Monetary Policy Spillovers through Invoicing Currencies

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Abstract

United States monetary policy affects macro-financial outcomes globally. I introduce heterogeneity in invoicing currencies into an open economy New Keynesian model that also allows for differences in country size and household preferences. Within the model, cross-sectional variation in U.S. monetary policy spillover effects is fully captured by heterogeneity in countries' shares of dollar invoiced trade. Moreover, central banks of countries in which a larger share of exports are invoiced in dollars face a worse output-inflation trade-off, i.e., a steeper Phillips Curve. Using high frequency measures of monetary policy shocks, I find support for the model's predictions. Countries' shares of dollar invoiced trade explain cross-sectional heterogeneity in spillovers from U.S. monetary policy shocks onto foreign exchange rates and interest rates. Moreover, these spillover effects are not limited to the FRB and the U.S. dollar. Using high frequency measures of monetary policy shocks from the European Central Bank, I show countries with more euro invoiced trade suffer larger spillovers from the ECB. After controlling for currency invoicing in trade, the magnitude of U.S. monetary policy spillovers are not different from those of the ECB.

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1 Introduction

The actions of the Federal Reserve Bank of the United States (FRB) are a fundamental concern for central bankers around the world. A growing literature finds monetary policy actions of the United States significantly impact macroeconomic conditions (Rey, 2013; Miranda-Agrippino and Rey, 2015) and asset prices (Brusa et al., 2015) globally. What features of the global economy allow the FRB to influence global macroeconomic conditions, and to what extent do monetary policy actions from other central banks also influence outcomes abroad? Recent evidence from Gopinath (2015) shows that over 80 percent of traded goods are invoiced in dollars and euros. This paper studies the role of trade invoicing currencies in transmitting monetary policy abroad.

Building on Engel (2011), I develop an open economy New Keynesian model in which prices of traded goods are sticky in their currency of invoicing. Firms in each country invoice exports either in a domestic currency or in a global trade currency issued by a "center" country. [1] The prices of traded goods invoiced in a particular currency are sticky in that currency in the short run. I show countries with a larger share of consumption invoiced in the global trade currency suffer larger spillovers of the center country's monetary policy. Moreover, countries in which firms invoice more of their exports in the global trade currency should face a worse policy trade-off between output and inflation (i.e. a steeper Phillips curve). Using the model, I generate predictions characterizing monetary policy transmission from the center country to foreign exchange rates and interest rates. I find support for each of the model's predictions using high-frequency measures of monetary policy shocks. Countries with more dollar invoiced traded indeed suffer larger spillovers from U.S. monetary policy shocks. However, these effects are not unique the FRB or the U.S. dollar. I provide evidence that monetary policy spillover effects emanate from the European Central Bank (ECB) as well, and that the large magnitude of U.S. monetary policy spillover effects can be explained by the dollar's dominance as a global trade currency.

I begin with the theoretical analysis and characterize the effect of currency invoicing on monetary policy transmission across countries in a model where financial markets are complete.

[1] Invoicing in domestic currency is typically labeled "producer currency pricing". Within the model, I take the firm's invoicing currency as given. A concern with this approach is that some underlying factors expose a country to monetary policy shocks from the center country and also cause a country to invoice exports in the global trade currency. The international trade literature has identified some characteristics that influence the firm's invoicing currency choice (Devereux et al., 2004; Goldberg and Tille, 2008; Gopinath et al., 2010). Examples include the liquidity of the currency, industry structure and desired exchange rate pass-through. However, there is no consensus for why we observe the heterogeneity in invoicing currency use at the country level. In the empirical section of the paper, I control for alternative hypotheses for explaining heterogeneity in monetary policy spillovers beyond the role of currency invoicing itself.

I allow countries to differ in size, households to differ in their preferences for varieties of traded goods and firms to differ in their invoicing currency of exports. In other words, I allow countries to differ along a number of dimensions that may matter for explaining heterogeneity monetary policy transmission, and I use the model to characterize the impact of these various sources of heterogeneity on monetary policy transmission.

Within the model, variation in monetary policy spillovers from the center country to foreign households is fully captured by the share of each country's consumption invoiced in the global trade currency. A country's share of consumption invoiced in the global trade currency is a function of underlying household preferences and the fraction of all traded goods invoiced in the global trade currency. Contractionary monetary policy from the center country increases the value of the global trade currency relative to all other currencies. Countries with a larger share of consumption invoiced in the global trade currency suffer a larger increase to their cost of consumption. As a result, these countries decrease their aggregate consumption more in response to monetary contraction in the center country.

The model shows central banks in countries where more firms invoice their exports in the global trade currency face a worse output-inflation trade-off. Because prices of traded godos are sticky in their currency of invoicing, a central bank's monetary policy affects demand for all traded goods invoiced in its currency — domestically and abroad. When domestic firms invoice their exports in the global trade currency, a central bank loses influence over foreign demand for their exports. Hence, the central bank's monetary policy actions become less effective at influencing domestic outcomes. As a result, production in countries that export in the global trade currency are subject to the monetary policy actions of the center country's central bank, even if exchange rates are freely floating.

Rather than try to estimate precise relationships between monetary policy and low frequency macroeconomic outcomes, I use the model to derive empirical predictions that can be tested using high frequency asset pricing data. In response to monetary contraction from the center country, foreign countries with a larger share of consumption invoiced in the global trade currency observe their currency appreciate and their interest rate increase relative to countries with a smaller share of consumption invoiced in the global trade currency. In the benchmark complete markets model, countries with more consumption invoiced in the global trade currency consume less and their marginal utility of consumption increase more in response to monetary contractions in the center country. This increase in the marginal utility of consumption directly translates into a real exchange rate appreciation (an increase in the real price of consumpiton) and an increase in real interest rates. Changes in nominal exchange rates and nominal interest rates mirror the changes in real exchange rates and real interest rates.

Moreover, these spillover effects are not unique to the FRB. Within the model, I show the magnitude of monetary policy spillover effects onto exchange rates and interest rates should be the across countries after controlling for shares of consumption invoiced in their currencies. In other words, the model suggests that the FRB has an outsized influence over global macro-financial conditions because of the dollar's preeminent position as an invoicing currency.

After showing the main insights of the paper in the complete asset markets model, I show the theoretical predictions of the model continue to hold in an incomplete markets extension. The incomplete markets extension shows that the forces identified in the model carry over to a broader class of models with more elaborate frictions. Moreover, the addition of incomplete markets breaks the tight link between exchange rates, interest rates and aggregate consumption, which is counterfactual to the data (Backus and Smith, 1993).

In the empirical section of the paper, I focus on testing the following predictions in which I use high frequency measures of monetary policy shocks to identify the effects of monetary policy. In response to a surprise U.S. monetary contraction, countries with a larger share of dollar invoiced imports should observe (1) their nominal exchange rates depreciate less and (2) their nominal interest rates increase more relative to other countries. Finally, (3) Monetary policy spillover effects from the United States are no larger than the magnitude of monetary policy spillover effects from the ECB, after controlling for the share of traded goods invoiced in dollars and euros.

Using high frequency measures of U.S. monetary policy shocks from Nakamura and Steinsson (2015), I show that countries with a higher share of dollar invoiced consumption do suffer larger spillovers from the FRB, as predicted by the model. I use currency invoicing data from Gopinath (2015) to construct countries' shares of consumption invoiced in dollars. If a country experiences a one standard deviation increase in its share of dollar invoiced consumption, a 100 basis point FRB contraction causes its nominal exchange rate to depreciate by 167 fewer basis points and causes its nominal interest rate to increase by an additional 54 basis points. These effects are economically large and robust to a number of alternative hypothesis. In particular, I show dollar invoicing explains heterogeneity in FRB monetary policy spillovers after accounting for a country's distance from the United States, trade intensity with the U.S., and dollar denominated debt position. Moreover, these results are not the result of systematic monetary policy reactions from foreign central banks to the FRB.

Finally, I show spillovers through invoicing currencies are not unique to the FRB and the U.S. dollar. Using high frequency measures of European Central Bank monetary policy shocks from Leombroni et al. (2017), I show countries with more euro invoiced consumption suffer larger spillovers from ECB monetary policy prior to the Great Recession. Moreover, I use tick-level eurodollar and euribor data to construct a consistent measure of monetary policy shocks across the FRB and the ECB, and I show that spillovers from FRB monetary policy shocks are not significantly larger once I control for countries' consumption invoiced in dollars and euros. In this sense, both the theoretical model and the empirical results highlight the role of invoicing currencies of traded goods in explaining variation in monetary policy transmission across countries.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model, and section 4 characterizes the equilibrium and derives the theoretical predictions of the model. Section 5 presents the data and empirical methodology. Section 6 presents empirical results and section 7 concludes.

2 Related Literature

This paper contributes to a large literature on the transmission of monetary policy shocks by characterizing how invoicing imports and exports in a global trade currency effects monetary policy transmission within and across borders. The existing literature has largely focused on models in which countries are symmetric (Clarida et al., 2002; Bacchetta and van Wincoop, 2005; Floden and Wilander, 2006; Engel, 2011) or in which all countries are of measure zero and each country's monetary policy has no externalities on other countries (Gali and Monacelli, 2005, 2008; Farhi and Werning, 2013). Notable exceptions are Casas et al. (2017) who allow for "dominant currency invoicing" (invoicing in one particular country's currency) in a small open economy setting, and Corsetti et al. (2018) who analyze optimal monetary policy in a dynamic New-Keynesian framework with dominant currency invoicing. This paper contributes to this literature by introducing other forms of cross-country heterogeneity and studying the consequences of monetary policy tranmission to financial variables.

This paper also relates to a literature on exchange rate pass through by identifying and analyzing effects of exchange rate movements. In general, this literature focuses on measuring long run consequences of exchange rate movements in macroeconomic variables (Gopinath and Rigobon, 2008; Burstein and Gopinath, 2014; Gopinath, 2015). Within this literature, my paper is most similar to Boz et al. (2017) who use currency invoicing data from Gopinath (2015) to understand heterogeneity in exchange rate pass through in the medium- to long-run. Relative to this literature, I provide new empirical results showing spillover effects in high frequency asset pricing variables and from multiple central banks.

This paper contributes to a growing literature measuring the consequences of monetary policy using high frequency measures of monetary policy shocks (Kuttner, 2001; Gurkaynak et al., 2005; Gertler and Karadi, 2015; Gorodnichenko and Weber, 2016; Leombroni et al., 2017; Ozdagli and Weber, 2017; Wiriadinata, 2017). The methodology in my paper is most similar to that of Nakamura and Steinsson (2015). This literature primarily focuses on measuring the effects of U.S. monetary policy shocks on U.S. macroeconomic and financial variables. Notable exceptions include Rey (2014) and Wiriadinata (2017). Rey (2014) measures the effect of U.S. monetary policy shocks on macroeconomic variables in four foreign countries. Wiriadinata (2017) shows that credit constrained countries with more dollar denominated debt are more exposed to U.S. monetary policy shocks. This paper focuses on a sample of advanced economies with well developed financial markets, which are unlikely to credit constrained. Shah (2017) uses high frequency asset pricing data to argue for an incomplete asset markets explanation of joint movements in exchange rates and long-term bond yields. [2]

Finally, this paper is related to a growing literature in international finance that studies the effects of heterogeneity across countries on exchange rates, currency returns and capital accumulation (Martin, 2012; Hassan, 2013; Maggiori, 2013; Richmond, 2015; Farhi and Gabaix, 2015; Hassan et al., 2016, 2017). This paper studies a different form of heterogeneity that influences how shocks transmit across countries, and drives variation in the properties of asset prices across countries.

3 The Model

I develop a static open economy New Keynesian model to capture key features of the distribution in currency invoicing. The model builds on canonical open economy New Keynesian models (Floden and Wilander, 2006; Engel, 2011) by allowing for heterogeneity in currency invoicing across countries. In each country, an exogenous fraction of firms invoice their exports in domestic currency, whereas the remainder of firms invoice their exports in a global trade currency. I also

[2] A separate literature looks at returns on asset prices on monetary policy announcement days (Savor and Wilson, 2013; Lucca and Moench, 2015; Brusa et al., 2015; Mueller et al., 2017).

allow for heterogeneity in country sizes and heterogeneity in household preferences for varieties of traded goods.

I develop intuition for the role of currency invoicing in monetary policy transmission in a benchmark model with complete financial markets. However, the complete markets model has well known empirical shortcomings. Hence, I extend the complete markets model to allow for segmented markets where only a subset of households within each country have access to complete asset markets. The remaining households in each country hold nominal bonds denominated in domestic currency. I show the empirical predictions ofthe complete markets model continue to hold in this segmented markets environment.

3.1 Households

Two time periods exist: $t = 1, 2$. A unit mass of households is partitioned into three countries. For expositional purposes, I label these countries the United States (US), Japan (JP) and Europe (EU). Let μ^n denote the mass of households residing in country n. Households in each country gain utility from consumption and disutility from providing labor services. In country n, a mass μ^n of firms also exists, and each firm produces a unique intermediate traded good. Households supply labor to all firms within their own country in the production of intermediate traded goods, and share in the profits of all sales of intermediate traded goods. In this manner, all households within a country are identical.

Households in country n exhibit CRRA utility,

$$U^n = \mathbb{E} \sum_{t=1,2} \beta^{t-1} \left\{ \frac{\left(C_t^n\right)^{1-\gamma}}{1-\gamma} - \frac{\left(N_t^n\right)^2}{2} \right\}. \tag{1}$$

N_t^n is the labor supplied by each household in country n, and C_t^n is aggregate consumption bundle consumed by a household in country n. This aggregate consumption bundle comprises traded and non-traded goods,

$$C_t^n = \left(C_{T,t}^n\right)^{\tau} \left(C_{N,t}^n\right)^{1-\tau}.$$

τ governs the share of the country n household's aggregate consumption comprised of traded goods. $C_{N,t}^n$ represents the household's consumption of its country specific non-traded good, and $C_{T,t}^n$ represents country n household's consumption of an aggregate traded good comprising intermediate traded goods from each country:

$$C^n_{T,t} = \left(C^n_{T,US,t}\right)^{\alpha^n_{US}} \left(C^n_{T,JP,t}\right)^{\alpha^n_{JP}} \left(C^n_{T,EU,t}\right)^{\alpha^n_{EU}}.$$

$C^n_{T,m,t}$ represents a consumption bundle of intermediate traded goods produced by firms in country m and consumed in country n for $m \in \{US, JP, EU\}$. The parameters α^n_m determine what fraction of country n's traded consumption is comprised of intermediate traded goods produced in country m. These parameters can be thought of as capturing differences in trade costs or preferences across countries. I assume $\alpha^n_{US} + \alpha^n_{EU} + \alpha^n_{JP} = 1$. Each consumption bundle of intermediate traded goods is a CES aggregate of differentiated intermediate traded goods from within each country:

$$C^n_{T,m,t} = \left[\int_0^{\mu^m} \left(C^n_{T,m,j,t}\right)^{\varepsilon} dj\right]^{\frac{1}{\varepsilon}}. \tag{2}$$

j indexes firms in country m, and $\varepsilon < 1$ determines the elasticity of substitution between the different varieties of country m's intermediate traded goods.

At the start of each period, each household in country n is endowed with $Y^n_{N,t}$ a unit of the non-traded good.

3.2 Firms

In country m, firm j's output of its variety of intermediate traded good is

$$Y_{T,m,j,t} = \mu^m N_{m,j,t},$$

where $N_{m,j,t}$ is the quantity of labor firm j demands in country m from each household.

The following pricing assumptions capture key features of the international price system described in Gopinath (2015): The nominal price of traded goods are fixed in their currency of invoicing in the short run. Firms either invoice their exports (sales to foreign households) in domestic currency or in U.S. dollars. In country m, a fraction $X^m_\$$ of firms invoice their exports in dollars. The remaining fraction $1 - X^m_\$$ of firms invoice their exports in domestic currency. Domestic sales of intermediate traded goods are invoiced in domestic currency.

Nominal prices in the first period are set prior to the realization of productivity shocks, endowments of non-traded goods and monetary policy actions. Nominal prices of intermediate traded goods are fixed in their currency of invoicing in the first period, and the law of one price holds for traded goods. Hence, the nominal price of any intermediate traded good in yen is equal to the nominal price of the traded good in dollars multiplied by the yen - dollar exchange rate:

$$\tilde{P}^{JP}_{T,m,j,t} = \tilde{E}^{JP,US}_t \tilde{P}^{US}_{T,m,j,t}. \tag{3}$$

$\tilde{P}^{n}_{T,m,j,t}$ is the nominal price of intermediate traded good j in units of country n currency. $\tilde{E}^{JP,US}$ is the yen - dollar nominal exchange rate given as units of yen per dollar. Throughout this paper, tildes present nominal prices.

I view the second period as the long run and assume nominal prices adjust freely in the second period. Thus, monetary policy actions only affect real allocations in the first period, and they are interpreted as temporary shocks.

3.3 Monetary Policy

Since prices of intermediate traded goods are fixed in the first period, central banks can affect first-period allocations. I assume the monetary policy action of the central bank in each country is to choose the domestic aggregate nominal price level. Because the final consumption bundle is a Cobb-Douglas aggregate of traded and non-traded goods, Appendix A.1 shows the aggregate nominal price level, $\tilde{P}^n_t$, in country n can be written as

$$\tilde{P}^n_t = \frac{\left(\tilde{P}^n_{T,t}\right)^{\tau} \left(\tilde{P}^n_{N,t}\right)^{1-\tau}}{(\tau)^{\tau} (1-\tau)^{1-\tau}}.$$

$\tilde{P}^n_{T,t}$ is the nominal price index for the consumption of intermediate traded goods for households in country n. I assume the central bank of country n chooses $\tilde{P}^n_t$. The nominal prices of traded goods, $\tilde{P}^n_{T,t}$, and non-traded goods, $\tilde{P}^n_{N,t}$ adjust once the central bank chooses the aggregate price level in the country.

Intuitively, one can think of an increase in the aggregate nominal price level as a currency devaluation. When the central bank increases $\tilde{P}^n_t$, more units of country n currency are required to purchase a unit of final consumption in country n. Hence, each unit of country n currency purchases fewer real consumption goods. This form of monetary policy provides a convenient modeling shorthand for other forms of monetary policy. For example, it is equivalent to introducing money into the model, forcing households to purchase goods using cash holdings, and allowing the central bank to control the money supply.

3.4 Financial Markets and Budget Constraints

Households trade a complete set of state-contingent claims denominated in U.S. dollars. The state of world at date t is determined by the monetary policy actions, $\tilde{P}_t^n$. Since, households must use domestic currency to purchase domestic consumption, they cannot perfectly insure against the monetary policy actions of all central banks.

Households receive wages for supplying labor in the production of intermediate traded goods and profits from owning intermediate traded goods firms within their country. The country n household faces the following budget constraint:

$$\mathbb{E}\left\{\sum_{t=1,2} Q_t \tilde{E}_t^{US,n}\left[\tilde{P}_{N,t}^n C_{N,t}^n + \sum_{i\in\{US,JP,EU\}}\left(\int_0^1 \tilde{P}_{T,i,j,t}^n C_{T,i,j,t}^n dj\right)\right]\right\} = \tag{4}$$
$$\mathbb{E}\left\{\sum_{t=1,2} Q_t \tilde{E}_t^{n,US}\left(\tilde{P}_{N,t}^n Y_{N,t}^n + \tilde{W}_t^n N_t^n + \tilde{\Pi}_t^n\right)\right\} + \kappa^n.$$

$\tilde{W}_t^n$ is country n's nominal wage. $\tilde{\Pi}_t^n$ is the household's share of nominal profits earned from owning country n intermediate traded goods firms. $\tilde{P}_{N,t}^n$ and $\tilde{P}_{T,i,j,t}^n$ are the nominal prices of the non-traded good and the jth variety of the intermediate traded good produced in country i. The n superscript above the prices denote these prices are denominated in country n currency.

Q_t is the price of a state-contingent claim (in dollars). I multiply Q_t by the nominal exchange rate $\tilde{E}_t^{US,n}$ to translate the domestic currency value of household wealth into dollars. κ^n is a transfer across households, which decentralizes a Social Planner's problem with unit Pareto weights, and allows me to abstract from wealth effects that result from introducing heterogeneity across countries.

The market clearing condition for intermediate traded good j produced in country m is $Y_{T,m,j,t} = A_t^m \mu^m N_{m,j,t} = \sum_{n\in\{US,JP,EU\}} \mu^n C_{T,m,j,t}^n$. Households consume their endowment of non-traded goods, $Y_{N,t}^n = C_{N,t}^n$. The labor market clearing condition for households in country m is, $N_t^m = \int_0^{\mu^n} N_{m,j,t} dj$.

An equilibrium in this economy is a set of consumption and labor supply allocations for each household $\left\{C_{T,m,j,t}^n, C_{N,t}^n, N_t^n\right\}$, a set of intermediate output, labor demand and nominal prices for each intermediate firm $\left\{Y_{T,m,j,t}^n, N_{m,j,t}, \tilde{P}_{T,m,j,t}^n\right\}$, a set of traded good prices and non-traded good prices $\left\{\tilde{P}_{T,t}^n, \tilde{P}_{N,t}^n\right\}$, a set of nominal wages for each country and a nominal exchange rate $\left\{\tilde{W}_t^n, \tilde{E}_t^{n,m}\right\}$, such that households maximize utility subject to their budget constraints, and the resource constraints are satisfied [3].

4 Theoretical Results

The model characterizes how heterogeneity in currency invoicing influences monetary policy transmission across countries. I use the model to provide intuition and to derive empirical predictions. I log-linearize the model around the deterministic solution. [4] Lowercase variables denote logs. I focus on the first period because nominal rigidites only affect allocations in the short-run. Hence, I drop the time t subscript whenever possible. All variables represent first-period prices and allocations unless otherwise noted.

Monetary policy shocks from the United States affect the demand for all dollar invoiced intermediate traded goods. Suppose the United States engages in contractionary monetary policy and decreases the (log) U.S. nominal price level, $\tilde{p}^{US}$. This increases the nominal value of the dollar relative to all other currencies. Fewer dollars are needed to purchase a unit of the final consumption bundle in the United States, and each dollar becomes more valuable. Since the nominal price of dollar invoiced goods are fixed in dollar units, all dollar invoiced traded goods become more expensive to consume. In response, foreign households decrease their traded consumption and as well as aggregate consumption. The response of country n household consumption to U.S. contractionary policy can be expressed as,

$$-\frac{\partial c^n}{\partial \tilde{p}^{US}} = -D_C^{US} - \Gamma \times M_\$^n \tag{5}$$

where D_C^{US}, $M_\$^n$ and Γ are both positive constants [5].

The constant D_C^{US} is the common effect of U.S. monetary policy on both Japanese and European consumption,

$$D_C^{US} = \frac{\gamma\tau^2(1+(\gamma-1)\tau)\left(\left(1-\alpha_{EU}^{EU}\right)\left(1-\alpha_{JP}^{JP}\right)-\left(1-X_\$^{JP}\right)\left(1-X_\$^{EU}\right)\alpha_{JP}^{EU}\alpha_{EU}^{JP}\right)}{\nu}$$

ν is a positive constant discussed in the Appendix A.4. The expression for D_C^{US} is positive and shows contractionary U.S. monetary policy decreases foreign consumption on average. As the the price of traded goods increases for foreign households, all households decrease aggregate

[3] See Appendix A.2 for additional details and the derivation of the first order conditions.

[4] See Appendix A.3 for additional details regarding the log-linearization of the model.

[5] $\Gamma = \frac{(1-\tau)(1+(\gamma-1)\tau)}{\nu} > 0$.

consumption. Moreover, the magnitude of this common effect increases as more Japanese and European firms invoice in dollars (as $X_{\$}^{JP}$ and $X_{\$}^{EU}$ increase).

All heterogeneity in effect of U.S. monetary policy on foreign consumption in Equation (5) is captured by $M_{\$}^{n}$. For Japan,

$$M_{\$}^{JP} = \tau \left(\alpha_{US}^{JP} + X_{\$}^{EU} \alpha_{EU}^{JP}\right),$$

which can be interpreted as the total share of Japanese consumption invoiced in dollars. $M_{\$}^{JP}$ comprises the share of imported consumption (τ), multiplied by the share of imported consumption invoiced in dollars ($\alpha_{US}^{JP} + X_{\$}^{EU} \alpha_{EU}^{JP}$). The expression for $M_{\$}^{EU}$ is defined analogously, $M_{\$}^{EU} = \tau \left(\alpha_{US}^{EU} + X_{\$}^{JP} \alpha_{JP}^{EU}\right)$.

Within this model, $M_{\$}^{n}$ is the sufficient statistic that fully captures the effect of different form of heterogeneity on U.S. monetary policy transmission abroad. Countries with a larger share of dollar invoiced consumption suffer larger spillovers from the FRB. Households that consume more dollar invoiced intermediate traded goods observe a price increase over a larger share of their consumption bundle. Hence, their aggregate consumption falls more. In the empirical section of this paper, I will measure $M_{\$}^{n}$ in the data and explore how heterogeneity in $M_{\$}^{n}$ impacts international monetary policy transmission.

4.1 Monetary Policy Effectiveness

A central equation governing monetary policy in the New Keynesian literature is the trade-off between output and inflation. In this section, I characterize the effect of invoicing currencies on this trade-off and the effectiveness of monetary policy in Europe and Japan.

Proposition 1. *Holding foreign monetary policy constant, the response of domestic traded production to domestic monetary policy is decreasing in the share of the country's dollar invoiced exports* $\left(X_{\$}^{n}\right)$. *Hence, central banks of countries where a larger share of firms invoice their exports in foreign currency face a steeper Phillips Curve.*

Proof. See Appendix A.5 □

A country's central bank policy affects both domestic and foreign demand for its products. Suppose the central bank of Japan engages in contractionary policy. Part of the contractionary effect comes from households decreasing consumption of Japanese goods in the rest of the world, because contractionary policy increases the relative cost of yen invoiced traded goods. This effect of Japanese monetary policy on foreign consumption of Japanese goods is directly proportional to the share of Japanese firms that invoice their exports in yen. In the limiting case, if no firms in Japan invoice in yen, then changes to relative value of the yen have no impact on the relative price of Japanese traded goods abroad. In this case, Japanese monetary policy would only affect domestic demand. Hence, the effect of Japanese monetary policy on its domestic output, as well as other macroeconomic outcomes, diminishes as more firms invoice in U.S. dollars.

Data from Gopinath (2015) show that a number of countries invoice the majority of their exports in foreign currencies, e.g., the U.S. dollar. Proposition 1 suggests the central banks of these countries incur greater inflationary costs when trying to manage their domestic economies. This is true even in this environment with complete asset markets and freely floating exchange rates. In this sense, central banks in countries where a large fraction of firms invoice their exports in foreign currency are less effective at implementing monetary policy.

The macroeconomic implications of the widespread use of the U.S. dollar as an invoicing currency are best summarized by Equation (5) and Proposition 1. However, empirical estimates of Phillips curves and of the impact of monetary policy on macroeconomic aggregates are strongly subject to specification and sampling uncertainty. For example, empirical studies of the New Keynesian Phillips Curve often give rise to conflicting results depending on how authors account for expected inflation (Mavroeidis et al., 2014). Therefore, I use the model to characterize the effect of invoicing currencies on monetary policy transmission to high frequency asset pricing outcomes, and I derive empirical predictions that can be tested using high frequency asset pricing data.

4.2 Exchange Rates and Interest Rates

A country's nominal dollar exchange rate is comprised of its real dollar exchange rate and the differences in nominal price levels between countries,

$$\tilde{e}^{n,US} = \underbrace{-\gamma \left(c^n - c^{US}\right)}_{\text{real exchange rate}} - \left(\tilde{p}^n - \tilde{p}^{US}\right).$$

U.S. monetary policy affects three components of this equation: nominal price level in the United States $\left(\tilde{p}^{US}\right)$, consumption in the United States $\left(c^{US}\right)$, and consumption in country n (c^n). However, heterogeneity in the transmission of U.S. monetary policy shocks only comes from the heterogeneous impact of U.S. monetary policy on consumption in country n (c^n) .

Prediction 1. *Holding foreign monetary policy constant, U.S. contractionary monetary policy* *should cause foreign currencies to depreciate against the U.S. dollar. Countries with higher $M^n_\$$ depreciate less than countries with lower $M^n_\$$.*

Equation (6) describes the effect of contractionary U.S. monetary policy on the nominal exchange rate of country n with respect to the dollar,

$$-\frac{\partial \tilde{e}^{n,US}}{\partial \tilde{p}^{US}} = D^{US}_E - \gamma \Gamma M^n_\$. \tag{6}$$

It is immediately clear that the transmission of U.S. monetary policy to nominal exchange rates comprises the same components as the transmission of U.S. monetary policy to foreign consumption. The term D^{US}_E is common across countries,

$$D^{US}_E = \frac{(1-\tau)(1+(\gamma-1)\tau)(1+\left(\gamma\left(2-\alpha^{JP}_{JP}-\alpha^{EU}_{EU}\right)-1\right)\tau)}{\nu}$$

Since the nominal exchange rate is given as units of foreign currency per dollar, the constant D^{US}_E shows more units of foreign currency are needed to purchase a dollar. On average, foreign currencies depreciate against the dollar in response to U.S. monetary contractions. The transmission effect captured by D^{US}_E captures two effects: (1) A mechanical increase in the nominal value of each dollar and (2) An appreciation of the real price level in the U.S. due to decreased domestic consumption.

Equation (6) shows all heterogeneity in the transmission of U.S. monetary policy to exchange rates is also captured by $M^n_\$$. Countries with higher shares of dollar invoiced consumption appreciate with respect to countries with lower shares of dollar invoiced consumption. Contractionary U.S. monetary policy decreases aggregate consumption more in foreign countries with a larger share of dollar invoiced consumption. Their marginal utility of consumption increases relative to countries with a smaller share of dollar invoiced consumption, which increases the real price level in countries with more dollar invoiced consumption relative to countries with less dollar invoiced consumption.

Prediction 2. *Holding foreign monetary policy constant, contractionary monetary policy from the United States should increase foreign nominal interest rates. Countries with higher $M^n_\$$ observe larger increases in nominal interest rates than countries with lower $M^n_\$$.*

The nominal interest rate, $\tilde{R}^n$, in country n is pinned down by the following consumption

Euler equation in levels:

$$\frac{1}{\tilde{R}^n} = \beta \mathbb{E}\left[\left(\frac{C_2^n}{C_1^n}\right)^{-\gamma} \frac{\tilde{P}_1^n}{\tilde{P}_2^n}\right]. \tag{7}$$

Holding foreign monetary policy fixed, the U.S. monetary policy action affects foreign real interest rates by changing the foreign household's marginal utility of consumption in the first period, C_1^n. Foreign consumption in the long-run , C_2^n, is unaffected because nominal prices are flexible in the second period.

Contractionary U.S. policy increases the foreign marginal utility of consumption today, which increases both real and nominal interest rates in the foreign country. Equation (5) showed all heterogeneity in the transmission of U.S. policy to foreign consumption is captured by $M_{\$}^n$. Hence, all heterogeneity in the transmission of U.S. policy to foreign interest rates is also captured by $M_{\$}^n$. Interest rates will increase more in countries with a higher share of dollar invoiced consumption, because households in these countries consume less and feel poorer in the short-run.

4.3 Policy Spillovers from Other Central Banks

Although my discussion primarily focuses on U.S. monetary policy transmission, spillover effects should clearly emanate from all countries.

Prediction 3. *Controlling for the share of consumption that is invoiced in each country's currency, the magnitude of spillover effects from Japanese monetary policy and the European monetary policy are the same as the magnitude of the spillover effects from U.S monetary policy.*

The magnitude of monetary policy spillover effects only differ according to the share of international trade that is invoiced in each currency. For example, equation (8) shows the effect of Japanese contractionary monetary policy on yen exchange rates,

$$-\frac{\partial e^{n,JP}}{\partial \tilde{p}^{JP}} = D_E^{JP} - \gamma \Gamma M_{¥}^n \tag{8}$$

where,

$$D_E^{JP} = \frac{(1-\tau)(1+(\gamma-1)\tau)(1+\left(\gamma\left(2-\alpha_{US}^{US}-\alpha_{EU}^{EU}-X_{\$}^{JP}\alpha_{JP}^{US}-X_{\$}^{EU}\alpha_{EU}^{US}\right)-1\right)\tau)}{\nu},$$

and $M_{¥}^n = \tau\alpha_{JP}^n\left(1-X_{\$}^{JP}\right)$ is the share of country n's total consumption that is invoiced in yen.

By comparing equations (6) and (8), it is evident that the heterogeneity in monetary policy transmission from the United States and Japan are the same after controlling for currency invoicing. For each increase in a country's share of consumption invoiced in yen ($M^n_{¥}$), the country's currency appreciates by an additional $\gamma\Gamma$ units against the yen. This is exactly the same effect as dollar invoicing on U.S. monetary policy transmission. I derive results showing the symmetry in monetary policy transmission to consumption and interest rates in Appendix A.6.

This symmetry result provides a rationale for why we observe large effects from U.S. monetary policy transmission. The widespread use of the dollar as an invoicing currency, coupled with the fact that traded goods prices are sticky in their currency of invoicing, theoretically explain why the Federal Reserve is able to influence global asset markets while other central banks do not. Moreover, the results in this section highlight the role of invoicing currencies and trade in explaining heterogeneity in monetary policy transmission across countries as well as across central banks.

4.4 Incomplete Asset Markets

Although the complete asset markets model is an important benchmark, it has some well known empirical shortcomings. For example, the model predicts a perfect negative correlation between appreciations in the real exchange rate and aggregate consumption (Backus and Smith, 1993). As a result, many authors have argued for a segmented markets model that breaks the tight link between consumption and asset prices (Alvarez et al., 2002).

I extend the model in section 3 to characterize the effect of currency invoicing on monetary policy transmission when markets are segmented. [6] Within each country, only a fraction ϕ of households (labeled "active") trade a complete set of state-contingent securities. The remaining $1-\phi$ of households (labeled "inactive") are excluded from trading in financial markets. Instead, inactive households cede the claims to their firm profits and their endowments of non-traded goods to active households. In return, inactive households receive a nominal bond that makes a fixed nominal payment $\tilde{B}^n_t$ in each period.

Exchange rates and interest rates are determined by the marginal utility of consumption of active households only. Equations (29) and (30) provide the incomplete markets counterparts to equations (6) and (5). In the incomplete markets model, the transmission mechanism from changes in U.S. monetary to foreign countries is unchanged. In particular, contractionary U.S. monetary policy decreases the consumption of active households more in countries with a higher

[6] See Appendix A.7 for the formal setup, model solution and derivation of the theoretical results in this section.

share of dollar invoiced consumption relative to countries with a lower share of dollar invoiced consumption. As a result, Proposition 1 and Predictions 1 through 3 continue to hold in the incomplete markets model.

The introduction of incomplete asset markets breaks the tight link between changes in aggregate consumption and movements in asset prices across countries. In response to U.S. monetary contractions, a decrease in demand for dollar invoiced intermediate traded goods leads to a decrease in labor demand and wages. In particular, labor income decreases more for inactive households in countries with more dollar invoiced exports. As long as cross-sectional differences in countries' shares of dollar invoiced imports and exports are not perfectly correlated, the relative change in inactive household consumption will not be perfectly correlated with the relative change in active household consumption.

As a result, equation (32) in Appendix A.7 shows changes in the consumption of active households and changes in aggregate consumption are disconnected in many regions of the parameter space. [7] Thus, the incomplete markets model extension to the model in section 3 achieves two goals. First, the incomplete markets model reinforces the insights from the model with complete financial markets. Second, incomplete markets improves the quantitative implications of the model by breaking the perfectly negative correlations between asset prices and changes in aggregate consumption across countries.

5 Empirical Analysis and Data

The model in section 3 predicts heterogeneity in monetary policy transmission is captured by heterogeneity in currency invoicing.In the rest of the paper, I investigate Predictions 1 through 3 for the FRB as well as the ECB. To re-iterate, The model yields the following testable implications. In response to contractionary U.S. monetary policy, countries with higher shares of dollar invoiced imports (1) should observe their currencies appreciate and (2) should observe their nominal interest rates increase relative to countries with lower shares of dollar invoiced imports. Furthermore, (3) controlling for heterogeneity in currency invoicing, the magnitude of monetary policy spillover effects from any one central bank foreign onto exchange rates and interest rates should be the same.

When testing the implications of the model, I use high frequency measures of monetary policy shocks and I measure daily changes in interest rates an exchange rates for outcome variables. A

[7]For example, $\phi \in [1/2, 2/3]$ and $\gamma > \frac{\phi}{\varepsilon\tau(1-(3/2)\phi}$.

literal interpretation of model in section 3 requires household consumption to change instantly in response to monetary policy. These dramatic changes in consumption at high frequencies are unlikely. Instead, I take the stance that participants in financial markets understand appreciations in the value of the dollar will affect countries more, in the short run, when a larger share of the country's consumption is invoiced in dollars. Hence, the interpretation for changes in interest rates and exchange rates relies on participants in financial market changing their expectations in response to monetary policy shocks, based on knowledge of countries' exposures to various currencies through trade.

5.1 Monetary Policy Shock Data

I use high frequency measurements of FRB monetary policy shocks from Nakamura and Steinsson (2015) to test for monetary policy transmission from the U.S. These are available from the authors' websites. I use their time series for U.S. "policy news shocks" from 1995 to 2014 as my independent variables. The authors measure changes in interest rate futures of differing maturities in a 30 minute window around FRB policy announcements and take the first principle component of these changes in interest rate futures.

High frequency measurements of monetary policy shocks provide an estimate of the unexpected component of U.S. monetary policy. The identifying assumption is that all other sources of macroeconomic news and investor expectations are already incorporated into prices 10 minutes prior to the FOMC meeting. Hence, changes in futures prices in a narrow minute window around FOMC meetings only capture the unexpected news from the FOMC meeting itself. Nakamura and Steinsson (2015) use a composite measure of monetary policy shocks from five interest rate futures to capture the possible effects of forward guidance along with FRB actions that target shorter maturity yields.

I study two additional sets of monetary policy shocks in order to understand monetary policy transmission from the ECB. The first set comes from Leombroni et al. (2017), in which authors measure changes in interest rate swaps around ECB target rate announcements as well as press conferences. Second, I construct a new data set of monetary policy shocks across both the FRB and the ECB in a consistent manner. Using data provided by Tick Data, I measure changes in three month interest rate futures denominated in dollars and euros currency around monetary policy announcements. By choosing interest rate futures with the same maturity, I attempt to consistently define monetary policy shocks across central banks in order to test Prediction 3. Figure 1 compares the FRB shocks I generate with three month eurodollar futures data against the policy news shocks from Nakamura and Steinsson (2015). The correlation between these two measures is 0.87, which indicates I am capturing a significant portion of the variation in the policy news shocks. Additional details regarding the construction of my monetary policy shocks can be found in Appendix C.1.

5.2 Currency Invoicing and Other Data

I use currency invoicing data come from Gopinath (2015) to measure empirical counterparts to M_f^n and X_f^n from section 3. These data are available from the author's website, and measure the average share of each country's imports and exports that are invoiced in various currencies between 1999-2014. In this sense, the currency invoicing data provides a snapshot of the use of various currencies in trade during this period of time. I use annual import and consumption data from the Penn World Tables to estimate the average share of each country's consumption comprised of imports and I calculate the average share of consumption invoiced in each currency (M_f^n).

The data describe cross-sectional variation in the currency invoicing of imports and exports, because the currency invoicing data are reported at the country - currency level and averaged over the years 1999 to 2014. However, Gopinath (2015) notes these invoicing shares are fairly stable over time. Across the countries in my sample the mean dollar invoiced share of consumption, $M_\$^n$, is 0.358 and its standard deviation is 0.187. In this sense, there is large variation in countries' exposures to the U.S. dollars through international trade [8].

My dependent variables are changes in nominal exchange rates and nominal interest rates on monetary policy announcement days. I gather these data from Thomson Reuters Datastream. I use forward data to construct nominal interest rates to address concerns that nominal interest rates derived from nominal bond yields would reflect differences in country specific risk premia. Given covered interest rate parity (CIP), the nominal interest rates of the United States and a foreign country, n, are related as follows:

$$\left(1 + \tilde{r}_{t,t+n}^{USA}\right)^n = \left(1 + \tilde{r}_{t,t+n}^{n}\right) \frac{F_{t,t+n}^{USA,n}}{E_t^{USA,n}}. \tag{9}$$

$\tilde{r}_{t,t+n}^{f}$ is the (annualized) nominal interest rate in country f of maturity n years. $F_{t,t+n}^{USA,f}$ is a forward exchange rate contract of maturity n years in units of dollars per unit of foreign currency, and $\tilde{E}_t^{USA,f}$ is the spot exchange rate in units of dollars per unit of foreign currency. I use U.S.

[8]See Appendix C.2 for additional details about the currency invoicing variables and the construction of M_f^n.

government bond yields from FRED as my time series of U.S. nominal interest rates [9].

I focus on a set of economies with developed financial markets for which I also observe currency invoicing data as my main sample [10]. Table 8 lists the countries in my sample along with their data availability. These countries are Australia, Canada, Denmark, Iceland, Israel, Japan, Norway, South Korea, Sweden, Switzerland, the United Kingdom and the United States. I include the Eurozone as a single "country" for studying changes in exchange rates and interest rates. I drop observations at the country-date level of countries engaged in a hard or soft currency peg with the central bank that issues the monetary policy shock [11]. Tables 9 provides summary statistics for monetary policy shocks and country - central bank level characteristics.

6 Empirical Strategy and Results

To test for the effect of currency invoicing on FRB monetary policy transmission, equation (6) suggests I run the following regression,

$$\Delta y_t^n = \delta_t + \beta_M M_{\$}^n + \gamma_M \left(\Delta i_{\$,t} \times M_{\$}^n\right) + \epsilon_t^n. \tag{10}$$

Δy_t^n is the change in country n's outcome variable on date t, which is either the country's nominal exchange rate or the country's nominal interest rate. δ_t is a date t fixed effect that captures D_E^{US} from equation (6). It is the average response of the dependent variable to the monetary policy on date t. $\Delta i_{\$,t}$ is the monetary policy shock from the Federal Reserve on date t. $M_{\$}^n$ is the value of country n's dollar invoiced imports normalized by consumption. The coefficient of interest in equation (10) is γ_M. γ_M corresponds to the coefficient in front of $M_{\$}^n$ from equation (6) and captures the effect of heterogeneity in dollar invoicing on the magnitude of monetary policy transmission. I do not include the monetary policy shock itself into the regression, because it would absorbed by the time fixed effect. I cluster my standard errors by date.

6.1 U.S. Monetary Policy Transmission

Table 1 presents results from estimating equation (10) with changes in nominal exchange rates as the outcome variable. Column (1) provides a benchmark estimate: a point estimate of -8.467

[9] For additional details regarding the measurement of nominal interest rates, see Appendix C.3.

[10] Countries in the sample are designated as advanced economies by the IMF's 2015 Economic Outlook

[11] I want to observe changes in interest rates and exchange rates that are a result of financial market participants, rather than reactions to other central banks' monetary policies. I use data from Shambaugh (2004) to measure which pairs of currencies are pegged in each month. See Appendix C.6 for details.

implies if a country experiences a one standard deviation increase in $M_{\$}^{n}$, then its currency depreciates by 167 fewer basis points in response to a 100 basis point contractionary U.S. monetary policy shock. The effect of variation in currency invoicing on monetary policy transmission is economically large as well as statistically significant.

A country's total share of dollar invoiced consumption comprises of (a) the share of its imports invoiced in dollars as well as (b) the share of its consumption coming from imports. The theory suggests both of these components should matter for explaining heterogeneity in monetary policy transmission. Column (2) of Table 1 splits $M_{\$}^{n}$ into these two components and shows this is indeed the case. Countries are more affected by U.S. monetary policy shocks if imports make up a larger share of their consumption and if a larger share of the imports they consume are invoiced in dollars.

A major concern with this empirical exercise is that another underlying characteristic of countries is driving co-movement in the outcome variables, and this underlying characteristic is also correlated with firms' decisions to invoice in dollars. For example, Canada is geographically close to the United States. Therefore, one might expect Canadian dollar to react more to U.S. monetary policy, and also conduct more trade invoiced in U.S. dollars. In the remainder of Table 1 attempt to control for these additional mechanisms that may drive dollar invoicing and the outcome variables.

Columns (3) - (5) in Table 1 shows the main result is robust to controls for the distance between countries, the share of each country's imports that originates from the United States and a country's dollar denominated net debt position [12]. By itself, distance significantly affects the magnitude of monetary policy spillovers from the United States (not shown). Countries that are further from the U.S. depreciate more against the dollar in response to U.S. monetary contractions. However, when I control for both distance and currency invoicing, only the currency invoicing variable significantly explains cross-sectional variation in U.S. monetary policy transmission. The same is true for the share of trade each country conducts with the United States, and each country's dollar denominated net debt asset position.

Table 2 provides the corresponding regression results using changes in one-year nominal interest rates as the outcome variable. A benchmark point estimate of 2.749 implies that if a country experiences a one standard deviation increase in $M_{\$}^{n}$, its nominal interest rate will increase by an additional 54 basis points in response to a 100 basis point contractionary U.S. monetary policy

[12] Wiriadinata (2017) shows that a country's dollar denominated net debt position characterizes exchange rate responses to U.S. monetary policy shocks among credit constrained countries. This paper focuses on advanced countries, which are unlikely to be credit constrained.

shock. This effect is economically large in response to a one percent increase in U.S. nominal interest rates as well as statistically significant. Column (2) further supports the theoretical mechanism that monetary policy transmission is explained by the combination of the share of each country's imports that are invoiced in dollars as well as each country's import to consumption ratio. Finally, columns (3) - (5) show dollar invoicing significantly explains cross-sectional variation in U.S. monetary policy spillovers even after controlling for additional measures of trade costs and net foreign assets.

Overall, the results in Tables 1 and 2 provide strong empirical support for Predictions 1 and 2 of the model. Dollar invoicing in international trade appears important for explaining cross-sectional variation in monetary policy spillover effects.

6.2 Price Stickiness and Alternative Specifications

Tables 3 and 4 provide additional tests of transmission mechanism. In all specifications, the outcome variables are changes in exchange rates or interest rates on U.S. monetary policy announcement days. All regressions include date fixed effects and the interaction term between the monetary policy shock and $M^n_{\$}$. Standard errors are always clustered by date.

The theoretical mechanism depends on the fact that nominal prices of intermediate traded goods are fixed in the short run. However, not all goods prices are equally sticky (Burstein and Gopinath, 2014). Commodities such as crude oil are often invoiced in dollars, but their prices are highly flexible. Hence, the theory predicts that countries that primarily import traded goods with flexible prices should not be affected by foreign monetary policy shocks.

I verify that my results are not being driven by imports of dollar invoiced traded goods with highly flexible prices. Using the Rauch (1999) classification of traded goods, Burstein and Gopinath (2014) show the nominal prices of goods traded on an organized exchange or are priced off of a reference magazine are more flexible than the nominal prices of differentiated products (i.e. clothing). I use dis-aggregated Comtrade data and to calculate the share of each country's imports categorized as differentiated products (Diff. Share) based off of the Rauch (1999) classification. [13] The "Diff. Share" variable estimates the degree of price stickiness of a country's imports in the sense that the nominal prices of imports in countries that import more differentiated products should be more rigid.

Table 3 estimates equation (10), controlling for the share of differentiated products in each country's imports. I find suggestive evidence that countries with more differentiated products as

[13] See Appendix C.9 for details about the Rauch (1999) classifications as well as my treatment of the data

a share of imports are more affected by U.S. monetary policy shocks. In particular, column (4) of Table 3 shows that the triple interaction term between the degree of import price stickiness (Diff. Share), the share of dollar invoiced consumption ($M^n_\$$) and the monetary policy shock captures more of the variation in monetary policy transmission to interest rates than $M^n_\$$ alone. This result suggests price stickiness is important for the transmission of monetary policy shocks. The results in the previous section are driven by imports of products with rigid prices.

Table 4 estimates equation 10 using alternative cuts of the data or alternative measures of dollar invoicing. Looking across the top rows of Panels A and B, the regression results are all quantitatively and qualitatively similar to the results in Tables 1 and 2.

Recent papers suggest deviations from covered interest rate parity were large during and after the Great Recession (Du et al., 2016), which may introduce measurement error into my construction of interest rates. Column 1 in Table 4 shows the results of estimating equation (10) using observations of monetary policy shocks prior to the height of the Great Recession (July, 2008). Another concern about the countries in my sample is that a few highly risky countries drive the empirical results. In another attempt to mitigate the effects of U.S. monetary policy on country specific risk premia, I collect monthly short term debt ratings from Fitch, and, in Column 2, I drop countries if their debt was ever rated below BBB-. Column 3 in Table 4 normalizes the value of each country's dollar invoiced imports by GDP, rather than consumption. The point estimates are larger in column 3 because GDP is a larger denominator than private consumption. However, the direction of the spillover effects remains the same. Finally, column 4 in Table 4 uses a week long window around each monetary policy shock for measuring changes in outcome variables. I show the main empirical findings are not just temporary phenomenon. Overall, I find empirical support for Predictions 1 and 2 across many alternative specifications.

6.3 Foreign Policy Reactions to the FRB

Throughout this paper, I have interpreted movements in foreign interest rates and exchange rates as being determined by financial markets. However, one concern is that my empirical results are instead driven by the actions of foreign central banks reacting to U.S. monetary policy. Specifically, the concern is that central banks in countries with higher shares of dollar invoiced consumption systematically increase their policy rates more in response to U.S. monetary policy contractions.

I find no evidence, in the data, that my results are driven by foreign monetary policy reactions. I gather data on central bank policy rates of foreign central banks in each of the countries in my sample from Global Financial Data [14]. I measure changes in foreign central bank policy rates in the 30 days and the 60 days after each Federal Reserve monetary policy announcement. Table 5 estimates equation (10) using these 30 day and 60 day changes in foreign central bank policy rates as the dependent variable. In both regressions, I fail to find evidence that countries with higher shares of dollar invoiced consumption systematically respond more to U.S. monetary policy.

6.4 Is the Federal Reserve Special?

The theory predicts that spillovers through invoicing currencies should exist for all central banks, and that after controlling for invoicing currencies, the magnitude of monetary policy spillovers should be the same across central banks. In this section, test the predictions of the model using monetary policy shocks from the European Central Bank. First, I use measures of ECB target rate shocks and communications shocks from Leombroni et al. (2017) to measure the international transmission of ECB monetary policy. Afterwards, I construct a new data set of monetary policy shocks consistently from both the FRB and the ECB, and to test whether the effect of currency invoicing on monetary policy transmission differs between the two central banks.

I use both the target rate shocks and the communication shocks from Leombroni et al. (2017) to estimate the effect of monetary policy transmission from the ECB through invoicing currencies. The target rate shock measures the change in interest rate swaps in a 45-minute window that brackets the announcement of the target rate itself. The communication shock measures the change in interest rate swaps in a 65-minute window that brackets the ECB press conference, which occurs after the target rate announcement to explain the policy decision. A positive target rate shock or a positive communication shock means interest rates increased and is interpreted as a contractionary policy surprise. Leombroni et al. (2017) show changes in bond yields are driven mostly by the communication shock.

I augment equation (10) to include both target rate and communication shocks,

$$\Delta y_t^n = \delta_t + \beta_M M_{\$}^n + \gamma_M^{tgt} \left(\Delta i_{€,t}^{tgt} \times M_{€}^n \right) + \gamma_M^{com} \left(\Delta i_{€,t}^{com} \times M_{€}^n \right) + \epsilon_t^n.$$

$\Delta i_{€,t}^{tgt}$ is the target rate shock on date t and $\Delta i_{€,t}^{com}$ is the communication shock on date t. γ_M^{tgt} and γ_M^{com} are the coefficients of interest that capture how euro invoicing affects the reaction of foreign interest rates and exchange rates to ECB monetary policy shocks. Table 6 shows the

[14] See Appendix C.10 for additional details

regression results.

The results suggest countries with more euro invoiced consumption were more exposed to ECB monetary policy shocks prior to the Great Recession. Columns (3) and (4) of Table 6 show that prior to the Great Recession, the communication shock was responsible for the heterogeneity in monetary transmission across countries. Just like the earlier results with dollar invoicing, countries with more euro invoiced consumption saw their exchange rates depreciate less and their interest rates increase more in response to ECB monetary contractions. However, the results are weaker for the ECB monetary policy shocks. Columns (1) and (2) show that when I use the entire sample of ECB policy announcements, the estimated coefficients have the right sign but are not statistically significant. These results suggest that the effect of euro trade invoicing on monetary policy transmission has decreased since the Great Recession.

The results in Columns (1) and (2) of Table (6) may reflect the diminishing role of the euro as an international currency in recent years. For example, Maggiori et al. (2018) shows the use of the euro to denominate corporate decreased dramatically from 2009 to 2015. As discussed previously, Gopinath (2015) provides the average share of imports and exports invoiced in various currencies from 1999 to 2014. However, the data coverage is more sparse in the later years and the invoicing data for some countries stop in 2012. As a result, the average use of the euro in trade invoicing may not reflect the more recent decline of the euro and would bias my estimates of γ_M^{tgt} and γ_M^{com} towards zero.

Finally, I measure monetary policy shocks consistently measured across the Federal Reserve Bank of the United States and the European Central Bank to test Prediction 3. I augment equation (10) with an indicator variable $D_{FRB,t}$, which equals 1 if the monetary policy shock on date t was from the FRB,

$$\Delta y_t^n = \delta_t + \beta_M M_f^n + \gamma_M \left(\Delta i_{f,t} \times M_f^n\right) + \gamma_{M,FRB} \left(\Delta i_{f,t} \times M_f^n \times D_{FRB,t}\right) + \epsilon_t^n. \tag{11}$$

The coefficient of interest in this regression is $\gamma_{M,FRB}$. Prediction 3 says the coefficient on the triple interaction between the monetary policy shock, the import currency variable, and the FRB indicator is zero, $\gamma_{M,FRB} = 0$.

Table 7 estimates equation (11) using the pooled sample of monetary policy shocks from both central banks, and provides support for Prediction 3. Table 7 shows that regardless of the sample period or the outcome variable, the effect dollar invoicing on FRB monetary policy transmission is not different from the effect of euro invoicing on ECB monetary policy transmission. The estimate of the coefficient on the triple interaction term is either not significant or it is weakly significant with the wrong sign. Similar to the earlier results with the ECB monetary policy shocks, the empirical results are stronger in the pre-crisis sample period.

The results from Tables 6 and 7 show monetary policy spillovers are not unique to the FRB. ECB monetary policy actions also affect interest rates and exchange rates, globally, and countries that have a larger share of euro invoiced consumption are more exposed to the ECB. The FRB has a large influence over the global economy. However, the empirical suggest the widespread use of the U.S. dollar in trade invoicing can account for the FRB's outsized influence over global financial conditions.

7 Conclusion

This paper analyzed the implications of heterogeneity in currency invoicing on monetary policy transmission. I present a model of currency invoicing that captures key features of the international price system: the vast majority of imports and exports in the world are invoiced in very few currencies, and goods prices tend to be sticky in their currency of invoicing in the short run. As a result, the central banks of countries where a larger fraction of firms invoice their exports in foreign currency should face a worse trade-off between output and inflation. The empirical part of the paper found support for each of the theoretical predictions of the model. I show heterogeneity in the invoicing currencies explains the heterogeneity in monetary policy transmission from the Federal Reserve to foreign nominal exchange rates and nominal interest rates across advanced economies. Additionally, I provide evidence that monetary policy spillovers emanate from the ECB as well.

The focus of this paper has been on characterizing monetary policy spillovers through trade channels within a group of advanced economies. This leaves a number of avenues for future research: First, it would be useful to begin characterizing how other forms of heterogeneity influence monetary policy transmission across a larger set of countries. A notable contribution towards this goal is Wiriadinata (2017), which shows how a country's dollar denominated debt position is important for monetary policy transmission across credit constrained countries. Second, the paper leaves out an interesting discussion of risk premia. In particular countries that invoice their exports in a global trade currency should be more exposed to monetary policy risk from abroad. Finally, the empirical section of the paper leaves open the larger task of measuring and characterizing the effects of monetary policy transmission from other central banks. This paper provides evidence that monetary policy actions from other central banks are an important source of shocks.

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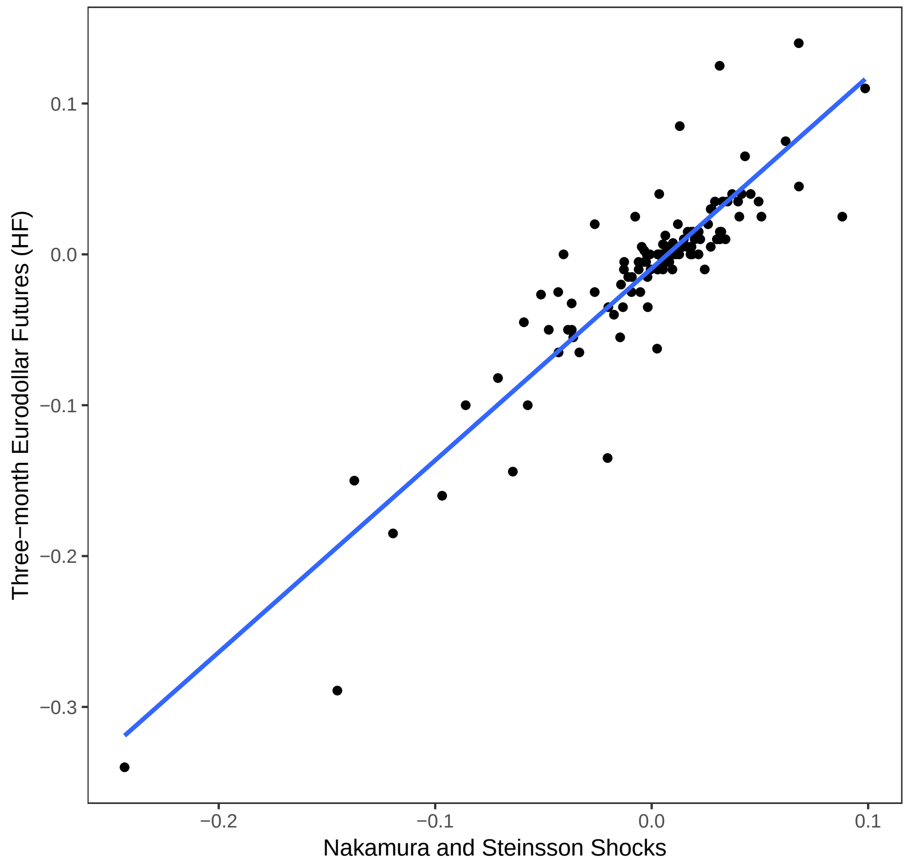
Figure 1: U.S. Monetary Policy Shock Comparison



Notes: The figure plots U.S. monetary policy shocks derived from changes in three-month Eurodollar futures in on U.S. monetary policy announcement days against those from Nakamura and Steinsson (2015). The slope of the regression line is 1.732 and the intercept is -0.011. The correlation between the two measures of monetary policy shocks is 0.87.

Table 1: Nominal Exchange Rate Response: Nakamura and Steinsson (2015) Shocks

	Dependent variable: Nominal Exchange Rates				
	(1)	(2)	(3)	(4)	(5)
$M^n_\$ \times \Delta i_{f,t}$	-8.467^{***}		-8.112^{***}	-7.616^{***}	-14.960^{***}
	(2.157)		(2.117)	(1.413)	(3.029)
(Imports / Cons.) $\times \Delta i_{f,t}$		-2.372^{**}			
		(1.035)			
(Dollar Import Share) $\times \Delta i_{f,t}$		-10.730^{***}			
		(3.112)			
Distance $\times \Delta i_{f,t}$			0.738	0.733	1.137
			(0.469)	(0.472)	(1.016)
US Import Share $\times \Delta i_{f,t}$				-3.958	-1.783
				(11.203)	(10.169)
$(\text{NetDebt}_{\$,t-1}/Y) \times \Delta i_{f,t}$					-0.050
					(0.041)
Observations	1,662	1,662	1,662	1,662	1,488
Adjusted R^2	0.644	0.645	0.644	0.644	0.626

Notes: This table presents regression results from estimating equation (10). The dependent variable is changes in nominal exchange rates. $M^n_\$$ is the value country n's dollar invoiced imports normalized by consumption. (Imports / Cons.) is country n's total imports as share of its consumption and (Dollar Import Share) represents the share of country n's imports that is invoiced in dollars. $M^n_\$ =$ (Dollar Import Share) $\times$ (Imports / Cons.). Distance is the log of the populaiton weighted distance between country n and the United States. US Import Share is the value of each country's imports from the U.S. normalized by consumption. $(\text{NetDebt}_{\$,t-1}/Y)$ is the country's dollar denominated net debt position. Standard errors in parentheses are clustered by date. *p<0.1; **p<0.05; ***p<0.01

Table 2: Nominal Interest Rate Response: Nakamura and Steinsson (2015) Shocks

	Dependent variable: Nominal Interest Rates (1Y)				
	(1)	(2)	(3)	(4)	(5)
$M^n_\$ \times \Delta i_{f,t}$	2.749***		2.822***	2.643***	1.847***
	(0.796)		(0.816)	(0.876)	(0.677)
(Imports / Cons.) $\times \Delta i_{f,t}$		0.995***			
		(0.376)			
(Dollar Import Share) $\times \Delta i_{f,t}$		2.009***			
		(0.615)			
Distance $\times \Delta i_{f,t}$			0.239*	0.396	0.515*
			(0.143)	(0.247)	(0.296)
US Import Share $\times \Delta i_{f,t}$				1.290	2.888
				(1.327)	(1.763)
$(NetDebt_{\$,t-1}/Y) \times \Delta i_{f,t}$					0.008
					(0.005)
Observations	1,374	1,374	1,374	1,374	1,286
Adjusted R^2	0.160	0.158	0.161	0.162	0.180

Notes: This table presents regression results from estimating equation (10). The dependent variable is changes in one-year nominal interest rates. $M^n_\$$ is the value country n's dollar invoiced imports normalized by consumption. (Imports / Cons.) is country n's total imports as share of its consumption and (Dollar Import Share) represents the share of country n's imports that is invoiced in dollars. $M^n_\$ =$ (Dollar Import Share) $\times$ (Imports / Cons.). Distance is the log of the populaiton weighted distance between country n and the United States. US Import Share is the value of each country's imports from the U.S. normalized by consumption. $(\text{NetDebt}_{\$,t-1}/Y)$ is the country's dollar denominated net debt position. Standard errors in parentheses are clustered by date. $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 3: Nakamura and Steinsson (2015) Shocks, Controlling for Product Pricing

	Nominal Exchange Rate		*Nominal Interest Rate (1Y)*	
	(1)	(2)	(3)	(4)
$M^n_\$ \times \Delta i_{f,t}$	-6.815^{**}	-6.628	2.550^{***}	-0.260
	(3.209)	(12.695)	(0.777)	(1.225)
Diff. Share $\times \Delta i_{f,t}$	-4.918		0.684^{*}	
	(4.082)		(0.375)	
Diff. Share $\times M^n_\$ \times \Delta i_{f,t}$		-2.109		3.689^{**}
		(12.614)		(1.614)
Observations	1,662	1,662	1,374	1,374
Adjusted R^2	0.644	0.643	0.161	0.164

Notes: Table 3 estimates equation (10) controlling for how products are priced according to Rauch (1999). The outcome variable in columns (1) and (2) is the nominal exchange rate. The outcome variable in columns (3) and (4) is the one year nominal interest rate. Diff. Share is the share of each country's imports that are differentiated products (not traded on an organized exchange or reference priced). Standard errors in parentheses are clustered by date. *p<0.1; **p<0.05; ***p<0.01

Table 4: Nakamura and Steinsson (2015) Shocks, Alternative Specifications

	(1)	(2)	(3)	(4)
Panel A:		*Nominal Exchange Rates*		
$M^n_\$ \times \Delta i_{\$,t}$	-8.062^{***}	-5.626^{***}	-11.426^{***}	-4.645^{*}
	(2.099)	(2.140)	(2.741)	(2.526)
Observations	1,188	1,226	1,662	1,662
Adjusted R^2	0.610	0.649	0.643	0.610
Panel B:		*Nominal Interest Rates (1Y)*		
$M^n_\$ \times \Delta i_{\$,t}$	2.771^{***}	3.176^{***}	4.241^{***}	2.942^{***}
	(0.790)	(0.896)	(1.310)	(0.617)
Observations	900	1,072	1,374	1,357
Adjusted R^2	0.255	0.318	0.160	0.147

Notes: This table estimates equation (10) using various alternative specifications. The dependent variable in Panel A is the change in nominal exchange rate and the dependent variable in Panel B is the change in the one year nominal interest rate. Column (1) only uses observations monetary policy shocks prior to the height of the financial crisis, June 2007. Column (2) restricts the sample of countries to those that maintained a Fitch sovereign bond credit rating above BBB- throughout the entirety of the sample. Column (3) normalizes $M^n_\$$ by GDP rather than consumption. Column (4) measures changes outcome variables from one day prior to the monetary policy announcement day to three days after the announcement day. Standard errors are clustered by date. *p<0.1; **p<0.05; ***p<0.01

Table 5: Foreign Central Bank Response

	30 Day Response	60 Day Response
	(1)	(2)
$M^n_{\$} \times \Delta i_{f,t}$	-0.022	-0.043
	(0.070)	(0.068)
Observations	1,133	1,124
Adjusted R^2	0.017	0.048

Notes: This table estimates equation (10) using responses in foreign central bank monetary policy rates to Federal Reserve monetary policy shocks as the dependent variable. I measure changes in foreign central bank policy rates in the 30 days and the 60 days after Federal Reserve monetary policy announcements. Standard errors are clustered by date. *p<0.1; **p<0.05; ***p<0.01

Table 6: ECB Monetary Policy Transmission

	Full Sample		*Pre-Crisis Sample*	
	$(\Delta s^{n,f}_t)$	(Δy^n_t)	$(\Delta s^{n,f}_t)$	Δy^n_t
$M^n_{€} \times \Delta i^{tgt}_{€,t}$	-5.135	0.082	-1.113	0.479
	(4.809)	(0.428)	(8.772)	(6.625)
$M^n_{€} \times \Delta i^{com}_{€,t}$	-11.403	0.067	-20.726^{***}	1.636^{***}
	(9.159)	(0.996)	(7.706)	(0.397)
Observations	1,022	1,030	554	521
Adjusted R^2	0.368	0.197	0.375	0.288

Notes: Table 6 presents regression results from estimating equation (10) using ECB monetary policy shocks from Leombroni et al. (2017). $\Delta i^{tgt}_{€,t}$ is the target rate shock on date t, and $\Delta i^{com}_{€,t}$ is the communication window shock. $M^n_{€}$ is the value country n's imports invoiced in euros and normalized by consumption. Nominal exchange rates are denominated in the currency of the central bank that issues the monetary policy shock. Standard errors in parentheses are clustered by date. *p<0.1; **p<0.05; ***p<0.01

Table 7: Pooled Regressions: ECB and FRB

	Full Sample		*Pre-Crisis Sample*	
	$(\Delta s_t^{n,f})$	(Δy_t^n)	$(\Delta s_t^{n,f})$	Δy_t^n
$M_f^n \times \Delta i_{f,t}$	-10.750^{***}	0.470	-12.237^{**}	1.303^{***}
	(4.093)	(0.423)	(4.828)	(0.281)
$M_f^n \times \Delta i_{f,t} \times FRB_t$	8.285^{*}	1.017	9.903^{*}	0.184
	(4.966)	(0.660)	(5.547)	(0.568)
Observations	2,371	2,166	1,503	1,256
Adjusted R^2	0.532	0.142	0.531	0.246

Notes: Table 7 presents regression results from estimating equation (10) using data from both the ECB and the FRB. $\Delta i_{f,t}$ is the measurement of the monetary policy shock from central bank f on date t. M_f^n is the value country n's imports invoiced in country f currency and normalized by consumption. Nominal exchange rates are denominated in the currency of the central bank that issues the monetary policy shock. Standard errors in parentheses are clustered by date. *p<0.1; **p<0.05; ***p<0.01

Appendix

A Appendix to the Theory Section

A.1 Deriving the Nominal Price Indices

In this section, I derive expressions for country n price indices for intermediate traded goods produced in country m, $\tilde{P}^n_{T,m}$, for aggregate consumption of traded goods, $\tilde{P}^n_T$, and for the aggregate consumption bundle of traded and non-traded goods, $\tilde{P}^n$.

In country n, the cost of a unit of traded consumption from country m is

$$\tilde{P}^n_{T,m} = \arg\min \int_0^{\mu^m} \tilde{P}^n_{T,m,j} C^n_{T,m,j} dj \text{ s.t. } \left[\int_0^{\mu^m} \left(C^n_{T,m,j}\right)^{\varepsilon} dj\right]^{\frac{1}{\varepsilon}} = 1$$

All country m intermediate traded goods invoiced in dollars have the same nominal price in country n, and all country m intermediate traded goods invoiced in domestic currency have the same nominal price in country n. Hence, I can re-write the optimization problem as,

$$\begin{aligned} \tilde{P}^n_{T,m} &= \arg\min \mu^m X^m_{\$} \tilde{P}^n_{T,m,\$} C^n_{T,m,\$} + \mu^m \left(1 - X^m_{\$}\right) \tilde{P}^n_{T,m,h} C^n_{T,m,h} \\ &\text{s.t. } \left[\mu^n X^m_{\$} \left(C^n_{T,m,\$}\right)^{\varepsilon} + \mu^n \left(1 - X^m_{\$}\right) \left(C^n_{T,m,h}\right)^{\varepsilon}\right]^{\frac{1}{\varepsilon}} = 1 \end{aligned}$$

I take first order conditions with respect to $C^n_{T,m,\$}$ and $C^n_{T,m,h}$ and take the ratio to solve for $C^n_{T,m,h}$,

$$C^n_{T,m,h} = C^n_{T,m,\$} \left(\frac{\tilde{P}^n_{T,m,\$}}{\tilde{P}^n_{T,m,h}}\right)^{\frac{1}{1-\varepsilon}}.$$

I solve for $C^n_{T,m,\$}$ by plugging this equation back into the constraint,

$$\begin{aligned} C^n_{T,m,j} &= \left(\mu^m X^m_{\$} + \mu^m \left(1 - X^m_{\$}\right) \left(\frac{\tilde{P}^n_{T,m,\$}}{\tilde{P}^n_{T,m,h}}\right)^{\frac{1}{1-\varepsilon}}\right)^{-\frac{1}{\varepsilon}} \\ &= \left(\tilde{P}^n_{T,m,\$}\right)^{-\frac{1}{1-\varepsilon}} \left(\mu^m X^m_{\$} \left(\tilde{P}^n_{T,m,\$}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \mu^m \left(1 - X^m_{\$}\right) \left(\tilde{P}^n_{T,m,h}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right)^{-\frac{1}{\varepsilon}}. \end{aligned}$$

I use this equation to solve for $C^n_{T,m,h}$, plug $C^n_{T,m,\$}$ and $C^n_{T,m,h}$ into the objective and derive the following expression for $\tilde{P}^n_{T,m}$,

$$\tilde{P}^n_{T,m} = \left(\mu^m X^m_{\$} \left(\tilde{P}^n_{T,m,\$}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \mu^m \left(1 - X^m_{\$}\right) \left(\tilde{P}^n_{T,m,h}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right)^{\frac{\varepsilon-1}{\varepsilon}}. \quad (12)$$

In country n, the cost of a unit of the final traded consumption comprised of intermediate traded goods from each country is,

$$\tilde{P}^n_T = \arg\min \sum_{m=\{US,JP,EU\}} \tilde{P}^n_{T,m} C^n_{T,m} \text{ s.t. } \left(C^n_{T,US}\right)^{\alpha^n_{US}} \left(C^n_{T,JP}\right)^{\alpha^n_{JP}} \left(C^n_{T,EU}\right)^{\alpha^n_{EU}} = 1$$

First order conditions imply,

$$C^n_{T,JP} = \frac{\alpha^n_{JP}}{\alpha^n_{US}} \frac{\tilde{P}^n_{T,US}}{\tilde{P}^n_{T,JP}} C^n_{T,US} \text{ and } C^n_{T,EU} = \frac{\alpha^n_{EU}}{\alpha^n_{US}} \frac{\tilde{P}^n_{T,US}}{\tilde{P}^n_{T,EU}} C^n_{T,US}.$$

Plugging these values into the constraint, using the identity $\sum_{m=US,JP,EU} \alpha^n_m = 1$ and solving for $C^n_{T,US}$ yields,

$$\tilde{P}^n_{T,US} C^n_{T,US} = \frac{\alpha^n_{US}}{\left(\alpha^n_{US}\right)^{\alpha^n_{US}} \left(\alpha^n_{JP}\right)^{\alpha^n_{JP}} \left(\alpha^n_{EU}\right)^{\alpha^n_{EU}}} \left(\tilde{P}^n_{T,US}\right)^{\alpha^n_{US}} \left(\tilde{P}^n_{T,JP}\right)^{\alpha^n_{JP}} \left(\tilde{P}^n_{T,EU}\right)^{\alpha^n_{EU}}.$$

Plugging this equation into the objective yields,

$$\tilde{P}^n_T = \frac{\left(\tilde{P}^n_{T,US}\right)^{\alpha^n_{US}} \left(\tilde{P}^n_{T,JP}\right)^{\alpha^n_{JP}} \left(\tilde{P}^n_{T,EU}\right)^{\alpha^n_{EU}}}{\left(\alpha^n_{US}\right)^{\alpha^n_{US}} \left(\alpha^n_{JP}\right)^{\alpha^n_{JP}} \left(\alpha^n_{EU}\right)^{\alpha^n_{EU}}}. \quad (13)$$

In country n, the cost of a unit of the aggregate consumption bundle comprised of final traded goods and non-traded goods is,

$$\tilde{P}^n = \arg\min \tilde{P}^n_T C^n_T + \tilde{P}^n_N \text{ s.t. } \left(C^n_T\right)^{\tau} \left(C^n_N\right)^{1-\tau} = 1$$

Repeating the same steps as above yields the following expression for the aggregate nominal price level in country n,

$$\tilde{P}^n = \frac{\left(\tilde{P}^n_T\right)^{\tau} \left(\tilde{P}^n_N\right)^{1-\tau}}{(\tau)^{\tau} (1-\tau)^{1-\tau}}. \quad (14)$$

A.2 Deriving the Conditions of Optimality

In this subsection, I provide additional details about solving the model in Section 3. Households maximize utility (1) subject to their budget constraints (4). Let Ψ^n denote the Lagrange multiplier on the budget constraint for households in country n. By assumption, the wealth transfer, κ^n, in equation 4 equalizes $\Psi^n = \Psi^m$ for all n, m. For a household in country n, the first order condition with respect to the consumption of intermediate traded good j produced in country m, $C^n_{T,m,j,t}$, yields,

$$\alpha^n_m \tau \left(\frac{(C^n_t)^{1-\gamma}}{C^n_{T,m,t}} \right) \left(\frac{C^n_{T,m,t}}{C^n_{T,m,j,t}} \right)^{1-\varepsilon} = \Psi^n Q^n_t \tilde{P}^n_{T,m,j,t}.$$

The first order condition with respect to the consumption of non-traded goods is,

$$(1-\tau) \frac{(C^n_t)^{1-\gamma}}{C^n_{N,t}} = \Psi^n Q^n_t \tilde{P}^n_{N,t}.$$

The first order condition with respect to labor supplied by the household is,

$$N^n_t = \Psi^n Q^n_t \tilde{W}^n_t.$$

Finally, I take a first order condition with respect to aggregate consumption,

$$(C^n_t)^{-\gamma} = \Psi^n Q^n_t \tilde{P}^n_t \tag{15}$$

I take the ratio between the households's first order conditions with respect to intermeditate traded consumption, non-traded consumption and labor supply with the household's first order condition with respect to aggregate consumption.

$$\alpha^n_m \tau \frac{C^n_t}{C^n_{T,m,t}} \left(\frac{C^n_{T,m,t}}{C^n_{T,m,j,t}} \right)^{1-\varepsilon} = \frac{\tilde{P}^n_{T,m,j,t}}{\tilde{P}^n_t}. \tag{16}$$

The first order condition with respect to non-traded consumption determines the price of the non-traded good

$$(1-\tau) \frac{C^n_t}{C^n_{N,t}} = \frac{\tilde{P}^n_{N,t}}{\tilde{P}^n_t} \tag{17}$$

Household labor supply is pinned down by the first order condition with respect to N_t^n,

$$C_t^{\gamma} N_t^n = \frac{\tilde{W}_t^n}{\tilde{P}_t^n}. \tag{18}$$

These first order conditions describe the demand side of the economy and hold in each period regardless of the nominal rigidity in the supply side of the economy.

For any two countries n and m, the nominal exchange rate can be derived as the ratio of the countries' marginal utilities of consumption divided by their nominal price levels,

$$\frac{(C_t^n)^{-\gamma}}{\tilde{P}_t^n} = \frac{(C_t^m)^{-\gamma}}{\tilde{E}_t^{n,m} \tilde{P}_t^m} \tag{19}$$

I derive equation (19) by taking the ratio between the first order condition with respect to aggregate consumption for country n and country m, given by equation (15). I can cancel out the Lagrange multipliers, because $\Psi^h = \Psi^f$. Since I do not introduce and frictions into financial markets, $Q_t^n = \tilde{E}_t^{n,m} Q_t^m$.

In the remainder of this subsection, I solve the firm's problem. These equations are not shown in the main text. In both periods, the intermediate traded good firm maximizes profits by choosing the price of its variety of intermediary traded good. Firm j in country m faces the following demand function in country n,

$$C_{T,m,j,t}^n = \left(\tilde{P}_{T,m,j,t}^n\right)^{\frac{1}{\varepsilon 1}} \left(\frac{\alpha_m^n}{\tau}\right)^{\frac{1}{\varepsilon-1}} \left(\frac{C_t^n}{\tilde{P}_t^n}\right)^{\frac{1}{\varepsilon-1}} \left(C_{T,m,t}^n\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{20}$$

In the second period, nominal prices are flexible and the firm j in country m chooses the nominal price of their traded good in each country and in each state of the world to maximize discounted profits,

$$\max_{\tilde{P}_{T,m,j,t}^n} \sum_{n \in \{US,JP,EU\}} \mu^n \left\{ \tilde{P}_{T,m,j,2}^n C_{T,m,j,2}^n - \tilde{W}_2^m \left(\frac{C_{T,m,j,2}^n}{A_2^m} \right) \right\} \tag{21}$$

I plug equation (20) into the equation (21), take first order conditions with respect to $\tilde{P}_{T,m,j,t}^n$, and I solve for $\tilde{P}_{T,m,j,t}^n$ This yields the typical result where the firm sets it's nominal price as a constant markup over marginal cost in domestic currency,

$$\tilde{P}_{T,m,j,2}^m = \frac{1}{\varepsilon} \frac{\tilde{W}_2^n}{A_2^n} \text{ and } \tilde{P}_{T,m,j,2}^n = \tilde{E}_2^{n,m} \tilde{P}_{T,m,j,2}^n$$

In the first period, firms set prices before the realization of shocks to maximize expected discounted first period profits. Furthermore, firms that invoice their exports in a foreign currency need to set a domestic currency price as well as a foreign currency price.

If the firm only invoices in domestic currency (producer currency pricing), then it faces the following problem,

$$\max_{\tilde{P}^m_{T,m,j,1}} \mathbb{E}\left[Q^m_1 \sum_{n\in\{US,JP,EU\}} \mu^n \left\{\tilde{P}^m_{T,m,j,1} C^n_{T,m,j,1} - \tilde{W}^m_1 \left(\frac{C^n_{T,m,j,1}}{A^m_1}\right)\right\}\right].$$

Demand in each country each country is given by,

$$C^n_{T,m,j,1} = \left(\tilde{E}^{n,m}_1 \tilde{P}^m_{T,m,j,1}\right)^{\frac{1}{\varepsilon 1}} \left(\frac{\alpha^n_m}{\tau}\right)^{\frac{1}{\varepsilon-1}} \left(\frac{C^n_1}{\tilde{P}^n_1}\right)^{\frac{1}{\varepsilon-1}} \left(C^n_{T,m,1}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

I plug in the demand function into the firm's problem, take the first order condition with respect to $\tilde{P}^m_{T,m,j,1}$ and solve for the price,

$$\tilde{P}^m_{T,m,j,1} = \frac{\mathbb{E}\left[Q^m_1\left(\tilde{W}^m_1 Y_{T,m,j,1}\right)/A^m_1\right]}{\mathbb{E}\left[Q^m_1 Y_{T,m,j,1}\right]}$$

where $Y_{T,m,j,1} = \sum_{n\in\{US,JP,EU\}} \mu^n C^n_{T,m,j,1}$ is the total output by firm j located in country m.

If the firm invoices in dollars, then the firm chooses a domestic currency price, $\tilde{P}^m_{T,m,j,1}$, as well as a dollar price, $\tilde{P}^{US}_{T,m,j,1}$, in order to maximize expected discounted first p

$$\max_{\tilde{P}^m_{T,m,j,1},\tilde{P}^{US}_{T,m,j,1}} \mathbb{E}\left\{Q^m_1\left[\mu^m \tilde{P}^m_{T,m,j,1} C^m_{T,m,j,1} + \sum_{n\neq m} \mu^n \tilde{E}^{m,US}_1 \tilde{P}^{US}_{T,m,j,1} C^m_{T,m,j,1} - \tilde{W}^m_1\left(\frac{\sum_n \mu^n C^n_{T,m,j,1}}{A^m_1}\right)\right]\right\}.$$

I plug in the demand function into the firm's problem, take the first order conditions and solve for the optimal prices,

$$\tilde{P}^m_{T,m,j,1} = \frac{\mathbb{E}\left[Q^m_1\left(\tilde{W}^m_1 C^m_{T,m,j,1}\right)/A^m_1\right]}{\mathbb{E}\left[Q^m_1 C^m_{T,m,j,1}\right]} \text{ and } \tilde{P}^m_{T,m,j,1} = \frac{\mathbb{E}\left[Q^m_1\left(\tilde{W}^m_1 \sum_{n\neq m}\mu^n C^n_{T,m,j,1}\right)/A^m_1\right]}{\mathbb{E}\left[Q^{US}_1 \sum_{n\neq m}\mu^n C^n_{T,m,j,1}\right]}$$

where $Q^{US}_1 = Q^m_1 \tilde{E}^{m,US}_1$.

A.3 Log-Linear Approximation of the Equilibrium

I log-linearize the model around the following deterministic steady state:

$$\begin{aligned}
A_t^n &= \sqrt{(\varepsilon\mu^n)^{-1}\sum_m \mu^m \alpha_n^m} \\
C_{T,m,j,t}^n &= \frac{\alpha_m^n}{\mu^m},\ C_{N,t}^n = Y_{N,t}^n = \prod_m \left[(\mu^m)^{-\frac{\alpha_m^n(1-\varepsilon)\tau}{\varepsilon(1-\tau)}} (\alpha_m^n)^{-\alpha_m^n \frac{\tau}{1-\tau}}\right] \\
Y_{T,m,j,t} &= \frac{1}{\mu^m}\sum_n \mu^n \alpha_m^n,\ N_{m,j,t} = Y_{T,m,j,t}/A_t^m,\ N_t^n = \mu^n N_{n,j,t} \\
\tilde{P}_{T,m,j,t}^n &= 1,\ \tilde{W}_t^n = \varepsilon A_t^n,\ \tilde{P}_{N,t}^n = (1-\tau)(\tau)^{-1+\frac{1}{(\gamma-1)(1-\tau)}} \prod_m \left[(\mu^m)^{-\frac{\alpha_m^n(1-\varepsilon)\tau}{\varepsilon(1-\tau)}} (\alpha_m^n)^{-\alpha_m^n \frac{\tau}{1-\tau}}\right] \\
\tilde{P}_t^n &= (\tau)^\gamma \gamma - 1,\ \tilde{E}_t^{n,m} = 1
\end{aligned}$$

for all countries n and m and time periods t.

A.4 Properties of Constant ν

$$\nu = (1-\tau)^3 + \gamma\tau(1-\tau)\left(\gamma\tau\Xi_1 + (1-\tau)\left(\left(1-X_\$^{EU}\right)\alpha_{EU}^{US} + \left(1-X_\$^{JP}\right)\alpha_{JP}^{US} + \alpha_{US}^{JP} + \alpha_{EU}^{JP} + \alpha_{US}^{EU} + \alpha_{JP}^{EU}\right)\right) \tag{22}$$

where,

$$\begin{aligned}
\Xi_1 = &\left(1-X_\$^{JP}\right)\alpha_{JP}^{US}\left(\left(1-X_\$^{EU}\right)\alpha_{EU}^{JP} + \alpha_{US}^{EU} + \alpha_{JP}^{EU}\right) + \\
&\left(1-X_\$^{EU}\right)\alpha_{EU}^{US}\left(\alpha_{US}^{JP} + \alpha_{EU}^{JP} + \left(1-X_\$^{JP}\right)\alpha_{JP}^{EU}\right) + \\
&\alpha_{US}^{JP}\left(\alpha_{US}^{EU} + \alpha_{JP}^{EU}\right) + \alpha_{EU}^{JP}\left(\alpha_{US}^{EU} + \alpha_{JP}^{EU}\left(X_\$^{JP} + X_\$^{EU} - X_\$^{JP}X_\$^{EU}\right)\right)
\end{aligned}$$

I prove two propositions characterizing the constant ν.

Remark. The constant $\nu > 0$.

Proof. This is straightforward to show, because $\gamma > 0$, $\tau \in (0,1)$, $X_\$^n \in (0,1)$ and $\alpha_m^n > 0$ for all n, m. Plugging these assumptions into equation (22) shows that $\nu > 0$. □

Remark. ν is increasing in α_m^n for $n \neq m$.

Proof. This is straightforward to check. Equation 22 displays ν as the product and sum of positive constants and each α_m^n where $n \neq m$. Hence, all derivatives of ν with respect to α_m^n are positive. □

ν captures the degree of risk-sharing across countries, and a positive ν allows me to sign analytic expressions. The fact that ν increases with α_m^n for $n \neq m$ means increased risk sharing across countries dampens monetary policy spillover effects. α_m^n determines the share of country n household's traded consumption that is comprised of imports from country m. If $\alpha_m^n = 0$ for all $n \neq m$, countries only consume traded goods produced domestically. No country trades with any other. Positive α_m^n allow countries to share risk by shipping traded goods to households with high marginal utilities of consumption.

A.5 Proof of Proposition 1

If $\tau^n = \tau$ is constant across all countries n, then it is straightforward to show that the derivative $\frac{\partial y_T^n}{\partial \tilde{p}^n}$ for $n = JP, EU$ is decreases with the share of country n's dollar invoiced exports, $X_\$^n$. For $n = JP$,

$$\frac{\partial^2 y_T^{JP}}{\partial \tilde{p}^{JP} \partial X_\$^{JP}} = -\frac{(1+(\gamma-1)\tau)\left(1+\left(\gamma\left(\left(1-X_\$^{EU}\right)\alpha_{EU}^{US}+1-\alpha_{EU}^{EU}\right)-1\right)\tau\right)}{(1-\tau)\left(\sum_m \mu^m \alpha_{JP}^m\right)(\Xi_4)^2} \times \left(\Xi_2 + \left(\mu^{US}\alpha_{JP}^{US} + \mu^{EU}\alpha_{JP}^{EU}\right)\Xi_3\right) < 0,$$

where Ξ_2 and Ξ_3 are positive constants. Ξ_4 is a constant that is not displayed here.

$$\begin{aligned}\Xi_2 =& \alpha_{JP}^{US}\alpha_{JP}^{EU}(\gamma-1)(1-\tau)(1-\mu^{JP})\left(1+\left(\gamma(1-\alpha_{JP}^{JP})-1\right)\tau\right)+\mu^{JP}\alpha_{JP}^{JP}\gamma\tau^2 \\ &\times\left((1-\tau)(1-X_\$^{EU})\left(\alpha_{US}^{JP}\alpha_{JP}^{US}+\alpha_{EU}^{JP}\alpha_{JP}^{EU}\right)+(1-\alpha_{JP}^{JP})\left(\gamma\tau\left(\alpha_{JP}^{US}\alpha_{US}^{EU}+\alpha_{JP}^{EU}\right.\right.\right. \\ &+\left.\left.\left.\left(1-\alpha_{US}^{US}-\alpha_{EU}^{US}X_\$^{EU}\right)\right)\alpha_{JP}^{US}(1-\tau)X_\$^{EU}\right)\right).\end{aligned}$$

$$\begin{aligned}\Xi_3 =& \left(1+\left(\gamma(1-\alpha_{JP}^{JP})-1\right)\tau\right)\left(\left(1+(1-\alpha_{JP}^{US})\tau\right)\left(1+\left(\alpha_{US}^{EU}\gamma-1\right)\tau\right)\right. \\ &+\alpha_{EU}^{US}\gamma(1-\tau)\tau(1-X^{EU})\$)+\left(\alpha_{JP}^{EU}\tau\left(1+\left(\gamma(1-\alpha_{US}^{US}-\alpha_{EU}^{US}X_\$^{EU})-1\right)\tau\right)\right)\right).\end{aligned}$$

The analogous expressions for $n = EU$ are similar except the superscripts and subscripts for EU and JP are reversed.

Domestic expansionary monetary policy increases aggregate traded production, $\frac{\partial y_T^n}{\partial \tilde{p}^n}$. Hence, an increase in the share of country n's dollar invoiced exports decreases the magnitude of the effect of domestic monetary policy on domestic aggregate production.

A.6 Proof of Prediction 3

The effect of contractionary Japanese monetary policy on aggregate consumption in the U.S. and Europe is

$$-\frac{\partial c^n}{\partial \tilde{p}^{JP}} = -\frac{\gamma\tau^2(1+(\gamma-1)\tau)\left(1-X_{\$}^{JP}\right)\left((1-\alpha_{US}^{US})(1-\alpha_{EU}^{EU})-\alpha_{EU}^{US}\left(\alpha_{US}^{EU}+\alpha_{JP}^{EU}X_{\$}^{EU}\right)\right)}{\nu}$$
$$-\frac{(1-\tau)(1+(\gamma-1)\tau)}{\nu}M_{¥}^{n}.$$

The first line of the previous equation corresponds with the first line of equation (5). The second line of the previous equation exactly mirrors the second line of equation (5) with $M_{¥}^{n}$ taking the place of $M_{\$}^{n}$.

Since nominal interest rates are determined by the consumption Euler equation given by equation (7), the effect of of contractionary Japanese monetary policy on nominal interst rates in the U.S. and Europe is

$$-\frac{\partial \tilde{r}^n}{\partial \tilde{p}^{JP}} = -\gamma\left(-\frac{\partial c^n}{\partial \tilde{p}^{JP}}\right).$$

Hence, the effect of Japanese monetary policy on foreign nominal interest rates is symmetric to the effect of U.S. monetary policy due to the symmetry in the monetary policy spillovers to consumption.

The corresponding expressions for European monetary policy shocks simply involve replacing the JP superscripts in the expressions above with EU superscripts.

A.7 Model Extension with Incomplete Asset Markets

In this section, I extend the model in the paper to include incomplete asset markets. Within each country, only a fixed proportion ϕ of households (labelled "active") trade a complete set of state-contigent securities. The remaining $1-\phi$ of households (labelled "inactive") are excluded from trading in financial markets. Instead, inactive households cede the claims to their firm profits and their endowments of non-traded goods to active households. In return, inactive households receive a nominal bond that makes a fixed nominal payment $\tilde{B}_t^n$ in each period.

Active households maximize expected utility (1) subject to the following modified budget constraint

$$\mathbb{E}\left\{\sum_{t=1,2} Q_t^n \left[\tilde{P}_{N,t}^n C_{N,t}^n + \sum_{i\in\{US,JP,EU\}} \left(\int_0^1 \tilde{P}_{T,i,j,t}^n C_{T,i,j,t}^n dj\right) - \left(\frac{1-\phi}{\phi}\right)\tilde{B}_t^n\right]\right\} =$$
$$\mathbb{E}\left\{\sum_{t=1,2} Q_t^n \left(\frac{1}{\phi}\left(\tilde{P}_{N,t}^n Y_{N,t}^n + \tilde{\Gamma}_t^n\right) + \tilde{W}_t^n N_t^n\right)\right\} + \kappa^n.$$

The active household's budget constraint differs from equation (4) in two ways: Active households pay $((1-\phi)/\phi)\,\tilde{B}_t^n$ units of currency to inactive households in each period, and active households receive $1/\phi$ shares of claims to the non-traded endowment and the profits of the firms in each period.

Although the budget constraint has changed from the model Section 3, the first order conditions for active households are unaffected. Equations (16), (17) and (18) still determine the active household's allocation of intermediate traded goods, non-traded goods and labor supply. The wealth transfer κ^n equates the marginal utility of wealth across active households such that equation (19) continues to hold.

Inactive households also maximize expected utility (1) in each period. However, their problem simply involves choosing how to use their nominal bond payment and their labor income to purchase consumption. Inactive households face the following budget constraint in each period t,

$$\tilde{P}_{N,t}^n \hat{C}_{N,t}^n + \sum_{i\in\{US,JP,EU\}} \left(\int_0^1 \tilde{P}_{T,i,j,t}^n \hat{C}_{T,i,j,t}^n dj\right) \le \tilde{B}_t^n + \tilde{W}_t^n \hat{N}_t^n$$

$\hat{C}_N^n$ and $\hat{C}_{T,m,j}^n$ denote the non-traded consumption and intermediate traded consumption of inactive households in country n. $\hat{N}^n$ denotes the labor supplied by inactive households in country n.

Since household utility (1) is a Cobb-Douglas aggregate of traded and non-traded goods, it is straightforward to write the inactive household's consumption of traded and non-traded goods as a function of market prices and their total income. The inactive household's consumption of intermediate good j produced in country m is

$$\hat{C}_{T,m,j}^n = \alpha_m^n \tau \left(\frac{\tilde{P}_{T,m}^n}{\tilde{P}_{T,m,j}^n}\right)^{\frac{1}{1-\varepsilon}} \left(\frac{\tilde{B}_t^n + \tilde{W}_t^n \hat{N}_t^n}{\tilde{P}_{T,m}^n}\right). \tag{23}$$

$\tilde{P}_{T,m}^n$ is the nominal price index in country n for the aggregate traded good produced in country

m, which is derived in Appendix A.1. The inactive household's consumption of the non-traded good is

$$\hat{C}^n_{N,t} = (1-\tau)\left(\frac{\tilde{B}^n_t + \tilde{W}^n_t \hat{N}^n_t}{\tilde{P}^n_{N,t}}\right). \tag{24}$$

The inactive household's labor supply is determined by the first order condition with respect to $\tilde{N}^n_t$

$$\left(\hat{C}^n_t\right)^{\gamma} \hat{N}^n_t = \frac{\tilde{W}^n_t}{\tilde{P}^n_t} \tag{25}$$

The resource constraints in the economy need to account for the consumption and labor supply of both active and inactive households. The market clearing condition for non-traded goods is

$$Y^n_{N,t} = \phi C^n_{N,t} + (1-\phi)\hat{C}^n_{N,t}. \tag{26}$$

The market clearing condition for intermediate traded good j produced in country m is

$$Y_{T,m,j,t} = A^m_t \mu^m \left(\phi N_{m,j,t} + (1-\phi)\hat{N}_{m,j,t}\right) = \sum_{n \in \{US,JP,EU\}} \mu^n \left(\phi C^n_{T,m,j,t} + (1-\phi)\hat{C}^n_{N,t}\right). \tag{27}$$

Finally, the labor market clearing conditions for active and inactive households in country m are,

$$N^m_t = \int_0^{\mu} N_{m,j,t} dj \text{ and } \hat{N}^m_t = \int_0^{\mu} \hat{N}_{m,j,t} dj \tag{28}$$

An equilibrium in this economy is a set of consumption and labor supply allocations for each household, $\left\{C^n_{T,m,j,t}, C^n_{N,t}, N^n_t, \hat{C}^n_{T,m,j,t}, \hat{C}^n_{N,t}, \hat{N}^n_t\right\}$; a set of intermediate output, labor demand and nominal prices for each intermediate firm, $\left\{Y^n_{T,m,j,t}, N_{m,j,t}, \hat{N}_{m,j,t}, \tilde{P}^n_{T,m,j,t}\right\}$; a set of traded goods prices and non-traded goods prices, $\left\{\tilde{P}^n_{T,t}, \tilde{P}^n_{N,t}\right\}$; a set of nominal wages for each country and a nominal exchange rate, $\left\{\tilde{W}^n_t, \tilde{E}^{n,m}_t\right\}$; such that active and inactive households maximize utility subject to their budget constraints and the markets clearing conditions are satisfied.

I log-linearize the model around the same deterministic steady state described in Appendix A.3. The addition of market segmentation significantly complicates the analytic expressions in the model. Hence, I make the simplifying assumptions in order to derive expressions analogous to the results from Section 4. I assume households consume an equal share of traded goods from all countries, $\alpha^n_m = \alpha = 1/3$ and countries are all the same size, $\mu^n = 1/3$, for all countries n and m. Furthermore, I assume all firms in the Europe invoice in Euros, $X^{EU}_{\$} = 0$.

I derive the effects of a contractionary U.S. monetary policy shock on nominal exchange rates, consumption and production and show that Predictions 1 through **??** hold in the incomplete markets model. First, I define the constant $\hat{\nu}$

$$\hat{\nu} = \frac{(1-\tau)(1+(\gamma-1)\tau)\hat{\Xi}_1\hat{\Xi}_2}{3\hat{\Xi}_3\hat{\Xi}_4},$$

where

$$\hat{\Xi}_1 = \phi(1-\tau) + \gamma\tau(1+\varepsilon(1-\tau)(2-\phi)) + \varepsilon\gamma^2\tau^2(2\phi-1)$$
$$\hat{\Xi}_2 = 2\gamma\tau(1+\varepsilon(1-\tau)) + \gamma\tau\hat{\Xi}_4(1-X_{\$}^{JP}) + 3\phi(1-\tau) + \varepsilon\gamma\tau(\phi(1-\tau)+2\gamma\tau(2\phi-1))$$
$$\hat{\Xi}_3 = \phi(1-\tau) + \gamma(1-(1-\tau)\phi) + \gamma\varepsilon\tau(1+(\gamma-1)\tau)(2\phi-1)$$
$$\hat{\Xi}_4 = 1 + \varepsilon\left(4(1-\tau)(1-\phi) + \gamma\tau(2\phi-1)\right)$$

It is straightforward to show that ν is positive whenever $\phi > 1/2$. Hence, I assume $\phi > 1/2$ for the remainder of this section.

Equation (29) provides the incomplete markets analog to equation (6) in Section (4),

$$-\frac{\partial\tilde{e}^{n,US}}{\partial\tilde{p}^{US}} = \hat{\delta}_e - \frac{\gamma(1-\tau)(1+(\gamma-1)\tau)}{\hat{\nu}}M_{\$}^n. \tag{29}$$

$\hat{\delta}_e$ is constant that is common across countries, $M_{\$}^{JP} = \alpha\tau$ and $M_{\$}^{EU} = \left(\alpha + X_{\$}^{JP}\alpha\right)\tau$. Equation (30) provides the incomplete markets analog to equation (5) in Section (4),

$$-\frac{\partial c^n}{\partial\tilde{p}^{US}} = \hat{\delta}_c - \frac{(1-\tau)(1+(\gamma-1)\tau)}{\hat{\nu}}M_{\$}^n. \tag{30}$$

Active household consumption in Europe decreases relative to active household consumption in Japan, because Europe has a larger share dollar invoiced consumption than Japan.

Equation (31) shows the effect of a contractionary Japanese monetary policy shock on an active household's consumption in the U.S. and Europe,

$$-\frac{\partial c^n}{\partial\tilde{p}^{JP}} = \hat{\delta}_c^{JP} - \frac{(1-\tau)(1+(\gamma-1)\tau)}{\hat{\nu}}M_{¥}^n. \tag{31}$$

In the incomplete markets extension, monetary policy shocks from Japan have the same magnitude at those from the U.S. after controlling for the share of consumption invoiced in each currency.

Finally, I show that the incomplete markets model breaks the tight link between a country's aggregate consumption and changes in its exchange rates and interest rates. Aggregate consumption in country n (denoted by $\bar{C}^n$) is the weighted sum of active household consumption and inactive household consumption,

$$\bar{C}^n = \phi C^n + (1-\phi)\,\hat{C}^n$$

Equation (32) shows the effect of a contractionary U.S. monetary policy shock on the difference in aggregate consumption between Japan and Europe

$$-\frac{\partial}{\partial \tilde{p}^{US}}\left(\bar{c}^{JP} - \bar{c}^{EU}\right) = -\frac{(1-\tau)(1+(\gamma-1)\tau)\,(2\gamma\varepsilon\tau(1-2\phi)+\phi(\gamma\varepsilon\tau-1))\,\alpha\tau X_{\$}^{JP}}{\nu\hat{\Xi}_4}. \tag{32}$$

It is straightforward to find an example where the right-hand side of equation 32 is negative. For example, this is true whenever $\phi \in [1/2, 2/3]$ and the coefficient of risk aversion is sufficiently large ($\gamma > \phi/(\varepsilon\tau(1-(3/2)\phi))$). Then, the decrease in Japanese aggregate consumption is greater than the decrease in European aggregate consumption even though European households consume more dollar invoiced traded goods as a share of total consumption.

B Appendix to the Empirical Section

This section of the Appendix provides summary statistics provides summary statistics for the data, empirical results and robustness checks for the main empirical results.

B.1 Summary Statistics

Table 8: Data Availability

Country	Alpha3	Exchange Rates	Interest Rates
Australia	AUS	1999 - 2014	1999 - 2014
Canada	CAN	1999 - 2014	1999 - 2014
Cyprus	CYP	1999 - 2007	2004 - 2007
Denmark	DNK	1999 - 2014	1999 - 2014
Estonia	EST	1999 - 2010	2004 - 2010
Eurozone		1999 - 2014	1999 - 2014
Greece	GRC	1999 - 2000	1999 - 2000
Iceland	ISL	1999 - 2014	2004 - 2014
Israel	ISR	1999 - 2014	2004 - 2014
Japan	JPN	1999 - 2014	1999 - 2014
Latvia	LVA	1999 - 2013	2004 - 2013
Lithuania	LTU	1999 - 2014	2004 - 2014
Malta	MLT	1999 - 2007	2004 - 2007
Norway	NOR	1999 - 2014	1999 - 2014
Slovenia	SVN	1999 - 2006	2004 - 2006
South Korea	KOR	1999 - 2014	2002 - 2014
Sweden	SWE	1999 - 2014	1999 - 2014
Switzerland	CHE	1999 - 2014	1999 - 2014
United Kingdom	GBR	1999 - 2014	1999 - 2014

Table 9: Summary Statistics

	Obs	Mean	Std Dev	Max	Min
Monetary Policy Shocks (bps)					
FRB (Nakamura and Steinsson, 2015)	114	0.463	3.367	9.863	-11.948
ECB (Leombroni et al. (2017), Target)	149	-0.035	2.174	7.262	-18.527
ECB (Leombroni et al. (2017), Communication)	149	0.185	3.305	18.025	-14.330
FRB (3M Future Only)	75	-0.361	5.332	17.500	-28.000
ECB (3M Future Only)	175	0.368	4.409	22.500	-19.500
Dollar Invoicing					
Dollar Imports / Imports	18	0.446	0.212	0.808	0.125
Imports / Consumption	18	0.868	0.325	1.441	0.308
Dollar Imports / Consumption $\left(M^n_\$\right)$	18	0.358	0.187	0.843	0.134
Euro Invoicing					
Euro Imports / Imports	12	0.253	0.184	0.535	0.035
Imports / Consumption	12	0.774	0.248	1.079	0.308
Euro Imports / Consumption $\left(M^n_{€}\right)$	12	0.228	0.191	0.566	0.011

Notes: The table summarizes monetary policy shocks on regularly scheduled policy announcements days for each central bank between 1999 to 2014. I drop days where more than one central bank issued a monetary policy announcement, and I drop the height of the financial crisis (July 2008 - June 2009). The number of observations vary, because I try to incorporate all available data in each set of monetary policy shocks. The Nakamura and Steinsson (2015) data set covers the period Jan. 1999 to Mar. 2014. The ECB data from Leombroni et al. (2017) covers the period Feb. 2001 to Dec. 2014. Finally, the monetary policy shocks from Tick Data cover non-overlapping monetary policy announcement between Jan. 1999 to Dec. 2014.

C Data Appendix

C.1 Monetary Policy Shocks

This subsection provides additional details for how I contructed monetary policy shocks for the FRB and the ECB using high frequency data from Tick Data. Table 10 summarizes the scheduled monetary policy announcement dates in my sample. I list the initial date, the final date, the number of monetary policy announcement days per year and link to the data.

Table 10: Scheduled Monetary Policy Announcement Days

Central Bank	Data Availability	Announcements per Year	Source
FRB	1999 - 2014	For the relevant part of the sample (1999 - 2014) there have been eight meetings a year. However, during the Great Recession, we also observe "Intermeeting Press Releases" on: Aug 10, 2007; Aug 17, 2007; Jan 22, 2008; Oct 8, 2008 and May 9, 2010.	Link
ECB	1999 - 2014	For the most part, the ECB meets monthly. However, the number of meetings increased when the Euro was first created, and during the Eurozone Crisis.	Link

I gather minute level measures of three month interest rate futures from Tick Data. The series symbols for three month interest rate futures for the dollar and euro are "ED" and "UR", respectively. My measure of FRB monetary policy shocks involves calculating the change in price of the 3-month eurodollar future between the last trade more than 10-minutes before the monetary policy announcement time and the first trade at least 20-minutes after the monetary policy announcement time. For my measure of ECB monetary policy shocks, I include both the target rate shock and the communications shock in one measure. Therefore, I calculate the change in price of the 3-month euribor future between the last trade more than 10-minutes before the target rate announcement time and the first trade at least 145-minutes after the target rate announcement time.

C.2 Import and Export Invoicing Currencies

I use data on import and export invoicing currencies from Gopinath (2015). The data are available on her website:

http://scholar.harvard.edu/gopinath/publications/international-price-system

The data provide the average share of each countries imports and exports that are invoiced in each currency over the period 1999 - 2014.

I need to perform additional calculations for the Eurozone, because I only observe one nominal interest rate for the Eurozone as a whole but I obseve currency invoicing data for the countries of the Eurozone individually. Hence, I need to estimate the currency composition of the Eurozone's exports and the Eurozone's imports with respect to the rest of the world. I use bilateral goods trade data to estimate the share of each country's imports and exports with respect to other Eurozone countries, and I assume that all trade conducted within the Eurozone is done using Euros. These assumptions allow me to estimate each country's imports and exports with respect to non-Eurozone countries that are invoiced in each currency. Finally, I aggregate the country level data to calculate the invoicing shares of the Eurozone's imports and exports with respect to the rest of the world.

The details of my procedure for estimating currency invoicing shares for the Eurozone are as follows. For each country n in the Eurozone in year t, I observe the share of the country's imports invoiced in currency c. Denote this as $S^n_{M,c}$. I denote the total current dollar value of country n's imports as V^n_M, and the total current dollar value of country n's imports invoiced in currency c as $V^n_{M,c}$. Furthermore, denote the value of country n's imports from the Eurozone as $V^{n,EUR}_M$ and the value of country n's imports with the non-Eurozone countries as $V^{n,RoW}_M$. All values are denoted in current U.S. dollars and these values are calculated annually. I have chosen to drop the time t notation to simplify the exposition.

I estimate the value of each country's imports from non-Eurozone countries that are invoiced in currency c is, $V^{n,RoW}_{M,c}$ as

$$S^n_{M,c} = \left(\frac{V^{n,RoW}_{M,c}}{V^{n,RoW}_M}\right)\left(\frac{V^{n,RoW}_M}{V^n_M}\right) + \left(\frac{V^{n,EUR}_{M,c}}{V^{n,EUR}_M}\right)\left(\frac{V^{n,EUR}_M}{V^n_M}\right). \tag{33}$$

Where $\left(V^{n,RoW}_{M,c}/V^{n,RoW}_M\right)$ is the share of country n's imports from non-Eurozone countries that are invoiced in currency c and $\left(V^{n,RoW}_M/V^n_M\right)$ is the share of country n's total imports that come from non-Eurozone countries. Analogously, $\left(V^{n,EUR}_{M,c}/V^{n,EUR}_M\right)$ is the share of country n's imports from Eurozone countries that are invoiced in currency c and $\left(V^{n,EUR}_M/V^n_M\right)$ is the share of country n's total imports that come from Eurozone countries.

I use equation (33) to solve for the share of country n's imports from non-Eurozone countries that are invoiced in currency c, $\left(V^{n,RoW}_{M,c}/V^{n,RoW}_M\right)$. I observe the left-hand side, and I assume that for all countries inside the Eurozone, all intra-Eurozone trade is conducted in Euros. I need to estimate the share of each country's total imports that originate from other Eurozone countries. To do so, I use bilateral bilateral **goods** trade data from the OECD Library. I use "Bilateral Trade in Goods by Industry and End-use (BTDIxE), ISIC Rev. 4" data. See Appendix C.5 for additional details.

For each Eurozone country and year, I calculate the share of that country's goods imports originating from other Eurozone countries. I do not have bilateral services trade data. Hence, I assume that the share of total imports originating from Eurozone countries is equal to the share of total goods imports originating from Eurozone countries.

These assumptions allow me to estimate $\left(V_{M,c}^{n,RoW}/V_{M}^{n,RoW}\right)$ for each country n. For a small number of observations this value is negative, which means I over-estimated the share of that country's total imports from the Eurozone that were invoiced in Euros. If $\left(V_{M,c}^{n,RoW}/V_{M}^{n,RoW}\right) < 0$, I set it to zero.

Next, I estimate the share of the Eurozone's imports invoiced in currency c by taking a weighted average of $\left(V_{M,c}^{n,RoW}/V_{M}^{n,RoW}\right)$ over the Eurozone countries,

$$\left(\frac{V_{M,c}^{EUR,RoW}}{V_{M}^{EUR,RoW}}\right) = \frac{\sum_n V_{M}^{n,RoW}\left(V_{M,c}^{n,RoW}/V_{M}^{n,RoW}\right)}{\sum_n V_{M}^{n,RoW}}.$$

where $V_{M}^{n,RoW}$ is still the value of country n's imports that originate from non-Eurozone countries. I estimate $V_{M}^{n,RoW}$ using bilateral goods import data by multiplying the share of country n's goods imports from non-Eurozone countries with the total value of country n's imports. I use import data from the OECD Library. See Appendix C.7 for more details.

Thus, I estimate the share of the Eurozone's imports from the rest of the world that are invoiced in currency c. I repeat the same calculations for exports.

C.3 Spot and Forward Rates

I use data for dollar-based spot and forward exchange rates from Datastream to construct nominal interest rates for each country. I use the World Markets PLC/ Reuters (WM/R) data provider within Datastream for my spot and forward exchange rate data. Identifiers for the forward rate data usually follow the form *USisomm*, where the U.S. dollar is the base currency, "iso" is the other currency's ISO code, and "mm" is a two letter identifier for the maturity of the forward rate. The identifiers for the three month, six month, one year and two year forward rates are

“3F”, “6F”, “1Y” and “2Y”, respectively. The identifiers for the spot rate data are of the form *cccccc*$, which is a six digit currency identifier followed by a dollar sign.

C.4 Industrial Production Data

I use the OECD index for Industrial Production, which is part of their Key Economic Indicators (KEI) series. The data are available at the monthly level. The indicators are prepared by national statistical agencies. Measures of industrial production typically include output from manufacturing, mining, electricity, gas and water industries.

C.5 Bilateral Trade Data

I use bilateral trade data to aggregate currency invoicing data, as well as other measure trade intensity, for the Eurozone countries. I use data from the “Bilateral Trade in Goods by Industry and End-use (BTDIxE), ISIC Rev. 4” database in the OECD Statistics Library. I use data from 1999 to 2014 for all countries within the Eurozone. Unforturnately, these data only cover goods trade and I would ideally like to observe bilateral trade in both goods and services. These data are mostly drawn from the UN Comtrade database. Their documentation is located:

http://www.oecd.org/trade/bilateraltradeingoodsbyindustryandend-usecategory.htm

C.6 Exchange Rate Peg Data

I use exchange rate peg data using classification mechanism from Shambaugh (2004). The data are available from Shambaugh’s website and define whether or not countries have fixed exchange rates from 1960 to 2014. The data are available at the annual frequency. Among other variables, the data classify whether countries engage in hard exchange rate pegs or soft exchange rate pegs.

C.7 Imports, Exports and GDP

I construct measures of the import share of GDP and export share of GDP for each country in my sample. I use annual import, export, and GDP data from the Penn World Tables version 9. I use the following series: cgdpo for GDP, csh_x for the export share of GDP, and csh_m for the import share of GDP.

For each country in the Eurozone, I estimate the share of the country's imports (exports) with non-Eurozone countries using bilateral goods trade data. Implicitly, I assume that the pattern of each country's trade in goods and services matches that country's pattern of trade in goods alone. These calculations are analogous to those for the currency invoicing data, described in Appendix C.2.

For each country, I calculate the share of its imports origination from each country, using bilateral trade data from the OECD. Next, I multiply these shares by the total value of goods and services imports in each country, from the Penn World Tables data, to estimate the value of each country's total imports from each country. Using these data, I calculate the total value of imports into the Eurozone originating from non-Eurozone countries. I normalize the Eurozone's imports by the total GDP of all countries within the Eurozone in each year. Finally, I average the annual data over all years in the sample period (1999 - 2014). I perform the analogous calculations for exports.

C.8 Trade Costs

I use measures of trade costs from Mayer and Zignago (2011). The data are located in the GeoDist database on the CEPII website:

http://www.cepii.fr/CEPII/en/bdd_modele/presentation.asp?id=6

I use the following variables from this database: *distw* and *contig*. I use Germany for the geographic region for the Euro. See the documentation in Mayer and Zignago (2011) for additional details.

C.9 Price Stickiness Data

I download the Rauch (1999) classification of goods from the author's website:

http://econweb.ucsd.edu/ jrauch/rauch_classification.html

The classification distinguishes between traded goods that are traded on an organized exchange, reference priced or differentiated products. The classification is provided at the 4-digit SITC rev. 2 level.

I merge the Rauch classification with UN Comtrade data. From the UN Comtrade database, I download import and export data for each country in my sample for each year in my sample period (1999 - 2014) at the 4-digit SITC rev. 2 level. For each country, I calculate the value of each country's imports classified as differentiated products and normalize by the country's total imports. The "Diff. Share" variable is the average share of each country's imports classified as differentiated products, where I take the average across years in the sample (1999 - 2014). For the Eurozone, I first calculate the share of all Eurozone imports from outside of the Eurozone that are differentiated products for each year in the sample, and then I average across years.

C.10 Foreign Monetary Policy Rates

I use foreign central bank policy rates from Global Financial Data. The data are provided by the Central Bank Interest Rate series. The data are found under the ticker code "IDcccD", where "ccc" represents a three digit country code. The data are provided at the daily frequency. I observe central bank policy rate data for the following countries: Australia, Canada, Switzerland, Cyprs, Denmark, Estonia, the United Kingdom, Greece, Iceland, Israel, Japan, Korea, Lithuania, Latvia, Malta, Norway, Slovenia, Sweden and the Eurozone.