Bond Finance, Bank Finance, and Bank Regulation

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Abstract

A dynamic general equilibrium model of bank regulation that omits bond financing is imprecise because such a model prevents firms from raising credit via alternative channels, and thus artificially lowers the price elasticity of demand for bank loans. In this paper, I build a continuous-time macrofinance model in which firms can use both bond credit and bank credit. Risky firms appreciate bank credit because banks are efficient at liquidating assets for troubled firms. However, risky firms must pay a risk premium for banks’ exposure to aggregate risks. This paper shows that a model that does not allow for bond financing overestimates both the welfare benefits of tightening bank capital requirements and the rate at which the banking sector recovers after a recession. In addition, I show that the optimal bank regulation highly depends on the efficiency of the bankruptcy procedure in an economy and the risk profile of its real sector.

Keywords: bank credit, bond credit, capital requirement, and macro-prudential regulation

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Introduction

Like bank loans, bond finance is an important source of external credit for firms. For instance, during the 2007-2009 financial crisis when the supply of bank loans declined substantially, firms, especially those with relatively high credit ratings, largely substituted bank credit with bond credit (Adrian et al., 2012). Nevertheless, the implication of direct bond finance for optimal bank regulation in dynamic general equilibrium frameworks has rarely been acknowledged in the literature, even though many papers have assessed the welfare-maximizing role of bank regulation in such frameworks (Van den Heuvel, 2008; Repullo and Suarez, 2012; Christiano and Ikeda, 2013; Martinez-Miera and Suarez, 2014; Nguyen, 2014; Derviz et al., 2015; Begeau, 2018; Elenev et al., 2018; Phelan, 2016; Davydiuk, 2017; Corbae and D’Erasmo, 2018; Mendicino et al., 2018; Pancost and Robatto, 2018). In this paper, I will show that a general equilibrium bank regulation model that omits the bond market generate imprecise results. Moreover, I highlight that the socially optimal level of the capital ratio requirement for banks largely depends on the efficiency of the bankruptcy system and the risk profile of the real sector in an economy because both factors affect the aggregate demand for bank credit.

I propose a continuous-time macro-finance framework with a productive expert sector, a less productive household sector, and an explicit banking sector. The production sector comprises safe firms and risky firms. Both types of firms can access the bond market and the loan market. The difference between bond finance and bank finance is that banks can liquidate troubled firms’ assets in a more efficient fashion (Bolton and Freixas, 2000). The net interest spread charged by banks compensate for their exposure to the aggregate risk that they assume via loan lending. Households can both hold corporate bonds directly and deposit their savings into banks.

In my framework, risky firms prefer bank credit while safe firms rely mainly on bond credit. Since banks can liquidate troubled firms’ assets in a more efficient way, banks request less compensation for bankruptcy costs relative to bondholders. The liquidation efficiency of bank credit is more important for risky firms than for safe firms because safe firms are less likely to face costly liquidation. This setting is consistent with empirical findings in Rauh and Sufi (2010) and Becker and Josephson (2016). Bank credit does not always dominate bond credit for risky firms. Since risky firms must pay bank a risk premium for the aggregate risk that banks are exposed to, risky firms will replace bank finance with bond finance when the risk premium increases. The risk premium in the model is the net interest spread earned by banks.

The net interest spread depends on the leverage of the intermediary sector, the aggregate risk of the economy, and the capital requirement faced by banks. Given the same amount of aggregate risk, banks with low leverage have low risk exposure. Therefore, the risk premium required by banks tends to be low. Hence, bank credit is relatively cheap when the banking sector has adequate equity capital. The capital ratio requirement also affects the net interest spread because a tightening of the capital requirement would lower the supply of bank loans. When there is excess demand for bank loans, the loan spread increases, as does the net interest spread earned by banks.

The impacts of exogenous aggregate shocks on the economy vary over time because the effects of financial amplification depend on the balance sheets of both banks and experts (Bernanke et al., 1999; Kiyotaki and Moore, 1997). Suppose a series of adverse shocks hit the economy. Both bank capital’s and productive experts’ net worth decline disproportionately due to their use of leverage. As a result, the supply of bank loans shrinks, leading to a decrease in experts’ holdings of assets, aggregate productivity, and asset prices.

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1 See Thakor (2014) for a review of the literature on the capital ratio requirement using microeconomic models of banking.
The depreciation of asset prices hurts balance sheets of both banks and experts, and further lowers the loan supply and experts’ holdings of assets. I label the effect of the financial amplification as endogenous risk.

The first key result of this paper concerns economic dynamics. In a model where the real sector does not issue bonds, the predicted recovery of the banking sector after a negative shock is overly swift. Suppose the banking sector shrinks due to some negative shock. The supply of bank loans declines, and the loan spread increases. If loans are the only source of external finance that can be accessed by the real sector, then the demand for bank loans is not very elastic. Hence, if the real sector cannot access bond financing, bank profitability can increase substantially due to a significant increase in the loan spread and a mild decline in loan origination. As a consequence, the banking sector recovers more quickly after adverse shocks in a model that omits bond financing than it would in a model with bond financing.

Bank regulation in my framework can improve social welfare because my model is subject to pecuniary externalities that are common in incomplete market models (Hart, 1975; Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986). In particular, experts and bankers in my model do not internalize the impact of their leverage decisions on asset prices and endogenous risks. Hence, bank regulation such as the capital ratio requirement can adjust bankers’ leverage, lower the loan supply, and raise the net interest spread. In this way, bank regulation can increase the profitability of banking and strengthen the banking sector to lower endogenous risks and improve social welfare.

The second key result of this paper is that a model that omits bond financing overemphasizes the benefit of bank capital requirements. The intuition is also related to the elasticity of the aggregate demand for bank loans. If capital requirement rises, there will be excess demand for bank loans. Thus, loan spread increases and loan demand declines. If the magnitude of the decline in loan demand is small enough, bank profitability could increase, and the banking sector can expand after accumulating more and more profit. A larger banking sector can contribute to the growth of the real sector as well as the increase in aggregate productivity. These are the ways in which tightening capital requirement improves social welfare. Consider two otherwise identical economies: one has a bond market and the other does not. Obviously, the aggregate demand for bank loans is much more elastic in the economy where firms can raise credit from the bond market. In this economy, when loan spread increases, the demand for bank loans declines more substantially, as does bank profitability. Therefore, tightening capital requirement is more likely to cause the banking sector to shrink, and social welfare to decline. Hence, the optimal capital ratio requirement should be more lenient if we consider a model that allows for bond financing.

The previous discussion shows that the loan spread elasticity of the demand for bank loans plays a crucial role in the welfare implication of capital requirement. In light of this property, I explore three factors that affect the elasticity of bank loan demand: the efficiency of the bankruptcy system in an economy, as well as the mean and skewness of the distribution of firms’ idiosyncratic default risks. The more efficiently bankruptcy cases are processed, the smaller the advantage of banks over bondholders in terms of liquidating insolvent firms’ assets. In an efficient bankruptcy system, bondholders enjoy higher recovery value ex post and request smaller premium ex ante. From the perspective of firms, replacing bank credit with bond credit is less costly, and thus firms’ demand for bank loans is more price elastic. Hence, tightening capital requirement can cause a substantial decline in bank loans, and a decrease in bank profits. Overall, the optimal capital requirement should be more lenient in an economy with a more efficient bankruptcy system.

The mean and skewness of firms’ default risk distribution also influence the elasticity of demand for bank loans. Since bondholders demand higher default premium for firms that are more likely to fail, riskier firms find it costly to switch from bank credit to bond credit. Hence, the demand for loans is less elastic if
firms in an economy tend to be risky. Subsequently, the optimal capital requirement should be tighter. The proportion of risky firms in an economy also matters. The greater the concentration of risky firms that rely mainly on bank credit, the less elastic the aggregate demand for loans. Hence, the rise in loan spread is less likely to cause a decline in bank profits. Again, the optimal capital requirement ought to be tighter in an economy with a high proportion of risky firms.

Related Literature. My paper is related to four strands of literature. First, I use a continuous-time macro-finance framework that emphasizes the financial amplification mechanism (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012; Di Tella, 2017). The major contribution of this paper is that I explicitly model a financial intermediary sector rather than grouping the real sector and financial intermediary sector together. With my proposed framework, I can explicitly analyze the macroeconomic implications of bank regulation. This framework highlights two layers of financial amplification — one at the firm level and the other at the intermediary level.

Second, since the 2007–2009 financial crisis, a number of papers have investigated the macro-prudential role of banking regulation in a dynamic general equilibrium framework (see, e.g., Begenau 2018; Elenev et al. 2018). Most of these papers are quantitative, and typically incorporate many ingredients, ranging from the liquidity premium of bank debt to the risk-shifting problem caused by either deposit insurance or implicit government guarantees. The framework proposed in this paper is rather simple as it is meant to highlight a feature that is currently missing in the literature, that is, the effectiveness of banking regulation highly depends on the elasticity of demand for bank loans, which in turn relies on the presence of the bond market.2 In my model, banking regulation mitigates pecuniary externalities and improves social welfare via the distributive effects emphasized by Dávila and Korinek (2017).

Thirdly, my paper contributes to a strand of macroeconomic literature that highlights the capital structure of firms (see, e.g., De Fiore and Uhlig 2011, 2015; Crouzet 2017). These papers model the surge in the cost of bank financing as an exogenous shock. Therefore, these papers are missing the rich characterizations of the dynamics of bank financing and bond financing that are captured in my paper. In this regard, my paper is similar to Rampini and Viswanathan (forthcoming), who also endogenize the cost of financial intermediation. However, they do not address the substitution between bank credit and bond credit. My paper shows that the dynamics of both the real sector and the intermediary sector would be significantly different if bond financing is absent in an economy.

Finally, there is a large corporate finance and banking literature that investigates firms’ choices of bond finance and bank finance (Chemmanur and Fulghieri, 1994; Bolton and Freixas, 2000). My paper highlights the dynamic properties of firms’ capital structure, and explores the general equilibrium effects of firms’ financing choices. In addition, my paper stresses that the cost of bank financing fluctuates over business cycles, and this fluctuation has important effects on financial stability and economic growth.

The structure of the rest of the paper is as follows. Section 1 describes the set-up of the model and defines the equilibrium. In Section 2, I characterize the optimal choice of individual agents and the Markov equilibrium. I highlight that the presence of bond financing has distinctive impacts on an economy’s dynamics. Section 3 shows that the optimal level of capital requirement depends heavily on the existence of a

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2Two recent papers, Xiang (2018) and Dempsey (2018), acknowledge the role of bond finance for bank capital requirements. Although both papers are intended to be quantitative, firms are modeled as short-lived. Hence, neither paper captures the dynamic interaction between the real sector and the banking sector, which turns out to have profound effects on the general equilibrium implication of bank capital requirements, as shown by my paper.
bond market, its development, and the distribution of borrowers’ risk characteristics. Section 4 concludes.

1 Model

In this section, I build an infinite-horizon continuous-time general equilibrium model, in which firms can issue corporate bonds as well as raise credit via financial intermediaries. The economy has two types of goods: perishable final goods (the numéraire) and durable physical capital goods. Three types of agents populate the economy: experts, bankers, and households. All agents have the same logarithmic preferences and the same time discount factor $\rho$. None of them accepts negative consumption. Although all three types of agents are able to hold physical capital goods and produce final goods, experts are the most productive while bankers specialize in financial intermediation.

For the purpose of exposition, I present the discrete-time version of the model with the length of each period being a positive constant $\Delta$.\(^3\) The continuous-time model that I actually solve is the limit of the discrete-time version when $\Delta$ becomes arbitrarily small.

1.1 Technology

In period $t$, an expert can produce $ak_t\Delta$ units of final goods with $k_t$ efficiency units of physical capital. Households and bankers, who are less productive than experts, also have linear production technologies: $y_t = a_h k_t\Delta$ for households and $y_t = a_b k_t\Delta$ for bankers, where $a_b < a_h < a$. All three types of agents can convert $\iota_t k_t \Delta$ units of final goods into $k_t \Phi(\iota_t) \Delta$ units of physical capital, where

$$\Phi(\iota_t) = \frac{\log(\iota_t \phi + 1)}{\phi}.$$  

Thus, there is technological illiquidity on the production side. In each period, physical capital in the possession of experts depreciates by $\delta\Delta$ percent, physical capital in the possession of households depreciates by $\delta^h\Delta$ percent, and physical capital in the possession of bankers depreciates by $\delta^b\Delta$ percent.

Exogenous aggregate shocks are driven by an i.i.d. process \{z_t, t = 1, 2, ...\}, and $z_t$ is normally distributed with mean 0 and variance $\Delta$.\(^4\) In the absence of any idiosyncratic shock, physical capital managed by an expert evolves according to

$$k_{t+1} = k_t + (\Phi(\iota_t) - \delta)k_t \Delta + \sigma k_t z_t,$$

where $\sigma$ is a positive constant that captures the direct impact of the exogenous shock on physical capital. Similarly, physical capital managed by households follows

$$k_{t+1} = k_t + (\Phi(\iota_t) - \delta^h)k_t \Delta + \sigma k_t z_t,$$

and physical capital managed by bankers follows

$$k_{t+1} = k_t + (\Phi(\iota_t) - \delta^b)k_t \Delta + \sigma k_t z_t.$$

At the beginning of each period, an expert becomes a safe expert with probability $\alpha$ or a risky expert

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\(^3\)In a typical discrete-time macroeconomics model, the length of a period is one.

\(^4\)As $\Delta$ converges to 0, the limit of $\sum_{u=1}^{t/\Delta} z_u$ is a Brownian motion.
with probability $1 - \alpha$. Whether an expert becomes risky within a period is independent across the time. Within a period, an exogenous default event may occur to a risky firm (a firm managed by a risky expert) with probability $\lambda$ after the firm has made its investment, production, and financing decisions. Since the default risk is independent across different firms, a risky expert establishes an infinite number of firms to diversify this idiosyncratic risk. Safe firms do not experience such adverse idiosyncratic shocks.

1.2 Corporate Bond, Bank Loan, and Liquidation

A firm can raise credit either from issuing corporate bonds or from obtaining a bank loan. In addition, assume that no firm can issue outside equity, and all firms have limited liability.

Both corporate bonds and bank loans are collateralized short-term contingent debt. Collateralized borrowing implies that if a firm raises $L$ dollars from creditors, it must put down physical capital worth $L$ dollars as collateral. If a risky firm defaults on the loan, the firm’s creditors will seize the collateral and liquidate physical capital. No liquidation is involved if a firm is self-financed.

Bondholders are assumed to be less efficient than banks in terms of liquidating physical capital. This is because it is harder and more time-consuming to achieve a collective decision for a number of bondholders during the liquidation process than it is for a single bank. In particular, assume that the depreciation rate of physical capital is $\kappa + \delta$ if banks liquidate the collateral, while the depreciation rate rises to $\kappa_d + \delta$ if bondholders seize the collateral, where $\kappa < \kappa_d$.

For simplicity, assume that there is a passive mutual fund that serves the intermediary in the corporate bond market. The fund charges its borrowers the risk-free rate plus the expected loss due to costly liquidation, and promises the risk-free rate $r_t$ to its investors. Any loss or profit realized by the mutual fund is driven by the aggregate shock $z_t$. Assume that the loss or profit realized in each period is instantly shared by all agents via lump-sum transfers. Thus, the unit borrowing cost of bond-financing is $r_t + \lambda \kappa$ for a risky firm.

Similar to the mutual fund, banks raise funds from households, and promise the risk-free rate $r_t$. Unlike the passive mutual fund, banks require a risk premium because their equity capital is exposed to the aggregate risk. Overall, risky firms’ unit borrowing cost of bank financing is $r_t^\lambda + \lambda \kappa$, and the net interest spread is $r_t^\lambda - r_t$.

1.3 The Expert’s Problem

I conjecture that the law of motion for the equilibrium price of physical capital can be approximated by

$$q_{t+1} = q_t + \mu^q_t q_t \Delta + \sigma^q_t q_t z_t,$$

where both $\mu^q_t$ and $\sigma^q_t$ are equilibrium objects that I will solve for. A nice property of the continuous-time approach is that I can decompose the dynamics of the stochastic process $(q_{t+1} - q_t)/q_t$ into the linear combination of a deterministic part $\mu^q_t \Delta$ and a stochastic part $\sigma^q_t z_t$. As in the macro-finance literature, I

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5The micro-foundation for creditors’ optimal decision is as follows. We can think of the default event as a publicly-known adverse signal, which increases the information asymmetry of the quality of collateral. As a result, it becomes easier for the firm’s owner to steal the collateral, leaving nothing to creditors. Therefore, given the negative signal, the optimal decision for creditors is to seize the collateral.
Therefore, a banker’s net worth label $\sigma_t^b$ the endogenous risk. An expert’s rate of net return from holding physical capital is

$$\frac{q_{t+1}k_{t+1} + ak_t\Delta - \mu_t b_t \Delta - q_t k_t}{q_t k_t} = R_t \Delta + (\sigma + \sigma_t^b)z_t + o(\Delta),$$

where $R_t \equiv \frac{a - \mu_t}{q_t} + \Phi(\mu_t) - \delta + \mu_t^b + \sigma_t^b$,

and $o(\Delta)$ denotes terms whose order is higher than one. Hereafter, I will drop the term $o(\Delta)$ when it is involved because it will vanish in the limit as $\Delta$ converges to zero. The derivation above uses the fact that $E[z_t^2] = \Delta$.\textsuperscript{6} Since costly liquidation does not happen to a safe expert, he or she raises external funds only through bond financing, and thus his/her dynamic budget constraint is

$$w_{t+1} = w_t + w_t(R_t \Delta + (\sigma + \sigma_t^b)z_t) + w_t b_t^b \left((R_t - r_t)\Delta + (1 - \lambda)(\sigma + \sigma_t^b)z_t\right) + w_t m_t(\sigma + \sigma_t^b)z_t - c_t \Delta, \quad (3)$$

where $b_t^b$ is the bond-to-equity ratio and $m_t(\sigma + \sigma_t^b)z_t$ denotes the lump-sum transfer from the bond mutual fund per unit of net worth.

A risky expert will choose among corporate debt, bank loans, and self-financing. Since all of the expert’s firms are identical prior to the realization of the liquidity shock, the financing decisions of all firms managed by the expert are the same. Thus, the debt-to-equity ratios of these firms are also the same, which is exactly the expert’s debt-to-net-worth ratio. The law of motion for the risky expert’s net worth is

$$w_{t+1} = w_t + w_t(R_t \Delta + (\sigma + \sigma_t^b)z_t) + w_t b_t^b \left((R_t - \lambda \kappa - r_t)\Delta + (1 - \lambda)(\sigma + \sigma_t^b)z_t\right) + w_t l_t \left((R_t - \lambda \kappa - r_t^b)\Delta + (1 - \lambda)(\sigma + \sigma_t^b)z_t\right) + w_t m_t(\sigma + \sigma_t^b)z_t - c_t \Delta, \quad (4)$$

where $b_t^b$ is the firms’ bond-to-equity ratio, and $l_t$ is the firm’s loan-to-equity ratio. By the Law of Large Numbers, creditors seizes a proportion $\lambda$ of the expert’s physical capital due to default. As a result, the risky expert partially unloads his/her exposure to the aggregate risk, $\lambda(\sigma + \sigma_t^b)z_t$.

Taking $\{q_t, r_t, r_t^b, m_t, t \geq 0\}$ as given, an expert chooses $\{c_t, b_t^b, b_t^b, l_t, t \geq 0\}$ to maximize his/her life-time expected utility

$$E_0 \left[ \sum_{t=0}^{\infty} e^{-\rho \Delta t} \ln(c_t) \right],$$

given that his/her net worth evolves in each period according to either equation (3) or (4) depending on his/her type.

### 1.4 The Banker’s Problem

The instant rate of return from holding physical capital for a banker is

$$R_t^b \Delta + (\sigma + \sigma_t^b)z_t,$$

where $R_t^b \equiv \frac{a_b - \mu_t}{q_t} + \Phi(\mu_t) - \delta + \mu_t^b + \sigma_t^b$.

Therefore, a banker’s net worth $n_t$ evolves according to

$$n_{t+1} = n_t + n_t x_t^b \left(R_t^b \Delta + (\sigma + \sigma_t^b)z_t\right) + n_t x_t (r_t^b \Delta + \lambda(\sigma + \sigma_t^b)z_t) + n_t (1 - x_t^b - x_t) r_t \Delta + n_t m_t(\sigma + \sigma_t^b)z_t - c_t \Delta, \quad (5)$$

\textsuperscript{6}I use Ito’s Lemma in the continuous-time setting.
where $x_t^l$ denotes the physical-capital-to-equity ratio and $x_t$ the loan-to-equity ratio for the bank. When $x_t > 1$, the bank absorbs deposits, and transfers funds from households to experts. When $x_t \leq 1$, the bank puts some of its equity capital in the mutual fund. The banker is exposed to the aggregate risk $n_t x_t^l \lambda (\sigma + \sigma_t^q) z_t$ because he or she takes over and resell the physical capital that backs her lending. I consider the time-invariant capital ratio requirement, which imposes an upper bound on banks’ loan-to-equity ratio, that is, $x_t \leq \bar{x}$. Taking $\{q_t, r_t, q_t^l, m_t, t \geq 0\}$ as given, a banker chooses $\{c_t, x_t^l, x_t^h, t \geq 0\}$ to maximize her life-time expected utility

$$E_0 \left[ \sum_{t=0}^{\infty} e^{-\rho \Delta t} \ln(c_t) \right] ,$$

subject to the dynamic budget constraint (5) and the capital ratio requirement.

### 1.5 The Household’s Problem

The rate of return from holding physical capital for a household is

$$R_t^h \Delta + (\sigma + \sigma_t^q) z_t, \quad \text{where} \quad R_t^h = \frac{a_k - t_k}{q_t} + \Phi(t_k) - \delta + \mu_t^h + \sigma \sigma_t^q.$$

The law of motion for a household’s net worth $w_t^h$ is

$$w_{t+1}^h = w_t^h + \Delta_t x_t^h (R_t^h \Delta + (\sigma + \sigma_t^q) z_t) + w_t^h (1 - x_t^h) r_t \Delta + w_t^h m_t (\sigma + \sigma_t^q) z_t - c_t , \quad (6)$$

where $x_t^h$ is the portfolio weight of physical capital. Taking $\{q_t, r_t, m_t, t \geq 0\}$ as given, a household maximizes life-time expected utility

$$E_0 \left[ \sum_{t=0}^{\infty} e^{-\rho \Delta t} \ln(c_t) \right] ,$$

by choosing $\{c_t, x_t^l, t \geq 0\}$ that satisfy the dynamic budget constraint (6).

### 1.6 Equilibrium

The aggregate shock $\{z_t\}_{t=0}^{\infty}$ drives the evolution of the economy. $I = [0,1)$ denotes the set of experts, $J = [1,2]$ the set of bankers, and $H = [2,3]$ the set of households. Given the idiosyncratic shock in period $t$, $I_t^s$ is the set of safe experts in period $t$ and $I_t^r$ the set of risky experts.

**Definition 1** Given the initial endowments of physical capital $\left\{ k_0^i, k_0^j, k_0^h, i \in I, j \in J, h \in H \right\}$ possessed by experts, bankers, and households such that

$$\int_0^1 k_0^i di + \int_1^2 k_0^j dj + \int_2^3 k_0^h dh = K_0,$$

an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{z_t\}_{t=0}^{\infty}$: the price of physical capital $\{q_t\}_{t=0}^{\infty}$, the risk-free rate $\{r_t\}_{t=0}^{\infty}$, the interest rate of bank loans $\{r_t^l\}_{t=0}^{\infty}$, wealth $\left\{ W_t^i, N_t^j, W_t^h, i \in I, j \in J, h \in H \right\}_{t=0}^{\infty}$, investment decisions $\left\{ c_t^i, c_t^j, c_t^h, i \in I, j \in J, h \in H \right\}_{t=0}^{\infty}$, asset...
holding decisions \( \{x_t^i, x_t^h, j \in J, h \in H_t^i\}_{t=0}^{\infty} \) of bankers and households, corporate debt financing decisions \( \{b_t^{i,0}, b_t^{i,\lambda}, i \in I_t\}_{t=0}^{\infty} \) of experts, bank financing decisions \( \{l_t^i, i \in I_t^r\}_{t=0}^{\infty} \) of risky experts, bank lending, \( \{x_t^{\lambda,j}, j \in J\}_{t=0}^{\infty} \) and consumption \( \{c_t^i, c_t^j, c_t^h, i, j \in J, h \in H\}_{t=0}^{\infty} \); such that

1. \( W_0^i = k_0^i q_0, N_0^i = k_0^i q_0, \) and \( W_0^h = k_0^h q_0 \) for \( i \in I, j \in J, \) and \( h \in H; \)
2. Each expert, banker, and household solves for his/her problem given prices;
3. Markets for final goods and physical capital clear, that is,

\[
\int_0^3 c_t^i \, dt = \frac{1}{q_t} \int I^1_t (a^b - c_t^i) n_t^i x_t^i \, dt + \frac{1}{q_t} \int I^2_t (a^h - c_t^i) w_t^h x_t^h \, dt + \frac{1}{q_t} \int I^1_t (a - c_t^i) w_t^i (1 + b_t^{i,0}) \, dt + \frac{1}{q_t} \int I^1_t (a - c_t^i) w_t^i (1 + b_t^{i,\lambda} + l_t^i) \, dt
\]

for the market of final goods, and

\[
\frac{1}{q_t} \int \int I^1_t w_t^i (1 + b_t^{i,0}) \, dt + \frac{1}{q_t} \int \int I^1_t w_t^i (1 + b_t^{i,\lambda} + l_t^i) \, dt + \frac{1}{q_t} \int I^2_t n_t^i x_t^i \, dt + \frac{1}{q_t} \int I^2_t w_t^h x_t^h \, dt = K_t
\]

for the market of physical capital goods, where \( K_t \) evolves according to

\[
\frac{K_{t+1} - K_t}{\Delta} = \frac{1}{q_t} \int I^1_t (\Phi(c_t^i) - \delta n_t^i x_t^i) \, dt + \frac{1}{q_t} \int I^2_t (\Phi(c_t^h) - \delta w_t^h x_t^h) \, dt + \frac{1}{q_t} \int I^1_t (\Phi(c_t^i) - \delta) w_t^i (1 + b_t^{i,0}) \, dt + \frac{1}{q_t} \int I^1_t (\Phi(c_t^i) - \delta) w_t^i (1 + b_t^{i,\lambda} + l_t^i) - \lambda s^d w_t^i b_t^i - \lambda \kappa w_t^j l_t^j \, dt.
\]

4. The bank loan market clears:

\[
\int \int I^1_j w_t^i l_t^j \, dt = \int I^1_j n_t^i x_t^{\lambda,j} \, dt.
\]

5. The bond mutual fund assumes no gains or losses, i.e., the lump-sum transfer between the mutual fund and all agents perfectly hedges the fund’s risk exposure to the aggregate risk

\[
\int_0^1 w_t^i m_t \, dt + \int_1^2 n_t^i m_t \, dt + \int_2^3 w_t^h m_t \, dt = \int \int I^1_j \lambda^\lambda w^\lambda t^j \, dt.
\]

The credit market for corporate bonds clears automatically by Walras’ Law.

2 Solving for the Equilibrium

Both experts’ net worth and bank capital are crucial for the allocation of physical capital and financial resources in the equilibrium. We expect the price of physical capital to decline as experts’ net worth and bank capital shrink due to adverse exogenous shocks.

To solve for the equilibrium, I first derive first-order conditions with respect to the optimal decisions of experts, bankers, and households. Next, I solve for the law of motion for endogenous state variables, wealth shares of different types of agents based on market clearing conditions and first-order conditions. Lastly,
I use first-order conditions and state variables’ law of motion to define partial differential equations that are satisfied by endogenous variables such as the price of physical capital. At the end of this section, I will characterize the dynamics of the economy and show that economic dynamics would be significantly different if the bond market is shut down.

2.1 Households’ Optimal Choices

Households have logarithmic preferences. In the following discussion, I will take advantage of two well-known properties with respect to logarithmic preferences in the continuous-time setting: (i) a household’s consumption $c_t$ is $\rho$ proportion of her wealth $w^h_t$ in the same period, i.e.,

$$c_t = \rho w^h_t;$$

(ii) a household’s portfolio weight on a risky investment is such that the Sharpe ratio of the risky investment equals the percentage volatility of the household’s wealth.

A household’s investment rate $\iota_t$ always maximizes $\Phi(\iota_t) - \iota_t/q_t$. The first-order condition implies that

$$\Phi'(\iota_t) = \frac{1}{q_t},$$

which defines the optimal investment as a function of the price of physical capital $\iota(q_t)$.

Given the second property, it is straightforward to derive a household’s optimal portfolio weight on physical capital $x^h_t$, which satisfies

$$x^h_t + m_t \geq \frac{R^h_t - r_t}{(\sigma + \sigma^q_t)^2} \text{ with equality if } x^h_t > 0.$$

2.2 Experts’ Portfolio Choices

According to the second property highlighted above, it is straightforward to characterize a safe expert’s optimal bond-to-equity ratio

$$1 + b^0_t + m_t \geq \frac{R_t - r_t}{(\sigma + \sigma^q_t)^2} \text{ with equality if } b^0_t > 0.$$

For a risky expert, both bond-to-equity ratio $b^\lambda_t$ and loan-to-equity ratio $l_t$ affect the percentage volatility of her wealth $(1 + (1 - \lambda)b^\lambda_t + (1 - \lambda)l_t + m_t)(\sigma + \sigma^q_t)$. Hence, optimal $b^\lambda_t$ and $l_t$ must satisfy

$$1 + (1 - \lambda)b^\lambda_t + (1 - \lambda)l_t + m_t \geq \frac{R - \lambda \kappa d - r_t}{(1 - \lambda)(\sigma + \sigma^q_t)^2} \text{ with equality if } b^\lambda_t > 0;$$

$$1 + (1 - \lambda)b^\lambda_t + (1 - \lambda)l_t + m_t \geq \frac{R - \lambda \kappa - r^\lambda_t}{(1 - \lambda)(\sigma + \sigma^q_t)^2} \text{ with equality if } l_t > 0.$$

When the cost of bond financing equals the cost of bank financing, i.e., $\lambda \kappa d + r_t = \lambda \kappa + r^\lambda_t$, individual risky experts are indifferent between bond financing and bank financing, and their portfolio choices are

---

8Sharpe ratio is $(R^h_t - r_t)/(\sigma + \sigma^q_t)$. The percentage volatility of the household’s wealth is $(x^h_t + m_t)(\sigma + \sigma^q_t)$.

9In this case, the Sharpe ratio is $(R_t - r_t)/(\sigma + \sigma^q_t)$. The percentage volatility of the safe expert’s wealth is $(1 + b^0_t)(\sigma + \sigma^q_t)$. 

indeterminate. Without loss of generality, I assume that portfolio weights of both bond-financing and bank-financing, \( b_t^\lambda \) and \( \lambda_t \), are the same across all risky experts.

### 2.3 Banker’s Optimal Choices

A banker’s optimal portfolio weights on holdings of physical capital and loans satisfy

\[
x_t^\lambda + \lambda x_t + m_t \geq \frac{R_t^b - r_t}{(\sigma + \sigma_t^q)^2}, \text{ with equality if } x_t^\lambda > 0
\]

and

\[
x_t^\lambda + \lambda x_t + m_t \leq (>) \frac{r_t^\lambda - r_t}{\lambda(\sigma + \sigma_t^q)^2}, \text{ with equality if } 0 < x_t < \bar{x} \text{ (if } x_t = 0)\]

The loan rate \( r_t^\lambda \) depends on banks’ exposure to aggregate risk \( \lambda(\sigma + \sigma_t^q) \), banks’ leverage \( x_t \) and \( x_t^\lambda \) and also whether the capital requirement constraint is binding or not. If the constraint is binding, i.e., \( x_t = \bar{x} \), then the positive Lagrange multiplier of the constraint implies that the loan \( r_t^\lambda \) is larger or equal to the level it would be if the constraint is not binding. The financing cost of bank loans for firms fluctuates endogenously for two reasons: the price volatility of physical capital changes over time, and banks’ leverage varies across business cycles.

### 2.4 Market Clearing

Let \( W_t \) denote the total wealth that experts have in period \( t \) and \( N_t \) the total bank capital. Hence, the total bank loans issued in equilibrium denoted by \( x_tN_t \) satisfies

\[
x_tN_t = (1 - \alpha)W_t\lambda_t.T
\]

The demand for final goods comprises consumption and investment. The aggregate consumption of households is \( \rho q_t K_t \). Therefore, the market clearing condition with respect to final goods is

\[
\rho q_t K_t = \alpha \frac{W_t}{q_t} (a - \iota_t)(1 + b_t^0) + (1 - \alpha) \frac{W_t}{q_t} (a - \iota_t)(1 + b_t^\lambda + \lambda_t)
\]

\[
+ \frac{N_t}{q_t} (a_h - \iota_t) x_t^\lambda + \frac{q_t K_t - W_t - N_t}{q_t} (a_h - \iota_t) x_t^h
\]

The market for physical capital clears if

\[
\alpha \frac{W_t}{q_t} (1 + b_t^0) + (1 - \alpha) \frac{W_t}{q_t} (1 + b_t^\lambda + \lambda_t) + \frac{N_t}{q_t} x_t^\lambda + \frac{q_t K_t - W_t - N_t}{q_t} x_t^h = K_t.
\]

Finally, the bond mutual fund’s exposure to the aggregate risk must be shared by all agents \( m_t q_t K_t = (1 - \alpha) \lambda b_t^\lambda W_t \).

### 2.5 Wealth Distribution

Two endogenous state variables that characterize the dynamics of the economy are experts’ wealth share \( \omega_t = W_t/(q_t K_t) \) and bankers’ wealth share \( \eta_t = N_t/(q_t K_t) \). The decline of experts’ wealth share naturally leads
to a fall in average productivity since financial markets are incomplete and households are less productive. If bankers’ wealth share declines, then the supply of bank loans shrinks, and the interest rate on bank loans rises, which in turn lowers the aggregate productivity of the economy due to the increased financing cost for experts.

Given dynamic budget constraints of individual experts and bankers, it is straightforward to derive laws of motion for both \( W_t \) and \( N_t \)

\[
W_{t+1} = W_t + W_t \left( R_t + \alpha b^0_t (R_t - r_t) + (1 - \alpha) b^\lambda_t (R_t - \lambda \kappa^d - r_t) + (1 - \alpha) l_t (R_t - \lambda \kappa - r_t^\lambda) \right) \Delta \\
- c_t \Delta + W_t \left( 1 + \alpha b^0_t + (1 - \alpha) (b^\lambda_t + l_t) (1 - \lambda) + m_t \right) (\sigma + \sigma^q_t) z_t \\
N_{t+1} = N_t + N_t \left( x^l_t R^b_t + x_t r_t^\lambda + (1 - x_t^l - x_t) r_t - \frac{c_t}{N_t} \right) \Delta + N_t (x^l_t + x_t \lambda + m_t) (\sigma + \sigma^q_t) z_t. 
\]

Dynamics of state variables in equilibrium also depend on the law of motion of the aggregate physical capital, which is

\[
K_{t+1} = K_t + K_t \mu^\kappa_t \Delta + K_t \sigma z_t, \quad \text{where} \\
\mu^\kappa_t \equiv \Phi(\nu_t) - \delta - (1 - \omega_t - \eta_t) x_t (\delta - \delta^h) - \eta_t x_t (\sigma - \delta^h) - (1 - \alpha) \omega_t \lambda (b^\kappa_t \kappa^d + l_t \kappa). 
\]

Given laws of motion of \( W_t, N_t, q_t, \) and \( K_t \), we can derive laws of motion for \( \omega_t \) and \( \eta_t \) in equilibrium, which are summarized in the following lemma.\(^\text{10}\)

**Lemma 1**  
*In equilibrium, experts’ wealth share \( \omega_t \) evolves according to*

\[
\omega_{t+1} = \omega_t + \omega_t \mu^\omega_t \Delta + \sigma^\omega_t z_t, \tag{20}
\]

*where*

\[
\mu^\omega_t = R_t - \mu^\omega_t - \sigma^\omega_t + \alpha b^0_t (R_t - r_t) + (1 - \alpha) b^\lambda_t (R_t - \lambda \kappa^d - r_t^\lambda) \\
+ (1 - \alpha) l_t (R_t - \lambda \kappa - r_t^\lambda) - (1 - \alpha) b^0_t + (1 - \alpha) b^\lambda_t (1 - \lambda) + (1 - \alpha) l_t (1 - \lambda) + m_t \right) (\sigma + \sigma^q_t)^2 - \rho \\
\sigma^\omega_t = (\alpha b^0_t + (1 - \alpha) b^\lambda_t (1 - \lambda) + (1 - \alpha) l_t (1 - \lambda) + m_t \right) (\sigma + \sigma^q_t).
\]

*The state variable \( \eta_t \) evolves according to*

\[
\eta_{t+1} = \eta_t + \eta_t \mu^\eta_t \Delta + \sigma^\eta_t z_t, \tag{21}
\]

*where*

\[
\mu^\eta_t = (x^l_t + \lambda x_t + m_t) (x^l_t - 1) (\sigma + \sigma^q_t)^2 + x_t (r_t^\lambda - r_t) + r_t - \mu^\eta_t - \mu^\kappa - \sigma^\eta_t + (\sigma + \sigma^q_t)^2 - \rho \\
\sigma^\eta_t = (x^l_t + \lambda x_t + m_t - 1) (\sigma + \sigma^q_t)
\]

The proof of Lemma 1 is in the appendix.\(^\text{10}\)

\(^{10}\) I apply Ito’s Lemma for this derivation in the continuous-time setting.
2.6 Markov Equilibrium

Like other continuous-time macro-finance models (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012), my framework also has the property of scale-invariance with respect to total physical capital $K_t$. I focus on the equilibrium that is Markov in state variables $\omega_t$ and $\eta_t$. In the Markov equilibrium, dynamics of endogenous variables such as $q_t$ can be characterized by laws of motion of $\omega_t$ and $\eta_t$ and functions $q(\omega, \eta)$.

To solve for the full dynamics of the economy, I derive a partial differential equations with respect to $q(\omega, \eta)$. The partial differential equation as well as its boundary conditions originate from equilibrium conditions and Ito’s formula with $q(\omega, \eta)$. Ito’s lemma with respect to the volatility of the price of physical capital implies that

$$q_t \sigma_q^2 = q(\omega_t, \eta_t) \omega_t \sigma_q^2 + q(\omega_t, \eta_t) \eta_t \sigma_q^2.$$  \hspace{1cm} (22)

Given $(q, \omega, \eta)$, we can solve the equilibrium and derive all endogenous choice variables $(c, b^0, b^1, l, x, x^b)$ and endogenous price variables $(r, r^f, \mu^q, \sigma^q)$ as well as the lump-sum transfer related to the bond mutual fund $m$. Therefore, volatility terms of two state variables $(\sigma^q, \sigma^\omega)$ are also known. Hence, equation (22) is a well-defined partial differential equation with respect to $q(\omega, \eta)$.

In addition to the differential equation, we need boundary conditions to solve for $q(\omega, \eta)$. There are three boundary conditions that correspond to three boundaries for the domain of $q(\omega, \eta)$: \{$(\omega, \eta) : \omega = 0, 0 \leq \eta \leq 1$\}, \{$(\omega, \eta) : 0 \leq \omega \leq 1, \eta = 0$\}, and \{$(\omega, \eta) : 0 \leq \omega \leq 1, 0 \leq \eta \leq 1, \omega + \eta = 1$\}. For any of the three boundaries, one of the three agents has zero net worth and the economy now has only two types of agents. Accordingly, differential equation (22) on boundaries reduces to an ordinary differential equation, which is straightforward to characterize.

2.7 Dynamics

In this subsection, I highlight that the dynamics of an economy highly depends on whether risky firms can directly issue bonds or not. I use numerical examples to illustrate this point. The choice of parameter values is $\rho = 3\%$, $a = 0.16$, $a^b = 0$, $a^h = 0$, $\delta = 0.01$, $\delta^b = 0.1$, $\delta^h = 0.01$, $\phi = 5$, $a = 0.2$, $\lambda = 0.3$, $\kappa^d = 0.4$, $\kappa = 0.2$, $\sigma = 0.1$, and $\bar{x} = 7$. To illustrate the role of bond financing in the aggregate economy, I consider the dynamics of both the benchmark economy with bond financing and a second economy without the bond market. In the second economy, safe firms obtain risk-free bank loans from banks. To ensure that the two economies are comparable, capital requirement imposes an upper bound on the risky loan-to-equity ratio.\(^{12}\)

To illustrate the economic dynamics, I borrow the idea of impulse response and characterize the stochastic dynamics of an economy after it experiences a relatively large exogenous shock.\(^{13}\) In particular, I set the initial state of the economy at the highest density in the long-run stationary distribution. The economy is then hit with a shock that is $1.58$ times the standard deviation within an interval of length $\Delta = 0.001$. In the aftermath of the initial shock, the economy will keep receiving stochastic aggregate shocks over time. Hence, we keep track of the dynamics of a spectrum of 10,000 economies. This type of characterization differs from the standard impulse response analysis. Figure 1 shows the dynamics of the median of nine key endogenous variables over four quarters after the initial negative shock. Figures 7 and 8 in Appendix C show the dynamics of the 45th and 55th percentiles of the relevant endogenous variables, respectively.

\(^{11}\)At this stage given $(q, \omega, \eta)$, I can only solve for $r - \mu^q$. However, it is straightforward to solve for $r_t$ and $\mu^q$ after I derive the entire $q(\omega, \eta)$.

\(^{12}\)Safe firms obtain risk-free bank loans in the second economy.

\(^{13}\)Readers who are familiar with the continuous-time macro-finance model can refer to Appendix B on the discussion of the global dynamics of the economy.
Before discussing economic dynamics in detail, let us review the *transmission mechanism* of the model. When a negative shock hits the economy, experts’ dynamic budget constraints (3) and (4) imply that their net worth will decline disproportionately due to the leverage effect. On top of the exogenous shock, the decline in the price of physical capital causes additional losses to experts’ net worth, as indicated by equations (3) and (4). The exogenous shock also affects bankers’ net worth, which is the other state variable. Bankers’ exposure to the aggregate risk comes from the collateral that backs their loans. When banks liquidate risky firms’ physical capital, the exogenous shock affects the (efficient) units of physical capital seized by banks, and also the price at which they can sell the physical capital in the secondary market. Note that banks also take on high leverage and thus have high risk exposure to the exogenous shock as shown by equation (5). The decline in the net worth of both productive experts and financial intermediaries has persistent effects on the productivity, investment, asset prices, and external financing in the economy.

![Graph showing the dynamics of the mean of nine key aggregate variables in two economies after being hit by an aggregate capital quality shock with a magnitude of 1.58 times the standard deviation: experts’ wealth share (top left), bankers’ wealth share (top middle), TFP (top right), consumption-to-physical capital ratio (middle left), investment-to-capital ratio (center), risky firms’ liability (middle right), outstanding bonds (bottom left), outstanding loans (bottom middle), loan spread (bottom right). The horizontal axis depicts the number of calendar quarters following the shock. Each line indicates the percentage change relative to the initial state of the variable over time. The solid lines refer to an economy with bond financing, and the dashed lines refer to an economy without bond financing. The initial states of the two economies prior to the aggregate shock are at the highest probability density of the long-run stationary distribution.](image)
The key message of Figure 1 is that the banking sector will recover more quickly after a negative shock if risky firms cannot access the bond market (see the top middle panel). Given the initial adverse shock, the wealth shares of experts and bankers decline by a similar magnitude in both types of economies. However, the two types of economies experience quite different dynamics in the aftermath of the shock. The top left and middle panels in Figure 1 as well as in Figures 7 and 8 in the Appendix show that both state variables, experts’ wealth share and bankers’ wealth share, tend to recover more quickly in the economy without bond financing. The difference is more significant for bankers’ wealth share. Since experts are the most productive type of agents in the economy, and bankers provide relatively cheap credit for productive agents, the economy without bond financing would also experience much faster recovery in its average productivity, consumption, and investment (see the top right, middle left, and center panels in Figure 1). This numerical exercise emphasizes a crucial point that models that do not permit direct bond financing are unable to precisely capture the dynamics of an economy with an active bond market.

I now explain why the banking sector can recover faster when the bond market is shut down in an economy. In the aftermath of an adverse shock, both the firm sector and the banking sector shrink. Since the decline in loan demand is more significant, the loan spread decreases right after the initial shock. However, as the firm sector recovers much faster than the banking sector, the demand for bank credit outgrows the supply, and thus the loan spread gradually increases (see the bottom right panel in Figure 1). Notice that the rise in the loan spread would be much less significant if there were no capital ratio requirement. Intuitively, the growth in the loan spread increases bank profitability and thus enhances the banking sector’s recovery. This effect is prominent in the economy without bond financing. Bankers’ wealth share is restored fairly quickly, as are risky firms’ total liability and outstanding loans (see the middle right and bottom middle panels in Figure 1).

However, the same effect that triggers the fast recovery of the banking sector could be dampened by the presence of bond financing. As bank loans become gradually more expensive, firms switch to bond credit if it is available. Hence banks’ profitability might not increase much as the aggregate demand for bank loans could decline due to the rise in the loan spread. Consequently, bankers’ wealth share grows at a much slower rate in an economy with a bond market relative to an economy without a bond market. Similarly, risky firms’ overall liability and outstanding loans are restored more slowly in an economy with a bond market relative to an economy without a bond market (see the middle right and bottom middle panels in Figure 1).

3 Optimal Capital Requirement

In this section, I emphasize that the socially optimal level of capital ratio requirement highly depends on (i) whether bond financing is present in a model, (ii) the efficiency of an economy’s bankruptcy procedure, and (iii) the distribution of borrowing firms’ idiosyncratic default risk. All these results are connected in the sense that the elasticity of aggregate demand for bank loans is the key factor that determines the general equilibrium costs and benefits of bank capital requirements.

The welfare of an individual agent is the weighted sum of the agent’s lifetime expected utility over all possible states of the economy. The weight of each state is the density of the long-run stationary distribution at that state. The social welfare is the equal-weighted sum of the welfare of all agents.
3.1 The Consequences of Omitting Bond Financing

An economic model that omits bond financing overstates the benefit of capital ratio requirements, and thus prescribes an optimal requirement that is overly tight. We compare the social welfare of two economies — one with bond financing and the other without bond financing — under different degrees of capital requirement. Figure 2 clearly shows that the cap on the loan-to-equity ratio \( \bar{x} \) that maximizes social welfare is higher in the economy with a bond market than in the economy without a bond market. In other words, the socially optimal capital requirement should be more lenient in the presence of bond financing. This statement holds regardless of whether we focus on the welfare of experts, bankers, or households (see the second, third, and fourth panels from the left in Figure 2). Before expounding why this difference exists, I explain the channel through which capital requirement influences social welfare.

![Figure 2: Welfare](image)

This figure shows the relationship between banks’ maximum leverage \( \bar{x} \) (horizontal axis) and the welfare of different types of agents in economies with and without bond financing. Solid lines refer to an economy with bond financing and dashed lines refer to an economy without bond financing. The aggregate welfare shown in the left panel is the sum of the welfare of the three types of agents. For the values of parameters other than \( \bar{x} \), see Section 2.7.

Elenev et al. (2018) highlights that tightening the capital requirement shifts wealth from savers to borrowers. Here, I emphasize that part of the wealth is actually diverted to financial intermediaries. To illustrate this effect more clearly, first consider an economy without bond financing. The dashed lines in the two left panels in Figure 3 show that the wealth share of both experts and bankers rises as the maximum leverage ratio declines from 9 to 6. Dashed lines in the top middle and upper right panels in Figure 3 clearly show why bankers’ wealth share increases. Tightening the capital requirement lowers the supply of bank loans. Therefore, the loan spread that banks can charge increases accordingly. To some extent, the overall effect leads to the increase in bank profitability as shown by the dashed line in the bottom right panel in Figure 3. The cumulative effect of high bank profits naturally leads to an increasingly stronger banking sector, which translates to an improvement in bankers’ welfare.

Tightening the capital requirement increases experts’ wealth share as well as the welfare of both experts and households. Lowering the maximum leverage of bankers limits the supply of overall credit. Given the excessive credit supply from less productive households, the overall borrowing costs decrease, and thus experts’ wealth share increases. In sum, the strengthening of both the firm sector and the banking sector increases the average productivity of the economy as highlighted in the lower middle panel in Figure 3. The rise in the average TFP results in the improvement of households’ welfare (see the right panel in figure 3).
Furthermore, we should notice that tightening the capital requirement does not always lead to positive effects. This is because if the capital ratio requirement is too tight bank profitability ultimately declines due to the substantial decrease in loans that banks can originate (see the bottom right panel in Figure 3). This reasoning also applies to social welfare.

Figure 3: Wealth Distribution
This figure shows the relationship between banks’ maximum leverage $\bar{x}$ (horizontal axis) and the moments of six financial variables in the long-run stationary distribution: median experts’ wealth share (upper left), average net interest spread (upper middle), average outstanding loans (upper right), median bankers’ wealth share (lower left), average total factor productivity (lower middle), and average bank profit (lower right). For the values of parameters other than $\bar{x}$, see Section 2.7.

The presence of bond financing, however, can significantly dampen the wealth transfer effect of the capital ratio requirement. We now turn to an economy with a bond market. Solid lines in the two left panels in Figure 3 display that as the cap on the loan-to-equity ratio declines from 9 to 7, the wealth shares of both experts and bankers increase. This increase is similar to their reactions to the regulatory change in an economy without bond financing. Nevertheless, if the cap further decreases, the wealth share of the intermediary sector shrinks drastically until it completely vanishes. The wealth share of experts also declines in the same dramatic fashion.

Why does the financial intermediary sector react so differently in the two economies? The key underlying reason is that firms have an alternative way of raising external credit in an economy with bond financing. Thanks to the alternative channel, firms can resort to bond financing when loan spreads rise. Hence, when bond financing is feasible, the decline in loan demand would be more substantial than in an economy where loan financing is the only option for the real sector. Therefore, bank profits are more likely to decline in an economy where firms have a second option for raising external credit. The decline in bank profitability, in turn, hurts bankers’ wealth share and loan supply, which ultimately lowers experts’ wealth share and the economy’s average productivity. In sum, tightening capital requirement in the presence of bond financing is more inclined to hurt the financial intermediary sector and the entire economy.
3.2 Policy Experiments

The previous section shows that the discussion on the optimal capital requirement could be misleading if bond financing is omitted from the model. In this subsection, I conduct three policy experiments, and discuss whether and how the optimal capital requirement depends on the structure of the bond market. In the first experiment, I vary the liquidation cost of bondholders \( \kappa^d \), and characterize the relationship between the optimal capital requirement and the development of the bond market (Djankov et al., 2008; Becker and Josephson, 2016). In the second and third policy experiments, I investigate the policy implication of the risk profile of borrowing companies in the bond market.

3.2.1 Development of the Bond Market

Figure 4: Development of bond market
This figure shows the welfare implications of a change in banks’ maximum leverage \( \bar{x} \) (horizontal axis) for economies with different degrees of bond market development: more developed bond market (\( \kappa^d = 0.3 \)), benchmark (\( \kappa^d = 0.4 \)), less developed bond market (\( \kappa^d = 0.6 \)). The bottom middle and right panels display effects of a change in \( \bar{x} \) on the median wealth shares of experts and bankers. For the values of parameters other than \( \bar{x} \) and \( \kappa^d \), see Section 2.7.

Becker and Josephson (2016) emphasize that the efficiency differences in the processing of insolvency and bankruptcy cases (e.g., bankruptcy recoveries) can explain the cross-firm and also cross-country heterogeneity regarding the use of bond financing and bank financing. Their empirical evidence as well as theoretical results show that inefficient bankruptcy procedures in an economy is associated with less bond financing by risky firms. The efficiency of bankruptcy procedures, in turn, can be traced back to the legal origin and income per capita according to Djankov et al. (2008). Here, I treat bondholders’ liquidation cost \( \kappa^d \) as an exogenous parameter that captures the efficiency of bankruptcy procedures in an economy. A lower liquidation cost \( \kappa^d \) signifies a more efficient bankruptcy system and a more developed bond market. Based on this assumption, I investigate how the optimal capital ratio requirement in a country depends on how developed its bond market is.

The top left panel in Figure 4 shows that the socially optimal capital requirement ought to be more stringent in an economy with a less developed bond market (i.e., higher \( \kappa^d \)). The intuition is the same
as that in the previous analysis on the absence of bond financing. If the cap on bank leverage decreases, the loan spread increases; at the same time, the amount of loans originated by banks also declines. In an economy with a less developed bond market, (i.e., higher liquidation cost \( \kappa^d \)), risky firms find it more costly to switch from bank financing to bond financing. Hence, the decrease in the amount of bank loans is not so significant; in fact, banks’ overall profitability may actually increase when the loan spread increases. When the bankruptcy cost parameter is between 0.4 and 0.6, lowering the maximum bank leverage actually increases bankers’ wealth share in the economy. However, the bottom right panel in Figure 4 shows that if the maximum bank leverage is less than 6, the banking sector tends to vanish in an economy with a developed bond market (\( \kappa^d = 0.4 \)). The reason is that when risky firms switch to bond financing, there is a substantial decline in the quantity of bank loans and also a sizable drop in bank profitability. When the banking sector vanishes, the borrowing cost of the firm sector increases substantially, leading to a decline in the firm sector. Since the average productivity of the economy depends on the wealth share of the firm sector, the capital ratio requirement affects social welfare through its impact on experts’ wealth share.

3.2.2 Average Firm Riskiness

![Figure 5: Riskier firms](attachment:image.png)

This figure shows the welfare implications of a change in banks’ maximum leverage \( \bar{x} \) (horizontal axis) for economies in which firms have different degrees of riskiness: less risky (\( \lambda = 0.25 \)), benchmark (\( \lambda = 0.3 \)), and more risky (\( \lambda = 0.35 \)). The bottom middle and right panels display effects of a change in \( \bar{x} \) on the median wealth shares of experts and bankers. For the values of parameters other than \( \bar{x} \) and \( \lambda \), see Section 2.7.

Other than the efficiency of the bankruptcy process in an economy, the risk profile of its ultimate borrowers also affects the use of bond financing and bank financing. I consider two experiments by varying the distribution of experts’ idiosyncratic riskiness. First, I investigate how the average riskiness of firms affects the optimal capital requirement. In particular, I keep all parameters unchanged and only adjust the value of individual firms’ bankruptcy probability \( \lambda \). Notice that a change in \( \lambda \) does not change the skewness of firms’ idiosyncratic default risk distribution. In light of this property, I vary the fraction of safe firms (i.e., \( \alpha \)) to adjust the skewness in the second experiment.
The top right panel in Figure 5 shows that the socially optimal capital requirement is tighter in an economy where firms are riskier on average. The same conclusion holds regardless of whether we focus on the welfare of experts, bankers, or households (the top middle, top right, and bottom left panels in Figure 5). When a risky firm switches from bank financing to bond financing, it has to pay an additional premium $\lambda(\kappa^d - \kappa)$ to compensate creditors for their loss in the event of a firm liquidation. This switch cost is increasing in the likelihood of firm failure, i.e., $\lambda$. Hence, relative to safe firms, risky firms find it more costly to replace bank loans with bonds. When the capital requirement tightens, the decrease in the amount of bank loans is less significant in an economy with riskier firms. In such an economy, bank profitability is less likely to decline given the rise in the loan spread. Consequently, the banking sector and the firm sector are less likely to shrink (see the bottom middle and bottom right panels in Figure 5). Hence, the optimal capital requirement ought to be tighter in an economy with riskier firms.

### 3.2.3 Skewness of Firms’ Idiosyncratic Risk Distribution

![Figure 6: Greater proportion of risky firms](image)

This figure shows the welfare implications of a change in banks’ maximum leverage $\bar{x}$ (horizontal axis) for economies with different proportions of risky firms: greater proportion of risky firms ($1 - \alpha = 0.85$), benchmark ($1 - \alpha = 0.8$), and smaller proportion of risky firms ($1 - \alpha = 0.75$). The bottom middle and right panels display effects of a change in $\bar{x}$ on the median wealth shares of experts and bankers. For the values of parameters other than $\bar{x}$ and $\alpha$, see Section 2.7.

The skewness of firms’ default risk distribution also affects the optimal capital requirement in an economy. To illustrate this, I explore how varying the fraction of risky firms affects the socially optimal capital ratio requirement. Figure 6 shows that the optimal cap on banks’ leverage ratio should be tighter in an economy with a greater proportion of risky firms (i.e., higher $1 - \alpha$). The intuition for this result is related to the elasticity of aggregate demand for bank loans. Consider two economies that only differ in their proportions of risky firms. Suppose the capital requirement tightens in the two economies. Consequently, the loan spreads increase in both economies. The decline in the amount of bank loans is smaller in the economy with a greater proportion of risky firms. Hence, this economy will experience greater bank profitability. The
lower right panel in Figure 6 shows that bankers’ wealth share is less inclined to fall in an economy with a greater proportion of risky firms when the cap on bank leverage decreases. As a result, the optimal capital requirement is also tighter when the distribution of firms’ default risk is skewed towards the risky end.

4 Conclusion

In this paper, I point out that bond financing is a critical feature in a dynamic general equilibrium framework analyzing the welfare implications of bank capital regulations. A model that omits the bond market overemphasizes the benefit of capital requirements. In addition, I highlight three factors that affects the optimal level of bank capital requirements via their influences on the demand elasticity of bank loans: the efficiency of the bankruptcy system in an economy, as well as the mean and skewness of the distribution of firms’ idiosyncratic default risks.

References


Appendix

A  Proofs

Proof of Lemma 1.

The laws of motion for the price of physical capital (2) and the efficiency units of physical capital (19) imply

\[ q_{t+1}K_{t+1} = (q_t + \mu_t^q q_t \Delta + \sigma_t^q q_t z_t)(K_t + K_t \mu_t^K \Delta + K_t \sigma z_t) \]

\[ = q_t K_t + q_t K_t (\mu_t^\sigma + \mu_t^K + \sigma_t^\sigma) \Delta + q_t K_t (\sigma + \sigma_t^\sigma) z_t. \]

I omit all terms of order above \( \Delta \) and use the property that \( E[z_t^2] = \Delta \). The equation above, together with
increase. Since bankers lower the financing costs of risky experts, the price of physical capital and the other physical capital. Consequently, the price of physical capital as well as the investment in physical capital dynamics of the economy, i.e., the laws of motion for variables including the drift and volatility terms of the two state variables. Hence, we know the exact global dynamics of the two state variables. Figures 9 and 10 in Appendix C show the solution of fifteen key endogenous variables including the drift and volatility terms of the two state variables. Hence, we know the exact global dynamics in an economy. Notice that while solving equation (17), lead to

\[ \frac{W_{t+1}}{q_{t+1}K_{t+1}} = \frac{1}{q_{t+1}K_{t+1}} \left( W_t + W_t \left( R_t + \alpha b_t^0(R_t - r_t) + (1 - \alpha)b_t^1(R_t - \lambda \kappa^d - r_t) + (1 - \alpha)l_t(R_t - \lambda \kappa - r_t^d) \right) \Delta \right) \]

\[ - \frac{c_t}{q_{t+1}K_{t+1}} \Delta + \frac{W_t}{q_{t+1}K_{t+1}} \left( 1 + \alpha b_t^0 + (1 - \alpha)(b_t^1 + l_t)(1 - \lambda) + m_t \right) (\sigma + \sigma_q^t) z_t \]

\[ = \frac{W_t}{q_tK_t} \left( R_t + \alpha b_t^0(R_t - r_t) + (1 - \alpha)b_t^1(R_t - \lambda \kappa^d - r_t) + (1 - \alpha)l_t(R_t - \lambda \kappa - r_t^d) - \frac{c_t}{W_t} \right) \Delta \]

\[ - \frac{W_t}{q_tK_t} \left( \alpha^t + \mu^K_t + \sigma \sigma_q^t \right) \Delta - \frac{W_t}{q_tK_t} \left( 1 + \alpha b_t^0 + (1 - \alpha)(b_t^1 + l_t)(1 - \lambda) + m_t \right) (\sigma + \sigma_q^t) z_t \]

\[ + \frac{W_t}{q_tK_t} (\sigma + \sigma_q^t)^2 \Delta + \frac{W_t}{q_tK_t} \left( 1 + \alpha b_t^0 + (1 - \alpha)(b_t^1 + l_t)(1 - \lambda) + m_t \right) (\sigma + \sigma_q^t) z_t - \frac{W_t}{q_tK_t} (\sigma + \sigma_q^t) z_t \]

\[ \omega_{t+1} = \omega_t + \omega_t \mu^\omega_t \Delta + \sigma^\omega_t z_t, \]

I use the approximation \( \frac{a}{b+x} = \frac{a}{b} - \frac{a}{b} x^2 + o(x^2) \) for \( x \) close to zero. In addition, I also omit all terms of order above \( \Delta \), and use the property that \( E[z^2_t] = \Delta \).

Given one of the bankers’ Euler equation (13), the law of motion for \( W_t \) can be rewritten as

\[ N_{t+1} = N_t + N_t \left( x_t^1(x_t^1 + \lambda x_t + m_t)(\sigma + \sigma_q^t)^2 + x_t(r_t^d - r_t) + r_t - \frac{c_t}{N_t} \right) \Delta + N_t(x_t^1 + \lambda x_t + m_t)(\sigma + \sigma_q^t) z_t. \]

Hence,

\[ \frac{N_{t+1}}{q_{t+1}K_{t+1}} = \frac{1}{q_{t+1}K_{t+1}} \left( N_t + N_t \left( x_t^1(x_t^1 + \lambda x_t + m_t)(\sigma + \sigma_q^t)^2 + x_t(r_t^d - r_t) + r_t - \frac{c_t}{N_t} \right) \Delta \right) \]

\[ + \frac{N_t}{q_{t+1}K_{t+1}} (x_t^1 + \lambda x_t + m_t)(\sigma + \sigma_q^t) z_t \]

\[ = \frac{N_t}{q_tK_t} \left( x_t^1(x_t^1 + \lambda x_t + m_t)(\sigma + \sigma_q^t)^2 + x_t(r_t^d - r_t) + r_t - \frac{c_t}{N_t} \right) \Delta - \frac{N_t}{q_tK_t} (\mu^K_t + \mu^K_t + \sigma \sigma_q^t) \Delta \]

\[ - \frac{N_t}{q_tK_t} (x_t^1 + \lambda x_t + m_t)(\sigma + \sigma_q^t)^2 \frac{\Delta}{\sigma q} + \frac{N_t}{q_tK_t} (\sigma + \sigma_q^t)^2 \Delta \]

\[ + \frac{N_t}{q_tK_t} (x_t^1 + \lambda x_t + m_t)(\sigma + \sigma_q^t) z_t - \frac{N_t}{q_tK_t} (\sigma + \sigma_q^t) z_t \]

\[ \frac{dq}{q_t} = \mu^q_t dt + \sigma^q_t dZ_t. \]

\[ \square \]

**B Global Dynamics**

In this section, I briefly review the property of the global dynamics in an economy. Notice that while solving for the equilibrium object \( q(\omega, \eta) \), we also obtain the value of all other endogenous variables as functions of the two state variables. Figures 9 and 10 in Appendix C show the solution of fifteen key endogenous variables including the drift and volatility terms of the two state variables. Hence, we know the exact global dynamics of the economy, i.e., the laws of motion for \( \omega \) and \( \eta \), which are depicted in equations (20) and (21).

The top plots in Figure 9 show that if the productive experts’ wealth share rises, they will hold more physical capital. Consequently, the price of physical capital as well as the investment in physical capital increase. Since bankers lower the financing costs of risky experts, the price of physical capital and the other
two relevant terms also increase when bankers’ wealth share rises. The middle right panel in Figure 9 depicts the scenario when experts’ wealth share is relatively small; the volatility of the price of physical capital is high when the magnitude of asset fire-sale is large. The same plot reveals another interesting fact: the increase in bankers’ wealth share does not necessarily mitigate the financial amplification (see the bottom right corner of the plot). The intuition is as follows. The magnitude of asset fire-sale ultimately depends on the real sector. When experts’ wealth share is low, excess supply of bank credit allows the real sector to take excess leverage, and amplifies asset fire-sale effects.

The bottom middle panel in Figure 9 shows that the leverage of risky firms highly depends on bankers’ wealth share as risky firms depend mainly on bank credit for external financing. The bottom right panel in Figure 9 shows that in the case where risky firms’ demand for bank loans is still high, bank leverage naturally declines when bankers’ wealth share declines. The top left panel in Figure 10 shows that risky firms only issue bonds when the banking sector is poorly capitalized. In this scenario, experts’ wealth share is low, bankers’ wealth share is high, and the net interest spread is high (the top middle panel in Figure 10). The price volatility of physical capital is high, which raises bankers’ exposure to aggregate risks.

The density of the stationary distribution of \((\omega, \eta)\) is displayed by the bottom right panel in Figure 10. The economy considered in our numerical example is mainly anchored the states where \(\omega = 0.054\) and \(\eta = 0.021\).

C Figures
Figure 7: Dynamics (45th percentile)
This figure shows the dynamics of nine key aggregate variables in two economies after being hit by an aggregate capital quality shock with a magnitude of 1.58 times the standard deviation: experts' wealth share (top left), bankers' wealth share (top middle), TFP (top right), consumption-to-physical capital ratio (middle left), investment-to-capital ratio (center), risky firms' liability (middle right), outstanding bonds (bottom left), outstanding loans (bottom middle), loan spread (bottom right). The horizontal axis shows the number of calendar quarters following the shock. Each line indicates the percentage change relative to the initial state of the variable over time. The solid lines refer to an economy with bond financing, and the dashed lines refer to an economy without bond financing. The initial states of the two economies prior to the aggregate shock are at the highest probability density of the long-run stationary distribution.
Figure 8: Dynamics (55th percentile)
This figure shows the dynamics of nine key aggregate variables in two economies after being hit by an aggregate capital quality shock with a magnitude of 1.58 times the standard deviation: experts' wealth share (top left), bankers' wealth share (top middle), TFP (top right), consumption-to-physical capital ratio (middle left), investment-to-capital ratio (center), risky firms' liability (middle right), outstanding bonds (bottom left), outstanding loans (bottom middle), loan spread (bottom right). The horizontal axis depicts the number of calendar quarters following the shock. Each line indicates the percentage change relative to the initial state of the variable over time. The solid lines refer to an economy with bond financing, and the dashed lines refer to an economy without bond financing. The initial states of the two economies prior to the aggregate shock are at the highest probability density of the long-run stationary distribution.
Figure 9: Global Dynamics I. This figure shows the value of nine key endogenous variables over all possible states of the economy. The variables (from top to bottom and from left to the right) are the price of physical capital $q$, the fraction of physical capital held by experts $\psi$, the investment to capital ratio $t$, the volatility of physical capital price $\sigma^p$, experts' risk exposure $\omega\sigma^x$, bankers' risk exposure $\eta\sigma^y$, safe firms' leverage $b^s$, risky firms' leverage $b^z + l$, and bank leverage $x$. The horizontal axis depicts bankers' wealth share $\eta$, and the vertical axis the experts' wealth share $\omega$. 
Figure 10: Global Dynamics II. This figure shows the value of six key endogenous variables over all possible states of the economy. The variables (from top to bottom and from left to the right) are risky bond to loan ratio $\frac{\lambda}{\lambda}$, net interest spread $\eta - r$, outstanding loans $(1 - \alpha)\omega l$, the drift of experts’ wealth share $\omega \mu \tilde{\gamma}$, the drift of bankers’ wealth share $\eta \mu \tilde{\gamma}$, and the density of the stationary distribution. The horizontal axis depicts bankers’ wealth share $\eta$, and the vertical axis the experts’ wealth share $\omega$. 