The Impact of Social Media on Belief Formation*

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Abstract

Social media are becoming increasingly important in our society and change the way people communicate, how they acquire information, and how they form beliefs. Experts are concerned that the rise of social media may make interaction and information exchange among like-minded individuals more pronounced and therefore lead to increased disagreement in a society. This paper analyzes a learning model with endogenous network formation in which people have different types and live in different regions. I show that when the importance of social media increases, the amount of disagreement in the society first decreases and then increases. Simultaneously, people of the same type hold increasingly similar beliefs. Furthermore, people who find it hard to communicate with people in the same region may interact with similar people online and consequently hold extreme beliefs. Finally, I propose a simple way to model people who neglect a potential correlation of signals and show that these people may be made worse off by social media.

JEL Codes: C72, D72, D83, D85, Z10, Z19

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1 Introduction

Social media are becoming an increasingly important part of many people's lives and our society. The number of Facebook's active users worldwide rose from 431 million at the beginning of 2010 to 1.71 billion in the second quarter of 2016.[1] On average, users spend more than 50 minutes per day on Facebook, Instagram, and Facebook messenger.[2] This is changing the way people communicate, how they acquire information, and how they form beliefs.[3] Social media advertisement revenue in the U.S. is estimated to exceed twelve billion USD, and the traffic to news sites from Facebook surpassed the traffic from Google in early 2015, while it had been less than half of the traffic from Google as late as 2013.[4] In fact, 48% of Americans with Internet access received news about politics from Facebook in 2014.[5] At the same time, research has found that people tend to communicate with similar people (McPherson, Smith-Lovin and Cook 2001, Gentzkow and Shapiro 2011). Moreover, the algorithms of some social media platforms make it easier to see what some individuals post and filter out other content. Sunstein (2001, 2009), Van Alstyne and Brynjolfsson (2005), and Pariser (2011) argue that social media and the Internet in general allow people to mostly communicate with similar individuals, which would expose these people to only certain kinds of information and may lead to increased segregation and polarization. Indeed, Americans disagree about Obama's religion, the existence of weapons of mass destruction in pre-2003 Iraq, and the reality of global warming.[6]

To answer the question of how social media impact belief formation, this paper combines a model of strategic network formation with learning: Agents have different types, live in different regions and form links to acquire additional information from other agents in order to make a decision in

[1] See `http://www.statista.com/statistics/264810/number-of-monthly-active-facebook-users-worldwide/` (accessed Sept.1, 2016).

[2] See `http://www.nytimes.com/2016/05/06/business/facebook-bends-the-rules-of-audience-engagement-to-its-advantage.html` (accessed Sept.1, 2016).

[3] See e.g., Bailey, Cao, Kuchler and Stroebel (2016) for geographically-distant friends' influence on real estate purchasing behavior.

[4] See `http://www2.biakelsey.com/webinars/biakelsey-u-s-local-advertising-forecast-for-2016-key-findings.pdf` and `http://fortune.com/2015/08/18/facebook-google/` (both accessed Sept.1, 2016); media outlets such as Wired, The Atlantic, Reuters, and The Daily Telegraph belong to Parse.ly's clients.

[5] See `http://www.journalism.org/2014/10/21/section-2-social-media-political-news-and-ideology` (accessed Oct.2, 2016).

[6] See `https://d3n8a8pro7vhmx.cloudfront.net/aai/pages/11126/attachments/original/1450651184/2015_American_Attitudes_Toward_Arabs_and_Muslims.pdf?1450651184`, `http://view2.fdu.edu/publicmind/2015/150107/`, and `www.eenews.net/assets/2016/04/27/document_cw_01.pdf` (all accessed Sept. 2nd, 2016).

the near future. The cost of links to other agents depends on two dimensions: The costs are lower if (i) the agents have more similar types and (ii) if the agents live in the same region. If they do not live in the same region, agents can form inter-regional links, for example by interacting online. While the links are formed for decisions in the near future, information that agents gather through these links and over time also more and more through indirect links influences agents' opinions in the long-run as well. This allows me to analyze the opinion formation in the long-run. I show that the extent and the speed of information aggregation for opinion formation follows an inverse-U-shape with respect to the cost of inter-regional links, i.e., up to a certain point, the increasing importance of social media leads to more and faster information, but beyond that point it leads to less and slower information aggregation. This implies that the growing influence of social media and progress in information and communication technology in general decreases disagreement in a society first, but facilitates disagreement if it grows even further. Social algorithms like Facebook's Top Stories accelerate and amplify these effects. At the same time, social media make people more likely to connect with similar people and let them disagree with different people for longer (or possibly forever). This may increase tensions between different parts of a society and prevent people from learning from each other. People who have more incentives to acquire information or who find it more difficult to find similar people in their own region form a higher fraction of links via social media. In addition, I explain how social media can help previously isolated people to form links with other people, but also how social media may make them more likely to hold extreme beliefs.

By incorporating correlation neglect (i.e., the unawareness that information may be correlated) in a simple portable way, I demonstrate how social media can hurt the decisions of people with correlation neglect and how social media can lead to more disagreement within a country, but less disagreement across countries. While rational people always benefit from a decrease in inter-regional linking cost, people with correlation neglect might be worse off.

The intuition for why disagreement in a society is U-shaped with respect to the importance of social media is as follows. If inter-regional communication becomes less costly, agents will form links to agents of the same type in other regions and will form fewer links within their own region. This has two effects: First, more links to other regions allow an agent to observe signals in other regions earlier and therefore reduce the heterogeneity of beliefs across regions. Second, fewer links within an agent's own region mean that the agent forms fewer links to different types. So it takes the agent longer to observe the signals of very different types and therefore the heterogeneity of beliefs across types increases. At a high inter-regional linking cost, when not many links to agents in other regions are formed, the first effect dominates and disagreement decreases. Only when there exist many links across regions and the cost of interacting with agents in other regions is further reduced, the second effect dominates and leads to more disagreement. Along with the disagreement, the inaccuracy of opinions in the long-run follows a U-shape with respect to the inter-regional linking cost. Interestingly, disagreement arises despite rational updating and ex ante symmetry with respect to agents' beliefs and their expected signals.

Social algorithms like Facebook's Top Stories accelerate and amplify the above effects. These algorithms make information of similar types more visible and information of different types (whom people might know from their region but also communicate with via Facebook) less visible. This is equivalent to further reducing the inter-regional linking cost and simultaneously increasing the cost of forming links to different types. As a consequence, agents form more inter-regional and fewer domestic, i.e., intra-regional, links. This reduces disagreement and makes opinions more accurate if only few inter-regional links were formed without social algorithms, but intensifies disagreement and inaccuracy of opinions if agents obtain most of their information via similar types even without social algorithms. Similarly, when disagreement between types leads to a higher cost of communicating with other types, a vicious circle may emerge that leads to more segregation and disagreement: More disagreement leads to a higher cost of inter-type communication, and this in turn leads to more disagreement. Indeed, evidence suggests that communication costs with other types might have increased: According to the Pew Research Center (2014), the fractions of Democrats and Republicans who see the other party as "a threat to the nation's well-being" have increased to 27% and 36%, respectively, and 15% of Democrats and 17% of Republicans would be unhappy if a family member married a person of the other party.

Sorting by types increases with decreasing inter-regional linking cost, because agents form more inter-regional links to agents of their own type and fewer links with other types. Then, agents observe signals from similar types more often and signals from different types less often. If types reflect different ideologies and agents receive type-specific signals, then agents' beliefs become more consistent with an ideology, which is what the Pew Research Center (2014) finds: The share of consistent liberals rose from 5% in 1994 to 23% in 2014, and the share of consistent conservatives rose from 13% to 20%.

For agents with high stakes such as professionals sorting is more pronounced, because they have higher incentives to acquire information and form more links.[7] At least some of these additional links are likely to be inter-regional, because linking to different types becomes increasingly expensive for more distant types, while inter-regional links are available at a uniform cost. Similarly, agents who are very different from most other agents make use of inter-regional links at a higher inter-regional linking cost than agents with moderate types and they form a higher fraction of inter-regional links. The reason is that for agents with very different types, it is more expensive to talk to agents in their own region, because the types of those agents are further away on average. As a real-world example, 47% of consistently conservatives and 32% of consistently liberals say that posts about politics on Facebook are mostly or always in line with their own views, while among all respondents only 23% say so.[8] Relatedly, agents with extreme types might form no links at all, because the number of domestic links required to potentially change their decision is too expensive. When the inter-regional linking cost is low enough for them to form links, their decision-making is better most of the time. However, there is a chance that they observe a higher number of wrong signals, which leads to confident wrong beliefs and thus makes them more likely to take extreme actions, with potentially negative consequences for other people.[9]

An extension of the model with correlation neglect predicts that social media can lead to an increased disagreement within countries, but reduced disagreement across countries. If people watch a lot of news on national TV, the correlation of information in a given city is probably lower than the correlation of information in a political movement, i.e., there are fewer region-specific signals than type-specific signals, and so increased segregation by types will lead to more disagreement. At the same time, the correlation of information in a given country may be higher than the correlation of information of people with a certain political preference, so reduced segregation by regions will lead

[7] See e.g., Poole and Rosenthal (1984, 1985, 1997), Layman and Carsey (2002), Harbridge and Malhotra (2011), Noel (2014) and McCarty, Poole and Rosenthal (2016) for evidence of the polarization of and sorting in Congress. Furthermore, Rosenblat and Möbius (2004) find that the collaboration between distant similar economists has increased from the 80s to the 90s, while the collaboration between close dissimilar economists has decreased.

[8] See `http://www.journalism.org/2014/10/21/political-polarization-media-habits` (accessed Oct.2, 2016).

[9] For example, in many past mass shooting incidents, the perpetrators radicalized themselves by interacting with similar people in online forums, and ISIS is well-known for recruiting in Western societies online. Social media are not the only reason for this development, of course, but they probably are a promoting factor.

to less disagreement. Another prediction is that when type-specific correlation is strong, agents with correlation neglect can be worse off when the inter-regional link formation cost decreases, while rational agents are better off. As a consequence, the difference in utilities between people with correlation neglect and rational people increases.

Long-run beliefs play an important role for voting: If the society does not aggregate all available information, it is more likely to make suboptimal choices. Even though forming additional links is too costly for an individual because each individual is unlikely to be pivotal, the aggregate effect of incomplete social learning and disagreement for the society might be large. Beyond politics and voting behavior, social learning and disagreement matter in a variety of other settings. First, if agents hold very different beliefs about the value of an asset, they may engage in belief-neutral inefficient trading according to Brunnermeier, Simsek and Xiong (2014). Moreover, in such a setting the realization of an outcome may lead to more jumps and therefore a higher volatility. Second, if doctors with different specializations fail to aggregate information, it might take them longer to learn about an adequate medical treatment or they might even never learn about it (Eyster and Rabin 2014). Third, different agents may not adopt new products or technologies such as fertilizer (Rosenblat and Möbius 2004) for a longer time when they do not learn about all the available signals. This may also affect marketing strategies and distort incentives for innovation.

The remainder of the paper is organized as follows. Section 2 relates the paper to the literature. Section 3 introduces the model. Section 4 derives and discusses the main results. Section 5 presents some extensions, including the case of agents with correlation neglect. Section 6 concludes. All proofs can be found in the appendix.

2 Related Literature

This paper contributes to five strands of literature. First, this paper extends a new and still small literature which combines learning with endogenous network formation.[10] The most closely-related

[10] There are several papers that look at either just learning in networks or just network formation. For learning in networks, see e.g., DeGroot (1974), Bala and Goyal (1998), DeMarzo, Vayanos and Zwiebel (2003), Gale and Kariv (2003), Golub and Jackson (2010), Mueller-Frank (2011), Acemoglu, Dahleh, Lobel and Ozdaglar (2011), Golub and Jackson (2012), Jadbabaie, Molavi, Sandroni and Tahbaz-Salehi (2012), Jadbabaie, Molavi and Tahbaz-Salehi (2013), and Mossel, Sly and Tamuz (2015). For network formation, see e.g., Jackson and Wolinsky (1996), Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002), Jackson and Rogers (2005), Hojman and Szeidl (2008), and Galeotti and Goyal (2010).

paper from a technical perspective is Acemoglu, Bimpikis and Ozdaglar (2014). In their paper, which focuses on the importance of so-called information hubs (some highly connected agents) for social learning, agents start with free links to other agents in exogenously given groups and linking to other agents is possible at a constant cost. By contrast, in my paper, there are no exogenous groups, and linking costs have more than just two different levels. As a consequence, the arising network structures are different. Song (2016) looks at a sequential decision process in which agents can decide to observe some of their predecessors' actions and finds that agents' influence must be infinitesimally influential for maximal learning to occur. In my paper agents can observe signals rather than actions and all agents decide simultaneously. Tan (2015) also combines opinion formation with endogenous network formation and, furthermore, a preference of people to talk to people with the same beliefs. She finds that there is a steady state with either consensus or extreme polarization. In contrast to my paper, however, she looks at a very different network formation process, in which in every period some randomly selected agent can delete or form a certain link. Moreover, there is no learning but opinions are chosen. None of the above papers looks at a gradual change in inter-regional linking cost, which is the focus of my paper. Further, none of them analyzes a framework in which agents have different types and live in different regions.

Second, this paper adds to the literature which investigates the consequences of progress in inter-regional communication technologies. Rosenblat and Möbius (2004) find that such technological progress always decreases the separation between two individuals when the number of agents is large (which is equivalent to decreasing disagreement in my paper), while Van Alstyne and Brynjolfsson (2005) find that technological progress always leads to more separation (unless agents have a preference for diversity). By contrast, my paper finds a U-shaped relationship between technological progress and the degree of separation; while Rosenblat and Möbius (2004) do not find this result because of the combination of randomness and only two types in their model, Van Alstyne and Brynjolfsson (2005) do not look at long-run effects.

Third, this paper provides another explanation of how agents can become more polarized, hold inaccurate beliefs, or agree to disagree (Aumann 1976). The explanations in the literature include different priors and bimodal preferences (Dixit and Weibull 2007), different priors and uncertain signal interpretations (Glaeser and Sunstein 2013, Acemoglu, Chernozhukov and Yildiz 2016), multi-dimensional beliefs and private information (Andreoni and Mylovanov 2012, Glaeser and Sunstein 2013), confirmatory bias (Rabin and Schrag 1999), correlation or selection neglect (Glaeser and Sunstein 2013), wishful thinking (Bénabou 2013, Le Yaouang 2016), learning with a misspecified model of the world (Bohren 2016, Heidhues, Kőszegi and Strack 2016), not taking into account strategic behavior of others (Eyster and Rabin 2010, Glaeser and Sunstein 2013, Gagnon-Bartsch and Rabin 2016), and reputation-concerned journalists (Shapiro 2015). While all of those explanations might be factors that play a role for forming more extreme beliefs, this paper focuses on rational updating and updating with correlation neglect in a setting of common priors and unidimensional beliefs. It investigates the change in polarization due to technological progress.

Fourth, this work speaks to an empirical literature that looks at the polarization of voters. So far, the evidence for increased polarization beyond sorting is mixed at best. (See for example Fiorina, Abrams and Pope (2005), Abramowitz and Saunders (2008), Hetherington and Weiler (2009), Gentzkow and Shapiro (2011), Falck, Gold and Heblich (2014), Lelkes, Sood and Iyengar (forthcoming), Fiorina (2016), and Poy and Schüller (2016).) My paper offers an explanation for why it might take a while before significanty increased polarization can be observed.

Fifth, this paper contributes to the literature of modeling correlation neglect and selection neglect and examining the consequences and implications.[11] Implicitly, all papers using models that build on DeGroot (1974) and DeMarzo et al. (2003) incorporate correlation neglect. More explicitly, correlation neglect is modeled by a series of papers by Levy and Razin.[12] The most closely related one of those papers in terms of application is Levy and Razin (2015b). It finds that correlation neglect may help to aggregate information in elections, because voters are more likely to vote according to their information rather than their political preference. Ortoleva and Snowberg (2015) show that correlation neglect leads to overconfidence, more extreme beliefs, and a higher voter turnout. In my paper, correlation neglect also leads to overconfidence but is never beneficial. Similar to those papers, correlation neglect in this paper is modeled in such a way that the belief of agents with correlation neglect coincides with the rational belief when correlation is absent. In this paper correlation neglect is modeled in an even simpler way and therefore more portable.

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[11] For experimental evidence on correlation neglect and selection neglect, see Eyster and Weizsäcker (2011), Enke and Zimmermann (2015), and Enke (2015). For evidence that people may avoid double-counting in other situations, see Möbius, Phan and Szeidl (2015).

[12] Levy and Razin (2015a, 2015b, 2015c, 2016a, 2016b).

3 Model

This section presents the basic model with rational agents and explains how agents form links, what signals they observe, and how they make their decisions.

There are $n \in \mathbb{N}$ regions with $m \in \mathbb{N}$ different agents each. Let agent θk be an agent of type $\theta \in M = \{1, ..., m\}$ in region $k \in N = \{1, ..., n\}$. I assume that both the number of types m and the number of regions n are large enough so that agents can always form another link to an agent in the same region or to an agent with the same type. There are $\bar{t} + 1$ periods: $t = 0, 1, ..., \bar{t}$, where $\bar{t} \geq 2$.

Decision problem. There is an unknown true state $r \in \{0, 1\}$, where $Prob(r = 1) = Prob(r = 0) = 0.5$. At the end of period $t = 1$, each agent decides between $d_{\theta k} = 1$ and $d_{\theta k} = 0$ and tries to match the state. She receives a gross utility of $\overline{u}_{1\theta}$ if $d_{\theta k} = r = 1$, $\underline{u}_{1\theta}$ if $d_{\theta k} = 1 \neq r$, $\overline{u}_{0\theta}$ if $d_{\theta k} = r = 0$, and $\underline{u}_{0\theta}$ if $d_{\theta k} = 0 \neq r$, where $\overline{u}_{i\theta} > \underline{u}_{j\theta}$ for all $i \neq j \in \{0; 1\}$. For example, an agent optimizes her links for decision problems in which her utility depends on whether something is good or popular. More concretely, she might wonder whether to watch a movie. If the movie is going to win an Academy Award ($r = 1$), an agent would like to watch the movie ($d_{\theta k} = 1$), while she would rather not watch the movie ($d_{\theta k} = 0$) if it is not going to win an Academy Award ($r = 0$).

Link formation. At the beginning of $t = 0$ each agent can form directed links to other agents.[13] These links are long-run relationships and cannot be changed later. It is more costly for an agent to form a link to an agent who does not live in the same region and it is the costlier the further away that agent's type (e.g., ideology or income) is. If agent θk wants to form a link to agent ηl, this costs her $c_T|\theta - \eta| + c_R \mathbb{1}_{l \neq k}$, where $c_T > 0$ is the type-specific linking cost and $c_R > 0$ is the inter-regional linking cost. I assume that an agent rather forms an inter-regional link to a close-by region than to a region further away, potentially because of cultural or language differences.[14] This is a conservative assumption, because a link to a region further away would allow information to spread even faster.

The network consisting of agents and links can be represented by an *adjacency matrix* A, of

[13] A link is *directed* if θk could have no link to ηl, even if ηl had a link to θk. One interpretation would be that ηl pays attention to θk, but θk does not pay attention to ηl.

[14] One could model this as an extra cost of $\varepsilon|k - l|$ per link, where $\varepsilon > 0$ is very small.

which entry $a_{\theta k,\eta l} = 1$ if there is an (unweighted) link from agent θk to agent ηl and 0 otherwise. The total cost of link formation for agent θk is the sum of the cost of all her links, $C(\mathbf{a_{\theta k}}) := \sum_{\eta\mathbf{l} \in \mathbf{M}\times\mathbf{N}:\mathbf{a_{\theta k,\eta l}=1}} (\mathbf{c_T}|\theta-\eta| + \mathbf{c_R}\mathbb{1}_{\mathbf{l}\neq\mathbf{k}})$, where $\mathbf{a_{\theta k}}$ is row θk of the adjacency matrix $A \in \{0;1\}^{m\cdot n\times m\cdot n}$. I interpret the increasing importance and availability of social media in people's lives as a decrease in the inter-regional cost.

Signals and beliefs. In $t = 0$, after having formed links, each agent observes a set of signals, $S^1_{\theta k}$. This set consists of up to three different types of signals: $\overline{s}_U \geq 1$ signals uniquely obtained by one agent, $\overline{s}_T \geq 0$ type-specific signals, and $\overline{s}_R \geq 0$ region-specific signals. Each $s^i_{\theta k} \in S^1_{\theta k}$, $i \in \{1, ..., \overline{s}_U + \overline{s}_T + \overline{s}_R\}$, is either 0 or 1 and equal to r with probability $\alpha \in (0.5, 1)$. Sometimes I will refer to the signals as *high* and *low*. Unless noted otherwise, signals are (conditional on the true state) independent and identically distributed. Signals uniquely obtained by one agent are always (conditional on the true state) independent and identically distributed. Every agent obtains some information no one else obtains by herself. Type-specific signals are the same for all agents of the same type, i.e., $s^i_{\theta k} = s^i_{\theta l}$ for all $i \in \{\overline{s}_U + 1, ..., \overline{s}_U + \overline{s}_T\}$. For example, agents of the same type might watch the same TV programs. Similarly, region-specific signals are the same for all agents in the same region, i.e., $s^i_{\theta k} = s^i_{\eta k}$ for all $i \in \{\overline{s}_U + \overline{s}_T + 1, ..., \overline{s}_U + \overline{s}_T + \overline{s}_R\}$. For example, agents in the same region might read the same local newspaper.

In $t = 1$, an agent also observes the signals of the agents she has formed links to. For example, if an agent has formed a link to an agent with the same type but in a different region, she will observe $\overline{s}_T$ independent type-specific signals (because the other agent obtains the same type-specific signals), $2\overline{s}_R$ region-specific signals (because the other agent obtains different region-specific signals), and $2\overline{s}_U$ uniquely obtained signals. These are in total $\overline{s}_T + 2\overline{s}_R + 2\overline{s}_U$ (conditional on the true state) independent and identically distributed signals.

Opinions. An agent also uses the links she formed to make an optimal decision in $t = 1$ to form an opinion about other questions that barely affect her utility. Exactly as in her decision problem above, there is a true state $r \in \{0, 1\}$, where $Prob(r = 1) = Prob(r = 0) = 0.5$, each agent observes $\overline{s}_U + \overline{s}_T + \overline{s}_R$ that can be uniquely obtained by one agent, type-, or region-specific in $t = 0$, and in $t = 1$, she also observes the signals of the agents she has formed links to. In addition to that, in period $t > 1$, an agent also observes the signals the agents she has formed links to have observed in the previous period $t-1$, i.e., the signals of her contacts at a geodesic distance of t.[15] She then forms her beliefs in a Bayesian way. Consequently, opinion formation differs from the belief formation regarding her decision problem in the following two ways. First, opinion formation is an ongoing process that can take place for a longer time and also takes into account the information of indirect contacts. Second, her links are not formed in order to hold accurate opinions, e.g., because the utility differences between different opinions are far less important than the utility differences that are possible for her decision problems or because she does not take into account the indirect effects of links when forming links. I prefer the first interpretation because it allows for the agents' behavior to be considered rational. This means that I will investigate opinion formation as a byproduct of links that were formed for information acquisition.[16]

Let $S^t_{\theta k}$ be the set of signals agent θk has observed up until and including period t. For example, $S^1_{\theta k} = \{s^i_{\theta k} : i \in \{1, ..., \overline{s}_U + \overline{s}_T + \overline{s}_R\}\} \cup \{s^i_{\eta l} : a_{\theta k,\eta l} = 1 \text{ and } i \in \{1, ..., \overline{s}_U + \overline{s}_T + \overline{s}_R\}\} = \bigcup_{\eta l: a_{\theta k,\eta l}=1} S^0_{\eta l} \cup S^1_{\theta k}$. Furthermore, let $\underline{S}^t_{\theta k}$ be the set of conditionally independent signals agent θk has observed up until and including period t.

Timing. In $t = 0$, agents form links and then receive a signal. In $t = 1$, agents observe the signals of the agents they have formed links to and then make a decision. In each $t > 1$, agents observe the signals of agents at a geodesic distance of t, which allows us to look at how agents' opinions develop.

Formal maximization problem. In $t = 1$ each agent maximizes her expected utility from the decision given the signals she observed:

$$\mathbb{E}[u_\theta(r, d_{\theta k}(S^1_{\theta k}))] := \tag{1}$$

$$\max_{d_{\theta k}(S^1_{\theta k})\in[0,1]} d_{\theta k} \quad [Prob(r = 1|S^1_{\theta k})\overline{u}_{1\theta} + (1 - Prob(r = 1|S^1_{\theta k}))\underline{u}_{1\theta}]$$
$$-(d_{\theta k} - 1) \quad [Prob(r = 1|S^1_{\theta k})\underline{u}_{0\theta} + (1 - Prob(r = 1|S^1_{\theta k}))\overline{u}_{0\theta}] \,.$$

[15] A *path* from agent θk to agent ηl is a sequence of links that connects θk to ηl, i.e., such that $a_{\theta k,\sigma(11)} = a_{\sigma(11),\sigma(12)} = a_{\sigma(12),\sigma(13)} = ... = a_{\sigma(\zeta i),\eta l} = 1$ for some permutation σ on the set of agents in the network, where ζi is an agent in the network. The *shortest path* is the path with the minimum number of such links. The *geodesic distance* is the length of the shortest path.

[16] Similarly, Allen and Gale (2000) look at contagion in a network that was formed for risk-sharing.

Obviously, if the expression following $d_{\theta k}$ is strictly greater than the expression following $-(d_{\theta k}-1)$, it is optimal for the agent to choose $d_{\theta k}=1$, and $d_{\theta k}=0$ if it is strictly smaller. In case the two expressions are equal, i.e., the agent is indifferent, I assume that she chooses each alternative with probability one half. For the sake of brevity, I will not state that the agent mixes with equal probability for each alternative in lemmas, propositions, nor corollaries. Her (unconditional) *expected utility from the decision* is equal to $\mathbb{E}[\mathbb{E}[u_\theta(r, d_{\theta k}(S^1_{\theta k}))]]$. In $t=0$, each agent forms links that maximize her expected utility which consists of the expected utility from the decision minus the cost for the links:

$$\max_{\mathbf{a}_{\theta k}\in\{0,1\}^{mn}:a_{\theta k\theta k}=0} \mathbb{E}[U_{\theta k}] := \max_{\mathbf{a}_{\theta k}\in\{0,1\}^{mn}:a_{\theta k\theta k}=0} \mathbb{E}[\mathbb{E}[u_\theta(r, d_{\theta k}(S^1_{\theta k}))]] - C(\mathbf{a}_{\theta k}). \tag{2}$$

4 Analysis

This section starts by calculating how agents decide in $t=1$ and then what links they form in $t=0$. After that it will look at the consequences for the resulting decisions in $t=1$. Finally, this section will show what opinions will be formed.

4.1 Belief Formation and Decision

When an agent observed i high and j low conditionally independent signals, her belief that $r=1$ is equal to $\frac{\binom{i+j}{i}\alpha^i(1-\alpha)^j}{\binom{i+j}{i}\alpha^i(1-\alpha)^j+\binom{i+j}{i}(1-\alpha)^i\alpha^j} = \frac{\alpha^{i-j}}{\alpha^{i-j}+(1-\alpha)^{i-j}}$. Because this expression only depends on the difference between high and low conditionally independent signals for a given α, this difference is a sufficient statistic for the agent's belief. Let $z^t_{\theta k}$ be the difference between high and low conditionally independent signals agent θk observed up until period t.

Lemma 1. *(i) $z^t_{\theta k}$ is a sufficient statistic for agent θk's belief in period t.*
(ii) In period $t=1$ the agent chooses $d_{\theta k}=1$ if and only if $z^1_{\theta k} > z^*_\theta := \frac{\log\left(\frac{\overline{u}_{1\theta}-\underline{u}_{0\theta}}{\overline{u}_{0\theta}-\underline{u}_{1\theta}}\right)}{\log\left(\frac{1}{\alpha}-1\right)}$.

How agents decide at the time they have to make a decision depends on their beliefs about the state of the world and what they can gain by matching the state (or lose by not matching it), depending on each state. This can be captured by a threshold of the difference between high and low observed conditionally independent signals. This threshold also has (among others) the following expected properties: First, when the utilities in the different states are symmetric, the agent chooses $d_{\theta k} = 1$ if and only if she received more high than low conditionally independent signals. Second, when the utility in any state increases after $d_{\theta k} = 1$ (or the utility in any state decreases after $d_{\theta k} = 0$), the necessary threshold for the difference between high and low conditionally independent signals z^*_θ decreases. Third, when the precision of the signal increases, i.e., $\alpha \to 1$, the threshold decreases, too.

The model captures the simple intuition that agents benefit from more independent signals. An exception is the case $|\underline{S}^t_{\theta k}| < |z^*_\theta|$. Then, signals are not of any value to an agent, because her decision will be independent of the signals as those will never be enough to exceed the threshold. Another exception may occur when the threshold of the difference between high and low signals, z^*_θ, is an integer. Then agents may only benefit from every two additional signals. For example, imagine a situation in which the different utility levels are exactly symmetric with respect to the states, so $z^*_\theta = 0$. If an agent has an odd number of signals, one additional signal will not help her, because it either will not influence her decision (if the threshold is either exceeded or not with and without the additional signal) or make the agent indifferent and not provide a strictly higher utility (if the threshold is met exactly with the additional signal). Consequently, she does not benefit from just one additional signal.[17]

Lemma 2. *(i) The expected utility from the decision,* $\mathbb{E}[\mathbb{E}[u_\theta(r, d_{\theta k}(S^1_{\theta k}))]]$*, is increasing in* $|\underline{S}^t_{\theta k}|$*. For* $|\underline{S}^t_{\theta k}| \geq |z^*_\theta|$*, it is strictly increasing in every additional conditionally independent signal if* $z^*_\theta \notin \mathbb{Z}$ *and strictly increasing in every two additional conditionally independent signals if* $z^*_\theta \in \mathbb{Z}$*.*
(ii) The additional expected utility from the decision approaches zero for $|\underline{S}^t_{\theta k}| \to \infty$*.*

Lemma 2 states that usually agents value additional information, but the value-added is decreasing. Therefore, the willingness to pay for two additional links is positive and decreasing and the number of links agents want to form is limited for a given cost per link. This helps us to avoid corner solutions if link formation is not prohibitively costly (so that no agents would want to form links), link formation is not free (so that agents would want to link to everybody), and the number of regions, n, is large (so that agents have a choice whom to link to).

[17] For brevity and clarity, I will ignore the non-generic cases in which $z^*_\theta \in \mathbb{Z}$ and assume $z^*_\theta \notin \mathbb{Z}$ for the rest of this paper after Lemma 2.

4.2 Link Formation

This section investigates how agents form links. A first observation is captured by the following lemma.

Lemma 3. *Agents form inter-regional links only with agents of their own type.*

Forming links to other types is cheaper within an agent's own region and forming links to an agent in another region is cheapest if this agent has the same type. Recall that we assumed that the number of regions is large, and so the agent can always link to another agent of her own type. Note that Lemma 3 also holds if agents receive type- or region-specific signals, despite an agent with a different type in a different region providing potentially more conditionally independent signals at a first glance. However, an agent rather forms links to that type in her own region and to an agent of her own type in a different region.

Lemma 3 simplifies the analysis a lot, because it allows us to focus on the trade-off between the most expensive domestic link versus a link to an agent of the same type in another region.

Moreover, Lemma 3 implies that there will be sorting by types when inter-regional links are formed. If an agent forms a link to agents of the same type, she will observe more signals (and their realizations) of agents of her type. Thus, her beliefs and also her decisions will become more similar to those of other agents of her type. As an example, if types are ideologies, people who form inter-regional links will make decisions more similar to the decisions of people of the same ideology and become more consistent with one ideology. This is in line with the finding of the Pew Research Center (2014) that the share of consistent liberals rose from 5% in 1994 to 23% in 2014, and the share of consistent conservatives rose from 13% to 20%.

The following lemma captures the intuition that if inter-regional links are cheaper, agents will generally form more links, but will also substitute domestic links with inter-regional links.

Lemma 4. *(i) Suppose $\overline{s}_T \geq \overline{s}_R$. Agents form (weakly) more links when the inter-regional linking cost, c_R, decreases.*
(ii) Conditional on forming a positive number of links, agents form (weakly) fewer domestic links when the inter-regional linking cost, c_R, decreases.

There are two subtleties. First, the condition that the number of type-specific signals should not be lower than the number of region-specific signals, $\overline{s}_T \geq \overline{s}_R$, ensures that replacing domestic links with inter-regional links does not make fewer links optimal. If the condition did not hold, so $\overline{s}_T < \overline{s}_R$, it could happen that two domestic links are replaced with just one inter-regional link, because an inter-regional link provides access to more information. A second inter-regional link might not be worth the cost due to decreasing value-added of additional signals. Second, the number of domestic links might increase with a decreasing cost of inter-regional links if an agent did not form links at all at a higher cost of inter-regional links, as the next proposition will show.

Let me first define what it means for a type to be less central.

Definition 1. *Type θ is called less central than η, if it is further away from the median type $\frac{m+1}{2}$, i.e., if $|\theta - \frac{m+1}{2}| > |\eta - \frac{m+1}{2}|$.*

I may also refer to very low and very high types as *extreme* types. For example, these could be people with radical left or right views. They would probably find it hard to communicate with each other, but also people with more moderate views would not share a lot of common ground with them and therefore might find it hard to form a relationship with them. It is not important for the model that the types can be ordered on a line. The intuition for the results would continue to hold as long as there are some more central types with whom most other types share some common ground for a relationship and some less central types that other less central and not close types would find it quite difficult to talk to.

Proposition 1. *(i) Conditional on forming a positive number of links, the less central an agent's type, the (weakly) higher the fraction of her inter-regional links.*
(ii) For all types θ, there exist $c_T, c_R > 0$ and utility parameters $\overline{u}_{\theta i} = \overline{u}_{\eta i}$ and $\underline{u}_{\theta i} = \underline{u}_{\eta i}$ for $i \in \{0, 1\}$ such that only types more central than θ form links to other agents and some types less central form links when c_R is reduced.

The first part of the proposition means that agents who are not at the center of the society tend to have a higher fraction of inter-regional links. For them, it is more expensive to form links within a region, because they cannot necessarily link to higher and lower types in equal numbers as more central types can do. Therefore they find forming inter-regional links relatively more attractive.

The second part of the proposition shows that people with extreme types may not form links to other people, but may be isolated. As in part (i), it is more expensive for them to form links within a region. So it might not make sense to form links at all, if a lower number of signals would never let them influence their decision anyway. When inter-regional communication becomes cheaper, it also becomes cheaper to acquire this minimum number of signals that could make a difference for the decision, and so it might make sense to form several links. The present model predicts that some of these links should be inter-regional and some domestic at first (when c_R is just low enough), which would mean that social media might help to reintegrate people also in their local communities. However, it might also be possible that the differences between types are not uniform as in the present model, but that some extreme types may feel more distant and therefore face even higher costs when they want to form links to other types. Then it may be the case that these extreme types just form links to agents with the same type online.

Not only more extreme types, but also agents with higher stakes tend to form a higher fraction of inter-regional links.

Proposition 2. *Let $h > 0$ be a factor that multiplies an agent's gross utility in every state and every decision.*
(i) The fraction of inter-regional links this agent forms (weakly) increases in h.
(ii) For all c_R, there exists an $\overline{h} > 0$ such that the agent with $h > \overline{h}$ forms inter-regional links.

Proposition 2 says that agents with higher stakes form inter-regional links at a higher inter-regional linking cost than agents with lower stakes would and for the same cost, they form a higher fraction of such links. If the inter-regional linking cost decreases because of technological progress, agents with high stakes start forming inter-regional links earlier. This also implies that the sorting of politicians and other professionals with a high demand for information should start earlier and also be more extensive (at least before there is perfect sorting for everyone). The intuition is that agents with high stakes want to gather more information for their decision and therefore want to form more links. Intra-regional links, however, become increasingly costly as the types become less similar, so these agents rather form inter-regional links to agents with the same type at a high cost than domestic links to agents with very different types at an even higher cost.

4.3 Learning in endogenous networks

Now we look at the outcomes of learning in endogenous networks. This section does so for decisions in $t = 1$.

Remark 1. *Agents are (weakly) better off with a decrease in the inter-regional linking cost.*

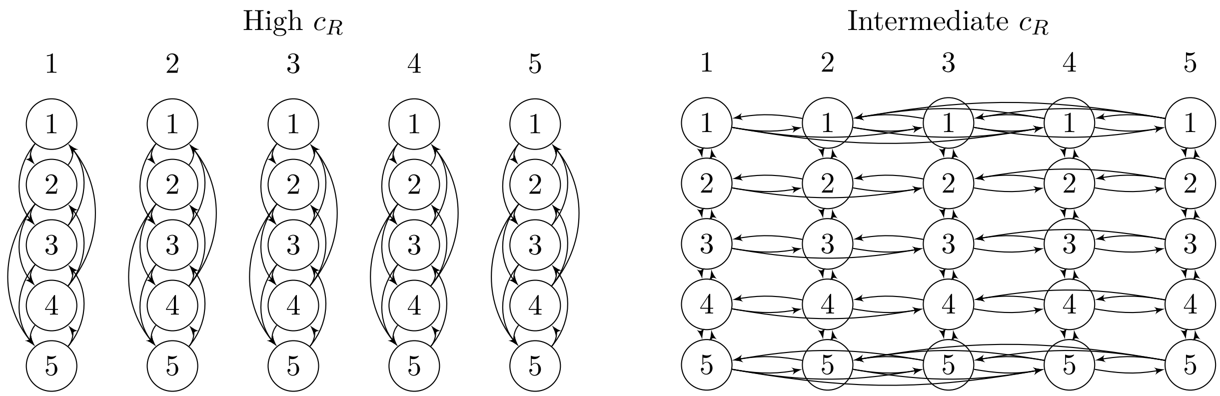
Obviously, utility-maximizing rational agents become weakly better off as the cost of inter-regional links decreases and they become strictly better off when they form inter-regional links and the cost of inter-regional links decreases even further. However, the quality of their information, i.e., their expected utility before the link formation cost, does not necessarily increase monotonically. For example, if there are more type-specific signals than region-specific signals, i.e., $\overline{s}_T > \overline{s}_R$, it may happen that a domestic link is replaced by a less informative inter-regional link when the latter is sufficiently cheaper, and the agents form no additional links. Then the number of conditionally independent signals she observes is lower. Section 5.1 shows that agents with correlation neglect may be worse off.

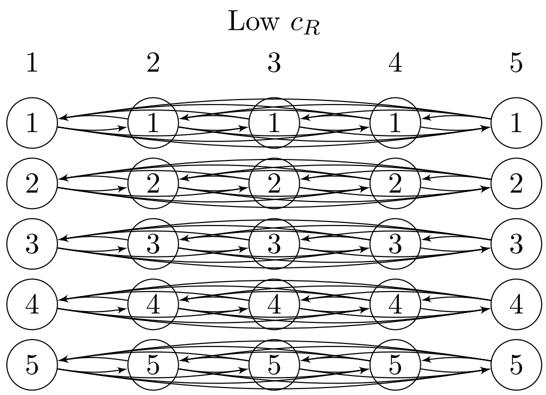
4.4 Opinions in Endogenous Networks

Now we look at beliefs in $t > 1$. We say that there is a *consensus* if all agents hold the same belief. When there is no consensus, people disagree, and disagreement is stronger when the sum of differences between agents' beliefs is higher or it takes longer to reach a consensus (for a given number of agents). A *component* is a collection of agents in the network and all their links in which there is a (undirected) path between every pair of agents in this collection of agents and no path to agents not in this collection of agents. In a *strongly connected* component or graph, there is a directed path from every agent to every other agent.

Proposition 3. *(i) For all $c_T > 0$, there exist $\overline{c_R} > \underline{c_R} > 0$ such that for all $c_R > \overline{c_R}$ and all $c_R < \underline{c_R}$ the society consists of several components that are not connected with each other. Consensus occurs only by chance, even if $\bar{t} \to \infty$.*
(ii) For all sufficiently small $c_T > 0$, there exist $\overline{c_R} \geq \overline{c_R}^* > \underline{c_R}^* \geq \underline{c_R} > 0$ such that for all $c_R \in (\underline{c_R}^*, \overline{c_R}^*)$ the society consists of one strongly connected component. There is consensus if $\bar{t} \to \infty$.

Part (i) says that when the cost of inter-regional links is too high, agents will only form links within their region. Then there will be no connection between different regions. A consensus between two different regions, i.e., that all agents in these regions hold the same belief, therefore only occurs when the agents received the same combination of signals by chance. If the cost of inter-regional links is very low, agents will only link to agents of the same type. Then there will be no connection between different types. Again, consensus only occurs by chance. Part (ii) says that





The numbers in the circles are the different types, the numbers in the top row are the different regions. If c_R is high, agents only link within their region, and so agents will disagree across regions, even if $t \to \infty$. If c_R is intermediate, there will be a consensus from $t = 6$ on. If c_R is low, agents only link agents of the same type, and so agents will disagree across types, even if $t \to \infty$.

Figure 1: Example with five regions and five types.

for an intermediate cost range of inter-regional linking, there is a directed path from every agent to every other agent in the society, so eventually all agents will have observed all signals and hold the same belief. (They might still decide differently, though, because their utility is type-specific.) To ensure that agents want to form some domestic links, c_T has to be sufficiently low. As we know from Proposition 1, agents of different types may differ in their link formation behavior. Therefore, the cost thresholds in part (i) do not necessarily have to coincide with the cost thresholds in part (ii). It may happen that an agent with a central type forms links both domestically and inter-regionally, while another agent does not form links at all or only domestically. Figure 1 illustrates Proposition 3 with five regions and five types.

Even if the society eventually reaches a consensus, this may happen quite fast or the society might stay polarized for a long time before reaching a consensus. How fast beliefs converge, depends on the length of shortest paths in the society. The minimal number of periods after which a consensus is reached for sure is equal to the maximal shortest path length plus one, because then all agents have observed all signals.

Lemma 5. *Suppose that the society is strongly connected, $\overline{s}_R = \overline{s}_T$, and $\underline{u}_{i\theta} = \underline{u}_{i\eta}, \overline{u}_{i\theta} = \overline{u}_{i\eta}$ for all $i \in \{0,1\}, \theta, \eta \in M$. Then there exists a threshold $c_R^* > 0$ such that the maximal shortest path length weakly decreases in c_R for $c_R < c_R^*$ and weakly increases in c_R for $c_R > c_R^*$.*

This means that the amount of time the society needs to reach a consensus first decreases when the cost of inter-regional links decreases, reaches a minimum, and then increases again. The reason is as follows. When the cost of inter-regional links decreases, more inter-regional links and fewer domestic links are formed. This speeds up the inter-regional exchange of information and slows down the exchange of information between types. When the cost of inter-regional links is high, there are only few inter-regional links, so the effect of faster inter-regional exchange of information dominates. When the cost of inter-regional links is low, the inter-regional exchange of information is already fast and the speed of exchange of information between types becomes the limiting factor and is reduced by replacing domestic links with inter-regional links. While there are many exceptions and special cases to consider if the gross utilities are type-dependent or the number of type-specific signals is not equal to the number of region-specific signals, the basic intuition remains the same. Together with Proposition 3, Lemma 5 shows that polarization follows a U-shape when the cost of inter-regional links decreases.

Proposition 4. *Suppose $\overline{s}_R = \overline{s}_T$ and $\underline{u}_{i\theta} = \underline{u}_{i\eta}, \overline{u}_{i\theta} = \overline{u}_{i\eta}$ for all $i \in \{0,1\}, \theta, \eta \in M$. For all sufficiently small $c_T > 0$, there exists a threshold $c_R^* > 0$ such that the maximal shortest path length weakly decreases in c_R for $c_R < c_R^*$ and weakly increases in c_R for $c_R > c_R^*$.*

5 Extensions and Discussion

5.1 Correlation Neglect

Experiments (Eyster and Weizsäcker 2011, Enke and Zimmermann 2015) have found that people have difficulties taking correlation of signals into account when they aggregate information and may double-count signals. This section investigates the case in which agents have correlation neglect, i.e., in which agents treat (conditionally) correlated signals as (conditionally) independent signals.

It shows that most results continue to hold with a slight modification that lies in the nature of correlation neglect: Agents form their beliefs and base their decisions on signals they perceive as (conditionally) independent, but those do not have to be actually (conditionally) independent. However, unlike rational agents, agents with correlation neglect may be worse off when the inter-regional linking cost decreases.

An agent with correlation neglect, whom I may also call *naive*, treats all signals she observes as conditionally independent signals. Let $\hat{\underline{S}}_{\theta k}^{t} = S_{\theta k}^{t}$ be the set of *perceived* conditionally independent signals agent θk has observed up until and including period t. Partial naivete, i.e., $\underline{S}_{\theta k}^{t} < \hat{\underline{S}}_{\theta k}^{t} < S_{\theta k}^{t}$ can also be modeled easily, but I will focus on complete naivete here. I conjecture partial naive agents to behave and form beliefs in between (completely) naive and rational agents.[18] Note that naive agents (whether completely or partially) cannot be told apart from rational agents if all signals indeed are conditionally independent.

In $t = 1$ each naive agent maximizes her *perceived* expected utility from the decision given the signals she observed:

$$\begin{aligned} &\hat{\mathbb{E}}[u_\theta(r, d_{\theta k}(S_{\theta k}^1))] := \\ \max_{d_{\theta k}(S_{\theta k}^1) \in [0,1]} d_{\theta k} \quad &\left[\widehat{Prob}(r = 1 | S_{\theta k}^1) \overline{u}_{1\theta} + (1 - \widehat{Prob}(r = 1 | S_{\theta k}^1)) \underline{u}_{1\theta}\right] \\ -(d_{\theta k} - 1) \quad &\left[\widehat{Prob}(r = 1 | S_{\theta k}^1) \underline{u}_{0\theta} + (1 - \widehat{Prob}(r = 1 | S_{\theta k}^1)) \overline{u}_{0\theta}\right], \end{aligned} \tag{3}$$

where $\hat{\mathbb{E}}$ and $\widehat{Prob}$ use the perceived probabilities as if all signals were conditionally independent.

In $t = 0$, each naive agent forms links that maximize her perceived expected utility which consists of the perceived expected utility from the decision minus the cost for the links:

$$\max_{\mathbf{a}_{\theta k} \in \{0,1\}^{mn} : a_{\theta k \theta k} = 0} \hat{\mathbb{E}}[U_{\theta k}] := \max_{\mathbf{a}_{\theta k} \in \{0,1\}^{mn} : a_{\theta k \theta k} = 0} \hat{\mathbb{E}}[\hat{\mathbb{E}}[u_\theta(r, d_{\theta k}(S_{\theta k}^1))]] - C(\mathbf{a}_{\theta k}). \tag{4}$$

Let furthermore $\hat{z}_{\theta k}^{t}$ be the difference between high and low perceived conditionally independent signals agent θk observed up until period t, which is equal to the difference between high and low signals. This difference is a sufficient statistic for a naive agent's belief and she forms links and decides just like a rational agent, but on grounds of $\hat{z}_{\theta k}^{t}$ instead of $z_{\theta k}^{t}$.

[18] There is no reason to expect a discontinuity as often found with partially naive present-biased agents (DellaVigna and Malmendier 2004, Eliaz and Spiegler 2006, Heidhues and Kőszegi 2010, Englmaier, Fahn and Schwarz 2016), because for partial correlation neglect, the equations determining the beliefs and actions are influenced gradually.

Remark 2. *All previous lemmas and propositions for rational agents also hold for naive agents when one replaces* $z_{\theta k}^t$ *with* $\hat{z}_{\theta k}^t$*,* $\underline{S}_{\theta k}^t$ *with* $\hat{\underline{S}}_{\theta k}^t$*, "expected" with "perceived expected", and "conditionally independent" with "perceived conditionally independent."*

The reason is that agents' decisions and links depend on their expectations. Consequently, that the patterns of link and opinion formation do not change compared to the case of rational agents, if all agents are naive. This does not mean, however, that the thresholds for naive agents are the same as for rational agents. In general, the thresholds could be higher or lower. To see that, assume first that both $\overline{s}_T$ and $\overline{s}_R$ are large, i.e., there are many type-specific and region-specific signals. Then agents with correlation neglect think that they already observe many independent signals when they only link to a few agents within their region. By contrast, rational agents are aware that they do not have that many independent signals and are willing to link inter-regionally at a higher cost than agents with correlation neglect. Assume then that $\overline{s}_T$ is very large and $\overline{s}_R$ is small. A rational agent prefers to link domestically as she is aware that many of the signals would be correlated if she linked to agents with the same type. Agents with correlation neglect, however, are not aware of this, and will link inter-regionally at higher inter-regional linking costs than rational agents will.

In societies with both rational and naive agents, it could theoretically happen that rational agents only form domestic links and naive agents only form inter-regional links or the other way around. Consequently, societies with both rational agents and agents with correlation neglect may have more difficulties reaching a consensus for two reasons: Firstly and obviously, rational agents and agents with correlation neglect agree only by chance. Secondly, if rational agents and agents with correlation neglect are not distributed uniformly, the society, i.e., the graph that includes all agents, is less likely to be strongly connected.

Remark 2 implies that naive agents think they are better off with a lower inter-regional linking cost, but what about their actual utility? The following two lemmas state how their actual utility is affected by additional correlated signals.

Lemma 6. *Suppose an agent with correlation neglect observes a set of conditionally independent signals* S_0 *and a set of perfectly correlated signals* S_1*, where all signals in* S_1 *are conditionally independent from all signals in* S_0*. The expected utility from the decision is decreasing in* $|S_1|$*. It is strictly decreasing for* $|S_1| < |S_0| - |z_\theta^*|$*.*

Furthermore, the expected utility from the decision decreases when both $|S_1|$ *and* $|S_0|$ *increase by the* *same amount. It strictly decreases for $|S_1| < |S_0| - |z_\theta^*|$. The reduction in expected utility converges to zero for $|S_0| \to \infty$.*

Lemma 6 says that an agent with correlation neglect is in general hurt by correlated signals. The decrease in expected utility even exceeds the gain by an additional independent signal. To see the intuition, imagine the agent observes an equal number of independent and perfectly correlated signals and chooses 1 if the difference between high and low signals is strictly positive. When the correlated signals are high, even though the state of the world is 0, only if all independent signals are low, the agent makes the (ex-post) right choice. This is similar when the agent observes more correlated and equally more independent signals. However, the probability that all independent signals are low decreases with the number of signals.

However, there are some situations in which an additional correlated signal does not hurt or even benefits the agent. First, if the condition $|S_1| < |S_0| - |z_\theta^*|$ is not met, the decision is based on only the correlated signals anyway, so one additional correlated signal does not make it worse. Second, naive agents can potentially benefit from more correlated signals when there is another set of perfectly correlated signals with a higher cardinality, as Lemma 7 below shows. Then, if these other correlated signals are wrong, the first set of correlated signals is more effective in making up for it. At the same time, however, the independent signals less often make a difference and which effect dominates depends on $|S_0|, |S_1|, |S_2|, \alpha$, and the different utility levels. An example of a situation where the agent benefits from more correlated signals is when $|S_1|$ and $|S_2|$ are already very high compared to $|S_0|$ so that the independent signals do not play a role for the decision when the correlated signals are all high or all low. Then one additional correlated signal does not hurt the agent when the correlated signals are all high or all low, but makes the independent signals more decisive when the signals of one group of correlated signals are high and the signals of the other group are low.

Furthermore, Lemma 7 below says that an agent with correlation neglect benefits from replacing a signal that is perfectly correlated with one set of signals with a signal that is perfectly correlated with another set of signals if the cardinality of the latter set is lower than the cardinality of the first set plus one, i.e., if one group of signals becomes less dominant for the decision.

Lemma 7. *Suppose an agent with correlation neglect observes a set of conditionally independent signals S_0, a set of perfectly correlated signals S_1, and another set of perfectly correlated signals S_2,*

where all signals in S_i are conditionally independent from all signals in S_0 and S_j, $i \neq j \in \{1, 2\}$.
(i) The expected utility from the decision can strictly increase in $|S_1|$ if $|S_1| < |S_2|$ and $|S_2| < |S_1| + |S_0| - |z^*_\theta|$. If $|S_1| > |S_2|$, and $|S_1| < |S_2| + |S_0| - |z^*_\theta|$, it strictly decreases in $|S_1|$.
(ii) The expected utility from the decision strictly increases if $|S_1|$ is decreased by one and $|S_2|$ is increased by one if and only if $|S_1| > |S_2| + 1$. It strictly decreases if and only if $|S_1| \leq |S_2|$.

Lemma 6 implies that the U-shape of disagreement with respect to the inter-regional linking cost is amplified if agents are naive: If agents only link domestically, they are exposed to many region-specific signals of that region and therefore likely to be overly influenced by these signals, because they treat them as conditionally independent. Similarly, if agents only link inter-regionally, they are exposed to many type-specific signals of their type and therefore are likely to be overly influenced by these signals. Because of this overweighting of certain signals, the distribution of beliefs has a greater variance compared to rational agents' beliefs, i.e., disagreement in these cases is stronger. The following corollary assumes $s^i_{\theta k} = s^j_{\theta k} \forall i, j = \overline{s}_U + 1, ..., \overline{s}_T$ and $s^i_{\theta k} = s^j_{\theta k} \forall i, j = \overline{s}_U + \overline{s}_T + 1, ..., \overline{s}_U + \overline{s}_T + \overline{s}_R$, i.e., all type-specific signals of each type are perfectly correlated and all region-specific signals are perfectly correlated in each region.

Corollary 1. *Suppose all type-specific signals of each type are perfectly correlated and all region-specific signals are perfectly correlated in each region. Further suppose all agents only link inter-regionally or all agents only link domestically. Disagreement is stronger if all agents are naive than if all agents are rational.*

We are now ready to say how agents with correlation neglect are affected when inter-regional link formation becomes cheaper. It is difficult to make general statements as for example replacing the last domestic link can make those agents better off when the number of region-specific signals is high enough. Similarly, already the first inter-regional link or forming an additional inter-regional link could make those agents worse off when the number of type-specific signals is high enough. Part (ii) of Proposition 5 shows that naive agents are likely to suffer from worse decisions due to a decreasing inter-regional linking cost if there are many type-specific signals.

Proposition 5. *Suppose all type-specific signals of each type are perfectly correlated and all region-specific signals are perfectly correlated in each region.*
(i) Then the expected utility from the decision of a naive agent increases when c_R decreases if the

number of inter-regional links does not exceed the number of domestic links, the number of total links is unchanged, and $\overline{s}_T \leq \overline{s}_R$. The expected utility from the decision of a naive agent decreases when c_R decreases if the number of domestic links does not exceed the number of inter-regional links, the number of total links is unchanged, and $\overline{s}_T \geq \overline{s}_R$.
(ii) If $\overline{s}_T \geq \frac{\overline{s}_U+\overline{s}_T+\overline{s}_R}{2}$, the expected utility from the decision of a naive agent decreases when c_R decreases if the number of domestic links does not exceed the number of inter-regional links.

Agents with correlation neglect tend to benefit from a lower inter-regional link formation cost when they replace the first domestic links with inter-regional ones. However, when the number of inter-regional links is already high, those agents' decisions become worse. This is true in particular when there are at least as many type-specific signals as other signals. Also note that agents who make worse decisions when they observe more signals have more 'extreme' beliefs in general. They make ex-post wrong decisions because some signal realizations do not coincide with the state of the world and are over-weighted; similarly, these signal realizations would have been over-weighted if they had coincided with the state of the world and would have led to a more confident belief.

Corollary 2. *A decrease in c_R can make agents with correlation neglect worse off, while it always makes rational agents better off.*

This means that a decrease in inter-regional link formation cost may increase the difference in utilities between rational agents and agents with correlation neglect. Consequently, if limiting the difference in utilities is among a policy maker's goals, the rise of social media provides an additional incentive to protect or educate naive citizens.

Proposition 6. *Suppose $\overline{s}_R = 0$ and $\overline{s}_T \geq \overline{s}_U$ and all type-specific signals are perfectly correlated for each type. Then the expected utility from the decision of a naive agent decreases when c_R decreases.*
Suppose $\overline{s}_T = 0$ and $\overline{s}_R \geq 0$. Then the expected utility from the decision of a naive agent increases when c_R decreases.

Proposition 6 shows that when type-specific correlation is strong, a decrease in inter-regional link formation cost leads to worse decisions of agents with correlation neglect and at the same time therefore also to more extreme beliefs and more differences across types. For example, when within a country, people with the same hobby or sympathizing with the same party also have a similar media consumption behavior, they will be more likely to buy the same products or share the same opinions when the inter-regional link formation cost decreases, whereas people with a different hobby or sympathizing with a different party will be more likely to buy completely different products or have very different opinions.

When region-specific correlation is strong, a decrease in inter-regional link formation cost leads to better decisions and smaller differences across regions. For example, people in different countries watch different news programs and read different newspapers. Via social media people now also get more exposure to news and perspectives in other countries. This leads to better decisions and more similar beliefs across countries.

5.2 Social Algorithms and Vicious Circles

Social algorithms, which filter content on some social media platforms, allow an agent to more easily observe signals of the same type on a social media platform, but at the same time make it more difficult to observe the signals of a different type. I model this as a decrease in inter-regional linking cost combined with an increase in the type-specific linking cost. As these two changes do not interact directly and the paper already analyzed the decrease in inter-regional linking cost, it remains to be shown what an increase in the type-specific linking cost entails.

Lemma 8. *If the type-specific linking cost, c_T, increases, agents form (weakly) fewer domestic links. Agents also form (weakly) more inter-regional links, if they form a positive number of links after the increase in c_T.*

Lemma 8 states that an increase in the type-specific linking cost usually leads to an increase in inter-regional links and a decrease in domestic links. This is very similar to the effect of a decrease in the inter-regional linking cost. An exception occurs, if the increase in the type-specific linking cost makes the links so expensive that an agent does not want to form links at all anymore. In combination with a decrease in the inter-regional linking cost, this seems unlikely, though. Consequently, in general social algorithms accelerate and amplify the effects of an increasing importance of social media.

Lemma 8 also implies that, if increasing disagreement across types leads to increasing costs to link to different types, this leads to fewer links to different types, which in turn leads to more disagreement across types. A vicious circle may emerge.

5.3 Strategic Communication

I have assumed that every agent can observe the signals of the agents she has linked to, which is equivalent to other agents telling the truth. This is plausible if an individual's utility is not affected by the choices of others, if individuals have a strong preference for truthful communication towards their contacts, or if individuals do not take potential consequences of strategic communication into account. In another interpretation of the model, the assumption that every agent can observe the signals of the agents she has linked to is a quite innocent one: If signals are hard evidence and agents know how many signals other agents observe, concealing a signal would not be effective, because the realization of that signal could be inferred. Consequently, the results are robust to strategic behavior in some interpretations of the model. However, because situations in which signals are hard evidence may be rare and correlation neglect seems implausible in such situations, there is some room for future research.

5.4 Decisions with Several Alternatives

In many situations, people do not only have to pick one of two alternatives, but face several alternatives. These can either be ordered or not. As an example of ordered alternatives, an agent could face the question of how much to invest in a certain project. Her decision will depend on how confident she is that the project will be successful. In this case, the different alternatives can easily be represented by different thresholds. As an example of alternatives that cannot be ordered, an agent might have to choose between a yellow, blue, or red car. In this case, the problem can be split up into several questions between two alternatives, in which the states of the world might be correlated. In both of these cases, the link formation problem becomes more involved, but the intuition for the results does not change.

6 Conclusion

In this paper I combine an endogenous network formation model with learning to investigate the question of how social media impact belief formation. In the model, agents live in different regions and have different types that can be ordered on a line. I find that agents with less central types and agents with higher stakes form a higher fraction of inter-regional links. These are also the agents who are willing to form inter-regional links at a higher cost than other agents. Moreover, I show that if these links and indirect links also influence agents' opinions, disagreement in a society is U-shaped with respect to the increasing importance of social media, which I model as a decrease in inter-regional linking cost. As an extension, I propose a portable model of correlation neglect and demonstrate that correlation neglect may amplify the U-shape and that agents with correlation neglect might be worse off with a lower inter-regional linking cost due to increasing importance of social media. Furthermore, I identify situations in which agents with correlation neglect may benefit from perfectly correlated signals.

It is hard to empirically identify whether the increasing importance of social media is still helping us to reduce disagreement in a society or is already increasing it, but the mechanism in this paper shows that the disagreement is likely to go up in the future. While the policy maker will probably be unable and unwilling to stop technological progress, easing domestic communication by subsidizing local events or clubs might be helpful to counteract this increasing disagreement. I also demonstrate that social media increases the need to integrate all people in society, because with social media they are more likely to radicalize themselves online. Moreover, the model suggests that educating citizens with correlation neglect becomes more important with growing influence of social media for two reasons. First, increasing disagreement in a society is stronger when citizens have correlation neglect. Second, people with correlation neglect might be worse off with social media, while rational people are better off.

In general, networks can be difficult to analyze. As this paper demonstrates, imposing a simple and plausible structure on link formation costs allows one to derive a variety of results and predictions, and thus this seems promising for future research. For example, this might allow to investigate how a social network changes when a new technology or product is introduced or how people could adjust their interactions with other people if they do not want to be infected by a spreading disease. The advantages of an approach similar to the one in this paper are that the network changes endogenously due to strategic considerations (in contrast to models with a random network formation process) and that the structure is less simplified than in other models of strategic network formation but still tractable.

This paper assumed that agents are uniformly distributed across regions. While this seems to be a reasonable simplification to analyze a model of network formation and learning and to derive some new mechanisms and insights, this is certainly not a realistic assumption and further research is needed. It might be interesting to see the interaction between social media and the incentives for people to move to a different region. On the one hand, the increasing number of possibilities provided by social media makes moving less desirable, because inter-regional communication becomes easier. On the other hand, the growing influence of social media may promote alienation (as discussed in Section 5.2) and thus make moving more desirable. Furthermore, the model in this paper, but with a different distribution of agents across regions, could serve as a basic framework to analyze in which regions investments in communication technologies have the highest impact.

Appendix: Proofs

Proof of Lemma 1.

This proof is slightly more general. Let the probability that $r = 1$ be p and the probability that $r = 0$ be $1 - p$. (i) When an agent observed i high and j low conditionally independent signals, her belief that $r = 1$ is $\frac{p\binom{i+j}{i}\alpha^i(1-\alpha)^j}{p\binom{i+j}{i}\alpha^i(1-\alpha)^j+(1-p)\binom{i+j}{i}(1-\alpha)^i\alpha^j} = 1 - \frac{(1-p)\binom{i+j}{j}\alpha^j(1-\alpha)^i}{(1-p)\binom{i+j}{j}\alpha^j(1-\alpha)^i+p\binom{i+j}{j}(1-\alpha)^j\alpha^i} = \frac{p\alpha^{i-j}}{p\alpha^{i-j}+(1-p)(1-\alpha)^{i-j}}$. Because this expression only depends on the difference between high and low conditionally independent signals for a given α, this difference is a sufficient statistic for the agent's belief.

(ii) It follows from equation 3 that an agent picks $d_{\theta k} = 1$ if and only if $Prob(r = 1|S^1_{\theta k})\overline{u}_{1\theta} + (1 - Prob(r = 1|S^1_{\theta k}))\underline{u}_{1\theta} > Prob(r = 1|S^1_{\theta k})\underline{u}_{0\theta} + (1 - Prob(r = 1|S^1_{\theta k}))\overline{u}_{0\theta} \Leftrightarrow Prob(r = 1|S^1_{\theta k}) > \frac{\overline{u}_{0\theta}-\underline{u}_{1\theta}}{\overline{u}_{1\theta}-\underline{u}_{1\theta}+\overline{u}_{0\theta}-\underline{u}_{0\theta}}$. This is equivalent to $z^1_{\theta k} > \frac{\log\left(\frac{p}{1-p}\frac{\overline{u}_{1\theta}-\underline{u}_{0\theta}}{\overline{u}_{0\theta}-\underline{u}_{1\theta}}\right)}{\log\left(\frac{1}{\alpha}-1\right)}$. □

Proof of Lemma 2.

First, observe that

$$\begin{aligned}
&\max_{d_{\theta k}(S^1_{\theta k})} d_{\theta k} \quad \left[Prob(r = 1|S^1_{\theta k})\overline{u}_{1\theta} + (1 - Prob(r = 1|S^1_{\theta k}))\underline{u}_{1\theta}\right] \\
&\quad -(d_{\theta k} - 1) \quad \left[Prob(r = 1|S^1_{\theta k})\underline{u}_{0\theta} + (1 - Prob(r = 1|S^1_{\theta k}))\overline{u}_{0\theta}\right] \\
&= \frac{1}{2}\Big[\sum_{i=\frac{|S^t_{\theta k}|+z^*_\theta}{2}+1}^{|S^t_{\theta k}|} \binom{|S^t_{\theta k}|}{i}(\alpha^i(1-\alpha)^{|S^t_{\theta k}|-i}\overline{u}_{1\theta} + \alpha^{|S^t_{\theta k}|-i}(1-\alpha)^i\underline{u}_{1\theta}) \\
&\quad + \sum_{i=0}^{\frac{|S^t_{\theta k}|+z^*_\theta}{2}} \binom{|S^t_{\theta k}|}{i}(\alpha^i(1-\alpha)^{|S^t_{\theta k}|-i}\underline{u}_{0\theta} + \alpha^{|S^t_{\theta k}|-i}(1-\alpha)^i\overline{u}_{0\theta})\Big],
\end{aligned}$$

where the first sum adds up the probability-weighted terms for $d_{\theta k} = 1$ and the second sum adds up the probability-weighted terms for $d_{\theta k} = 0$. For example, if the agent has observed no high signals and the threshold z^*_θ is not negative, she will choose $d_{\theta k} = 0$. If $r = 1$ (which occurs with probability 0.5), the probability of zero high signals is $(1-\alpha)^{|S^t_{\theta k}|}$, which is multiplied by $\underline{u}_{0\theta}$. If $r = 0$ (which also occurs with probability 0.5), the probability of zero high signals is $\alpha^{|S^t_{\theta k}|}$, which is multiplied by $\overline{u}_{0\theta}$.

Second, assume for now $z^*_\theta = \frac{\log\left(\frac{\overline{u}_{1\theta}-\underline{u}_{0\theta}}{\overline{u}_{0\theta}-\underline{u}_{1\theta}}\right)}{\log\left(\frac{1}{\alpha}-1\right)} \notin \mathbb{Z}$. We look at the expected utility from the decision, if we increase $|S^1_{\theta k}$ by one: There are two cases: either (i) $\lfloor\frac{|S^t_{\theta k}|+1+z^*_\theta}{2}\rfloor = \lfloor\frac{|S^t_{\theta k}|+z^*_\theta}{2}\rfloor$ or (ii)

$\lfloor\frac{|\underline{S}^t_{\theta k}|+1-z^*_\theta}{2}\rfloor = \lfloor\frac{|\underline{S}^t_{\theta k}|-z^*_\theta}{2}\rfloor + 1$.

Case (i): Because $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$ and with an index shift for one term,

$$\begin{aligned} 2[\mathbb{E}[U_{\theta k}||\underline{S}^t_{\theta k}|+1] - \mathbb{E}[U_{\theta k}||\underline{S}^t_{\theta k}|]] &= \\ \sum_{i=\lfloor\frac{|\underline{S}^t_{\theta k}|+1+z^*_\theta}{2}\rfloor+1}^{|\underline{S}^t_{\theta k}|+1} \binom{|\underline{S}^t_{\theta k}|+1}{i}\alpha^i(1-\alpha)^{|\underline{S}^t_{\theta k}|+1-i}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) &+ \sum_{i=0}^{\lfloor\frac{|\underline{S}^t_{\theta k}|+1+z^*_\theta}{2}\rfloor} \binom{|\underline{S}^t_{\theta k}|+1}{i}\alpha^{|\underline{S}^t_{\theta k}|+1-i}(1-\alpha)^i(\overline{u}_{0\theta}-\underline{u}_{1\theta}) \\ - \sum_{i=\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}^{|\underline{S}^t_{\theta k}|} \binom{|\underline{S}^t_{\theta k}|}{i}\alpha^i(1-\alpha)^{|\underline{S}^t_{\theta k}|-i}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) &- \sum_{i=0}^{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor} \binom{|\underline{S}^t_{\theta k}|}{i}\alpha^{|\underline{S}^t_{\theta k}|-i}(1-\alpha)^i(\overline{u}_{0\theta}-\underline{u}_{1\theta}) \\ = \binom{|\underline{S}^t_{\theta k}|}{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor}(\alpha^{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}(1-\alpha)^{|\underline{S}^t_{\theta k}|-\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\ -(\alpha^{|\underline{S}^t_{\theta k}|-\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor}(1-\alpha)^{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}(\overline{u}_{0\theta}-\underline{u}_{1\theta})) \end{aligned}$$

This is positive by the definition of $\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor$ in case (i). More generally, the difference in expected utilities from the decision going from $|\underline{S}^t_{\theta k}|+2j$ to $|\underline{S}^t_{\theta k}|+2j+1$ signals, where $j\in\mathbb{N}$ is

$$\begin{aligned} \binom{|\underline{S}^t_{\theta k}|+2j}{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+j}\alpha^j(1-\alpha)^j(\alpha^{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}(1-\alpha)^{|\underline{S}^t_{\theta k}|-\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\ -(\alpha^{|\underline{S}^t_{\theta k}|-\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor}(1-\alpha)^{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}(\overline{u}_{0\theta}-\underline{u}_{1\theta})), \end{aligned}$$

which goes to zero for $j\to\infty$, because $\alpha(1-a\alpha)<\frac{1}{4}$.

Case (ii): Similarly,

$$\begin{aligned} 2[\mathbb{E}[U_{\theta k}||\underline{S}^t_{\theta k}|+1] - \mathbb{E}[U_{\theta k}||\underline{S}^t_{\theta k}|]] &= \\ \sum_{i=\lfloor\frac{|\underline{S}^t_{\theta k}|+1+z^*_\theta}{2}\rfloor+2}^{|\underline{S}^t_{\theta k}|+1} \binom{|\underline{S}^t_{\theta k}|+1}{i}\alpha^i(1-\alpha)^{|\underline{S}^t_{\theta k}|+1-i}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) &+ \sum_{i=0}^{\lfloor\frac{|\underline{S}^t_{\theta k}|+1+z^*_\theta}{2}\rfloor+1} \binom{|\underline{S}^t_{\theta k}|+1}{i}\alpha^{|\underline{S}^t_{\theta k}|+1-i}(1-\alpha)^i(\overline{u}_{0\theta}-\underline{u}_{1\theta}) \\ - \sum_{i=\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}^{|\underline{S}^t_{\theta k}|} \binom{|\underline{S}^t_{\theta k}|}{i}\alpha^i(1-\alpha)^{|\underline{S}^t_{\theta k}|-i}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) &- \sum_{i=0}^{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor} \binom{|\underline{S}^t_{\theta k}|}{i}\alpha^{|\underline{S}^t_{\theta k}|-i}(1-\alpha)^i(\overline{u}_{0\theta}-\underline{u}_{1\theta}) \\ = -\binom{|\underline{S}^t_{\theta k}|}{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}(\alpha^{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}(1-\alpha)^{|\underline{S}^t_{\theta k}|-\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\ -(\alpha^{|\underline{S}^t_{\theta k}|-\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor}(1-\alpha)^{\lfloor\frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2}\rfloor+1}(\overline{u}_{0\theta}-\underline{u}_{1\theta})) \end{aligned}$$

This is negative by the definition of $\lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor$ in case (ii). More generally, the difference in expected utilities from the decision going from $|\underline{S}^t_{\theta k}| + 2j$ to $|\underline{S}^t_{\theta k}| + 2j + 1$ signals, where $j \in \mathbb{N}$, without taking into account link formation cost is

$$\binom{|\underline{S}^t_{\theta k}| + 2j}{\lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor + 1 + j} \alpha^j (1-\alpha)^j (\alpha^{\lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor + 1} (1-\alpha)^{|\underline{S}^t_{\theta k}| - \lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor} (\overline{u}_{1\theta} - \underline{u}_{0\theta})$$
$$- (\alpha^{|\underline{S}^t_{\theta k}| - \lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor} (1-\alpha)^{\lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor + 1} (\overline{u}_{0\theta} - \underline{u}_{1\theta})),$$

which again goes to zero for $j \to \infty$.

Third, consider $z^*_\theta \in \mathbb{Z}$: If the agent is indifferent, choosing either alternative with probability $\frac{1}{2}$ has the same expected utility as always choosing 0. With this in mind the expressions are the same as for $z^*_\theta \notin \mathbb{Z}$. However, $\alpha^{\lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor + 1} (1-\alpha)^{|\underline{S}^t_{\theta k}| - \lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor} (\overline{u}_{1\theta} - \underline{u}_{0\theta}) - (\alpha^{|\underline{S}^t_{\theta k}| - \lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor} (1-\alpha)^{\lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor + 1} (\overline{u}_{0\theta} - \underline{u}_{1\theta})$ is zero in case (ii) as $2\lfloor \frac{|\underline{S}^t_{\theta k}|+z^*_\theta}{2} \rfloor + 1 - |\underline{S}^t_{\theta k}| = z^*_\theta$, so not every additional signal is beneficial, but only every two additional conditionally independent signals are beneficial.

□

Proof of Lemma 3.

Assume this is not true, but an agent has optimally formed a link to an agent with type $\eta \neq \theta$ via the Internet. Then an agent would rather connect to an agent of type η in her own region and an agent of her own type at the different region at the same total cost, but obtaining more independent signals. If she has already linked to the agent of type η in her own region, too, then linking to an agent of type θ via the Internet gives her (at least) the same number of independent signals as the link to an agent of type η at a strictly lower cost. This is also possible if n is sufficiently large. This is a contradiction to the optimality of the link to an agent with type $\eta \neq \theta$ via the Internet. □

Proof of Lemma 4.

I will prove a slightly more general version of Lemma 4, also dealing with $z^*_\theta \in \mathbb{Z}$.

Lemma 9. *(i) Suppose $\overline{s}_T \geq \overline{s}_R$. Agents form (weakly) more links when the inter-regional linking cost, c_R, decreases.*

(ii) Conditional on forming a positive number of links, if $z^*_\theta \notin \mathbb{Z}$, agents form (weakly) fewer domestic links when the inter-regional linking cost, c_R, decreases. If $z^*_\theta \in \mathbb{Z}$, agents form at most one additional domestic link when c_R decreases.

(i) When the cost of inter-regional linking decreases, an agent might form an additional inter-regional link, make no change, or substitute one or several of her domestic links with one or several inter-regional links. We have to show that in the last case, a number of domestic links will be replaced with at least as many inter-regional links. We know from Lemma 3 that we have to compare domestic links to agents of different types with inter-regional links to agents of the same type. The former provide access to $\overline{s}_U + \overline{s}_T$ conditionally independent signals, while the latter provide access to $\overline{s}_U + \overline{s}_R$ conditionally independent signals. Hence, if $\overline{s}_T \geq \overline{s}_R$, inter-regional links provide access to weakly fewer signals, and therefore an agent will substitute a domestic with at least as many inter-regional links.

(ii) If an agent forms links at a certain cost c_R, lowering c_R does not change the cost of domestic links. The benefit of forming an additional domestic link is unchanged if $z^*_\theta \notin \mathbb{Z}$, so it is not optimal to form additional domestic links. If $z^*_\theta \in \mathbb{Z}$, it might make sense to form an additional domestic link together with an inter-regional link when an odd number of additional signals provides a higher benefit of creating a new link. Even then, it will never make sense to form two or more additional domestic links, because then it would have made sense to form these links at a higher c_R, too. □

Proof of Proposition 1.

(i) Within their region, agents link to the most similar types, i.e., an agent who forms $2i$ domestic links forms these links with i agents of a lower type and i agents of a higher type, if possible (in the case of an odd number, she would be indifferent about one link). If this is not possible, because e.g., there are only $j < i$ agents of a lower type available, the agent would have to pay more for those $i - j$ domestic links as those agents would have a more distant higher type. Therefore, an agent further away from the central type $\frac{m+1}{2}$ forms weakly fewer domestic links and replaces them with inter-regional links at a weakly higher c_R.

For forming inter-regional links, all agents face the same cost. The benefits of agents further away from the central type are weakly higher if $z^*_\theta \notin \mathbb{Z}$. If $z^*_\theta \in \mathbb{Z}$, it can happen that a more central type forms one domestic link more than a less central type and one inter-regional link, while the less central type does not form any inter-regional link (see proof of of Lemma 2).

(ii) Let $c_T > 0$ and $\overline{u}_1, \overline{u}_0, \underline{u}_1, \underline{u}_0$ be such that an agent of type $\eta^* + 1$ it is just optimal to form only $2\eta^*$ domestic links to receive exactly $z^*_{\eta^*+1} = z^*_\theta$ conditionally independent signals and let $c_R > c^*_R = \eta^* c_T$. For all other agents with types $\eta \in \{\eta^* + 1, ..., m - \eta^*\}$, it is optimal to form at least the same number of links, too. For agents with types $\eta \notin \{\eta^* + 1, ..., m - \eta^*\}$, it is optimal to not form any links, because by assumption it is not optimal for them to form the same number of links (because this would be more expensive), and forming a positive but lower number of links is not optimal, because the signals could never change the decision. If $c_R < c_R^*$, types η^* and $m + 1 - \eta^*$ would form links, because they could form $2\eta^* - 1$ domestic links and one inter-regional link at costs below the total linking costs of agents of type $\eta^* + 1$ when c_R was greater than c_R^* (and forming links was optimal for them by assumption). □

Proof of Proposition 2.

If h increases, the benefit of additional signals increases, while the costs for additional signals stay the same. Therefore, the optimal number of links weakly increases with h. Suppose an agent with $h = 1$ does not form any inter-regional links. If $h \to \infty$, the benefit of an additional inter-regional link approaches infinity. Therefore there exists an $\overline{h} > 1$ such that an agent with $h = \overline{h}$ forms inter-regional links.

□

Proof of Proposition 3.

(i) We know from Lemma 2 that conditionally independent signals improve an agent's decision. Therefore, if c_R is sufficiently small, all agents will form at least two links. Call the inter-regional linking cost which ensures this c'_R. The number of conditionally independent signals an agent receives from forming a domestic link is $\overline{s}_U + \overline{s}_T$ and the number of conditionally independent signals an agent receives from forming an inter-regional link (with an agent of the same type) is $\overline{s}_U + \overline{s}_R$. If $c_R < \min\{c_T, c_T \frac{\overline{s}_U + \overline{s}_R}{\overline{s}_U + \overline{s}_T}\} =: c''_R$, agents will only form inter-regional links, because they are cheaper and cheaper per signal than domestic links. Set $\underline{c_R} = \min\{c'_R, c''_R\}$. For $c_R < \underline{c_R}$, all agents form no domestic links and at least links to the agents of the same type in the two closest regions. Hence, there is a path from every agents to all agents of the same type, but no path to an agent of a different type. Consequently, even if $\bar{t} \to \infty$, agents will never of observe the signals of other types. Only by chance have the signals the same realizations and consensus occurs.

If the inter-regional link cost is high enough, agents will not form any inter-regional links. Such a high enough cost exists, because the value-added of signals is decreasing in the number of conditionally independent signals as we know from Lemma 2. Call the threshold $\overline{c_R}$. If agents do not form any inter-regional links, even if $\bar{t} \to \infty$, they will never of observe the signals of agents in other regions. Only by chance have the signals the same realizations and consensus occurs.

(ii) It is sufficient to show that there exist according parameters such that agents form at least two inter-regional links and at least two domestic links: Then each agent forms links to agents of the same type in the at least two closest regions and to at least two agents in the same region with the closest types. This means that there exists a path from every agent to every other agent. Consequently, for $\bar{t} \to \infty$, agents will observe all signals and then hold the same beliefs.

Let c_R be very high and c_T be such that each agent optimally forms domestic links to observe at least $6\overline{s}_U + 3\overline{s}_R + 4\overline{s}_T$ conditionally independent signals in addition to her own signals. This is possible because of Lemma 2. Then reduce c_R so that agents will form inter-regional links and replace domestic links (see Lemma 4). Do so till the point where the first agent is indifferent between giving up her second to last domestic link (this will be agents of type 1 and m). For a neighborhood of slightly higher inter-regional linking costs, all agents have at least two domestic links. Because all agents previously observed $6\overline{s}_U + 3\overline{s}_R + 4\overline{s}_T$ conditionally independent signals in addition to their own signals, all agents also have at least two inter-regional links in this neighborhood of inter-regional linking costs. $\square$

Proof of Lemma 5.

That the component is strongly connected implies that all agents have domestic links: Suppose one agent did not have a domestic link. Then inter-regional links must be cheaper than domestic links or the agent could not observe any other agent's signals. If inter-regional links are cheaper than domestic links, no agent would form domestic links. This is a contradiction. $\overline{s}_R = \overline{s}_T$ implies that agents replace each domestic link one by one with a domestic link. Because the gross utilities are the same for all agents, an agent has at most one inter-regional link more and at most one domestic link less then a slightly more central agent. Therefore, the maximal path length is determined by the path from agent 11 to agent mn via agent $1n$. Suppose c_R is so high that when increasing it the component would no longer be strongly connected. Let $i_1(c_R) > 0$ the number of inter-regional links agents of type 1 form and $i_1(c_R) > 0$ be the number of domestic links they form. Then the path length from 11 to $1n$ is equal to $\lceil \frac{2(n-1-i_1)}{i_1} \rceil$, because 11's link will go to agent $1(i_1+1)$ and this agent's link will go to agent $1(i_1 + 1 + 0.5i_1)$ (because she forms links in both directions) and so on. The path length from agent $1n$ to agent mn is between $\lceil \frac{m-1-i_2}{i_2} \rceil$ and $\lceil \frac{2(m-1-i_2)}{i_2-1} \rceil$, depending on how many inter-regional links central agents form. If they form inter-regional links, then the path length is $\lceil\frac{m-1-i_2}{i_2}\rceil$, because then they will form twice as many domestic links as agents of type 1. For now, assume all agents form inter-regional links. Because domestic links are replaced one by one with inter-regional links (as long as the component is strongly connected) when c_R decreases, the number of links $b := i_1 + i_2$ stays constant. The maximal path length is $\lceil\frac{2(n-1-i_1)}{i_1}\rceil + \lceil\frac{m-1-i_1-b}{b-i_1}\rceil$. This function has one minimum on $i_1 \in \{1, b-1\}$ and is decreasing before the minimum and increasing after it. If some relatively central agents do not form any inter-regional links for high c_R, then the maximal path length decreases even faster when decreasing c_R, because they do not change their links before they start forming inter-regional links. Because types start forming inter-regional links one type after another, this will increase the path length from agent $1n$ to agent mn by not more than 2 at a time, and so there is no inflection point for the maximal path length. Consequently, the maximal path length follows a U-shape with respect to the inter-regional linking cost. □

Proof of Proposition 4.

This immediately follows from Proposition 3 and Lemma 5 and the fact the maximal path length is ∞ if the society is not strongly connected. □

Proof of Lemma 6.

We will show that the expected utility from the decision decreases when both $|S_0|$ and $|S_1|$ increase by the same amount. This together with Lemma 2 implies that the expected utility from the decision decreases in $|S_1|$.

Similar to the proof of Lemma 2, assume that $z_\theta^* \notin \mathbb{Z}$. We separate the terms into cases in which the signals in S_1 are correct (which happens with probability α) and those in which they are incorrect.

$$
\begin{aligned}
& 2[\mathbb{E}[U_{\theta k}||S_0|+1,|S_1|+1]-\mathbb{E}[U_{\theta k}||S_0|,|S_1|]] \\
= \; & (\overline{u}_{1\theta}-\underline{u}_{0\theta})(1-\alpha)\left(\sum_{i=\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor+2}^{|S_0|+1}\binom{|S_0|+1}{i}\alpha^i(1-\alpha)^{|S_0|+1-i}-\sum_{i=\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor+1}^{|S_0|}\binom{|S_0|}{i}\alpha^i(1-\alpha)^{|S_0|-i}\right) \\
+ \; & (\overline{u}_{0\theta}-\underline{u}_{1\theta})\alpha\left(\sum_{i=0}^{\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor+1}\binom{|S_0|+|S_1|+1}{i}\alpha^{|S_0|+1-i}(1-\alpha)^i-\sum_{i=0}^{\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor}\binom{|S_0|}{i}\alpha^{|S_0|-i}(1-\alpha)^i\right)
\end{aligned}
$$

$$
\begin{aligned}
= & \binom{|S_0|}{\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor+1}(\alpha^{\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor+1}(1-\alpha)^{|S_0|-\lfloor\frac{|S_0|+z^*_\theta}{2}\rfloor+1}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\
& -(\alpha^{|S_0|-\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor+1}(1-\alpha)^{\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor+1}(\overline{u}_{0\theta}-\underline{u}_{1\theta})),
\end{aligned}
$$

which is smaller than zero, because $\frac{\overline{u}_{1\theta}-\underline{u}_{0\theta}}{\overline{u}_{0\theta}-\underline{u}_{1\theta}}>\left(\frac{1-\alpha}{\alpha}\right)^{\lfloor\frac{|S_0|+|S_1|+z^*_\theta}{2}\rfloor-|S_0|}$.

Dealing with $z^*_\theta\in\mathbb{Z}$ and showing convergence to zero for $|S_0|$ is analogous to that part in the proof of Lemma 2. □

Proof of Lemma 7.

(i) There are two cases to deal with: (a) $|S_0|+|S_1|+|S_2|+z^*_\theta\in\bigcup_{j\in\mathbb{Z}}[2j,2j+1)$ and (b) $|S_0|+|S_1|+|S_2|+z^*_\theta\notin\bigcup_{j\in\mathbb{Z}}[2j,2j+1)$.

(a) In this case, $\lfloor\frac{|S_0|-|S_1|-|S_2|+z^*_\theta}{2}\rfloor>\lfloor\frac{|S_0|-|S_1|-|S_2|+z^*_\theta-1}{2}\rfloor$ and $\lfloor\frac{|S_0|-|S_1|+|S_2|+z^*_\theta}{2}\rfloor>\lfloor\frac{|S_0|-|S_1|+|S_2|+z^*_\theta-1}{2}\rfloor$. Proceeding as in the proof of Lemma 6 yields

$$
\begin{aligned}
& \binom{|S_0|}{\lfloor\frac{|S_0|-|S_1|-|S_2|+z^*_\theta}{2}\rfloor}(\alpha^{\lfloor\frac{|S_0|-|S_1|-|S_2|+z^*_\theta}{2}\rfloor+2}(1-\alpha)^{|S_0|-\lfloor\frac{|S_0|-|S_1|-|S_2|+z^*_\theta}{2}\rfloor}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\
& \qquad -(\alpha^{|S_0|-\lfloor\frac{|S_0|-|S_1|-|S_2|+z^*_\theta}{2}\rfloor}(1-\alpha)^{\lfloor\frac{|S_0|-|S_1|-|S_2|+z^*_\theta}{2}\rfloor+2}(\overline{u}_{0\theta}-\underline{u}_{1\theta})) \\
+ & \binom{|S_0|}{\lfloor\frac{|S_0|-|S_1|+|S_2|+z^*_\theta}{2}\rfloor}(\alpha^{\lfloor\frac{|S_0|-|S_1|+|S_2|+z^*_\theta}{2}\rfloor+1}(1-\alpha)^{|S_0|-\lfloor\frac{|S_0|-|S_1|+|S_2|+z^*_\theta}{2}\rfloor+1}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\
& \qquad -(\alpha^{|S_0|-\lfloor\frac{|S_0|-|S_1|+|S_2|+z^*_\theta}{2}\rfloor+1}(1-\alpha)^{\lfloor\frac{|S_0|-|S_1|+|S_2|+z^*_\theta}{2}\rfloor+1}(\overline{u}_{0\theta}-\underline{u}_{1\theta})).
\end{aligned}
$$

The first term is always smaller than zero for $|S_1|,|S_2|>1$. The second term is smaller than zero if $|S_1|>|S_2|$, but greater than zero if $|S_1|+1<|S_2|$. If $|S_0|-|S_1|+z^*_\theta$ is small, the second term dominates.

(b) In this case, $\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor < \lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*+1}{2}\rfloor$ and $\lfloor\frac{|S_0|+|S_1|+|S_2|+z_\theta^*}{2}\rfloor < \lfloor\frac{|S_0|+|S_1|+|S_2|+z_\theta^*+1}{2}\rfloor$. Proceeding as in the proof of Lemma 6 yields

$$\begin{aligned}
&-\binom{|S_0|}{\lfloor\frac{|S_0|+|S_1|+|S_2|+z_\theta^*}{2}\rfloor+1}\Big(\alpha^{\lfloor\frac{|S_0|+|S_1|+|S_2|+z_\theta^*}{2}\rfloor+1}(1-\alpha)^{|S_0|-\lfloor\frac{|S_0|+|S_1|+|S_2|+z_\theta^*}{2}\rfloor+1}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\
&\qquad -(\alpha^{|S_0|-\lfloor\frac{|S_0|+|S_1|+|S_2|+z_\theta^*}{2}\rfloor+1}(1-\alpha)^{\lfloor\frac{|S_0|+|S_1|+|S_2|+z_\theta^*}{2}\rfloor+1}(\overline{u}_{0\theta}-\underline{u}_{1\theta})\Big) \\
&-\binom{|S_0|}{\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor+1}\Big(\alpha^{\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor+2}(1-\alpha)^{|S_0|-\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\
&\qquad -(\alpha^{|S_0|-\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor}(1-\alpha)^{\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor+2}(\overline{u}_{0\theta}-\underline{u}_{1\theta})\Big).
\end{aligned}$$

The first term is always smaller than zero for $|S_1|, |S_2| > 1$. The second term is smaller than zero if $|S_1| > |S_2|$, but greater than zero if $|S_1| + 1 < |S_2|$. If $|S_0| - |S_1| + z_\theta^*$ is big, the second term dominates.

(ii) Proceeding as in the proof of Lemma 6 yields

$$\begin{aligned}
&-\binom{|S_0|}{\lfloor\frac{|S_0|-|S_1|+|S_2|+z_\theta^*}{2}\rfloor+1}\Big(\alpha^{\lfloor\frac{|S_0|-|S_1|+|S_2|+z_\theta^*}{2}\rfloor+2}(1-\alpha)^{|S_0|-\lfloor\frac{|S_0|-|S_1|+|S_2|+z_\theta^*}{2}\rfloor}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\
&\qquad -(\alpha^{|S_0|-\lfloor\frac{|S_0|-|S_1|+|S_2|+z_\theta^*}{2}\rfloor}(1-\alpha)^{\lfloor\frac{|S_0|-|S_1|+|S_2|+z_\theta^*}{2}\rfloor+2}(\overline{u}_{0\theta}-\underline{u}_{1\theta})\Big) \\
&+\binom{|S_0|}{\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor}\Big(\alpha^{\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor+1}(1-\alpha)^{|S_0|-\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor+1}(\overline{u}_{1\theta}-\underline{u}_{0\theta}) \\
&\qquad -(\alpha^{|S_0|-\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor+1}(1-\alpha)^{\lfloor\frac{|S_0|+|S_1|-|S_2|+z_\theta^*}{2}\rfloor+1}(\overline{u}_{0\theta}-\underline{u}_{1\theta})\Big).
\end{aligned}$$

Both terms are smaller than zero for $|S_2| > |S_1|$ and greater than zero for $|S_2| + 1 < |S_1|$. If $|S_2| + 1 = |S_1|$ the sum of both terms is zero. □

Proof of Corollary 1.

This follows directly from Lemma 6. □

Proof of Proposition 5.

(i) This follows directly from Lemma 7.

(ii) This follows directly from Lemma 6. □

Proof of Corollary 2.

This follows from Remark 1 and Proposition 5 together with a situation in which a rational agent forms at least one inter-regional link and a slight decrease in the inter-regional linking cost leads to a change in a naive agent's link formation. While the naive agent can be strictly (and bounded away from zero) worse off according to Proposition 5, a rational agent is slightly better off.

□

Proof of Proposition 6.

Both parts follow directly from Lemma 6.

□

Proof of Lemma 8.

If the type-specific linking cost increases, the number of domestic links will obviously not increase. An agent might replace a domestic link with an inter-regional link. In this case, the number of inter-regional links increases. It will not decrease because the costs are constant and the benefits weakly increasing for $z_\theta^* \notin \mathbb{Z}$. □

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