A Study of the Long-Run Substitutability Between Men and Women

Pallab Ghosh. Department of Economics, University of Oklahoma.

Abstract

Using a variable elasticity of substitution (VES) framework, this study examines the long-run elasticity of substitution between US male and female workers, specifically the slope of the inverse demand curve for male workers relative to female workers. Our SLS estimates exploit possible exogenous sources of change in state employment induced by national employment growth. We find that the long-run elasticity of substitution between male and female workers is close to 1.72 and show that this estimate is robust across a wide range of model specifications. This estimated elasticity of substitution parameter is consistent with an increase in the relative female labor supply during the period 1980-2014.

Theoretical Framework

Let us consider the following aggregate production function with only two inputs, male (\(N_{mst}\)) and female (\(N_{fst}\)) workers:

\[
Q = F(N_{mst}, N_{fst})
\]

(1)

where \(N_{mst}\) and \(N_{fst}\) are the number of employed male and female workers in state \(s\) at period \(t\). Suppose \(F()\) follows the CES specification, then equation (1) can be written as:

\[
Q = \left[ \frac{1}{\sigma} (\alpha N_{mst} + \beta N_{fst})^\sigma \right]^{1/\sigma}
\]

(2)

where \(\alpha\) and \(\beta\) represent gender-augmented technological change, \(\theta\) is a time-varying technology parameter, and \(\sigma\) is a time-invariant production parameter. The elasticity of substitution between male and female workers \(\sigma\) for the CES production function is given by

\[
\sigma = \frac{1}{1 - \alpha} + \frac{\alpha}{1 - \beta}
\]

(3)

We assume that male and female workers are paid according to their marginal product, and this assumption leads to the following log-linear relationship between male-female wages and labor supply ratio:

\[
\ln \left( \frac{w_{mst}}{w_{fst}} \right) = \frac{\beta}{\lambda} N_{mst} - \frac{(1 - \beta)\lambda}{\lambda - 1} N_{fst}
\]

(4)

When \(F()\) follows the VES specification, we obtain from equation (1):

\[
Q = \gamma N_{mst}^{\psi} N_{fst}^{1-\psi}(\theta N_{mst})^{\lambda}
\]

(5)

The elasticity of substitution parameter for the VES production function is:

\[
\sigma = 1 + \left( \frac{\psi - \lambda}{\lambda - 1} \right) N_{mst} + 1 + \psi \left( \frac{\lambda}{\lambda - 1} \right) N_{fst}
\]

(6)

where \(\psi = (\sigma - 1)/(1 - \beta)\). Again, the assumption that in market equilibrium, workers are paid according to their marginal product of labor yields the following relationship between wage rates and labor supply ratio:

\[
\frac{w_{mst}}{w_{fst}} = \left( \frac{\beta - 1}{\beta \gamma} \right) \left( \frac{N_{mst}}{1 - \beta} \right)^{\lambda} \left( \frac{N_{fst}}{1 - \beta} \right)^{1-\lambda}
\]

(7)

Following the Katz and Murphy (1992) approach, we substitute the observed demand shifts by using a linear time trend (\(\Delta t\)) to estimate equation (7). Thus, our model specification is given by:

\[
\frac{w_{mst}}{w_{fst}} = \beta_0 + \beta_1 N_{mst} + \beta_2 \Delta t + \beta_3 N_{fst} + \eta_1 + \epsilon_{st}
\]

(8)

A Comparison Between the CES and VES Frameworks

Consider the following relationship between male-female wages and labor supply ratio:

\[
\left( \frac{w_{mst}}{w_{fst}} \right)^{1/\lambda} = \delta_0 + \delta_1 N_{mst}^{1/\lambda} + \delta_2 N_{fst}^{1 - 1/\lambda} + \eta_2 + \epsilon_{st}
\]

(9)

where \(\lambda\) is the transformation parameter. Equation (9) defines a whole class of production functions, two of which are CES and VES. As \(\lambda \rightarrow 0\), equation (9) approaches (4), which is the CES cost minimization side relation, if \(\lambda = 1\), equation (7) reduces to (9), which is the VES side relation. We can rewrite equation (9) as:

\[
\left( \frac{w_{mst}}{w_{fst}} \right)^{1/\lambda} = \delta_0 + \delta_1 N_{mst}^{1/\lambda} + \delta_2 N_{fst}^{1 - 1/\lambda} + \eta_2 + \epsilon_{st}
\]

(10)

By adding a disturbance term to equation (10), we can estimate \(\lambda\) in a non-linear least squares setting to choose the appropriate model specification between CES and VES.

Data and Results

Figure 1: Relative Female/Male Annual Wage and Labor Supply Ratios from 1980 - 2014

The CPS data consist of US-born women and men aged 16-65, with positive annual earnings and hours worked in the preceding year and a normal sampling weight for the period 1980-2014. Thus, we have 51 observations in each year and a total 1,785 state-level observations.

Using this estimated elasticity of substitution parameter, we find that the long-run elasticity of substitutability between US male and female workers is close to 1.72 and show that this estimate is robust across a wide range of model specifications. This estimated elasticity of substitution parameter is consistent with an increase in the relative female labor supply during the period 1980-2014.

Conclusion

The main contribution of this study is to provide an estimate of the long-run elasticity of substitution between male and female workers for the period 1980-2014 by exploiting the region-specific variations in industry-level changes in employment induced by national employment growth.

The SLS point estimate varies between 1.67 and 1.72, and the estimate is robust to a wide range of model specifications. I show that the wage gap fell approximately 7% during the period 1980-2014 due to a 12% increase in the relative female labor supply.