The Predictive Power of Tail Risk Premia on Individual Stock Returns

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The impact of tail events on returns is well documented at the aggregate market level, but not so much is known about its impact at the individual stock level. This paper introduces a novel, option-free, methodology to directly calculate the tail risk premium for individual stocks, and then examines the characteristics of this premium in the cross section of stock returns. The existence of a premium for bearing negative tail risk is significantly associated with negative returns up to one month in the future. The same cannot be said for the premium for bearing positive tail risk, which seems to have no predictive power at the monthly level. Further, the larger in magnitude is the premium for bearing negative tail risk, the greater and longer lasting is its impact on expected future returns.

Keywords: tail risk, asymmetry, cross-section of stock returns, return prediction, empirical asset pricing. JEL Classification: G11, G12, G17

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I. Introduction

Compensation for extreme, tail event risk is formally referred to in the academic finance literature as a "tail risk premium." Bollerslev, Todorov and Xu (2015) shows that a majority of the predictability in the variance risk premium is attributed to this premium for bearing jump tail risk, and that, specifically, it is negative tail risk and not positive tail risk that seems to be priced[1]. The impact of tail events on returns is well documented at the aggregate market level, but not so much is known about its impact at the individual stock level. One reason why is that out of the money call and put options are required to determine the tail risk in the risk neutral probability space. Although out of the money options are prevalent for an index such as the S&P500, they either do not exist or are illiquid for most stocks. For this reason, to date no paper has *directly* examined both the impact of tail risk and its premium on the cross section of individual stock returns. Given that the return distribution for individual stocks will likely exhibit a greater proclivity for extreme events than the return distribution for a diversified market portfolio (where extreme negative events in some securities might be tempered by extreme positive events in others) one would expect that tail risk should play a more prominent role in the returns for individual stocks than it would for a market portfolio. Consequently, a careful study of tail risk premia for individual stocks may yield new and heretofore unseen insights into their predictive power for future returns.

Kelly and Jiang (2014) is the first to examine tail risk in the cross section of individual stock returns. This important paper employs an aggregate measure of time-varying tail risk that

[1] Variance risk premium of market index possesses return predictive power was first documented in Bollerslev, Tauchen and Zhou (2009).

relies on panel estimation from the cross section of stock returns. It then measures a stock's sensitivity to this measure of tail risk by sorting portfolios into quintiles based on tail beta-exposure, and documents that the lowest tail beta quintile is associated with the lowest future returns, while the highest tail beta quintile is associated with the highest future returns. Although Kelly and Jiang provides strong evidence that tail risk is priced in individual stocks, it does not directly calculate the tail risk premium nor does it examine any asymmetry in the way positive and negative tail risk premia affect future returns.

The current paper differentiates itself from Kelly and Jiang (2014) in three critical ways. First, rather than using an aggregate measure of tail risk and indirectly examining the sensitivity of a stock's return to this aggregate measure, the current paper directly calculates the tail risk premium for individual stocks and examines how this premium varies across the cross-section of stock returns. Specifically, the paper introduces a novel, non-parametric approach to directly determine the tail risk premium. The approach avoids the need for the use of liquid out of the money stock options (which don't exist for most stocks). The second contribution is that this new approach allows stocks to be sorted by their exposure to tail risk, so that the impact of positive and negative tail risk premia on future returns can be examined separately. Stocks with exposure to negative tail risk require a tail risk premium that is positive (investors demand a higher return today than otherwise expected for bearing negative tail risk) while those with positive tail risk require a tail risk premium that is negative (investors are willing to accept a lower return today when there is a chance for extreme positive events). Third, this paper documents that almost all of the extreme jumps are concentrated in the first and tenth deciles; consequently, an analysis of deciles, and even percentiles, rather than the quintiles examined in prior studies, is necessary if researchers are to better understand how extreme jump tail risk is priced.

The results in this paper provide evidence on the differential pricing of information related to negative and positive tail risk. Bollerslev, Todorov and Xu (2015) and Kelly and Jiang (2014) find evidence of pricing for negative tail risk, but neither fully examines the extent to which positive tail risk is priced. The current paper documents the existence of a premium for bearing negative tail risk today is associated with significantly lower future monthly returns, but that the existence of a premium for positive tail risk does not have statistically significant predictive power in the cross section of individual stock returns. In addition, the current paper presents evidence that it is not only the sign of the tail risk premium that matters in predicting future returns, but also its magnitude. The larger and more positive the current tail risk premium (that is, the greater the concerns about a big negative jump), the more negative and persistent will be the association with future returns.

The paper's empirical methodology controls for several explanations previously offered in the literature for the existence and pricing of tail risk including momentum (Lehman, 1990 and Jegadeesh, 1993), lottery effects (Barberis and Huang, 2008 and Bali, Cakci and Whitelaw 2011), idiosyncratic volatility (Ang, Hoderick, Xing, and Zhang, 2006), illiquidity (Amihud, 2002), market beta (Sholes and Williams, 1977 and Dimson 1979), maximum and minimum monthly return (Bali, Cakici and Whitelaw, 2011). The predictive power of the premium for bearing negative tail risk on future returns survives the inclusion of these control variables.

This paper is organized as follows: Section II contains the literature review; Section III demonstrates individual stock level tail risk premium estimation and the data; Section IV contains the tail risk premium cross-sectional pricing characteristics and cross-sectional return tests; Section V includes robustness checks; and Section VI concludes the paper.

II. Literature Review

In addition to the papers mentioned in the introduction, there are several recent papers that examine the pricing of downside risk which are related to the current paper. Ang, Chen and Xing (2006) finds that stocks that covary strongly with the market during periods of market decline tend to have higher average returns than other stocks. Investors are downside risk averse and therefore require a premium to hold these assets. Bali, Cakici and Whitelaw (2014) introduces a hybrid tail covariance risk measure that measures stock return tail covariance risk. The measure is based on the basic form of lower partial moments. The paper documents a significant positive premium for bearing negative tail risk captured in the cross section.

This paper is also related to the literature on crash risk. Kelly and Jiang (2014) is among the first papers that examine extreme crash risk on stock returns. The paper find that stocks with high loadings on market tail risk earn higher abnormal returns. Chabi-Yo, Ruenzi and Weigert (2015) finds that investors are crash-averse; that is, they receive positive compensation for holding crash-sensitive stocks through the measure of "lower tail dependence" from individual stock price distributions. The findings in these papers are consistent with the downside risk literature (Ang, Chen and Xing (2006) and Bali, Cakici and Whitelaw (2014)) that investors are downside risk averse and require a positive premium for holding the crash risk sensitive stocks.

Bali, Cakici and Whitelaw (2014) constructs a firm-specific tail risk measure based on lower partial moments of stock returns and finds that it negatively predicts future stock returns. Almeida et al. (2017) adopts a risk-neutral excess expected shortfall approach to construct a nonparametric tail risk measure. The paper finds that the risk-neutral tail risk measure possesses negative predictive power for intermediate horizon stock returns. Lu and Murray (2017) constructs a proxy for bear-market risk and finds it to be negatively priced; that is, stocks with a high sensitivity to bear-market risk are found to underperform their low-sensitivity counterparts.

Our paper is also related to the asset pricing literature on higher moments. Traditional finance theory assumes a normal distribution of asset returns, for which mean and variance together are sufficient to characterize the entire return distribution. The capital asset pricing model (Sharpe (1964), Lintner (1965) and Mossin (1966)) predicts that market volatility is a determinant of the market equity premium. Contrary to this notion, Ang, et al. (2006) examines whether aggregate volatility innovation is priced in cross-section of stock returns, and concludes that high sensitivity stocks have subsequent lower average returns. Given this controversy, it is natural to ask is whether other return distributional characteristics are also priced in the cross section. Chang, Christofferson and Jacobs (2013) shows that the cross-section of stock returns has substantial exposure to higher moments. Cremers, Halling and Weinbaum (2015) finds that although both jumps and volatility are priced in cross section, jumps seem to have larger impact on returns than does volatility. Bali, Cakici and Whitelaw (2011) finds that stocks with maximum returns have a significant negatively return in the following month. These pricing findings are consistent with the erroneous probability weighting of investors as modeled in Barberis and Huang (2008) and optimal belief framework of economic agents modeled in Brunnermeier, Gollier and Parker (2007).

III. Calculation of the Tail Risk Premium in the Cross Section of Individual Stock Returns

A. Methodology

This section discusses the construction of the tail risk premium associated with jumps in returns for individual stocks. The methodology is an innovation on the well-established notion - Bollerslev, Todorov and Xu (2015), Carr and Wu (2009) and others - that the jump tail risk premium can be calculated as the difference between the expectation of the tail variation in the physical probability space ($\mathbb{P}$-space) and its counterpart in the risk-neutral probability space ($\mathbb{Q}$-space).

To this end, we define the infinite-order polynomial variation of log returns, which captures not only the second-order (quadratic) variation (see Carr and Wu, 2009), but also the higher-order (third-order and up) variations, which Jiang and Oomen (2008) has shown to be associated with jumps in stock returns. We denote the simple return $R_{t+1} = \frac{S_{t+1}-S_t}{S_t}$ and logarithmic return $r_{t+1} = \ln\left(\frac{S_{t+1}}{S_t}\right)$ over a period from t to $t+1$. Formally, based on Merton (1976)'s jump diffusion process, the realized infinite-order *polynomial variation* ($\mathbb{PV}$) for individual asset i at time $t+1$ can be expressed as follows:

$$\begin{aligned}
\mathbb{PV}_{i,[t,t+1]} = 2\left(R_{i,t+1} - r_{i,t+1}\right) \quad & \underbrace{= 2 \times \left(\frac{1}{2!} \times r_{i,t+1}^2 + \frac{1}{3!} \times r_{i,t+1}^3 + \cdots + \frac{1}{n!} \times r_{i,t+1}^n\right)}_{Polynomial\ form\ of\ \log return\ variations} \\
& = \int_t^{t+1} \sigma_{i,t}^2 dt \ + \ \sum\nolimits_{n=2}^{\infty} \frac{2}{n!} \int_t^{t+1} \int_{\mathbb{R}^0} x_i^n \, \mu(dx_i, dt) \\
& = \ \mathbb{CV}_{i,[t,t+1]} \ + \ \mathbb{JPV}_{i,[t,t+1]}. \qquad (1)
\end{aligned}$$

where σ is the volatility. $\mu(dx_i, dt)$ is the Poisson random measure for the compound Poisson process with compensator equal to $\lambda \frac{1}{\sqrt{2\pi\sigma_J^2}} e^{-\frac{1}{2}(x-\alpha)^2}$, with λ as the jump intensity. $\mathbb{CV}$ is the integral of the continuously instantaneous variance (often referred to as the *integrated volatility*), and $\mathbb{JPV}$ represents the realized jump component of the infinite-order *polynomial variation.* Analogously, the second-order polynomial variation (the realized quadratic variance, denotes $\mathbb{QV}$) can be written by the following equation:

$$\mathbb{QV}_{i,[t,t+1]} = r_{i,t+1}^2 = \int_t^{t+1} \sigma_{i,t}^2 \, dt + \int_t^{t+1} \int_{\mathbb{R}^0} x_i^2 \mu(dx_i, dt) = \ \mathbb{CV}_{i,[t,t+1]} + \mathbb{JQV}_{i,[t,t+1]}. \quad (2)$$

By subtracting Equation 2 from Equation 1, we then have the realized tail-jump variation at time *t* such that

$$\begin{aligned}\mathbb{TV}_{i,[t,t+1]} &= 2\left(R_{i,t+1} - r_{i,t+1}\right) - r_{i,t+1}^2 = \mathbb{PV}_{i,[t,t+1]} - \mathbb{QV}_{i,[t,t+1]} \\ &= \sum_{n=3}^{\infty} \frac{2}{n!} \int_t^{t+1} \int_{\mathbb{R}^0} x_i^n \, \mu(dx_i, dt). \end{aligned} \tag{3}$$

Now that we have the unconditional realized tail-jump variation, we next present the conditional ex-ante estimation of the tail-jump variation and then will develop a proxy for the tail-risk premium.

Following Bollerslev, Tauchen and Zhou (2009), under the assumption that $\mathbb{TV}$ is a martingale[2], the $\mathbb{P}$-space expected tail-variation of returns at time t can be expressed as follows:

$$E_t^{\mathbb{P}}\left(\mathbb{TV}_{i,[t,t+1]}\right) = 2\left(R_{i,t} - r_{i,t}\right) - r_{i,t}^2. \tag{4a}$$

Then, the difference between expected $\mathbb{TV}$ in the $\mathbb{P}$-space and that in the risk-neutral $\mathbb{Q}$-space, $E_t^{\mathbb{P}}\left(\mathbb{TV}_{i,[t,t+1]}\right) - E_t^{\mathbb{Q}}\left(\mathbb{TV}_{i,[t,t+1]}\right)$, serves as a proxy for the tail-risk premium. The advantages of using $E_t^{\mathbb{P}}\left(\mathbb{TV}_{i,[t,t+1]}\right)$ as a tail risk measure in the physical space are threefold. First, it is non-parametric, i.e., it does not require the estimation of a cutoff value as in Kelly and Jiang (2014) or Bollerslev and Todorov (2011b). Second, it does not require the estimation of a jump compensator in order for the instantaneous arithmetic stock return to be a semi-martingale process as in Bollerslev and Todorov (2011a), Bollerslev and Todorov (2015), or Bollerslev, Todorov and Xu

[2] Under Merton (1976) jump diffusion model assumption, the compensated compound Poisson process $\mu(dx_i, dt)$, with compensator $\lambda \frac{1}{\sqrt{2\pi\sigma_J^2}} e^{-\frac{1}{2}(x-\alpha)^2}$, is a martingale process; consequently, $\mathbb{PV}_{i,[t,t+1]}$ is also a martingale. Todorov and Tauchen (2011) provides empirical evidence that the VIX index is a pure-jump process without a continuous component, which supports the notion that $\mathbb{PV}_{i,[t,t+1]}$ is a martingale process. (Du and Kapadia (2012) demonstrates that $\mathbb{PV}_{i,[t,t+1]}$ is the underlying process of the VIX index). Furthermore, it is widely accepted in both theory and practice that $\mathbb{QV}_{i,[t,t+1]}$ is a martingale process, consequently, the claim that $\mathbb{TV}_{i,[t,t+1]}$ is a martingale process has both theoretical and practical support.

(2015). Third, $E_t^{\mathbb{P}}(\mathbb{TV}_{i,[t,t+1]})$ only relies on stock price information and can be easily calculated using databases such as the WRDS CRSP database and the TAQ database. Thus, relative to the measures used in the afformentioned papers, our measure not only lessens estimation error, but also shortens the calculation time, and because it only relies on prices, it is broadly applicable to other asset classes. Once the $\mathbb{P}$-space tail risk measure is calculated, then one can examine the innovations in this measure

$$\Delta E_t^{\mathbb{P}}(\mathbb{TV}_{i,[t,t+1]}) = \Delta[2(R_{i,t} - r_{i,t}) - r_{i,t}^2] \qquad (4b)$$

In contrast to the $\mathbb{P}$-space tail variation measure which is easy to calculate, the corresponding calculation of Equation 4a in the $\mathbb{Q}$-space for individual stocks is much more problematical, since the necessary data is not readily available. If, instead of examining individual stocks, one were interested in calculating the $\mathbb{Q}$-space tail variation of the market as a whole, then the methodology would be relatively easy to implement. For example, Carr and Wu (2009) shows that the CBOE VIX index is a measure of moment combinations, and therefore a polynomial variation in the risk neutral probability space. Specifically, that paper argues that the VIX index measures the risk-neutral expectation of the polynomial variation process for the S&P 500 market index,[3]

$$E_t^{\mathbb{Q}}(\mathbb{PV}_{M,[t,t+1]}) = VIX_t^2 \qquad (5)$$

Once Equation 5 has been calculated, Du and Kapadia (2012) and Chow, Jiang and Li (2014) show that the $\mathbb{Q}$-space measure of tail variation for the market can be calculated as the difference

[3] Note that, Equation 5 clearly indicates that the literature-prevalent variance risk premium estimation methodology, which takes the difference between the VIX index and physical probability space quadratic variation is biased. Specifically, the VIX, because it includes higher order moments, undervalues (overvalues) volatility when the market return is expected to be negatively (positively) skewed.

between the square of the VIX and the centralized Bakshi, Kapadia, and Madan's (2003) volatility measure, V^C_{BKM} such that [4]

$$E^{\mathbb{Q}}_t\left(\mathbb{TV}_{M,[t,t+1]}\right) = VIX^2_t - V^C_{BKM,t} \tag{6}$$

It is important to note that the calculation of both the VIX and V^C_{BKM} rely on highly liquid out-of-the-money put and call options[5], which fortunately are prevalent on the S&P 500 index. Recent papers by Gao, Gao and Song (2018) and Gao, Lu and Song (2018) estimate tail risk based on the $\mathbb{Q}$-space tail variation measure (in Equation 6) for the market index and portfolio of assets where there exists liquid option trading. Unfortunately, these options often either do not even exist for individual stocks or, if they do exist, are not frequently traded, and thus an analogue of the aforementioned methodology to calculate the $\mathbb{Q}$-space tail variation for individual stocks is impossible to implement. Consequently, an alternative is required.

To this end, we propose a methodology for the estimation of the $\mathbb{Q}$-space tail variation that is based on the groundbreaking work on tracking portfolios presented in Breeden, Gibbons and Litzenberger (1989) and Lamont (2001). According to Lamont (2001), "A tracking portfolio for any variable *y* can be obtained as the fitted value of a regression of *y* on a set of base asset returns. The portfolio weights for the economic tracking portfolio for *y* are identical to the coefficients of an OLS regression." Ang, et al. (2006) applies the tracking portfolio technology and uses returns to capture innovations in the VIX index. We employ a modified version of Ang, et al. (2006); specifically, we use first-order difference in the $\mathbb{Q}$-space tail variation measures, rather than the

[4] $V^C_{BKM} = V_{BKM} - \mu^2_{BKM}$, where $\mu_{BKM} = \ln\left(\frac{K_0}{S_0}\right) + \left(\frac{F_0}{K_0} - 1\right) - e^{rT}[\int_{K_0}^{\infty} \frac{1}{K^2} C_T(K)dK + \int_0^{K_0} \frac{1}{K^2} P_T(K)dK]$ and $V_{BKM} = \ln^2\left(\frac{K_0}{S_0}\right) + 2\ln\left(\frac{K_0}{S_0}\right)\left(\frac{F_0}{K_0} - 1\right) + 2e^{rT}[\int_{K_0}^{\infty} \frac{[1-\ln(\frac{K}{S_0})]}{K^2} C_T(K)dK + \int_0^{K_0} \frac{[1+\ln(\frac{K}{S_0})]}{K^2} P_T(K)dK]\}$ as in Bakshi, Kapadia, and Madan (2003).

[5] Demeterfi, Derman, Michael and Zou (1999) present a methodology for estimating the $\mathbb{Q}$-space measure of implied volatility for individual securities that is based on the variance swap concept, which requires highly liquid out of the money put and call options.

raw measures themselves, in order to capture innovations in $\mathbb{P}$-space tail variation measures. Accordingly, we estimate the following ordinary least squares regression for each stock in each month to obtain our portfolio weights, β_i[6]

$$\Delta[2(R_{i,t} - r_{i,t}) - r_{i,t}^2] = \alpha_i + \beta_i \cdot \Delta(VIX_{t-22}^2 - V_{BKM,t-22}^C) + \varepsilon_{i,t} \tag{7}$$

Where $VIX_{t-22}^2 - V_{BKM,t-22}^C$ represents the tail variation in the $\mathbb{Q}$-space, as delineated in Equation 6, and $2(R_{i,t} - r_{i,t}) - r_{i,t}^2$ represents the tail variation in the $\mathbb{P}$-space, as shown in Equation 4a. Note that we follow the precedent set by Bekaert and Hoerova (2014) which estimates $\mathbb{P}$-spaced conditional realized variation utilizing a 22-day lag. Their approach is based on the notion that options-based $\mathbb{Q}$-spaced measures, such as the VIX, are forward-looking, and thus there is a time lag error of one month (22 trading days) that must be corrected.

Once the β_i coefficients have been obtained, then $\beta_i \cdot \Delta(VIX_{t-22}^2 - V_{BKM,t-22}^C)$ represents the $\mathbb{Q}$-space tracking portfolio that mimics innovations in tail variation that occur in the $\mathbb{P}$-space. Formally,

$$\Delta E_t^{\mathbb{Q}}(\mathbb{TV}_{i,[t,t+1]}) = \beta_i \cdot \Delta(VIX_{t-22}^2 - V_{BKM,t-22}^C) \tag{8}$$

Now that the daily innovations in the $\mathbb{P}$-space and $\mathbb{Q}$-space tail variation measures (Equations 4b and 8, respectively) have been obtained, then the daily tail risk premium for any asset can be estimated by taking their difference

$$TRP_{i,[t,t+1]}^{daily} = \Delta E_t^{\mathbb{P}}(\mathbb{TV}_{i,[t,t+1]}) - \Delta E_t^{\mathbb{Q}}(\mathbb{TV}_{i,[t,t+1]}) \tag{9}$$

Since there are 22 trading days in a month, the corresponding monthly tail risk premium for each individual stock can be estimated as

$$TRP_i^{Monthly} = 22 \cdot TRP_{i,[t,t+1]}^{daily} \tag{10}$$

[6] Stocks must have at least 17 observations in any given month to be included in that month's regression.

B. Data

We run the baseline regression model in Equation 7 for all common stocks on AMEX, NASDAQ, and NYSE, with more than 17 daily observations in any given month. Daily stock returns come from the WRDS CRSP dataset, over the sample period from January 1990 to September 2014. S&P index option data are obtained from IVolatility.com, which provides end-of-day and high frequency option data on major stock market indices across countries.

[Insert Table 1 Here]

Table 1 reports the summary statistics for portfolios sorted into deciles by the tail risk premium. Definitions for all the variables can be found in the Appendix. Panel A presents the decile portfolio firm-specific characteristics sorted by the tail risk premium. Firms with a higher tail risk premium tend to have lower lagged 1-month returns (short-term return reversal effect). Firms that fall into the extreme first and 10th decile also tend to be smaller firms that have higher market betas, higher idiosyncratic volatility, more illiquidity, higher maximum monthly returns, lower minimum monthly returns, lower trading volumes and lower prices.

To examine the correlation structure among the explanatory variables, we report in percentage form Pearson correlation coefficients of the variables in Table 2.

[Insert Table 2 Here]

Idiosyncratic volatility (Ang, et al. (2006)) is negatively correlated with size (correlation coefficient of -49.70%), which is consistent with the findings in Fu (2009). Moreover, idiosyncratic volatility is also correlated with maximum and minimum monthly returns (Bali, Cakici and Whitelaw (2011)), with correlation coefficients of 89.72% and 80.77%, respectively). Maximum and minimum monthly returns (Bali, Cakici and Whitelaw (2011)) are correlated with size, but to a much lesser extent (-37.24% and 39.00%, respectively).

IV. Predictive Power of the Tail Risk Premium on Future Returns

A. Portfolios Sorted by the Tail Risk Premium

To investigate the predictive power of the tail risk premium in the cross section, we first calculate the tail risk premium for each of the stocks in our sample, and then sort the stocks into decile portfolios by the magnitude of their monthly tail risk premium. We next calculate the one month forward buy and hold returns for each decile portfolio. We term these returns as the $1/0/1$ (sort in one month, examine the one-month return for the following month) return. The results are reported in Panel A of Table 3.

[Insert Table 3 Here]

The lowest (decile 1) tail risk premium portfolio earns the highest return of 2.02% in the following month, while the highest (decile 10) portfolio earns the lowest return of 0.27%. The difference between the lowest and highest quintile portfolio is 1.75% monthly, and has a t-statistic of -11.26. After a Newey-West (1987) adjustment for heteroskedasticity and autocorrelation, the t-statistic is still strongly significant with a value of -7.30.

We next examine the length of time it takes for the market to correct this pricing error, by comparing the results for the to $1/0/1$ (sort in one month, examine the one-month return for the following month) portfolio strategy discussed in the previous paragraph with $1/1/1$ and $1/2/1$ (sort in one month, examine the one-month return starting two months from now, and three months from now, respectively) portfolio strategies.

These results are reported in Panels B and C of Table 3. The existence of a tail risk premium at time *t* possesses virtually no impact on future returns moving from the second-next month into the future. This suggests that the adjustment period for market perception of tail risk seems to be somewhere between one month and two months, after which the market fully incorporates information about tail risk into the price.

B. Cross-Sectional Return Test for the Predictive Power of the Tail Risk Premium

The above evidence suggests that tail risk is priced at the individual stock level. Consistent with prior studies, we perform a more thorough firm-level cross-sectional returns and examine whether the predictive power of the tail risk premium remains. Specifically, we estimate the following monthly regression:

$$R_{i,t+1} = \gamma_{0,t+1} + \gamma_{1,t+1} \times TRP_{i,t}^{Monthly} + \phi_{t+1}' \times Z_{i,t} + \varepsilon_{i,t+1} \tag{11}$$

where $R_{i,t+1}$ is the monthly stock return for stock i in month $t+1$. $TRP_{i,t}^{Monthly}$ is the individual stock tail risk premium, delineated by Equation 10. $Z_{i,t}$ represents a vector of characteristics and controls for firm i at the end of month t such as size, book-to-market ratio and market beta. Controls are also provided for illiquidity following Amihud (2002), idiosyncratic volatility following Ang, et al. (2006), lagged 1-month return for short-term return reversal effect following Jegadeesh (1990) and Lehmann (1990), lagged 12-month return accounting for the momentum effect, and maximum and minimum monthly return following Bali, Cakici and Whitelaw (2011).

Table 4 reports the time-series average of γ and ϕ coefficients for the cross-sectional regressions.

[Insert Table 4 Here]

Column 1 provides univariate results and Column 2 adds firm-specific control variables.[7] The coefficient for the tail risk premium is negative and is statistically significant in both the univariate and multivariate regressions, with coefficients of -0.801 and -1.155 and Newey-West (1987) t-statistic equal to -5.60 and -5.50, respectively. Specifically, stocks with tail risk require a premium in the current month, and this premium is associated with lower returns the following month.

[7] See Appendix for variable definitions.

The results for the impact of overall tail risk on one-month future returns are interesting, but tail risk involves concerns about both extreme positive events and extreme negative ones. Consequently, it may be of interest to examine, separately, the impact of positive and negative tail risk on future returns.

1. The Monthly Predictive Power of Positive versus Negative Tail Risk Premia

To investigate the extent to which positive and negative tail risk may be priced differentially in the cross-section of returns, we again perform firm-level cross-sectional monthly regressions, but this time we include dummy variables to identify those stocks in the top and bottom deciles when sorted by their tail risk premia. The regression is specified in Equation 12.

$$\begin{aligned} R_{i,t+n} = \gamma_{0,t+n} + \gamma_{1,t+n} \times TRP_{i,t}^{Monthly} \times I_{[Decile\ 1\ TRP_{i,t}^{Monthly}]} \\ + \gamma_{2,t+n} \times TRP_{i,t}^{Monthly} \times I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]} + \phi_{t+n}^{'} \times Z_{i,t} + \varepsilon_{i,t+n} \end{aligned} \tag{12}$$

Where $I_{[Decile\ 1\ TRP_{i,t}^{Monthly}]}$ is a dummy variable that equals 1 if $TRP_{i,t}^{Monthly}$ is in decile 1 and equals 0 otherwise, and $I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]}$ is the corresponding dummy variable for decile 10. $R_{i,t+n}$ is monthly stock return for stock i in month $t+n$, where $N = 1,2$. $TRP_{i,t}^{Monthly}$ is the individual stock tail risk premium calculated in Equation 10. $Z_{i,t}$ represents a vector of characteristics and controls for firm i at the end of month t such as size, B/M ratio, market beta, illiquidity, etc.

[Insert Table 5 Here]

Table 5 presents the results. In the t+1 regression, the coefficient on the interacted variable $TRP_{i,t}^{Monthly} \times I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]}$ is statistically significant with γ coefficient of -1.971 and Newey-West (1987) t-statistic of -5.09. The negative coefficient implies that the greater the tail risk premium today (i.e, the greater the negative tail risk) the more negative will be the next month's return. The coefficient on the decile 1 interacted variable is also negative with a γ coefficient of -0.830, but is statistically significant at only the 10% level, thus we refrain from making any claims about its impact on future returns. In the t+2 regression, neither the Decile 1 nor the Decile 10 interracted dummies are significant different from zero.

2. The Daily Predictive Power of Positive versus Negative Tail Risk Premia

In order to more fully examine the relationship between negative tail risk premia and future returns, we replicate the study conducted in the previous section using daily returns. Specifically, we conduct a firm-level cross-sectional predictive regression as in Equation 13. We add the caveat that, at the daily level, there is likely to be some noise in our estimates, thus any conclusions should be tempered somewhat.

$$R_{i,t+n}^{Daily} = \gamma_{0,t+n} + \gamma_{1,t+n} \times TRP_{i,t}^{Daily} \times I_{[Decile\ 10\ TRP_{i,t}^{Daily}]} + \phi_{t+n}^{'} \times Z_{i,t} + \varepsilon_{i,t+n} \qquad (13)$$

[Insert Table 6 Here]

Panel 1 of Table 6 presents the results of the regression of the relationship in Equation 13, day into the future. The coefficient on the interacted variable for Decile 10 shows that the existence of a premium for bearing negative tail risk continues to have predictive power for about 10 days. Even on day 10, the coefficient for the interacted variable for decile 10 is -5.128 with NW t-stat -2.43. The daily results corroborate the findings of the previous section and offer additional evidence on the way that concerns about extreme negative tail events impact future returns.

C. Do Larger Tail Risk Premia Have More Predictive Power?

The results in the previous section suggest that the larger the tail risk premium, the greater its impact on future returns. This suggests that using a finer grid to sort stocks, for example, sorting the stock by tail risk premia into percentiles rather than deciles, and then redoing the earlier analysis may yield interesting results. To this end, we sort and then split the stocks contained in deciles 1 and 10 into deciles once again; that is, we effectively create ten extreme high and low *percentile* porfolios, with percentiles 1-10 belonging to decile Portolio 1 and percentiles 91-100 belonging to decile Porfolio 10. We then apply the 1/0/1 (sort in one month, examine the one-month return for the following month) portfolio strategy for percentile 1 and 100 porfolios, 2 and 99 portolios and 3 and 98 porfolios and report the results in Table 7.

[Insert Table 7 Here]

The results are consistent with the notion that there is a monotonic relationship between the magnitude of the tail risk premium and the impact on future returns. The t+1 return difference is most negative when comparing the two most extreme (1 and 100) portfolios, and decreases monotonically thereafter. The return difference (in percentage) for the 100-1 portfolio is -3.91, for the 99-2 portfolio is -2.50, and the 98-3 portfolio is -2.44, respectively. All are significant at the 1% level.

As a robustness check, we combine percentiles 98-100 into one portfolio and percentiles 1-3 into another portfolio, and then do the same for percentiles 97-99 and 2-4. The t+1 return difference for the (98-100)-(1-3) portfolio is -2.95, for the (97-99)-(2-4) portfolio is -2.35, with both being significant at the 1% level, once again lending support to the notion that predictive power of the current premium for bearing negative tail risk should be directly related to its magnitude.

V. Robustness Checks

We perform a variety of robustness checks in order to ensure that our results are not being driven by other factors.

A. Monte Carlo Analysis of Regression Beta

The first robustness check is on the beta of the baseline regression model in Equation 7. There may be a concern that the beta may not be statistically different from zero both cross-sectionally and in the time series. The standard unidimensional t-test cannot capture this possibility. Instead, to capture both the time series and cross-sectional properties of the regression beta, we use Monte Carlo simulation to test whether beta is statistically different from zero. Monte Carlo simulation has two advantages. First, it is a distributional-free approach. Second, it allows us to make statistical inferences on both the cross sectional and time-series dynamics of the regression beta in Equation 7.

In our sample, the number of firms that have more than 17 trading days in a given month ranges from 3626 to 7471. We denote sample size as S, number of random draws as N. For a given month in a given year, we perform the following simulation,

1) Random draw (with placement) $\beta_1, \beta_2, \beta_3, \cdots, \beta_S$ and compute the mean of $\beta_1, \beta_2, \beta_3, \cdots,$ β_S, denote $\overline{\beta_n}$.
2) Repeat 1) N times and get $\overline{\beta_1}, \overline{\beta_2}, \overline{\beta_3}, \cdots, \overline{\beta_N}$.
3) Compute t-statistic for $\overline{\beta_1}, \overline{\beta_2}, \overline{\beta_3}, \cdots, \overline{\beta_N}$.

We then compute the average of the (time series) year-month t-statistic to get the simulated t-statistics.

[Insert Figure 1 Here]

From panels A through B in Figure 1, we observe that the bootstrapped t-statistic is statistically and significantly different from zero even if we limit to sample size to only 500 firms in each independent random draw[8]. This indicates that the beta of the baseline regression model in

[8] In figure 1 we limit the number of random draws to 10000. We also perform the Monte Carlo simulation by varying number of random draws from 1000 to 10000 and sample size 500 to 3000, results are similar.

Equation 7 is both statistically and economically important that carries important pricing information. In other words, our tail risk premium estimation methodology indeed captures the difference between the $\mathbb{P}$- and $\mathbb{Q}$- spaced expectations of tail risk variation.

B. Sensitivity to Market Aggregate Tail Risk Premium

The second robustness check is to ensure that our results are not being driven by the sensitivity the individual stock's loadings to the market tail risk premium. To this end, we follow Ang, et al. (2006) which adopts a "beta approach." They obtain an individual stock's sensitivity (beta) to innovation in market aggregate volatility (specifically ΔVIX), and then determine whether this beta has predictive power for the next-month's stock returns. We run the following regression model,

$$r_t^i = \beta_0 + \beta_{MKT}^i MKT_t + \beta_{\Delta TRP^{Market}}^i \Delta TRP_t^{Market} + \varepsilon_t^i \tag{14}$$

where MKT is the market excess return and ΔTRP_t^{Market} is estimated tail risk premium for the S&P 500, which is our proxy for innovations in the market aggregate tail risk compensation, that is, factor loading, $\beta_{\Delta TRP^{Market}}^i$, captures the sensitivity of individual stock monthly returns to the change in market aggregate tail risk premium. The results are reported in Table 8.

[Insert Table 8 Here]

The value weighted mean *t+1* return for deciles 1 and 10 are 1.16 and 1.31, respectively. This difference is not statistically significant, which implies that sensitivity to the market aggregate tail risk premium has no predictive power for these stocks. Moreover, it implies that our tail risk premium estimation methodology captures an individual stock's idiosyncratic tail risk premium, which provides pricing information beyond the individual stock's loadings to the market tail risk premium.

C. Contemporaneous Regression

Our paper uses a 22-day lag adjustment between the risk neutral and the physical probability space measures. However, in the literature on the variance risk premium normally does not require a lag adjustment for the $\mathbb{Q}$- spaced variables calculation in the baseline regression model in Equation 7. For example, Bollerslev, Tauchen and Zhou (2009) is among the first to document the variance risk premium's return predictability at the quarterly horizon. They compute the variance risk premium using a relatively "conventional" approach, where the risk premium of return variation is defined as the difference between the time series conditional expected future return variation in the (options based) risk-neutral ($\mathbb{Q}$-spaced) framework and that in the physical probability ($\mathbb{P}$-) space in a contemporaneous manner; however, this approach is inherently biased in that it assumes the risk neutral measures are "backward-looking".

As a robustness check, we employ the non-lagged methodology of Bollerslev, Tauchen and Zhou (2009) and redo the analysis presented in Secion V.A. We sort the stocks into ten equal groups (decile portfolios) by $TRP_i^{Monthly,nolag}$ calculated on the following regression model,

$$\Delta[2(R_{i,t} - r_{i,t}) - r_{i,t}^2] = \alpha_i + \beta_i \cdot \Delta(VIX_t^2 - V_{BKM,t}^C) + \varepsilon_{i,t} \tag{15}$$

The results are reported in Table 9.

[Insert Table 9 Here]

As can be seen by comparing the results of Table 9 to those presented in Table 3, using contemporaneous rather than 22-day lagged $\mathbb{Q}$-spaced measures makes little qualitative difference.

VI. Conclusion

This paper introduces a novel methodology to directly determine the tail risk premium for individual stocks, and then employs this measure to examine the impact of equity tail risk in the cross section of stock returns. Bollerslev, Todorov and Xu (2015) and Kelly and Jiang (2014) find evidence of pricing for negative tail risk, however, neither examines the pricing of positive tail risk, nor the impact on the magnitude of the tail risk on the predictive power of its associated premium. The current paper controls for a variety of variables associated with positive and negative return skewness, and finds at the monthly level, the existence of a premium for bearing positive tail risk today holds no statistically significant power for lower future returns, while its counterpart for bearing negative tail risk has significant predictive power for lower future returns.

At the daily level, the predictive power associated with a premium for bearing negative tail risk lasts for 10 trading days. Moreover, the size of the current premium for bearing negative tail risk matters. The larger the premium associated with exposure to negative tail risk, the more negative and longer lasting is its impact on expected future returns.

The methodology in this paper can be easily extended to other asset classes and to investor behavior in different countries, for example, bond markets, foreign exchange markets, commodity markets in both US and foreign markets. As future research, it would be interesting to investigate how tail risk is priced in these other asset classes; especially in the presence of liquidity risk.

References

Almeida, C., Ardison, K., Garcia, R. and Vicente, J., 2017. Nonparametric Tail Risk, Stock Returns, and the Macroeconomy. Journal of Financial Econometrics, p.nbx007.

Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. Journal of financial markets, 5(1), pp.31-56.

Ang, A., Chen, J. and Xing, Y., 2006. Downside risk. Review of Financial Studies, 19(4), pp.1191-1239.

Ang, A., Hodrick, R.J., Xing, Y. and Zhang, X., 2006. The cross‐section of volatility and expected returns. The Journal of Finance, 61(1), pp.259-299.

Bakshi, G., Kapadia, N. and Madan, D., 2003. Stock return characteristics, skew laws, and the differential pricing of individual equity options. The Review of Financial Studies, 16(1), pp.101-143.

Bali, T.G., Cakici, N. and Whitelaw, R.F., 2011. Maxing out: Stocks as lotteries and the cross-section of expected returns. Journal of Financial Economics, 99(2), pp.427-446.

Bali, T.G., Cakici, N. and Whitelaw, R.F., 2014. Hybrid tail risk and expected stock returns: When does the tail wag the dog?. Review of Asset Pricing Studies, p.rau006.

Barberis, N. and Huang, M., 2008. Stocks as lotteries: The implications of probability weighting for security prices. The American Economic Review, 98(5), pp.2066-2100.

Bekaert, G. and Hoerova, M., 2014. The VIX, the variance premium and stock market volatility. Journal of Econometrics, 183(2), pp.181-192.

Bollerslev, T., Tauchen, G. and Zhou, H., 2009. Expected stock returns and variance risk premia. Review of Financial studies, 22(11), pp.4463-4492.

Bollerslev, T. and Todorov, V., 2011a. Estimation of jump tails. Econometrica, 79(6), pp.1727-1783.

Bollerslev, T. and Todorov, V., 2011b. Tails, fears, and risk premia. The Journal of Finance, 66(6), pp.2165-2211.

Bollerslev, T., Todorov, V. and Xu, L., 2015. Tail risk premia and return predictability. Journal of Financial Economics, 118(1), pp.113-134.

Breeden, D.T., Gibbons, M.R. and Litzenberger, R.H., 1989. Empirical tests of the consumption‐oriented CAPM. The Journal of Finance, 44(2), pp.231-262.

Brunnermeier, M.K., Gollier, C. and Parker, J.A., 2007. Optimal beliefs, asset prices, and the preference for skewed returns. American Economic Review, 97(2), pp.159-165.

Burger, J.M. and Burns, L., 1988. The illusion of unique invulnerability and the use of effective contraception. Personality and Social Psychology Bulletin, 14(2), pp.264-270.

Campbell, J., Greenauer, N., Macaluso, K. and End, C., 2007. Unrealistic optimism in internet events. Computers in human behavior, 23(3), pp.1273-1284.

Carr, P. and Wu, L., 2009. Variance risk premiums. Review of Financial Studies, 22(3), pp.1311-1341.

Chabi-Yo, F., Ruenzi, S. and Weigert, F., 2018. Crash Sensitivity and the Cross Section of Expected Stock Returns. Journal of Financial and Quantitative Analysis, 53(3), pp.1059-1100.

Chang, B.Y., Christoffersen, P. and Jacobs, K., 2013. Market skewness risk and the cross section of stock returns. Journal of Financial Economics, 107(1), pp.46-68.

Chow, V., Jiang, W. and Li, J., 2014. Does VIX Truly Measure Return Volatility?. Handbook of Financial Econometrics, Mathematics, Statistics, and Technology Forthcoming.

Cremers, M., Halling, M. and Weinbaum, D., 2015. Aggregate Jump and Volatility Risk in the Cross‐Section of Stock Returns. The Journal of Finance, 70(2), pp.577-614.

Demeterfi, K., Derman, E., Michael, K., and Zhou, J., 1999, More than you ever wanted to know about volatility swaps, Quantitative Strategies Research Notes, Goldman Sachs.

Dimson, E., 1979. Risk measurement when shares are subject to infrequent trading. Journal of Financial Economics, 7(2), pp.197-226.

Du, J. and Kapadia, N., 2012. Tail and volatility indices from option prices. Unpublished working paper. University of Massachusetts, Amhurst.

Fama, E.F. and French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of financial economics, 33(1), pp.3-56.

Fama, E.F. and MacBeth, J.D., 1973. Risk, return, and equilibrium: Empirical tests. Journal of political economy, 81(3), pp.607-636.

Fu, F., 2009. Idiosyncratic risk and the cross-section of expected stock returns. Journal of Financial Economics, 91(1), pp.24-37.

Gao, G.P., Gao, P. and Song, Z., 2018. Do hedge funds exploit rare disaster concerns? The Review of Financial Studies, 31(7), pp.2650-2692.

Gao, G.P., Lu, X. and Song, Z., 2018. Tail risk concerns everywhere. Management Science.

Jegadeesh, N., 1990. Evidence of predictable behavior of security returns. The Journal of Finance, 45(3), pp.881-898.

Jegadeesh, N. and Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. The Journal of finance, 48(1), pp.65-91.

Jiang, G.J. and Oomen, R.C., 2008. Testing for jumps when asset prices are observed with noise–a “swap variance” approach. Journal of Econometrics, 144(2), pp.352-370.

Kelly, B. and Jiang, H., 2014. Tail risk and asset prices. Review of Financial Studies, 27(10), pp.2841-2871.

Lamont, O.A., 2001. Economic tracking portfolios. Journal of Econometrics, 105(1), pp.161-184.

Lehmann, B.N., 1990. Fads, martingales, and market efficiency. The Quarterly Journal of Economics, 105(1), pp.1-28.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. The review of economics and statistics, pp.13-37.

Merton, R.C., 1976. Option pricing when underlying stock returns are discontinuous. Journal of financial economics, 3(1-2), pp.125-144.

Mossin, J., 1966. Equilibrium in a capital asset market. Econometrica: Journal of the econometric society, pp.768-783.

Newey, W.K. and West, K.D., 1986. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix.

Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, 19(3), pp.425-442.

Scholes, M. and Williams, J., 1977. Estimating betas from nonsynchronous data. Journal of financial economics, 5(3), pp.309-327.

Todorov, V. and Tauchen, G., 2011. Volatility jumps. Journal of Business & Economic Statistics, 29(3), pp.356-371.

Tversky, A. and Kahneman, D., 1992. Advances in prospect theory: Cumulative representation of uncertainty. Journal of Risk and uncertainty, 5(4), pp.297-323.

Appendix Variable Definitions

Tail risk premium: We compute tail risk premium as in Equation 10 in Section III A.

Log (Size): Following Fama and French (1993), size is computed each June as stock price times number of shares outstanding (in hundreds). Size is measured in hundred thousand. We control for size effect by taking natural logarithm of Size.

Log (B/M): Following Fama and French (1993), book-to-market is computed as the ratio of book common equity over market capitalization (size). Book common equity is calculated using Compustat's book value of stockholders' equity plus balance-sheet deferred taxes and investment tax credit minus the book value of preferred stock. The ratio is computed as the book common equity at the end of fiscal year over size as the December end of fiscal year end.[9]

Market beta: We follow Sholes and Williams (1977) and Dimson (1979) to address nonsynchronous trading in beta estimation. We run regression including lag, current and lead market risk premium as independent variables as in Equation 16,

$$\begin{aligned} R_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i}\left(R_{market,t-1} - r_{f,t-1}\right) + \beta_{2,i}\left(R_{market,t} - r_{f,t}\right) \\ + \beta_{3,i}\left(R_{market,t+1} - r_{f,t+1}\right) + \varepsilon_{i,t} \end{aligned} \tag{16}$$

where $R_{i,t}$ is return for stock i on day t. $R_{market,t}$ is market return on day t and $r_{f,t}$ is risk free rate on day t. We estimation the above equation for each stock using daily returns within each month. For each month, the market beta is estimated as follows in Equation 17 for each stock i,

$$\widehat{\beta_i} = \widehat{\beta_{1,i}} + \widehat{\beta_{2,i}} + \widehat{\beta_{3,i}} \tag{17}$$

Idiosyncratic Volatility: Following Ang, Hodrick, Xing and Zhang (2006), idiosyncratic volatility is calculated as

[9] To avoid issues with extreme values, the book-to market ratios are winsorized at the 1% and 99% levels.

$$Idiosyncratic\ volatility = \sqrt{var(\varepsilon_{t,t})} \tag{18}$$

where $\varepsilon_{t,t}$ is the error term from the three-factor Fama and French (1993) regression. The regression is estimated monthly with more than 17 daily observations in a month.

Lagged 1-month return: Following Jegadeesh (1990) and Lehmann (1990), we use lagged 1-month return to account for short-term return reversal effect, the reversal variable for each stock in month m is defined as the return on the stock over the previous month, i.e., the return in month $m-1$.

Lagged 12-month return: Jegadeesh and Titman (1993) documented intermediate-term momentum effect, we use lagged 12-month return to account for momentum effect, it is defined as return $m-12$ for each stock in month m.

Illiquidity: Following Amihud (2002), we compute stock illiquidity for each stock i in each month m as the ratio of the absolute monthly stock return to its dollar trading volume:

$$Illiquidity_{i,m} = \frac{|R_{i,m}|}{|Volumn_m \times Price_m|} \tag{19}$$

Maximum (Minimum) monthly return: Following Bali, Cakici and Whitelaw (2011), we control for maximum (minimum) monthly return for each stock i in month m as the maximum (minimum) daily return within month m.

$$Maximum\ monthly\ return_m = \max\{R_{i,t}\}, t = 1, \cdots, T \tag{20}$$

$$Minimum\ monthly\ return_m = \min\{R_{i,t}\}, t = 1, \cdots, T \tag{21}$$

where T is the maximum number of daily observations in month m. These are estimated monthly with more than 17 daily observations in a month.

Log (trading volume): trading volume is the sum of the trading volumes during that month. We control for size effect by taking natural logarithm of Size.

Price: the price on the last trading date of the month.

Table 1. Characteristics of Portfolios Sorted by Tail Risk Premium

Each week, stocks in CRSP database are ranked by their respective tail risk premium. The equal-weighted characteristics of each quintile are computed over the same week. The procedure is repeated for every month from January 1990 to September 2014. Tail risk premium and illiquidity are in 10^{-6}. Lagged 1-month return, Lagged 12-month return, Maximum monthly return, Minimum monthly return are in percentages. Log (Size), Log (B/M), Market beta, Idiosyncratic Volatility, Log (trading volume), Price are in absolute values. See Appendix for variable definitions.

Characteristics of Portfolios Sorted by Tail Risk Premium

Deciles	1	2	3	4	5	6	7	8	9	10
Tail risk premium	-88.01	-4.33	-1.27	-0.44	-0.08	0.16	0.53	1.42	4.53	89.05
Log (Size)	4.00	4.87	5.56	6.10	6.49	6.54	6.14	5.59	4.88	3.95
Log (B/M)	-0.53	-0.59	-0.60	-0.59	-0.57	-0.57	-0.60	-0.61	-0.59	-0.53
Market beta	0.91	0.96	0.91	0.81	0.71	0.74	0.82	0.99	0.94	0.81
Idiosyncratic volatility	6.15	3.73	2.76	2.10	1.66	1.66	2.12	2.77	3.76	6.35
Lagged 1-month return	2.47	2.05	1.97	1.88	1.61	1.48	0.88	0.54	-0.23	-2.44
Lagged 12-month return	0.18	0.96	1.51	1.20	1.50	1.34	1.34	1.27	1.11	0.37
Illiquidity	123.92	28.31	19.43	9.67	7.85	5.32	10.36	15.29	37.74	125.65
Maximum monthly return	15.11	8.93	6.64	5.05	4.04	4.10	5.17	6.81	9.30	16.71
Minimum monthly return	-12.19	-7.59	-5.74	-4.46	-3.60	-3.60	-4.48	-5.71	-7.51	-11.63
Log (trading volume)	9.02	9.31	9.58	9.71	9.80	9.84	9.75	9.60	9.34	8.99
Price	7.92	16.95	28.76	40.00	58.04	48.50	44.57	25.91	16.46	8.13
Number of stocks	13749	14237	13977	13307	12515	12635	13221	14023	14273	13792

Table 2. Pearson Correlation Matrix

Pearson correlation coefficients reported as percent for firm characteristics of all CRSP stocks from January 1990 to September 2014. See Appendix for variable definitions.

	Tail risk premium	Log (Size)	Log (B/M)	Market beta	Idiosyncratic volatility	Lagged 1-month return	Lagged 12-month return	Illiquidity	Maximum monthly return	Minimum monthly return	Log (trading volume)
Tail risk premium	100.00	-0.09	-0.33	0.28	1.56	-2.55	0.26	-0.35	3.74	0.78	0.01
Log (Size)		100.00	-27.87	12.54	-49.70	5.59	4.67	-2.82	-37.25	39.00	76.96
Log (B/M)			100.00	-6.32	4.90	3.91	-7.55	1.62	3.72	-1.22	-28.95
Market beta				100.00	-0.12	0.58	1.29	-0.38	3.44	-2.71	17.09
Idiosyncratic volatility					100.00	-10.41	-5.05	3.27	89.72	-80.77	-19.94
Lagged 1-month return						100.00	0.06	-0.58	-10.29	9.54	2.65
Lagged 12-month return							100.00	-0.20	-4.20	4.19	1.61
Illiquidity								100.00	2.53	-3.26	-2.46
Maximum monthly return									100.00	-60.25	-11.99
Minimum monthly return										100.00	11.32
Log (trading volume)											100.00

Table 3. Portfolios Returns Sorted into Deciles by Tail Risk Premium

We form value-weighted decile portfolios every month by sorting stocks based on tail risk premium in Equation 10. Portfolios are formed every month, based on tail risk premium in Equation 10 computed using daily data over the previous month. Panel A displays the 1/0/1 portfolio strategy (sort in one month, examine the one-month return for the following month), Panel B for 1/1/1 portfolio strategy strategy (sort in one month, examine the one-month return starting two month from now) and Panel C for 1/2/1 portfolio strategy (sort in one month, examine the one-month return starting three month from now). Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) previous month tail risk premium. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to the total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "10-1" refers to the difference in monthly returns between portfolio 10 and portfolio 1. NW t-stat refers to robust Newey-West (1987) t-stat. Pre-formulation TRP is reported in basis points. The sample period is January 1990 to September 2014.

Panel A: Portfolios Sorted by Tail Risk Premium, 1/0/1 Portfolio Strategy

						Factor Loadings (bps)
Rank	**Mean**	**Std. Dev.**	**%Mkt Share**	**Size**	**B/M**	**Pre-Formation TRP**
1	2.02	7.19	1.76%	4.03	0.94	-1.023
2	1.70	6.25	4.45%	4.92	0.79	-0.042
3	1.61	5.31	8.72%	5.53	0.73	-0.013
4	1.49	4.52	14.48%	6.03	0.70	-0.004
5	1.43	3.98	19.92%	6.40	0.70	-0.001
6	1.31	4.10	19.73%	6.39	0.70	0.002
7	1.22	4.71	15.20%	6.07	0.70	0.005
8	1.09	5.48	9.44%	5.56	0.73	0.014
9	1.02	6.47	4.40%	4.93	0.78	0.044
10	0.27	7.39	1.88%	4.03	0.93	1.063
10-1	-1.75					
t-stat	(-11.26)					
NW t-stat	(-7.30)					

Panel B: Portfolios Sorted by Tail Risk Premium, 1/1/1 Portfolio Strategy						Factor Loadings (bps)
Rank	**Mean**	**Std. Dev.**	**%Mkt Share**	**Size**	**B/M**	**Pre-Formation TRP**
1	0.76	7.51	1.76%	4.05	0.94	-0.982
2	0.97	6.36	4.45%	4.93	0.79	-0.042
3	1.11	5.54	8.75%	5.53	0.73	-0.013
4	1.12	4.62	14.46%	6.04	0.70	-0.004
5	1.06	4.13	19.97%	6.40	0.70	-0.001
6	1.06	4.19	19.75%	6.40	0.70	0.002
7	1.06	4.66	15.22%	6.07	0.70	0.005
8	1.06	5.48	9.41%	5.56	0.72	0.014
9	0.97	6.52	4.37%	4.93	0.78	0.044
10	0.84	7.28	1.85%	4.04	0.94	0.997
10-1	0.08					
t-stat	(0.73)					
NW t-stat	(0.76)					

Panel C: Portfolios Sorted by Tail Risk Premium, 1/2/1 Portfolio Strategy

Rank	Mean	Std. Dev.	%Mkt Share	Size	B/M	Factor Loadings (bps) Pre-Formation TRP
1	0.96	7.58	1.72%	4.06	0.95	-0.934
2	1.24	6.60	4.44%	4.93	0.79	-0.042
3	1.16	5.60	8.78%	5.53	0.73	-0.013
4	1.24	4.70	14.48%	6.04	0.70	-0.004
5	1.16	4.18	19.96%	6.41	0.70	-0.001
6	1.18	4.11	19.79%	6.40	0.70	0.002
7	1.18	4.67	15.21%	6.08	0.70	0.005
8	1.21	5.38	9.43%	5.57	0.73	0.014
9	1.14	6.41	4.37%	4.94	0.78	0.044
10	0.99	7.33	1.82%	4.06	0.94	0.969
10-1	0.03					
t-stat	(0.18)					
NW t-stat	(0.18)					

Table 4. Fama-MacBeth Cross-Sectional Regression
Results of a Fama-MacBeth cross-sectional regression of stock returns for the following:

$$R_{i,t+1} = \gamma_{0,t+1} + \gamma_{1,t+1} \times TRP_{i,t}^{Monthly} + \phi'_{t+1} \times Z_{i,t} + \varepsilon_{i,t+1} \quad (11)$$

$R_{i,t+1}$ is monthly stock return for stock i in month $t+1$. $TRP_{i,t}^{Monthly}$ is individual stock tail risk premium for stock i in month t, calculated in Equation 10. $Z_{i,t}$ represents a vector of characteristics and controls for firm i at the end of month t such as size, B/M ratio, market beta, illiquidity, etc. The control variables are described in the Appendix. The Newey-West (1973) HAC robust t-statistic is reported in parentheses. Specification (1) is univariate regression, Specification (2) is multiple regression adding control variables. The sample period is from January 1990 to September 2014.

	(1)	(2)
Intercept	0.009	0.021
	(2.42)	(6.16)
Tail risk premium	-0.801	-1.155
	(-5.60)	(-5.50)
Log (Size)		0.000
		(0.19)
Log (B/M)		0.003
		(3.52)
Market beta		0.000
		(0.74)
Illiquidity		19.715
		(1.11)
Idiosyncratic volatility		-0.001
		(-1.90)
Lagged 1-month return		0.003
		(0.85)
Lagged 12-month return		-0.001
		(-0.40)
Maximum monthly return		-0.032
		(-2.02)
Minimum monthly return		-0.019
		(-1.06)
Log (trading volume)		-0.001
		(-0.80)
Adjusted R^2 (%)	0.133	4.765

Table 5. Fama-MacBeth Regression including dummy variables for Decile 1 and Decile 10 Tail Risk Premia

Results of a Fama-MacBeth cross-sectional regression of stock returns for the following:

$$R_{i,t+n} = \gamma_{0,t+n} + \gamma_{1,t+n} \times TRP_{i,t}^{Monthly} \times I_{[Decile\ 1\ TRP_{i,t}^{Monthly}]} + \gamma_{2,t+n} \times TRP_{i,t}^{Monthly} \times I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]} + \phi_{t+n}^{'} \times Z_{i,t} + \varepsilon_{i,t+n} \quad (12)$$

Where $I_{[Decile\ 1\ TRP_{i,t}^{Monthly}]}$ is a dummy variable that equals 1 if $TRP_{i,t}^{Monthly}$ is in decile 1 and equals 0 otherwise, and $I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]}$ is the corresponding dummy variable for decile 10. $R_{i,t+n}$ is monthly stock return for stock i in month $t+n$, where $N = 1,2$. $TRP_{i,t}^{Monthly}$ is individual stock tail risk premium for stock i in month t, calculated in Equation 10. $Z_{i,t}$ represents a vector of characteristics and controls for firm i at the end of month t such as size, B/M ratio, market beta, illiquidity, etc. The control variables are described in the Appendix. The Newey-West (1973) HAC robust t-statistic is reported in parentheses. Specification (1) is the multiple regression adding control variables for monthly return $R_{i,t+1}$, specification (2) is the multiple regression adding control variables for monthly return $R_{i,t+2}$. The sample period is from January 1990 to September 2014.

	(1)	(2)
Return	t+1	t+2
Intercept	0.020	0.022
	(5.92)	(6.51)
$TRP_{i,t}^{Monthly} \times I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]}$	-1.971	-0.022
	(-5.09)	(-0.05)
$TRP_{i,t}^{Monthly} \times I_{[Decile\ 1\ TRP_{i,t}^{Monthly}]}$	-0.803	-0.668
	(-1.86)	(-1.39)
Log (Size)	0.000	0.001
	(0.00)	(0.63)
Log (B/M)	0.003	0.003
	(3.46)	(3.19)
Market beta	0.000	0.000
	(0.55)	(0.23)
Illiquidity	21.788	14.994
	(1.73)	(1.33)
Idiosyncratic volatility	-0.002	-0.002
	(-2.73)	(-2.73)
Lagged 1-month return	0.003	0.014
	(0.91)	(4.47)
Lagged 12-month return	-0.002	-0.004
	(-0.97)	(-1.72)
Maximum monthly return	-0.026	0.005
	(-1.98)	(0.40)
Minimum monthly return	-0.025	0.020
	(-1.62)	(1.58)
Log (trading volume)	-0.000	-0.001
	(-0.52)	(-1.41)
Adjusted R^2 (%)	4.063	3.947

Table 6. Fama-MacBeth Regression of Positive and Negative Tails using Daily Returns

Results of daily Fama-MacBeth cross-sectional regression of stock returns for the following:

$$R_{i,t+n}^{Daily} = \gamma_{0,t+n} + \gamma_{1,t+n} \times TRP_{i,t}^{Daily} \times I_{[Decile\ 10\ TRP_{i,t}^{Daily}]} + \phi_{t+n}^{'} \times Z_{i,t} + \varepsilon_{i,t+n} \quad (13)$$

Where $I_{[Decile\ 10\ TRP_{i,t}^{Daily}]}$ is a dummy variable that equals 1 if $TRP_{i,t}^{Daily}$ is in decile 10 and equals 0 otherwise. $R_{i,t+n}^{Daily}$ is one-day holding period return for stock i from day $t+n-1$ to day $t+n$, where $n = 1,2 \cdots, 18$. A maximum of 18 days is used since we require stocks to have a minimum of 18 days to be included in the sample. $TRP_{i,t}^{Daily}$ is individual stock tail risk premium for stock i in month t, calculated in Equation 10. $Z_{i,t}$ represents a vector of characteristics and controls for firm i at the end of month t such as size, B/M ratio, market beta, illiquidity, etc. The control variables are described in the Appendix. The table reports predictive regression using next-month daily return (day 1, 7, 10, 14, 18) as dependent variable. The Newey-West (1973) HAC robust t-statistic is reported in parentheses. Decile portfolio sort section, similar to table 3, reports decile 10 return minus decile 1 return difference in basis points, as well as t-statistic and Newey-West (1973) HAC robust t-statistic associated with it (see table 3 for detailed testing methodology). For ease of reading, significant coefficients are highlighted in bold.

	(1)	(2)	(3)	(4)	(5)
Return	Day 1	Day 7	Day 10	Day 14	Day 18
Intercept	-0.005	0.000	-0.000	0.001	0.001
	(-5.92)	(0.74)	(-0.07)	(2.05)	(1.34)
$TRP_{i,t}^{Monthly} \times I_{[Decile\ 10\ TRP_{i,t}^{Monthly}]}$	**-23.931**	**-5.609**	**-5.91**	0.531	4.511
	(-5.80)	**(-2.34)**	**(-2.68)**	(0.16)	(1.53)
Log (Size)	0.001	-0.000	0.000	-0.000	-0.000
	(2.95)	(-1.07)	(1.68)	(-1.88)	(-0.24)
Log (B/M)	0.001	0.000	0.000	0.000	0.000
	(4.69)	(1.54)	(2.21)	(2.59)	(1.05)
Market beta	0.000	-0.000	-0.000	-0.000	0.000
	(0.30)	(-0.64)	(-0.06)	(-2.26)	(1.05)
Illiquidity	12.796	5.576	12.852	4.756	-1.524
	(3.12)	(1.76)	(2.67)	(1.78)	(-1.47)
Idiosyncratic volatility	0.001	0.000	0.000	0.000	0.000
	(1.62)	(2.15)	(2.02)	(1.85)	(1.52)
Lagged 1-month return	-0.001	-0.000	0.001	0.002	0.000
	(-0.56)	(-0.02)	(0.82)	(2.07)	(0.58)
Lagged 12-month return	-0.001	-0.001	-0.001	-0.000	-0.000
	(-1.11)	(-1.73)	(-1.15)	(-0.03)	(-0.16)
Maximum monthly return	-0.010	-0.004	-0.008	-0.008	-0.002
	(-1.52)	(-0.81)	(-1.95)	(-1.88)	(-0.48)
Minimum monthly return	-0.017	0.002	0.001	0.001	0.001
	(-2.37)	(0.39)	(0.12)	(0.28)	(0.21)
Log (trading volume)	0.000	0.000	-0.000	-0.000	-0.000
	(0.95)	(0.02)	(-0.98)	(-0.49)	(-0.10)
Adjusted R^2 (%)	3.989	2.360	2.115	2.095	1.832

Table 7. Extreme Tail: Percentile Portfolio Returns Sorted by Tail Risk Premium

We form value-weighted percentile portfolios every month by sorting stocks based on tail risk premium in Equation 10. Portfolios are formed every month, based on tail risk premium in Equation 10 computed using daily data over the previous month. The table displays the 1/0/1 portfolio strategy (sort in one month, examine the one-month return for the following month). Portfolio 1 (100) is the portfolio of stocks with the lowest (highest) previous month tail risk premium. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to the total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "100-1" refers to the difference in monthly returns between portfolio 100 and portfolio 1, the row "99-2" refers to the difference in monthly returns between portfolio 99 and portfolio 2, and the row "98-3" refers to the difference in monthly returns between portfolio 98 and portfolio 3. The row "98-100 minus 1-3" stands for the difference between mean monthly returns of portfolio 98 through 100 and mean monthly returns of portfolio 1 through 3. The row "97-99 minus 2-4" represents for the difference between mean monthly returns of portfolio 97 through 99 and mean monthly returns of portfolio 2 through 4. NW t-stat refers to robust Newey-West (1987) t-stat. Pre-formulation TRP is reported in basis points. The sample period is January 1990 to September 2014.

Percentile Portfolio Returns Sorted by Tail Risk Premium

Rank	Mean	Std. Dev.	%Mkt Share	Size	B/M	Factor Loadings (bps) Pre-Formation *TRP*
1	2.74	9.19	6.18%	3.28	1.10	-8.330
2	2.20	8.54	6.68%	5.53	1.06	-1.072
3	2.14	8.21	6.70%	3.74	1.01	-0.570
4	2.32	7.97	8.25%	3.91	0.97	-0.368
5	1.83	7.81	8.63%	4.04	0.93	-0.264
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
96	0.44	8.39	8.23%	4.02	0.93	0.268
97	0.20	7.91	8.03%	3.89	0.96	0.373
98	-0.30	7.84	7.05%	3.74	1.00	0.572
99	-0.30	8.56	6.58%	3.54	1.04	1.074
100	-1.17	8.81	6.29%	3.26	1.12	8.504
100-1	-3.91					
t-stat	(-8.34)					
NW t-stat	(-7.21)					
99-2	-2.50					
t-stat	(-7.14)					
NW t-stat	(-7.24)					
98-3	-2.44					
t-stat	(-7.01)					
NW t-stat	(-6.35)					
98-100 minus 1-3	-2.95					
t-stat	(-13.02)					
NW t-stat	(-9.14)					
97-99 minus 2-4	-2.35					
t-stat	(-12.11)					
NW t-stat	(-9.58)					

Table 8. Portfolios Returns Sorted into Deciles by Sensitivity to Market Aggregate Tail Risk Premium
We form value-weighted decile portfolios every month by sorting stocks based on sensitivity to market tail risk premium, β^i_{MKT}, in Equation 14.

$$r^i_t = \beta_0 + \beta^i_{MKT} MKT_t + \beta^i_{\Delta TRP^{Market}} \Delta TRP^{Market}_t + \varepsilon^i_t \tag{14}$$

Portfolios are formed every month, based on sensitivity to market tail risk premium, β^i_{MKT}, in Equation 14 computed using daily data over the previous month. The table displays the 1/0/1 portfolio strategy (sort in one month, examine the one-month return for the following month), Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) previous month tail risk premium. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to the total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "10-1" refers to the difference in monthly returns between portfolio 10 and portfolio 1. NW t-stat refers to robust Newey-West (1987) t-stat. Pre-formulation TRP is reported in basis points. The sample period is January 1990 to September 2014.

						Factor Loadings
Rank	**Mean**	**Std. Dev.**	**%Mkt Share**	**Size**	**B/M**	**Pre-Formation TRP**
1	1.16	7.05	2.31%	4.20	0.90	-24.86
2	1.34	5.71	6.73%	5.17	0.76	-10.00
3	1.26	5.01	11.29%	5.65	0.73	-5.73
4	1.35	4.66	14.23%	5.92	0.72	-3.08
5	1.31	4.42	14.98%	6.03	0.72	-1.03
6	1.33	4.37	15.28%	6.00	0.73	0.83
7	1.33	4.58	14.54%	5.95	0.72	2.86
8	1.38	5.06	11.41%	5.68	0.74	5.47
9	1.45	5.83	6.89%	5.20	0.77	9.71
10	1.31	6.84	2.34%	4.20	0.92	24.64
10-1	0.15					
t-stat	(1.08)					
NW t-stat	(0.95)					

Table 9. Portfolios Returns Sorted into Deciles by Contemporaneous Tail Risk Premium

We form value-weighted decile portfolios every month by sorting stocks by tail risk premium estimated based on Equation 15. Portfolios are formed every month, based on tail risk premium in Equation 15 computed using daily data over the previous month. Panel A displays the 1/0/1 portfolio strategy (sort in one month, examine the one-month return for the following month), Panel B for 1/1/1 portfolio strategy strategy (sort in one month, examine the one-month return starting two month from now) and Panel C for 1/2/1 portfolio strategy (sort in one month, examine the one-month return starting three month from now). Portfolio 1 (10) is the portfolio of stocks with the lowest (highest) previous month tail risk premium. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to the total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "10-1" refers to the difference in monthly returns between portfolio 10 and portfolio 1. NW t-stat refers to robust Newey-West (1987) t-stat. Pre-formulation TRP is reported in basis points. The sample period is January 1990 to September 2014.

Panel A: Portfolios Sorted by Tail Risk Premium, 1/0/1 Portfolio Strategy						**Factor Loadings (bps)**
Rank	**Mean**	**Std. Dev.**	**%Mkt Share**	**Size**	**B/M**	**Pre-Formation TRP**
1	2.00	7.23	1.83%	4.03	0.94	-1.006
2	1.69	6.31	4.46%	4.92	0.79	-0.043
3	1.59	5.23	8.89%	5.54	0.73	-0.013
4	1.52	4.38	14.92%	6.05	0.70	-0.005
5	1.40	4.02	19.56%	6.39	0.70	-0.001
6	1.30	4.14	19.84%	6.39	0.70	0.001
7	1.26	4.75	15.16%	6.07	0.70	0.005
8	1.19	5.44	9.05%	5.55	0.72	0.014
9	0.91	6.46	4.41%	4.92	0.78	0.045
10	0.34	7.45	1.89%	4.03	0.94	1.108
10-1	-1.66					
t-stat	(-9.92)					
NW t-stat	(-6.96)					

Panel B: Portfolios Sorted by Tail Risk Premium, 1/1/1 Portfolio Strategy

Rank	Mean	Std. Dev.	%Mkt Share	Size	B/M	Factor Loadings (bps) Pre-Formation TRP
1	0.72	7.60	1.82%	4.04	0.94	-0.973
2	0.89	6.48	4.48%	4.93	0.79	-0.043
3	1.13	5.54	8.91%	5.55	0.73	-0.013
4	1.08	4.72	14.93%	6.06	0.70	-0.004
5	1.06	4.13	19.62%	6.39	0.70	-0.001
6	1.08	4.12	19.85%	6.39	0.70	0.001
7	1.14	4.55	15.14%	6.07	0.70	0.005
8	1.12	5.45	9.01%	5.55	0.72	0.014
9	0.99	6.45	4.36%	4.92	0.78	0.045
10	0.80	7.24	1.87%	4.05	0.94	1.030
10-1	0.08					
t-stat	(0.68)					
NW t-stat	(0.73)					

Panel C: Portfolios Sorted by Tail Risk Premium, 1/2/1 Portfolio Strategy

Rank	Mean	Std. Dev.	%Mkt Share	Size	B/M	Factor Loadings (bps) Pre-Formation TRP
1	1.03	7.55	1.78%	4.06	0.94	-0.922
2	1.27	6.70	4.46%	4.93	0.79	-0.043
3	1.19	5.62	8.91%	5.55	0.73	-0.013
4	1.23	4.78	14.96%	6.06	0.70	-0.005
5	1.21	4.18	19.64%	6.40	0.70	-0.001
6	1.20	4.07	19.85%	6.40	0.70	0.001
7	1.18	4.62	15.15%	6.08	0.70	0.005
8	1.11	5.30	9.03%	5.56	0.72	0.014
9	1.11	6.37	4.37%	4.92	0.79	0.044
10	0.99	7.31	1.86%	4.06	0.95	1.010
10-1	-0.04					
t-stat	(-0.33)					
NW t-stat	(-0.37)					

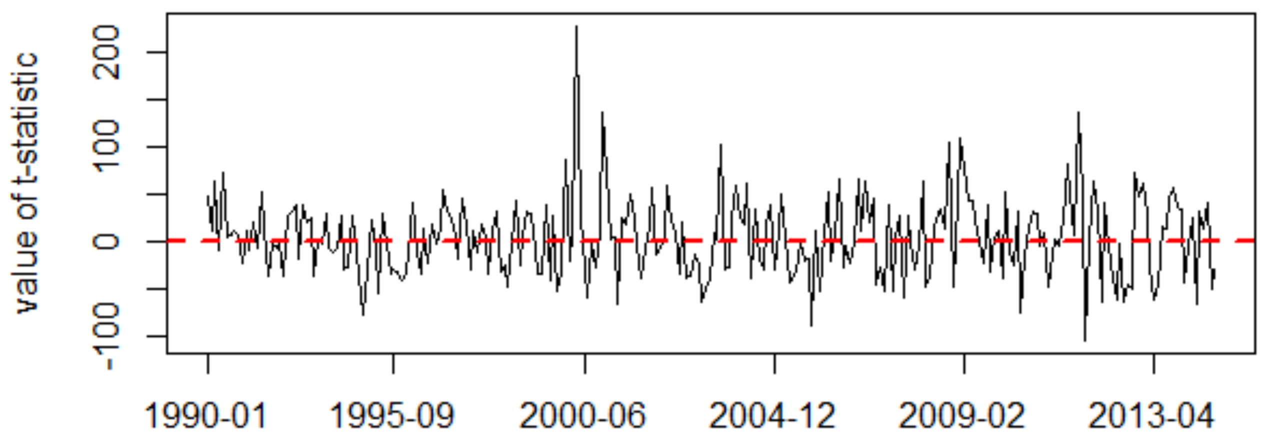
Figure 1. Monte Carlo Analysis of Regression Beta

The Monte Carlo analysis is performed to test whether the baseline regression (7) beta equal to zero. Denote sample size as S, number of random draws as N. For a given month in a given year, we perform the following,

1) Random draw (with placement) $\beta_1, \beta_2, \beta_3, \cdots, \beta_S$ and compute the mean of $\beta_1, \beta_2, \beta_3, \cdots, \beta_S$, denote $\overline{\beta_n}$.
2) Repeat 1) N times and get $\overline{\beta_1}, \overline{\beta_2}, \overline{\beta_3}, \cdots, \overline{\beta_N}$.
3) Compute t-statistic for $\overline{\beta_1}, \overline{\beta_2}, \overline{\beta_3}, \cdots, \overline{\beta_N}$.

We then compute the average of the (time series) year-month t-statistic to get the simulated t-statistic. The sample period is January 1990 to September 2014. The following graphs, panels A through C plots the Monte Carlo simulated regression beta values for different combinations sample size (S=500, 1000) and number of random draw values (N=10000).

Panel A: Sample Size (S) = 500, Number of Random Draw (N) = 10000. Average t-statistic=3.684.



Panel B: Sample Size (S) = 1000, Number of Random Draw (N) = 10000. Average t-statistic=5.152.