A Plucking Model of Business Cycles

Stéphane Dupraz  Emi Nakamura  Jón Steinsson
Banque de France  UC Berkeley  UC Berkeley

December 20, 2018

Abstract

The dynamics of unemployment fit what Milton Friedman labeled a plucking model: a rise in unemployment is followed by a fall of similar amplitude, but the amplitude of the rise does not depend on the previous fall. We develop a microfounded plucking model of the business cycle to account for these phenomena. The model features downward nominal wage rigidity within an explicit search model of the labor market. Our search framework implies that downward nominal wage rigidity is consistent with optimizing behavior and equilibrium. We reassess the costs of business cycle fluctuations through the lens of the plucking model. Contrary to New-Keynesian models where fluctuations are cycles around an average natural rate, the plucking model generates fluctuations that are gaps below potential (as in Old-Keynesian models). In this model, business cycle fluctuations raise not only the volatility but also the average level of unemployment, and stabilization policies can reduce the average level of unemployment and therefore yield sizable welfare benefits. A positive inflation target, by easing the adjustment of wages to aggregate shocks, is one such policy. The benefits of positive inflation go beyond stabilizing aggregate fluctuations since it also eases the adjustment to sectoral shocks.

Keywords: Downward Nominal Rigidity, Stabilization Policy, Labor Search.

JEL Classification: E24, E30, E52

*We would like to thank Gadi Barlevy, Marco Bassetto, François Gourio, John Leahy, Julien Matheron, Andreas Mueller, Ricardo Reis, and Johannes Wieland for valuable discussions. Emi thanks the National Science Foundation (grant SES-1056107) and the George S. Eccles Research Award in Finance and Economics at Columbia Business School for financial support.
1 Introduction

The unemployment rate in the United States displays a striking asymmetry: much of the time, it hovers around 5%, but occasionally it rises far above this level, peaking each time at a different maximum before falling back. Milton Friedman proposed a “plucking model” analogy to describe this behavior of the economy: “In this analogy, [...] output is viewed as bumping along the ceiling of maximum feasible output except that every now and then it is plucked down by a cyclical contraction” (Friedman, 1964, 1993). Friedman highlighted one manifestation of these asymmetric dynamics: economic contractions are followed by expansions of a similar amplitude—as if the economy is recovering back to its maximum level—while the amplitude of expansions are not related to the previous contractions—each pluck seems to be a new event.

Workhorse models of the business cycle do not capture this asymmetry in unemployment and output. Instead, they see the business cycle as symmetric ups and downs of unemployment and output around an average level. An important implication of this view is that stabilization policy cannot affect the average level of output or unemployment. At best, stabilization policy can reduce inefficient fluctuations around the average. As a consequence, in these models the welfare gains of stabilization policy are trivial (Lucas, 1987, 2003).

Friedman’s plucking model view of the business cycle potentially has very different implications for the welfare gains from stabilization policy. In this view, economic contractions involve drops below the economy’s full-potential “ceiling,” rather than symmetric cycles around a “natural rate.” Eliminating such drops increases average output and decreases average unemployment, which raises welfare by non-trivial amounts (De Long and Summers, 1988).

We consider what model of the labor market can account for the plucking property of the unemployment rate. We find that a standard version of the Diamond-Mortensen-Pissarides search model cannot. The aggregate labor-demand schedule of the model features some non-linearities that can generate some plucking, but much less than in the data. We therefore add a key ingredient to generate the plucking property: downward nominal wage rigidity. We depart from the previous literature by introducing downward nominal wage rigidity within our search model of the labor market. The search framework rationalizes unemployment as an equilibrium phenomenon and, most importantly, makes the downward rigidity of wages consistent with optimizing behavior, and thus robust to Barro’s (1977) critique that wage rigidity should neither interfere with the efficient formation of employment matches nor lead to inefficient job separations.
Our plucking model also reproduces other asymmetries in the dynamics of unemployment. Empirically, the distribution of unemployment has a longer right tail than left tail, as emphasized by Sichel (1993). The unemployment rate spends much time around 5%, occasionally rises much higher, but never falls much below. Our plucking model can generate this asymmetry in the distribution of unemployment. Empirically, unemployment also rises faster in contractions than it falls during expansions, as emphasized by Neftçi (1984), among others. Our plucking model can generate contractions faster than expansions, although the asymmetry is not as pronounced as in the data.

Intuitively, our model reproduces the plucking property because good shocks mostly lead to increases in wages, while bad shocks mostly lead to increases in unemployment. The source of this asymmetry is our assumption of downward nominal wage rigidity. This notion has a long history within macroeconomics going back at least to Tobin (1972). The main theoretical challenge for this line of thinking has been how to justify the notion that wages do not fall in recessions despite obvious incentives of unemployed workers to bid wages down.

To make downward nominal wage rigidity robust to this critique, we build on the recent insights from the labor search literature. Hall (2005) pointed out that, once a search and matching model is purged of its ad hoc assumption of Nash-bargaining, wages are not uniquely pinned down. They are only constrained to lie within a wage-band, making some amount of wage-rigidity consistent with individual rationality and equilibrium. Intuitively, because of search frictions, unemployed workers cannot freely meet with firms and offer to replace employed workers at a lower wage. Instead, unemployed workers and potential employers must engage in a costly matching process. But after the worker and employer have matched, the worker has some monopoly power and therefore no longer has any reason to bid the wage down.

The plucking nature of our model has important normative implications. It implies that fluctuations in unemployment are fluctuations above a resting point of low unemployment, not symmetric fluctuations around a natural rate. As a consequence, a reduction in the volatility of aggregate shocks not only reduces the volatility of the unemployment rate, but also reduces its average level. Eliminating all shocks in our model reduces the average unemployment rate from 5.8% to 4.6%. The welfare benefits of stabilization policy are therefore an order of magnitude larger in our model than in standard models in which stabilization policy cannot affect the average level of output and unemployment. Our reliance on a search framework implies that the natural-rate

\footnote{Recall that Lucas (2003) shows that the consumption equivalent welfare loss of business cycle fluctuations in

| 2 |
view of business cycles—and its corollary that stabilization policy cannot affect mean output and unemployment—is not a necessary implication of imposing the discipline of optimizing behavior, equilibrium analysis, and rational expectations.

In our model, a modest amount of inflation can “grease the wheels of the labor market” by allowing real wages to fall in response to adverse shocks even though nominal wages are downward rigid. Increasing the average inflation rate from 2% (our baseline calibration) to 4% yields a drop in average unemployment from 5.8% to 4.4%. The benefits of inflation diminish at higher levels of inflation but are quite large at low levels. Reducing the average inflation rate from 2% to 1% increases the average unemployment rate from 5.8% to 7.5%.

Our work is related to several strands of existing literature. A large literature documents business cycles asymmetries. Two asymmetries of the unemployment that have received particular attention are the skewness in the unemployment rate, which Sichel (1993) calls “deepness”, and the fact that unemployment raises faster in expansions than it falls during contractions, as emphasized by Neftci (1984), among others. Friedman’s plucking property is related to the former but conceptually distinct from the latter.

Kim and Nelson (1999) offer one of the few modern attempts to assess the specific asymmetry emphasized by Friedman. They decompose US output and unemployment between trend and cycle through an unobserved components model that allows for asymmetric, discrete shocks—plucks—in the cyclical component. They find the data speaks in favor of the plucking view. Sinclair (2010) extends their model to allow for correlation between the permanent and transitory innovations, and finds that allowing for the asymmetry of the plucking model restores an important role of transitory fluctuations. Fatas and Mihov (2015) make the closely related case that an economic expansion is at least in part a recovery from the previous contraction, and offer methods to decompose an economic cycle in not two but three phases: expansion, recession, and recovery. Bordo and Haubrich (2012) consider whether larger contractions are followed by faster recoveries, and whether recoveries from financial crises are faster than others.

On the theoretical side, Kim and Ruge-Murcia (2009, 2011), Benigno and Ricci (2011) and Daly and Hobijn (2014) assume downward nominal wage rigidity in models in which employment is restricted to fluctuate at the intensive margin only, i.e., they dispense with unemployed workers altogether. Akerlof, Dickens, and Perry (1996) and Schmitt-Grohe and Uribe (2016) close the consumption over the period 1947-2001 is 0.05% if consumers are assumed to have log-utility and face trend stationary fluctuations in output with normally distributed innovations.
labor market with some variant of a short-side rule—i.e., assuming the labor-market is demand-constrained when wages need to fall—but without explaining explicitly why unemployed workers do not bid down the wage of employed workers. In contrast, Ferraro (2017) relies on an explicit search-and-matching model of the labor market to replicate some of the asymmetries in the business cycles—Sichel’s skewness in employment and Neftçi’s asymmetry between the speeds of expansions and contractions—without relying on downward nominal wage rigidity. Abbritti and Fahr (2013) rely on a search model with asymmetric wage adjustment to generate Neftçi’s asymmetry that contractions are faster than expansions. Chodorow-Reich and Wieland (2018) show that a multi-area multi-sector search model with downward nominal wage rigidity can replicate the fact they document that industry labor reallocation increases unemployment in recessions but not in expansions.

Our assumption of downward nominal wage rigidity is motivated by microeconomic evidence on the existence of asymmetric nominal wage rigidity. For the US, studies based on worker survey data such as the Panel Study of Income Dynamics (McLaughlin (1994); Kahn (1997); Card and Hyslop (1997); Altonji and Devereux (2000)), the Current Population Survey (Card and Hyslop (1997)), the Survey of Income and Program Participation (Gottschalk (2005); Barattieri, Basu, and Gottschalk (2014)), or on firm data (Altonji and Devereux (2000); Lebow, Saks, and Wilson (2003); Kurmann and McEntarfer (2017)), find a fraction of wage freezes for workers paid by the hour between 7% and 55%, and a fraction of nominal wage cuts between virtually 0% and 20%. Of even more direct relevance for our focus on the business cycle, Daly and Hobijn (2014) and their publicly available Wage Rigidity Meter document how downward nominal wage rigidity—measured as the fraction of wage freezes and based on the CPS—counter-cyclically moves with the business cycle.\footnote{See also Elsby, Shin, and Solon (2015) using the same CPS data, and Fallick and Wascher (2016) using the Employment Cost Index firm data.}

Pissarides (2009) and Haefke, Sonntag, and van Rens (2013) argue that it is only wage rigidity for new hires that has allocative implications (see, however, Schoefer, 2015, for an alternative view). Haefke, Sonntag, and van Rens (2013) argue that wages of new hired are less rigid than those of existing workers. Gertler and Trigari (2009); Gertler, Huckfeldt, and Trigari (2016) argue that this result may be mainly due to a compositional effect. Using a dataset of online posted wages, Hazell and Taska (2018) find that nominal wages posted for the same job, for which the compositional bias does not apply, are rigid, especially downwardly so. Bewley (1999, ch. 9)
presents survey evidence that for reasons of equity and morale, employers seek to maintain internal equity between similar workers within the firm and therefore to tie the wage of new hires to existing workers in similar positions within the firm.

Recent work has explored several ways in which Lucas’s (1987, 2003) calculations may underestimate the costs of business cycle fluctuations and therefore the potential benefits of stabilization policy. Fluctuations are more costly when output is difference stationary (Obstfeld, 1994) and when shocks have fat tails (Barro, 2009). Uninsurable income risk also increases the cost of fluctuations (Krebs, 2007, Krusell et al., 2009). Barlevy (2004) argues that fluctuations can have a large effect on welfare because they can affect the long-term growth rate of the economy. Our work highlights another reason why fluctuations may be more costly than Lucas’s estimates: they may reduce the average level of output, even when they do not affect the long-term growth rate. Hairault, Langot, and Osotimehin (2010); Jung and Kuester (2011); Lepetit (2018) emphasize the same mechanism within a labor search framework. Because they assume symmetric real wage rigidity, they find an effect of fluctuations on the average level of unemployment an order of magnitude smaller than the one we find under downward nominal wage rigidity (0.10-0.15 percentage point). Consistently, we find that the non-linearity in the worker-flow equation of the search model can only generate a limited increase in the average unemployment rate, for the same reason that it cannot replicate the plucking property found in the data.

The existence of downward nominal wage rigidity—and the resulting usefulness of inflation to grease the wheels of the labor market—is a long-lived argument in favor of a positive inflation target. The economic environment of the past decade, with the overnight nominal rates in many developed countries up against the zero lower bound (ZLB), has revived a distinct argument in favor of a positive inflation target: reducing the probability of falling into a liquidity trap (Phelps, 1972; Summers, 1991). Formal quantitative models, such as Coibion, Gorodnichenko, and Wieland (2012), Andrade, Gali, Le Bihan, and Matheron (2016), and Blanco (2018) find that the ZLB constraint can rationalize an inflation target from 1.5% to 3%. While both downward nominal wage rigidity and the ZLB speak in favor of a positive inflation target, the interplay between the two is subtle: downward nominal rigidity can actually weaken the ZLB rationale for a higher inflation target, by tempering deflation during a ZLB episode.

3These optimal inflation targets are calculated ruling out forward guidance and fiscal policy during a liquidity trap. Such policies, if available, can substitute for a higher inflation target by alleviating the cost of a spell at the zero lower bound. On the other hand, Dordal i Carreras, Coibion, Gorodnichenko, and Wieland (2016) emphasize that the optimal inflation target is quite sensitive to how long ZLB episodes are modeled to be, since the cost of a ZLB episode increases sharply with the duration of the ZLB episode.
The paper proceeds as follows. Section 2 presents empirical evidence. Section 3 lays out our plucking model of business cycles. Section 4 analyses its ability to generate business cycles asymmetries. Section 5 shows that fluctuations increase the average level of unemployment and higher inflation reduces the average level of unemployment. Section 6 concludes.

2 Three Business Cycles Asymmetries: Friedman’s, Sichel’s, and Neftçi’s

In this section, we document the salient asymmetry in the dynamic of the post-WWII US unemployment rate that Friedman called the plucking property: the amplitude of a contraction forecasts the amplitude of the subsequent expansion, while the amplitude of an expansion does not forecast the amplitude of the subsequent contraction. We then document two other asymmetries that have received more attention in the literature. The distribution of the unemployment rate is right-skewed, as emphasized by Sichel (1993). The unemployment rate rises more quickly than it falls, as emphasized by Neftçi (1984), among others. In addition, we highlight the long duration of expansions and contractions in the data. Finally, we review micro-data evidence on asymmetric wage adjustments.

2.1 Defining Expansions and Contractions

The data that we use are the seasonally adjusted monthly unemployment rate for workers over 16 years old. Our sample period is January 1948 to September 2018. We define business cycles peaks and troughs as follows. We start taking the first month of our sample as a candidate for a business cycle peak. If, in all following periods until unemployment becomes \( x \) percentage point higher than in the first month, unemployment is higher than in the first month, we call the first month a business cycle peak. Otherwise, the first period we encounter in which unemployment is less than in the first month becomes our new candidate for a business cycle peak. Once we have identified a peak, we switch to looking for a trough, and so on until we reach the end of the series. We set \( x = 1.5 \) percentage points, the decrease in unemployment during the 1970-1973 expansion, which is, to one exception, the smallest variation in unemployment in any of the expansions and contractions identified by the National Bureau of Economic Research (NBER) Business Cycle Dating Committee over our sample. The one exception is the 0.6 percentage point decrease in unemployment in 1980-1981 that the NBER identifies as an expansion, but which we consider to be part of a 1980-1982 double-dip recession. Any value for \( x \) between 0.8 and 1.5
percentage point identifies exactly the same cycles. Notice that a business cycle peak is a trough in the unemployment rate and vice-versa. In what follows, a peak will always refer to a business cycle peak as opposed to a peak in the unemployment rate.

Figure 1 plots the unemployment rate over our sample period with vertical lines indicating the times that we identify as business cycle peaks and troughs. The algorithm described above identifies ten peaks and ten troughs. To these we add a peak at the end of our sample in September 2018 when unemployment was lower than in any month since the previous trough.

Table I presents the peak and trough dates we identify. We also present the peaks and troughs identified by the National Bureau of Economic Research (NBER) Business Cycle Dating Commit-
Table 1: Business Cycle Peaks and Troughs

<table>
<thead>
<tr>
<th>Unemployment Peak</th>
<th>NBER Peak</th>
<th>Unemployment Trough</th>
<th>NBER Trough</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 [9/2018]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Business cycle peaks and troughs defined solely based on the unemployment rate and, for comparison, business cycle peaks and troughs as defined by the Business Cycle Dating Committee of the National Bureau of Economic Research.

The NBER peaks and troughs are based on a broader set of business-cycle indicators than only the unemployment rate. However, our procedure identifies the same number of peaks and troughs—up to the double-dip recession of the early 1980s. In addition, the times of peaks and troughs that we identify based solely on the unemployment rate are in most cases quite similar to the times identified by the NBER. The NBER peaks tend to lag our peaks by a few months and the NBER troughs tend to precede our troughs by a few months. This implies that our estimate of the average duration of contractions is about one year longer than what results from the NBER’s dating procedure.

2.2 The Plucking Property

Figure 2 presents scatter plots illustrating the plucking property of the unemployment rate. The left panel plots the amplitude of an expansion against the amplitude of the previous contraction. The amplitude of expansions is defined as the percentage point decrease in the unemployment rate from the business cycle trough to the next peak. The amplitude of contractions is defined analogously. There is clearly a strong positive relationship between the amplitude of an expansion and the amplitude of the previous contraction in our sample period. In other words, the size of a contraction strongly forecasts the size of the subsequent expansion. We have included an OLS
regression line in the panel. Table 2 reports the regression coefficient from this regression. The relationship is roughly one-for-one. For every percentage point increase in the amplitude of a contraction, the amplitude of the subsequent expansion increases by 1.09 percentage points on average. Despite the small number of data points, the relationship is highly statistically significant (t-statistic of 3.4). This hints at a large explanatory power. Indeed, the $R^2$ of the regression is high, at 0.58.

The right panel plots the amplitude of a contraction against the amplitude of the previous expansion. In sharp contrast to the left panel, there is no relationship in this case. The size of an expansion does not forecast the size of the next contraction. In Friedman’s language, each contractionary pluck that the economy experiences is independent of what happened before. The linear regression line in the panel is actually slightly downward sloping. But the association is far from statistically significant. The $R^2$ of the regression is low, at 0.22.

Overall, the two panels in Figure 2 strongly indicate that Milton Friedman was right: The am-

Jackson and Tebaldi (2017) suggest that the duration (not size) of an expansion is predictive of the size of the following contraction, which they explain through an analogy with forest fires: the longer the expansion, the more “underbrush” inputs there are to burn during the contraction. We find no evidence of the forest fire theory at the aggregate level: the duration of an expansion is no more predictive of the size of the following contraction than the size of the expansion is. The relationship is actually negative (but not significant), driven by the fact that the three longest post-WW2 expansions (1961-1968, 1982-1989, 1992-2000) were followed by among the mildest recessions.
plitude of contractions forecasts the amplitude of the subsequent expansions, but the amplitude of expansions does not forecast the amplitude of the subsequent contractions. Unemployment reverts strongly from high values, but mildly from low values.

2.3 The Positive Skewness of Unemployment

The plucking property is an asymmetry in the predictive power of contractions and expansions. A related but distinct asymmetry is what [Sichel (1993)] called “deepness”: “when troughs are deeper than peaks are tall”. This informal definition relies on an unobserved reference level to define troughs and peaks, but Sichel suggests to assess deepness by testing for negative skewness in the distribution of the level of a business cycle variable.

For the unemployment rate, deepness corresponds to positive skewness. We confirm that the unemployment rate is positively skewed in post-WWII US data. Figure 3 plots a histogram of the distribution of the unemployment rate over our sample period. The unemployment rate is noticeably right skewed. Much of the mass of the distribution is close to 5% (median of 5.6% and mean of 5.8%). However, the right tail reaches quite a bit further out than the left tail. The maximum value of the unemployment rate in our sample is 10.8% in 1982, 5.2 points above the median value, while the minimum value is 2.5% in 1953, only 3.1 points below the median. The skewness of the distribution is 0.63, with 95% bootstrapped confidence interval [0.51, 0.75].

2.4 Contractions are Faster than Expansions

A third business cycles asymmetry has received much more attention than the previous two: the asymmetry in growth rates between contractions and expansions. Applied to the unemployment rate, it is the idea that unemployment rises much more quickly during contractions than it falls.

<table>
<thead>
<tr>
<th></th>
<th>β</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expansion on previous contraction</td>
<td>1.09</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td></td>
</tr>
<tr>
<td>Contraction on previous expansion</td>
<td>-0.38</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(0.27)</td>
<td></td>
</tr>
</tbody>
</table>

Note: The first row reports the coefficient in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second row reports the coefficient in an analogous regression of the size of a contraction on the size of the previous expansion.
during expansions. The observation of this asymmetry dates back at least to Burns and Mitchell, and Keynes. We refer to it as Neftçi’s asymmetry, because Neftçi (1984) was one of the first to draw statistical attention to it by providing a statistical test of “sudden jumps and slower drops” in unemployment. Neftçi’s asymmetry in speeds is quite distinct from both Friedman’s asymmetry in predictive power, and Sichel’s asymmetry in levels. Sichel (1993) refers to it as “steepness” to contrast it to deepness. McKay and Reis (2008) refer to it as the greater “violence” of contractions, in reference to Mitchell (1927).

The asymmetry in speeds can be assessed by testing for skewness in the growth rate of unemployment. Yet, a particularly simple way to illustrate it is to calculate the average speed of expansions and contractions in percentage points of unemployment per year. Table 3 reports two sets of estimates of the average speed of expansions and contractions. The first set weights expansions and contractions by their length, while the second set weights all expansions and contractions equally. (See the table note for details.) We find that the unemployment rate rises roughly twice

---

Mitchell (1927) notes: “Business contractions appear to be briefer and more violent than business expansions”. Keynes (1936) writes: “The substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning point when a upward is substituted for a downward tendency”.

McKay and Reis (2008) document this asymmetry in speed in employment, but much less so, if at all, in output. Abbritti and Fahr (2013) show that a search model with downward nominal wage rigidity can generate this steepness or violence. Pizzinelli, Theodoridis, and Zanetti (2018) argue for an asymmetry based not on whether unemployment is increasing or decreasing, but on whether employment is high or low—they argue unemployment is more volatile when it is low.
Table 3: Speed of Expansions Versus Recoveries

<table>
<thead>
<tr>
<th></th>
<th>Year Weighted</th>
<th>Spell Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of expansions</td>
<td>0.77</td>
<td>0.88</td>
</tr>
<tr>
<td>Speed of contractions</td>
<td>1.67</td>
<td>1.89</td>
</tr>
<tr>
<td>P-value for equal speed</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

Note: The speed of expansion and contractions is measured in percentage points of unemployment per year. For each expansion and contraction, we calculate the change in unemployment over the spell and the length of time the spell lasts for. We then calculate the speed of expansions and contractions in two ways: 1) First calculate the speed in each spell and then take a simple average across spells. We refer to this as spell weighting. 2) Sum the change in unemployment across spells and sum the length of time across spells and then calculate the average speed by dividing the aggregate change in unemployment by the aggregate length of time. We refer to this as year weighting. We also regress the absolute value of the speed of adjustment for both expansions and contractions on a dummy for contractions and report the p-value for this dummy.

as quickly during contractions (1.9 percentage points per year) as it falls during expansions (0.9 percentage points per year). This difference is highly statistically significant. We run a regression of the absolute value of the speed of expansions and contractions on a dummy variable for a spell being a contraction and find that the p-value for the dummy is 0.002.

2.5 The Duration of Expansions and Contractions

Given that contractions and expansions are of about the same average size (3.7 percentage points), the fact that contractions are faster than expansions implies that contractions are also shorter than expansions. Yet, a distinct fact about the dynamics of the unemployment rate that we would like to highlight is the long durations of both expansions and (to a lesser extent) contractions. Looking back at Figure 1, we can clearly see that when the unemployment rate starts falling, it usually falls steadily for a long time. Table 4 lists the duration of all expansions and contractions over our sample period. The average length of expansions is 57.9 months, or almost five years. Contractions are also quite persistent. The average length of contractions in our sample is 26.9 months, or more than two years. Perhaps most strikingly, in a few cases—the 1960s, 1980s, 1990s, and the current expansion—the unemployment rate has fallen steadily for six to nine years without reversal. We will argue that these long and steady expansions place interesting restrictions on the types of models or shock processes that drive business cycles.

2.6 Wage Rigidity

In the next section, we seek to account for the four previous empirical facts through a model of the labor market embedding downward nominal wage rigidity. Regardless of its ability to generate
Table 4: The Duration of Expansions and Contractions

<table>
<thead>
<tr>
<th>Dates</th>
<th>Length in Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>Trough</td>
</tr>
<tr>
<td>1</td>
<td>[1/1948]</td>
</tr>
<tr>
<td>2</td>
<td>5/1953</td>
</tr>
<tr>
<td>3</td>
<td>3/1957</td>
</tr>
<tr>
<td>6</td>
<td>10/1973</td>
</tr>
<tr>
<td>7</td>
<td>5/1979</td>
</tr>
<tr>
<td>10</td>
<td>10/2006</td>
</tr>
<tr>
<td>11</td>
<td>[9/2018]</td>
</tr>
</tbody>
</table>

Mean 57.9 26.9

the plucking property of the unemployment rate, here we refer to direct evidence for downward nominal wage rigidity and its impact on unemployment. In principle, it is straightforward to document the fraction of workers experiencing wage freezes and wage decreases in panel surveys of workers’ earnings. In practice, the values of these two statistics have been subject to disagreement, arising from the difficulty to account for measurement errors in reported wages. To focus on the year-to-year changes in the wages of job-stayers paid by the hour, McLaughlin (1994), Kahn (1997), Card and Hyslop (1997) report raw figures of about 7 to 17% wage freezes and 10 to 20% wage decreases based on the Panel Study of Income Dynamics (PSID) and the Current Population Survey (CPS) over periods comprised between 1970 and 1993—a period including many high-inflation years. Although measurement errors could in principle either overstate or understate the extent of nominal wage rigidity, Altonji and Devereux (2000) report up to 43% wage freezes and virtually no wage cuts in the PSID after correcting for measurement errors. Correcting for measurement errors in a different way (and relying instead on the Survey of Income and Program Participation), Gottschalk (2005) and Barattieri, Basu, and Gottschalk (2014) report similar

8Measurement errors in worker survey data can be argued to either underestimate the extent of wage rigidity if zero wage-changes are misreported as small wage-changes (Akerlof, Dickens, and Perry (1996)), or overstate it because of rounding errors (Smith (2000)).

9Firm data are in principle less subject to measurement errors. Altonji and Devereux (2000) confirm their results on the virtual absence of wage cuts (0.5%) using the personal records of a large financial corporation. Lebow, Saks, and Wilson (2003) also argue that firm data display more evidence of wage rigidity. They report 18% of wage freezes and 14.5% of wage cuts. Kurmann and McIntarfer (2017) challenge the idea of more wage rigidity in firm datasets using data from the Longitudinal Employer Household Dynamic program. They report 10% wage freezes and 25% wage cuts over the period 1998-2014.
Both more relevant to whether downward nominal wage rigidity affects fluctuations in unemployment, and less prone to measurement errors, is how the fraction of wage freezes evolves along the business cycles. Recently, Daly and Hobijn (2014) have documented the strong countercyclicality of the fraction of wage freezes using the CPS. Figure 4 plots their Wage Rigidity Meter—the fraction of wage freezes of job-stayers with respect to the wage one year prior, with no correction for measurement errors—since 1997, the last discontinuity in the series and the date at which inflation reached levels inferior to 3%. The figure superimposes the unemployment rate and (one minus) the employment rate (the ratio of the employed to the working age population). The correlation between wage freezes and activity is striking. It is less pronounced from 1980 to 1997, when the fraction of wage freezes includes an increasing trend plausibly related to the decreasing trend in inflation. Interestingly, the correlation is even stronger with the employment rate. In particular, both the Wage Rigidity Meter and the employment rate have only slowly recovered during the current expansion.

3 An Equilibrium Model of Downward Nominal Wage Rigidity

What model of the labor market can generate the plucking property exhibited in US data? We consider two versions of the Diamond-Mortensen-Pissarides (DMP) search model of unemployment and assess whether either can replicate the empirical facts we documented in section 2. The first model is a canonical DMP model with symmetric real wage rigidity to get around the Shimer puzzle. The second model replaces symmetric real wage rigidity with downward nominal wage rigidity. We allow the aggregate productivity process to follow either an AR(1) or an AR(2) process, and allow for sectoral productivity shocks. We present both models in their most general versions, with sectoral heterogeneity.

3.1 Heterogeneous labor inputs, separated labor markets

There is a continuum $i \in [0, 1]$ of labor types. To each labor type $i$ corresponds a sector $i$ in which identical firms (or equivalently, a representative firm) have access to a decreasing-returns-to-scale...
technology that uses labor type $i$ as its single input:

$$Y_t^i = A_t^i F(N_t^i),$$

where $Y_t^i$ is output, $N_t^i$ is employment, and $A_t^i$ is an exogenous productivity shifter. It can be taken to represent any change in labor productivity in sector $i$, or more broadly as a stand-in for any shifter of labor demand in sector $i$. Sectoral heterogeneity is restricted to labor markets: consumers perceive goods produced in different sectors as identical and therefore value them equally. All goods are sold in a competitive market at a common price $P_t$.

We focus instead on labor-market heterogeneity. A given worker provides only one type of labor, and can therefore only seek to work at a firm in one sector: there is a distinct labor market for every type of labor and workers cannot flow across labor markets. A type of labor is to be thought...
of as, for instance, lawyers in Houston. The time necessary to accumulate human capital makes is costly for a lawyer to retrain as an electrician. Mobility constraints limit the willingness of the Texan lawyer to seek work in New York. At the business cycles frequency we are concerned with, we simply rule out such career changes. The literature on the China trade shock (Autor, Dorn, and Hanson (2013); Autor, Dorn, Hanson, and Song (2014)) provides evidence that reallocation of workers across sectors and regions can be very slow.\footnote{The assumption of such differentiated labor inputs is standard in the New-Keynesian literature. There, differentiated labor inputs are an important source of strategic complementarity in pricing relative to the case of a unique homogeneous labor input. See e.g. Woodford (2003), chapter 3.}

3.2 Workers

There is a fixed supply of workers in all sectors, equal across sectors and normalized to one. Because of search frictions, not all workers in sector $i$ are employed: only $N_i^t$ workers are, while $U_i^t = 1 - N_i^t$ are unemployed. Employed workers in sector $i$ earn the nominal wage $W_i^t$ while unemployed workers earn nothing. We denote the real wage in sector $i$ as $w_i^t = W_i^t/P_t$. We abstract from the intensive-margin labor-supply decision of workers. Workers supply (or at least try to supply) an exogenous quantity of labor—which we normalize to 1—to firms. They do so whatever the wage: their labor supply is fully inelastic, i.e. their valuation of leisure is zero.\footnote{The assumption is inessential. Workers’ valuation of leisure imposes a positive lower-bound on an equilibrium wage. In our main model of downward nominal wage rigidity however, the risk is that wages could be too high, not too low. We therefore assume a zero valuation of leisure to avoid lengthening the exposition with the peripheral problem of job-seekers. As we explain further on, preventing firms from hiring more workers than exist imposes another lower bound on wages.}

Because we do not assume Nash bargaining, households play an essentially passive role in our model. We do not need to specify their preferences further, but assume all households are risk-neutral with discount factor $\beta \in (0, 1)$ to rationalize that firms discount future profits with the risk-neutral stochastic discount factor $\beta$.

3.3 Labor-Demand

A firm’s workforce is constantly depleted by exogenous job separations: each period a fraction $s \in (0, 1)$ of a firm’s workforce leaves the firm. We denote the number of workers hired by firm $i$ at $t$ by $H_i^t$. Firm $i$’s workforce therefore evolves according to:

$$N_i^t = (1 - s)N_{i-1}^t + H_i^t.$$  \hspace{1cm} (2)

This implicitly assumes that workers hired at $t$ start to work at $t$.\footnote{The assumption of such differentiated labor inputs is standard in the New-Keynesian literature. There, differentiated labor inputs are an important source of strategic complementarity in pricing relative to the case of a unique homogeneous labor input. See e.g. Woodford (2003), chapter 3.}
To hire workers, firm \( i \) must post vacancies. Posting a vacancy costs \( A_i^t c \) units of the unique good of the economy, where \( c \) is a constant. A vacancy translates into a hire if it matches with a job-seeker. A match happens with probability \( q_i^t \), which firm \( i \) takes as given. The determination of \( q_i^t \) is described below. We assume that the firm is big enough that it can abstract from the randomness in seeking a worker: hiring one worker requires the firm to post \( 1/q_i^t \) vacancy and has the certain real cost \( A_i^t c/q_i^t \).

Firm \( i \)'s real profits at \( t \) are real revenues, minus real labor costs, minus real hiring costs:

\[
A_i^t F(N_i^t) - w_i^t N_i^t - \frac{A_i^t c}{q_i^t} H_i^t I|_{H_i^t \geq 0}.
\]  

(3)

Firm \( i \) chooses employment and hires in order to maximize intertemporal real profits, discounting them using the risk-neutral discount factor \( \beta \), and subject to the workforce flow equation (2). If firm \( i \) hires every period—which we will impose in equilibrium—firm \( i \)'s labor demand (equivalently hiring decision) is characterized by the first-order condition:

\[
A_i^t F'(N_i^t) = w_i^t + \frac{c A_i^t}{q_i^t} - \beta (1 - s) E_t \left( \frac{c A_{i+1}^t}{q_{i+1}^t} \right),
\]  

(4)

which equates the marginal productivity of a worker to his cost to the firm, which is equal to the wage, plus the hiring cost, minus the expected savings of having a worker next period without having to hire him next period. For firms not to be willing to fire workers in equilibrium, it must be that the marginal value of an (already hired) worker is positive. This imposes the following upper-bound on the wage:

\[
w_i^t \leq A_i^t F'(N_{i-1}^t) + \beta (1 - s) E_t \left( \frac{c A_{i+1}^t}{q_{i+1}^t} \right).
\]  

(5)

The probability of filling a vacancy \( q_i^t \) is determined in equilibrium through an exogenous matching function \( q(\theta_i^t) \), where labor market tightness in labor-market \( i \), \( \theta_i^t = H_i^t/(q_i^t S_i^t) \), denotes the ratio of the number of vacancy posted \( H_i^t \) to the number \( S_i^t \) of job-seekers at the beginning of the period. The probability for an unemployed worker of type \( i \) to find a job is equal to the ratio of hires to job-seekers \( f(\theta_i^t) = H_i^t/S_i^t = \theta_i^t q(\theta_i^t) \). We assume that a worker losing his job between periods \( t-1 \) and \( t \) gets a chance to find a new job at the beginning of period \( t \) and therefore to work in period \( t \), spending no period without a job. Thus, the number of job-seekers in labor-market \( i \) at \( t \) is \( S_i^t = 1 - (1 - s) N_{i-1}^t \). The employment flow equation (2) can therefore be rewritten using

\[13\] The number \( S_i^t \) of job seekers at \( t \), although it can be seen as the number of unemployed at the beginning of period \( t \), is not equal to what we defined as the unemployment rate \( U_i^t \) at \( t \), which only counts those job seekers who did not find a job at \( t \).
the tightness ratio \( \theta_t \) instead of hires \( H_t^i \):

\[ N_t^i = 1 - (1 - f(\theta_t))(1 - s)N_{t-1}^i. \]  \hfill (6)

### 3.4 Wage-Setting

Because of search frictions, unemployed workers cannot instantly meet with firms to offer to replace employed workers at a lower wage. Instead, an unemployed worker only meets a firm after a match has occurred, at which point he has some monopoly power and no longer has any reason to bid the wage down. As a result the wage has no reason to be driven to market-clearing level, nor to be uniquely pinned down to any level: nothing forces the equilibrium to be at the crossing of the labor-demand curve \( (4) \) and labor supply curve \( N_t^i = 1 \). Instead, there are only upper and lower bounds on an equilibrium wage. The upper bound is defined by the no-firing condition \((5)\). Since we assume workers do not value leisure, there is no lower bound coming from workers’ unwillingness to work for too low a wage. However, an equilibrium wage must prevent firms from being willing to collectively hire more workers than exist in the economy\[14\]. Using the labor demand equation \((4)\), the condition of no excess labor demand \( N_t^i \leq 1 \) translates into the following lower-bound on the wage:

\[ w_t^i \geq A_t^iF'(1) - \frac{cA_t^i}{q} + \beta(1 - s)E_t \left( \frac{cA_{t+1}^i}{q_{t+1}} \right), \]  \hfill (7)

where \( q \) is the value of the vacancy-filling rate when the job-finding rate \( f \) is equal to 1.

In-between these two bounds, all wages are consistent with individual optimality. This continuum of wages defines an infinity of equilibria, each characterized by an assumption on wage-setting. We consider two wage-setting assumptions: symmetric real wage rigidity, and downward nominal wage rigidity. Both assumptions of wage rigidity are in reference to a benchmark of flexible wages. Following [Blanchard and Gali (2010)] and [Michaillat (2012)], we specify wage flexibility through the short-cut assumption that real wages follow productivity:

\[ w_t^i = \bar{w}A_t^i, \]  \hfill (8)

where \( \bar{w} \) is a constant. The short-cut is justified by the fact that this is the dynamics of wages under market-clearing, and a very close approximation to the dynamics of wages in the search model under Nash bargaining.

\[14\] The necessity of imposing such a condition depends on the functional form of the matching function. It is necessary with the Cobb-Douglas specification we will rely on because nothing in the Cobb-Douglas matching function imposes the job-finding probability \( f \) to be less than 1. But a matching function that restricts \( f \) to lie between 0 and 1 would make hiring infinitely costly as \( N_t^i \) tends to 1, killing any incentive for firms to hire the whole labor supply.
As Shimer (2005) shows, when wages follow productivity closely (e.g. because of Nash bargaining), the DMP model cannot replicate the volatility of unemployment in the data. If all the effect of shocks goes to prices, quantities stay mostly unchanged. To circumvent the Shimer puzzle, we take as our benchmark search model the model under the assumption that real wages adjust slowly (but symmetrically) to productivity. Specifically, we follow Shimer’s (2010) specification that the real wage is a weighted average of the past real wage and the present flexible wage:

$$\log(w_i^t) = \rho \log(w_{t-1}^i) + (1 - \rho) \log(A_i^t),$$  \hspace{1cm} (9)

where $\rho$ is a weight between 0 and 1. (Because we will assume a symmetric process for the $\log$ of $A_i^t$, we take the average to be geometric—arithmetic for the $\log$ of wages—in order not to introduce an ad hoc source of asymmetry in the model).

In our second model, we replace symmetric real wage rigidity with downward nominal wage rigidity. We assume that the nominal wage is set to the flexible wage, except if this requires the nominal wage to fall: $W_i^t = \max\{P_t A_i^t, W_{t-1}^i\}$. Expressed in terms of real wages, and denoting the inflation rate by $\Pi_t = P_t / P_{t-1}$, the wage-setting equation becomes:

$$w_i^t = \max\left\{\bar{w} A_i^t, \frac{w_{t-1}^i}{\Pi_t}\right\}.$$ \hspace{1cm} (10)

Inflation relaxes the constraint on downward real wage adjustments: it greases the wheels of the labor market.

The two specifications of wage-setting do not explicitly impose that the wage remains within the wage band defined by the no-firing condition (5) and no excess demand condition (7). However, we will check that they almost always do in our simulations.

### 3.5 Equilibrium

To close the model, we assume that the good market clears. This implies that production is equal to households’ demand for consumption, plus firms’ demand for hiring services:

$$\int_0^1 Y_t^i\,di = C_t + \int_0^1 \frac{cA_t^i}{q(\theta_t^i)} \left[N_t^i - (1 - s)N_{t-1}^i\right].$$ \hspace{1cm} (11)

Yet under risk-neutrality the market-clearing conditions only residually gives consumption and we can abstract from it (as well as from production) in defining an equilibrium.

An equilibrium is given by processes for the three idiosyncratic variables $N_t^i, \theta_t^i, w_t^i$ for each sector $i \in [0, 1]$, and aggregate inflation $\Pi_t$, such that in all sectors $i$, firm $i$ is on its labor demand...
schedule (4), the employment flow equation (6) holds, the no-firing condition (5) and no-excess-demand condition (7) hold, and wages are set according to the downward nominal wage rigidity wage-setting rule (10) or the alternative benchmark of symmetric real wage rigidity (9). We are interested in the aggregate unemployment rate, defined as the average unemployment rate across sectors:

\[ U_t = \int_0^1 (1 - N^i_t) di \]  

(12)

An equilibrium is conditional on exogenous processes for productivity \( A^i_t \), initial conditions for employment \( N^i_0 ) \), and a monetary policy. We specify monetary policy as directly setting a path for the inflation rate \( \Pi_t \), which we take to be constant to some target value \( \Pi \). As for productivity, we assume that sectoral productivity \( \log(A^i_t) \) in sector \( i \) is the sum of a time trend at growth rate \( g \), a stationary aggregate component \( \log(A_t) \), and an integrated idiosyncratic component \( \log(Z^i_t) \), assumed to be independent from the aggregate component and from the idiosyncratic components in other sectors:

\[ \log(A^i_t) = g \times t + \log(A_t) + \log(Z^i_t), \]

(13)

We assume the idiosyncratic component follows an AR(1) in growth rates with Gaussian innovations:

\[ \Delta \log(Z^i_t) = \rho_{\Delta z} \Delta \log(Z^i_{t-1}) + \varepsilon^\Delta z^i_t \sim N(0, \sigma_{\varepsilon^\Delta z}). \]

(14)

We assume the aggregate component follows either an AR(1) or an AR(2) in levels with Gaussian innovations:

\[ \log(A_t) = (I - \rho_1^a)^{-1} \varepsilon^a_t, \varepsilon^a_t \sim N(0, \sigma_{\varepsilon^a}), \]

(15)

or

\[ \log(A_t) = (I - \rho_1^a)^{-1}(I - \rho_2^a)^{-1} \varepsilon^a_t, \varepsilon^a_t \sim N(0, \sigma_{\varepsilon^a}). \]

(16)

### 3.6 Calibration

We calibrate the model to a monthly frequency. We calibrate the discount factor \( \beta \) to correspond to an annual interest rate of 4%. We assume a constant-elasticity production function \( F(N) = N^\alpha \) and set decreasing returns to \( \alpha = 2/3 \). We assume a Cobb-Douglas matching function \( q(\theta) = \mu \theta^{-\eta} \) and set the elasticity of the matching function to \( \eta = 0.5 \), in the middle of the range reported
in Petrongolo and Pissarides (2001)’s survey. The parameters $\mu$ and $c$ jointly determine hiring costs. One of the two is redundant: only $c\mu^{1-\eta}$ is identified. (Details are provided in appendix [A].) We normalize $\mu$ to 1. We set $c$ so that steady-state hiring costs $c/q$ are 10% of the monthly steady-state wage $\bar{w}$, in line with what Jose and Manuel (2009) report based on the Employer Opportunity Pilot Project survey in the US. We calibrated the constant monthly separation rate to its value reported by Shimer (2005) based on CPS data: $s = 3.4\%$. We set the growth rate of productivity $g$ to 2.3% annually, the average growth of US labor productivity from 1948 to 2018. These parameters determine the steady-state of the model up to $\bar{w}$, which we calibrate to match the average unemployment rate, as explained below.

The other parameters only affect the response of the economy to (aggregate and idiosyncratic) shocks. In what follows we consider three version of the model: the model of symmetric real wage rigidity with a single sector and an AR(1) in productivity, and both the models of symmetric real and downward nominal wage rigidity with heterogeneous sectors and an AR(2) in productivity. We calibrate the homogeneous model in a standard way. We set the auto-regressive root of the AR(1) process $\rho_1$ to 0.98 and the persistence in real wages $\rho$ to 0.9, following Shimer (2010). We calibrate $\bar{w}$ to get a steady-state level of unemployment equal to the average level of unemployment in the data (5.8%) and $\sigma_\varepsilon$ to get the standard deviation of unemployment equal to its value in the data (1.6 percentage point). This gives $\bar{w} = 0.6895$ and $\sigma_\varepsilon^2 = 5.37 \times 10^{-3}$ (so that $\sigma^2 = 2.7\%$).

We calibrate the heterogeneous models as follows. We calibrate the persistence of the idiosyncratic productivity process based on KLEMS annual data on US sectoral productivity from 1947 to 2010 (Jorgensen, Ho, and Samuels (2012)). The KLEMS dataset provides labor productivity series (value added per hour) for 31 sectors. We take $\log(Z_i^t)$ to be the log difference between the sectoral labor productivity series and the BLS series for aggregate labor productivity. We apply an MA(1) filter to the series in level to abstract from high-frequency variations in $\log(Z_i^t)$, and estimate AR(1)s on the first-difference $\Delta \log(Z_i^t)$ in each sector. The average estimated autoregressive root across sectors is $\rho_{\Delta z} = 0.62$ at an annual frequency. We therefore calibrate $\rho_{\Delta z} = 0.62^{1/12} = 0.96$ in our monthly calibration. We calibrate the volatility of idiosyncratic productivity growth $\sigma_{\Delta z}$ in order to get a steady-state fraction of constrained firms of 7.5%. This sets $\sigma_{\Delta z} = 7 \times 10^{-4}$.

We calibrate the roots of the aggregate process with the objective of reproducing the frequency of unemployment cycles in the data. Because there is little internal propagation in our model, we calibrate the roots to the ones obtained from estimating an AR(2) directly on the US unemployment rate series. We find them to be $\phi_1^a = 1.803$ and $\phi_2^a = -0.810$, i.e. roots $\rho_1^a = 0.96$ and
\( \rho_2^a = 0.84^{15} \) (Again, we first apply an MA(1) filter to the monthly series to abstract from very-high frequency variations. We get the same results when estimating the roots on the quarterly unemployment series without any smoothing and converting them to a monthly frequency).

Under downward nominal wage rigidity, we are left with three parameters to calibrate: the inflation rate \( \bar{\Pi} \), the steady-state wage \( \bar{w} \) and the standard deviation of the innovations to the aggregate process \( \sigma^a_{\epsilon} \). We set inflation to 2% yearly. We calibrate the last two in order to match the 5.8% average level of unemployment in the data and its 1.6 percentage point standard deviation. This gives \( \bar{w} = 0.6736 \) and \( \sigma^a_{\epsilon} = 1.8 \times 10^{-3} \) (so that \( \sigma^a = 3.7\% \)).

Under symmetric real wage rigidity, we maintain our calibration of wage-rigidity of \( \rho = 0.9 \). We reset the steady-state wage and volatility of aggregate shocks to match the 5.8% average level of unemployment in the data and its 1.6 percentage point standard deviation. This gives \( \bar{w} = 0.6890 \) and \( \sigma^a_{\epsilon} = 1.1 \times 10^{-3} \) (so that \( \sigma^a = 2.3\% \)). Table 5 provides a summary of our calibration.

<table>
<thead>
<tr>
<th>Table 5: Calibration</th>
<th>Homogenous, AR(1)</th>
<th>Heterogenous, AR(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogenous, AR(1)</td>
<td>SRWR</td>
<td>SRWR</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.96(^{\frac{1}{12}})</td>
<td>0.96</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>2/3</td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>st. c/q is 10% of ( \bar{w} )</td>
<td>3.4%</td>
</tr>
<tr>
<td>( g )</td>
<td>0.023(^{\frac{1}{12}})</td>
<td></td>
</tr>
<tr>
<td>( \rho_{\Delta z} )</td>
<td>–</td>
<td>0.96</td>
</tr>
<tr>
<td>( \sigma_{\Delta z}^2 )</td>
<td>–</td>
<td>( 7 \times 10^{-4} )</td>
</tr>
<tr>
<td>( \rho^1_a )</td>
<td>0.98</td>
<td>0.96</td>
</tr>
<tr>
<td>( \rho^2_a )</td>
<td>–</td>
<td>0.84</td>
</tr>
<tr>
<td>( \sigma^a_{\epsilon} ) st. ( \sigma^a = 2.7% )</td>
<td>st. ( \sigma^a = 2.3% )</td>
<td>st. ( \sigma^a = 3.7% )</td>
</tr>
<tr>
<td>( \bar{\Pi} )</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>( \bar{w} )</td>
<td>0.6895</td>
<td>0.6890</td>
</tr>
</tbody>
</table>

\(^{15}\) The autoregressive coefficients \( \phi^a_1 \) and \( \phi^a_2 \) are related to the roots \( \rho^a_1 \) and \( \rho^a_2 \) through: \( I - \phi^a_1 L - \phi^a_2 L^2 = (I - \rho^a_1 L)(I - \rho^a_2 L) \).
3.7 Solution Method

We introduced sectoral heterogeneity in such a way that a firm does not need to expect any endogenous aggregate variable in order to decide how many workers to hire. Therefore, we do not need to keep track of the endogenous aggregate state of the economy in order to solve for the hiring decision of a firm. The hiring decision of a firm is a function of four (if aggregate productivity follows an AR(1)) or five (if it follows an AR(2)) state variables: aggregate productivity $A_t$ (and lagged aggregate productivity $A_{t-1}$ under an AR(2)), idiosyncratic productivity growth $\Delta Z^i_t$, the wage $w^i_t$, and lagged employment in sector $i$, $N^i_{t-1}$.

Given the asymmetries and non-linearities our model is intended to capture, we rely on global methods to numerically solve for the equilibrium. A solution to the problem of a firm can be described as policy functions for $1/q^i$ and $N^i$ as a function of the firm’s state variables. We form a discrete grid over the state-space, approximate the stochastic processes for the exogenous productivity variables using Rouwenhorst (1995) discretization method, and solve the model by iteration on the policy function. In simulating the heterogeneous model, we assume 1000 sectors. Details are provided in appendix B.

An issue arises in solving the model: some points of the grid of the state space necessarily feature high wages and low productivity. Firms would like to fire workers in such states, violating the no-firing condition. This does not mean, however, that the no-firing constraint is likely to be violated on an equilibrium path. These states are very unlikely to occur: we check ex post that our simulated paths remain away from these states. Solving the equilibrium in these extreme states is nevertheless necessary to calculate expectations in states that do occur with reasonable probability on the equilibrium path. We adopt the following approach: in a state where the no-firing condition fails, we assume that firms do not fire workers and simply do not hire.\footnote{The symmetric problem can occur with the no-excess-demand condition under symmetric real wage rigidity: wages may be so much below productivity that firms are willing to hire more workers than there are. We deal with such cases in the same way: we assume that firms hire all workers but no more.}

4 Generating Plucking

In this section, we consider whether our search models can replicate the business cycles asymmetries documented in section 2. We first consider the model of symmetric real wage rigidity with a single sector and an AR(1) in productivity. It generates some skewness in unemployment though less than in the data, much less plucking, and no asymmetry in speeds. Because the search model
has very little internal propagation, it also generates much too short contractions and expansions.

We then consider the model of downward nominal wage rigidity, with an AR(2) in productivity to avoid too brief business cycles, and with sectoral productivity shocks to let downward nominal wage rigidity have realistic implications. It generates significantly more plucking than the baseline search model, in line with the data. It also considerably increases the skewness of the unemployment rate, actually beyond its value in the data. It also creates contractions that are shorter and faster than expansions, also the asymmetry is less pronounced than in the data.

4.1 Can the Baseline Search Model Generate Plucking?

We consider first the search model under symmetric real wage rigidity. Our motivation with this model is to assess whether a baseline model of search unemployment can generate Friedman’s plucking property, Sichel’s skewness in unemployment, and Neftçi’s asymmetry in speeds. We therefore consider the model first in its most standard version: a single-sector model hit by AR(1) productivity shocks.

The second column of table 6 provides the business cycles statistics for this model. It generates some plucking: the coefficient in a regression of the size of an expansion on the size of the previous contraction is 0.42, greater than the coefficient in a regression of the size of a contraction on the size of the previous expansion, 0.22. In addition, the former regression has more explanatory power than the latter: the $R^2$ is 0.18 against 0.05. The difference between expansions and contractions is however much smaller than in the data. Appendix C displays the scatter plots associated to the regressions. The model also generates some skewness in the unemployment rate, at 0.29, but again less than in the data. Finally, it generates no asymmetry between the speeds of expansions and contractions, nor in their durations.

These results follow from the fact that there are some, but little, asymmetry in our search model under symmetric real wage rigidity. Given our symmetric assumption on the wage-setting rule, non-linearities in the model need to arise from non-linearities in the labor-demand schedule of a firm (4), in the matching-function relationship between the vacancy-filling probability $q_t$ and the job-finding probability $f_t$, or from the worker-flow relationship (2) between the job-finding probability and aggregate employment. All three can be summed up into an aggregate labor demand schedule. For a Cobb-Douglas matching function with elasticity $\eta$, $q(\theta) = \mu \theta^{-\eta}$, it is
Table 6: Plucking Property, Skewness, Speed and Duration

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Homogenous, AR(1)</th>
<th>Heterogenous, AR(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SRWR</td>
<td>SRWR</td>
</tr>
<tr>
<td>Expansion on previous contraction, $\beta$</td>
<td>1.09</td>
<td>0.42</td>
<td>0.54</td>
</tr>
<tr>
<td>Contraction on previous expansion, $\beta$</td>
<td>-0.38</td>
<td>0.22</td>
<td>0.25</td>
</tr>
<tr>
<td>Expansion on previous contraction, $R^2$</td>
<td>0.58</td>
<td>0.18</td>
<td>0.30</td>
</tr>
<tr>
<td>Contraction on previous expansion, $R^2$</td>
<td>0.22</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.63</td>
<td>0.29</td>
<td>0.35</td>
</tr>
<tr>
<td>Speed of expansions (pp/year)</td>
<td>0.88</td>
<td>3.95</td>
<td>2.66</td>
</tr>
<tr>
<td>Speed of contractions (pp/year)</td>
<td>1.89</td>
<td>3.74</td>
<td>2.49</td>
</tr>
<tr>
<td>Duration of Expansions (months)</td>
<td>57.9</td>
<td>13.9</td>
<td>21.5</td>
</tr>
<tr>
<td>Duration of Contractions (months)</td>
<td>26.9</td>
<td>14.4</td>
<td>22.5</td>
</tr>
</tbody>
</table>

*Note:* The table compares real world data with data from our symmetric real wage rigidity model (SRWR) and our downward nominal wage rigidity model (DNWR) along four dimensions. The first (third) row reports the coefficient ($R^2$) in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second (fourth) row report the coefficient ($R^2$) in an analogous regression of the size of a contraction on the size of the previous expansion. The fifth row reports the skewness of the distribution of the unemployment rate. The next two rows report the average speed of expansion and contractions, measured in percentage points of unemployment per year. The final two rows report the average duration of expansions and contractions, measured in months. The statistics are obtained by simulating a sample of 100 000 months, with 1000 sectors in the heterogeneous model.

Given by:

$$\frac{w_t}{A_t} = F'(N_t) - C(N_t, N_{t-1}) + \beta (1 - s)E_t \left( \frac{A_{t+1}}{A_t} C(N_{t+1}, N_t) \right), \quad (17)$$

where

$$C(N_t, N_{t-1}) = \frac{c}{q_t} = c \mu^{\frac{1}{1-\eta}} \left( \frac{N_t - (1 - s)N_{t-1}}{1 - (1 - s)N_{t-1}} \right)^{\frac{\eta}{1-\eta}} \quad (18)$$

is the aggregate marginal (search) cost of hiring one more worker.

Because there are hiring costs, this labor demand schedule is forward-looking: what matters to the hiring decision of a firm is the real wage and marginal productivity of a worker not only today but also in the future. However, it is insightful to first abstract from the intertemporal dimension by considering the steady-state labor demand schedule induced by the model, which corresponds to the demand for labor should the real wage remain at its current level and productivity grow deterministically forever after. Because the DMP model has little internal propagation, it is a

---

18 To see this formally, iterate the labor-demand equation forward.
reasonable first approximation to the dynamic labor demand schedule. It is given by:

\[
\frac{w}{A} = F'(N) - K(N)(1 - \beta(1 - \phi)e^\theta)
\]

where \(K(N) \equiv C(N, N) = c\mu^{\frac{1}{1-\eta}} \left( \frac{sN}{1 - (1 - \phi)N} \right)^{\frac{\phi}{1-\eta}}
\]

This equation makes apparent the two reasons why labor-demand is downward-slopping in our search model. First, there are decreasing returns to scale in production (red term). This is what makes labor-demand downward-slopping in a model without search frictions. Second, the marginal search cost is increasing in employment (blue term). This is the motive that is specific to a search model.

These two determinants of labor demand shape it in very different ways. Decreasing returns tend to make labor-demand log-linear: with no search frictions and provided a constant-elastic production function, employment is a log-linear function of the wage-to-flexible-wage ratio \(w/\bar{w}A\) (equivalently, of the wage-to-productivity ratio \(w/A\)). In contrast, search costs make the log of employment a convex function of the log of the wage-to-flexible-wage ratio \(^{19}\)

It turns out that quantitatively, the decreasing-returns determinant of labor demand trumps the search-cost determinant in shaping the aggregate demand schedule. As a result, labor-demand inherits much of its linearity. This is illustrated in figure 5 which plots both the labor-demand schedule under no search frictions and the steady-state labor-demand schedule in our search model. (We plot unemployment \(u\) on the y-axis. This gives virtually the same results as plotting \(-\log(N) \approx u\). On the same figure, we add a scatter plot of the relationship between the log of the wage-to-flexible-wage ratio and unemployment away from steady-state. It is more convex than the steady-state relationship but still too linear to generate as much plucking and unemployment skewness as in the data.

A corollary is that the extent of non-linearity in the search model depends a lot on the assumptions of constant or decreasing returns to scale. Under constant returns to scale, the aggregate labor-demand schedule would be shaped by the search costs function only. We show in appendix D that in this case, a search model under symmetric real wage rigidity generates more plucking and a more skewed unemployment rate. Consistently, Petrosky-Nadeau, Zhang, and Kuehn

\(^{19}\)Where does the non-linearity come from? The individual hiring-cost function is independent of (a firm’s own) employment so it is a limited source of non-linearities. The assumption of a Cobb-Douglas matching function implies that the relationship between the vacancy-filling rate \(q_t\) and the job-finding rate \(f_t\) is log-linear. The non-linearity comes from the worker-flow equation. It captures the fact that a one-percent increase in the job-finding rate empties the pool of unemployed workers less than a one-percent decrease in the job-finding rate increases it. Hairault, Langot, and Osotimehin (2010) and Jung and Kuester (2011) emphasize this last source of non-linearity.
Figure 5: Aggregate Labor Demand Schedule

Note: The figure plots the relationships between the log of the wage-to-flexible-wage ratio and unemployment in the one-sector model of symmetric real wage rigidity with AR(1) productivity shocks.
show how the search model under constant returns to scale features asymmetries that can generate business-cycles disasters—large drops in production—despite symmetric shocks. The appendix also shows that the increased extent of plucking under constant returns to scale is driven by such extreme events: excluding from the simulated sample the recessions and expansions that are larger than the largest expansion in our empirical sample, the extent of plucking is similar to the one under decreasing returns to scale.

4.2 The Frequency of Business Cycles

A last fact to take out of the second column of table 2 concerns the speed and duration of cycles, regardless of the difference between expansions and contractions. Both are very short-lived—they last an average of 14 months. Since the model gets the average size of contractions and expansions right (3.7 percentage points), it means they are very fast—unemployment changes by about 4 percentage points per year on average. This reflects the lack of internal propagation in the DMP model emphasized e.g. by [Fujita and Ramey (2007)]. When a productivity shock hits, employment responds very quickly and therefore inherits much of the high-frequency fluctuations of the shock process. This feature of the model is orthogonal to its ability to generate business cycles asymmetries. It is however desirable to assess the business cycles properties of the models for business cycles that bear some resemblance with the ones in the data.

There are two possible reactions to the lack of internal propagation. One can take the view that in the real world much of the propagation of shocks occurs within the labor market. In this case, it is important to amend the DMP model in a way that strengthens its ability to propagate shocks. For example, [Fujita and Ramey (2007)] show that making the cost of opening a vacancy non-zero and increasing in the number of new vacancies opened makes vacancy creation sluggish. Alternatively, one can take the view that propagation arises from outside the labor market, for instance through the capital accumulation process, through information frictions, or through financial frictions.

To get closer to the frequency of business cycles in the data, in what follows we assumes that the aggregate labor productivity process follows an AR(2) instead of an AR(1). This implicitly leans toward the second view, but we are not committed to one view or the other. We take it as a

---

20Capital accumulation in itself does not solve the problem: a lack of internal propagation is also a feature of early real business cycles models [Cogley and Nason (1995)]. A popular solution in the DSGE literature is to assume investment adjustment costs that make changing investment costly, following [Christiano, Eichenbaum, and Evans (2005)]. Note the similitude between this solution and the one of Fujita and Ramey.
practical approach to generate business cycles of about the same frequency as in the data.

4.3 Downward Nominal Wage Rigidity and the Plucking Property

If not from non-linearities in the relationship between wages and employment, business cycles asymmetries can arise from non-linearities in the relationship between shocks and wages. We turn to the model of downward nominal wage rigidity to assess whether asymmetric wage adjustments can account for the business cycles asymmetries of the data. We do so in the model with sectoral heterogeneity, for two reasons. First, a single-sector model would have the unappealing feature that the constraint on falling wages would be binding for either all or no firms at any given time. Second, it would underplay the importance of the inability of wages to fall because it would underestimate the probability of hitting the constraint. Indeed, from the labor-demand equation (4), what matters for the hiring decision of a firm is the ratio of real wages to flexible wages \( \frac{w_i}{\bar{w}A_i} \). From (10), under downward nominal wage rigidity this ratio evolves according to:

\[
\frac{w_i}{\bar{w}A_i} = \max \left\{ 1, \frac{w_i}{\bar{w}A_i} \prod \frac{A_i}{A_{i-1}} \right\}.
\]  

(21)

Productivity growth \( \frac{A_i}{A_{i-1}} \) plays the same role as inflation in alleviating the constraint on downward wage adjustments. It, too, greases the wheels of the labor market. Real wages can fall by the sum of the inflation rate and the productivity growth rate. Therefore, with 2% inflation, only falls in productivity of more than 2% over a year make the constraint bind. If productivity is aggregate productivity, these are rare events. In contrast, sectoral productivity shocks make it possible for productivity to be falling in some sectors even when aggregate productivity is growing.

Sectoral heterogeneity makes much less difference to the model of symmetric real wage rigidity. The third column of table 6 gives the business cycles statistics for the model of symmetric real wage rigidity with sectoral shocks and an AR(2) in aggregate productivity. The results are virtually unchanged with respect to the homogeneous model with AR(1) shocks, except for the fact that the AR(2) process is making expansions and contractions longer and slower. However, they are still shorter and faster than in the data.

The fourth column of table 6 provides the business cycles statistics for the model of downward nominal wage rigidity with sectoral shocks and an AR(2) in aggregate productivity. Thanks to the AR(2) process for aggregate shocks, the duration of expansions and contractions is now in
line with the data. More importantly, downward nominal wage rigidity generates substantial plucking. After a contraction of \( n \) percentage points an expansion of \( 0.82 \times n \) percentage points can be expected, whereas the size of an expansion is of no help in forecasting the size of the following contraction. The \( R^2 \) is 0.73 in the first case and zero in the second. Appendix displays the scatter plots associated to the regressions. The model also makes the unemployment rate substantially right-skewed, with a skewness of 2.16, actually higher than in the data. Our model of downward nominal wage rigidity also manages to get some asymmetry between the duration of expansions (44 months on average) and the durations of contractions (29 months on average). Since the model gets the average size of expansions and contractions right (3.8 percentage points vs. 3.7 in the data), this means it also gets some asymmetry between the speed of expansions (1.39 percentage point per year on average) and the speed of contractions (2.11 percentage points per year). This asymmetry is however less pronounced than in the data.

Under downward nominal wage rigidity, fluctuations in unemployment are driven by fluctuations in the number of firms that are constrained, and by how much below wages productivity falls in sectors where the constraint is or becomes binding. As figure illustrates through a simulated sample path, there is a strong comovement between unemployment and wage freezes in the model, as documented in the data by the Daly-Hobijn Wage-Rigidity Meter (figure 4). The fraction of constrained firms is however more volatile than documented by the Daly-Hobijn meter. It typically increases from below 10% to above 50% during a severe contraction. Appendix checks that even if such severe contractions, the constraint on falling wages almost never violates the no-firing condition (5).

5 Costs of Business Cycles and Benefits of Stabilization Policy

We now turn to the normative implications of our model. We first present the view that our plucking model with downward nominal wage rigidity takes on unemployment fluctuations—fluctuations above a resting point of low unemployment—and contrast it with the view that the unemployment rate fluctuates symmetrically around a natural rate. We then reassess the costs of business cycle fluctuations through the lens of our model. Finally, we consider how monetary policy can achieve the benefits of stabilization implied by the model through a higher inflation target.
Figure 6: Simulated Paths for the Unemployment Rate and the Fraction of Wage Freezes

Note: The figure plots a sample path for the unemployment rate and the fraction of wage freezes of 70 years (the same length as our empirical sample), for the AR(2) heterogeneous model of downward nominal wage rigidity. A wage freeze is defined as a wage that has remained constant over the past twelve months.

5.1 An Elastic String Glued Lightly to a Board

Our plucking model with downward nominal wage rigidity takes a view on unemployment fluctuations that contrasts sharply with the one of workhorse models of business cycles. To illustrate this view, figure 7 plots simulated paths for the unemployment rate in both our plucking model of downward nominal wage rigidity and our benchmark model of symmetric real wage rigidity. We superimpose on these paths the steady-state rates of unemployment—the rates that would prevailed absent any aggregate shock $\sigma^n = 0$—in both models.

The figure illustrates the sharp contrast between two views of the business cycle. The model of symmetric real wage rigidity is an example of a natural-rate model. Unemployment fluctuates roughly symmetrically above and below its steady-state level of 5.7%, which is close to its average
In contrast, with downward nominal wage rigidity unemployment most often lies above its steady-state level of 4.6%. Decreases in aggregate productivity increase unemployment. Increases in productivity can decrease employment back to its steady-state level during a recovery, but rarely decrease it much further because wages adjust easily upward. As a result the average unemployment rate (5.8%) is above the steady-state rate of unemployment (4.6%).

The view on business cycles that our plucking model takes was part of Milton Friedman’s own interpretation of his plucking model. In Friedman (1964), he described business cycle fluctuations as—flip figure 7 upside-down to go from the unemployment rate to output—“an elastic string glued lightly to a board, and plucked at a number of points chosen more or less at random”. This view of business cycles was also implicit in the Old-Keynesian faith in the benefits of stabilization policies, and still lies today as a vestige in the terminology of “potential output”—which suggests the potential is higher—and “output gap”—which suggests the gap between output and potential output is generically negative. Potential output was indeed defined by Okun (1962) as the answer to the question: “how much output can the economy produce under condition of full employment?” Although he qualified that the concept should be understood as maximum production “without inflationary pressure; or, more precisely, as aiming for a point of balance between more output and greater stability, with appropriate regard for the social valuation of these two objectives,” his use of the concept had little resemblance with either a natural rate or the NAIRU.

5.2 First-Order Effect of Economic Fluctuations

The natural-rate view has drastic implications for the costs of business cycles and the benefits of stabilization policies. In a thought-provoking exercise, Lucas (1987, 2003) asked whether a reasonable estimate of the costs of business cycles justifies the attention that the design of stabilization policies receives. He answered negatively: replacing the stochastic stream of consumption of a

---

21 We use the term natural rate to refer to the steady-state level of unemployment, constant in the model and to be thought of as slow-moving in practice. We do not use it to refer to the Non Accelerating-Inflation Rate of Unemployment (NAIRU)—the unemployment rate consistent with a constant inflation rate. In our model of symmetric real wage rigidity the NAIRU is simply the unemployment rate since money neutrality holds in this case. The natural rate is also different from the unemployment rate that would prevail absent any form of wage rigidity or absent any (aggregate and idiosyncratic) shocks. This last rate is also constant but slightly lower (5.6%) due to the existence of idiosyncratic shocks.

22 “Potential output” is now used as a synonym of “natural output”, and a “gap” is no longer meant to be generically negative. As Nelson (2017) documents in his study of Friedman’s contributions to the economic debate, Friedman himself later contributed to shifting the understanding of “potential output” and “output gap” toward the natural-rate view: “Much more than he used “the natural level of output”, Friedman deployed the term “potential output” to describe [the] baseline level of output […] It was clear, however, than Friedman was using these terms [of potential output and capacity output] as synonyms for the natural rate of output” (p.411).
representative agent by a constant stream with the same mean would yield extremely small welfare gains, unlikely to compensate for the costs of stabilization. Assuming log-utility and trend-stationary fluctuations with Gaussian innovations, Lucas (2003) finds that the representative agent would be willing to forgo no more than 0.05% of his consumption to be rid of fluctuations.

Subsequent literature has considered whether Lucas’s result is robust to alternative assumptions on preferences toward risk (Obstfeld (1994), Dolmas (1998), Tallarini (2000)), or to removing the assumption of perfect insurance against idiosyncratic shocks induced by the existence of complete markets (Imrohoroglu (1989), Atkeson and Phelan (1994), Krusell and Smith (1999)). Most of these papers show that such extensions can beef up the costs of business cycles, and thus the benefits of stabilization policy. Yet, because Lucas’s initial estimate is so small, finding bigger estimates...
does not necessarily overcome the general conclusion that fluctuations do not matter much: these papers still find small—although not as small as Lucas’s—costs of business cycle fluctuations.

The robustness of Lucas’s result is not necessarily surprising. The contrary intuition which prevailed before Lucas’s that stabilization policies can do much relies on the presumption that they can eliminate slumps without getting rid of the booms—that they affect not only the volatility, but also the mean level of unemployment and output. Because Lucas assumes away the possibility for policy to change the mean, in itself his result does not refute the earlier view.

Part of the answer to whether business cycles are costly is then whether they affect the mean of output and unemployment. We answer this question in our plucking model with downward nominal wage rigidity. Figure 8 gives the average level of unemployment as a function of the volatility of aggregate shocks. Reducing the standard deviation of aggregate shocks from its calibrated value of 3.7% to zero would decrease the unemployment rate from 5.8% to its steady-state value of 4.6%. Conversely, more volatile aggregate shocks would increase unemployment beyond 5.8%: for a standard deviation of aggregate shocks of 5% instead of 3.7%, unemployment would be 6.9%.

5.3 Greasing the Wheels of the Labor Market

Lucas’s thought experiment of eliminating all fluctuations is intended to give an upper-bound of the benefits of stabilization policies, abstracting from the constraints that may exist on what outcomes policy can achieve. Our microfounded model permits to consider specific policies, and to derive and not assume their effects. We consider one specific policy: monetary policy, and specifically the choice of the inflation target. In our model with downward nominal wage rigidity, inflation greases the wheels of the labor market by easing the downward adjustment of real wages. The reliance on monetary policy to alleviate the inefficiency created by downward nominal wage rigidity is as old as the early emphasis on downward nominal wage rigidity by [Tobin (1972)].

Table 9 gives the average unemployment rate as a function of the inflation target in our model. Increasing the inflation target from 2% to 4% decreases unemployment from 5.8% to 4.4%. This is more than eliminating aggregate fluctuations does, because it eases the adjustment not only to aggregate shocks, but to idiosyncratic shocks too. As inflation increases further, average unemployment asymptotes to its values absent any frictions on wage adjustments or absent any (idiosyncratic and aggregate) shocks: 4.1%. The marginal benefit of higher inflation is decreasing with inflation. This implies that, conversely, lowering inflation below 2% is very costly in terms
Figure 8: Average Unemployment as a Function of the Volatility of Aggregate Shocks

Note: The figure gives the average rate of unemployment as a function on the standard deviation of aggregate shocks in our model of downward nominal wage rigidity with sectoral heterogeneity and an AR(2) process for productivity.

of average unemployment. In our model, reducing the inflation target from 2% to 1% increases unemployment to 7.5%.

These estimates of the effect of the inflation target on unemployment rely on the assumption that wage-setting remains unchanged in the face of the new monetary policy. For high enough an inflation target, it is however likely that workers would shift to thinking in real terms. Any reluctance to wage cuts would manifest itself through downward real wage rigidity, and inflation would no longer have an effect on unemployment. The greasing-the-wheels benefits of inflation should then be thought of as U-shaped: some inflation eases the downward adjustment of real wages; too much inflation not only does not, but also undoes the initial benefits of inflation. The behavior of US unemployment during the high-inflation period of the 1970s seemed to still display
the plucking property. Seen through the lens of the model, this suggests that at 10% inflation, the constraint on downward adjustment bears on real, not nominal, wages.

Although the distinction between real and nominal downward rigidity is critical when assessing the benefits of a higher inflation target, it is not for the ability of our model to replicate the plucking property. Indeed, our wage-setting equation (10) reduces to the case of downward real wage rigidity for an inflation target of zero (up to a reinterpretation of the parameters). Nor do the implications of our model for the welfare costs of business cycles depend on whether the constraint on wage cuts bears on nominal or real wages. If real wages cannot fall as easily as they can rise, it is still the case that more volatile shocks increase average unemployment.

We find the benefits of inflation in greasing the wheels of the labor market to be large. Estimates vary widely in the literature. Different assumptions on whether and how firms and/or
workers can circumvent the constraint on falling wages are likely to account for different results. Schmitt-Grohe and Uribe (2016) find large benefits. They model downward nominal wage rigidity in a walrasian labor market, but in the same way as we do: in their model, the wage either adjusts to clear the market, or, if this requires nominal wage to fall, stays constant.

In contrast, Kim and Ruge-Murcia (2009, 2011) find that the existence of downward nominal wage rigidity justifies an optimal inflation target of no more than 0.75%. They model downward nominal wage rigidity through monopolistic workers who face asymmetric adjustment costs to wages—it is more costly to adjust wages downward than upward. Because the rigidity is modeled through adjustment costs, wages can never be too misaligned with flexible wages, as workers would rather pay the adjustment cost. In other words, workers can already grease the wheels of the labor market by themselves by paying the adjustment cost. Therefore, there is little room for monetary policy to improve on the allocation. In our search model, large misalignment between wages and flexible wages can be sustained even though workers and firms face no adjustment costs on wage changes and, from an individual perspective, behave optimally.

The real effect of downward nominal wage rigidity can also be attenuated if firms and workers preemptively moderate wage increases in booms, in order to reduce the probability of a painful adjustment during a downturn. Such forward-looking wage moderation is present in Kim and Ruge-Murcia’s model of wage-setting under asymmetric adjustment costs but is not specific to an adjustment-cost model. It is also present in Elsby (2009)’s efficiency-wage model, where firms set nominal wages understanding that wage cuts deteriorate morale and therefore productivity, or in Benigno and Ricci (2011)’s model, where firms set nominal wages subject to the constraint that they cannot fall. In all models, workers or firms set wages understanding that the wage they set today is the one that may constrain them tomorrow, and have therefore an incentive to preemptively moderate wage increases today. Our assumption of downward nominal wage rigidity does not incorporate such a behavior. Yet this does not mean firms in our model are myopic. They forward-lookingly maximize intertemporal profits. What they preemptively moderate in anticipation of a fall in productivity is hires, not wages. Either prices or quantities can respond to concerns about the future.

---

23 This is if the inflation rate is restricted to be constant over time. If the inflation rate is allowed to be state-dependent, they find that the state-dependent optimal policy has an even lower average rate of inflation, at 0.35%. In their single-sector model, inflation is beneficial only when the economy is hit by shocks for which downward nominal wage rigidity is costly. A state-dependent policy allows to use inflation to grease the wheels of the labor market only when needed.

24 Elsby (2009) empirically documents that downward nominal wage rigidity compresses both wage decreases and wage increases, and present it as evidence for forward-looking wage moderation. Compression of wage increases is
Finally, as we emphasized, sectoral shocks significantly increase the stringency of the constraint posed by the inability of nominal wages to fall. Benigno and Ricci (2011) similarly find that in a model of downward nominal wage rigidity more volatile idiosyncratic shocks significantly increases the long-run trade-off between output and inflation. In this context, they find potentially large benefits of inflation to grease the wheels of the labor market. This is despite the fact that they model downward nominal wage rigidity through wage-setting workers who preemptively moderate wage increases in view of the possibility of a binding constraint in the future.

6 Conclusion

We build a plucking model of the business cycle that captures the asymmetry in the predictive power of contractions and expansions emphasized by Milton Friedman. In our model, the asymmetry arises from downward nominal wage rigidity. In contrast to earlier models of downward nominal wage rigidity, our model is consistent with optimizing behavior and therefore robust to the Barro (1977) critique.

We show that in our model eliminating business cycles has large welfare benefits since it lowers the average unemployment rate. Our simulations imply that eliminating all aggregate fluctuations could lower the average unemployment rate by about 1.2 percentage points. Downward nominal wage rigidity provides one rationale for a positive inflation rate. Our results imply that moving from a 2% inflation target to a 4% inflation target would lower the average unemployment rate by 1.4 percentage points by easing the adjustment to both idiosyncratic and aggregate shocks. Lowering the inflation target to 1% would raise the average unemployment rate by 1.7 percentage points.

---

no evidence of forward-looking wage moderation however. Appendix shows that our model is able to replicate the compression of wage increases that Elsby documents.
A Normalization of \( \mu \)

We show that only the calibration of \( c \mu^{1-\eta} \)—not of \( \mu \) and \( c \) separately—matters. The two parameters \( c \) and \( \mu \) only show up as \( c/q(\theta) \) and \( f(\theta) \) in the system characterizing the equilibrium. Because
\[
q = \mu^{1-\eta} f^{1-\eta},
\]
we have that:
\[
\frac{c}{q} = \left( c \mu^{1-\eta} \right) f^{1-\eta},
\]
so that only \( c \mu^{1-\eta} \) is identified. Assuming that a vacancy is costly to open but fills with a high probability is the same as assuming that a vacancy is cheap to open but fill with a low probability. We thus normalize \( \mu \) to one.

B Solution Algorithm

We describe the details of our solving method, here in the case of our assumption of downward nominal wage rigidity and an AR(2) process for aggregate productivity—the method is similar for symmetric real wage rigidity or an AR(1) in productivity. It is convenient to work with the wage-to-flexible-wage ratio \( R^i_t = w^i_t/(\bar{w} A^i_t) \) rather than the wage. Incorporating the assumption of a constant-elasticity production function, of a Cobb-Douglas matching function, and of a monetary policy of constant inflation \( \bar{\pi} \), solving for the equilibrium consists, given the exogenous productivity perturbations \( \log(A_t) \) and \( \log(\Delta Z^i_t) \), in solving for the two endogenous variables \( N^i_t \) and \( 1/q^i_t \) that solve the system:

\[
\alpha (N^i_t)^{\alpha-1} = \bar{w} R^i_t + c \frac{1}{q^i_t} - \beta (1-s)cE_t \left( \frac{1}{q^i_{t+1}} e^{\log(A_{t+1})-\log(A_t)+\Delta \log(Z^i_{t+1})} \right),
\]

where
\[
R^i_t = \max \left\{ 1, \frac{R^i_{t-1}}{\bar{\pi} e^{A_t-\log(A_{t-1})}-\Delta \log(Z^i_t) + g} \right\},
\]

\[
\frac{1}{q^i_t} = \left( \frac{N^i_t - (1-s)N^i_{t-1}}{1 - (1-s)N^i_{t-1}} \right)^\frac{1}{1-\eta},
\]

\[
\log(A_{t+1}) = (I - \rho_1^0)^{-1} (I - \rho_2^0)^{-1} \varepsilon^o_{t+1},
\]

\[
\Delta \log(Z^i_{t+1}) = \rho_{\Delta Z} \Delta \log(Z^i_t) + \varepsilon^\Delta z^i_{t+1},
\]

39
B.1 Steady-State

A non-stochastic steady-state equilibrium with $A_t = 1$ and $\Delta \log(Z_t) = 0$ is a value for $N$ that solves the equation:

$$\alpha N^{\alpha-1} = \bar{w} R + c \frac{1}{q} [1 - \beta (1 - s) e^q], \quad (B.4)$$

where $\frac{1}{q} = \left( \frac{s N}{1 - (1 - s) N} \right)^{\frac{\alpha}{1-\alpha}},$

where $R = 1$ under downward nominal wage rigidity, and $R = \exp(-\frac{g}{1-\rho} g) < 1$ under symmetric real wage rigidity, since wages lag the deterministic growth in productivity.\[25\]

B.2 Change of variables when discretizing the AR(2)

A solution to the model can be described as policy functions $N^i$ and $1/q^i$ of the 5-variable state: the exogenous present and lagged aggregate productivity levels $\log(A)$ and $\log(A_{-1})$, the exogenous idiosyncratic productivity growth rate $\Delta \log(Z^i)$, the (exogenous) wage-to-flexible-wage ratio $R = w/(\bar{w} A)$, and the endogenous lagged level of employment $N_{-1}$. To discretize the AR(2) process for $(A_t)_t$, we first make the following change of variable to the representation of the state space along the $(A, A_{-1})$ dimensions. Define the AR(1) process:

$$\eta_t \equiv (1 - \rho_2^2)^{-1} \epsilon_t^\alpha, \quad (B.5)$$

so that:

$$\log(A_{t+1}) = \rho_1^\alpha \log(A_t) + \eta_{t+1}. \quad (B.6)$$

We represent the five-dimensional state by $(\log(A), \eta, \Delta \log(Z^i), R, N_{-1})$. We use the Rouwenhorst (1995) method to discretize the AR(1) process for $(\eta_t)_t$. The resulting approach to adapting the Rouwenhorst method to an AR(2) process is close to the one of Galindev and Lkhagvasuren (2010), who consider the more general case of generalizing it to a VAR(1).

B.3 Iteration Method

We form a discrete grid of the state-space with 11 points along each dimension, approximate the AR(1) processes for the exogenous variables $\eta_t$ and $\Delta \log(Z^i)$ using Rouwenhorst (1995) dis-\[25\]For the downwardly-rigid nominal-wages equilibrium, we assume $\log(\bar{\Pi}) + g \geq 0$, otherwise there is no steady-state equilibrium.
cretization method, and solve for the policy functions at each point of the grid by policy function iteration. Specifically, we start from an initial guess on the policy functions $N$ and $1/q$. Then, at each point of the grid, we use these guesses to calculate the expectation term in equation (B.1). In calculating the expectation term, we need to evaluate the policy function at points that are not on the grid. We do so through linear interpolation. Given this expectation term, we solve for the $N^i$ (and resulting $1/q^i$) that solves equation (B.1), and store the solution in new policy functions. Done in all states of the grid, this provides a new guess for the policy functions. We repeat until convergence of the policy functions.

### B.4 Dealing with the Constraints

A solution $N^i$ to equation (A.1) needs to lie between $(1 - s)N^i_{t-1}$ and 1. Otherwise, the firm would need to fire people and the no-firing condition (5), or no excess-demand condition (7) would fail. In the unlikely states—which do not occur on the sample paths in our simulations—where the no-firing constraint fails, we assume that the firm does not hire nor fire workers and thus set $N^i_t = (1 - s)N^i_{t-1}$. In the unlikely states—which do not occur on the sample paths in our simulations—where the no-excess demand constraint fails, we assume that firms hire all the available workers and thus set $N^i_t = 1$.

### C Plucking Scatter Plots

Figure C.1 displays the scatter plots associated to the results of the plucking regressions reported in table 6, paralleling the empirical counterpart figure 2.

### D Constant vs. Decreasing Returns to Scale

The extent of non-linearity in the aggregate labor-demand schedule of the search model depends significantly on the assumptions of constant or decreasing returns to scale. In this appendix, we show how the assumption of constant instead of decreasing returns to scale affects business cycles asymmetries in our AR(1) homogeneous model of symmetric real wage rigidity. We calibrate the constant-returns model like our decreasing-returns model, except that we recalibrate $\bar{\omega}$ and

---

Figure C.1: Plucking Scatter Plots

Note: The figure displays the scatter plots associated to the plucking regressions in the homogeneous AR(1) model of symmetric real wage rigidity (SRWR) and the heterogeneous AR(2) model of downward nominal wage rigidity (DNWR). OLS regression lines are plotted in each panel.
the volatility of shocks $\sigma^\varepsilon$ in order to still get a steady-state level of unemployment equal to the average level of unemployment in the data (5.8%) and to still get a standard deviations of unemployment equal to its value in the data (1.6 percentage point). Because employment responds much more strongly to changes in the wage-to-flexible-wage ratio under constant returns to scale, this requires to significantly reduce the volatility of shocks: we calibrate $\sigma^\varepsilon$ to 0.9% under constant returns (vs. 2.7% under decreasing returns).

Table D.1 provides the business cycles statistics for the model under constant and decreasing returns to scale. The constant-returns model generates substantially more plucking than under decreasing returns: the coefficient in a regression of the size of an expansion on the size of the previous contraction is now 0.60, substantially larger than the coefficient in a regression of the size of a contraction on the size of the previous expansion, 0.12. In addition, the difference in the explanatory powers of the two regressions is now larger: the $R^2$ is 0.35 against 0.01. The model also generates more skewness in the unemployment rate, at 0.79. Expansions and contractions are a bit longer and therefore slower, but still much shorter and faster than in the data. The model still generates no asymmetry between expansions and contractions in terms of speed and duration.

To provide insights for the greater asymmetry of the search model under constant returns to scale, figure D.2 compares the relationship between the log of the wage-to-flexible-wage ratio and unemployment under decreasing returns to scale (left panel) and constant returns to scale (right panel). Under constant returns, only search costs shape the aggregate labor-demand schedule.

Figure D.2 also illustrates that under constant returns to scale, the unemployment rate can easily take extreme values (despite being calibrated to match the standard deviation of unemployment in the data), a property stressed by Petrosky-Nadeau and Zhang (2017); Petrosky-Nadeau, Zhang, and Kuehn (2018). The greater extent of plucking in the constant-returns-to-scale model is actually mostly driven by these extreme events. To verify this, table D.1 also provides the results of the plucking regressions on the “top-truncated” samples that only include expansions and contractions of less than 6.3 percentage points, which is the size of the largest expansion or contraction over our sample—namely the 2009-2018 expansion. The extent of plucking under constant returns to scale is considerably reduced. Therefore, although the baseline DMP model under constant returns to scale can generate some plucking, it can do so only through counter-factually large expansions and contractions.
Figure D.2: Aggregate Labor Demand Schedule under Constant vs. Decreasing Returns to Scale

Note: The figure plots the relationships between the log of the wage-to-flexible-wage ratio and unemployment in the one-sector model of symmetric real wage rigidity with AR(1) productivity shocks. The left panel considers the case of decreasing returns to scale (DRS). The right panel considers the case of constant returns to scale (CRS).
Table D.1: Plucking Property, Skewness, Speed and Duration: The effect of Decreasing vs. Constant Returns to Scale

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>SRWR, Homogenous, AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DRS</td>
<td>DRS Top-truncated</td>
</tr>
<tr>
<td></td>
<td>CRS</td>
<td>CRS Top-truncated</td>
</tr>
<tr>
<td>Expansion on previous contraction, $\beta$</td>
<td>1.09</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.39</td>
</tr>
<tr>
<td>Contraction on previous expansion, $\beta$</td>
<td>-0.38</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.10</td>
</tr>
<tr>
<td>Expansion on previous contraction, $R^2$</td>
<td>0.58</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td>Contraction on previous expansion, $R^2$</td>
<td>0.22</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.63</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>Speed of expansions</td>
<td>0.88</td>
<td>3.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>Speed of contractions</td>
<td>1.89</td>
<td>3.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>Duration of Expansions</td>
<td>57.9</td>
<td>13.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>Duration of Contractions</td>
<td>26.9</td>
<td>14.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

Note: The table compares data from our model of symmetric real wage rigidity under decreasing returns to scale (DRS) and constant returns to scale (CRS) along four dimensions. The first (third) row reports the coefficient ($R^2$) in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second (fourth) row report the coefficient ($R^2$) in an analogous regression of the size of a contraction on the size of the previous expansion. The fifth row reports the skewness of the distribution of the unemployment rate. The next two rows report the average speed of expansion and contractions, measured in percentage points of unemployment per year. The final two rows report the average duration of expansions and contractions, measured in months. For both constant and decreasing returns to scale, we also report the results of the OLS regressions for the “top-truncated” samples that only include expansions and contractions of less than 6.3 percentage points.

E No Violation of Individual Optimality

We check that in the simulations of our model of downward nominal wage rigidity, the wage almost always remains within the wage band defined by the no-firing constraint and the constraint of no excess demand for labor. For a given firm, the wage ventures out of the wage band every 23,000 months, or about 1900 years.

To illustrate why the no-firing constraint imposes only a weak constraint on the wage, figure E.3 gives a simulated sample path for the wage and wage band in an instance when a sector experiences an extreme contraction—at its peak sectoral wages are 27% above sectoral productivity and sectoral unemployment is 53%—yet the no-firing condition remains satisfied. To understand why, notice that as productivity (and therefore the flexible wage) decreases, the upper bound of the wage band falls by less than the flexible wage does. What accounts for this is the decreasing marginal productivity of labor. Exogenous separation is 3.4% per month in our calibration. There-
Figure E.3: Simulated path for wages and the wage band under downward nominal wage rigidity.

Note: The figure plots a simulated path for the nominal wage, the flexible nominal wage, and the nominal wage band in one sector at a time the sector goes through an extreme contraction that leaves wages 27% above productivity and reduces employment by half (53% unemployment) in the sector.

Therefore, by simply not replacing the workers who are leaving the firm, a firm’s workforce already decreases by 3.4% monthly. But with decreasing marginal productivity of labor, the marginal productivity of the remaining workers increases by $1 - \alpha = 1/3$ percentage point for any one percentage point decrease in employment. Therefore, if the firm does not hire any worker, the marginal value of a remaining worker (rigorously, the part of the marginal value that comes from what he produces today) increases by $3.4/3 \approx 1.1\%$ monthly. As a result, it requires a productivity drop of more than 1.3% monthly for the no-firing constraint to be violated. Only large sudden changes in the gap between wages and productivity threaten the no-firing condition. In themselves, high levels of the gap between wages and productivity do not, because, as employment contracts, the marginal value of the remaining employees increases.
Table F.2: Compression of Wage Increases

<table>
<thead>
<tr>
<th>Fraction of Real Wage Increases</th>
<th>Flexible Wages</th>
<th>Π = 4%</th>
<th>Π = 3%</th>
<th>Π = 2%</th>
<th>Π = 1%</th>
<th>Π = 0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible Wages</td>
<td>79.4%</td>
<td>79.3%</td>
<td>78.8%</td>
<td>77.7%</td>
<td>74.2%</td>
<td>68.6%</td>
</tr>
</tbody>
</table>

Note: The table gives the fraction of real wage increases among all real wage changes in the cross-sectional steady-state distribution of our heterogeneous model of downward nominal wage rigidity. The fraction is given for different inflation rates, and in the case of flexible wages.

F Compression of Wage Increases

Elsby (2009) argues that the distribution of wage changes in worker panel data provides evidence of forward-looking wage moderation. Elsby uses the CPS and PSID in the US and the New Earning Survey in the UK. By comparing periods of low inflation (when downward nominal wage rigidity is likely to bind often) with periods of high inflation (when it is not), he documents that downward nominal rigidity compresses not only real wage decreases, but also real wage increases. Elsby argues this is evidence in favor of forward-looking wage moderation, and therefore against large economic effects of downward nominal wage rigidity. However, as Elsby explains, such a compression of wage increases also mechanically arises from the fact that, if downward nominal wage rigidity prevents some wage decreases when productivity decreases, it mechanically needs to also mitigate the subsequent wage increases when productivity recovers. Censoring the fall censors the rebound too. This second mechanism is present absent any forward-looking wage moderation, and can by itself account for the compression of wage increases found in the data.

It does in our model. Table F.2 gives the fraction of real wage increases among all real wage changes in the cross-sectional steady-state distribution of our model, for different inflation rates as well as and under flexible wages. Lower inflation—and therefore a more binding constraint on downward wage adjustment—comes with fewer real wage increases.
References


