A Study of the Long-Run Substitutability Between Men and Women

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Abstract

Using a variable elasticity of substitution (VES) framework, this study estimates the long-run elasticity of substitution between US male and female workers, specifically, the slope of the inverse demand curve for male workers relative to female workers. Our 2SLS approach exploits possible exogenous sources of change in state employment induced by national employment growth. We find that the long-run elasticity of substitution between male and female workers is close to 1.7 and show that this estimate is robust across a wide range of model specifications. Using this estimated elasticity of substitution parameter, we find that approximately 7% of the fall in the gender wage gap can be explained by a steep increase in the relative female labor supply during the period 1980-2014.

Theoretical Framework

Let us consider the following aggregate production function with only two inputs, male (N_m) and female (N_f) workers:

$$Q_{st} = F\Big(N_{mst}, N_{fst}\Big) \tag{1}$$

where N_{mst} and N_{fst} are the number of employed male and female workers in state s at period t. Suppose $F(\cdot)$ follows the CES specification; then equation (1) can be written as

$$Q_{st} = \Big(\theta_{st}\big(a_{st}N_{mst}\big)^{\kappa} + (1-\theta_{st})\big(b_{st}N_{fst}\big)^{\kappa}\Big)^{1/\kappa} \tag{2}$$

where a_{st} and b_{st} represent gender-augmented technological change, θ_{st} is a time-varying technology parameter, and κ is a time-invariant production parameter. The elasticity of substitution between male and female workers σ for the CES production function is given by

$$\sigma = \frac{1}{1-\kappa} \tag{3}$$

We assume that male and female workers are paid according to their marginal product, and this assumption leads to the following log-linear relationship between male-female wages and labor supply ratios:

$$\ln\left(\frac{\mathrm{w}_{mst}}{\mathrm{w}_{fst}}\right) = \frac{1}{\sigma}\left[D_{st} - \ln\left(\frac{N_{mst}}{N_{fst}}\right)\right] \tag{4}$$

When $F(\cdot)$ follows the VES specification, we obtain from equation (1)

$$Q_{st} = \gamma N_{mst}^{\alpha(1-\delta\rho)}\Big[N_{fst} + (\rho-1)N_{mst}\Big]^{\alpha\delta\rho} \tag{5}$$

The elasticity of substitution parameter for the VES production is

$$\sigma = 1 + \left(\frac{\rho-1}{1-\delta\rho}\right)\frac{N_{mst}}{N_{fst}} = 1 + \psi \times \left(\frac{N_{mst}}{N_{fst}}\right) \tag{6}$$

where $\psi = (\rho-1)/(1-\delta\rho)$. Again, the assumption that in market equilibrium, workers are paid according to their marginal product of labor yields the following relationship between relative wages and relative labor supply ratios:

$$\begin{aligned}\frac{\mathrm{w}_{mst}}{\mathrm{w}_{fst}} &= \left(\frac{\rho-1}{1-\delta\rho}\right) + \left(\frac{1-\delta\rho}{\delta\rho}\right)\left(\frac{N_{fst}}{N_{mst}}\right)\\ &= \beta_0 + \beta_1\left(\frac{N_{fst}}{N_{mst}}\right)\end{aligned} \tag{7}$$

where $\beta_0 = (\rho-1)/1-\delta\rho$ and $\beta_1 = (1-\delta\rho)/\delta\rho$. Thus, substituting β_0 and β_1 in equation (6), we obtain,

$$\sigma = 1 + \left(\frac{\beta_0}{\beta_1}\right)\left(\frac{N_{mst}}{N_{fst}}\right) \tag{8}$$

A Comparison Between the CES and VES Frameworks

Consider the following relationship between male-female wage ratios and labor supply ratio

$$\left(\frac{\mathrm{w}_{mst}}{\mathrm{w}_{fst}}\right)^{\lambda} = a_0 + a_1\left(\frac{N_{fst}}{N_{mst}}\right)^{\lambda} \tag{9}$$

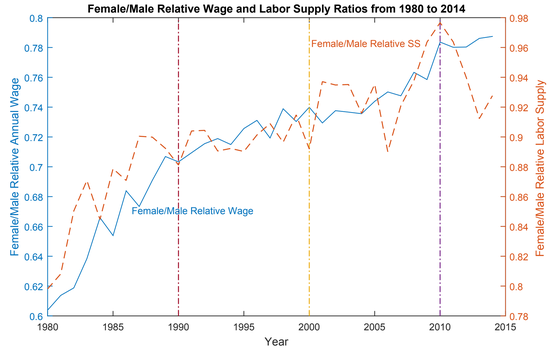
where λ is the transformation parameter. Equation (9) defines a whole class of production functions, two of which are CES and VES. As $\lambda \to 0$, (9) approaches (4), which is the CES cost minimization side relation; if $\lambda = 1$, (9) reduces to (7), which is the VES side relation. We can rewrite equation (9) as

$$\left(\frac{(\mathrm{w}_{mst}/\mathrm{w}_{fst})^{\lambda}-1}{\lambda}\right) = b_0 + b_1\left(\frac{(N_{fst}/N_{mst})^{\lambda}-1}{\lambda}\right) \tag{10}$$

By adding a disturbance term to equation (10), we can estimate λ in a non-linear least squares setting to choose the appropriate model specification between CES and VES.

Data and Results

Figure 1: Relative Female/Male Annual Wage and Labor Supply Ratios from 1980 - 2014



The CPS data consist of US-born women and men, aged 18-65, with positive annual earnings and hours worked in the preceding year and a nonzero sampling weight for the period 1980-2014. Thus, we have 51 observations in each year and a total 1,785 state-level observations for the period 1980-2014.

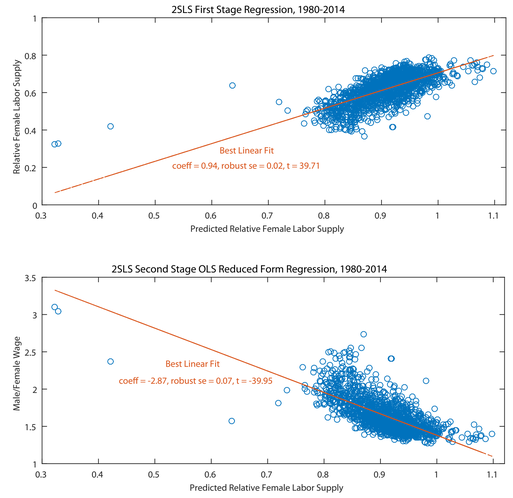
Table 1: Specification Test for the Selection of Production Function By Using the Box-Cox Transformation

	(1)			(2)		
	Estimate	Lower Bound	Upper Bound	Estimate	Lower Bound	Upper Bound
Box-Cox Transformation	0.943***	0.915	0.971	1.293***	1.191	1.394
Parameter (λ)	(0.014)			(0.052)		
Female/Male	-1.182***	-1.229	-1.135	-0.830***	-1.014	-0.646
Relative Supply	(0.024)			(0.094)		
Control Variables	No	No	No	Yes	Yes	Yes

Following the Katz and Murphy (1992) approach, we substitute the unobserved demand shifts by using a linear time trend (D_t) to estimate equation (7). Thus, our model specification is given by,

$$\frac{\mathrm{w}_{mst}}{\mathrm{w}_{fst}} = \beta_0 + \beta_1\left(\frac{N_{fst}}{N_{mst}}\right) + \beta_2 D_t + \beta_4 X_{st} + \eta_s + \varepsilon_{st} \tag{11}$$

Figure 2: Impact of Relative Female Labor Supply on Relative Wage Ratios: Plots of First Stage and Reduced Form Estimates



Our modified version of the Bartik (1991) instruments are the predicted change in employment ($\Delta\widehat{E}_{st}$) in state s at year t is given by

$$\Delta\widehat{E}_{st} = \sum_{k=1}^{K}\varphi_{skt_0-5} \times \vartheta_{skt_1} \tag{12}$$

where $\varphi_{sk(t_0-5)}$ is the employment share in industry k in state s at period t_0-5. This measure of predicted regional employment growth in period t can also work as an instrument because it is mainly determined by local labor market composition five years prior to t. Therefore, we expect it to be correlated with the long-run component of the relative female labor supply ratio but uncorrelated with contemporaneous unobserved local labor market conditions reflected in ε_{st}.

Table 2: Elasticity of Substitutions Between Men and Women from the VES Production Function, 1980-2014

Dependent Variable: Composition Adjusted Male/Female Relative Annual Earnings Ratio

	(1)		(2)		(3)		(4)		(5)	
	OLS	2SLS	OLS	2SLS	OLS	2SLS	OLS	2SLS	OLS	2SLS
Female/Male Lab Supply	-2.282*** (0.200)	-5.149*** (0.271)	-0.497*** (0.158)	-3.314*** (0.450)	-0.348*** (0.083)	-1.480*** (0.231)	-0.338*** (0.103)	-1.158*** (0.250)	-0.335*** (0.094)	-1.171*** (0.239)
State FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Demographic Controls	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Constant	3.853*** (0.164)	7.082*** (0.304)	2.749*** (0.127)	5.229*** (0.461)	1.277*** (0.124)	1.914*** (0.169)	0.482 (1.241)	2.806*** (0.941)	0.631 (1.267)	2.896*** (0.906)
Elasticity of Substitution	0.859	0.514	5.083	0.737	3.041	0.424	0.570	1.669	1.072	1.724
Sargan Test Stat (p-value)		31.117 (0.000)		36.986 (0.000)		25.739 (0.018)		17.899 (0.161)		13.517 (0.409)
Adjusted R^2	0.402	0.010	0.701	0.453	0.812	0.755	0.825	0.801	0.824	0.799
Observations	1,785	1,778	1,785	1,778	1,596	1,589	1,647	1,640	1,596	1,589

Figure 3: Scatter plot of Relative Female/Male Annual Wage and Labor Supply Ratios and the Marginal Effects of Relative Labor Supply on Relative Wage

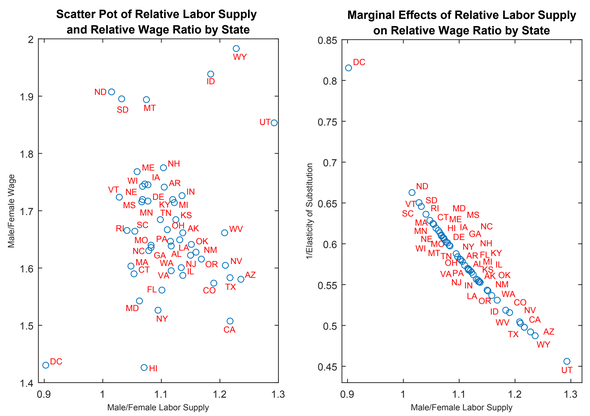


Table 3: Elasticity of Substitutions Between Relatively Younger (Age 18-35) and Older (Age 36-65) Men and Women

Dependent Variable: Composition Adjusted Male/Female Relative Annual Earnings Ratio

	Age Group: 18-35			Age Group: 36-65		
	(1)	(2)	(3)	(1)	(2)	(3)
Female/Male Relative Labor Supply	-1.366*** (0.361)	-1.286*** (0.379)	-1.241*** (0.375)	-1.113*** (0.232)	-0.913*** (0.268)	-0.779*** (0.233)
Time	-0.010*** (0.001)	0.004 (0.004)	0.005 (0.004)	-0.012*** (0.001)	-0.005** (0.002)	-0.004 (0.003)
Poverty	0.001 (0.003)		0.001 (0.003)	0.000 (0.002)		-0.000 (0.002)
Constant	2.244*** (0.268)	5.848*** (1.765)	5.895*** (1.832)	2.040*** (0.191)	1.836 (1.148)	1.239 (1.012)
Elasticity of Substitution	0.849	4.122	4.350	1.043	1.242	0.774
Adjusted R^2	0.428	0.477	0.490	0.488	0.572	0.602
No of Observations	1,518	1,568	1,518	1,467	1,515	1,467

Conclusion

- The main contribution of this study is to provide an estimate of the long-run elasticity of substitution between male and female workers for the period 1980-2014 by exploiting the regional variations in industry-level changes in employment induced by national employment growth.
- The 2SLS point estimate varies between 1.67 and 1.72, and the estimate is robust to a wide range of model specifications.
- I show that the wage gap fell approximately 7% during the period 1980-2014 due to a 12% increase in the relative female labor supply.

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