Credit Lines and the Liquidity Insurance Channel*

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Abstract

We suggest a new mechanism—the liquidity insurance channel—based on the widespread reliance of high credit quality firms on bank credit lines for liquidity management. Our model matches the patterns of usage of loans and credit lines in the cross-section of firms, and defines the conditions under which shocks to bank health affect primarily low or high credit quality firms. Our framework can explain why credit line origination is more cyclical than loan origination. Overall, we uncover a novel interaction between bank health and economic activity through the provision of bank credit lines to high credit quality firms.

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1 Introduction

The corporate sector in the U.S. has access to a large amount of liquidity in the form of bank credit lines, with undrawn credit lines representing over 10% of net assets (Acharya, Almeida, Ippolito, and Perez-Orive, 2018). Drawdowns on credit lines represent a large percentage of bank lending to businesses and new issuance of credit lines is almost twice as large as that of loans (Cornett, McNutt, Strahan, and Tehranian, 2011). In this paper, we seek to understand what determines the ability of financial intermediaries to satisfy corporate demand for liquidity through the provision of credit lines. How does the supply of liquidity through credit lines vary with changes in bank financial health? How does it vary over the business cycle? How does it interact with the provision of term loans?

Credit lines represent an important and still relatively unexplored channel of bank finance for the corporate sector. While bank financing is usually associated with low credit quality firms, bank credit lines are more commonly used by large, profitable, high credit quality firms (Sufi, 2009; Acharya, Almeida, Ippolito, and Perez-Orive, 2014). Small, financially constrained firms tend to rely on cash for their liquidity management. Bank credit lines are distinct from standard term loans in that they can provide liquidity insurance, allowing firms to access bank financing in states of world in which their financial performance deteriorates. Although firms that rely on credit lines tend not to draw down on credit lines often, credit line access can be very important for them in bad states of the world. It is well known that shocks to bank health affect the ability of banks to provide loans, and that this has an impact on financially constrained firms. However, negative shocks to bank health also affect the ability of banks to honor credit line drawdowns, which means that high credit quality firms may also be affected by such shocks (Huang, 2010; Acharya, Almeida, Ippolito, and Perez-Orive, 2018). We call the link between bank health and the real economy, which operates through liquidity insurance provision, the liquidity insurance channel.

We provide a model of this channel. The model considers both the standard motivation for bank lending to financially weak firms and a framework for liquidity management. The bank lending model is based on Holmstrom and Tirole (1997), while the liquidity insurance framework is similar to that in Holmstrom and Tirole (1998). The key innovation is that we consider the decision to borrow from a bank or not and the decision of how to hold liquidity simultaneously. In the model, the benefit of bank lending is to increase pledgeable income
through monitoring. Since monitoring is costly, firms borrow from a bank only when the marginal benefit of increasing investment is high. Besides funding current investment, firms must also plan for a future liquidity shortfall by holding pre-arranged financing (liquidity insurance). Using cash to insure liquidity is costly because of a liquidity premium, while relying on credit lines exposes the firm to the risk of credit line revocation.

The key intuition of the model is that firms with high credit quality have both a lower benefit of bank monitoring and a lower cost of credit line revocation. These firms are less financially constrained and thus invest at higher levels. Under decreasing returns to scale, the marginal benefit of using bank monitoring to increase investment is lower. At the same time, these firms also have lower liquidity risk, either because their probability of facing a liquidity shock is lower, or because the losses associated with not having sufficient liquidity are smaller. Thus, high credit quality firms rely on credit lines for liquidity insurance but do not benefit from using bank debt for regular borrowing. Low credit quality firms rely on bank debt, and are also more likely to choose cash for their liquidity management.¹

We then use the model to study banks’ optimal liquidity allocation between term loans and credit lines. Term lending to low quality firms against their future cash flows increases their investment. Saving bank liquidity, instead, increases banks’ ability to honor credit line drawdowns in adverse future states, which, in turn, reduces expected bankruptcy costs for high credit quality firms. This analysis also allows us to predict when shocks to bank health will affect primarily low or high credit quality firms. For example, if expected bankruptcy costs are large, banks find it optimal to allocate liquidity to the provision of credit lines, so that shocks to bank capital or liquidity primarily affect high credit quality firms through the liquidity insurance channel. The model also has implications for how shocks to bank health affect ex-ante liquidity management. Starting from an equilibrium in which high credit quality firms rely on credit lines, a negative shock to bank capital or liquidity may cause these firms to switch from credit lines to cash for their liquidity management. Because holding cash is costly, there can be negative consequences for investment by high quality firms as a result. To our knowledge, this channel of bank finance is new to the theoretical literature and not hitherto explored in empirical work.

¹If a firm relies on credit lines for liquidity management, there will be states of the world in which it will draw on the credit line and use bank debt. But for high credit quality firms these states of the world do not happen very often. Thus, at any point of time they will have less bank debt than lower credit quality firms that rely on banks for their regular financing.
Our third contribution is to analyze how the allocation of bank liquidity between loans and credit lines changes with the state of the economy. Because term loans are used by relatively weaker firms, in bad times the marginal value of allocating bank liquidity to term loans increases relative to the marginal value of saving cash to honor future drawdowns on credit lines. Aggregate evidence that we provide on the dynamics of the stocks of undrawn credit lines and total business loans outstanding are consistent with this novel testable prediction.

Taken together, the results of our theoretical work and descriptive empirical analysis shed light on how the choice of banks to allocate liquidity between term loans and credit lines affects liquidity and investment by high quality and low quality (or large versus small) firms, and how this varies over the economic cycle. Our framework helps in the understanding of the macroeconomic consequences of regulatory initiatives such as bank capital requirements or liquidity coverage ratios in a setting where there is heterogeneity among firms in the corporate sector in the nature of their usage of bank finance.

Related Literature

An important strand of the theoretical banking literature has focused on the role of banks in improving resource allocation by creating pledgeable income through a reduction in private benefits, improved project screening, and other mechanisms (Fama, 1985; Houston and James, 1996; Holmstrom and Tirole, 1997) and analyzed the macroeconomic implications of bank financial constraints that limit their ability to perform this role (Brunnermeier and Sannikov, 2014; Gertler and Kiyotaki, 2011; Adrian and Shin, 2013). Another strand of the literature has instead addressed the liquidity provision by banks to the corporate sector through credit lines (Boot, Greenbaum, and Thakor, 1993; Kashyap, Rajan and Stein, 2002; Gatev and Strahan, 2006). Our theory contributes to these literatures by bringing together both aspects of financial intermediation and considering firms’ external financing and liquidity management problems in the same framework, and also by exploring the macroeconomic implications of banks’ liquidity provision role.

Our work is also related to the empirical literature that shows that a deterioration of the financial health of banks affects bank dependent borrowers through a contraction in credit supply (Kashyap and Stein, 2000; Peck and Rosengren, 2000; Kishan and Opiela, 2000; Ashcraft, 2006; Khwaja and Mian, 2008; Paravisini, 2008; Jimenez, Ongena, Peydró
and Saurina, 2012). We contribute to this literature by showing that the liquidity insurance channel is an additional reason why financial health of banks may matter for corporate finance and the real economy.

Our work also builds on the growing empirical literature on the role of credit lines in corporate finance (Sufi, 2009; Yun, 2009; Campello, Giambona, Graham, and Harvey, 2012; Acharya, Almeida and Campello, 2013; Acharya, Almeida, Ippolito, and Perez-Orive, 2014, 2018). Sufi (2009) shows that firms with low profitability and high cash flow risk are less likely to use credit lines and more likely to use cash for liquidity management because they face a greater risk of covenant violation and credit line revocation. Acharya, Almeida, Ippolito, and Perez-Orive (2014) demonstrate that credit line revocations following negative profitability shocks can be an optimal way to incentivize firms to not strategically increase liquidity risk and to provide incentives for the bank monitoring that can contain the illiquidity transformation problem. The most relevant paper for us is Acharya, Almeida, Ippolito, and Perez-Orive (2018), who find that banks facing liquidity shortages are significantly less willing to supply capital to borrowers under previously committed credit lines. We contribute to this literature by providing a theoretical foundation for the role of bank financial health and general economic conditions in determining the ex-post access to credit lines and the ex-ante liquidity management policy of firms.

The paper is organized as follows. In Section 2 we introduce the evidence that guides our theoretical work. In Section 3 we provide a model of the liquidity insurance channel, which we use in Section 4 to show how it matches the cross sectional distribution of bank financing observed in the data, and in Section 5 to study the impact of bank health shocks. Section 6 analyzes how the allocation of bank liquidity between loans and credit lines changes with the state of the economy. Finally, Section 7 concludes.

## 2 Empirical Evidence

We start by documenting four empirical facts that motivate our paper.

1. Usage of bank credit lines for liquidity management and of non-bank external finance is positively associated with firm size and credit quality, and usage of cash for liquidity management and of bank loans for external finance is negatively associated with firm size and credit quality.
The usage of credit lines and loans is strongly correlated with the credit worthiness of firms: Credit lines are more commonly used by larger, more profitable, higher credit quality firms (Sufi, 2009; Acharya, Almeida, Ippolito, and Perez-Orive, 2014). Instead, small, financially constrained firms tend to use cash rather than credit lines for their liquidity management. At the same time, loans rather than bonds are the main source of financing for financially constrained firms (Petersen and Rajan, 1994; Berger, Klapper and Udell, 2001), mostly because they have limited access to capital markets.

We provide additional evidence in support of this fact. Following Colla, Ippolito and Li (2013) and Acharya, Almeida, Ippolito, and Perez-Orive (2014), we use Capital IQ to obtain data on debt structure and on the use of credit lines, in conjunction with Compustat for other data on firm characteristics. The sample consists of non-utility (excluding SIC codes 4900-4949) and non-financial (excluding SIC codes 6000-6999) U.S. firms covered by both Capital IQ and Compustat from 2002 to 2011. We remove firm-years with 1) negative revenues, and 2) negative or missing assets. We regress several measures of bank loan and credit line usage on three proxies of firm credit worthiness, controlling for firm characteristics that have been shown to be important for capital structure and cash holdings (Bates, Kahle, and Stulz, 2009; Graham and Leary, 2011). As proxies for firm creditworthiness, we use firm size, age, and access to bond financing (Hadlock and Pierce, 2010).

The results of our regressions are reported in Table 1. Bank debt usage, measured as bank debt over either assets or debt, is significantly higher for young and small firms with no access to bond financing. The amount of outstanding undrawn credit lines over either assets or liquidity –where liquidity is measured as the sum of undrawn credit lines plus cash– is significantly larger for older and bigger firms with access to bond financing. Taken together, this evidence shows that firms with low credit quality use relatively more bank than market debt, and use relatively more cash than credit lines.

2. Violations of credit line covenants are frequent and lead to significant restrictions in the access to unused lines of credit.

A strand of the empirical corporate finance literature documents that debt covenants are triggered often and that violations of these covenants have significant negative effects for
firms’ access to external funds and investment. Nini, Smith, and Sufi (2012) find that, on average, between 10% and 20% of U.S. publicly listed firms breach a financial covenant in a debt contract every year, consistent with evidence, based on different samples, in Dichev and Skinner (2002), Chava and Roberts (2008), and Roberts and Sufi (2009a). Following a debt covenant violation, net debt issuance suffers a large and persistent drop (Roberts and Sufi, 2009) and investment falls significantly (Chava and Roberts, 2008; Nini, Smith, and Sufi, 2009, 2012). Although most of these studies cover not just credit line covenant violations but also bond and term loan covenant violations, cross-default provisions in credit line contracts and the common bundling of credit line agreements in larger loan packages mean that most of these breaches will affect access to outstanding lines of credit. Moreover, these estimates are likely an underestimation of the true frequency and effects of covenant violations because term loans and credit lines are often renegotiated (see Mian and Santos, 2017), possibly in anticipation of covenant violations (Roberts and Sufi, 2009b).

Some recent studies have focused specifically on violations of credit line contracts. Sufi (2009) finds that 35% of firms in his sample of firms that have access to a line of credit are in violation of a credit line covenant. Furthermore, he finds that covenant violations are associated with a loss of access to, on average, around 15 to 25% of the credit line limit. Acharya, Almeida, Ippolito, and Perez-Orive (2018) also focus explicitly on credit line covenant violations and, using hand collected detailed information on the consequences of covenant violations, find that a small fraction of these are fully waived without any consequence. A large majority of covenant violations generate a negative consequence such as an increase in spreads, an increase in collateral requirements, a reduction in the limit, or a decrease in maturity. Importantly, while large or total reductions in credit line limits are not very common, other negative consequences are very common and impose substantial restrictions on the usage of credit lines that generate reductions in usage of credit lines of a similar magnitude as the ones observed for firms that suffer a large explicit limit cut.

3 - Bank financial health affects liquidity provision through credit lines to businesses.

A large literature has documented that a deterioration of bank financial health is associated with a contraction in the supply of lending (Kashyap and Stein, 2000; Peek and Rosengren, 2000; Kishan and Opiela, 2000; Ashcraft, 2006; Khwaja and Mian, 2008; Par-

\(^2\) Importantly, most of these papers control for firm characteristics that are associated with the likelihood of a covenant violation by using techniques such as regression discontinuity design.
avisini, 2008; Jimenez, Ongena, Peydró and Saurina, 2012). While most of this literature does not test separately for effects through term loans or credit lines, a large majority of bank lending to businesses in the U.S. is in the form of credit line drawdowns (around 75%, according to Sufi (2009) and Demiroglu and James (2011)). As a result, it is very likely that these important effects of changes in bank financial health operate to a large extent through changes in the supply of liquidity through credit lines. Along these lines, Acharya, Almeida, Ippolito, and Perez-Orive (2018) provide evidence of the transmission of shocks to bank financial health to the provision of credit under precommitted lines of credit in the context of covenant violations. They show that banks that suffered an increase in their liquidity risk renegotiated significantly tougher conditions on the outstanding credit lines offered to borrowers in violation of a covenant, with capital structure and real implications for the affected firms. Huang (2010) also focuses on credit lines and shows that borrowers who had access to lines of credit with distressed banks used the lines less than those borrowers whose banks were in a better financial shape, consistent with the interpretation that distressed banks were able to restrict usage of the credit lines they had issued.

4 - The share of term loans over total bank credit (term loans plus used and unused credit lines) is countercyclical.

The share of bank lending in the form of term loans versus credit lines varies with the state of the economy. Using aggregate Call Report data of U.S. commercial banks, Gilchrist and Zakrajsek (2012) and Bassett, Chosak, Driscoll and Zakrajsek (2014) show that unused credit lines contract strongly during the early stages of economic downturns, while loans contract only at a later stage of the downturn. Mian and Santos (2017) document a similar pattern. Using the supervisory Shared National Credit database, they find that the availability of unused lines of credit is pro-cyclical.

In Panel A of Figure 1 we plot the time series of the aggregate volume of undrawn business lines of credit as a ratio of the aggregate volume of business credit, calculated as the sum of term loans and drawn and undrawn credit lines. The black solid line displays this ratio based on data for commercial and industrial (C&I) loans from quarterly Call Report filings by U.S. banks with the Federal Reserve. The red dashed line displays this ratio based on an aggregation of annual U.S. firm-level data from the S&P-Capital IQ database, after excluding utilities (SIC codes 4900-4949) and financial firms (excluding SIC codes
credit line availability growth contracts in recessions—especially in the Great Recession of 2007-09—and accelerates in booms.

The sharp decrease in unused credit lines can be driven both by lower issuance of credit lines and higher drawdowns. Ivashina and Scharfstein (2010) show, however, that the issuance of new credit lines dropped significantly during the crisis. Le (2013) shows, similarly, that banks that suffered liquidity shocks during the 2007-2009 financial crisis were more likely to reduce their exposure to credit lines than to reduce their lending. To the extent that economic downturns are associated with a weakening of banks’ financial strength, this would be additional indirect evidence in support of the idea that in downturns banks shift the allocation of liquidity to the provision of loans.

We use again the data on loans and drawdowns contained in Capital IQ to further distinguish between loan issuance and credit line drawdowns. We aggregate the data at the annual level, and compute two new measures: the ratio of outstanding drawn and undrawn lines of credit over total credit (term loans and drawn and undrawn lines of credit), and the ratio of outstanding undrawn lines of credit plus the growth in drawn lines of credit over total credit. These two measures should control for the possibility that the variation in unused lines of credit is driven by drawdown behavior. We plot the time series of these two ratios in Panel B of Figure 1. Both time series are again consistent with the procyclicality of line of credit availability.

3 A Model of Bank Lending and Liquidity Insurance Provision

In this section, we describe a model that incorporates, in the same framework, liquidity management through banks and bank lending subject to monitoring.

3.1 Firms

There is a measure 1 of firms, and each of them can invest $I$ at date 0. Entrepreneurs’ date-0 wealth is $A > 0$. Investment produces a payoff equal to $R(I)$ with probability $p$ if it
is continued until the final date, where the function $R(\cdot)$ exhibits decreasing returns to scale ($R' > 0$, $R'' < 0$). With probability $1 - p$, the project produces nothing. The probability $p$ depends on entrepreneurial effort. High effort produces a probability $p_H$, while low effort produces $p_L < p_H$ but also produces private benefits $BI$.\(^3\)

Given this setup, the entrepreneur will only put high effort if her share of the cash flow (call it $R_E$) is greater than a minimum amount:

$$p_H R_E \geq p_L R_E + BI,$$

so that the project’s pledgeable income is:

$$\rho_0(I) = p_H \left[ R(I) - \frac{BI}{\Delta p} \right],$$

where $\Delta p = p_H - p_L$. Firms can raise $(I - A)$ at date-0 from a bank, or directly from individuals (call this “market financing”). There is no discounting (the required rate of return is one).

The investment opportunity also requires an additional investment at date 1, which represents the firms’ liquidity need at date 1. The date 1 investment can be either equal to $\rho I$, with probability $\lambda$, or 0, with probability $(1 - \lambda)$. If the date-1 investment is not made the project is liquidated (no partial liquidation). Liquidation at date-1 produces a payoff equal to $\tau I \geq 0$. We assume that it is efficient to continue the investment in state $\lambda$:

$$p_H R(I) - \rho I > \tau I,$$

in the relevant range for $I$. However, firms will not have sufficient pledgeable income to continue the project in state $\lambda$ when:

$$\rho_0(I) < \rho I.$$  

In this case, to continue the project in state $\lambda$ firms need to bring liquidity from the good state of the world $1 - \lambda$. We denote the firm’s total demand for liquidity by $l(I)$, for a given investment level $I$:

$$l(I) = \rho I - \rho_0(I).$$

The firm can hold liquidity by buying treasury bonds at date-0 (e.g., cash), or by securing a bank credit line. Holding cash implies a liquidity premium $q_0$. Credit lines require a firm

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\(^3\)Bank monitoring will reduce private benefits to $b < B$, at a cost $\phi I$, as we model below.
to make a payment to the bank in state $1 - \lambda$, in exchange for the right to draw down in state $\lambda$. We describe the frictions associated with these liquidity management choices below.

Firms are potentially heterogeneous with respect to their initial wealth $A$ and liquidation payoff $\tau$. We introduce specific assumptions about the distribution of these variables below.

### 3.2 Banks

There is a single bank in this economy. The bank plays two distinct roles: monitored lending and liquidity insurance provision. We analyze them in turn.

#### 3.2.1 Monitored lending

First, the bank provides monitored financing to firms, as in Holmstrom and Tirole (1997). By paying a cost $\phi I$, which is proportional to the size of the investment, the bank can reduce managerial private benefits from $BI$ to $bI$. Because monitoring is costly, the bank must retain a stake $R_b$ in the project:

$$p_H R_b - \phi I \geq p_L R_b, \text{ or}$$

$$R_b \geq \frac{\phi I}{\Delta p}.$$  \hspace{1cm} (6)

This constraint will generally bind, and thus the income that can be pledged to investors other than the bank is now:

$$p^b_0(I) = p_H \left[ R(I) - \left( b + \frac{\phi}{\Delta p} I \right) \right]. \hspace{1cm} (7)$$

The bank is endowed with initial capital equal to $K_0$, which it uses to make loans $i_b$. The bank’s ex-ante budget constraint for a given loan level $i_b$ is:

$$i_b + \phi I \leq p_H R_b = \frac{p_H \phi I}{\Delta p}.$$  \hspace{1cm} (8)

An increase in $i_b$ transfers the bank’s monitoring rent $p_H R_b - \phi I$ to the firm, and allows the firm to invest more. This link between $i_b$ and $I$ is the channel through which bank capital affects the equilibrium in the Holmstrom and Tirole (1997) framework. Notice that bank lending cannot be higher than $(\frac{p_H}{\Delta p} - 1)\phi I$. We denote this value by:

$$i_b^{\max} = (\frac{p_H}{\Delta p} - 1)\phi I$$  \hspace{1cm} (9)
3.2.2 Liquidity insurance provision

Second, the bank provides liquidity insurance to firms through credit lines. In order to explain banks’ advantage in providing insurance to firms, we put additional structure on the distribution of liquidity shocks. At date-1, an aggregate state realizes, which determines the probability that a firm faces a liquidity shock (which is also the fraction of firms that face a liquidity shock). This probability is $\lambda_\theta$ in state $\theta$, and $\lambda_{1-\theta} < \lambda_\theta$ otherwise. Thus, the unconditional date-0 probability of a liquidity shock ($\lambda$) must obey:

$$\lambda = \theta \lambda_\theta + (1 - \theta) \lambda_{1-\theta}.$$  \hspace{1cm} (10)

We assume that in the aggregate state $\theta$ the corporate sector is not self-sufficient to meet liquidity needs, because the aggregate demand for liquidity is greater than aggregate pledgeable income:\footnote{The reverse is true in state $1 - \theta$.}

$$\lambda_\theta \rho I > \rho_0(I),$$ \hspace{1cm} (11)

in the relevant range for $I$. Thus, one should think of state $\theta$ as a state with an aggregate liquidity shock.

Banks’ advantage in providing credit lines arises from a contingent source of outside liquidity in state $\theta$, which it can use to honor credit line drawdowns. Let the amount of contingent liquidity be denoted by $D_1$. This outside liquidity can arise from the bank’s deposit-taking activities, as in Kashyap, Rajan, and Stein (2002). The bank must hold excess cash to honor deposit drawdowns, and can use this cash to honor credit line drawdowns as well. Alternatively, contingent liquidity in a bad aggregate state can arise from the mechanism in Gatev and Strahan (2006), who show that cash may flow into the banking sector following negative aggregate shocks.\footnote{Pennacchi (2006) highlights the central role that government guarantees and deposit insurance play in the role of banks as providers of liquidity simultaneously through deposits and credit lines.} Credit lines enable firms to access this contingent liquidity.

Because there is limited liquidity in state $\theta$, it is possible that the bank will revoke access to credit lines in this state. For example, in a situation in which all firms use credit lines, revocation is necessary when $D_1 < \lambda_\theta \rho I - \rho_0(I)$. We denote the probability of credit line revocation in state $\theta$ by $\mu_\theta$.

Finally, in state $(1 - \theta)$ the bank does not require excess liquidity to meet credit line...
drawdowns. In particular, this means that the probability of credit line revocation is always zero ($\mu_{1-q} = 0$).

3.2.3 Trade-off between liquidity insurance and lending

The values of $\mu_\theta$ and $i_b$ will be determined in equilibrium, as we discuss below. Essentially, the bank faces a trade-off between using capital to provide loans (increasing $i_b$), or to support credit line drawdowns (reducing $\mu_\theta$).

To increase $i_b$, the bank can choose to borrow against future contingent liquidity $D_1$, and use the proceeds to make loans. Given the assumptions above, $D_1$ can generate up to $\theta D_1$ in date-0 cash. Borrowing against contingent liquidity increases $\mu_\theta$.

The bank can also carry date-0 cash into date-1 to support additional drawdowns. In this case, the bank must also pay the liquidity premium $q_0$. Thus, $K_0$ can generate $\frac{K_0}{q_0}$ in date-1 cash. Saving cash into date-1 requires the bank to reduce date-0 lending, $i_b$. We explore how the bank decides between lending and saving in the discussion of the equilibrium below.

3.3 Optimal Decisions

The firm must decide how much to invest $I$, how much liquidity to hold, and whether to use the bank for monitored lending and/or liquidity insurance provision.

We assume that there are no additional frictions in the bank’s optimization problem, such that the bank implements the optimal solution. Given this, the bank allocates capital $K_0$ and liquidity $D_1$ to loans and credit lines, in order to maximize the aggregate welfare of all firms and the bank.

3.3.1 Feasible choices

Given the menu of choices that firms face, there are four distinct possibilities.

I. Market financing and liquidity insurance through cash In this case, the firm does not use the bank. As in Holmstrom and Tirole (1998), holding cash entails a liquidity premium, which can be thought of as a date-0 price for treasury bonds which is greater than one. Thus, if the price of treasury bonds is $q_0$, we have $q_0 > 1$. Given this, the firm’s optimization problem is:

$$\max p_H R(I) - (1 + \lambda \rho)I - (q_0 - 1)c(I) \text{ s.t.}$$

(12)
where $c(I) = \rho I - \rho_0 I$.

The optimal investment level is given by:

$$p_H R'(I_{c, \text{max}}) = 1 + \lambda \rho + (q_0 - 1)c'(I_{c, \text{max}}),$$

(13)

if the constraint does not bind at $I_{c, \text{max}}$, and:

$$A + \rho_0 (I') = (1 + \lambda \rho) I' + (q_0 - 1)c(I')$$

(14)

if the constraint binds at $I_{c, \text{max}}$. The firm’s payoff is:

$$U^c = p_H R(I^c) - (1 + \lambda \rho) I^c - (q_0 - 1)c(I^c),$$

(15)

where $I^c$ is the optimal investment level. Clearly, the payoff $U^c$ decreases with the liquidity premium in this case.

II. Market financing and liquidity insurance through credit lines

In this case, the firm uses the bank only for liquidity management. This means that the firm faces the risk of credit line revocation in the negative aggregate state $\theta$ ($\mu_\theta > 0$). Given $\mu_\theta$, the ex-ante probability of credit line revocation is $\theta \lambda_\theta \mu_\theta$, which we denote by $\mu$ to economize notation:

$$\mu \equiv \theta \lambda_\theta \mu_\theta.$$  

(16)

We take this probability as given for now, it will be derived in equilibrium below.

The firm’s optimization problem is:

$$\max (1 - \mu)p_H R(I) + \mu \tau I - [1 + (\lambda - \mu)\rho] I \text{ s.t.}$$

$$A + (1 - \mu)\rho_0 (I) + \mu \tau I \geq [1 + (\lambda - \mu)\rho] I$$

(17)

As above, we define the optimal investment as $I^{LC}$, and the associated payoff:

$$U^{LC} = (1 - \mu)p_H R(I^{LC}) + \mu \tau I^{LC} - [1 + (\lambda - \mu)\rho] I^{LC}.$$  

(18)

The payoff $U^{LC}$ decreases with the probability of credit line revocation $\mu$ given assumption (3), and it increases with the liquidation payoff $\tau$. 

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III. Bank financing and liquidity insurance through cash

As explained above, the solution in this case depends on the amount that the bank allocates to monitored lending, \( i_b \). We take this amount as given for now, and derive it in equilibrium below. The firm’s maximization problem is:

\[
\max \ p_H R(I) - (1 + \lambda \rho)I - (q_0 - 1)c^b(I) - \frac{p_H}{\Delta p} \varphi I + i_b \text{ s.t.} \]

\[ A + \rho_0^b(I) + i_b \geq (1 + \lambda \rho)I + (q_0 - 1)c^b(I), \]

where \( \rho_0^b(I) \) is defined above in equation (7) and \( c^b(I) = \rho I - \rho_0^b(I) \).

We define the optimal firm investment by \( I_{cb} \), and the associated payoff by:

\[
U_{cb} = p_H R(I_{cb}) - (1 + \lambda \rho)I_{cb} - (q_0 - 1)c^b(I_{cb}) - \frac{p_H}{\Delta p} \varphi I_{cb} + i_b. \tag{20}
\]

The payoff to the bank in this case is equal to:

\[
U_{cb}^b = p_H \varphi I_{cb} - i_b - \varphi I_{cb} \geq 0. \]

This payoff is strictly positive whenever bank loan supply is constrained and \( i_b < i_b^{max} \).

IV. Bank financing and liquidity insurance through credit lines

If the firm uses bank monitoring and the credit line, the solution will depend both on the amount that the bank allocates to monitored lending \( (i_b) \), and the probability of credit line revocation \( \mu \).

The firm’s maximization problem is:

\[
\max (1 - \mu) p_H R(I) + \mu \tau I - [1 + (\lambda - \mu)\rho]I - (1 - \mu)\frac{p_H}{\Delta p} \varphi I + i_b \text{ s.t.} \]

\[ A + (1 - \mu)\rho_0^b(I) + \mu \tau I + i_b \geq [1 + (\lambda - \mu)\rho]I \]

We define the optimal firm investment by \( I_{cb} \), and the associated payoff by:

\[
U_{cb} = (1 - \mu)p_H R(I_{cb}) + \mu \tau I_{cb} - [1 + (\lambda - \mu)\rho]I_{cb} - (1 - \mu)\frac{p_H}{\Delta p} \varphi I_{cb} + i_b. \tag{22}
\]

The payoff to the bank in this case is equal to:

\[
U_{cb}^b = (1 - \mu)p_H \varphi I_{cb} - i_b - \varphi I_{cb} \geq 0. \]

This payoff is strictly positive whenever bank loan supply is constrained and \( i_b < i_b^{max} \), where \( i_b^{max} \) in the case of liquidity insurance through credit lines is given by:
\[ i_{b,LC}^{\text{max}} = \left( \frac{(1 - \mu)p_H}{\Delta p} - 1 \right) \varphi I. \]  

3.4 Equilibrium

Firms choose the solution that maximizes their payoff, given their characteristics and the key variables \( q_0 \) (liquidity premium), \( i_b \) (the amount of funds that they can borrow from the bank), and \( \mu \) (the probability of credit line revocation). In order to characterize the solution, we proceed as follows. First, we discuss some intuitive properties of the solution, taking the bank variables \( \mu \) and \( i_b \) as given. Second, we provide a definition of the equilibrium in our framework. Third, we investigate how the bank will choose \( \mu \) and \( i_b \) to implement equilibria.

3.4.1 Properties of the equilibrium

Consider first the choice of whether to use bank monitoring. As in Holmstrom and Tirole (1997), the main benefit of using bank monitoring is to increase the firm’s pledgeable income. Consider for example equations (15) and (20). By choosing bank financing, the firm reduces its payoff for a given investment level, by an amount equal to \( \frac{p_H}{\Delta p} \varphi I_b - i_b \), which is positive by equation (8). Thus, the firm will only choose bank monitoring if it leads to a higher investment level (\( I_c^b \) must be higher than \( I_c \)). It follows then that the benefit of using bank monitoring will be higher when financial constraints are tight and the investment level is low. Since there are decreasing returns to scale, highly constrained firms (for example, firms with very low \( A \)) will benefit the most from bank monitoring.

Consider now the choice between cash and credit lines. The obvious trade-off is between the cost of credit line revocation, and the liquidity premium. Take the difference between the payoffs in (18) and (15) to obtain:

\[
U^{LC} - U^c = p_H \left[ R(I^{LC}) - R(I^c) \right] - (1 + \lambda \rho)(I^{LC} - I^c) + (q - 1)c(I^c) - (1 + \lambda \rho)(I^{LC} - I^c) + (q - 1)c(I^c) - (24)
\]

\[-\mu[p_H R(I^{LC}) - (\rho + \tau)I^{LC}].
\]

The credit line reduces cash holdings, and thus avoids the liquidity premium \((q - 1)c(I^c)\). However, the credit line also introduces a risk of credit line revocation which is associated with value loss \( \mu[p_H R(I^{LC}) - (\rho + \tau)I^{LC}] \), which is positive by assumption (3). In particular, notice that the value loss with liquidation \( \mu[p_H R(I^{LC}) - (\rho + \tau)I^{LC}] \) is decreasing with the
parameter $\tau$. Firms with high $\tau$ have lower liquidity risk, since they lose less value conditional on credit line revocation.

When firms are financially constrained, increases in $A$ also reduce the loss of value conditional on credit line revocation. Since the production function has decreasing returns to scale, an increase in $A$ that raises the firm’s ability to invest lowers the marginal return to investment. The benefits of liquidation $(\rho + \tau)I^LC$ increase linearly with investment, which means that the loss of value $\mu[p_H R(I^LC) - (\rho + \tau)I^LC]$ decreases with $A$ when firms are financially constrained. In sum, high $\tau$ and high $A$ both increase the incentive to choose credit lines for liquidity management.

### 3.4.2 Equilibrium definition

An equilibrium can be defined as follows:

- Firms pick the highest payoff among $U^c$, $U^{LC}$, $U^b_c$ and $U^{LC}_b$, given their type $(A, \tau)$, and given $\mu = \mu^*$ and $i_b = i^*_b$.
- The bank’s optimization problem results in $\mu = \mu^*$ and $i_b = i^*_b$, given bank endowments $K_0$ and $D_1$, and given optimal choices by all firms in the economy.
- The market for treasury bonds clears at price $q^*_0$, that is, the demand for treasury bonds at $q^*_0$ is less or equal to the supply.

To simplify the problem, we will generally assume a constant liquidity premium $q^*_0 > 1$, and a supply of treasury bonds that is large enough so that all firms that demand cash can access bonds at a price $q^*_0$.

Since there are no frictions on the bank’s optimization problem, one can characterize this problem as choosing the best possible allocation of bank endowments to loan and credit line provision, given firms’ expected reactions to changes in $\mu$ and $i_b$. Effectively, the bank plays the role of "social planner."

### 3.4.3 Bank’s optimization problem

In order to better understand the bank’s problem, it is useful to work with a specific assumption for the distribution of firm types.
Take for example the simplest case in which all firms are identical (same $A$ and $\tau$), and thus make the same choice of financing or liquidity management in equilibrium. The main goal of this example is to illustrate how the equilibrium is determined in the model.

Suppose that $A$ is low enough, so that it is efficient to use monitored lending as we explained above ($U^c$ and $U^{LC}$ are low). We must still determine whether the bank will also provide credit lines in equilibrium or not (whether $U^{LC}_b$ is higher than $U^c_b$). We now show what is the trade-off that drives this choice.

If initial capital $K_0$ is not enough to fund lending to firms in date-0, given by $i^{\text{max}}_b$ or $i^{\text{max}}_{b,LC}$, then the bank can increase firms’ payoffs by borrowing against its date-1 contingent liquidity, and making additional loans at date-0. If we denote bank borrowing by $W_0$, we have:

$$i_b(W_0) = K_0 + W_0, \quad (25)$$

for $W_0 \leq \theta D_1$. The trade-off is that moving funds to date-0 will increase the probability of credit line revocation in state $\theta$. If firms choose to use credit lines, the total demand for drawdowns by firms that face a liquidity shock is $\lambda_\theta(\rho I - \rho_0(I))$, which by assumption (11) is larger than the total pledgeable income generated by firms that do not face a liquidity shock, $(1 - \lambda_\theta)\rho_0(I)$. The bank also has external liquidity equal to $D_1 - \frac{W_0}{\theta}$. If external liquidity is not enough, the bank will need to revoke access to credit lines with positive probability, such that:

$$[1 - \mu_\theta(W_0)]\lambda_\theta(\rho I - \rho_0(I)) = (1 - \lambda_\theta)\rho_0(I) + D_1 - \frac{W_0}{\theta}, \quad (26)$$

and thus $\mu_\theta$ increases with $W_0$. The ex-ante probability of revocation is then $\mu(W_0) = \theta\lambda_\theta\mu_\theta(W_0)$.

Equation (26) equates the demand for funds from credit line drawdowns to the sources of funds for such drawdowns. On the left hand side of the equation, we have the demand for liquidity that arises from the firms that face a liquidity shock and that are not subject to revocation. On the right hand side of the equation, liquidity can be obtained from the firms that do not suffer a liquidity shock, as well as from the amount $D_1$ that banks receive, minus the amount used by banks for lending in the previous date.

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6We assume that when the bank does not have enough liquidity to satisfy all credit line drawdowns, there is a random sequential servicing so that some firms are fully revoked and the rest get access to their entire credit line.

7It can also save additional funds into date-1, but since the bank pays the same liquidity premium that firms do ($q^*_0$), this action is equivalent to increasing cash holdings at the firm level.
The bank picks \( W_0 \) to maximize the aggregate payoff of firms and the bank. \( U_c^b + U_c^{bank} \) is maximized by making \( W_0 \) as high as possible, \( W_0 = \theta D_1 \). Let \( W^*_0 \) be the value that maximizes \( U_b^{LC} + U_{bank}^{LC} \). If

\[
U_c^b(W_0 = \theta D_1) + U_c^{bank}(W_0 = \theta D_1) > U_b^{LC}(W^*_0) + U_{bank}^{LC}(W^*_0),
\]

then the optimal bank action is to borrow against future liquidity up to the maximum amount \( \theta D_1 \), and make loans at date-0. If in contrast

\[
U_b^{LC}(W^*_0) + U_{bank}^{LC}(W^*_0) > U_c^b(W_0 = \theta D_1) + U_c^{bank}(W_0 = \theta D_1),
\]

then the bank borrows up to \( W^*_0 \), and firms use the bank both for lending and for liquidity insurance provision.

4 Cross Sectional Distribution in the Use of Bank Finance

We now introduce an example that illustrates an equilibrium with firm heterogeneity. Our goal is to show how the model can deliver an equilibrium that is consistent with the empirical fact discussed in the introduction. Profitable firms, firms with low cash flow variance and high credit ratings are more likely to use credit lines for their liquidity management (Sufi, 2009). But firms with high credit-quality (high credit rating, profitable firms) are less likely to be bank-dependent for their regular financing. In our model, this would correspond to an equilibrium in which low credit quality firms choose \( U_c^b \), and higher credit quality firms choose \( U^{LC} \).

The model is able to generate such an equilibrium under fairly general conditions. The only requirement is that the two sources of firm heterogeneity are not strongly correlated in a particular way. Specifically, firms with low net worth (low \( A \)) must not also have significantly lower liquidity risk (high \( \tau \)). As discussed above in Section 3.4.1, low \( A \) increases the benefit of bank monitoring by raising the marginal productivity of investment. Low \( A \) also increases the value of cash by raising the marginal product of capital and the bankruptcy costs of foregone output. And high liquidity risk (low \( \tau \)) increases the value of cash by mitigating the risk of credit line revocation.

The empirical evidence is consistent with a positive correlation between \( A \) and \( \tau \). Using the Compustat-Capital IQ sample of publicly listed U.S. firms for the period 2002-2011, we
find that firm size and cash-flow volatility, proxies respectively for $A$ and $\tau$, are negatively correlated. The average volatility of quarterly cash-flows, measured as a share of total assets, is 5.8% for firms in the lowest quartile of firm size and 1.2% for firms in the top quartile.\footnote{Cash-flow volatility is calculated as the standard deviation of operating income (Compustat item 13) over the previous 12 quarters, scaled by total assets (6).} Another proxy for $\tau$ is the likelihood that the firm needs to draw on its liquid reserves. To capture this, we define a liquidity event as a year in which firm profitability is negative and the sum of the decrease in cash holdings and the increase in drawn credit lines is positive. The likelihood of a liquidity event is 23.1% for firms in the lowest quartile of firm size, and only 1.3% for firms in the top quartile of firm assets. Other studies have suggested that large firms are on average more diversified (Rajan and Zingales (1995)), and have a lower likelihood of bankruptcy (Griffin and Lemmon (2002)).

To show this result, consider an example in which half of the firms have high $A$ and high $\tau$, and the other half has low $A$ and low $\tau$. We want to characterize an equilibrium in which the first type of firm (high) chooses $U_{LC}^t$, and the second type (low) chooses $U_{C}^t$. Let $I_{LC}^t$ and $U_{LC}^t$ represent the investment and payoff of the first type of firm, and $I_{C}^t$, $U_{C}^t$ represent the corresponding variables for the second type.

We characterize the demand for loans and credit line drawdowns from firms. The aggregate demand for date-0 loans is:

$$i_b(I_{b,low}^c) = (1 + \lambda \rho)I_{b,low}^c + (q_0 - 1)\epsilon(I_{b,low}^c) - A - \rho_0(I_{b,low}^c).$$

As long as $i_b(I_{b,low}^c) < i_b^{max}(I_{b,low}^c)$, $i_b$ is an increasing function of $I_{b,low}^c$. The bank can then increase $I_{b,low}^c$ by allocating funds to make date-0 loans:

$$\frac{1}{2}i_b(I_{b,low}^c) \leq K_0 + W_0,$$

for $W_0 \leq \theta D_1$. This equation shows that $I_{b,low}^c$ (and thus $U_{b,low}^c$) increases with $W_0$.

Alternatively, the bank can reduce $W_0$ to support credit line drawdowns:

$$\frac{1}{2}[1 - \mu_0(W_0)]\lambda_0(\rho I_{high}^{LC} - \rho_0(I_{high}^{LC})) = \frac{1}{2}(1 - \lambda_0)\rho_0(I_{LC}^t) + D_1 - \frac{W_0}{\theta}.$$ 

This equation implies that $\mu_0(W_0)$ increases with $W_0$, and thus $U_{LC,high}$ decreases with $W_0$. It follows that increasing $W_0$ will increase the payoff for low quality firms, but reduce the payoff for high quality ones. The bank will choose the optimal $W_0^*$ such that:

$$W_0^* = \arg \max \left[ \frac{1}{2} \left( U_{b,low}^c(W_0^*) + U_{bank,low}^c(W_0^*) \right) + \frac{1}{2} U_{LC,high}^t(W_0^*) \right].$$
Finally, for this situation to be an equilibrium, firms must not have an incentive to change their optimal choices:

\[ U_{b,low}(W_0^*) = \max\{U_{b,low}^c(W_0^*), U_{low}^c(W_0^*), U_{low}^{LC}(W_0^*), U_{b,low}^{LC}(W_0^*)\}, \quad (31) \]

\[ U_{high}^{LC}(W_0^*) = \max\{U_{b,high}^c(W_0^*), U_{high}^c(W_0^*), U_{high}^{LC}(W_0^*), U_{b,high}^{LC}(W_0^*)\}. \quad (32) \]

### 4.1 Excess Capital

Consider first an example in which there is sufficient capital \( K_0 \) in the economy to satisfy the maximum demand for date-0 loans:

\[ \frac{1}{2} \hat{I}_{b,low}^{max} (I^c_{b,low}) \leq K_0. \quad (33) \]

In this case, the bank’s decision is simple because there is no benefit of borrowing against future liquidity. This means that \( W_0^* = \frac{1}{2} \hat{I}_{b,low}^{max} (I^c_{b,low}) - K_0 \leq 0 \), minimizing the probability of credit line revocation.

Given this result, it suffices to show that \( U_{b,low}^c(W_0^*) = \frac{1}{2} \hat{I}_{b,low}^{max} (I^c_{b,low}) - K_0 \) is the highest payoff for low quality firms, and \( U_{high}^{LC}(W_0^*) = \frac{1}{2} \hat{I}_{b,low}^{max} (I^c_{b,low}) - K_0 \) is the highest possible payoff for high quality firms. We do this in the numerical example below.

### 4.2 Limited Capital

Suppose now that capital is limited, such that:

\[ \frac{1}{2} \hat{I}_{b,low}^{max} (I^c_{b,low}) > K_0. \quad (34) \]

Now, the bank must choose the optimal level of \( W_0^* \). As we characterized above, the trade-off is that increasing \( W_0 \) increases the probability of revocation (hurting high quality firms), but relaxes financial constraints for low quality firms (increasing \( I^c_{b,low} \)). As equation (30) suggests, the bank’s choice will depend on the marginal impact of \( W_0 \) on the payoffs of high- and low-quality firms. We characterize a solution in the example below.

### 4.3 Numerical Example

Assume that the production function is represented by:

\[ R(I) = kI^\alpha, \]
Liquidity Management | Financing | \( I_{low} \) | \( U_{low} \)  
--- | --- | --- | ---  
cash | bank | 3.65 | 7.31  
credit line | bank | 3.73 | 7.11  
cash | market | 1.86 | 6.70  
credit line | market | 1.86 | 6.38  

Table 2: Optimal Investment Levels and Payoffs of Different Liquidity Management and Financing Alternatives for Low Credit Quality Firms

where \( k = 8 \). We also assume that \( p_H = 0.8, p_L = 0.4, B = 1.8, \) and \( \rho = 1.4 \). The date-0 price of cash is \( q_0 = 1.05 \). The high credit-quality firm has high net worth \((A_{high} = 25)\), and high liquidation value \((\tau_{high} = 0.75)\). The low credit-quality firm has \( A_{low} = 0 \), and \( \tau_{low} = 0 \). There is an equal measure of low and high credit-quality firms. Our assumptions about the states are: \( \theta = 0.1, \lambda_0 = 1, \) and \( \lambda_{1-\theta} = 0.20 \). The bank parameters are as follows: the cost of monitoring \( \phi \) is \( 0.4 \), and the managerial private benefits with monitored financing, \( b_1 \), is \( 1.4 \).

4.3.1 Excess capital

We assume initially that capital is large so that date-0 loans are not constrained. Under our benchmark example this requires that

\[
K_0 \geq \frac{1}{2} i_b \max (I^{C}_{b,low}) = 0.731,
\]

so we set \( K_0 = 0.731 \) for simplicity. Finally, \( D_1 = 5 \).

We first compute \( U^{C}_{b,low} \), the payoff for low quality firms, given that capital is large (so that \( i_b = i_b^{\max} \)). In our example, \( U^{C}_{b,low} = 7.31 \) and \( I^{C}_{b,low} = 3.65 \). Table 2 shows that \( U^{C}_{b,low} \) is the highest possible payoff for low credit quality firms.\(^9\)

Firms are acting optimally by choosing to manage liquidity through cash and to access external financing through banks. Accessing market financing reduces significantly their ability to pledge output, obtain external finance, and invest. Managing liquidity through lines of credit exposes them to costly liquidation and low expected returns to investment due to their low liquidation value.

\(^9\)Throughout this exercise we evaluate firms’ alternative financing and liquidity management options under the equilibrium level of \( \mu^* = 4.72\% \) that obtains when low credit quality firms choose \( U^{C}_{LC} \), and higher credit quality firms choose \( U^{LC}_C \).
Turning to high quality firms, $I_{\text{high}}^{\text{LC}}$ and $\mu$ depend on each other so must be determined jointly through the equations:

$$
\frac{1}{2}(1 - \mu)\lambda \rho (I_{\text{high}}^{\text{LC}} - \rho_0(I_{\text{high}}^{\text{LC}})) = \frac{1}{2}(1 - \lambda)\rho_0(I_{\text{high}}^{\text{LC}}) + D_1 - \frac{W_0}{q_0}
$$

(36)

$$
\mu \equiv \theta \lambda \rho \mu
$$

(37)

and the firm’s optimization problem in 3.3.1. It is either the first-order condition if the budget constraint doesn’t bind:

$$
(1 - \mu)\rho H R'(I_{\text{high}}^{\text{LC}}) + \mu \tau_{\text{high}} I_{\text{high}}^{\text{LC}} - [1 + (\lambda - \mu)\rho] = 0.
$$

if the constraint binds. These two equations determine $\mu^*$ and $I_{\text{high}}^{\text{LC}}$, and thus $U_{\text{high}}^{\text{LC}}$.

Under our benchmark example, we get that the date-0 probability of revocation in date-1 is $\mu^* = 4.72\%$, and that $I_{\text{high}}^{\text{LC}} = 13.53$ and $U_{\text{high}}^{\text{LC}} = 11.64$. Table 3 shows that $U_{\text{high}}^{\text{LC}}$ is the highest possible payoff for high credit quality firms when $\mu^* = 4.72\%$.

High quality firms are acting optimally by choosing to manage liquidity through credit lines and to access external financing through arms length finance. High credit quality firms are financially unconstrained in equilibrium so they prefer market financing, which is cheaper than bank financing. On the other hand, managing liquidity through lines of credit exposes them to the risk of revocation, but their costs of liquidation are small and do not justify incurring costly cash accumulation to prevent liquidation.

We provide an analysis of firm optimal liquidity management and financing for a range of values for $A, \tau$. The results are in Figure 2. Low credit quality firms ($A_{\text{low}} = 0, \tau_{\text{low}} = 0$)
choose cash for liquidity management and bank financing. If only $A$ increases (vertical axis), firms first move away from bank monitoring but still use cash. Increasing $\tau$, on the other hand, provides the incentive for firms to switch into credit lines. Higher $A$ also increases the incentives to shift into credit lines. This result is due to decreasing returns to scale - the overall costs of liquidation (which include the foregone date-2 output) are lower on the margin for high net worth firms whose investment scale is larger. So high $A$ reduces the marginal cost of bankruptcy, and induces firms to switch to credit lines.

4.3.2 Limited capital

We now analyze the case in which capital $K_0$ is limited. Specifically, we assume that $K_0 = 0.25$. The rest of the parameters is set as in the previous section. The lower bound of the relevant range for $W_0^*$ is the value below which low credit quality firms choose not to use bank financing and thus there is no optimal allocation in which $W_0 > 0$. Under our calibration that value is $W_0^* = 0.2$. The upper bound of the relevant range for $W_0^*$ is the minimum of two values: the value that causes $i_b^*$ to reach $i_b^{\text{max}}$ and the maximum borrowing capacity of the bank. Under our calibration, the value of borrowing necessary to achieve $i_b^{\text{max}}$ is

$$0.5 i_b^{\text{max}} - K_0 = 0.48,$$

and the maximum amount the bank can borrow is $\theta D_1 = 0.5$. We will thus evaluate aggregate welfare in the range $[0.20, 0.48]$. We compute the payoffs of low quality firms, high quality firms, and the bank, respectively $U_{c,low}^c$, $U_{c,high}^{LC}$ and $U_{c,\text{bank}}^c$. For each value of $W_0^*$, we restrict lending to low quality firms to be:

$$i_b^* = 2 (K_0 + W_0^*).$$

We proceed to solve the equilibrium value for $I_{c,high}^{LC}$ and $\mu^*$ jointly as in the previous section, and also check that $U_{c,high}^{LC}$ and $U_{c,\text{low}}^{c}$ are the highest possible payoffs for the levels of $\mu^*$ and $i_b^*$ for all values of $W_0^*$ considered.

The results of this exercise are plotted in Panel A of Figure 3, where in the horizontal axes of the four panels we plot the range of values of $W_0^*$ considered. When $W_0^* = 0.2$, the

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\footnote{We check that, under our calibration, an allocation in which $W_0^* = -K_0$ and no firm uses bank financing is not optimal.}
unconditional probability of a revocation is $\mu = 0.069$, which implies that the conditional probability of revocation is $\mu_\theta = 0.69$ because $\mu \equiv \theta \lambda_\theta \mu_\theta$ with $\theta = 0.1$ and $\lambda_\theta = 1$. In other words, there is enough liquidity in state $\theta$ in date-1 to meet around 31% of credit line drawdowns. Increasing $W_0$ at that point is welfare improving as it increases date-0 lending to low credit quality firms. However, increased bank borrowing affects liquidity provision in date-1 and $\mu_\theta$ rises.\footnote{Increases in $\mu_\theta$ are associated with increases in investment by high credit quality firms because pledgeable output increases due to higher expected revenues from liquidation and lower liquidity costs.}

The bank trades off an increase in investment by low quality firms, against an increase in the probability of revocation for high quality firms in the bad aggregate state. The former turns out to be more important in our current example because bankruptcy costs, which include foregone output and capital destruction, are small for high quality firms ($\tau_{\text{high}} = 0.75$). As a result, the bank finds it optimal to increase $W_0$ to the maximum amount possible, which is $W_0^* = 0.48$. In this equilibrium, the unconditional probability of revocation is $\mu^* = 9.8\%$, which implies that $\mu_\theta = 98\%$.

Instead, if bankruptcy costs for high credit quality firms are high (low $\tau_{\text{high}}$), the bank has less of an incentive to borrow against its date-1 liquidity because it wants to maintain a low probability of credit line revocations. Consider the case in which $\tau_{\text{high}} = 0.25$. Panel B in Figure 3 displays the implications of different choices for $W_0$. The figure shows that the bank will find it optimal to borrow as little as possible from date-1, resulting in $\mu_\theta = 0.67$ and $W_0^* = 0.20$.

5 Impact of Bank Health Shocks

We begin with the comparative statics in the case of excess capital ($i^*_b = i^{\text{max}}_b$). Since banks already allocate the maximum possible amount to date-0 loans, shocks to bank health affect only credit line provision. In Panel A of Figure 4 we plot the effect of a variation in $D_1$ from its benchmark value of 5 down to a value of 0. We observe the following results. A decrease in $D_1$ increases the probability of credit line revocation, conditional on a negative aggregate shock ($\mu_{\theta}^*$). This result follows directly from equation (29). Notice that, in this case, $W_0 = 0$. In the model, the \emph{ex-post} impact of the revocation of credit lines is absorbed entirely by
continuation investment, which is $\rho I_{\text{high}}$. This effect holds for high-quality firms that suffer a liquidity shock in the negative aggregate state $\theta$. Thus, we have that a decrease in $D_1$ reduces date-1 investment by firms that rely on credit lines for liquidity management (high-quality firms), conditional on a negative aggregate shock, and conditional on a firm-level liquidity shock.\(^{13}\)

The model also has implications for how shocks to bank health affect ex-ante liquidity management. Starting from an equilibrium in which high quality firms rely on credit lines, a decrease in $D_1$ may cause these firms to switch from credit lines to cash for their liquidity management. While in the present example, this effect is not visible, under plausible alternative calibrations this could occur. In that case, there are negative consequences for investment by high quality firms because holding cash is costly.

In the case of limited capital ($i_b^* < i_b^{\text{max}}$), shocks to $K_0$ and $D_1$ have similar implications because the bank optimally allocates funds to loans and credit lines. There are two possible cases. If $W_0^* = W_0^{\text{max}}$ (which occurs when $\tau_{\text{high}}$ is sufficiently high), then we are in a corner solution in which the bank allocates all funds to make loans. Starting from such a situation, a decrease in bank health (lower $K_0$ or $D_1$) reduces bank loans and investment of low quality firms. This case is displayed in Panel B of Figure 4. High quality firms are not affected initially since the bank does not use liquidity to support credit lines. Following a sufficiently strong shock to $D_1$, however, low credit quality firms find it optimal to switch to market financing and the bank reallocates its liquidity to date-1 liquidity provision. High credit quality firms benefit from a decrease in the probability of revocation in that scenario.

The case of $W_0^* < W_0^{\text{max}}$ occurs when $\tau_{\text{high}}$ is sufficiently low, or in other words, when bankruptcy costs are sufficiently high for high credit quality firms. In this case, a shock to bank health can affect both low- and high-quality firms. In our example in Panel B of Figure 3, in which $\tau_{\text{high}} = 0.25$, we obtain a corner solution in the choice of $W_0^*$, which in our case corresponds to $W_0^* = 0.2$. A decrease in $D_1$ in this case has similar implications as in the case with excess bank capital displayed in Panel A of Figure 4: the probability of credit line revocation increases, reducing these firms’ payoffs, even though their investment increases. Loans to low quality firms remain unchanged.

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\(^{13}\)Note that ex-ante (date-0) investment of high-quality firms, displayed in the bottom left panel, increases as the probability of revocation increases. This happens when $1 + \lambda \rho < \rho + \tau$, which is the case in our calibration. Despite the increase in investment, welfare of high-quality firms decreases.
6 Loans and Credit Lines across the Business Cycle

In this section, we analyze how the allocation of bank liquidity to loans and credit lines changes with the state of the economy. We show that because loans are used by relatively weaker firms, when bad times hit, the marginal value of allocating bank liquidity to loans increases, relative to the marginal value of saving cash to honor future drawdowns. To keep the analysis focused, we simplify the setup by keeping the ex-ante investment of high quality firms ($I_{high}^{LC}$) fixed.\footnote{As the example of Section 5 shows, the main effect of bank liquidity on high quality firms operates via the change in the probability of credit line revocation and of the continuation investment.}

The bank’s optimization problem is described by equation (30). We assume that the budget constraint for low quality firms binds and thus:

$$U_{low} = p_HR(I_{low}) - (1 + \lambda \rho)I_{low} - (q_0 - 1)c(I_{low}) - \frac{p_H}{\Delta p} \varphi I_{low} + i_b$$  \hspace{1cm} (38)

$$I_{low} = A_{low} + \rho_0I_{low} + i_b - \lambda \rho I_{low} - (q_0 - 1)c(I_{low}).$$  \hspace{1cm} (39)

Also:

$$U_{bank} = \frac{p_H \varphi I}{\Delta p} - i_b - \varphi I.$$  \hspace{1cm} (40)

Thus we can write:

$$U_{low} + U_{bank} = v(I_{low}),$$  \hspace{1cm} (41)

where $v(I_{low})$ is an increasing and concave function of $I_{low}$ defined as $v(I_{low}) = p_HR(I_{low}) - (1 + \lambda \rho)I_{low} - (q_0 - 1)c(I_{low}) - \varphi I_{low}$, and $I_{low}$ is an increasing function of $i_b$ which we write as $I_{low}(i_b)$.

We can write $U_{high}$ as:

$$U_{high} = p_H R(I_{high}) - (1 + \lambda \rho)I_{high} - \mu[p_H R(I_{high}) - (\tau + \rho)I_{high}].$$  \hspace{1cm} (42)

Given the assumption that $I_{high}$ is fixed and does not depend on the bank’s liquidity choice, this utility can be written as:

$$U_{high} = \overline{U} - \mu Y,$$  \hspace{1cm} (43)

where $\overline{U} = p_H R(I_{high}) - (1 + \lambda \rho)I_{high}$ and $Y = p_H R(I_{high}) - (\tau + \rho)I_{high}$ represents the cost of financial distress for high quality firms (the payoff loss in case they are liquidated).
The bank’s allocation of liquidity will affect $U_{high}$ through the probability of revocation $\mu$. We assume $\lambda_\theta = 1$ (all firms face a liquidity shock in state $\theta$) and thus:

$$\mu \equiv \theta \mu_\theta,$$

(44)

where $\theta$ is the probability of an aggregate liquidity shock and:

$$\mu_\theta = 1 - \frac{D_1 - \frac{W_\theta}{\theta}}{\frac{1}{2}[\rho I_{high} - \rho_0(I_{high})]},$$

(45)

The expression in equation (45) is obtained from equation (26) setting $\lambda_\theta = 1$ under the assumption that high quality firms occur with probability $\frac{1}{2}$. The bank uses liquidity $D_1 - \frac{W_\theta}{\theta}$ to support credit line drawdowns and $\frac{1}{2}[\rho I_{high} - \rho_0(I_{high})]$ is the aggregate demand from high quality firms.

The bank’s optimization problem can then be written as:

$$W_0^* = \arg \max \frac{1}{2} v(I_{low}) + \frac{1}{2}(U - \mu Y),$$

s.t. (46)

$$K_0 + W_0^* = \frac{1}{2} i_b,$$

(47)

$$I_{low} = I_{low}(i_b),$$

$$\mu \equiv \theta \left(1 - \frac{D_1 - \frac{W_\theta}{\theta}}{\frac{1}{2}[\rho I_{high} - \rho_0(I_{high})]}\right).$$

(48)

An interior solution for $W_0$ must obey:

$$\frac{1}{2}[\rho I_{high} - \rho_0(I_{high})] = v'(I_{low}) I_{low}'(i_b).$$

(49)

The left hand side is the marginal benefit of saving liquidity in the bank, which is to reduce liquidation losses for high quality firms in the bad aggregate state. The right hand side is the marginal benefit of borrowing against contingent liquidity to make additional loans to low quality firms (this increases their investment and payoff).

To study how the allocation of bank liquidity to loans and credit lines changes with the state of the economy we introduce two comparative statics exercises meant to capture financial aspects of the business cycle: changes in firm net worth and changes in bank financial health. Firm net worth is considered by a large literature in macroeconomics, starting with the seminal paper of Bernanke and Gertler (1989), to be an important state variable driving the tightness of the external financing constraints of firms. In our setting, a decrease in firm net worth $A$ will increase $W_0^*$. Notice that $I_{low}$ is an increasing function of
Thus, a reduction in $A_{low}$ and increases the marginal benefit of investment by constrained firms ($v'(I_{low})$ goes up). The effect of $i_b$ on $I_{low}$ is determined by the budget constraint in equation (39). The function $\rho_{0,low}(I_{low})$ will inherit the concavity from $R(I_{low})$, thus a reduction in $A_{low}$ will increase the marginal effect of $I_{low}$ on pledgeable income. Similarly $c(I_{low}) = \rho I_{low} - \rho_{0,low}(I_{low})$, so $-c(I_{low})$ is concave. Given this, the effect of $i_b$ on $I_{low}$ should also increase as $A_{low}$ goes down. The term $v'(I_{low})I_{low}'(i_b)$ thus increases, increasing the marginal benefit of date-0 loans.

The marginal benefit of saving liquidity $\frac{Y}{2[\rho_1 h_{high} + \rho_2 h_{high}]}$ is independent of $A_{high}$ since we assumed that investment $h_{high}$ is fixed. Thus, a reduction in $A_{high}$ will not change the marginal benefit of bank liquidity, and thus bank liquidity goes down with $A$. If one interprets bad times as a situation when firm net worth is low, then bank liquidity should go down in bad times as the bank makes additional loans to low quality firms by shifting liquidity from credit line provision.

A contraction in bank credit supply, on the other hand, has been identified to be a key factor behind some of the recent recessions in developed countries, such as the Great Recession of 2007-2009 and the downturn of the early 1990s. We capture changes in bank financial health by changes in bank contingent liquidity $D_1$. A decrease in $D_1$ will not change $W^*_0$. Notice that $D_1$ affects $\mu$ in (48), but that $\mu$, in turn, does not affect the equilibrium choice of $W^*_0$ in (49). As a result, the provision of liquidity insurance (which is proportional to $\mu$) falls, while loan provision remains constant. If one interprets bad times as a situation when bank liquidity is low, then bank allocation of credit should shift in favor of loans to low quality firms at the expense of credit line provision.

6.1 Example

To illustrate this result, we go back to the numerical analysis of Sections 4 and 5 and solve for the equilibrium using the same parameter values that set above. We set $K_0 = 0.25$, and choose $(\tau_{high} = 0.75)$ so that the equilibrium value of $W^*_0$ is an interior solution within the admissible range of $W^*_0$ described in Section 4.3.2 $(\tau_{high} = 0.575)$.

The results for changes in $A_{low}$ are displayed in Figure 5 and confirm the intuition above: if one interprets bad times as a situation when firm net worth $A_{low}$ is low, then bank
borrowing should go up in bad times. The figure shows that bank borrowing drops from around 0.48 when firm net worth is low ($A_{low} = 0$) to around 0.28 when firm net worth is high ($A_{low} = 0.4$). Bank lending to low credit quality firms is negatively related to firm net worth $A_{low}$, so to keep their investment constant. The payoff of high credit quality firms increases with firm net worth $A_{low}$, while the payoff of low credit quality firms remains constant. The bank finds this solution optimal, because the welfare of high credit quality firms has lower sensitivity to changes in the allocation of bank liquidity.

The results for changes in $D_1$ are displayed in Figure 6 and also confirm the intuition above: if one interprets bad times as a situation when bank contingent liquidity $D_1$ is low, then the allocation of bank resources shifts toward lending to low-quality firms in bad times (see the middle-right panel). The figure shows that the share of total bank credit ($K_0 + W_0^* + D_1 - \frac{W_0}{\theta}$) dedicated to lending ($K + W_0^*$) goes from around 0.35 when bank contingent liquidity is high ($D_1 = 6$) to around 11.50 when bank contingent liquidity is low ($D_1 = 4$). Bank lending to low credit quality firms remains constant, which means that the residual liquidity available to satisfy credit line drawdowns decreases, raising the frequency of revocations.

7 Conclusions

Four empirical observations motivate this paper. First, financially weaker firms depend on loans for funding, while stronger firms do not depend on loans but use banks for liquidity insurance. Second, violations of credit line covenants are frequent and lead to significant restrictions in the access to unused lines of credit. Third, bank financial health affects liquidity provision through credit lines to businesses. Fourth, the share of bank lending that goes to term loans versus credit lines increases in bad times. We develop a model that can generate these results, which pertain to the cross section and to the time series. The model shows that these results are linked: because term loans go to low quality firms, the marginal benefit of loans relative to credit lines tend to go up in bad times.

The results of our theoretical work and descriptive empirical analysis shed light on how the choice of banks to allocate liquidity between term loans and credit lines affects liquidity
and investment by high quality and low quality (or large versus small) firms, and on how this choice varies over the economic cycle. Our framework has the potential to assess the macroeconomic consequences of regulatory initiatives, such as bank capital requirements or liquidity coverage ratios, in a setting where there is heterogeneity among firms in the corporate sector in the nature of their reliance on bank finance.
References


Figure 1: Time series of the aggregate outstanding stocks of undrawn business lines of credit and of term loans. Panel A displays the time series of the aggregate share of undrawn business lines of credit over total business credit (term loans and drawn and undrawn credit lines). The black solid line displays this ratio based on data for commercial and industrial (C&I) loans from quarterly Call Report filings by U.S. banks with the Federal Reserve. The red dashed line displays this ratio based on an aggregation of annual U.S. firm-level data from the S&P-Capital IQ database, after excluding utilities (SIC codes 4900-4949) and financial firms (excluding SIC codes 6000-6999). Panel B displays two additional ratios using the same Capital IQ data: the ratio of outstanding drawn and undrawn lines of credit over total credit (term loans and drawn and undrawn lines of credit) in the solid black line, and the ratio of outstanding undrawn lines of credit plus the growth in drawn lines of credit over total credit in the red dashed line.
Figure 2: Optimal liquidity management and financing for a range of values of $\tau$ and $A$. 

```
Liquidity: credit line
Financing: market
```

```
Liquidity: cash
Financing: bank
```

```
Liquidity: credit line
Financing: market
```

```
Liquidity: cash
Financing: bank
```
**Figure 3:** Optimal bank choice of liquidity provision when bank capital is limited

**PANEL A: Low bankruptcy costs ($\tau = 0.75$)**

- Probability of Revocation
- Loans ($i_b$) and LCs ($D_1 - W_0/\theta$)
- Investment (normalized)
- Welfare (normalized)

**PANEL B: High bankruptcy costs ($\tau = 0.25$)**

- Probability of Revocation
- Loans ($i_b$) and LCs ($D_1 - W_0/\theta$)
- Investment (normalized)
- Welfare (normalized)

Legend:
- **Low Credit Quality Firms**
- **High Credit Quality Firms**
- **Aggregate**
Figure 4: Effect of a shock to bank contingent liquidity $D_1$

PANEL A: Excess bank capital ($K_0 = 0.75$)

- Probability of Revocation
- Bank Lending
- Investment (normalized)
- Welfare (normalized)

PANEL B: Limited bank capital ($K_0 = 0.25$)

- Probability of Revocation
- Loans ($i_b$) and LCs ($D_1 - W_0/\theta$)
- Investment (normalized)
- Welfare (normalized)

Legend:
- Low Credit Quality Firms
- High Credit Quality Firms
**Figure 5:** Sensitivity of the bank's optimal liquidity allocation to variations in economic conditions (proxied by $A_{\text{low}}$).
Figure 6: Sensitivity of the bank’s optimal liquidity allocation to variations in economic conditions (proxied by $D_1$).
Table 1: Bank Loan and Credit Line Usage. This table presents Tobit regression results to study the relation between bank debt usage, credit line usage, and firm credit quality. The sample consists of non-utility (excluding SIC codes 4900-4949) and non-financial (excluding SIC codes 6000-6999) U.S. firms covered by both Capital IQ and Compustat from 2002 to 2011. We have removed firm-years with 1) negative revenues, and 2) negative or missing assets. All control variables are lagged. Undrawn Credit Ratio is calculated as undrawn credit lines over total liquidity, where total liquidity is equal to the sum of undrawn credit lines and cash holdings. Robust standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

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Robust t-statistics in parentheses. *** p<0.01, ** p<0.05, * p<0.1