Affordable Housing and City Welfare [*]

Jack Favilukis[†]   Pierre Mabille[‡]   Stijn Van Nieuwerburgh[§]

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Abstract

Housing affordability has become the main policy challenge for the major cities of the world. Two key policy levers are zoning and rent control. We build a new dynamic spatial equilibrium model to evaluate the effect of these policies on house prices, rents, labor supply, production, residential construction, income and wealth inequality, as well as the location decision of households within the city. The model incorporates risk, wealth effects, and dynamic spatial equilibrium. We calibrate the model to the New York MSA, incorporating current zoning and rent control policies. Our model suggests sizable welfare gains from relaxing zoning regulations in the city center. That policy is progressive; it reduces overall inequality but increases spatial inequality between neighborhoods. In contrast, policies that decrease the size of the rent control system are welfare reducing. The concentrated losses of the poor outweigh the efficiency gains from a better spatial allocation of labor and housing. The same trade-off is present for a housing voucher program.

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[†]Department of Finance, Sauder School of Business, University of British Columbia, 2053 Main Mall, Vancouver, BC, V6T 1Z2, Canada; jack.favilukis@sauder.ubc.ca; https://sites.google.com/site/jackfavilukis.

[‡]Department of Economics, Stern School of Business, New York University, 44 W. 4th Street, New York, NY 10012, USA; pmabille@stern.nyu.edu.

[§]Department of Finance, Graduate School of Business, Columbia University, Uris Hall 809, 3022 Braodway, New York, NY 10027, USA; NBER; CEPR; svnieuwe@gsb.columbia.edu; https://www0.gsb.columbia.edu/faculty/svannieuwerburgh.

1   Introduction

The increasing appeal of major urban centers has brought on an unprecedented housing affordability crisis. Ever more urban households are burdened by rents or mortgage payments that take up a large fraction of their paycheck and/or by long commutes.[1] Local politicians are under pressure to improve affordability. Economists have argued that restrictive zoning policies are a key culprit, and may even make our most productive cities smaller than they should be.

Policy makers have two main categories of tools to address housing affordability: rent control (RC) and zoning policies. RC, defined to include all government-provided or regulated housing, creates affordable rental housing by fiat. There are multiple policy levers: the size of the rent control program, the rent discount to the market rent, the income threshold for qualifying, the depth of the affordability, and the spatial distribution of the housing stock. Zoning governs land use and can be relaxed to increase the supply of housing, and all else equal, reduce its cost. Zoning changes are often associated with requirements on developers to set aside affordable housing units (inclusionary zoning). Each policy affects the quantity and price of owned and rented housing and its spatial distribution. It affects incentives to work, wages, and commuting patterns. Each policy also affects wealth inequality in the city and in each of its parts.

While there is much work, both empirical and theoretical, on housing affordability, what is missing is a general equilibrium model that can quantify the impact of such policies on prices and quantities, on the spatial distribution of households, on income inequality within and across neighborhoods, and ultimately on individual and city-wide welfare. This paper sets up and solves a model that is able to address these questions.

We model a metropolitan area (city), which consists of two zones, the city center or central business district (zone 1) and the rest of the metropolitan area (zone 2). Working-age households who live in zone 2 commute to zone 1 for work. The model is an overlapping generations model with risk averse households that face labor income risk during their life-cycle and make dynamic decisions on non-housing and housing consumption, savings in bonds and investment property, labor supply, tenure status (own or rent), and location. They face mortality risk in retirement. Since households cannot hedge labor income and longevity risk, markets are incomplete. This incompleteness opens up the possibility for housing affordability policies to provide insurance. We model a progres-

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[1]The share of cost-burdened renters in the United States has risen from 23.8% in the 1960's to 47.5% in 2016. During this same time, the median home value rose 112%, far more than the 50% increase in the median owner income. Joint Center for Housing Studies of Harvard University (2018)

sive tax system to capture the other government tax and transfer programs aside from housing policies. The model generates a rich cross-sectional distribution over age, labor income, tenure status, housing wealth, and financial wealth. This richness is paramount to understanding the distributional effects from housing affordability policies.

The city produces tradable goods and residential housing in each zone, subject to decreasing returns to scale. Construction is subject to zoning regulation which limits the maximum amount of housing that can be built and thereby the housing supply elasticity. The city has a rent control system in place. Qualifying households enter in a housing lottery that randomly allocates housing units with steeply discounted rents. Rent control affects rents earned by landlords, which distorts the average price of rental buildings, and thereby the incentives for residential development. Wages, house prices, and market rents are determined in the city's equilibrium.

We calibrate the model to the New York metropolitan area. Our calibration matches key features of the data, including the relative size of Manhattan versus the rest of the metropolitan area, the New York metro income distribution, commuting times and costs, the housing supply elasticity, current zoning laws, the current size and scope of the rent control system, and the current federal, state, and local tax and transfer system.

We use this model as a laboratory to explore various housing affordability policies. This allows us to highlight the channels through which rent control, an important feature of urban housing markets like New York and a policy pursued by other cities with renewed interest,[2] alters the mechanisms of general equilibrium models with housing.

Rent control provides an in-kind subsidy to households with a high marginal utility from housing services, creates tension between insiders and outsiders, and gives rise to a (spatial and sectoral) misallocation of labor and housing. The baseline model is calibrated to match the prevalence of rent control (RC) in each of the two zones in the New York metro, the observed discount of RC rents to market rents, and the typical size of a RC unit. The first set of policy experiments changes each of these policy levers one at the time, in order to study the welfare effects of reducing the generosity of the RC program.

Welfare results depend on which policy lever is chosen. Consistent with received wisdom[3], reducing the share of housing devoted to RC increases the quantity of housing, improving housing affordability. The policy also concentrates more high-productivity households with a high opportunity cost of time for commuting in the city center. Surprisingly maybe, these benefits are outweighed by the large welfare losses the policy in-

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[2]Fifteen cities in California have rent control. A November 2018 California state ballot initiative will decide on whether to overturn the 1995 Costa-Hawkins Act, clearing the way for more rent control.

[3]In a 1992 survey of professional economists by Alston, Kearl, and Vaughn (1992), 92.9% agreed that "A ceiling on rents reduces the quantity and quality of housing available." Micheli and Schmidt (2015)

flicts on households who were in RC units prior to the policy change. More generally, reducing RC is a regressive policy with welfare gains that increase in income and wealth.

To explore the spatial aspect of our model, we study a policy that concentrates all affordable housing units in zone 2 while keeping the overall size of the program unchanged. In equilibrium, fewer households end up in larger RC units. The removal of RC units from zone 1 frees up space for high productivity households; the spatial allocation of productivity improves. Improved incentives for residential development result in a larger housing stock and lower rents, but not in a larger population share in the city center. Instead, the model predicts higher demand for larger housing units from the rich in zone 1. This process of gentrification raises inequality across the two zones. Yet, overall housing affordability improves and the policy benefits households in the lowest quartile of the income distribution.

The third main policy experiment concerns zoning. The baseline model is calibrated to capture the relative population size of the two NYC zones. The policy makes it easier to build in zone 1 by increasing the allowed maximum residential buildable area. The current RC system is unaffected. There are substantial welfare gains from this policy. The policy is progressive and benefits the poor strictly more than the rich. It also lowers inequality between zones by expanding the population of the urban core. It lowers total commuting time, which benefits leisure, and creates a boom in the construction sector.

In a last main policy experiment, we study housing vouchers, cash subsidies provided through the tax and transfer system to low income households for housing expenditures. An expansion of the voucher system is welfare increasing because it reduces inequality, despite increasing labor market distortions and generating a less efficient spatial allocation of labor, ultimately resulting in a substantial fall in output.

Related Literature   Our work is at the intersection of the macro-finance and urban economics literatures. On the one hand, a large literature in finance solves partial-equilibrium models of portfolio choice between housing (extensive and intensive margin), financial assets, and mortgages. Examples are Campbell and Cocco (2003), Cocco (2005), Yao and Zhang (2004), and Berger, Guerrieri, Lorenzoni, and Vavra (2017). Davis and Van Nieuwerburgh (2015) summarize this literature. More recent work in macro-finance has solved such models in general equilibrium, adding aggregate risk, endogenizing house prices and sometimes also interest rates. E.g., Favilukis, Ludvigson, and Van Nieuwerburgh (2017) and Kaplan, Mitman, and Violante (2017). Imrohoroglu, Matoba, and Tuzel (2016) study the effect of the 1978 passage of Proposition 13 which lowered property taxes in California. They find quantitatively meaningful effects on house prices, moving rates,

and welfare. Like the former literature, our model features a life-cycle and a rich portfolio choice problem. It aims to capture key quantitative features of observed wealth accumulation and home ownership over the life-cycle. Like the latter literature, house prices, rents, and wages are determined in equilibrium. Because we model one city, interest rates and tradeable goods prices are naturally taken as given. Like the macro-finance literature, we aim to capture key features of house prices, income inequality, and wealth inequality. Our key contribution to this literature is to add a spatial dimension to these macro models by introducing a cost of commuting and differing housing supply elasticity.

On the other hand, a voluminous literature in urban economics studies the spatial location of households and firms in urban areas. Brueckner (1987) summarizes the Muth-Mills monocentric city model. Rosen (1979) and Roback (1982) introduce spatial equilibrium. This literature studies the trade-off between the commuting costs and housing expenditures. These models tend to be static and households tend to be risk neutral or have quasi-linear preferences. Recent work on spatial sorting across cities includes Van Nieuwerburgh and Weill (2010), Behrens, Duranton, and Robert-Nicoud (2014) and Eeckhout, Pinheiro, and Schmidheiny (2014). Rappaport (2014) introduces leisure as a source of utility and argues that the monocentric model remains empirically relevant. Guerrieri, Hartley, and Hurst (2013) study house price dynamics in a city and focus on neighborhood consumption externalities, in part based on empirical evidence in Rossi-Hansberg, Sarte, and Owens (2010). The lack of risk, investment demand for housing by local residents, and wealth effects makes it hard to connect these spatial models to the finance literature.

Hizmo (2015) and Ortalo-Magné and Prat (2016) bridge some of this gap when they study a portfolio choice problem where households make a once-and-for-all location choice between cities. Conditional on the location choice, they are exposed to local labor income risk and make an optimal portfolio choice. They have constant absolute risk aversion preferences and consume at the end of life. The model features absentee landlords. The models are complementary to ours in that they solve a richer portfolio choice problem in closed-form, and have a location choice across cities. We solve a within-city location choice, but allow for preferences that admit wealth effects, and allow for consumption and location choice each period. In short, studying the welfare effects of housing affordability policies on the local economy requires a model with wealth effects. Our model studies spatial equilibrium within a city. Households are free to move across neighborhoods each period, rent or own, and choose how much housing to consume. We close the housing market in that local landlords who own more housing than they consume rent to other locals.

Because it is a heterogeneous-agent, incomplete-markets model, agents choices and equilibrium prices depend on the entire wealth distribution. Because of the spatial dimension, households' location is an additional state variable that needs to be kept track of. We use state-of-the-art methods to solve the model. We extend the approach of Favilukis et al. (2017), which itself extends Gomes and Michaelides (2008) and Krusell and Smith (1998) before that. The resulting model is a new laboratory which can be used to explore many important questions like the impact of rent control and zoning laws on house prices, inequality, and housing affordability. Favilukis and Van Nieuwerburgh (2018) use a similar framework to study the effect of out-of-town investors on residential property prices. A key difference is that this paper adds a spatial dimension which dramatically increases the complexity of the model , adds a lot of moments for the model to match, and delivers an entirely new set of implications for spatial inequality.

Finally, our model connects to a growing empirical literature that studies the effect of rent control and zoning policies on rents, house prices, and housing supply. Autor, Palmer, and Pathak (2014, 2017) find that the elimination of the rent control mandate on prices in Cambridge increased the value of decontrolled units and neighboring properties in the following decade, by allowing constrained owners to raise rents and increasing the amenity value of those neighborhoods through housing market externalities. The price increase spurred new construction, increasing the rental stock. Diamond, McQuade, and Qian (2017) show that the expansion of the rent control mandate in San Francisco led to a reduction in the supply of available housing, by decreasing owners' incentives to rent below market prices, paradoxically contributing to rising rents and the gentrification of the area. While beneficial to tenants in rent control, it resulted in an aggregate welfare loss. In contrast, Diamond and McQuade (forthcoming) find that the use of the Low Income Housing Tax Credit, a financial incentive for landlords to rent their properties below market prices to low-income tenants, leads to house price appreciation and decreasing segregation in low-income neighborhoods, thereby increasing welfare. As in our paper, the nature of the rent control policy and its distributional consequences are essential. In contemporaneous work, Sieg and Yoon (2017) also study the welfare gains from affordable housing policies in New York. Different from our set up, they consider public housing and rent-stabilized housing as two distinct housing options aside from market rentals. They do not allow for home ownership. Since they only focus on Manhattan, their model does not consider the equilibrium spillover effects from changes in Manhattan housing policies on the rest of the metropolitan area.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 describes the calibration to the New York metropolitan area. Section 4 discusses its

main results and implications for quantities and prices, the distribution of households and affordability. Section 5 studies counterfactual policy experiments in which we vary the nature and strength of the rent control and zoning tools. Section 6 concludes.

2 Model

The model consists of two geographies, the "urban core" and the "periphery", whose union forms the "metropolitan area" or "city." The urban core, which we refer to as zone 1, is the central business district where all employment takes place. The periphery, or zone 2, contains the outer boroughs of the city as well as the suburban areas that belong to the metropolitan area. In the context of the New York calibration in Section 3, zone 1 will be Manhattan while zone 2 contains the other 24 counties that make up the metro area. While clearly an abstraction of the more complex production and commuting patterns in large cities, the monocentric city assumption captures the essence of commuting patterns (Rappaport, 2014). The two zones differ in size, as explained below. The city has a fixed population.[4]

2.1 Households

Preferences   The economy consists of overlapping generations of risk averse households. There is a continuum of households of a given age a. Each household maximizes a utility function u over consumption goods c, housing h, and labor supply n. Utility depends on location ℓ and age a, allowing the model to capture commuting time and amenity differences across locations.

The period utility function is a CES aggregator of c and h and leisure l:

$$U(c_t, h_t, n_t, \ell_t, a) = \frac{\left[\chi_t^{\ell, a} \mathcal{C}(c_t, h_t, l_t) \right]^{1-\gamma}}{1 - \gamma}, \tag{1}$$

$$\mathcal{C}(c_t, h_t, l_t) = \left[(1 - \alpha_n) \left((1 - \alpha_h) c_t^\epsilon + \alpha_h h_t^\epsilon \right)^{\frac{\eta}{\epsilon}} + \alpha_n l_t^\eta \right]^{\frac{1}{\eta}}$$

$$h_t \geq \underline{h} \tag{2}$$

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[4]Future work could study interactions between affordability policies and net migration patterns in an open-city model. Such a model would need to take a stance on a reservation utility of moving and on the moving costs. These reservation utilities would naturally differ by age, productivity, and wealth, leading to a proliferation of free parameters. The lack of guidance from the literature would pose a substantial challenge to calibration. Furthermore, the empirical evidence for the New York metropolitan area, discussed in Appendix B.6, suggests that the zero net migration assumption fits the data well. These two considerations motivate the closed-city model assumption.

$$n_t^a = \begin{cases} 1 - \phi_T^\ell - l_t \geq \underline{n} & \text{if } a < 65 \\ 0 & \text{if } a \geq 65 \end{cases} \tag{3}$$

$$\chi_t^{\ell,a} = \begin{cases} \chi^{all}\chi^W & \text{if } \ell = 1 \text{ and } a < 65 \text{ and } c_t \geq \underline{c} \\ \chi^{all} & \text{if } \ell = 1 \text{ and } a < 65 \text{ and } c_t < \underline{c} \\ \chi^{all}\chi^R & \text{if } \ell = 1 \text{ and } a \geq 65 \text{ and } c_t \geq \underline{c} \\ \chi^{all} & \text{if } \ell = 1 \text{ and } a \geq 65 \text{ and } c_t < \underline{c} \\ 1 & \text{if } \ell = 2 \end{cases} \tag{4}$$

The coefficient of relative risk aversion is γ. The Frisch elasticity of labor supply is affected by all utility parameters, but mostly governed by η. The parameter ϵ controls the intra-temporal elasticity of substitution between housing and non-housing consumption. We impose a minimum house size requirement ($\underline{h}$), capturing the notion that a minimum amount of shelter is necessary for a household.

Total non-sleeping time is normalized to 1 and allocated to work (n_t), leisure (l_t), and commuting time ϕ_T^ℓ. We normalize commuting time for zone 1 residents to zero: $\phi_T^2 > \phi_T^1 = 0$. Since we will match income data that exclude the unemployed, we impose a minimum constraint on the number of hours worked ($\underline{n}$) for working-age households. This restriction will also help us match the correlation between income and wealth. There is an exogenous retirement age of 65. Retirees supply no labor.

The age- and location-specific taste-shifter $\chi^{\ell,a}(c_t)$ is one for all zone 2 residents. The shifter $\chi^{all} \geq 1$ increase the utility of all zone 1 residents. The shifter χ^W (χ^R) increases the utility for working-age (retired) households consuming above a threshold $\underline{c}$. This creates a complementarity between living in zone 1 and high consumption levels. This modeling device captures that certain amenities such as high-end entertainment, restaurants, museums, or art galleries are concentrated in Manhattan. By assuming that the benefit from such luxury amenities only accrue above a certain consumption threshold, rich households will congregate in the city center beyond what is implied by the opportunity cost of commuting alone. Guerrieri et al. (2013) achieve a similar outcome through a neighborhood consumption externality. A special case of the model arises for $\chi^{all} = \chi^R = \chi^W = 1$; location choice is solely determined by commuting costs. Another special case is $\underline{c} = 0$, which gives the same amenity value of Manhattan to all households, rich or poor.

There are two types of households in terms of the time discount factor. One group of households have a high degree of patience β^H while the rest have a low degree of patience β^L. This preference heterogeneity helps the model match observed patterns of wealth inequality and wealth accumulation over the life cycle. A special case of the model sets $\beta^H = \beta^L$.

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Endowments   A household's labor income y_t^{lab} depends on the number of hours worked n, the wage per hour worked W, a deterministic component $G(a)$ which captures the hump-shaped pattern in average labor income over the life-cycle, and an idiosyncratic labor productivity z, which is stochastic and persistent.

After retirement, households earn a pension which is the product of an aggregate component $\overline{\Psi}$ and an idiosyncratic component $\psi^{a,z}$. The idiosyncratic component has cross-sectional mean of one, and is determined by productivity during the last year of work. Labor income is taxed linearly at rate τ^{SS} to finance retirement income. All other taxes and transfers are captured by the function $T(\cdot)$ which maps total pre-tax income into a net tax (negative if transfer). Net tax revenue goes to finance a public good which does not enter in household utility.

Households face mortality risk which depends on age, p^a. Although there is no intentional bequest motive, agents who die leave accidental bequests. We assume that the number of people who die with positive wealth leave a bequest to the same number of agents alive of ages 21 to 65. These recipient agents are randomly chosen, with one restriction. Patient agents (β^H) only leave bequests to other patient agents and impatient agents (β^L) only leave bequests to other impatient agents. One interpretation is that attitudes towards saving are passed on from parents to children. Conditional on receiving a bequest, the size of the bequest $\widehat{b}_{t+1}$ is a draw from the relevant distribution, which differs for β^H and β^L types. Because housing wealth is part of the bequest and the house price depends on the aggregate state of the economy, the size of the bequest is stochastic. Agents know the distribution of bequests, conditional on β type. This structure captures several features of real-world bequests: many households receive no bequest, bequests typically arrive later in life and at different points in time for different households, and there is substantial heterogeneity among bequest sizes for those who receive a bequest.

Rent Control   Every household in the model enters the rent control lottery every period. A household that wins the lottery has the option to move into a rent controlled (RC) unit in a zone assigned by the lottery, provided she qualifies.[5] A winning household can choose to turn down the RC unit, and rent or own the unit of its choice in the location of its choice on the free market. RC units rent at a fraction $\kappa_1 < 1$ of the free-market rent. If the household accepts the RC lottery win, it must abide by two conditions: (i) its income must be below a cutoff, expressed as a fraction κ_2 of area median income (AMI),

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[5]There is a single lottery for all RC units. A certain lottery number range gives access to RC housing in zone 1, while a second range gives access to housing in zone 2. Households with lottery numbers outside these ranges lose the housing lottery.

and (ii), the rent paid on the RC unit must be below a fraction κ_3 of AMI. Both of these conditions are consistent with NYC rent regulation rules. The latter condition effectively restricts the maximum size of the RC unit. The probability of winning the lottery for each zone is endogenously determined to equate the demand and supply of RC units in each zone. Households form beliefs about this probability. This belief must be consistent with rational expectations, and is updated as part of the equilibrium determination.

Rent control engenders four distortions. Since labor supply is endogenous, a household which wins the RC lottery may choose to reduce hours worked in order to qualify for a RC unit. This has adverse implications for the level of output. Second, a household may choose to consume a larger amount of housing, conditional on being in a RC unit, than it would on the free market. This may lead to misallocation of housing in the cross-section of households. Third, a household who would otherwise live in zone 2 but wins the housing lottery in zone 1 may choose to live in zone 1 and vice versa. Fourth, RC blunts developers' incentives to construct housing, as explained below.

Location and Tenure Choice   Denote by $p^{RC,\ell}$ the probability of winning the RC lottery and being offered a unit in zone ℓ. The household chooses whether to accept the RC option with value $V_{RC,\ell}$, or to turn it down and go to the private housing market with value V_{free}. The value function V is:

$$V = p^{RC,1} \max \left\{ V_{RC,1}, V_{free} \right\} + p^{RC,2} \max \left\{ V_{RC,2}, V_{free} \right\} + \left(1 - p^{RC,1} - p^{RC,2} \right) V_{free}.$$

A household that loses the lottery or wins it but turns it down, freely chooses in which location $\ell \in \{1, 2\}$ to live and whether to be an owner (O) or a renter (R).

$$V_{free} = \max \left\{ V_{O,1}, V_{R,1}, V_{O,2}, V_{R,2} \right\}.$$

The Bellman equations for $V_{RC,\ell}$, $V_{R,\ell}$ and $V_{O,\ell}$ are defined below.

Let S_t be the aggregate state of the world, which includes the wage W_t, as well as the housing price P_t^ℓ, the market rent R_t^ℓ and previous housing stock H_{t-1}^ℓ for each location ℓ. The household's individual state variables are its net worth at the start of the period x_t, its idiosyncratic productivity level z_t, and its age a. We suppress the dependence on β-type in the problem formulation below, but note here that there is one set of Bellman equations for each β-type.

Market Renter Problem A renter household on the free rental market in location ℓ chooses non-durable consumption c_t, housing consumption h_t, and working hours n_t to solve:

$$V_{R,\ell}(x_t, z_t, a, S_t) = \max_{c_t, h_t, n_t, b_{t+1}} U(c_t, h_t, n_t, \ell_t) + (1 - p^a)\beta\mathbb{E}_t[V(x_{t+1}, z_{t+1}, a+1, S_{t+1})]$$

s.t.
$$c_t + R_t^\ell h_t + Qb_{t+1} + \phi_{F,t}^\ell = \left(1 - \tau^{SS}\right) y_t^{lab} + \overline{\Psi}_t \psi^z + x_t - T\left(y_t^{tot}\right),$$
$$y_t^{lab} = W_t n_t G^a z_t,$$
$$y_t^{tot} = y_t^{lab} + \left(\frac{1}{Q} - 1\right) x_t,$$
$$x_{t+1} = b_{t+1} + \widehat{b}_{t+1} \geq 0,$$
and equations (1), (2), (3), (4).

The renter's savings in the risk-free bond, Qb_{t+1}, are obtained from the budget constraint. Pre-tax labor income y_t^{lab} is the product of wages W per efficiency unit of labor, the number of hours n, and the productivity per hour $G^a z$. Total pre-tax and transfer income, y^{tot}, is comprised of labor income and financial income, which is the interest income on bonds. Next period's financial wealth x_{t+1} consists of savings b_{t+1} plus any accidental bequests $\widehat{b}_{t+1}$. Housing and labor choices are subject to minimum constraints discussed above. In addition to a time cost, residents of zone 2 face a financial cost of commuting $\phi_{F,t}^2$. Like we did for the time cost, we normalize the financial cost of commuting in zone 1 to 0.

RC Renter Problem A renter household in the RC system in location ℓ chooses non-durable consumption c_t, housing consumption h_t, and working hours n_t to solve:

$$V_{RC,\ell}(x_t, z_t, a, S_t) = \max_{c_t, h_t, n_t, b_{t+1}} U(c_t, h_t, n_t, \ell_t) + (1 - p^a)\beta\mathbb{E}_t[V(x_{t+1}, z_{t+1}, a+1, S_{t+1})]$$

s.t.
$$c_t + \kappa_1 R_t^\ell h_t + Qb_{t+1} + \phi_{F,t}^\ell = \left(1 - \tau^{SS}\right) y_t^{lab} + \overline{\Psi}_t \psi^z + x_t - T\left(y_t^{tot}\right),$$
$$x_{t+1} = b_{t+1} + \widehat{b}_{t+1} \geq 0,$$
$$y_t^{lab} \leq \kappa_2 \overline{Y}_t,$$
$$h_t \leq \frac{\kappa_3 \overline{Y}_t}{\kappa_1 R_t^\ell},$$
and equations (1), (2), (3), (4).

The rent (per square foot) of a RC unit is a fraction κ_1 of the market rent R_t^ℓ. To qualify for RC, labor income must not exceed a fraction κ_2 of area median income (AMI), $\overline{Y}_t = Median[y_t^{lab,i}]$, the median across all residents in the metro are. The last inequality says

that expenditures on rent ($\kappa_1 R_t^\ell h_t$) must not exceed a fraction κ_3 of AMI.[6] We impose the same minimum house size constraint in the RC system.

Owner's Problem   An owner in location ℓ chooses non-durable consumption c_t, housing consumption h_t, working hours n_t, and investment property $\widehat{h}_t$ to solve:

$$V_{O,\ell}(x_t, z_t, a, S_t) = \max_{c_t, h_t, \widehat{h}_t, n_t, b_{t+1}} U(c_t, h_t, n_t, \ell_t) + (1 - p^a)\beta E_t[V(x_{t+1}, z_{t+1}, a+1, S_{t+1})]$$

s.t.
$$c_t + P_t^\ell h_t + Q b_{t+1} + \kappa_4^\ell P_t^\ell \widehat{h}_t + \phi_{F,t}^\ell = \left(1 - \tau^{SS}\right) y_t^{lab} + \overline{\Psi}_t \psi^z + x_t + \kappa_4^\ell R_t^\ell \widehat{h}_t - T\left(y_t^{tot}\right),$$
$$x_{t+1} = b_{t+1} + \widehat{b}_{t+1} + P_{t+1}^\ell h_t(1 - \delta - \tau^{P,\ell}) + \kappa_4^\ell P_{t+1}^\ell \widehat{h}_t(1 - \delta - \tau^{P,\ell}),$$
$$-Q_t b_{t+1} \leq P_t^\ell\left(\theta_{res} h_t + \theta_{inv}\kappa_4^\ell \widehat{h}_t\right) - \kappa_4^\ell R_t^\ell \widehat{h}_t - \left(y_t^{tot} - c_t\right),$$
$$\widehat{h}_t \geq 0,$$
$$\kappa_4^\ell = 1 - \eta^\ell + \eta^\ell \kappa_1,$$
and equations (1), (2), (3), (4).

Local home owners are the landlords to the local renters. For simplicity, we assume that renters cannot buy investment property and that owners can only buy investment property in the zone of their primary residence. Landlords earn rental income on their investment units.

Landlords in the model are required to buy η^ℓ square feet of RC property for every $1 - \eta^\ell$ square feet of free-market rental property. This captures the institutional reality of affordable housing programs in NYC and elsewhere.[7] The effective rent earned per square foot of investment property is $\kappa_4^\ell R_t^\ell$. Since the blended rent is a multiple $\kappa_4 \leq 1$ of the market rent, the blended price of rental property must be the same multiple of the market price, $\kappa_4^\ell P_t^\ell$. Because prices and rents scale by the same constant, the return on investing in regulated units is identical to the return on investing in market units. As a result, landlords are unaffected by rent regulation. However, the lower average price

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[6]In the implementation, we assume that the income and size qualification cutoffs for RC are constants. We then compute what fractions κ_2 and κ_3 of AMI they represent. This allows us to sidestep the issue that the AMI may change with RC policies.

[7]Examples of incentives provided for the development of affordable housing in NYC are (i) the 80/20 new construction housing program, a state program that gives low-cost financing to developers who set aside at least 20% of the units in a property for lower-income families; (ii) the 421a program, which gives tax breaks (up to 25 years) for the development of under-utilized or vacant sites often conditional on providing at least 20% affordable units (i.e., used in conjunction with the 80-20 program), (iii) the Federal Low Income Housing Tax Credits (LITCH) program, which gives tax credits to developers directly linked to the number of low-income households served, and (iv) Mandatory Inclusionary Housing, a New York City program that lets developers build bigger buildings and gives them tax breaks if they reserve some of the units for (permanently) affordable housing.

for rental property ($\kappa_4 < 1$) has important effects on housing supply/development, as discussed below.

The physical rate of depreciation for housing units is δ. The term $Ph\delta$ is a financial costs, i.e., a maintenance cost. As shown in equation (10) below, the physical depreciation $Ph\delta$ can be offset by residential investment undertaken by the construction sector.

Property taxes on the housing owned in period t are paid in year $t + 1$; the tax rate is $\tau^{P,\ell}$. Property tax revenue finances local government spending which does not confer utility to the households.[8]

Housing serves as a collateral asset for debt. For simplicity, mortgages are negative short-term safe assets. Households can borrow a fraction θ_{res} of the market value of their primary residence and a potentially different fraction θ_{inv} against investment property. The empirically relevant case is $\theta_{res} \geq \theta_{inv}$. We exclude current-period rental income and savings from the pledgable collateral. In light of the fact that one period is four years in the calibration, we do not want to include four years worth of (future) rental income and savings for fear of making the borrowing constraint too loose.

In the appendix we show that for renters, the choices of h_t and n_t are analytic functions of c_t, therefore the renter's problem can be rewritten with just two choices: consumption c_t and location ℓ. For owners, the choices of h_t and n_t are analytic functions of c_t and $\widehat{h}_t$, therefore the owner's problem can be rewritten with just three choices: consumption c_t, investment property size $\widehat{h}_t$, and location ℓ.

2.2 Firms

Goods Producers   There are a large number n_f of identical, competitive firms located in the urban core (zone 1), all of which produce the numéraire consumption good.[9] This good is traded nationally; its price is unaffected by events in the city and normalized to 1. The firms have decreasing returns to scale and choose efficiency units of labor to maximize profit each period:

$$\Pi_{c,t} = \max_{N_{c,t}} N_{c,t}^{\rho_c} - W_t N_{c,t} \tag{5}$$

We assume that these firms are owned by equity owners outside the city. In robustness

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[8]Alternatively, one could solve a model where the public good enters the utility function. That requires taking a stance on the intra-temporal elasticity of substitution between private and public consumption.

[9]We assume that the number of firms is proportional to the number of households in the city when solving the model. With this assumption, our numerical solution is invariant to the number of households. Due to decreasing return to scale, the numerical solution would depend on the number of households otherwise.

analysis, we consider a model where half of the profits accrue to locals.

Developers and Affordable Housing Mandate In each location ℓ there is a large number n_f of identical, competitive construction firms (developers) which produce new housing units and sell them locally. All developers are headquartered in the urban core, regardless of where their construction activity takes place. Like goods firms, construction firms are owned by equity owners outside the city. Again, we relax this assumption in robustness.

The cost of the affordable housing mandate is born by developers. Affordable housing regulation stipulates that for every square foot of market rental units built in zone ℓ, η^ℓ square feet of RC units must be built. Developers receive an average price per foot for rental property of $\kappa_4^\ell P_t^\ell$, while they receive a price per foot of P_t^ℓ for owner-occupied units. Given a home ownership rate in zone ℓ of ho_t^ℓ, developers receive an average price per foot of:

$$\overline{P}_t^\ell = \left(ho_t^\ell + (1 - ho_t^\ell)\kappa_4^\ell \right) P_t^\ell. \tag{6}$$

The cost of construction of owner-occupied and rental property in a given location is the same. After completion of construction but prior to sale, some of the newly constructed housing units are designated as rental units and the remainder as ownership units. The renter-occupancy designation triggers affordable housing regulation. It results in a lower rent and price than for owner-occupied units. Developers would like to sell ownership units rather than rental units, but the home ownership rate is determined in equilibrium. Developers are price takers in the market for space, and face an average sale price of $\overline{P}_t^\ell$.

A special case of the model is the case without rent control: $\kappa_4^\ell = 1$ either because $\eta^\ell = 0$ or $\kappa_1 = 1$. In that case, $\overline{P}_t^\ell = P_t^\ell$. Without rent control, the higher sale price for housing increases incentives to develop more housing.

Zoning Given the existing housing stock in location ℓ, H_{t-1}^ℓ, construction firms have decreasing returns to scale and choose labor to maximize profit each period:

$$\Pi_{h,t}^\ell = \max_{N_{\ell,t}} \overline{P}_t^\ell \left(1 - \frac{H_{t-1}^\ell}{\overline{H}^\ell} \right) N_{\ell,t}^{\rho_h} - W_t N_{\ell,t} \tag{7}$$

The production function of housing has two nonlinearities. First, as for consumption good firms, there are decreasing returns to scale because $\rho_h < 1$.

Second, construction is limited by zoning laws. The maximal amount of square footage zoned for residential use in zone ℓ is given by $\overline{H}^\ell$. We interpret $\overline{H}^\ell$ as the total land area

zoned for residential use multiplied by the maximum permitted number of floors that could be built on this land, the floor area ratio (FAR). This term captures the idea that, the more housing is already built in a zone, the more expensive it is to build additional housing. For example, additional construction may have to take the form of taller structures, buildings on less suitable terrain, or irregular infill lots. Therefore, producing twice as much housing requires more than twice as much labor. Laxer zoning policy, modeled as a larger $\overline{H^\ell}$, makes development cheaper, and all else equal, will expand the supply of housing.

When $\overline{H^\ell}$ is sufficiently high, the model's solution becomes independent of $\overline{H^\ell}$, and the supply of housing is governed solely by ρ_h. When $\overline{H^\ell}$ is sufficiently low, the housing supply elasticity depends on both $\overline{H^\ell}$ and ρ_h.[10]

2.3 Equilibrium

Given parameters, a competitive equilibrium is a price vector $(W_t, P_t^\ell, R_t^\ell)$ and an allocation, namely aggregate residential demand by market renters $H_t^{R,\ell}$, RC renters $H_t^{RC,\ell}$, and owners $H_t^{O,\ell}$, aggregate investment demand by owners $\widehat{H}_t^\ell$, aggregate housing supply, aggregate labor demand by goods and housing producing firms $(N_{c,t}, N_{\ell,t})$, and aggregate labor supply N_t such that households and firms optimize and markets clear.

The following conditions characterize the equilibrium. First, given wages and average prices given by (6), firms optimize their labor demand, resulting in the first-order conditions:

$$N_{c,t} = \left(\frac{\rho_c}{W_t} \right)^{\frac{1}{1-\rho_c}} \quad \text{and} \quad N_{\ell,t} = \left(\frac{\left(1 - \frac{H_{t-1}^\ell}{\overline{H^\ell}}\right) \overline{P}_t^\ell \rho_h}{W_t} \right)^{\frac{1}{1-\rho_h}}. \tag{8}$$

Second, labor demand equals labor supply:

$$n_f \left(N_{c,t} + \sum_\ell N_{\ell,t} \right) = N_t. \tag{9}$$

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[10]In this sense, the model captures that construction firms must pay more for land when land is scarce or difficult to build on due to regulatory constraints. This scarcity is reflected in equilibrium house prices.

Third, the housing market clears in each location ℓ:

$$(1 - \delta)H_{t-1}^{\ell} + n_f \left(1 - \frac{H_{t-1}^{\ell}}{\overline{H}^{\ell}}\right) N_{\ell,t}^{\rho_h} = H_t^{O,\ell} + \widehat{H}_t^{\ell}. \tag{10}$$

The left-hand-side is the supply of housing which consists of the non-depreciated housing stock and new residential construction. The right-hand-side is the demand for those housing units by owner-occupiers and landlords. Fourth, the supply of rental units in each location ℓ must equal the demand from market tenants and RC tenants:

$$\widehat{H}_t^{\ell} = H_t^{R,\ell} + H_t^{RC,\ell} \tag{11}$$

Fifth, total pension payments equal to total Social Security taxes collected:

$$\overline{\Psi}_t N_{ret} = \tau^{SS} N_t W_t, \tag{12}$$

where N_{ret} is the total number of retirees, which is a constant, and N_t are total efficiency units of labor. Sixth, the aggregate state S_t evolves according to rational expectations. Seventh, the value of all bequests received is equal to the wealth of all agents who die.

2.4 Welfare Effects of Affordability Policies

We compute the welfare effect of an affordability policy using the following procedure. Denote agent i's value function under benchmark policy θ_b as $V_i(x, z, a, S; \theta_b)$. Consider an alternative policy θ_a with value function $V_i(x, z, a, S; \theta_a)$. The alternative policy implies a change Δ_i in consumption equivalent units relative to the benchmark policy, where:

$$\mathcal{W}_i = \left(\frac{V_i(\theta_a)}{V_i(\theta_b)}\right)^{\frac{1}{(1-\gamma)(1-\alpha_n)}} - 1. \tag{13}$$

We compute aggregate welfare effects from a policy change by summing $\mathcal{W}_i$ across agents, calling the resulting aggregate welfare measure $\mathcal{W}$. We also sum separately among owners, market renters, and RC renters, for different age groups, and for different income and wealth groups. In addition to computing steady state welfare effects, we can also compute transition paths that take into account the (slow) adjustment of the endogenous state variables to the new policy.

3 Calibration

We calibrate the model to the New York metropolitan area. Data sources and details are described in Appendix B. Table 1 summarizes the chosen model parameters.

Geography   The New York metro consists of 25 counties located in New York (12), New Jersey (12), and Pennsylvania (1). We assume that Manhattan (New York County) represents zone 1 and the other 24 counties make up zone 2.[11] The zones differ in the maximum buildable residential square footage, captured by $\overline{H}$. Appendix B describes detailed calculations on the relative size of Manhattan and the rest of the metro area, which imply that $\overline{H^1} = 0.0238 \times \overline{H^2}$. We then choose $\overline{H^2}$ such that the fraction of households living in zone 1 equals 10.5% of the total, the fraction observed in in the data. Since the model has no vacancies, we equate the number of households in the model with the number of occupied housing units in the data.

Demographics   The model is calibrated so that one model period is equivalent to 4 years. Households enter the model at age 21, work until age 64, and retire with a pension at age 65. Survival probabilities $p(a)$ are calibrated to mortality data from the Census Bureau. People age 65 and over comprise 19.1% of the population age 21 and over in the data. In the model, we get 21.8%. The average New York metro resident is 47.6 years old in the data and 47.4 years old in the model.

Labor Income   Recall that pre-tax labor income for household i of age a is $y_t^{lab} = W_t n_t^i G^a z_t^i$, where the household takes wages as given and chooses labor supply n_t^i. The choice of hours is subject to a minimum hours constraint, which is set to 0.6 times average hours worked. This constraint rules out a choice of a positive but very small number of hours, which we do not see in the data given the indivisibility of jobs. It also rules out unemployment since our earnings data are for the (part-time and full-time) employed. This constraint binds for 5% of workers in equilibrium. The presence of the constraint enables the model to better match the observed correlation between wealth and income.

Efficiency units of labor $G^a z_t^i$ consist of a deterministic component that depends on age (G^a) and a stochastic component z^i that captures idiosyncratic income risk. The G^a function is chosen to enable the model to match the mean of labor earnings by age. We use

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[11]Alternative choices are to designate (i) New York City (five counties coinciding with the five boroughs of NYC) as zone 1 and the rest of the metro as zone 2, or (ii) Manhattan as zone 1 and the other four counties in New York City as zone 2. Both choices ignore that the dominant commuting pattern is from the rest of the metro area to Manhattan.

data from ten waves of the Survey of Consumer Finance (SCF) from 1983-2010 to estimate G^a.

The idiosyncratic productivity process z is chosen to both match earnings inequality in the NY metro data and to generate realistic persistence in earnings. We discretize z as a 4-state Markov chain. The grid points differ by age in order to capture that the variance of earnings rises in age. We use the SCF data to discipline this increase in variance by age. The four grid points at the average age are chosen to match the NY metro pre-tax earnings distribution. We choose annual household earnings cutoffs in the data of $41,000, $82,000, and $164,000. This results in four earnings groups with average earnings of $28,125, $60,951, $116,738, and $309,016. Average New York MSA household earnings are $124,091. The four point grid for productivity z is chosen to match the average earnings in each group. This is an iterative process since labor supply is endogenous and depends on all other parameters and features of the model.

The transition probability matrix for z^i is age-invariant, but is allowed to depend on β type. Specifically, the expected duration of the highest productivity state is higher for the more patient agents. There are five unique parameters governing transition probabilities which are pinned down by five moments in the data. The four income groups have population shares in the data of 16.1%, 29.8%, 34.2%, and 19.9%, respectively. Since the shares sum to 1, that delivers three restrictions on the transition matrix. Matching the persistence of labor income to 0.9 delivers a fourth restriction. Finally, the dependence on β is calibrated to deliver the observed correlation between income and wealth in the SCF data. Appendix C contains the parameter values and further details.

Taxation Since our model is an incomplete markets model in which housing affordability policies can act as an insurance device and "complete the market," it is important to realistically calibrate the redistribution provided through the tax code. We follow Heathcote, Storesletten, and Violante (2017) and choose an income tax schedule that captures the observed progressivity of the U.S. tax code in a parsimonious way:

$$T(y^{tot}) = y^{tot} - \lambda(y^{tot})^{1-\tau}$$

The parameter τ governs the progressivity of the tax and transfer system. We set $\tau = 0.17$ to match the average income-weighted marginal tax rate of 34% for the U.S. It is close to the value of 0.18 estimated by Heathcote et al. (2017). We set λ to match state and local

government spending to aggregate income in the NY metro area, equal to 15-20%.[12] This delivers $\lambda = 0.75$. Appendix D shows the resulting taxes and transfers along the income distribution.

Retirement Income   Social Security taxes and receipts are treated separately from the tax and transfer system. Social security taxes are proportional to labor earnings and set to $\tau^{SS} = 0.10$. Retirement income is increasing in the household's last productivity level prior to retirement, but is capped for higher income levels. We use actual Social Security rules to estimate each productivity group's pension relative to the average pension. The resulting pension income states are $\psi^z = [0.520, 1.147, 1.436, 1.436]$, where z reflects the last productivity level prior to retirement. They are multiplied by average retirement income $\overline{\Psi}$, which is endogenously determined in equation (12) to balance the social security budget. Average retirement income $\overline{\Psi}$ is \$43,100, which corresponds to 35% of average earnings.

Commuting Cost   We choose the time cost to match the time spent commuting for the average New York metro area resident. This time cost is the average of all commutes, including those within Manhattan. Since the model normalizes the commuting time within zone 1 to zero, we target the additional commuting time of zone 2 residents. The additional commuting time amounts to 25 minutes per trip for 10 commuting trips per week.[13] The 4.2 hours represent 3.7% of the 112 hours of weekly non-sleeping time. Hence, we set $\phi_T^2 = 0.037$.

The financial cost of commuting ϕ_F^2 is set to 1.1% of average labor earnings, or \$1,387 per household per year. This is a reasonable estimate for the commuting cost in excess of the commuting cost within Manhattan, which is normalized to zero.[14]

We assume that retirees have time and financial commuting costs that are 1/3 of those of workers. This captures that retirees make fewer trips, travel at off-peak hours, and

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[12]Depending on what share of NY State spending goes to the NY metro area, we get a different number in this range.

[13]The 25 minute additional commute results from a 15 minute commute within Manhattan and a 40 minute commute from zone 2 to zone 1. With 10.5% of the population living in Manhattan, the average commuting time is 37.4 minutes per trip or 6.2 hours a week. This is exactly the observed average for the New York metro from Census data.

[14]In NYC, an unlimited subway pass costs around \$1,400 per year per person. Rail passes from the suburbs cost around \$2400 per year per person, depending on the railway station of departure. The cost of commuting by car is at least as high as the cost of rail once the costs of owning, insuring, parking, and fueling the car and tolls for roads, bridges, and tunnels are factored in. If we assume that zone 1 residents need a subway pass while zone 2 residents need a rail pass, the cost difference is about \$1000 per person. With 1.66 workers per household, the cost difference is \$1660 per household, close to our parameter choice.

Table 1: Calibration

Description	Parameter	Value	Target
Panel A: Geography and Income			
Productivity states	Z	[0.255 0.753 1.453 3.522]	Income inequality - see App. C
Transition prob.	$(p_{11}, p_{22}, p_{33}, p_{44}, p^H)$	(.9299,.9216,.9216,.8198,.02)	Pop. shares of income groups, persistence of income - see App. C
Tax progressivity	τ	0.17	Heathcote et al. (16)
Tax burden	λ	0.75	Pension tax revenue
Average pension	$\overline{\Psi}$	0.1597	Average replacement rate of 35%
Pension distr.	ψ^z	[0.520 1.147 1.436 1.436]	U.S. pension rules
Available space	$(\overline{H^1}, \overline{H^2})$	(0.06,2.48)	Max. residential buildable area, pop. share Manhattan
Time-Commuting cost	ϕ^2_T	0.037	Average commuting time NY
Financial-Commuting cost	ϕ^2_F	0.005	Average commuting cost NY
Panel B: Preferences			Standard value
Risk aversion	γ	5	
Leisure weight	α_n	0.625	Average 42.8-hour workweek
Nondurable consumption weight	α_c	0.55	
Housing consumption weight	α_h	0.45	Avg. rent/avg. income of 23%
CES parameter consumption/housing	ϵ	-0.5	Elasticity of substitution consumption/housing of 0.60
Minimum hours worked	$\underline{n}$	0.15	Min. hours worked of 15% of available time
Time Preference (4yr)	(β^H, β^L)	(1.3080,1.0170)	Average wealth/income 5.69 and wealth Gini 0.80 in SCF
Cons. externality threshold	$\underline{\varsigma}$	0.45	House-price/income ratio of zones in NY data
Extra utility zone 1 workers	χ^W	1.00214	Income ratio of zones in NY data
Extra utility zone 1 retirees	χ^R	1.03636	Fraction of retirees ratio of zones in NY data
Extra utility zone 1 all	χ^{all}	1.09099	Rent/sqft ratio of zones in NY data
Panel C: Finance, Housing, Construction			
Bond Price (4yr)	Q	0.903	Annual real bond yield
Maximum residential LTV	θ_{res}	0.80	Mortgage underwriting standards
Maximum investment LTV	θ_{inv}	0.80	Mortgage underwriting standards
Property tax (4yr)	$(\tau^{P,1}, \tau^{P,2})$	(0.0292,0.0532)	Property tax rate in Manhattan and NY metro area
Depreciation rate	δ	.0954	Residential depreciation rate of 2.39% per year
Minimum housing size	$\underline{h}$	0.25	Legal minimum housing size
Panel D: Production			
Return to scale consumption sector	ρ_c	0.66	Labor income share of 2/3
Return to scale housing sector	ρ_h	0.66	Housing supply elasticity for NY
Panel E: Rent Control			
Fraction rent control	(η^1, η^2)	(24.26%,9.53%)	Fraction of households in RC of 13% and 4.7%
Rental discount	κ_1	50%	Observed rental discount
Income threshold for RC	κ_2	38%	Fraction of income Q1 hhs in RC
Rental share threshold for RC	κ_3	64%	Fraction of income Q3/Q4 hhs in RC

receive transportation discounts.

Preferences   The functional form for the utility function is given in equation (1). We set risk aversion $\gamma = 5$, a standard value in the asset pricing literature.

The observed average workweek for New York metro residents is 42.8 hours or 38.2% of available non-sleeping time. Since there are 1.66 workers on average per household, household time spent working is 38.2%$\times$ 1.66/2=31.7%. We set α_n to match household time spent working. The model generates 31.3% of time worked.

We set α_h in order to match the ratio of average market rent to metro-wide average income. Income data discussed above and rental data from Zillow, detailed in Appendix B, indicate that this ratio is 23% for the New York MSA in 2015. The model generates 23%.

We set $\beta^H = 1.3080$ (1.07 per year) and $\beta^L = 1.0170$ (1.00 per year). A 25% share of agents has β^H, the rest has β^L. This delivers an average β of 1.09, chosen to match the average wealth-income ratio which is 5.69 in the 1998-2010 SCF data. The model generates 5.64. The dispersion in betas delivers a wealth Gini coefficient of 0.80, exactly equal to the observed wealth Gini coefficient of 0.80 for the U.S.[15] Note that because of mortality, the effective time discount rate is $(1 - p(a))\beta$.

For the taste-shifter for zone 1, we choose $\chi^{all} = 1.09099$, $\chi^W = 1.00214$, $\chi^R = 1.03636$, $\underline{c} = 0.45$. The latter number implies that 10% of the population derives extra utility from living in Manhattan. We chose these four parameters to get our model to better match the following four ratios of zone 1 relative to zone 2 variables, given all other parameters: the relative fraction of retirees of 0.91, a relative household income ratio of 1.66, the relative ratio of market rents per square foot of 2.78, and the relative home ownership rate of 0.42. In the model, these ratios are 0.93, 1.64, 2.77, and 0.95, respectively.

Housing   We choose a price for the one-period (4-year) bond of $Q = 0.903$, to match the average house price to rent ratio for the New York metro, which is 17.4. The model delivers 18.6. Under the logic of the user cost model, the price-to-rent ratio depends on the interest rate, the depreciation rate, and the property tax rate.

We set the maximum loan-to-value ratio (LTV) for the primary residence at 80% ($\theta_{res} = 0.8$), implying a 20% down payment requirement. This is the median downpayment in the U.S. data on purchase mortgages. The LTV for investment property is also set at 80% ($\theta_{inv} = 0.8$).

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[15]No wealth data is available for the NY metro. We believe it is likely that wealth inequality is at least as high in the NY metro than in the rest of the U.S.

We set the property tax rate in Manhattan equal to $\tau^{P,1} = 0.0292$ or 0.73% per year, and the property tax rate in zone $\tau^{P,2} = 0.0532$ or 1.33% per year. These match the observed tax rates averaged over 2007-2011 according to the Brookings Institution. The zone-2 property tax rate is computed as the weighted average across the 24 counties, weighted by the number of housing units.

We assume that property depreciates at 2.39% per year and set $\delta = 0.0946$. This is the average depreciation rate for privately-held residential property in the BEA Fixed Asset tables for the period 1972-2016.

Finally, we impose a minimum housing size of 490 square feet, a realistic value for New York. This represents 35% of the average housing unit size of 1445 square feet.

Production and Construction We assume that the return to scale $\rho_c = 0.66$. This value implies a labor share of 66% of output, consistent with the data.

For the housing sector, we set $\rho_h = 0.66$ in order to match the housing supply elasticity, given the other parameters. The long-run housing supply elasticity in the model is given in appendix E. Saiz (2010) reports a housing supply elasticity for the New York metro area of 0.76. The model delivers 0.76. The housing supply elasticity is much lower in zone 1 (0.015) than in zone 2 (0.76), because in zone 1 the housing stock is much closer to $\overline{H}$ (15% from the constraint) than in zone 2 (73% from the constraint). Since the housing stock of the metro area is concentrated in zone 2, the city-wide elasticity is dominated by that in zone 2.

Rent Control Rent regulation plays a major role in the New York housing market. Using data from the New York City Housing and Vacancy Survey and county-level data on affordable housing for the New York metro area counties outside of New York City, we define RC housing as those units that are (i) rent controlled, (ii) public housing, (iii) Mitchell Lama housing, (iv) all other government-assisted or regulated housing. Appendix B.5 contains a detailed description of our rent regulation data and definitions. According to this definition, 13.0% of zone 1 households and 4.7% of zone 2 households live in RC units. The metro-wide average is 5.57%, the ratio of zone 1 to zone 2 is 2.77. We set the share of square feet of housing devoted to RC units, $\eta^1 = 24.26\%$ and $\eta^2 = 9.53\%$, to match the fraction of households that are in RC units in each zone. This fraction is endogenous since housing size is a choice variable.

According to the same definition and data sources, the average rent in RC units is 50% below that in all other rentals. We set $\kappa_1 = 0.50$ to the observed rent discount. It follows that $\kappa_4^1 = 0.88$ and $\kappa_4^2 = 0.95$, so that landlords earn 12% lower rents in zone 1 and 5%

lower rents in zone 2 than they would in an unregulated market.

We set the income qualification threshold to a fraction $\kappa_2 = 38\%$ of AMI. The target for this parameter is the fraction of households who are in RC housing, conditional on being in the first quartile of the metro-wide household income distribution. For the RC size threshold, we set $\kappa_3 = 64\%$ to match the fraction of households who are in RC, conditional on being in the top half of the income distribution. High-income agents in the model want to live in a house that is larger than the maximum allowed size under RC and would turn down a RC lottery win.

4   Baseline Model Results

We start by discussing the implications of the baseline model for the spatial distribution of population, housing, income, and wealth. We also discuss house prices and rents for the city as a whole and for the two zones. Then we look at the model's implications for income, wealth, and home ownership over the life-cycle.

4.1   Demographics, Income, and Wealth

Demographics   The first three rows of Table 2 show that the model matches basic demographic moments. One moment not targeted by the calibration is the relative age of zone 1 to zone 2 households. The average Manhattan resident is younger (41) than the average resident in the rest of the metro (48). The model generates the same age gap. On the one hand, younger households are in the working phase of life, when proximity to work is valuable. Younger agents also tend to have lower income, which increases the importance of not having to shoulder the financial cost of commuting. On the other hand, they may not be able to afford the high rents in Manhattan until later in their life-cycle when earnings are higher and they have been able to save more. The model strikes the right balance between these two effects.

In the data, retirees represent 17.6% of Manhattan residents, and the model matches this share. Retirees have lower time and financial costs of commuting, weakening their incentives to live in Manhattan. This captures the fact that they don't have to commute to a job. On the other hand, retirees tend to be wealthier making living in Manhattan financially feasible. Absent a taste for Manhattan, the commuting cost effect would allocate most retirees in zone 2. A fairly strong retiree preference for living in Manhattan is needed to offset the commuting effect ($\chi^{all} = 1.09099$, $\chi_R = 1.03636$).

Table 2: New York Metro Data Targets and Model Fit

|		Data		Model	
		metro	ratio zone 1/zone 2	metro	ratio zone 1/zone 2
1	Households (thousands)	7124.9	0.12	7124.9	0.12
2	Avg. hh age, cond over 20	47.6	0.95	47.4	0.90
3	People over 65 as % over 20	19.1	0.91	21.8	0.93
4	Avg. house size (sqft)	1445	0.59	1451	0.64
5	Avg. pre-tax lab income ($)	124091	1.66	124395	1.64
6	Home ownership rate (%)	51.5	0.42	58.7	0.95
7	Median mkt price per unit ($)	510051	3.11	428067	2.09
8	Median mkt price per sqft ($)	353	5.24	394	3.01
9	Median mkt rent per unit (monthly $)	2,390	1.65	1919	1.92
10	Median mkt rent per sqft (monthly $)	1.65	2.78	1.76	2.77
11	Median mkt price/median mkt rent (annual)	17.35	1.72	18.59	1.09
12	Mkt price/avg. income (annual)	3.99	1.71	3.44	1.28
13	Avg. rent/avg. income (%)	23.0	1.00	23.1	1.03
14	Avg. rent/income ratio for renters (%)	42.0	0.93	33.4	1.29
15	Rent burdened (%)	53.0	0.89	35.9	0.09
16	Severely rent burdened (%)	31.0	0.88	10.4	3.92
17	% Rent regulated of all housing units	5.57	2.77	5.47	2.94

Notes: Columns 2-3 report the values for the data of the variables listed in the first column. Data sources and construction are described in detail in Appendix B. Column 3 reports the ratio of the zone 1 value to the zone 2 value in the data. Column 5 reports the same ratio in the model.

Housing Units In the data, the typical housing unit is much smaller in Manhattan than in the rest of the metro area. We back out the typical house size (in square feet) in each county from the median house value and the median house value per square foot, using 2015 year-end values. We obtain an average housing unit size of 1,445 sqft on average, but only 826 sqft in Manhattan. The ratio of zone 1 to zone 2 is 0.59. In the model, households freely choose their housing size subject to a minimum house size constraint. The model generates a ratio of house size in zone 1 to zone 2 of 0.64.

Figure 1 shows the distribution of house sizes in the model (left). It matches the data quite well (right), even though these moments are not targeted by the calibration. The model also captures the fact that the distribution of owner-occupied housing is shifted to the right from the distribution of renter-occupied housing units.

Income Average income in the metro area matches the data (row 5 of Table 2) by virtue of the calibration. The ratio of average income in zone 1 to zone 2 is 1.64 in the model and 1.66 in the data. The productivity distribution is substantially different in the two zones. Zone 1 contains workers that are on average 57% more productive than in zone 2. Productive working-age households have a high opportunity cost of time and prefer to live close to work given the time cost of commuting. The high opportunity cost of time is the dominant effect. Mitigating this effect is the high cost of living in Manhattan.

Figure 1: House size distribution in Model (L) and Data (R)
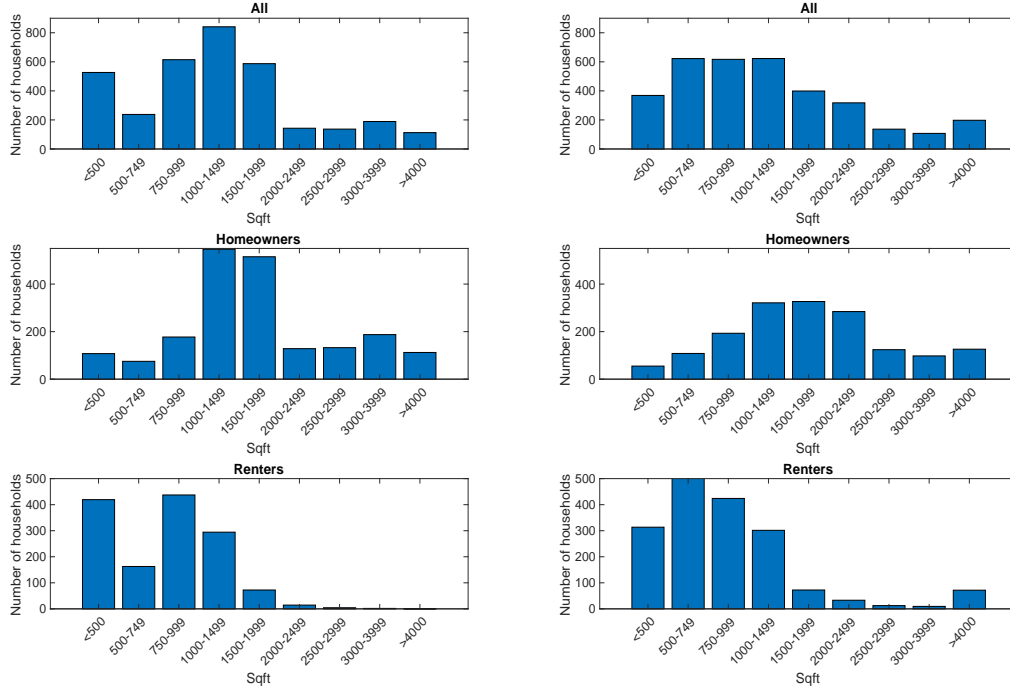
Notes: Left panel: model. Right panel: data. Data source: American Housing Survey for the New York MSA, U.S. Census Bureau, 2015.

indeed, some high-productivity workers may still be early in the life-cycle when earnings are lower and accumulated wealth smaller. Also contributing to the income gap between zone 1 and zone 2 is the luxury amenity value of living in Manhattan for working-age households, χ^W. Our calibration shows that its value is close to 1, implying that the opportunity cost of commuting effect alone is (more than) strong enough to generate the observed income gap.

The top left panel of Figure 2 shows household labor income over the life-cycle, measured as pre-tax earnings during the working phase and social security income in retirement. We plot average income in the bottom 25% of the income distribution, in the middle of the income distribution (25-50%), in the top 25% of the distribution, and the overall average income. Labor income has the familiar hump-shaped profile over the life-cycle inherited from the deterministic productivity process G^a.

The model generates a large amount of income inequality at every age. The model's earnings Gini of 0.46 matches the 0.47 value in the 2015 NY metro data.[16] Earnings in-

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[16]The Gini in the data is calculated by fitting a log-normal distribution to the mean and median of earnings.

equality in the model is similar within zone 1 (Gini of 0.44) and within zone 2 (Gini of 0.46).

The top right panel of Figure 2 plots average income profiles by zone. Average income is higher in Manhattan throughout the life cycle, for the reasons explained above.

Wealth   The model makes predictions for average wealth, the distribution of wealth across households, as well as how that wealth is spatially distributed. Average wealth to average total income (y^{tot}) in the metro area is 5.6. Wealth inequality is high, with a wealth Gini coefficient of 0.80. Both match the data by virtue of the calibration.

The probability of receiving a bequest equals the number of households between ages 21 and 65 divided by the number of dead households. It is equal to 10% over each 4-year period, and identical for β^H and β^L household types. Under our calibration, about 1.2% of wealth is bequeathed each year. This number matches the data. Some agents with low productivity who receive a bequest decide not to work and choose to live in zone 2.

The model predicts a ratio of average wealth in zone 1 to zone 2 of 1.20. While this number is not observable in the data, it seems reasonable to expect a geographic wealth inequality pattern that mirrors that in income inequality.

The middle left panel of Figure 2 shows household wealth over the life cycle at the same *income* percentiles as in the top panel. It shows that the model endogenously generates substantial wealth accumulation for the average New York resident and a very large amount of wealth inequality between income groups. This wealth inequality grows with age. The middle right panel shows how average wealth evolves differentially across zones.

Mobility   The model implies realistic moving rates from zone 1 to zone 2 and vice versa. Figure 7 in the Appendix shows that mobility is highest for the youngest agents (21-25) and for middle-aged households (35-40). For these two groups, the annual mobility rate is 7-8% annually. The overall mobility rate across neighborhoods in the model is about 3% annually. These data are consistent with the facts for NYC.[17]

4.2   Home Ownership, House Prices, and Rents

Next, we explore the model's predictions for home ownership, house prices, and rents. The model manages to drive a large wedge between house prices, rents, income, and

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[17]The 2011 Housing and Vacancy Survey shows that 40% of New York City residents moves in the past five years. About 40% of these moves are across boroughs and an additional 35% across neighborhoods, according to a report by Brick.

Figure 2: Income, Wealth, and Home Ownership over the Life-Cycle
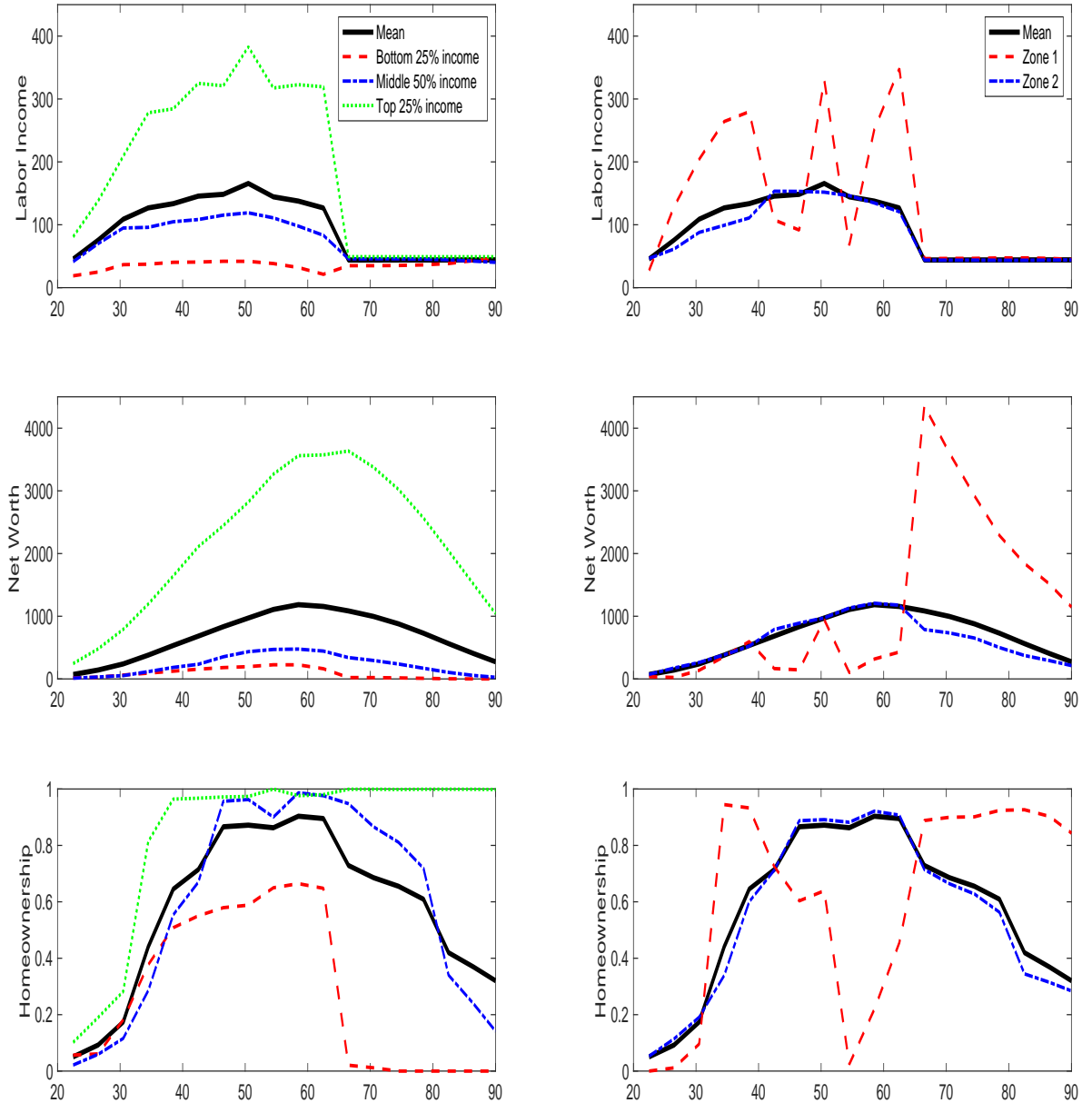
home ownership rates between zones 1 and 2 for realistic commuting costs. These mobility rates are not targeted by the calibration and are modest despite the absence of moving costs.

Home Ownership The model generates a home ownership rate of 58.7%, fairly close to the 51.5% in the New York metro. The bottom left panel of Figure 2 plots the home ownership rate over the life-cycle. It displays a hump-shape over the life-cycle with interesting variation across income groups. The life-cycle patterns broadly match the data.

Row 6 of Table 2 shows that the observed home ownership rate in Manhattan, at 23.1%, is far below that in the rest of the metro area, at 54.9%. The ratio of these two numbers is 0.42. The model generates a home ownership rate of 56.5% for Manhattan and a rate of 58.9% for the rest of the metro area. Thus the model's prediction for home ownership in zone 2, where 89.5% of the population lives is close to the data, but the model fails to generate the very low home ownership rate for the 10.5% that lives in Manhattan. This failure is closely connected to the model's failure to generate enough of a wedge in the price-rent ratios across zones. Intuitively, since owning is only a little bit more expensive than renting in zone 1 relative to zone 2, the home ownership rate in zone 1 is only a little bit lower than in zone 2. In the data, both wedges are substantially larger. We return to the price-rent ratio wedge below.

Market Prices and Rents Turning to rents and house prices, row 7 of Table 2 shows the median price per housing unit, row 8 the median price per square foot (the ratio of rows 8 and 2), row 9 the median rent per unit, and row 10 the median rent per foot. In the data, we use the Zillow home value index (ZHVI) to measure the median price of owner-occupied units, the Zillow median home value per square foot, and the Zillow rental index (ZRI) for the median rent. These indices are available for each county in the New York metro, and we use the year-end 2015 values. Zillow excludes non-arms' length transactions and rent-controlled rentals. To aggregate across the 24 counties in zone 2, we calculate the median price as the weighted average of the median prices in each county, where the weights are the shares of owner-occupied units. Similarly, for the median rent of zone 2, we average median rents of the 24 counties using renter-occupied unit shares as weights. Zillow uses a machine-learning algorithm that ensures that the ZHVI and ZRI pertain to the same, typical, constant-quality unit, in a particular geography. The ratio of the ZHVI to the ZRI in a county, is the price-rent ratio (row 11).

To ensure consistency with the empirical procedure, we calculate the median house size in each zone including both owner- and renter-occupied units (but excluding RC units) in the model. Call these $\bar{h}^\ell$. We form the median price per unit as the product of the market price times the typical unit size $P^\ell \bar{h}^\ell$. The market rent is $R^\ell \bar{h}^\ell$. The price-rent ratio is simply $P^\ell \bar{h}^\ell / R^\ell \bar{h}^\ell = P^\ell / R^\ell$.

The median house value in the NY metro area is \$510,051 in the data compared to

$428,067 in the model. The median is $1.3 million in Manhattan and $417,000 outside Manhattan in the data, a ratio of 3.11. This 3.11 house value ratio is the product of a house size ratio of 0.59 and a price per sqft ratio of 5.24. The model generates a ratio of prices of 2.09. It overstates the ratio of relative house sizes a bit (0.64) and understates the price per sqft ratio substantially (3.01).

The Zillow data indicate a monthly rent on a typical market-rate unit of $2,390 per month in the metro area. The model predicts $1,919. The ratio of rents per unit in zone 1 to zone 2 is 1.65 in the data, with a higher value of 1.92 obtained by the model. The ratio of rents per square foot in zone 1 to zone 2 is 2.78 in the data and closely matched by the 2.77 in the model.

The model is close to the metro-wide price/rent ratio level of 17.35 (row 11). A simple user cost model would imply a steady state 4-year price-rent ratio of $(1 - Q \times (1 - \delta - \tau^P))^{-1} = 4.36$ or 17.4 when rent is expressed in annual terms (with τ^P a weighted average of property tax rates in zones 1 and 2). The model has borrowing constraints so that housing has an additional collateral value component which increases its price. There also is a small housing risk premium, which lowers the price.

The proximity to jobs and proximity to amenities are the reasons why the model generates higher housing demand in Manhattan. Because of the highly inelastic housing supply in Manhattan, this translates into higher house prices and higher rents. The house price per sqft is 3.01 times as high in Manhattan than in the rest of the metro, while the rent per square foot is 2.77 times as high. In the data, the price-rent ratio in Manhattan is 26.65, or 1.72 times the 15.52 value in zone 2. In the model that ratio is 1.09. In other words, the model generates too little spatial variation in price-to-rent ratios. Only property taxes drive a wedge between house prices in the two regions. Outside of the model, houses in Manhattan may be less risky which would increase the price-rent ratio.[18] Another feature of the real world that is absent from the model is that owner-occupied housing in Manhattan may be of higher quality than in zone 2. In the absence of multiple types of housing, one way of capturing quality differences would be through a lower depreciation rate in zone 1 than in zone 2.

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[18]The model has no aggregate risk and risk aversion is only 5. To generate meaningful variation in housing risk premia would require, for example, changes in mortgage lending standards as in Favilukis et al. (2017). If homeowners in Manhattan are exposed less to changes in mortgage lending standards, housing in Manhattan would carry a lower risk premium. This would increase the price-rent ratio in Manhattan, and increase the wedge with the price-rent ratio in zone 2.

4.3 Housing Affordability and Rent Control

Price-Income and Rent-Income   Row 12 of Table 2 reports the ratio of the median value of owner-occupied housing to average earnings in each zone. Earnings are averaged among all working-age residents in a zone, both owners and renters. The median home price to the average income is an often-used metric of housing affordability. In the NY metro data, the median owner-occupied house costs 3.99 times average income. Price-income is 6.08 in Manhattan compared to 3.55 outside Manhattan, a ratio of 1.71. We use the same definition as in the data and compute average pre-tax labor earnings among the working-age population. The model generates a metro-wide price-income ratio of 3.44, which is fairly close, and understates the ratio across zones, which is 1.28. This is a direct consequence of not generating enough spatial variation in median house values.

Row 13 reports average rent paid by market renters divided by average income of all residents in a zone. This moment was the target for the housing preference parameter α^h. The model matches the 23% target.

To get at the household-level rent burden, we compute three additional moments reported in rows 14-16 of Table 2. One important caveat is that we only have these household-level data for the five counties in New York City, not for the 20 counties in zone 2 counties outside of NYC. The average rent data for NYC from the NYHVS household-level data are substantially below the corresponding Zillow rents, possibly resulting in overstated rent burden numbers. The first one computes household-level rent to income ratios for renters with positive income, caps the ratios at 101%, and takes the average across households. For this calculation, income is earnings for working-age households and social security income for retirees. The observed average share of income spent on rent by renters is 40% in Manhattan and 43% in New York City ex-Manhattan. The second moment computes the fraction of renters whose rent is between 30% and 50% of income. These households are known as rent-burdened. The third measure computes the fraction of renters whose rent exceeds 50% of income and includes the renters with zero income. This group is known as severely rent-burdened. In the Manhattan data, 48% of renters are rent burdened and another 28% are severely rent burdened. Outside of Manhattan, 54% of renters are rent-burdened and an additional 32% severely rent-burdened. The model generates a sufficiently higher rent burden for renters of 33.4% of income than the city-wide average among all residents of 23%. In the data, and subject to the caveats mentioned above, this number is even higher at 42%. the fraction of rent burdened households is 35.9% in the model with another 10.4% of households severely rent burdened. These numbers are below those in the data, but the NYC data are likely overstating the NY

metro-wide rent burden.

Rent Control By virtue of the calibration, the model generates the right share of rent-controlled households in the population in each zone (row 17 of Table 2). The calibration also targets the distribution of rent controlled households into the various income quartiles. In the model, high- and middle-income households who win the lottery turn down rent-regulated housing. The maximum rent or maximum income restrictions are too unappealing from a utility perspective (recall labor supply is endogenous). Some of these households may be altering their choice of hours worked in order to qualify. This has adverse implications for the city-wide labor supply and production. A second distortion is that low-income households who win the lottery and who are unconstrained by the maximum rent restriction demand more housing than they would under market conditions because of the large rent subsidy.

Rent control acts as an insurance device in our incomplete markets model. If it is difficult for a low-productivity household to get into the RC system, then the value of that insurance is low. The model predicts that the chance of winning the lottery in zone 1 is 6.2% and 18.5% in zone 2. Conditional on a win, about 22.5% accepts the RC unit in each zone. The probability of accepting the RC lottery conditional on winning is 72.4% for households in the lowest income quartile, and zero for all other income quartiles. Thus, RC units are well-targeted in the benchmark model.

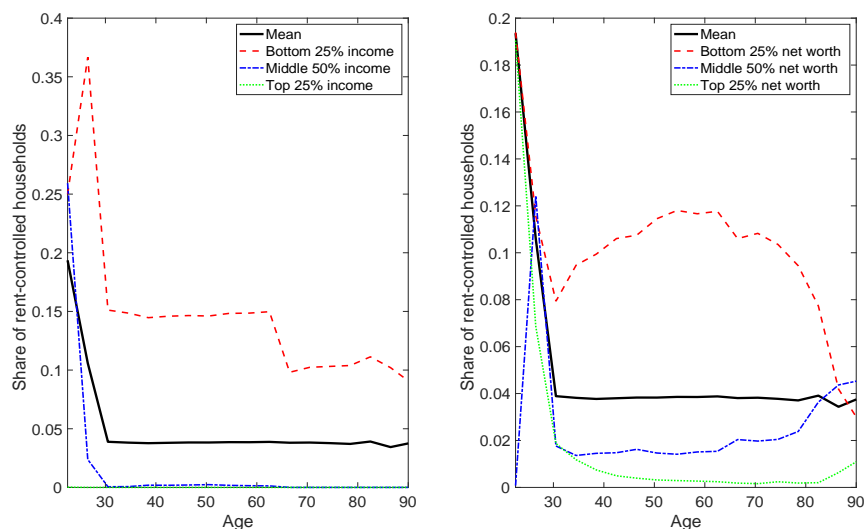Figure 3: Distribution of rent-controlled agents by age, and income and net worth quartiles.

Figure 3 plots the model's fraction of households that are in RC for the bottom 25%, middle 50% and top 25% of the income distribution against age in the left panel, and for the wealth distribution in the right panel. After age 30, only households in the bottom quartile of the income distribution are in RC. At younger ages, some households in the middle of the income distribution (for their age) also get RC. Across the wealth distribution (right panel), younger and poorer households tend to be more rent-controlled than older and richer ones. The one exception is that at older ages, some of the RC goes to the middle-50% of the wealth distribution.

5   Affordability Policies

This section studies our four main affordable housing policy experiments. Table 3 summarizes how key moments of the model change under the alternative policies. The first column reports the benchmark model, while the other columns report the percentage change in moments relative to the benchmark for each of the policy experiments. Figure 4 plots the associated welfare changes. Welfare changes are changes in value functions, expressed in consumption equivalent units; see equation (13).

Appendix G discusses three additional affordability policies. It also presents graphical evidence of how the policies change the cross-sectional distribution of households.

5.1   Reducing the Amount of Rent Controlled Housing

In the first experiment, we reduce the square foot amount of RC housing in each zone by half; the new η^ℓ values are half their benchmark values. The fraction of *households* in RC falls by 74% because of the relocation of households to zone 2 where the typical RC unit is larger. The distribution of RC units across the income distribution remains heavily concentrated in the first quartile of income.

The reduction in RC results in fewer distortions for developers; they receive a higher price for apartments they build (κ_4 is closer to 1). The empirical literature finds that increased incentives of landlords to renovate their properties and of developers to invest in new construction generate a modest housing boom in decontrolled areas (Autor et al. (2014), Diamond et al. (2017)). In our model, the developers' supply curve also shift upwards. Because of the low housing supply elasticity in Manhattan, housing supply in zone 1 only increases 0.18%. Housing supply in zone 2 increases 0.10%, despite the larger supply elasticity, because the distortions were smaller to begin with. Rents decrease by

Table 3: Main moments of the model under affordability policies that modify features of the RC system and the spatial allocation of housing.

		Benchmark	RC share	All RC in Z2	Zoning Z1	Vouchers
1	Rent to income Z1 (%)	41.6	-27.46%	-67.11%	-0.29%	-1.40%
2	Rent to income Z2 (%)	32.3	-3.85%	1.92%	-0.40%	0.02%
3	Frac. RC (%)	5.47	-74.27%	-9.99%	-0.35%	-1.81%
4	Frac. RC of those in income Q1 (%)	19.75	-72.39%	-8.96%	0.66%	-0.58%
5	Frac. sev. rent-burdened (%)	4.3	-42.54%	-24.91%	-0.44%	-0.10%
6	Avg. size RC unit (sqft)	695	0.90%	8.75%	0.43%	1.05%
7	Avg. size Z1 unit (sqft)	942	1.51%	5.04%	-1.39%	3.83%
8	Avg. size Z2 unit (sqft)	1510	-0.07%	-0.61%	0.72%	-0.87%
9	Frac. pop. Z1 (%)	10.5	-1.40%	-4.51%	9.24%	-4.48%
10	Housing stock Z1	–	0.18%	0.59%	7.94%	-0.56%
11	Housing stock Z2	–	0.10%	-0.08%	-0.44%	-0.37%
12	Rent/sqft Z1 ($)	4.15	-0.32%	-0.09%	-0.59%	-0.43%
13	Rent/sqft Z2 ($)	1.50	-0.38%	-0.03%	-0.71%	-0.43%
14	Price/sqft Z1 ($)	993	-0.49%	-0.23%	-0.91%	-0.68%
15	Price/sqft Z2 ($)	329	-0.69%	-0.29%	-1.20%	-0.84%
16	Home ownership Z1 (%)	56.0	11.80%	18.67%	5.94%	-0.93%
17	Home ownership Z2 (%)	59.0	0.79%	-1.39%	-0.25%	0.45%
18	Avg. income Z1 ($)	161323	4.46%	7.86%	-2.33%	-8.36%
19	Avg. income Z2 ($)	100651	0.18%	-0.42%	0.67%	1.44%
20	Frac. high prod. Z1 (%)	23.6	0.16%	0.42%	1.42%	-11.12%
21	Total hours worked	–	0.38%	-0.06%	0.03%	-1.32%
22	Total hours worked (efficiency)	–	-0.06%	-0.12%	-0.12%	-1.57%
23	Total output	–	-0.17%	-0.20%	-0.03%	-1.15%
24	Total commuting time	–	0.14%	0.65%	-1.28%	1.07%
25	Welfare change RC	–	-2.31%	-0.03%	0.26%	0.76%
26	Welfare change market renters	–	-0.07%	0.08%	0.30%	0.43%
27	Welfare change owners	–	0.03%	0.08%	0.04%	-0.00%
28	**Aggregate welfare change**	–	**-0.14%**	**0.07%**	**0.15%**	**0.19%**

Notes: Column "Benchmark" reports values of the moments for the benchmark model. Columns "RC share" to "Vouchers" report percentage changes of the moments in the policy experiments relative to the benchmark. Rows 1-8 report housing affordability moments , rows 9-24 aggregate moments across the two zones, and rows 25-26 welfare moments. Z1 stands for zone 1 (Manhattan), Z2 for the rest of the metro area. Row 20 reports what fraction of working age top-productivity households live in zone 1.

0.32% in zone 1 and by 0.38% in zone 2. The population in Manhattan decreases (-1.40%) because of the combination of a lower housing stock and larger housing units (+1.51%).

The policy results not only in a smaller but also in a *different* population in Manhattan. With the disappearance of 74% of the RC households, the average income in Manhattan rises by 4.46%. More productive working-age households end up in the urban core. Row 20 reports what fraction of households of working age in the highest productivity state live in zone 1. That fraction increases by 0.16%, a sign of the spatial reallocation of labor. The presence of more rich households in zone 1 explains the larger size of housing units, and also the higher home ownership rate in Manhattan, which rises 11.8%. Thus reducing RC symmetrically across zones has the asymmetric effect of attracting previously crowded-out, upper-middle-income households to the more desirable part of the city. This *gentrification* is consistent with the empirical evidence in Autor et al. (2014),

who show that richer households moved into units previously occupied by poorer RC tenants. In sum, the policy leads to more income inequality across zones.

With fewer high-productivity households remaining in zone 2 after the policy change, average housing unit size decreases by 0.07%. Home ownership only increases by 0.8% in the wake of a substantial reduction in house prices (-0.69%) which exceeds that of rents (-0.38%).

Housing affordability metrics improve. The fraction of severely rent-burdened renters falls by 43% metro-wide. Average rent-income ratios fall in both zones. The large decline in the rent-income ratio in zone 1 is mostly due to rising incomes rather than falling rents, however. House prices also fall in both zones, and more so in zone 2 (-0.69%) than in zone 1 (-0.49%).

Figure 4: Welfare effects of Affordability Policies by Age
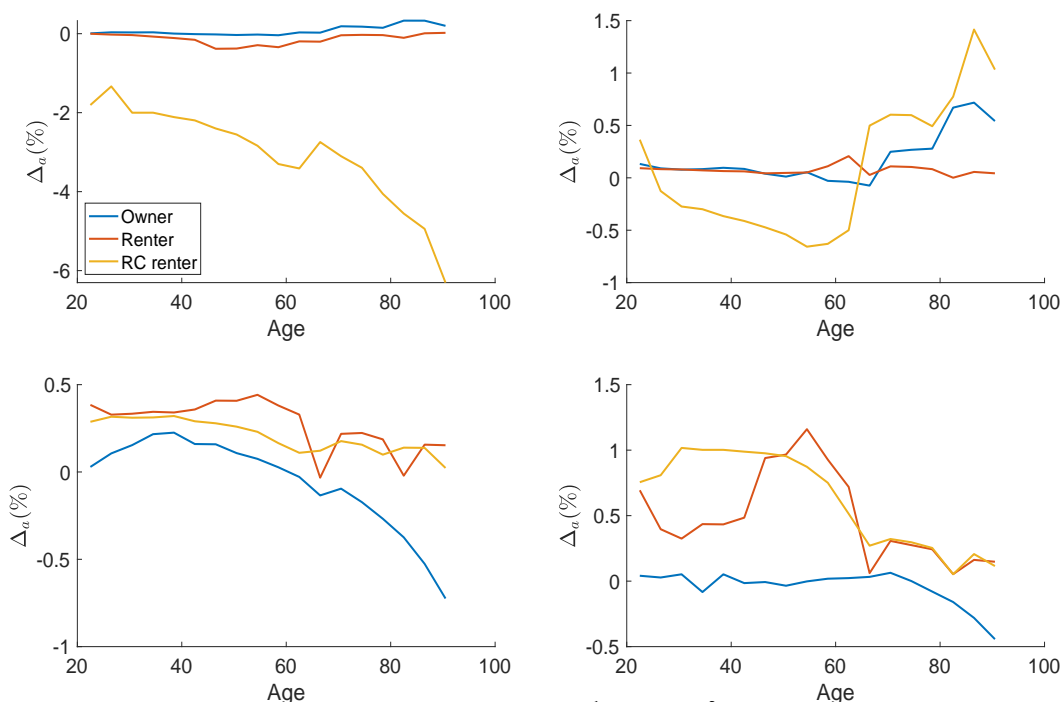
Notes: The baseline model has the following rent control parameters: $\eta^1 = .2426$, $\eta^2 = .0953$, $\kappa_1 = .5$, $\kappa_2 = .1110$, $\kappa_3 = .1869$. Top left panel: welfare changes from a decrease in the mandated fraction of RC sqft by 50% ($\eta^1 = .1213, \eta^2 = .0476$). Top right panel: shift of all RC sqft from zone 1 to zone 2 ($\eta^1 = 0, \eta^2 = .1133$). Bottom left panel: no change in RC parameters, zoning relaxation in zone 1 ($\overline{H}^1 = .0649$). Bottom right panel: no change in RC parameters, increase in amount of housing vouchers given to subsidized households. For each household, the welfare changes are measured as changes in the value function under the alternative policy relative to the value function under the benchmark policy, expressed in consumption equivalent units; see equation (13). These welfare changes are then aggregated across age and tenure status groups, where tenure status is the tenure status in the benchmark model, i.e., before the policy changes.

The average welfare effect of the reduction in the size of the RC program is negative at -0.14%. There are several sources of this aggregate welfare loss. More time is spent commuting because fewer households live in zone 1. This reduces leisure and uses re-

sources that could otherwise be consumed. Time spent working also increases (+0.38%), which further reduces leisure and utility. Despite the increase in hours worked, total output falls (-0.17%). This is because average productivity falls; total efficiency units of labor fall (-0.06%). While the share of high productivity households living in zone 1 increases modestly, the total productivity of zone 1 falls because of the substantial reduction in the population of zone 1. Partial equilibrium logic that fails to account for the endogenous changes in housing size would erroneously conclude that reducing RC would result in a better spatial allocation of labor. Instead, the policy mostly leads to rich households living in larger houses in zone 1.

The second key source of the welfare loss is that the policy aggravates inequality. It removes an important source of insurance for households who face a bad income shock. The policy is highly regressive. In untabulated results, we confirm that household in the top quartile of the wealth distribution gain +0.21% while households in he bottom quartile lose -0.38%. The main losers from the policy are households that are in RC units under the benchmark policy (-2.31%). As noted 74% of these households lose access to RC. While market rents go down, the rents that these households face go up sharply from the heavily subsidized rents they enjoyed in the benchmark model. The higher rents hit the old particularly hard as shown in the top left panel of Figure 4. Even those that keep RC often have to move from zone 1 to zone 2 and face longer and more expensive commutes. Consistent with the regressive nature of the policy, owners gain (+0.03%), and market renters lose modestly (-0.07%). Market renters lose slightly because many move to zone 2, where they tend to live in smaller units and further away from their jobs. Owners gain despite a reduction in house prices. Lower house prices are a double-edged sword. They impose losses on existing housing assets but make owned housing cheaper, allowing for larger dwellings. Unlike market renters, owners are more likely to live closer to their jobs after the policy.

One interesting take-away from this experiment is that improving standard housing affordability metrics and increasing welfare may be conflicting objectives. The distributional implications from the policy and how they affect the poor, high marginal utility households are a key driver of the aggregate welfare effect.

5.2 Spatial Allocation of RC Housing

In the second experiment, we explore the spatial dimension of our model and study the effects of a policy that shifts all RC units from Manhattan to zone 2 but holds fixed the overall amount of RC housing. That is η^1 is now set to zero and η^2 is increased from 0.0953

to 0.1133 to make up for the loss of RC housing in zone 1. This policy leads to a sizeable decrease (-10%) in the fraction of *households* in RC. The average RC unit is 8.75% larger, a composition effect because RC units in zone 1 were (endogenously) much smaller than in zone 2 in the benchmark. RC units in zone 2 become smaller than they were in the benchmark (-0.61%).

The removal of developer distortions in Manhattan, results in a meaningful expansion of the housing stock (+0.59%) and a small reduction in rents (-0.09%). On top of this new construction, the space formerly occupied by RC units is now available for market renters and owners, who roughly split the space equally. The increase in Manhattan housing supply is met by an increase in housing demand from the rich. The average dwelling size in Manhattan surges by 5.0% with small RC units replaced by larger market rentals and condos. The home ownership rate increases 18.7%. The population share of Manhattan falls by 4.5%. Average income in zone 1 is 7.9% higher as (many) low-income RC tenants are replaced by (fewer) high-income residents. Because of lower rents but especially because of higher average income, rent-income in zone 1 falls 67%.

The expansion of RC in zone 2 reduces incentives to build in zone 2. But demand for housing increases because of the households who move from zone 1 to zone 2. On net, the housing stock decreases modestly (-0.08%) as do rents (-0.03%) and average unit size (-0.61%). Average income in zone 2 is 0.42% lower, reflecting the new socio-economic make-up of zone 2. Average rent to income increases 1.9% in zone 2. As measured by the fraction of severely rent-burdened households metro-wide (-24.9%), housing affordability improves with this spatial reallocation of RC. With more working-age households in zone 2, total commuting time increases by 0.65%.

This policy generates an aggregate welfare gain of 0.07%. RC renters lose 0.03%, market renters gain 0.08%, and owners gain 0.08%. The top left panel of Figure 4 summarizes the welfare effects. Older households of all types gain the most, because they are more likely to live in zone 2 (as retirees) and benefit from the cheaper rent that comes with the spatial reallocation of rent control. Some richer owners also get to live in zone 1 (and without a commute) in a larger housing unit. This policy does not hurt RC households too much because it preserves the size of the RC system. Those who remain in RC enjoy strictly lower rents and larger housing units. The downside is that they commute more. Those who lose their RC unit do suffer welfare losses but can still take advantage of the lower rents.

This policy illustrates an interesting trade-off between increasing spatial inequality (between neighborhoods) and improving housing affordability. It also suggests that rather than reducing the amount of RC, as in the first experiment, a better policy may be to pre-

serve the amount of RC, and the insurance benefits it brings, but to address its spatial distortions. It is worth noting that these results may well be different were households to have an explicit preference for socio-economic diversity in every neighborhood.

5.3 Relaxing Zoning Laws in Zone 1

In the third experiment, we study a policy which is arguably the consensus solution for housing affordability among economists: allow for more housing in the city center. We think of this policy as a relaxation in zoning laws, often referred to as upzoning. Under the policy we consider, more construction is permitted in zone 1. But for every square foot of rental housing built, a fraction η^1 must be affordable. This policy leaves the RC system untouched. It has no direct cost to implement, but affects the entire equilibrium of the model. The results are shown in the third column of Table 3.

For our chosen increase in $\overline{H}^1$, the equilibrium housing stock in Manhattan increases by 8% and rents fall by 0.6%. The average unit size in zone 1 decreases by 1.4% so that the population share of Manhattan rises by 9.2%. Because of the population reallocation, average income in Manhattan decreases by 2.3%. Yet, a substantially higher fraction of highest-productivity households now lives in Manhattan (+1.4%), so that it is the younger high-productivity households that move in. House prices fall (-0.91%) by more than rents and home ownership increases (+6%). With more working-age households in Manhattan, aggregate commuting time falls by 1.28%.

The housing stock in zone 2 falls by 0.44% as developers shift their resources towards zone 1 where the population has swelled. Rents in zone 2 fall by 0.71% as demand for housing weakens due to the aforementioned demographic changes.

Housing affordability metrics improve surprisingly little. Average rent-to-income decreases slightly, by 0.29% in zone 1 and by 0.40% in zone 2. The fraction of severely rent-burdened households decreases slightly by 0.44%. What these metrics fail to capture is that more (high-productivity) households can now afford to live close to work.

This experiment leads to a 0.03% decrease in output. Output in the Manhattan construction sector increases substantially (+44.5%). But there is a loss in the larger construction sector of zone 2 (-2%) and a small loss in the output of the very large tradable sector (-0.17%). The latter is due to slightly higher wages (+0.09%), signalling a reduction in competitiveness. The aggregate time saved commuting goes into leisure rather than increasing aggregate hours worked (esp. in efficiency units), so that output falls.

Increasing housing supply in zone 1 generates an aggregate welfare gain of 0.15%. Not all groups gain by similar amounts: RC renters (+0.26%) and market renters (+0.30%) gain

the most, while the average owner gains the least (+0.04%). This policy is progressive, with gains that are strictly declining in wealth. This is in sharp contrast with the reduction in RC policy. The policy is not a Pareto improvement, however. As can be seen in the bottom left panel of Figure 4, older home owners lose. They suffer large house price declines, but have a shorter horizon to benefit from future reductions in the cost of living.

5.4 Vouchers

One important pillar of housing policy is the Section 8 voucher program, housing assistance provided by the federal government to low-income households. Since this policy is part of the tax and transfer system, it is already captured by the function $T(\cdot)$. As a final main policy exercise, we study the effects of increasing the size of the voucher program. We engineer the increase by increasing the tax progressivity parameter τ from 0.17 to 0.18 while lowering the overall size of the tax and transfer system from λ of 0.75 to 0.7415. This dual parameter change leaves the total net tax collected unchanged, so that there is no direct fiscal cost associated with the voucher expansion, like for the other policies. Because of the increased progressivity, the system generates both higher tax revenues and higher transfers. The policy translates to an increase in spending on the Section 8 voucher program in the New York metropolitan area from $2 billion to $3.4 billion annually.[19]

The fourth column of Table 3 shows the results. The policy leads to a large reduction of the population share of Manhattan (-4.5%), a decline in its housing stock (-0.56%), and an increase in commuting (+1.07%). It also substantially reduces the average income of zone 1 (-8.36%) and the fraction of top-productivity households who live in zone 1 (-11.1%). Total output falls by 1.15% along with total hours worked (-1.32% and -1.57% in efficiency units).

Economists typically favor given cash transfers to low-income households over RC under the presumption that the former are less distortionary. Vouchers are such transfers, but they are typically paid for with distortionary labor (and capital) income taxes, like in our model. The large reduction of average income in zone 1, where many of the

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[19]Data compiled from the Housing and Urban Development department show that the housing authorities responsible for the 25 counties in the New York MSA disbursed $2.06 billion in 246,000 Section 8 vouchers in the year 2013 (latest available). This amounts to an average of $8,300 per year per voucher. The policy exercise we consider raises an additional $1.4 billion in tax revenues. At the cost of $8,300 per voucher, this translates into 168,000 additional vouchers. In the model, the tax change increases the transfers of those who already received transfers before, and the transfer is higher the lower is household income. It also creates new beneficiaries who now receive a transfer while they were paying a tax in the benchmark. Thus, the policy both increases the amount of the existing vouchers and disburses additional smaller vouchers. We enforce that all households who receive an additional transfer spend at least the amount of the additional transfer on housing, to capture the institutional reality that vouchers must be used for housing expenses.

households who face the higher income taxes live, underscores this point. Some of these high-productivity households are no longer able to live in Manhattan, which reduces the efficiency of the spatial allocation of labor. In sum, while vouchers may be thought to enable lower-income households to live in zone 1, in equilibrium they crowd out higher productivity households instead.

Despite the labor tax distortions, reduced aggregate output, and increased commuting times, aggregate welfare increases when the housing voucher program is expanded. The gains come from the RC and market renters, while owners lose but are nearly indifferent. Both types of renters enjoy the reduced rents. The experiment shows that, for our parameters, more redistribution is welfare improving even if it creates additional distortions.

5.5 Other RC Policies

Appendix G discusses three more policies that change the other policy levers of the RC system: the rent subsidy, governed by κ_1, the income qualification threshold, governed by κ_2, and the size of the RC units, governed by κ_3. These three policies have lower average welfare effects than the policies discussed above and their economic logic is similar.

6 Conclusion

In a world with rising urbanization rates, the high cost of urban housing has surfaced as a daunting policy challenge. Existing policy tools affect the supply of housing, how the housing stock is used (owned, rented, rent controlled), and how it is distributed in space. Households of different tenure status, age, income, and wealth are differentially affected by changes in policy. This paper develops a novel spatial equilibrium model with wealth effects and rich household heterogeneity that allows for the first time to quantitatively assess the welfare implications of the main housing affordability policies.

The model is calibrated to the New York metropolitan area. It matches patterns of average earnings, wealth accumulation, and home ownership over the life-cycle, delivers realistic house prices, rents, and wages, and matches key facts on the spatial differences in income and rents between the urban core and the periphery. The model's rent control sector matches the key features of the New York rent control system as well as restrictions on residential land use (zoning).

We use the model to evaluate various policy changes to the rent control system as well as to zoning policy and the size of the housing voucher system. We trace out their

aggregate, distributional, and spatial implications. These policies have quantitatively important general equilibrium effects that are often at odds with partial equilibrium logic.

Contrary to conventional wisdom that highlights the distortions generated by various housing policies, our work finds important insurance benefits from such policies. Consistent with the redistributive role of housing policy, reducing the size of the RC system is welfare reducing while increasing the size of the housing voucher system is welfare increasing. Such policies disproportionately affect the well-being of the poorest households in society. Reducing their housing safety net creates welfare losses that exceed the improvements in efficiency from eliminating the distortions.

Highlighting the interaction between the housing market and space, a policy that shifts all rent controlled housing units from the city center to the rest of the metropolitan area generates a sizeable welfare gain. The policy prevents losses on low-income households while improving the spatial allocation of labor. Rising inequality between neighborhoods is the price to pay for a significant improvement in overall housing affordability.

Allowing for more construction in the city center, combined with the provision of additional affordable units there is also welfare improving. Like the previous policy, it improves the spatial allocation of labor without inflicting losses on the poor. A greater supply of housing lowers the cost of living, decreases commuting times, and increases leisure. The improved affordability spills over from the city center to the rest of the metro area as the population relocates.

Overall, these results underscore the need for rich models of household heterogeneity to understand both the aggregate and the distributional implications of housing affordability policies.

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A Appendix

A.1 Analytical solution for housing and labor supply choices

Preferences are a CES aggregator over leisure and a CES aggregator of nondurable consumption and housing. We will solve only the worker's problem here. A retiree's problem is analogous, but simpler because there is one fewer choice as $n_t = 0$.

The utility function is $U(c, h, n) = \frac{\mathcal{C}(c,h,n)^{1-\gamma}}{1-\gamma}$, where

$$\mathcal{C}(c, h, n) = \left[(1 - \alpha_N) \left[(1 - \alpha_H)c^\epsilon + \alpha_H h^\epsilon \right]^{\frac{\eta}{\epsilon}} + \alpha_N \left[1 - \Phi_T - n \right]^{\chi_0 \eta} \right]^{\frac{1}{\eta}}$$

when $|\eta|, |\epsilon| > 0$ (case (i)). χ_0 makes leisure nonlinear in hours (we set it to 1), and we impose a lower bound on hours, $n \geq n_{min}$.

When $|\eta| > 0, |\epsilon| = 0$, $u(c, h, n) = \left[(1 - \alpha_N) \left[c^{1-\alpha_H} h^{\alpha_H} \right]^\eta + \alpha_N \left[1 - \Phi_T - n \right]^{\chi_0 \eta} \right]^{\frac{1}{\eta}}$ (case (ii)).

When $|\eta| = 0, |\epsilon| > 0$, $u(c, h, n) = \left[(1 - \alpha_H)c^\epsilon + \alpha_H h^\epsilon \right]^{\frac{(1-\alpha_N)}{\epsilon}} \left[1 - \Phi_T - n \right]^{\chi_0 \alpha_N}$ (case (iii)).

When $|\eta| = |\epsilon| = 0$, $u(c, h, n) = \left[c^{1-\alpha_H} h^{\alpha_H} \right]^{1-\alpha_N} \left[1 - \Phi_T - n \right]^{\chi_0 \alpha_N}$ (case (iv)).

Renter First, consider the renter's problem and let λ_t be the Lagrange multiplier on the budget constraint, ν_t be the Lagrange multiplier on the borrowing constraint, and ξ_t be the Lagrange multiplier on the non-negativity labor constraint. The numerical strategy is to choose c_t in order to maximize the household's utility, and l_t to solve the non-linear equation for labor supply. Here we will show that the other choices (h_t and b_{t+1}) can be written as analytic functions of c_t and n_t. Denote $\mathcal{C}_t = \mathcal{C}(c_t, h_t)$. We ignore the taste shifter (which is multiplicative and raised to the power $1 - \gamma$ in the equations involving $\mathcal{C}_t$), and assume $\widehat{b}_{t+1} = 0$. The budget constraint simplifies to:

$$c_t + R_t^\ell h_t + Q_t b_{t+1} + \phi_{F,t}^\ell = \overline{\Psi}_t \psi^z + \left(2 - \frac{1}{Q_t} \right) x_t + \lambda \left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1 \right) x_t \right)^{1-\tau} - \tau^{SS} W_t G^a z_t n_t \tag{14}$$

The first order conditions for c_t, l_t, h_t, b_{t+1} are respectively:

$$\mathcal{C}_t^{1-\gamma-\eta}(1 - \alpha_n)(1 - \alpha_h)((1 - \alpha_h)c^\epsilon + \alpha_h h^\epsilon)^{\frac{\eta-\epsilon}{\epsilon}} c^{\epsilon-1} = \lambda_t$$

$$\mathcal{C}_t^{1-\gamma-\eta}\alpha_n(1 - \phi_T - n_t)^{\eta-1} = \lambda_t W_t G^a z_t \left[\lambda(1 - \tau) \left(W_t G^a z_t \left(1 - \phi_{T,t}^\ell - l_t \right) + \left(\frac{1}{Q_t} - 1 \right) x_t \right)^{-\tau} - \tau^{SS} \right] + \xi_t$$

$$\mathcal{C}_t^{1-\gamma-\eta}(1 - \alpha_n)\alpha_h((1 - \alpha_h)c^\epsilon + \alpha_h h^\epsilon)^{\frac{\eta-\epsilon}{\epsilon}} h^{\epsilon-1} = \lambda_t R_t^\ell$$

$$\lambda_t Q_t = (1 - p^a) \beta \mathbb{E}_t \left[\frac{\partial}{\partial x_{t+1}} V(x_{t+1}, z_{t+1}, a+1, S_{t+1}) \right] + \nu_t \tag{15}$$

where $\frac{\partial}{\partial x_t} V(x_t, z_t, a, S_t) = \lambda_t \left(2 - \frac{1}{Q_t} + \lambda(1 - \tau) \left(\frac{1}{Q_t} - 1 \right) \left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1 \right) x_t \right)^{-\tau} \right)$, since the marginal value of net worth is the same in the problem of renters, RC renters and owners, and the probabilities to receive rent control sum to 1.

Case 1: $\nu_t = 0$ and $\xi_t = 0$. In this case the household is unconstrained. Residential housing is obtained by combining the conditions for c_t and h_t: $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R_t^\ell} \right)^{\frac{1}{1-\epsilon}} c_t$.

Case (i) The nonlinear equation for hours n_t is obtained by combining the first order conditions for

c_t and n_t, and substituting for h_t as a function of c_t in the CES aggregator:

$$\chi_0 \alpha_N (1 - \Phi_T - n)^{\chi_0 \eta - 1} - (1 - \alpha_N)(1 - \alpha_H) \left[(1 - \alpha_H) + \alpha_H \left[\frac{\alpha_H}{(1 - \alpha_H)R} \right]^{\frac{\epsilon}{1-\epsilon}} \right]^{\frac{\eta - \epsilon}{\epsilon}} WG_a z$$
$$\left\{ \lambda(1 - \tau) (WG_a z n + rx)^{-\tau} - \tau^{SS} \right\} c^{\eta - 1} = 0 \tag{16}$$

The Jacobian is:

$$-\alpha_N \chi_0 (\chi_0 \eta - 1)(1 - \Phi_T - n)^{\chi_0 \eta - 2} + (1 - \alpha_N)(1 - \alpha_H) \left[(1 - \alpha_H) + \alpha_H \left[\frac{\alpha_H}{(1 - \alpha_H)R} \right]^{\frac{\epsilon}{1-\epsilon}} \right]^{\frac{\eta - \epsilon}{\epsilon}} (WG_a z)^2$$
$$\lambda(1 - \tau)\tau (WG_a z n + rx)^{-\tau - 1} c^{\eta - 1} \tag{17}$$

Absent HSV taxes, the analytic solution for hours is:

$$n = 1 - \Phi_T - \left[\frac{(1 - \alpha_N)(1 - \alpha_H)}{\chi_0 \alpha_N} \left[(1 - \alpha_H) + \alpha_H \left(\frac{\alpha_H}{(1 - \alpha_H)R} \right)^{\frac{\epsilon}{1-\epsilon}} \right]^{\frac{\eta - \epsilon}{\epsilon}} WG_a z (1 - \tau^{SS}) \right]^{\frac{1}{\chi_0 \eta - 1}} c^{\frac{\eta - 1}{\chi_0 \eta - 1}} \tag{18}$$

Case (ii) The nonlinear equation for hours is:

$$\chi_0 \alpha_N (1 - \Phi_T - n)^{\chi_0 \eta - 1} - (1 - \alpha_N)(1 - \alpha_H) \left(\frac{\alpha_H}{(1 - \alpha_H)R} \right)^{\eta \alpha_H} WG_a z$$
$$\left\{ \lambda(1 - \tau) (WG_a z n + rx)^{-\tau} - \tau^{SS} \right\} c^{\eta - 1} = 0 \tag{19}$$

The Jacobian is:

$$-\chi_0 \alpha_N (\chi_0 \eta - 1)(1 - \Phi_T - n)^{\chi_0 \eta - 2} + (1 - \alpha_N)(1 - \alpha_H) \left(\frac{\alpha_H}{(1 - \alpha_H)R} \right)^{\eta \alpha_H} (WG_a z)^2$$
$$\lambda(1 - \tau)\tau (WG_a z n + rx)^{-\tau - 1} c^{\eta - 1} \tag{20}$$

Absent HSV taxes, the analytic solution for hours is:

$$n = 1 - \Phi_T - \left[\frac{(1 - \alpha_N)(1 - \alpha_H)}{\chi_0 \alpha_N} \left(\frac{\alpha_H}{(1 - \alpha_H)R} \right)^{\eta \alpha_H} WG_a z (1 - \tau^{SS}) \right]^{\frac{1}{\chi_0 \eta - 1}} c^{\frac{\eta - 1}{\chi_0 \eta - 1}} \tag{21}$$

Case (iii) The nonlinear equation for hours is:

$$\chi_0 \alpha_N \left[(1 - \alpha_H) + \alpha_H \left(\frac{\alpha_H}{(1 - \alpha_H)R} \right)^{\frac{\epsilon}{1-\epsilon}} \right] c$$
$$-(1 - \alpha_N)(1 - \alpha_H)(1 - \Phi_T - n)WG_a z \left\{ \lambda(1 - \tau) (WG_a z n + rx)^{-\tau} - \tau^{SS} \right\} = 0 \tag{22}$$

The Jacobian is:

$$(1 - \alpha_N)(1 - \alpha_H)WG_a z \left[\left\{ \lambda(1 - \tau) (WG_a z n + rx)^{-\tau} - \tau^{SS} \right\} + \right.$$
$$\left. (1 - \Phi_T - n)\lambda(1 - \tau)\tau (WG_a z n + rx + \Pi)^{-\tau - 1} \right] \tag{23}$$

Absent HSV taxes, the analytic solution for hours is:

$$n = 1 - \Phi_T - \left[(1 - \alpha_H) + \alpha_H \left(\frac{\alpha_H}{(1 - \alpha_H)R} \right)^{\frac{\epsilon}{1-\epsilon}} \right] \frac{\chi_0 \alpha_N}{(1 - \alpha_N)(1 - \alpha_H)WG_a z (1 - \tau^{SS})} c \tag{24}$$

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Case (iv) The nonlinear equation for hours is:

$$\chi_0 \alpha_N c - (1-\alpha_N)(1-\alpha_H)(1-\Phi_T-n)WG_a z \left\{ \lambda(1-\tau)(WG_a zn + rx)^{-\tau} - \tau^{SS} \right\} = 0 \tag{25}$$

The Jacobian is:

$$
\begin{aligned}
&(1-\alpha_N)(1-\alpha_H)WG_a z \left[\left\{ \lambda(1-\tau)(WG_a zn + rx + \Pi)^{-\tau} - \tau^{SS} \right\} + \right. \\
&\left. (1-\Phi_T-n)\lambda(1-\tau)\tau(WG_a zn + rx)^{-\tau-1} \right]
\end{aligned}
\tag{26}
$$

Absent HSV taxes, the analytic solution for hours is:

$$n = 1 - \Phi_T - \frac{\chi_0 \alpha_N}{(1-\alpha_N)(1-\alpha_H)WG_a z(1-\tau^{SS})}c \tag{27}$$

Given c_t, we obtain l_t (hence n_t) by numerically solving the labor supply equation. Given c_t, n_t, h_t, we obtain b_{t+1} from the budget constraint:

$$b_{t+1} = \frac{1}{Q_t}\left[\overline{\Psi}_t \psi^z - \phi^\ell_{F,t} + \left(2 - \frac{1}{Q_t}\right)x_t + \lambda\left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1\right)x_t\right)^{1-\tau} - \tau^{SS}W_t G^a z_t n_t - c_t - R^\ell_t h_t \right] \tag{28}$$

Case 2: $\nu_t > 0$ and $\xi_t = 0$. In this case the borrowing constraint binds and $b_{t+1} = 0$ but the labor constraint does not. The first order conditions in the first three lines of equation 15 are still correct. It is still the case that conditional on choosing a location ℓ, $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t}\right)^{\frac{1}{1-\epsilon}} c_t$ and l_t is the solution to the nonlinear equation in Case 1. Given those, c_t can be obtained from the budget constraint:

$$c_t = \frac{\overline{\Psi}_t \psi^z - \phi^\ell_{F,t} + \left(2 - \frac{1}{Q_t}\right)x_t + \lambda\left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1\right)x_t\right)^{1-\tau} - \tau^{SS}W_t G^a z_t n_t}{1 + R^\ell_t \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t}\right)^{\frac{\epsilon}{1-\epsilon}}} \tag{29}$$

Case 3: $\nu_t = 0$ and $\xi_t > 0$. In this case the borrowing constraint does not bind, but the labor constraint does, implying $l_t = 1 - \phi^\ell_{T,t}$, hence $n_t = 0$. The first order conditions in the first, third, and fourth lines of equation 15 are still correct. As in Case 1, conditional on choosing a location ℓ, $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t}\right)^{\frac{1}{1-\epsilon}} c_t$. We obtain b_{t+1} from the budget constraint.

Case 4: $\nu_t > 0$ and $\xi_t > 0$. In this case both constraints bind, implying $n_t = 0$ and $b_{t+1} = 0$. The first order conditions in the first and third lines of equation 15 are still correct, so conditional on choosing a location ℓ, $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t}\right)^{\frac{1}{1-\epsilon}} c_t$. By plugging this into the budget constraint, we can explicitly solve for $c_t = \frac{\overline{\Psi}_t \psi^z - \phi^\ell_{F,t} + \left(2 - \frac{1}{Q_t}\right)x_t + \lambda\left(\left(\frac{1}{Q_t} - 1\right)x_t\right)^{1-\tau}}{1 + R^\ell_t \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t}\right)^{\frac{\epsilon}{1-\epsilon}}}$.

In the case with $\lambda = 1, \tau = 0$, we simply have $c_t = \frac{\overline{\Psi}_t \psi^z - \phi^\ell_{F,t} + x_t}{1 + R^\ell_t \left(\frac{\alpha_h}{(1-\alpha_h)R^\ell_t}\right)^{\frac{\epsilon}{1-\epsilon}}}$.

RC renter Next, the RC renter's problem is the same as the renter's problem, with additional Lagrange multipliers on the income and the rent restriction constraints, $\tilde{\zeta}^y_t$ and $\tilde{\zeta}^r_t$, and κ_1 the rent

control discount multiplying R_t^ℓ wherever it appears. Note that we cannot have both $\xi_t, \xi_t^y > 0$ (we rule out $\frac{\kappa_2 \overline{Y}_t}{W_t G^a z_t} = 0$). Because households are atomistic we ignore the derivative $\frac{\partial \overline{Y}_t}{\partial n_t}$.

Case 1: $\xi_t = 0$ and $\xi_t^y = 0$. There is an interior solution $n_{min} < n_t < \frac{\kappa_2 \overline{Y}_t}{W_t G^a z_t}$, which solves the same labor supply equation as the market renter (with $\kappa_1 R_t^\ell$ instead of R_t^ℓ).
If $\xi_t^r > 0$, then the residential housing choice is constrained: $h_t = \frac{\kappa_3 \overline{Y}_t}{\kappa_1 R_t^\ell}$. If $\xi_t^r = 0$, then combining the conditions for c_t and for h_t, we obtain $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)\kappa_1 R_t^\ell} \right)^{\frac{1}{1-\epsilon}} c_t$.
If $\nu_t > 0$, then savings are constrained and $b_{t+1} = 0$. If $\nu_t = 0$, then given c_t, n_t, h_t, we obtain b_{t+1} from the budget constraint.

Case 2: $\xi_t > 0$ and $\xi_t^y = 0$. The leisure choice is constrained at its upper bound, and $n_t = 0$. Choices for h_t and b_{t+1} as functions of c_t are identical to Case 1.

Case 3: $\xi_t = 0$ and $\xi_t^y > 0$. The labor choice is constrained at the upper bound implied by RC, and $n_t = \frac{\kappa_2 \overline{Y}_t}{W_t G^a z_t}$. Choices for h_t and b_{t+1} as functions of c_t are identical to Case 1.

Owner Finally, consider the owner's problem and let λ_t be the Lagrange multiplier on the budget constraint, ν_t be the Lagrange multiplier on the borrowing constraint, and ξ_t be the Lagrange multiplier on the labor constraint. The numerical strategy is to choose c_t and $\hat{h}_t$ in order to maximize the household's utility (therefore, we ignore the multiplier on the non-negativity constraint $\hat{h}_t \geq 0$), and n_t to solve the nonlinear equation for labor supply. Here we will show that the other choices (h_t and b_{t+1}) can be written as analytic functions of c_t and $\hat{h}_t$. The budget constraint simplifies to:

$$
\begin{aligned}
&c_t + P_t^\ell h_t + Q_t b_{t+1} + \kappa_4^\ell P_t^\ell \hat{h}_t + \phi_{F,t}^\ell \\
&= \overline{\Psi}_t \psi^z + \left(2 - \frac{1}{Q_t} \right) x_t + \kappa_4^\ell R_t^\ell \hat{h}_t + \lambda \left(W_t G^a z_t n_t + \left(\frac{1}{Q_t} - 1 \right) x_t \right)^{1-\tau} - \tau^{SS} W_t G^a z_t n_t
\end{aligned}
\tag{30}
$$

The first-order conditions for c_t, l_t are identical to the market renter. The first-order conditions for $h_t, \hat{h}_t, b_{t+1}$ are respectively:

$$
\begin{aligned}
&C_t^{1-\gamma-\eta}(1-\alpha_n)\alpha_h((1-\alpha_h)c^\epsilon + \alpha_h h^\epsilon)^{\frac{\eta-\epsilon}{\epsilon}} h^{\epsilon-1} + \\
&(1-p^a)\beta(1-\delta-\tau^P)\mathbb{E}_t \left[P_{t+1}^\ell \frac{\partial}{\partial x_{t+1}} V\left(x_{t+1}, z_{t+1}, a+1, S_{t+1} \right) \right] + \nu_t \theta_{res} P_t^\ell = \lambda_t P_t^\ell \\
&(1-p^a)\beta(1-\delta-\tau^P)\mathbb{E}_t \left[P_{t+1}^\ell \frac{\partial}{\partial x_{t+1}} V\left(x_{t+1}, z_{t+1}, a+1, S_{t+1} \right) \right] + \nu_t \theta_{inv} P_t^\ell = \lambda_t (P_t - R_t) \\
&\lambda_t Q_t = (1-p^a)\beta \mathbb{E}_t \left[\frac{\partial}{\partial x_{t+1}} V\left(x_{t+1}, z_{t+1}, a+1, S_{t+1} \right) \right] + \nu_t Q_t
\end{aligned}
\tag{31}
$$

Case 1: $\nu_t = 0$ and $\xi_t = 0$. In this case the household is unconstrained. Combining the conditions for h_t and $\hat{h}_t$, and combining the result with the condition for c_t, we can solve analytically for $h_t = \left(\frac{\alpha_h}{(1-\alpha_h)R_t^\ell} \right)^{\frac{1}{1-\epsilon}} c_t$, as in the market renter's case.
Then, the nonlinear labor supply equation for n_t is the same.

Given $c_t, \widehat{h}_t, h_t, n_t$, we obtain b_{t+1} from the budget constraint:

$$
\begin{aligned}
b_{t+1} = \tfrac{1}{Q_t} \Big[&\overline{\Psi}_t \psi^z - \phi^{\ell}_{F,t} + \left(2 - \tfrac{1}{Q_t}\right) x_t + \kappa^{\ell}_4 \left(R^{\ell}_t - P^{\ell}_t\right) \widehat{h}_t \\
&+ \lambda \left(W_t G^a z_t n_t + \left(\tfrac{1}{Q_t} - 1\right) x_t\right)^{1-\tau} - \tau^{SS} W_t G^a z_t n_t - c_t - P^{\ell}_t h_t \Big]
\end{aligned}
\tag{32}
$$

Case 2: $\nu_t > 0$ and $\xi_t = 0$. In this case the borrowing constraint binds implying $b_{t+1} = -\frac{\theta_{res} P^{\ell}_t h_t + \theta_{inv} \kappa^{\ell}_4 P^{\ell}_t \widehat{h}_t}{Q_t}$, but the leisure constraint does not bind. We solve for l_t as in Case 1, as the nonlinear equation for hours is unaffected. Given $c_t, \widehat{h}_t, n_t$, we use the budget constraint to solve for h_t analytically:

$$
\begin{aligned}
h_t = \tfrac{1}{P^{\ell}_t (1-\theta_{res})} \Big[&\overline{\Psi}_t \psi^z + \left(2 - \tfrac{1}{Q_t}\right) x_t + \kappa^{\ell}_4 R^{\ell}_t \widehat{h}_t + \lambda \left(W_t G^z z_t n_t + \left(\tfrac{1}{Q_t} - 1\right) x_t\right)^{1-\tau} \\
&- \phi^{\ell}_{F,t} - c_t - (1 - \theta_{inv}) \kappa^{\ell}_4 P^{\ell}_t \widehat{h}_t - \tau^{SS} W_t G^z z_t n_t \Big]
\end{aligned}
\tag{33}
$$

In the case with $\lambda = 1, \tau = 0$, we have

$$
h_t = \frac{1}{P^{\ell}_t (1 - \theta_{res})} \left[\overline{\Psi}_t \psi^z + x_t + \left(1 - \tau^{SS}\right) W_t G^a z_t n_t + \kappa^{\ell}_4 R^{\ell}_t \widehat{h}_t - \phi^{\ell}_{F,t} - c_t - (1 - \theta_{inv}) \kappa^{\ell}_4 P^{\ell}_t \widehat{h}_t \right]
$$

Case 3: $\nu_t = 0$ and $\xi_t > 0$. In this case the borrowing constraint does not bind, but the leisure constraint does, implying $n_t = 0$. Conditional on choosing a location ℓ, h_t is identical to Case 1. From the budget constraint, we deduce:

$$
b_{t+1} = \frac{\overline{\Psi}_t \psi^z - \phi^{\ell}_{F,t} + x_t \left(2 - \tfrac{1}{Q_t}\right) + \kappa_4 (R^{\ell}_t - P^{\ell}_t) \widehat{h}_t + \lambda \left(\left(\tfrac{1}{Q_t} - 1\right) x_t\right)^{1-\tau} - \left(1 + \left(\tfrac{\alpha_h}{(1-\alpha_h) R^{\ell}_t}\right)^{\frac{1}{1-\epsilon}} P^{\ell}_t\right) c_t}{Q_t}
\tag{34}
$$

Case 4: $\nu_t > 0$ and $\xi_t > 0$. In this case both constraints bind, implying $n_t = 0$ and $b_{t+1} = -\frac{\theta_{res} P^{\ell}_t h_t + \theta_{inv} \kappa^{\ell}_4 P^{\ell}_t \widehat{h}_t}{Q_t}$. Eliminating b_{t+1} and n_t from the budget constraint, we can solve analytically for h_t as a function of c_t and $\widehat{h}_t$ just as in case 2:

$$
h_t = \tfrac{1}{P^{\ell}_t (1-\theta_{res})} \left[\overline{\Psi}_t \psi^z + \left(2 - \tfrac{1}{Q_t}\right) x_t + \kappa^{\ell}_4 R^{\ell}_t \widehat{h}_t + \lambda \left(\left(\tfrac{1}{Q_t} - 1\right) x_t\right)^{1-\tau} - \phi^{\ell}_{F,t} - c_t - (1 - \theta_{inv}) \kappa^{\ell}_4 P^{\ell}_t \widehat{h}_t \right]
\tag{35}
$$

In the case with $\lambda = 1, \tau = 0$, we have

$$
h_t = \frac{1}{P^{\ell}_t (1 - \theta_{res})} \left[\overline{\Psi}_t \psi^z + x_t + \kappa^{\ell}_4 R^{\ell}_t \widehat{h}_t - \phi^{\ell}_{F,t} - c_t - (1 - \theta_{inv}) \kappa^{\ell}_4 P^{\ell}_t \widehat{h}_t \right]
$$

A.2 Special case which can be solved analytically

Here we use Cobb-Douglas preferences as a special case of the CES aggregator described earlier. Consider a perpetual renter who is facing a constant wage W and a constant rent R, who is not choosing location, who is not constrained, who faces no idiosyncratic shocks ($A = 1$), and whose productivity and utility are not age dependent ($G^a = 1$, $\alpha_{c,a} = \alpha_c$, and $\alpha_{h,a} = \alpha_h \; \forall a$). His problem

can be written as:

$$v(xs_t, a) = \max_{cs,hs,n} \tfrac{1}{1-\gamma} \left(cs_t^{\alpha_c} hs_t^{\alpha_h} (1-n_t)^{\alpha_n} \right)^{1-\gamma} + \beta E_t[v(xs_{t+1}, a+1)] \text{ s.t.}$$
$$xs_{t+1} = \tfrac{1}{Q}(xs_t + n_t W - cs_t - hs_t R) \tag{36}$$

As shown earlier, the optimal housing and labor choices satisfy: $hs_t = \tfrac{\alpha_h}{\alpha_c}\tfrac{1}{R}cs_t$ and $n_t = 1 - \tfrac{\alpha_n}{\alpha_c}\tfrac{1}{W}cs_t$. Redefining $\widehat{cs} = \tfrac{1}{\alpha_c}cs$ and plugging these into the maximization problem, the problem is rewritten as:

$$v(xs_t, a) = \max_{\widehat{cs}} \tfrac{\overline{U}}{1-\gamma}\widehat{cs}_t^{1-\gamma} + \beta E_t[v(xs_{t+1}, a+1)] \text{ s.t.}$$
$$xs_{t+1} = \tfrac{1}{Q}(xs_t + W - \widehat{cs}_t) \tag{37}$$

where $\overline{U} = \left(\alpha_c^{\alpha_c} \alpha_h^{\alpha_h} \alpha_n^{\alpha_n} R^{-\alpha_h} W^{-\alpha_n} \right)^{1-\gamma}$. Next we can guess and verify that the value function has the form $v(xs_t, a) = \tfrac{v_a}{1-\gamma} * \left(xs_t + \tfrac{1}{1-Q_a}W \right)^{1-\gamma}$ where v_a and Q_a are constants that depend on age a. Suppose this is true for $a+1$. Then the problem is:

$$
\begin{aligned}
v(xs_t, a) &= \max_{\widehat{cs}} \tfrac{\overline{U}}{1-\gamma}\widehat{cs}_t^{1-\gamma} + \tfrac{v_{a+1}}{1-\gamma}\beta Q^{-(1-\gamma)}(xs_t + W - \widehat{cs}_t + W\tfrac{Q}{1-Q_{a+1}})^{1-\gamma} \\
&= \max_{\widehat{cs}} \tfrac{\overline{U}}{1-\gamma}\widehat{cs}_t^{1-\gamma} + \tfrac{v_{a+1}}{1-\gamma}\beta Q^{-(1-\gamma)}(xs_t - \widehat{cs}_t + W\tfrac{1-Q_{a+1}+Q}{1-Q_{a+1}})^{1-\gamma}
\end{aligned}
\tag{38}
$$

Define $X_{a+1} = v_{a+1}Q^{-(1-\gamma)}\beta$. Then the first order condition is: $\overline{U} * \widehat{cs}_t^{-\gamma} = X_{a+1} * (xs_t - \widehat{cs}_t + W\tfrac{1-Q_{a+1}+Q}{1-Q_{a+1}})^{-\gamma}$. Rearranging, we can solve for optimal consumption:

$$\widehat{cs}_t = \frac{\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}}{1+\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}} \left(xs_t + W\tfrac{1-Q_{a+1}+Q}{1-Q_{a+1}} \right)$$
$$xs_{t+1} + \tfrac{1}{1-Q_{a+1}}W = \frac{1}{1+\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}} \left(xs_t + W\tfrac{1-Q_{a+1}+Q}{1-Q_{a+1}} \right) \tag{39}$$

Plugging this back into the original problem:

$$
\begin{aligned}
v(xs_t, a) &= \left(\overline{U}\left(\frac{\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}}{1+\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}}\right)^{1-\gamma} + X_{a+1}\left(\frac{1}{1+\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}}\right)^{1-\gamma} \right) \frac{\left(xs_t+W\tfrac{1-Q_{a+1}+Q}{1-Q_{a+1}}\right)^{1-\gamma}}{1-\gamma} \\
&= \overline{U}\left(1+\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}\right)^{-(1-\gamma)} \left(\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-(1-\gamma)/\gamma} + \left(\tfrac{X_{a+1}}{\overline{U}}\right)\right) \frac{\left(xs_t+W\tfrac{1-Q_{a+1}+Q}{1-Q_{a+1}}\right)^{1-\gamma}}{1-\gamma} \\
&= X_{a+1}\left(1+\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}\right)^{\gamma} \frac{\left(xs_t+W\tfrac{1-Q_{a+1}+Q}{1-Q_{a+1}}\right)^{1-\gamma}}{1-\gamma}
\end{aligned}
\tag{40}
$$

This verifies the conjecture. The age dependent constants take the following form:

$$
\begin{aligned}
v_a &= X_{a+1}\left(1+\left(\tfrac{X_{a+1}}{\overline{U}}\right)^{-1/\gamma}\right)^{\gamma} \\
&= \beta Q^{-(1-\gamma)}v_{a+1}\left(1+\left(v_{a+1}\beta Q^{-(1-\gamma)}\overline{U}^{-1}\right)^{-1/\gamma}\right)^{\gamma} \\
Q_a &= \tfrac{Q}{1+Q-Q_{a+1}}
\end{aligned}
\tag{41}
$$

Note that $Q_\infty = Q$ and $v_\infty = \overline{U}\left(1 - \beta^{\frac{1}{\gamma}} Q^{\frac{-(1-\gamma)}{\gamma}}\right)^{-\gamma}$.

A.3 Commuting costs and composition of Zone 1

From the household's FOC, we know that $\frac{\partial U}{\partial C} = \frac{\partial U}{\partial N} \times \frac{1}{w}$ where C is the numeraire, N is hours worked, and w is the wage. Suppose that moving one unit of distance towards center decreases the hourly commuting cost by ϕ_T and the financial commuting cost by ϕ_F. Also, suppose that the price is a function of distance from center $P(x)$.

First, consider time costs only ($\phi_F = 0$). The cost of decreasing the commute by d is $d \times H \times P'(x) \times \frac{\partial U}{\partial C}$, this is the amount of housing consumed H, multiplied by the price increase at the current location $P'(x) \times d$, multiplied by the marginal utility of the numeraire good. The benefit of decreasing the commute by d is $d \times \phi_T \times \frac{\partial U}{\partial N} = d \times \phi_T \times w \times \frac{\partial U}{\partial C}$, this is the marginal utility of leisure, multiplied by the extra leisure $d \times \phi_T$. Equating the cost to the benefit and rearranging: $P'(x) = \phi_T \frac{w}{H}$. The left hand side represents one's willingness to pay per square foot implying that agents with high $\frac{w}{H}$ are willing to pay a higher price. For a fixed amount of wealth, high income agents have higher $\frac{w}{H}$ because individual productivity is stationary, therefore high income agents tend to save relatively more and consume relatively less of their wealth ($\frac{w}{H}$ would be constant if individual productivity had permanent shocks). For a fixed income, high wealth agents have higher $\frac{w}{H}$ because, consistent with the Permanent Income Hypothesis, for a fixed w, high wealth agents are willing to spend more on housing.

Next, consider financial costs only ($\phi_T = 0$). The cost of decreasing the commute is the same as before $d \times H \times P'(x) \times \frac{\partial U}{\partial C}$. The benefit of decreasing the commute is $d \times \phi_F \times \frac{\partial U}{\partial C}$, this is the financial saving $d \times \phi_F$ multiplied by the marginal utility of the numeraire. Equating the cost to the benefit: $P'(x) = \phi_F \frac{1}{H}$. Low H agents are willing to pay a higher price. Agents who have low wealth or low income tend to have lower housing demand H and are willing to pay more per square foot to reduce their commute. The intuition is that the financial cost is fixed, thus agents with low housing demand are willing to pay a much higher price per square foot to 'ammortize' the benefit of not paying the fixed cost.

A.4 One-period case which can be solved analytically

There are m agents, m^c consumption producing firms, m^1 construction firms in zone 1, and m^2 construction firms in zone 2. There are two zones with sizes $m\overline{h^1}$ and $m\overline{h}^2$. Agents have initial wealth $W = 0$ and earn a wage w. They live for one period only, and there is no resale value for the housing that they buy.

Conditional on a zone, a household maximizes $U = c^{\alpha_c} h^{\alpha_h} (1 - \lambda - x)^{\alpha_n}$ subject to $c + P * h = W + w * x$ where λ is a zone specific time cost and P is a zone specific housing price ($\lambda = 0$ in zone 1). This can be rewritten as:

$$U = \max_{h,x}(W + w * x - P * h)^{\alpha_c} h^{\alpha_h}(1 - \lambda - x)^{\alpha_n} \tag{42}$$

The first order conditions imply the following solution:

$$
\begin{aligned}
c &= \alpha_c((1-\lambda)w + W) \\
h &= \alpha_h((1-\lambda)w + W) \\
x &= (\alpha_c + \alpha_h)(1-\lambda) - \alpha_n \frac{W}{w} \\
U &= \left(\frac{1}{P}\right)^{\alpha_h}\left(\frac{1}{w}\right)^{\alpha_n}\alpha_c^{\alpha_c}\alpha_h^{\alpha_h}\alpha_n^{\alpha_n}((1-\lambda)w + W)
\end{aligned}
\tag{43}
$$

Here we used $\alpha_c + \alpha_h + \alpha_n = 1$.

Each consumption producing firm chooses hours x_c to maximize $\pi_c = x_c^{\rho_c} - wx_c$ which implies that $w = \rho_c x_c^{\rho_c - 1}$. Each construction firm in zone 1 maximizes $\pi_1 = \left(1 - \frac{H^1}{\overline{mh^1}}\right) P_1 x_1^{\rho_h} - wx_1$ which implies that $w = \left(1 - \frac{H^1}{\overline{mh^1}}\right) P_1 \rho_h x_1^{\rho_h - 1}$. Each construction firm in zone 2 maximizes $\pi_2 = \left(1 - \frac{H^2}{\overline{mh^2}}\right) P_2 x_2^{\rho_h} - wx_2$ which implies that $w = \left(1 - \frac{H^2}{\overline{mh^2}}\right) P_2 \rho_h x_2^{\rho_h - 1}$. Here H^1 and H^2 are the total amount of housing built in each zone.

Equilibrium implies that the following equations must be satisfied.

$$P_2 = P_1(1 - \lambda)^{1/\alpha_h} \tag{44}$$

Equation 44 says that for households to be indifferent between the two zones, their utility of living in each zone must be the same.

$$n_1 = \frac{H^1 P^1}{\alpha_h w} \tag{45}$$

$$n_2 = \frac{H^2 P^2}{\alpha_h w(1 - \lambda)} \tag{46}$$

$$n_1 + n_2 = m \tag{47}$$

Equations 45 and 46 say that the total number of households in each zone (N_1 and N_2) must equal to the total housing in each zone, divided by the housing size an agent in that zone would demand. The housing size comes from the solution of the agent's problem. Equation 47 says that the sum of agents living in zones 1 and 2 must equal to the total number of agents.

$$w = \rho_c x_c^{\rho_c - 1} \tag{48}$$

$$w = \left(1 - \frac{H^1}{\overline{mh^1}}\right) P_1 \rho_h x_1^{\rho_h - 1} \tag{49}$$

$$w = \left(1 - \frac{H^2}{\overline{mh^2}}\right) P_2 \rho_h x_2^{\rho_h - 1} \tag{50}$$

Equations 48, 49, and 50 relate each firm's optimal behavior to the wage.

$$H^1 = \left(1 - \frac{H^1}{\overline{mh^1}}\right) m_1 x_1^{\rho_h} \tag{51}$$

$$H^2 = \left(1 - \frac{H^2}{\overline{mh^2}}\right) m_1 x_2^{\rho_h} \tag{52}$$

Equations 51 and 52 relate each firm's output to the total output of housing in each zone. They can be rewritten as $H^1 = \frac{\overline{mh^1} m_1 x_1^{\rho_h}}{\overline{mh^1} + m_1 x_1^{\rho_h}}$ and $H^2 = \frac{\overline{mh^2} m_2 x_2^{\rho_h}}{\overline{mh^2} + m_2 x_2^{\rho_h}}$.

$$(\alpha_c + \alpha_h)(n_1 + n_2(1 - \lambda)) = m_c x_c + m_1 x_1 + m_2 x_2 \tag{53}$$

Equation 53 relates labor supply, on the left side, to labor demand, on the right side.

This is 10 equations and 10 unknowns: prices P_1, P_2; labor demand for each firm type x_1, x_2, x_c; number of households living in each zone n_1, n_2; total housing in each zone H^1, H^2; and the wage w. This can can be reduced to a single equation.

First, plug H and P into equations (49) and (50): $w = P_1\rho_h\frac{m\overline{h^1}x_1^{\rho_h-1}}{m\overline{h^1}+m_1x_1^{\rho_h}} = P_2\rho_h\frac{m\overline{h^2}x_2^{\rho_h-1}}{m\overline{h^2}+m_2x_2^{\rho_h}}$

Second, plug the wage into equations (45) and (46): $n_1 = \frac{m_1x_1}{\alpha_h\rho_h}$ and $n_2 = \frac{m_2x_2}{\alpha_h\rho_h(1-\lambda)}$.

Third, plug n_1 and n_2 into equation (47) to solve for x_2 in terms of x_1: $x_2 = \frac{1-\lambda}{m_2}(m\alpha_h\rho_h - m_1x_1) = A_0 + A_1x_1$ where $A_0 = \frac{1-\lambda}{m_2}m\alpha_h\rho_h$ and $A_1 = -m_1\frac{1-\lambda}{m_2}$.

Fourth, plug $x_2 = A_0 + A_1x_1$ into the equality between zone 1 and zone 2 firms' wages derived earlier and use equation (44) to get rid of prices: $\frac{m\overline{h^1}x_1^{\rho_h-1}}{m\overline{h^1}+m_1x_1^{\rho_h}} = (1-\lambda)^{1/\alpha_h}\frac{m\overline{h^2}(A_0+A_1x_1)^{\rho_h-1}}{m\overline{h^2}+m_2(A_0+A_1x_1)^{\rho_h}}$. This is now one equation with one unknown and can be solved numerically.

Fifth, once we have x_1 we can immediately calculate x_2, n_1, n_2, H^1, H^2 but we still need to solve for w and P_1. We can solve for w as a function of P_1 using equation (49). We can then solve for x_c as a function of P_1 using equation (48). We can then plug this into equation (53) to solve for P_1.

B   Data Appendix: New York

B.1   The New York Metro Area

U.S. Office of Management and Budget publishes the list and delineations of Metropolitan Statistical Areas (MSAs) on the Census website (https://www.census.gov/population/metro/data/metrodef.html). The current delineation is as of July 2015. New York-Newark-Jersey City, NY-NJ-PA MSA (NYC MSA) is the most populous MSA among the 382 MSAs in the nation.

NYC MSA consists of 4 metropolitan divisions and 25 counties, spanning three states around New York City. The complete list of counties with state and zone information is presented in Table 4. As previously defined, only New York County (Manhattan borough) is categorized as zone 1 and the rest 24 counties are categorized as zone 2. For informational purposes, the five counties of New York City are appended with parenthesized borough names used in New York City.

B.2   Population, Housing Stock, and Land Area

The main source for population, housing stock and land area is US Census Bureau American FactFinder (http://factfiner.census.gov). American FactFinder provides comprehensive survey data on a wide range of demographic and housing topics. Using the Advanced Search option on the webpage, topics such as population and housing can be queried alongside geographic filters. We select the DP02 table (selected social characteristics) for population estimates, the DP04 table (selected housing characteristics) for housing estimates, and the GCT-PH1 table (population, housing units, area and density) for land area information. Adding 25 counties separately in the geographic filter, all queried information is retrieved at the county level. We then aggregate the 24 columns as a single zone 2 column.

Since the ACS (American Community Survey) surveys are conducted regularly, the survey year must be additionally specified. We use the 2015 1-year ACS dataset as it contains the most up-to-date numbers available. For Pike County, PA, the 2015 ACS data is not available and we use the 2014 5-year ACS number instead. Given that Pike County accounts only for 0.3% of zone 2 population, the effect of using lagged numbers for Pike County is minimal.

The ratio of the land mass of zone 1 (Manhattan) to the land mass of zone 2 (the other 24 counties of the NY MSA) is 0.0028. However, that ratio is not the appropriate measure of the

Table 4: Counties in the New York MSA

County	State	Zone
New York (Manhattan)	NY	Zone 1
Bergen	NJ	Zone 2
Bronx (Bronx)	NY	Zone 2
Dutchess	NY	Zone 2
Essex	NJ	Zone 2
Hudson	NJ	Zone 2
Hunterdon	NJ	Zone 2
Kings (Brooklyn)	NY	Zone 2
Middlesex	NJ	Zone 2
Monmouth	NJ	Zone 2
Morris	NJ	Zone 2
Nassau	NY	Zone 2
Ocean	NJ	Zone 2
Orange	NY	Zone 2
Passaic	NJ	Zone 2
Pike	PA	Zone 2
Putnam	NY	Zone 2
Queens (Queens)	NY	Zone 2
Richmond (Staten Island)	NY	Zone 2
Rockland	NY	Zone 2
Somerset	NJ	Zone 2
Suffolk	NY	Zone 2
Sussex	NJ	Zone 2
Union	NJ	Zone 2
Westchester	NY	Zone 2

relative maximum availability of housing in each of the zones since Manhattan zoning allows for taller buildings, smaller lot sizes, etc.

Data on the maximum buildable residential area are graciously computed and shared by Chamna Yoon from Baruch College. He combines the maximum allowed floor area ratio (FAR) to each parcel to construct the maximum residential area for each of the five counties (boroughs) that make up New York City. Manhattan has a maximum residential area of 1,812,692,477 square feet. This is our measure for $\bar{H}^1$. The other four boroughs of NYC combine for a maximum buildable residential area of 4,870,924,726 square feet. Using the land area of each of the boroughs (expressed in square feet), we can calculate the ratio of maximum buildable residential area (sqft) to the land area (sqft). For Manhattan, this number is 2.85. For the other four boroughs of NYC it is 0.62. For Staten Island, the most suburban of the boroughs, it is 0.32. We assume that the Staten Island ratio is representative of the 20 counties in the New York MSA that lie outside NYC since these are more suburban. Applying this ratio to their land area of 222,808,633,344 square feet, this delivers a maximum buildable residential square feet for those 20 counties of 71,305,449,967 square feet. Combining that with the four NYC counties in zone 2, we get a maximum buildable residential area for zone 2 of 76,176,377,693 square feet. This is $\bar{H}^2$. The ratio $\bar{H}^1/\bar{H}^2$ is 0.0238. We argue that

this ratio better reflects the relative scarcity of space in Manhattan than the corresponding land mass ratio.

B.3   Income

The main source for the income distribution data is again US Census Bureau `American FactFinder`. From table DP03 (selected economic characteristics), we retrieve the number of households in each of 10 income brackets, ranging from "less than $10,000" for the lowest to "$200,000 or more" for the highest bracket. The distribution suffers from top-coding problem, so we additionally estimate the conditional means for the households in each income bracket. For the eight income brackets except for the lowest and the highest, we simply assume the midpoint of the interval as the conditional mean. For example, for the households in $50,000 to $74,999 bracket, the conditional mean income is assumed to be $62,500. For the lowest bracket, (less than $10,000) we assume the conditional mean is $7,500. Then we can calculate the conditional mean of the highest income bracket, using the average household income and conditional means of the other brackets, since the reported unconditional mean is based on all data.

Our concept of income is household income before taxes. It includes income from (i) wages, salaries, commissions, bonuses, or tips, (ii) farm or non-farm business, proprietorship, or partnership, (iii) social security and railroad retirement payments, (iv) retirement, survivor, or disability pensions, (v) SSI, TANF, family assistance, safety net, other public assistance, or public welfare, (vi) interest, dividends, royalties, estates and trusts.

We aggregate the county-level income distribution into a zone 2 income distribution in two steps. First, the aggregate number of households included in each income bracket is the simple sum of county-level household numbers in the bracket. Second, we calculate the zone 2 conditional mean of the income brackets using the weighted average methods. For the lower nine income brackets, the conditional means are assumed to be constant across counties, so zone 2 conditional means are also the same. For the highest income bracket, we use the county-specific conditional mean of the highest bracket, and calculate its weighted average over the 24 counties. Using these conditional means, and the household distribution over 10 income brackets, the zone 2 average household income can be calculated.

B.4   House Prices, Rental Prices, and Home Ownership

Housing prices and rental prices data come from `Zillow` (http://www.zillow.com/research/data) indices. Zillow publishes Zillow Home Value Index (ZHVI) and Zillow Rent Index (ZRI) monthly. The main advantage of using Zillow indices compared to other indices is that it overcomes sales-composition bias by constantly estimating hypothetical market prices, controlling for hedonics such as house size. We use 2015 year-end data to be consistent with the ACS dataset. There are a few missing counties in ZHVI and ZRI. For the five counties with missing ZHVI index price, we search those counties from `Zillow` (http://www.zillow.com) website, and use the median listing prices instead. For the two counties with missing ZRI index price, we estimate the rents using the price/rent ratio of comparable counties.

Home ownership data is directly from `American FactFinder`. In table DP04 (selected housing characteristics), the *Total housing units* number is divided by *Occupied housing units* and *Vacant housing units*. *Occupied housing units* are further classified into *Owner-occupied* and *Renter-occupied* housing units, which enables us to calculate the home ownership ratio.

B.5   Rent Regulation

The main source for rent regulation data is US Census Bureau New York City Housing and Vacancy Survey (NYCHVS; http://www.census.gov/housing/nychvs). NYCHVS is conducted every three years to comply with New York state and New York City's rent regulation laws. We use the 2014 survey data table, which is the most recent survey data. In Series IA table 14, the number of housing units under various rent-control regulations are available for each of the five NYC boroughs. We define rent-regulated units as those units that are (i) rent controlled, (ii) public housing, (iii) Mitchell Lama housing, (iv) all other government-assisted or regulated housing.

We exclude rent-stabilized units from our definition. Rent stabilized units are restricted in terms of their annual rent increases. The vast majority of units built after 1947 that are rent stabilized are so voluntarily. They receive tax abatement in lieu of subjecting their property to rent stabilization for a defined period of time. Both rent levels and income levels of tenants in rent-stabilized units are in between those of rent-regulated and unregulated units.

We calculate the proportion of rent-regulated units among all the renter-occupied units. The proportion is 16.9% for Manhattan and 13.2% for the other four NYC boroughs.

We use a different data source for the other 20 counties outside of New York City. `Affordable Housing Online` (http://affordablehousingonline.com) provides various rent-related statistics at the county level. For each of the 20 counties outside NYC, we calculate the fraction of rent-regulated units by dividing *Federally Assisted Units* number by *Renter Households* number reported on each county's webpage. We then multiply these %-numbers with the renter-occupied units in ACS data set to calculate the rent-regulated units for the 20 counties. Along with the NYCHVS numbers for the four NYC boroughs, we can aggregate all the 24 counties in zone 2 to calculate the fraction of rent-regulated units. The four NYC boroughs have 1.53 million renter-occupied housing units while the rest of zone 2 has 1.30 million. The resulting fraction of rent-regulated units in zone 2 is 10.4%.

From the NYCHVS, we also calculate the percentage difference in average rent in New York City between our definition of regulated rentals and the others (unregulated plus rent-stabilized). That percentage difference is 49.9%. We apply the same percentage difference to all of the MSA in our model.

Finally, we calculate the percentage difference in average household income (Series IA - Table 9) in New York City between our definition of regulated rentals and the others (unregulated plus rent-stabilized). That percentage difference is 54.2%. This is a moment we can compute in the model and compare to the data.

B.6   Migration

We use county-to-county migration data for 2006-2010 and 2010-2014 from the 5-year American Community Survey for the 25 counties in the New York metropolitan area. For each county and survey wave, we compute net migration rates (inflow minus outflow divided by population). When one person enters the New York labor market and another one leaves, the model is unchanged, so net migration is the relevant concept for the model. We aggregate net migration for the 24 counties in zone 2. The net migration rate over the 5-year period between 2010-2014 for the entire MSA is -0.15%, or -0.03% per year. First, this net migration rate is minuscule: only about 30,000 people moved in over a 5-year period on a MSA population of 20 million. Of course, this masks much larger gross flows: about 824,000 came into the MSA and 854,000 left. Second, Manhattan (zone 1) saw a net inflow of 30,000 people coming from outside the MSA while the rest of the MSA (zone 2) saw a net outflow to the rest of the country/world of 60,000. This is the opposite

pattern than what we would expect if the out-of-town (OOT) purchases prompted migration of residents, since OOT purchases were much stronger in Manhattan than in the rest of the MSA (twice as large). Third, comparing the net migration in the 2010-2014 period to that in the 2006-2010 period, we find that the net migration rate rose, from -73,000 to -30,000. The net migration rate rises from -0.38% in 2006-2010 to -0.15% in the 2010-2014 period. The rise in OOT purchases over time did not coincide with a decline in net migration, but with an increase. In other words, not only are the relevant net migration rates tiny, they also have the wrong-sign cross-sectional correlation with the spatial OOT pattern, and with the time-series of OOT purchases. We conclude that there is little evidence in the New York data of substantial net migration responses to OOT purchases.

C   Earnings Calibration

Before-tax earnings for household i of age a is given by:

$$y_{i,a}^{lab} = W_t n_t^i G^a z^i$$

where G^a is a function of age and z^i is the idiosyncratic component of productivity. Since endogenous labor supply decisions depend on all other parameters and state variables of the model, exactly matching earnings in model and data is a non-trivial task.

We determine G^a as follows. For each wave of the Survey of Consumer Finance (SCF, every 3 years form 1983-2010), we compute average earnings in each 4-year age bucket (above age 21), and divide it by the average income of all households (above age 21). This gives us an average relative income at each age. We then average this relative age-income across all 10 SCF waves.

We also use SCF data to determine how the dispersion of income changes with age. We choose four grid points for income, corresponding to fixed percentiles (0-25, 25-75, 75-90, 90-100). To compute the idiosyncratic income $z_{a,i}$ of each group $i \in \{1, 2, 3, 4\}$ at a particular age a, relative to the average income of all households of that age we do the following:

Step 1: For each positive-earnings household, we compute which earnings group it belongs to among the households of the same age.

Step 2: For each 4-year age bucket, we compute average earnings of all earners in a group.

Step 3: We normalize each group's income by the average income in each age group, to get each group's relative income.

Step 4: Steps 1-3 above are done separately for each wave of the SCF. We compute an equal-weighted average across all 10 waves to get an average relative income for each age and income group. This gives us four 11x1 vectors $z_{a,i}$ since there are 11 4-year age groups between ages 21 (entry into job market) and 65 (retirement). Note that the average z across all households of a particular age group is always one: $E[z_{a,j}|a] = 1$.

Step 5: We regress each vector, on a linear trend to get a linearly fitted value for each group's relative income at each age. The reason we perform Step 5, rather stopping at Step 4 is that the relative income at age 4 exhibits some small non-monotonicities that are likely caused by statistical noise (sampling and measurement error). Step 5 smoothes this out.

We set the productivity states to $z^i \in Z = [0.255, 0.753, 1.453, 3.522]$ to match the observed mean NY household income levels, scaled by the NY metro area average, in the income groups

below $41,000, between $41,000 and $82,000, between $82,000 and $164,000, and above $164,000. Those bins respectively correspond to bins for individual earnings below $25,000, between $25,000 and $50,000, between $50,000 and $100,000, and above $100,000, adjusted for the number of working adults in the average New York household (1.64). The NY income data is top-coded. For each county in the NY metro area, we observe the number of individuals whose earnings exceed $100,000. Because we also observe average earnings (without top-coding), we can infer the average income of those in the top coded group.

The transition probability matrix for z is $\mathbb{P}$ for β^L agents. We impose the following restrictions:

$$\mathbb{P} = \begin{bmatrix} p_{11} & 1-p_{11} & 0 & 0 \\ (1-p_{22})/2 & p_{22} & (1-p_{22})/2 & 0 \\ 0 & (1-p_{33})/2 & p_{33} & (1-p_{33})/2 \\ 0 & 0 & 1-p_{44} & p_{44} \end{bmatrix}$$

For β^H types, the transition probability matrix is the same, except for the last two entries which are $1-p_{44}-p^H$ and $p_{44}+p^H$, where $p^H < 1-p_{44}$. We pin down the five parameters

$$(p_{11}, p_{22}, p_{33}, p_{44}, p^H) = (.9299, .9216, .9216, .8198, .02)$$

to match the following five moments. We match the population shares in each of the four income groups defined above: 16.1%, 29.8%, 34.2%, and 19.9%, respectively (taken from the individual earnings data). Given that population shares sum to one, that delivers three moments. We match the persistence of individual labor income to a value of 0.9, based on evidence form the PSID in Storesletten, Telmer, and Yaron (2006). Finally, we choose p^H to match the fraction of high-wealth households in the top 10% of the income distribution.

Table 5 summarizes the results we obtain in the model. Average earnings are reported annually. Earnings autocorrelation and volatility are reported for 4 years.

Table 5: Labor Earnings Calibration

	Group 1	Group 2	Group 3	Group 4	All
Avg earnings	23,950	66,870	123,110	339,630	
(Data)	(28,125)	(60,951)	(116,738)	(309,016)	
Pop shares	21.9%	28.7%	31.5%	17.9%	
(Data)	(16.1%)	(29.8%)	(34.2%)	(19.9%)	
Earnings autocorr.					0.77
Earnings vol.					0.125
Corr. (income,wealth)					0.25

Notes: Averages by bins in the data are obtained by multiplying average labor earnings ($124,091 annually) by the ratio of average earnings in each bin to the overall average (0.23, 0.50, 0.96, 2.42). Average household earnings and population shares in the data are denoted in parentheses and obtained from Census Bureau data for the year 2016.

D Progressive Taxation

Households with income $y^{tot} < y_0^{tot} = \lambda^{\frac{1}{\tau}}$ receive transfers $T(y^{tot}) < 0$, and those with $y^{tot} \geq y_0^{tot}$ pay taxes $T(y^{tot}) \geq 0$. As a result of our calibration, 26% of households are subsidized by the progressive tax system, and 14% receive a subsidy after subtracting Social Security taxes. Figure 5 describes the progressive taxation system. At low total income values, some households receive a subsidy, which progressively decreases. At higher incomes, taxes increase faster than income. This is reflected in households' post-tax income, shown in Figure 6.

Figure 5: Progressive Taxes
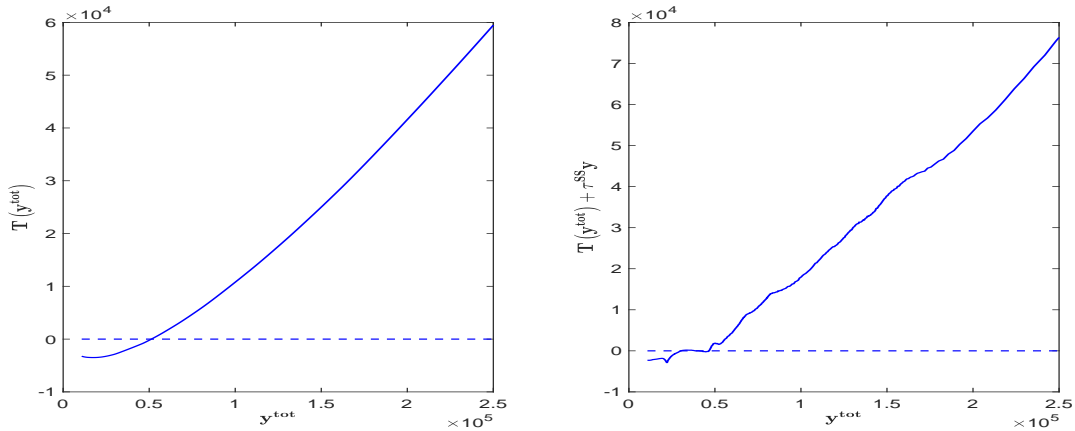
Notes: Horizontal axis: total income (in dollars, annual), measured as the sum of labor earnings, pensions, and financial income. Vertical axis: taxes minus transfers excluding Social Security taxes and transfers (in dollars, annual; left panel), total taxes minus transfers including Social Security taxes and not transfers (in dollars, annual; left panel). The dashed line plots the zero-tax case.

Figure 6: After-tax Total Income
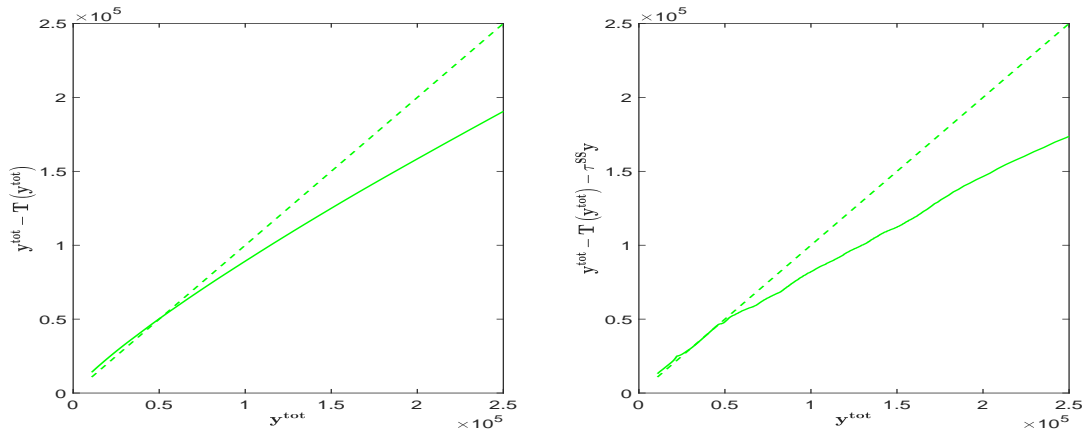
Notes: Horizontal axis: total income (in dollars, annual). Vertical axis: post-tax income excluding Social Security taxes (in dollars, annual; left panel), post-tax income including Social Security taxes (in dollars, annual; left panel). The dashed line is the 45 degree line.

E Housing Supply Elasticity Calibration

We compute the long-run housing supply elasticity. It measures what happens to the housing quantity and housing investment in response to a 1% permanent increase in house prices. Define housing investment for a given zone, dropping the location superscript since the treatment is parallel for both zones, as:

$$Y_t^h = \left(1 - \frac{H_{t-1}}{\overline{H}}\right) N_t^{\rho_h}.$$

Note that $H_{t+1} = (1 - \delta)H_t + Y_t^h$, so that in steady state, $Y^h = \delta H$. Rewriting the steady state housing investment equation in terms of equilibrium quantities using (8) delivers:

$$H = \frac{1}{\delta}\left(1 - \frac{H}{\overline{H}}\right)^{\frac{1}{1-\rho_h}} \rho_h^{\frac{\rho_h}{1-\rho_h}} \overline{P}^{\frac{\rho_h}{1-\rho_h}} W^{\frac{-\rho_h}{1-\rho_h}}$$

Rewrite in logs, using lowercase letters to denote logs:

$$h = -\log(\delta) + \frac{1}{1-\rho_h}\log(1 - \exp(h - \overline{h})) + \frac{\rho_h}{1-\rho_h}\overline{p} - \frac{\rho_h}{1-\rho_h}w$$

Rearrange and substitute for $\overline{p}$ in terms of the market price $\overline{p} = \log(ho + (1 - ho)\kappa_4) + p$:

$$p = \frac{1-\rho_h}{\rho_h}h - \frac{1}{\rho_h}\log(1 - \exp(h - \overline{h})) + k$$

where

$$k \equiv \frac{1-\rho_h}{\rho_h}\log(\delta) + w - \log(ho + (1 - ho)\kappa_4)$$

Now take the partial derivative of p w.r.t. h:

$$\frac{\partial p}{\partial h} = \frac{1-\rho_h}{\rho_h} + \frac{1}{\rho_h}\frac{\exp(h - \overline{h})}{1 - \exp(h - \overline{h})} + \frac{\partial k}{\partial h}$$

Invert this expression delivers the housing supply elasticity:

$$\frac{\partial h}{\partial p} = \frac{\rho_h}{1 - \rho_h + \frac{\exp(h-\overline{h})}{1-\exp(h-\overline{h})} + \left[\rho_h\frac{\partial w}{\partial h} - \rho_h\frac{(1-\kappa_4)}{ho+(1-ho)\kappa_4}\frac{\partial ho}{\partial h}\right]} \tag{54}$$

If (i) the elasticity of wages to housing supply is small ($\frac{\partial w}{\partial h} \approx 0$) and either the rent control distortions are small ($\kappa_4 \approx 1$) or the home ownership rate is inelastic to the housing supply ($\frac{\partial ho}{\partial h} \approx 0$), or (ii) if the two terms in square brackets are positive but approximately cancel each other out, then the last two terms are small. In that case, the housing supply elasticity simplifies to:

$$\frac{\partial h}{\partial p} \approx \frac{\rho_h}{1 - \rho_h + \frac{\exp(h-\overline{h})}{1-\exp(h-\overline{h})}}$$

Since, in equilibrium, $Y^h = \delta H$, $\partial y^h/\partial p = \partial h/\partial p$.

Note that $h - \overline{h}$ measures how far the housing stock is from the constraint, in percentage terms.

As H approaches $\overline{H}$, the term $\frac{\exp(h-\overline{h})}{1-\exp(h-\overline{h})}$ approaches $+\infty$ and the elasticity approaches zero. This is approximately the case in zone 1 for our calibration. If H is far below $\overline{H}$, that term is close to zero and the housing supply elasticity is close to $\frac{\rho_h}{1-\rho_h}$. That is approximately the case for zone 2 in our calibration. Since zone 2 is by far the largest component of the New York metro housing stock, zone 2 dominates the overall housing supply elasticity we calibrate to.

In the calibration, we use equation (54) to measure the housing supply elasticity and set $\frac{\partial w}{\partial h} = 0.25$ based on evidence from Favilukis and Van Nieuwerburgh (2018), who study a model with aggregate shocks to housing demand driven by out-of-town home buyers. We also set $\kappa_4 = 1$.

F Mobility Rates

Figure 7: Moving Rates
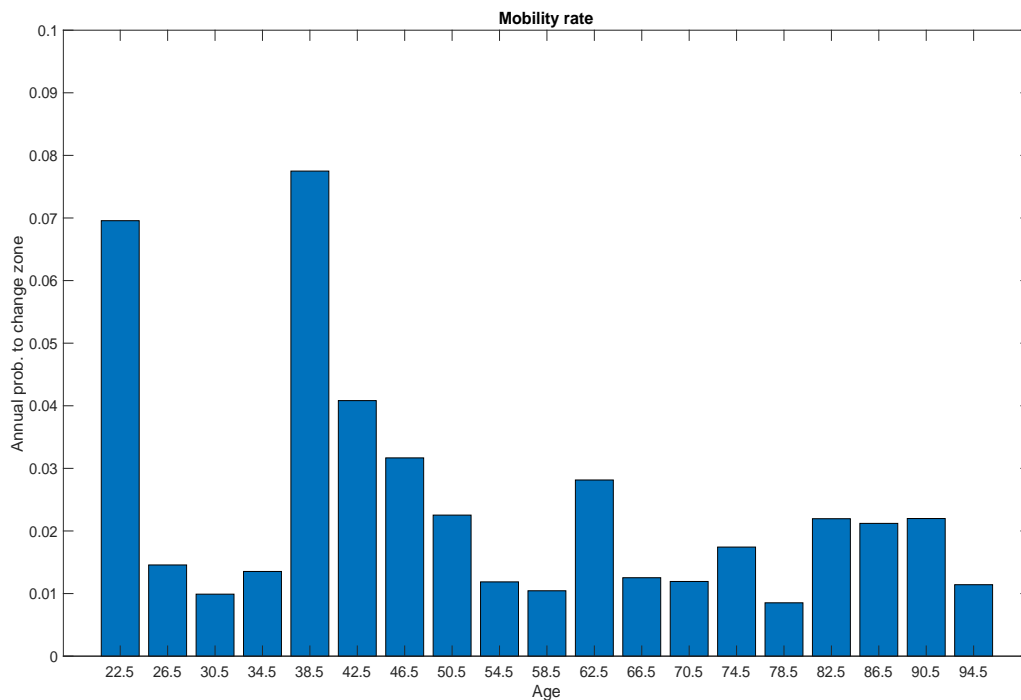
Notes: Mobility rates by age are measured as the annual probability to move across zones.

G Additional Affordability Policies

In this appendix, we study moving three additional policy levers of the RC system: the rent subsidy governed by κ_1, the income qualification threshold governed by κ_2, and the size of the RC units governed by κ_3. These three policies have lower average welfare effects than the four policies discussed in the main text. Table 6 and Figure 8 summarize the results.

G.1 Reducing the Rent Subsidy for RC Housing

In the first additional experiment, we reduce the rental discount that RC households enjoy. Specifically, we lower the rent discount parameter κ_1 from 50% to 25%, while keeping the share of square footage that goes to RC housing in each zone unchanged.

Surprisingly, the reduced generosity of the RC system leads to a 19% increase in the fraction of households in RC. The reason is that RC households choose a much smaller average RC apartment (-19%). Thus, this policy has similar effects as a policy that has more "micro units" in the RC system.[20] All the RC units continue to go to the bottom quartile of the income distribution. In that income group, nearly 25% of households obtain a RC unit compared to 20% in the benchmark.

The lower rent discount reduces development distortions; κ_4 increases from .88 to .94 in zone 1 and from .95 to .98 in zone 2. The upwards shift in the housing supply curve generates a 0.21% increase in the equilibrium housing stock in zone 1, and a 0.27% increase in zone 2. Rents fall 0.43% in zone 1 and 0.56% in zone 2.

The policy also causes a modest shift in the composition of the population in each zone. The population of Manhattan increases by 0.84% as more medium-productivity households move in. The fraction of top-productivity households falls, and so does average income in Manhattan (-0.86%). With more households in Manhattan, commuting time falls (-0.28%). Total hours worked and output both increase modestly.

Housing affordability metrics improve. While the average rent to average income ratio increase by 2% in zone 1, it decreases by 2% in zone 2 where it applies to a much larger population of renters. The fraction of severely rent-burdened households falls by 13%. This illustrates the power of general equilibrium effects. Partial equilibrium logic would imply that the reduced rent discount for RC would lead to increased rent burdens.

This policy reduces welfare by -0.07%, similar to the policy that reduces the amount of rent control by half. Owners gain (+0.10%) because house prices are lower, market renters gain (+0.28%) because they enjoy lower rents and commute less, and RC households lose substantially (-4.18%). The households who enjoyed RC before now live in much smaller units, accounting for the large welfare loss. Concerns of adverse redistribution dominate aggregate welfare.

G.2 Reducing the Income Qualification Threshold for RC Housing

In the second additional experiment, we tighten the income requirements to qualify for rent control, governed by the parameter κ_2. Households must make less than 33% of AMI to qualify, compared to 38% in the benchmark economy.

This policy results in a very small welfare gain of 0.00%, the smallest among all policies we consider. It hurts the tenants currently in RC (-0.05%) as well as home owners (-0.06%), while

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[20]Such a policy advocated by several policy institutes. For instance the NYU Furman Center (2018). New York City historically discouraged the development of small apartment units due to a variety of rules and regulations.

market renters gain (+0.11%). The middle panel of Figure 8 plots how the gains and losses are distributed across age.

Because households receiving RC must now have a lower income to qualify, they live on average in smaller units (-3%). For the same amount of RC square feet, the number of households in RC rises (+1.5%), all of them from the lowest quartile of the income distribution.

The policy change results in fewer households living in Manhattan (-0.85%). The policy change does not directly affects developer incentives. However, construction increases in Manhattan (+0.16%) as the demand for larger housing units trumps the reduction in population. The average income in zone 1 increases by 0.58%. But there are fewer top-productivity households in Manhattan (-0.9%), because the Manhattan population has shrunk.

Tightening the income qualification threshold essentially leaves welfare unchanged. RC renters lose modestly (-0.05%) since the average RC unit size shrinks. Market renters gain (+0.11%) while owners lose (-0.06%). This policy is progressive. When sorted on income in the benchmark, the welfare gains are strictly decreasing, and become negative beyond the second income quartile. The gains are largest in the lowest income quartile (+0.19%).

G.3 Reducing the Maximum Size of RC Housing

The third additional experiment changes how "deeply affordable" RC units must be, governed by the parameter κ_3. In the benchmark, affordable housing units have a rent expenditure cap of 64% of AMI on rent, or about \$1,650 per month. Here we change this cap to 32% of AMI or \$825 per month. In the model, this lowers the maximum size of a RC unit. The policy change lowers the average RC unit size by 0.6% and the fraction of households in RC by 0.35%.

This policy has small welfare losses. The overall welfare change is -0.01%, with RC renters gaining (+0.01%), renters gaining (+0.02%), owners losing (-0.03%). The bottom panel of Figure 8 plots how the gains and losses are distributed across age. This policy has small effects on equilibrium rents and prices, and the implications for the spatial allocation of households and housing are similar as in the previous experiment.

Table 6: Main moments of the model under affordability policies that modify features of the RC system and the spatial allocation of housing.

		Benchmark	RC discount	Inc. cutoff	RC housing size
1	Rent to income Z1 (%)	41.6	2.04%	2.77%	-0.25%
2	Rent to income Z2 (%)	32.3	-2.05%	0.29%	0.13%
3	Frac. RC (%)	5.47	18.73%	1.48%	-0.35%
4	Frac. RC of those in income Q1 (%)	19.75	21.20%	2.99%	0.45%
5	Frac. sev. rent-burdened (%)	4.3	-12.83%	9.52%	-1.05%
6	Avg. size RC unit (sqft)	695	-19.35%	-3.08%	-0.59%
7	Avg. size Z1 unit (sqft)	942	-0.68%	0.95%	0.75%
8	Avg. size Z2 unit (sqft)	1510	0.37%	0.07%	0.03%
9	Frac. pop. Z1 (%)	10.5	0.84%	-0.85%	-0.73%
10	Housing stock Z1	–	0.21%	0.16%	0.07%
11	Housing stock Z2	–	0.27%	0.17%	0.13%
12	Rent/sqft Z1 ($)	4.15	-0.43%	0.15%	0.17%
13	Rent/sqft Z2 ($)	1.50	-0.56%	0.20%	0.23%
14	Price/sqft Z1 ($)	993	-0.51%	-0.12%	-0.12%
15	Price/sqft Z2 ($)	329	-0.74%	-0.16%	-0.19%
16	Home ownership Z1 (%)	56.0	2.08%	0.56%	0.43%
17	Home ownership Z2 (%)	59.0	-2.33%	0.42%	0.44%
18	Avg. income Z1 ($)	161323	-0.86%	0.58%	0.57%
19	Avg. income Z2 ($)	100651	1.02%	0.76%	0.68%
20	Frac. high prod. Z1 (%)	23.6	-1.62%	-0.93%	-1.08%
21	Total hours worked	–	0.58%	0.11%	-0.06%
22	Total hours worked (efficiency)	–	0.17%	-0.03%	-0.16%
23	Total output	–	0.05%	-0.09%	-0.16%
24	Total commuting time	–	-0.28%	0.29%	0.23%
25	Welfare change RC	–	-4.18%	-0.05%	0.01%
26	Welfare change market renters	–	0.28%	0.11%	0.02%
27	Welfare change owners	–	0.10%	-0.06%	-0.03%
28	**Aggregate welfare change**	–	**-0.07%**	**0.00%**	**-0.01%**

Notes: Column "Benchmark" reports values of the moments for the benchmark model. Columns "RC discount" to "RC housing size" report percentage changes of the moments in the policy experiments relative to the benchmark. Rows 1-8 report housing affordability moments , rows 9-24 aggregate moments across the two zones, and rows 25-26 welfare moments. Z1 stands for zone 1 (Manhattan), Z2 for the rest of the metro area. Row 20 reports what fraction of working age top-productivity households live in zone 1.

Figure 8: Welfare effects of tightening RC
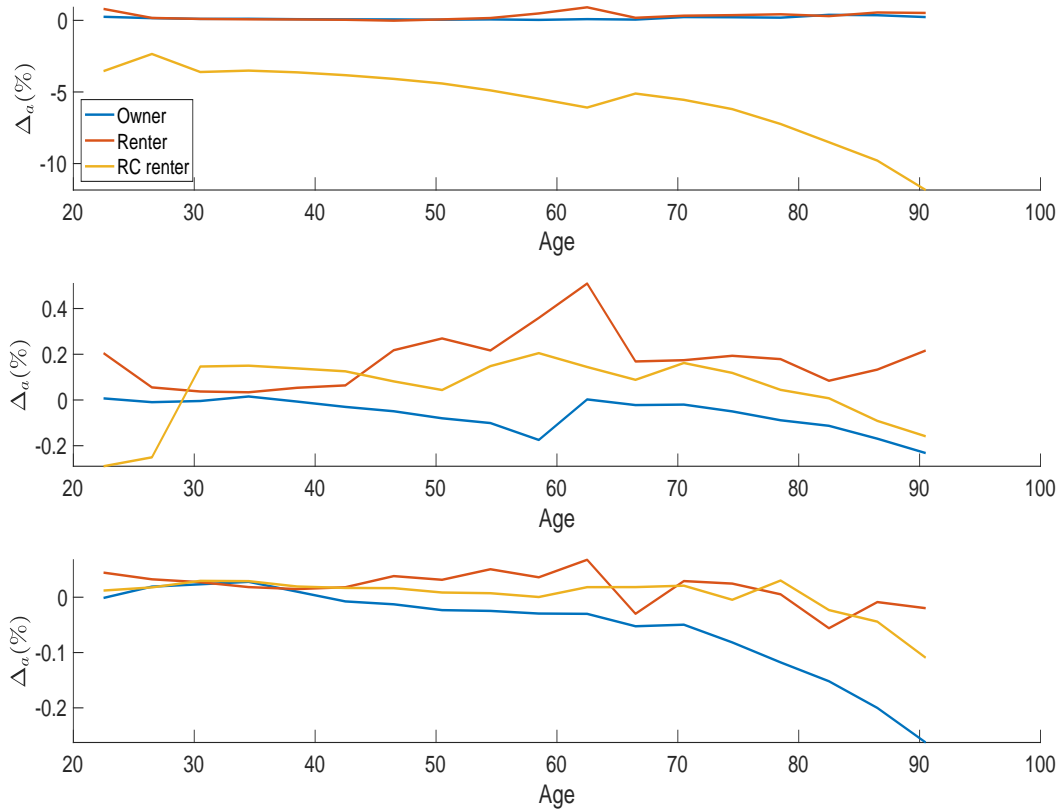
Notes: The baseline model has the following rent control parameters: $\eta^1 = .2426$, $\eta^2 = .0953$, $\kappa_1 = .5$, $\kappa_2 = .1110$, $\kappa_3 = .1869$. Top panel: welfare changes from a decrease in the RC discount on market rent by 50% ($\kappa_1 = 0.25$). Middle panel: decreasing the qualifying income cutoff for RC by 10% ($\kappa_2 = .0555$). Bottom panel: decreasing the maximum size of RC units by 50% ($\kappa_3 = .0935$). For each household, the welfare changes are measured as changes in the value function under the alternative policy relative to the value function under the benchmark policy, expressed in consumption equivalent units; see equation (13). These welfare changes are then aggregated across age and tenure status groups, where tenure status is the tenure status in the benchmark model, i.e., before the policy changes.

G.4 Graphing the Household Distribution Across Space, Productivity, and Tenure Status

Below, we present various graphs that graph the household distribution, first for the benchmark model and then for the various policy experiments. The vertical axes measure the total square footage devoted to the various types of *housing* in each zone (model units for readability, equal to the measure in sqft divided by 1,976): owner-occupied, rental, rent controlled housing. Values reported on the top of the bars correspond to the percentage of *households* in each category. Colors correspond to productivity levels: increasing from yellow (low) to red (high) for working-age households, green for retirees. Each bar stacks from the largest housing unit (in the bottom) to the smallest housing unit (at the top).

Figure 9: Benchmark model: zone 1 (left panel) and zone 2 (right panel).
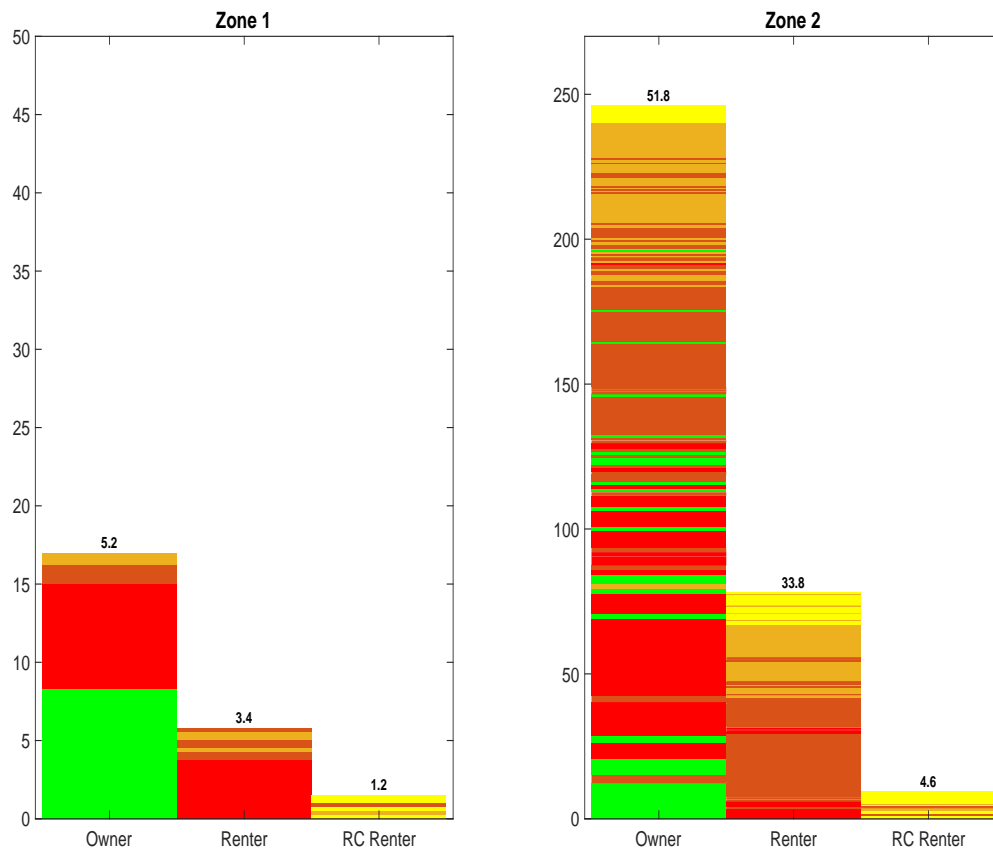
Figure 10: Reducing the fraction of rent controlled housing by 50%: zone 1 (left panel) and zone 2 (right panel).
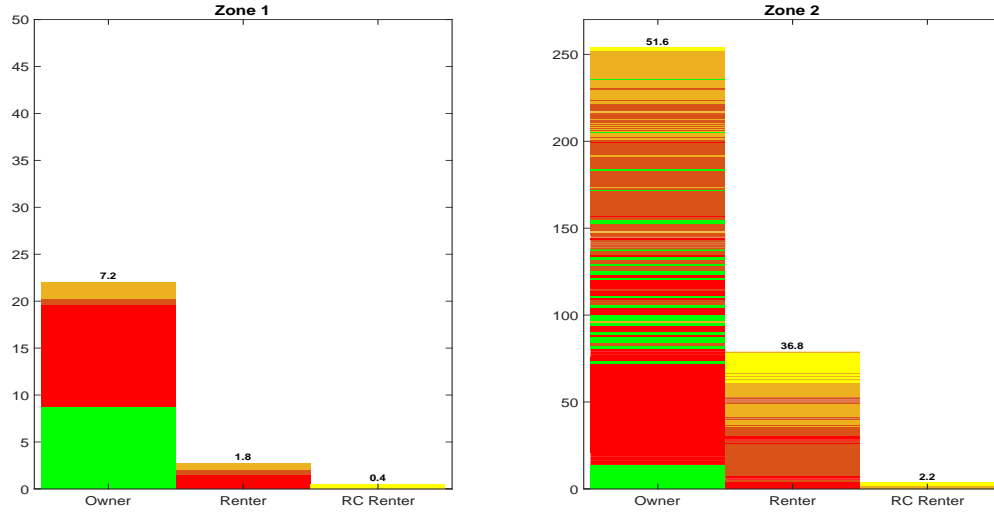
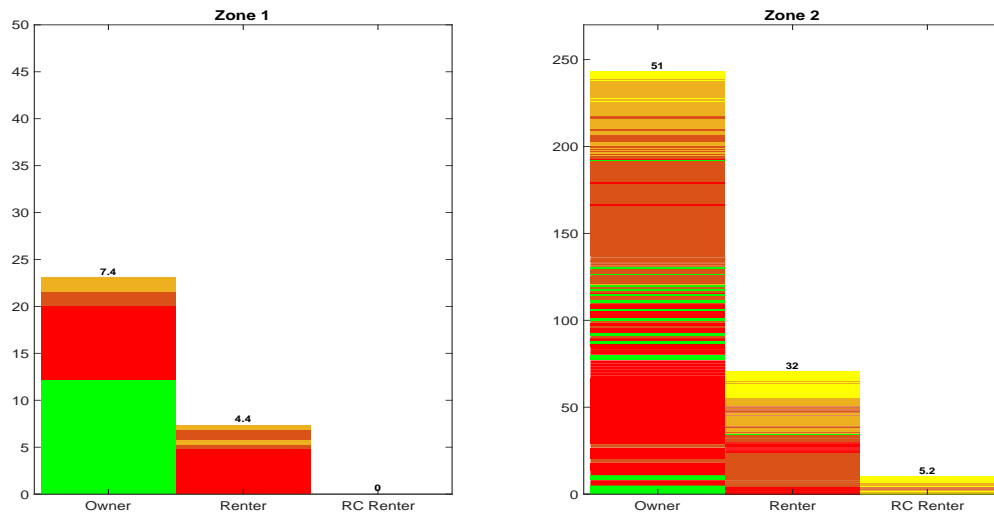Figure 11: Spatial allocation of RC housing: zone 1 (left panel) and zone 2 (right panel).

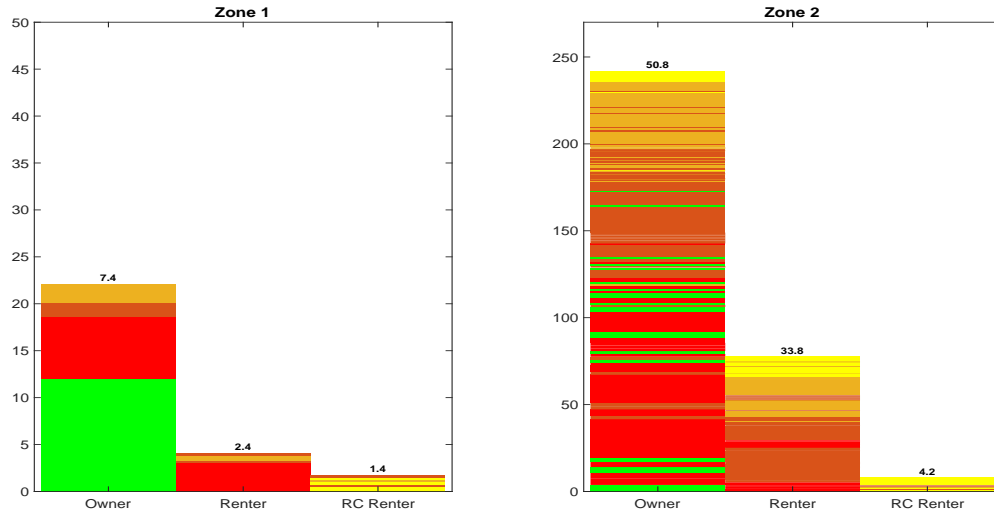Figure 12: Relaxing zoning laws in zone 1: zone 1 (left panel) and zone 2 (right panel).

Figure 13: Increasing the amount of housing vouchers: zone 1 (left panel) and zone 2 (right panel).
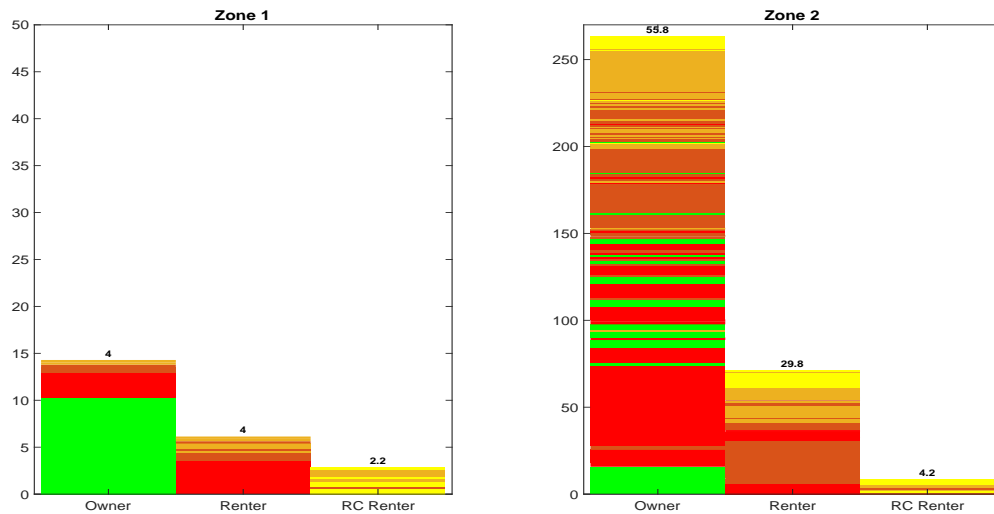
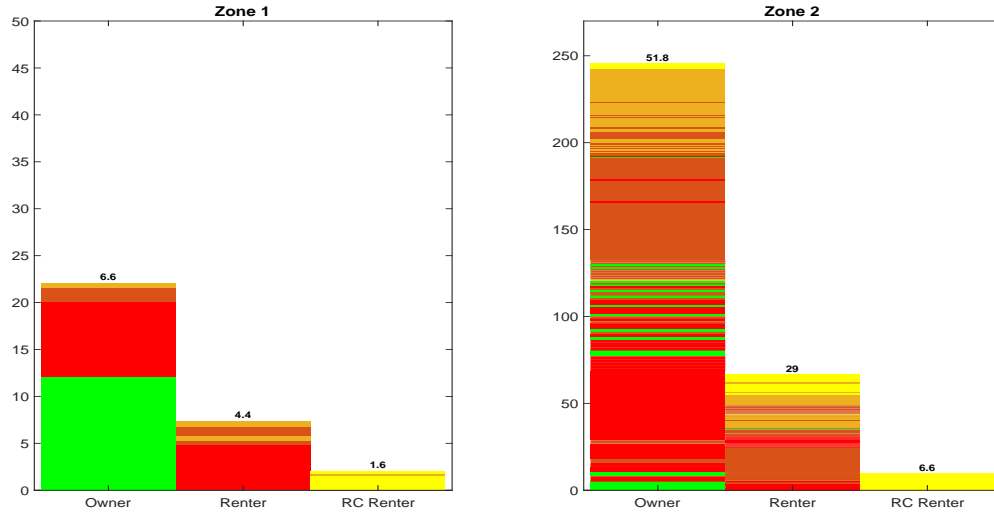Figure 14: Reducing the rent subsidy for RC households by 50%: zone 1 (left panel) and zone 2 (right panel).

Figure 15: Reducing the income qualification threshold for RC housing by 10%: zone 1 (left panel) and zone 2 (right panel).
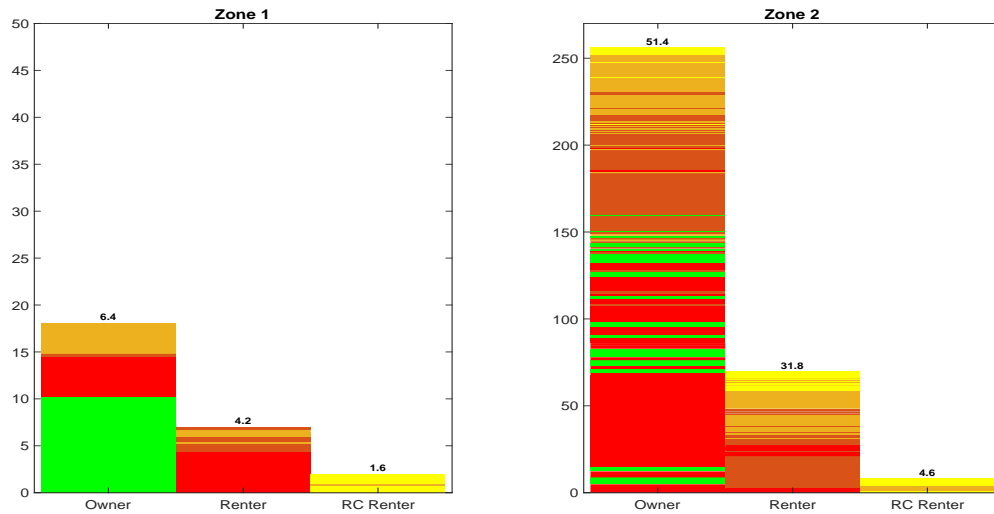
Figure 16: Reducing the maximum size of RC housing by 50%: zone 1 (left panel) and zone 2 (right panel).