dominates the downside from greater variance. Larger banks also pay higher dividends: $e'(n) \geq 0$. This is also because diminishing marginal returns. For larger banks, the value of retained earnings is smaller, and thus paying out more dividends is optimal.

Entrants Each period, there is an infinite mass of new banks that compete to enter the market. Entrants pay a fixed cost c^e to enter the industry, after which they obtain a randomly distributed seed capital $n^e \sim G(.)$ from the government. Expected value after entry is given as:

$$\mathbb{E}V^e = \int V(n^e) dG(n^e) \tag{2.0.6}$$

A potential entrant enters the market if and only if $\mathbb{E}V^e - c^e \geq 0$. However, free entry implies that the net value of entry cannot be positive, otherwise an infinite mass of entrants would enter the market, implying: $\mathbb{E}V^e - c^e \leq 0$. Together, these conditions imply that if the mass M_t of entrants is strictly positive, it must be that $\mathbb{E}V^e = c^e$.

Distribution of bank capital In the recursive formulation of the bank's problem, let $n \in \mathcal{N} = [0, \bar{n}]$ be the state space, where $\overline{n}$ be an upper bound of $\mathcal{N}$ (see Appendix 7.1). Also let $(s, d) : \mathcal{N} \to \mathcal{R}_+^2$ be the policy functions. Let the distribution of bank capital in the *middle* of a period – i.e. after bank entry and exit have occurred but before period decisions have been made – be given by μ_t. Specifically, let $\mu_t(N)$ denote the date-t *mass* of banks with capital $0 \le n \le N$. Then the evolution of μ_t is given as follows:

$$\mu_{t+1}(N) = M_{t+1} \int_0^N dG(n^e) + \int_0^{\bar{n}} \underbrace{\left(\int_{\underline{\psi}}^{\overline{\psi}} \mathbb{1}\Big[0 \le H\Big(R^k \psi' s(n) - Rd(n)\Big) \le N\Big] f(\psi') d\psi' \right)}_{W(n,N)} d\mu_t(n)$$

The first term captures the mass M_{t+1} of entrant banks that enter during period $(t+1)$ and have start-up capital less then N. The second term represents the flow of incumbent banks into the $[0, N]$ subset of the state space, net of those which fail. As shown, the integrand can be denoted concisely using a transition function $W(n, N)$ that denotes the probability of transition from point n on date t to subset $[0, N]$ on date $t+1$. Formally, it maps $\mathcal{N} \times \mathcal{B}(\mathcal{N}) \to \mathcal{R}_+$, where $\mathcal{B}(\mathcal{N})$ is the set of Borel subsets of $\mathcal{N}$.

I will later establish the existence of an invariant distribution of bank capital. For that, it is useful to express the evolution of bank capital distribution in terms of an operator T: $\mu_{t+1} = T(\mu_t, M_{t+1})$, and show that T admits the following property (see Appendix 7.3 for a proof).

Proposition 3. *T is linearly homogeneous in (μ, M), that is:*

$$\mu_M = T(\mu_M, M) \implies M'\mu_M/M = T(M'\mu_M/M, M').$$

The key to linear homogeneity is that the failure rate of banks does not change with the mass of entrants. As such, when the mass of entrants increases for a given steady state distribution, the mass of incumbents must increase in a way that the mass of failing banks matches the mass of entrants.

3 Stationary Equilibrium

Since there is no aggregate uncertainty, an equilibrium where aggregates such as total output, total physical capital stock, and bank capital distribution are time-invariant, can exist. I refer to such an equilibrium as the stationary equilibrium (SE), define it below, and later show that it exists.

Definition An SE consists of (i) bank value function $V(n)$, (ii) bank policy functions $s(n), d(n), e(n)$, (iii) bank capital distribution $\mu(n)$, (iv) entrant mass M, (v) aggregates namely consumption C, physical capital K, deposits D, labor L, investment I, Taxes T, and bank dividends E, (vi) prices namely wages W, interest rate R, and firm dividend rate Z, and (vii) the stochastic discount factor Λ such that:

1. $V(n), s(n), d(n)$ and $e(n)$ solve the bank's problem given prices and Λ;
2. Free entry condition is satisfied: $EV^e \leq c^e$; with equality if $M > 0$;
3. Labor market clears at wage rate W: $L = 1$;
4. Deposit market clears at interest rate R:

$$\int_0^{\bar{n}} d(n) d\mu(n) = D;$$

5. Capital market clears:

$$\int_0^{\bar{n}} \left(\int_{\underline{\psi}}^{\overline{\psi}} \psi' f(\psi') d\psi' \right) s(n) d\mu(n) = K;$$

6. Goods market clears:

$$\text{Output: } Y = K^\alpha L^{1-\alpha} = C + I + Mc^e + c^o;$$

$$\text{Consumption: } C = E + W + (R-1)D - O;$$

$$\text{Dividends: } E = \int_0^{\bar{n}} e(n) d\mu(n);$$

$$\text{Investment: } I = S - (1-\delta)K;$$

$$\text{Bank Assets: } S = \int_0^{\bar{n}} s(n) d\mu(n)$$

   and where c^o is the loss in output due to bank managers' limited span of control:

$$c^o = \int_0^{\bar{n}} \left(\int_{\underline{\psi}}^{\overline{\psi}} \left[\Big(R^k \psi' s(n) - Rd(n) \Big) - \mathcal{H}\Big(R^k \psi' s(n) - Rd(n) \Big) \right] f(\psi') d\psi' \right) d\mu(n);$$

7. The distribution of bank capital is the unique fixed point of the distribution evolution operator T given entrant mass M:

$$\mu = T(\mu, M)$$

8. Government's budget constraint is satisfied:

$$O = Mc^e + M \int_0^{\bar{n}} dG(n^e) + \int_0^{\bar{n}} \left(\int_{\underline{\psi}}^{\overline{\psi}} \Big(min\Big(0, R^k \psi' s - Rd\Big) \Big) f(\psi') d\psi' \right) d\mu(n)$$

   where the first term represents the cost of entry that the government bears on behalf of the entrants, the second term represents the total seed funding the government provides to the entrants, and the last term represents the shortfall in liabilities of failed banks that the government covers.

Parameters (value set individually)	Symbol	Value
Discount factor	β	0.96
Capital income share	α	0.36
Capital depreciation rate	δ	0.025
Benchmark regulation	$\hat{\chi}$	33
Expected value of quality shocks	μ_ψ	1
Expected value of seed funding	μ_n	1
Parameters (value set jointly using MM)	**Symbol**	**Value**
Span of control	A	47.0361
Span of control	B	-3.0635
S.t.d. of quality shocks	σ_ψ	0.0492
S.t.d. of seed funding	σ_n	0.4238
Cost of entry	c^e	8.2630
Moments	**Data**	**Model**
Interest income to asset ratio	0.0630	0.0530
Failure rate	0.0020	0.0310
Leverage	9.0909	10.301
Consumption to output ratio	0.6973	0.8387
KS distance between observed and model distribution	0	0.0309

Table 1: Summary of parameter values (first two blocks), and a comparison of data moments and model implies moments (third block). KS = Kolmogorov-Smirnov (KS) distance between the model and data implied cumulative distribution functions.

Solution strategy I show that an SE with positive entry exists by first solving for such an SE and then verifying that the equilibrium conditions are satisfied. The first step is to solve the bank's problem, for which I need R^*, Λ^*, and Z^*. R^* can be pinned down from the household's first-order conditions, which imply that $R^* = 1/\beta$ since C is time-invariant in an SE. This also implies that $\Lambda^* = \beta$. To find Z^*, I use the free entry condition (2.0.6). Basically, I assume a value of Z^*, use it to solve the bank's problem, and check if the free entry condition is satisfied. If not, I adjust Z^* and repeat the previous steps. The following proposition, which is proved in Appendix 7.4, ensures that this approach converges to the equilibrium Z^*.

Proposition 4. *Given an entry cost $c^e > \mathbb{E}[n^e]$, equilibrium firm dividend rate Z^* is uniquely determined.*

Once Z^* is known, I solve the bank's problem and obtain the policy functions. Then, to compute K^*, I proceed as follows. First note from the firm's first order condition, labor demand L^* is given as:

$$L^* = \left(\frac{(1-\alpha)}{W^*}\right)^{1/\alpha} K^* \tag{3.0.7}$$

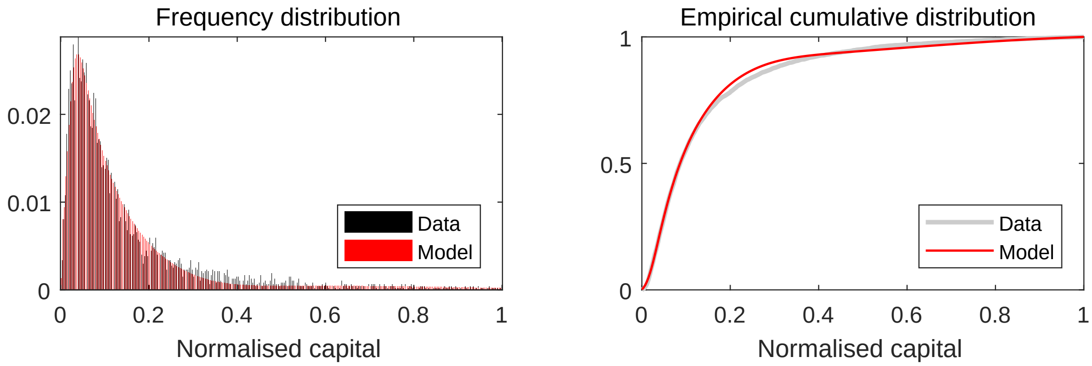

Figure 3: Distribution of bank capital.

Therefore, firm dividends are given as:

$$Z^* = \frac{Y^* - W^* L^*}{K^*} = \frac{\alpha Y^*}{K^*} = \alpha \left(\frac{L^*}{K^*}\right)^{1-\alpha} \implies K^* = \left(\frac{\alpha}{Z^*}\right)^{1/1-\alpha} L^* \tag{3.0.8}$$

Since the labor market clearing condition implies $L^* = 1$, K^* is obtained using (3.0.8), and subsequently, W^* is calculated using (3.0.7).

Next, I compute the invariant distribution μ and the entrant mass M. To calculate the equilibrium distribution μ^* and equilibrium entrant mass M^*, I use the capital market clearing condition as follows. I 'scale' μ^* such that aggregate bank assets corresponding to the 'scaled' distribution sum up to the total physical capital K^*. The scaling factor equals M^*/M, precisely because of the linear homogeneity property of μ. Intuitively, if there are too many (few) entrants in the equilibrium, mass of incumbents is proportionally larger (smaller), and so is the aggregate amount of bank shares relative to K^*. Since the aggregate amount of bank shares is monotonically increasing in the mass of entrants, it follows from the Intermediate Value Theorem that there does exist a mass of entrants that clears the capital market. Finally, assuming $\mathbb{E}\psi = 1$ implies $I^* = \delta K^*$. Subsequently, I compute all the equilibrium quantities using the conditions listed in the definition of SE, except the goods market clearing condition. By Walras' law, this condition is not needed when solving the equilibrium quantities – as such, it offers a suitable test of the validity of the solution obtained.

4 Calibration

I divide the model parameters into two sets. For parameters in the first block in Table 1, I set their values individually based on the literature. The discount factor β is set such that annual risk free interest rate is close to 4%. Firm production function parameter α is set such that the income share of physical capital equals 0.36. I set the depreciation rate δ to a value of 0.025, which is common in the macroeconomic literature. I set the benchmark

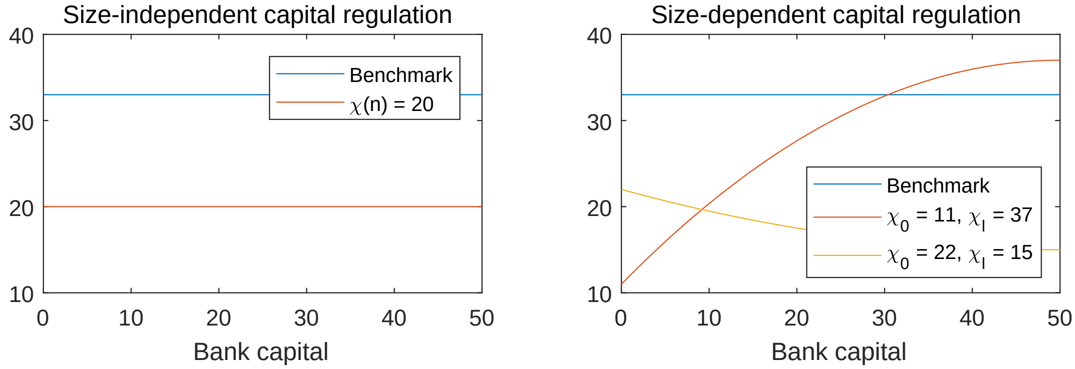

Figure 4: Examples of capital regulation profiles considered.

capital regulation as $\chi(n) \equiv \hat{\chi} = 33$, in line with the Basel III minimum leverage-ratio requirement of 3%.

With regards to functions used in the model, I assume that households have a log utility, and that the span of control $\mathcal{H}$ has the following form: $\mathcal{H}(n) = A(1 - e^{-n/(A+B)})$. The capital quality shock is assumed to be normally distributed with mean $\mu_\psi = 1$ and standard deviation σ_ψ. The distribution of seed capital for entrants is assumed to be log-normally distributed with mean $\mu_n = 1$ and standard deviation σ_n.[7] The unknown parameters herein, along with c^e, are denoted by $\mathcal{P} = (A, B, \sigma_\psi, \sigma_n, c^e)$ (second block in Table 1), and are calibrated jointly using the method of moments (MM).

For the MM, I begin with a guess for $\mathcal{P}$, and compute the corresponding SE. Based on this SE, I compute the values of the target moments listed in the third block in Table 1, and compare with their estimates in data. The idea is to find a set of parameters $\mathcal{P}^*$ that minimizes the Euclidean distance between model and data moment vectors.[8] Note that the fifth 'moment' nests several of them since the target is a cumulative distribution function. In this sense, the estimation of parameters in $\mathcal{P}$ is over-identified, and the model moments may not exactly match the data moments.

Empirical estimates of the moments are based on data on US commercial and savings banks. This data is sourced from: (a) the FDIC Call and Thrift Financial (CTR) reports, (b) the FDIC failed banks list, (c) FDIC Historical Statistics on Banking (HSOB) reports, and (d) S&P Global SNL database. In addition, I source US macroeconomic aggregates from the Federal Reserve Economic Database (FRED). As motivated in the introduction, I focus on 1995-2007 as the calibration period, while assuming that one period in the model corresponds to an year in the data. For interest-income to asset ratio, failure rate, and leverage, I compute the mean during 1995-2007, while for the ratio of personal consumption expenditure to GDP, and for the distribution of bank capital, I use data as of the start and end of 2007, respectively.

[7] Normalizing μ_ψ and μ_n to unity is a simplifying assumption. While the former would simply scale Z^*, the latter would scale the distribution.

[8] For this part, I use the Nelder-Mead simplex optimization algorithm which is available via the *fminsearch* function in MATLAB.

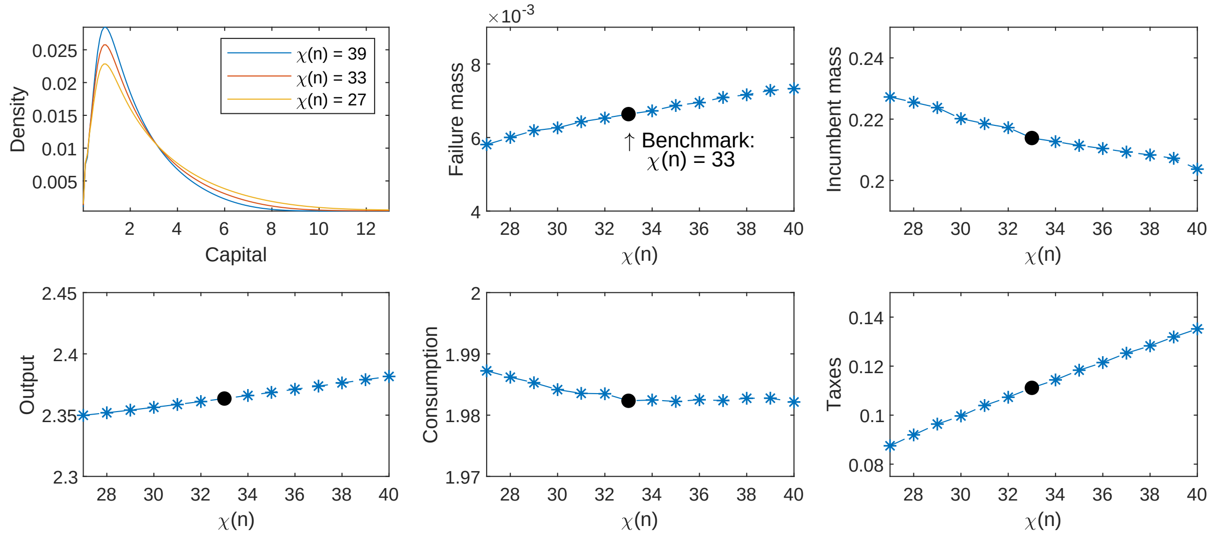

Figure 5: Response in bank capital distribution and macroeconomic aggregates due to changes in size-independent regulation.

Numerical solution Since analytical solution of the bank's problem is not possible given the heterogeneity involved, I use numerical methods. For the bank's problem, I use value function iteration. The state space for n has 576 log-spaced grid points in $[0.01, 50]$. I use linear interpolation to evaluate the value function at off-grid points. To obtain the invariant distribution of bank capital, I construct the transition matrix using the banks' policy functions, and compute the corresponding ergodic distribution. To handle entry and exit, I introduce a *dump* state in the transition matrix – a failing bank would enter the dump state from the actual state space, while an entrant would leave the dump state to enter the actual state space. This approach works precisely because in the stationary equilibrium, the mass of entrants equals the mass of failures, implying that the mass of banks in the dump state equals the mass of entrants (or failures). In Figure 3, I compare the model generated capital distribution with that in data. The comparison shows that the model is able to closely match the distribution observed in data. Other aspects of the numerical solution, such as the banks' value and policy functions, are presented in Appendix 7.5.

5 Optimal capital regulation

The rationale for capital regulation in this model is deposit insurance. Deposit insurance drives down the cost of deposit based funding for banks, and induces them to take more leverage than what is socially optimal. Thus far, I took capital regulation to be given exogenously, and to be the same for banks of different sizes. In this section, I characterize the capital regulation that maximizes social welfare, while considering both size-independent and size-dependent policies. I restrict attention to continuous, differentiable and monotonic functions $\chi(n)$ that satisfy the following conditions: $\chi(n) \geq 1, \lim_{n\to\overline{n}} \chi'(n) = 0$.

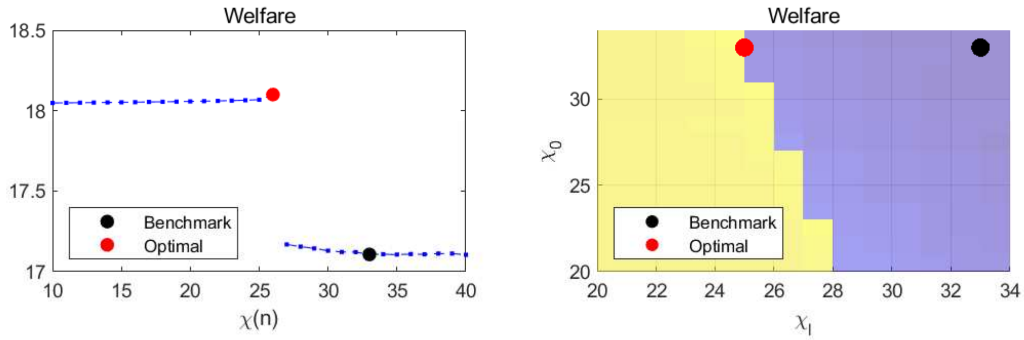

Figure 6: For size-independent policy, I consider a one-dimensional grid for the level of regulation (left-hand panel). For size-dependent policy, I consider a two-dimensional grid for (χ_0, χ_l) (center panel). Each point on this grid represents a specific capital regulation $\chi(n)$ as per equation (5.0.9).

Given that leverage cannot be less than 1, the first condition rules out absurd regulations. The second condition ensures that regulation does not diverge to $+\infty$ or $-\infty$.

For size-dependent regulation, I assume that $\chi(n)$ is of the quadratic form: $\chi(n) = \chi_0 + \chi_1 n + \chi_2 n^2$.[9] Note that although a quadratic function is generally described by three parameters, the limiting condition on the slope of $\chi(n)$ at $\overline{n}$ bring down the number of free parameters by one. Thus, $\chi(n)$ can be expressed in terms of regulation χ_0 for the smallest banks, and regulation χ_l for the limiting size banks, as follows:

$$\chi(n) = (\chi_0 - \chi_l)\Big(n/\overline{n}\Big)^2 - 2(\chi_0 - \chi_l)\Big(n/\overline{n}\Big) + \chi_0. \tag{5.0.9}$$

Examples of valid regulation functions $\chi(n)$ are given in Figure 4. Before posing the optimal regulation problem, to gain intuition, I discuss how banks in particular and the economy in general respond to changes in regulation around the benchmark.

Bank policy choices and capital distribution: As capital regulation tightens, overall, banks shrink their assets and liabilities. At the same time, they pay lower dividends to households. The distribution of bank capital shifts to the right (see Figure 5, top left-hand panel). This is driven in part by the lower failure rate (top center panel), which follows from the lower leverage banks assume. A lower failure rate drives up the mass of incumbent banks (top right-hand panel).

Macroeconomic aggregates: As capital regulation tightens, overall, banks are able to intermediate less. The physical capital stock shrinks, and so does total output (bottom left-hand panel). Yet, household consumption increases (bottom center panel). This in-

[9] I rule out more complicated regulation functions as optimizing over a richer class of of functions is a higher dimensional problem that is more difficult to handle computationally. Moreover, the choice of the quadratic form is not restrictive. This is because the Weierstrass Approximation Theorem implies that quadratic functions are *dense* in the space of bounded, differentiable and continuous functions on a closed set.

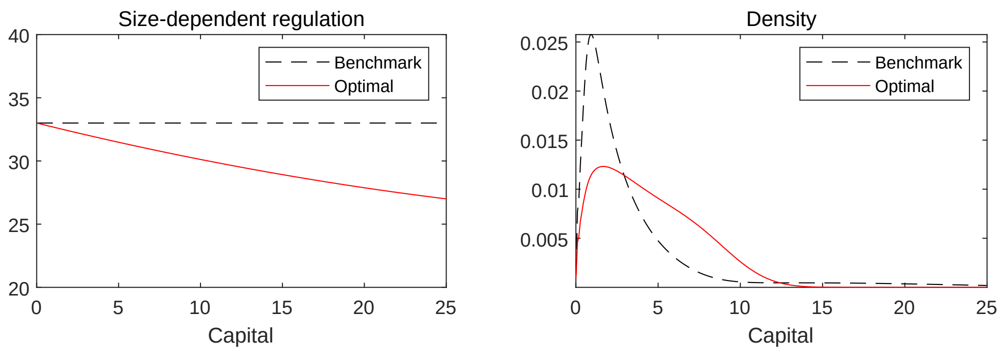

Figure 7: Optimal regulation and corresponding bank capital distribution.

crease is primarily driven by the lower need for funding entrants (there is less entry and exit, as discussed above), and the lower shortfall in the liabilities of failing banks – both of which reduce the tax bill on households (bottom right-hand panel).

Welfare I assume that the regulator is benevolent, and cares about the household's welfare in the stationary equilibrium. As such, the regulator chooses $\chi(n)$ in order to maximize the lifetime utility of the representative household:

$$\max_{\chi(n)} \frac{u(C)}{(1-\beta)}$$

Size-independent regulation: The optimal is tighter relative to the benchmark. While maximising welfare on a grid of potential capital requirements, I find that welfare is maximized when $\chi(n) = 26$ (see Figure 6, left-hand panel).[10] While not directly comparable, this result points in the same direction (which is that of tightening) as that proposed in other studies, including Admati and Hellwig [2014], Begenau [2016], and Nguyen [2014].

Size-dependent regulation: The optimal $\chi(n)$ in this case imposes a capital requirement of $\chi_0^* = 33$ for the smallest banks and of $\chi_l^* = 25$ for the largest banks, while varying monotonically for banks of intermediate sizes. Welfare as a function of regulation is shown in Figure 6 (right-hand panel), while the optimal regulation is plotted in the left-hand panel in Figure 7. This result reinforces the higher capital surcharges for larger, systemically important banks *a'la* the Basel III GSIB framework. In Figure 7, right-hand panel, I document the capital distribution under the optimal (size-dependent) capital regulation regime. The key insight is that optimal regulation leads to a larger mass of better capitalized banks, while the tail of the distribution is lighter. Below I provide intuition for the channels through which size-dependent capital regulation generates welfare gains.

[10] I work with a coarse instead of a fine grid because of computational constraints. The discontinuity in the welfare profile follows from the fact that when regulation tightens beyond a certain point, there is a substantial shift in the bindingness of the regulatory constraint for banks. Specifically, at this point, regulation induces a large fraction of the banks to delever substantially.

Channels of welfare gain The intuition for why the optimum regulation is tighter relative to the benchmark, is two-fold. First, the dead-weight costs associated with industry turnover is lower as tighter regulation leads to less levered banks that fail less often and necessitate lower entry. This *saving* more than compensates for the lower bank intermediation and thus lower overall output that tighter regulation entails. Second, tighter regulation increases the mass of banks in the stationary equilibrium, and also pushes the distribution of bank capital to the right. As a result, even though each bank is more constrained, there are more banks, and with more capital on an average.

The intuition for why the optimum regulation is tighter for large banks relative to small banks, is three-fold. First, by allowing the small banks to assume more leverage, they fail more often. However, the mean shock that fails these banks is lower compared to the mean shock entrant banks receive, which is unity. Formally, the mean shock of failed banks is given as:

$$\int_0^{\overline{n}} \left(\int_{\underline{\psi}}^{\psi^*(n)} \psi' f(\psi') d\psi' \right) d\mu(n) \Bigg/ \int_0^{\overline{n}} \left(\int_{\underline{\psi}}^{\psi^*(n)} f(\psi') d\psi' \right) d\mu(n) \tag{5.0.10}$$

Assuming that bank failure probability is less than half for all banks – a reasonable assumption given the calibration – implies that $\psi^*(n) < 1 \; \forall n$, which then leads to the claim given (5.0.10). In this sense, there is a *cleansing* of the less productive banks. Second, by allowing the smaller banks to take on more leverage, the regulator allows them to grow faster *conditional* on survival. Naturally, more small banks end up failing this way, but their failure is less costly. Finally, large banks are not born as such – smaller banks become large by growing over time, but in the process, incur a dead-weight loss in each period due to the limited managerial span of control. As such, large bank failures are more costly for tax-payers, and tighter regulation that reduces their probability of failure generates welfare gains.

Welfare gain in consumption equivalent terms The gain in welfare from imposing regulation can be expressed in terms of consumption equivalence. Let A denote the benchmark regime and let B denote the optimal regime. Then welfare gain in consumption equivalent (CE) units is defined as the fractional increase ω_U in consumption that the household would receive should it live in the optimal regime forever:

$$\frac{u\Big((1+\omega_U)C^A\Big)}{(1-\beta)} = \frac{u\Big(C^B\Big)}{(1-\beta)}$$

The lifetime welfare gain in CE terms from the optimal size-independent regulation is 4.05%, while that from the optimal size-dependent regulation is 4.12%. To gauge these magnitudes, I compare them to the estimated lifetime cost of the Great Financial Crisis (GFC) which, according to Luttrell et al. [2013] for instance, is in the range of $40 - 90\%$ of US GDP. Assuming that capital regulation can play a role in avoiding crises, a back of the envelope calculation suggests that capital requirements can reduce the lifetime cost of future crises by somewhere between $4 - 10\%$.

6 Conclusion

I provide a general equilibrium framework for studying the response of a dynamic and heterogeneous banking industry to changes in regulation. A key feature of the framework is that industry dynamics such the entry rate, the exit rate, and the size distribution of banks are determined endogenously. Despite being tractable, I show that the calibrated version of the model is able to match closely the observed distribution of bank capital in the US.

Another key feature of the framework is that there is an explicit rationale for capital regulation, which allows for a meaningful welfare analysis, and permits a characterization of optimal regulation, including size-dependent regulation. As such, the model provides a basis for the appraisal of size-dependent policies, such as the GSIB framework that imposes tighter regulation on large and systemically important banks. Quantitatively, the results suggest that there may be room for a general tightening of capital requirements relative to the levels envisioned under Basel III. This is in line with empirical research reported by Fender and Lewrick [2016].

The paper contributes to the literature by being one of the few papers to develop a model of a heterogeneous banking industry in order to study size-dependent policy. In addition, as opposed to the firm dynamics literature where the dynamics is generally driven by an exogenous process, this paper introduces leverage – which is central to banks' business models – as the key driver.

The model can be useful for addressing important questions of theoretical as well as policy interest. For one, it is possible to allow banks to choose the riskiness of its assets – this would not only make the bank's problem more realistic, but also facilitate the study of an interplay between risk-insensitive and risk-sensitive regulations, such as leverage and value-at-risk constraints, respectively. It is also possible to study anti-trust policies in this framework, and relatedly, characterize the optimal size-distribution of banks from a competition as well as sectoral efficiency perspective. This, I believe, is a fundamental question, one that has become particularly relevant for policy-makers in current times as the financial sector is becoming more concentrated in certain dimensions, not least because of the advent of financial-technology (FinTech) companies.

References

Admati, A. and M. Hellwig (2014). *The Bankers' New Clothes: What's Wrong with Banking and What to Do about It.* Princeton University Press.

Adrian, T. and N. Boyarchenko (2012). Intermediary leverage cycles and financial stability. *Becker Friedman Institute for Research in Economics Working Paper* (2012-010).

Albuquerque, R. and H. A. Hopenhayn (2004). Optimal lending contracts and firm dynamics. *The Review of Economic Studies 71*(2), 285–315.

BCBS (2011). Basel 3: A global regulatory framework for more resilient banks and banking systems. *Basel Committee for Banking Supervision.*

Begenau, J. (2016). Capital requirement, risk choice, and liquidity provision in a business cycle model. *Stanford University Working Paper*.

Benes, J. and M. Kumhof (2011). Risky bank lending and optimal capital adequacy regulation. Technical report.

Boissay, F. and F. Collard (2016). Macroeconomics of bank capital and liquidity regulations. *BIS Working Paper 596*.

Christensen, I., C. Meh, and K. Moran (2011). Bank leverage regulation and macroeconomic dynamic. *CIRANO-Scientific Publications 2011s-76*.

Christiano, L. and D. Ikeda (2013). Leverage restrictions in a business cycle model. Technical report, National Bureau of Economic Research.

Clementi, G. L. and H. A. Hopenhayn (2006). A theory of financing constraints and firm dynamics. *The Quarterly Journal of Economics*, 229–265.

Clementi, G. L. and B. Palazzo (2015). Entry, exit, firm dynamics, and aggregate fluctuations. Technical report, National Bureau of Economic Research.

Cooley, T. F. and V. Quadrini (2001). Financial markets and firm dynamics. *American Economic Review*, 1286–1310.

Corbae, D. and P. D'erasmo (2014). Capital requirements in a quantitative model of banking industry dynamics.

Covas, F. and J. C. Driscoll (2014). Bank liquidity and capital regulation in general equilibrium.

De Nicolo, G., A. Gamba, and M. Lucchetta (2014). Microprudential regulation in a dynamic model of banking. *Review of Financial Studies 27*(7), 2097–2138.

DeAngelo, H. and R. M. Stulz (2013). Why high leverage is optimal for banks. Technical report, National Bureau of Economic Research.

Fender, I. and U. Lewrick (2016). Adding it all up: the macroeconomic impact of basel iii and outstanding reform issues. *BIS Working Papers, no 591*.

Gertler, M. and P. Karadi (2011). A model of unconventional monetary policy. *Journal of Monetary Economics 58*(1), 17 – 34.

Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of Monetary Economics 3*, 547.

Hanson, S., A. Kashyap, and J. Stein (2010). A macroprudential approach to financial regulation. *Chicago Booth Research Paper* (10-29).

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, 1127–1150.

Hughes, J. P. and L. J. Mester (2013). Who said large banks do not experience scale economies? evidence from a risk-return-driven cost function. *Journal of Financial Intermediation 22*(4), 559–585.

Jayaratne, J. and P. E. Strahan (1997). The benefits of branching deregulation. *Economic Policy Review 3*(4).

Kalemli-Ozcan, S., B. Sorensen, and S. Yesiltas (2012). Leverage across firms, banks, and countries. *Journal of international Economics 88*(2), 284–298.

Lang, L., E. Ofek, and R. Stulz (1996). Leverage, investment, and firm growth. *Journal of financial Economics 40*(1), 3–29.

Lucas Jr, R. E. (1978). On the size distribution of business firms. *The Bell Journal of Economics*, 508–523.

Luttrell, D., T. Atkinson, and H. Rosenblum (2013). Assessing the costs and consequences of the 2007–09 financial crisis and its aftermath. *Economic Letter 8*.

Miao, J. (2005). Optimal capital structure and industry dynamics. *The Journal of finance 60*(6), 2621–2659.

Nguyen, T. T. (2014). Bank capital requirements: A quantitative analysis. *Available at SSRN 2356043*.

Stokey, N. L. (1989). *Recursive methods in economic dynamics*. Harvard University Press.

Van den Heuvel, S. J. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics 55*(2), 298–320.

Zhu, H. (2007). Capital regulation and banks' financial decisions. *BIS Working Paper*.

7 Appendix

7.1 Proof of proposition 1

Proof. I follow Stokey [1989] to prove that a unique value function exists. I reproduce the bank's problem below, while assuming that the economy is already in a stationary equilibrium so that $\Lambda = \beta$, $R = 1/\beta$, and $R^k = Z + 1 - \delta$ are time-invariant.

$$V(n) = \max_{s,d,e} \quad \left(e + \beta \int_{\psi^*} V(n')f(\psi')d\psi'\right)$$

$$n' = H\Big(R^k \psi' s - Rd\Big)$$

$$n + d = s + e$$

$$0 \leq s \leq \chi(n)(n-e); \qquad 0 \leq e \leq n; \qquad 0 \leq d.$$

I begin by noting that when the reward function in a Bellman equation is unbounded, existence of a fixed point of the Bellman operator is not guaranteed. The bank's problem above is subject to this issue because the dividend payment e is unbounded *per-se*. However, if I can show that the state space is bounded, then since dividends cannot exceed capital ($e \leq n$), the reward function would, *in effect*, be bounded. This can be achieved by assuming that H is bounded above by, say $\overline{n}$, that is, $H(n) \leq \overline{n}$, $\forall\, n \geq 0$.

Next, I verify the Blackwell sufficient conditions (BSC) to conclude that the Bellman operator is a contraction mapping. To this end, let $\mathcal{N}$ be the state space, let $\mathcal{C}$ be the class of continuous functions on $\mathcal{N}$, and define the Bellman operator for $q \in \mathcal{C}$ as:

$$\mathcal{Q}(q(n)) = \max_{s,d,e} \quad \left(e + \beta \int_{\psi^*} q(n')f(\psi')d\psi'\right)$$

subject to the constraints listed above. The first BSC is regarding the *Monotonicity* property of $\mathcal{Q}$:

$$\forall\, l, q \in \mathcal{C} \text{ s.t. } l(x) \leq q(x)\ \forall x \in \mathcal{N} \implies \mathcal{Q}(l(x)) \leq \mathcal{Q}(q(x))\ \forall x \in \mathcal{N}$$

To show this, consider $l(x) \leq q(x)\ \forall x \in \chi$. Let (s^l, d^l, e^l) and (s^q, d^q, e^q) be the maximands of $\mathcal{Q}(l(n))$ and $\mathcal{Q}(q(n))$ respectively for $n \in \mathcal{N}$. Since the feasibility set for the maximands does not depend on l or q, (s^l, d^l, e^l) is a feasible point for the Bellman operator on q at n. But this implies that:

$$\mathcal{Q}(l(n)) = \left(e + \beta \int_{\psi^*} l(n')f(\psi')d\psi'\right)\Bigg|_{\text{at } (s^l,d^l,e^l)} \leq \left(e + \beta \int_{\psi^*} q(n')f(\psi')d\psi'\right)\Bigg|_{\text{at } (s^l,d^l,e^l)} \leq$$

$$\left(e + \beta \int_{\psi^*} q(n')f(\psi')d\psi'\right)\Bigg|_{\text{at } (s^q,d^q,e^q)} = \mathcal{Q}(q(n))$$

The second BSC is regarding the *Discounting* property of $\mathcal{Q}$:

$$\exists \Delta \in (0,1) \text{ s.t. } \forall q \in \mathcal{C},\ \forall a \geq 0,\ \forall x \in \mathcal{N} \implies \mathcal{Q}(q(x)+a) \leq \mathcal{Q}(q(x)) + \Delta a$$

The left-hand-side of the last inequality is given as:

$$\mathcal{Q}(q(x)+a) = \max_{s,d,e} \quad \left(e + \beta \int_{\psi^*} \Big(q(n') + a\Big) f(\psi')d\psi'\right)$$

$$\leq \mathcal{Q}(q(x)) + \max_{s,d,e} \quad \left(e + \beta \int_{\psi^*} af(\psi')d\psi'\right)$$

The inequality follows from the fact that a maximization of the sum of two or more functions cannot be greater than the sum of the maximized values of the individual functions (Principle of the Maximum). In the second maximization, choosing $e = 0, d = 0, s = n$, which is a feasible choice, results in $\psi^* = 0$, which in turn implies that the second term is equal to βa. Since $0 < \beta < 1$, this verifies the second BSC.

The verification of the BSCs implies that there exists a unique fixed point of the Bellman operator. As such, I can use value function iteration to obtain an arbitrarily close estimate of the value function. ■

7.2 Proof of proposition 2

Proof. The failure probability of a bank with capital n is given as:

$$p(n) = \int_{\underline{\psi}}^{\psi^*} f(\psi')d\psi'$$

The failure cutoff $\psi^* = Rd/R^k s = R(s+e-n)/R^k s = R/R^k - R(n-e)/R^k s = R/R^k(1 - 1/x)$. It then follows trivially that the cutoff is increasing in leverage x. ■

7.3 Proof of proposition 3

Proof. Let μ_M be the stationary distribution corresponding to $M : \mu_M = T(\mu_M, M)$. Then,

$$\mu_M(N) = M \int_0^N dG(n_e) + \int_0^{\bar{n}} W(n,N)d\mu_M(n)$$

Multiplying both sides by M'/M gives the following

$$\mu_M(N)\frac{M'}{M} = M' \int_0^N dG(n_e) + \int_0^{\bar{n}} W(n,N)d\mu_M(n)\frac{M'}{M}$$

Interpreting $\left[\frac{M'\mu_M}{M}\right]$ as the new measure implies the proposition. ■

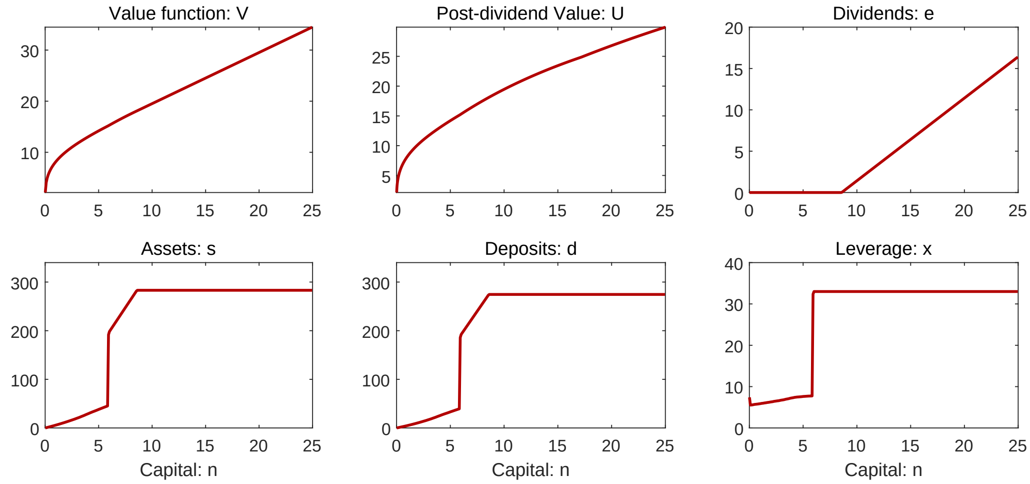

Figure 8: Bank value and policy functions.

7.4 Proof of proposition 4

Proof. An application of the envelope theorem to the Bellman equation in the bank's problem implies that $V(n;Z)$ is strictly increasing in Z. Define $g(Z)$ as:

$$g(Z) = \int_0^{\overline{n}} V(n^e;Z)dG(n^e)$$

which is the expected value of entry as a function of Z. Then

$$\frac{\partial V(n;Z)}{\partial Z} > 0 \implies \frac{dg(Z)}{dZ} > 0.$$

Now,

$$Z \to 0 \implies V(n^e;Z) \to n^e \implies g(Z) \to \int_0^{\overline{n}} n^e dG(n^e) = \mathbb{E}[n^e].$$

In addition, $Z \to \infty \implies g(Z) \to \infty$. Then, by the intermediate value theorem $\exists\ Z^* \ni g(Z^*) = c^e$. ■

7.5 Bank value and policy functions

In Figure 8, I plot the banks' value and policy functions in terms of their capital n. The value function V is (weakly) concave and increasing in n. The post-dividend value function U, defined as the optimized value of the bank after dividends have been paid but before asset and liabilities have been chosen, is (strictly) concave and increasing in n. Dividends exhibit a phase shift: when n is low, banks do not pay any dividend, while pay an increasingly larger fraction of their capital as dividends thereafter. Assets, deposits, and leverage are increasing in n, and also exhibit phase shifts: when n is low, banks maintain a leverage that is below the maximum allowed limit, but when n is high, banks operate at the maximum limit.