Partial Moment Momentum

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Abstract

Momentum profits benefit from persistent trends of the market, which can be predicted by market volatility. However, such strategies are unable to distinguish between upside and downside risk. We propose partial moments-based momentum trading strategies and find that they outperform plain momentum and volatility-scaled momentum strategies. We suggest that this greater profitability is due to the unexploited investment opportunities that arise from being able to distinguish between good and bad risk. We find strong outperformance for seven out of eight partial moments-based strategies during states of market downturn. The outperformance is robust across different time periods.

1. Introduction

Cross-sectional momentum strategies are employed by buying previous winners and selling previous losers[1]. The literature shows that cross-sectional momentum strategies (henceforth, plain momentum strategies) are profitable in different markets and asset classes across different sample periods[2]. Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) present evidence that scaling the weights of momentum portfolios increases the Sharpe ratio of the plain momentum strategy. It is well-known that volatility is larger when markets fall than when they rise. Furthermore, the economic consequences of volatility in falling or rising markets are not equal. Rather than scaling plain momentum portfolios, which does not distinguish between upside and downside risk, we construct two partial moments-based strategies, the partial moment momentum (PMM) strategy and the extended partial moment-decomposed (PMD) strategy. These two strategies use the partial moment decompositions of squared market returns to better capture market trends and reduce losses during market turbulence. We find that both strategies significantly outperform the plain momentum strategy. We further introduce an adapted Sortino ratio[3] as a measure of performance, more suited to our different volatility regimes.

We give more details of these two strategies. The first is the PMM strategy, which involves switching positions of the winner and loser portfolios during the holding periods, depending upon

[1] Jegadeesh and Titman (1993) find that momentum strategies are profitable in the US equities markets over the short to medium horizons (3 to 12 months) from 1965 to 1989. Jegadeesh and Titman (2001) continue to show similar results for the period 1990 to 1998. Israel and Moskowitz (2013) extend momentum evidence to two periods: from 1927 to 1965 and from 1990 to 2012.

[2] See Richards (1997) for evidence of momentum in stock market indexes; Asness, Liew, and Stevens (1997) for that in country indexes; and Rouwenhorst (1998, 1999) for that in emerging stock markets; Chan, Hameed, and Tong (2000) and Hameed and Yuanto (2002) for that in momentum in international equity markets. See Okunev and White (2003) for momentum in exchange rates; Erb and Harvey (2006) for that in commodities; and Moskowitz, Ooi, and Pedersen (2012) for that in futures contracts. Consistent with Asness, Moskowitz, and Pedersen (2013), Daniel and Moskowitz (2016) find momentum returns in markets across regions (European Union, Japan, UK, and US) and asset classes (fixed income, commodities, foreign exchange, and equity) from 1972 through 2013. For more detailed literature, see Daniel and Moskowitz (2016).

[3] The adapted Sortino ratio is defined in Appendix A.2. We also report a practitioner's version of Sharpe ratio in which we divide the return of winners minus the return of losers by the portfolio standard deviation instead of using the version by Sharpe (1994), in which the excess return of the strategy is the numerator. We find this to be relatively less conservative than the Sharpe (1994) ratio. For more details, see Appendix A.1.

current estimates of partial moments. The second is the PMD strategy, which can be viewed as an extension of the dynamic volatility-based momentum strategy in Barroso and Santa-Clara (2015), who do not differentiate between upside and downside risk. We extend this class of strategies by tilting our strategy long or short towards favorable/ unfavorable volatility signals and holding an offsetting position in cash. We take particular care to use non-overlapping data in our estimations of volatility thereby avoiding the artificial autocorrelation evident in other authors' volatility series.

We present strong evidence that our two partial moments-based strategies outperform benchmark plain momentum strategies[4], particularly during financial turbulence. During our market downturn period, our results reveal higher annualized adapted Sortino ratios for the PMM strategies compared to the benchmark 11×1 plain momentum strategy. Our robustness test results for the unconstrained PMD strategy on a winners-minus-losers WML basis (11×1) from 2000 to 2016 outperform those in Daniel and Moskowitz (2016) for dynamic WML strategies from 2000 to 2013, confirming the persistence of momentum returns beyond 2013. Further analyses show similar outperformance across four sub-periods within 1927 to 2016. Overall, five out of six PMM strategies and both PMD strategies (one with and another without leverage constraint) outperform the plain momentum benchmark strategy. The results suggest that partial moments-based strategies might be employed in the risk management of momentum-based strategies and, in particular, as an extension to the volatility-based risk management applications proposed by Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016).

Supporting evidence for the relevance of partial moment effects on momentum is given by Chordia and Shivakumar (2002), who demonstrate that economic expansionary periods might be important in explaining profits in the US equities market, but the literature is inconclusive on

[4] As the benchmark, we use an 11×1 plain momentum strategy, which represents an 11-month formation period and a 1-month holding period plain momentum strategy with a 1-month gap between formation and holding periods. We use this as the benchmark strategy to be consistent with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) for performance comparison. This plain strategy also yields the highest returns over the whole sample period among all five plain momentum strategies.

whether momentum profits are positive or negative during contractionary periods. Ali and Trombley (2006) find that the level of momentum returns of US stocks for the period from 1984 to 2001 is positively related to short sales constraints, and that loser portfolios rather than winner portfolios drive this result. In contrast, Cooper, Gutierrez, and Hameed (2004) find that momentum profits depend "critically" on the state of the market. They show that a 6-month momentum strategy is profitable only following periods of gains in the US market during the sample period, from 1929 to 1995, consistent with the predictions of overreaction models in Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999).

The literature has further shown that momentum returns are related to partial moments, especially lower partial moments [see Menkhoff and Schmeling (2006), Baltas and Kosowski (2012), and Daniel, Jagannathan, and Kim (2012)]. We employ the idea of downside realized partial moment (RPM^{-}) and upside realized partial moment (RPM^{+}) in the context of momentum trading strategies. We hypothesize that if volatility based momentum strategies, such as those in Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), have been shown to counter extreme momentum losses in times of market crashes, then further momentum profits and better risk management can be achieved by utilizing the connection between RS^{-} and future volatility shown by Barndorff-Nielsen, Kinnebrock, and Shephard (2010). Other applications of partial moments to finance are included, for example, in Bali, Cakici and Whitelaw (2014).

Findings outside the US in Gao and Leung (2017) support asymmetric momentum performance. They show that momentum returns of Australian stocks are negatively correlated to short sale restrictions and were less profitable during the global financial crisis (GFC) period of July 2007 through September 2009 compared to pre-GFC levels. The authors explain that the imposition of short selling restrictions by the Australian regulatory authority during the GFC might have moderated the ability of momentum traders to profit from the short sale of loser portfolios. This is supported by Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), who show that momentum strategies experience extreme losses during periods of economic upheaval following market crashes and high market volatility.

Recent literature has introduced a number of volatility-scaled momentum strategies, in both a cross-sectional momentum setting [Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016)] and in a time-series momentum setting [Moskowitz, Ooi and Pederson (2012)]. Both cases involve the notion of the use of target volatility to scale the risk exposure of plain momentum returns to produce risk-managed momentum returns. For instance, Barroso and Santa-Clara (2015) reveal that gains in momentum returns can be wiped out by momentum crashes over short periods. What is more surprising is the high level of predictability of the risk of momentum returns. The authors proceed to scale the long–short WML momentum portfolio by its prior 6 months' realized volatility to implement a constant volatility strategy that avoids forward-looking bias. This risk-managed momentum strategy results in negligible negative returns during the crashes, a doubling of the Sharpe ratio, and a reduction in both excess kurtosis and left skewness. Daniel and Moskowitz (2016) implement a dynamic momentum strategy based on conditional moments (mean and variance) and achieve twice the alpha and Sharpe ratio compared to the traditional static WML strategy over multiple time periods and different equity markets[5]. This evidence suggests that momentum strategies, which dynamically account for past volatility, act as a hedging mechanism for the extreme momentum losses following sudden market downturns. However, the unconstrained leverage implicit in such strategies makes these strong results questionable as a practical investment strategy. We consider, in our analyses, leverage-constrained strategies to address this issue.

Barndorff-Nielsen, Kinnebrock, and Shephard (2010) show that future volatility is related more to past negative returns than past positive returns. In doing so, the authors develop a volatility measure called "realized semi-variance," which decomposes the direction of the quadratic variation

[5] Both Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) construct momentum portfolios with an 11-month formation period and a 1-month holding period. This strategy uses value-weighted holding period returns with a 1-month gap between the formation period and the holding period. We refer to this strategy in particular as (static) *WML* strategy.

in asset prices, termed “realized variance.” Andersen, Bollerslev, Diebold, and Ebens (2001) and Barndorff-Nielsen and Shephard (2002) define the realized variance as the sum of the squared returns to estimate the quadratic variation in high frequency asset prices. As such, the negative and positive returns of asset prices are used to compute a downside (RS^{-}) and an upside (RS^{+}) realized semi-variance, respectively. A review of the semi-variance literature is presented in Sortino and Satchell (2001). Further development of the economic theory underpinning this risk measure is set out in Pedersen and Satchell (2002). These directional volatility measures are found to capture the asymmetrical properties of volatility experienced by asset prices. Hedge funds might employ downside realized semi-variance in the context of risk management. These investors might have short positions in the market, and a drop in price therefore yields a positive return, with the corresponding measure of risk being RS-. Baruník, Kočenda, and Vácha (2016) extend this idea to construct asymmetric volatility spillover indexes and reveal high levels of asymmetrical spillover among the most liquid US stocks in seven sectors. Linkages between downside risk and momentum are explored by Min and Kim (2016). As we argue in Section 3, it is more appropriate that this semi-variance is called partial moments of order 2. Upper and lower partial moments are what has been termed upside and downside realized semi-variance, respectively.

2. Momentum in the US equity market

2.1 Data

The data used in this study are sourced from the Centre for Research in Security Prices (CRSP) and the Kenneth R. French Data Library[6]. The monthly and daily US equity data for the period January 1926 to December 2016 are sourced from the CRSP. Our sample includes common stocks (CRSP share code 10 or 11) of all firms listed on the NYSE, Amex, and Nasdaq (CRSP exchange code 1, 2 or 3). We use the value-weighted index of all listed firms in the CRSP and the 1-month

[6] These data are available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Treasury bill rate as the proxy for the market portfolio and the risk-free rate, respectively. We obtain the 1-month Treasury bill rate for the same period as that of the US equity data from the Kenneth R. French Data Library.

2.2 Momentum portfolio construction

We follow a method similar to Jegadeesh and Titman's (1993) $J \times K$ trading model to construct our momentum portfolio with a zero net position. First, in month t, all valid sample stocks are ranked based on their past *J-month* formation period adjusted returns from month $t\text{-}J$ to $t\text{-}1$, and then sorted into ten decile portfolios according to the NYSE breakpoints as each decile portfolio contains an equal number of NYSE firms[7]. For any construction month t, we define a valid sample stock as one which has share price, number of shares outstanding, and a minimum of $2J/3$ (rounding up to the nearest integer) monthly returns during the *J-month* formation period. For instance, for a momentum strategy with an 11-month formation period, we require a stock with at least eight monthly returns during this period. Subsequently, we buy the best-performing portfolio (winners) and short the worst-performing portfolio (losers) for the *K-month* holding period from month t to $t+K$. The value-weighted holding period logarithmic adjusted returns are calculated for computing our momentum profits. In the meantime, this strategy rebalances at the end of every month. Thus, in any month t, one certain strategy holds not only the winner and loser portfolios constructed in month t, but also those portfolios in the previous $K\text{-}2$ months. There is a 1-month lag between the formation period and the holding period to avoid short-term reversals [see Jegadeesh (1990) and

[7] Using NYSE breakpoints rather than all-firm breakpoints can limit the side effects of small and illiquid stocks that are barely traded in practice. This is consistent with Barroso and Santa-Clara (2015) and how the Kenneth R. French Data Library constructs its "10 Portfolios Formed on Momentum." Our ten decile portfolio returns are almost identical to those of (2-12) momentum returns on the Kenneth R. French Data Library. We note that Daniel and Moskowitz (2016) differ from us, as the authors constructed the decile portfolios using all-firm breakpoints rather than NYSE breakpoints. See http://www.kentdaniel.net/data.php for a detailed comparison of portfolio returns using these two breakpoints.

Lehmann (1990)]. This allows us to be consistent with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), while noting that they differ from us in other respects[8].

2.3 Performance of plain momentum strategies in US equity markets

We report the performance of all momentum based strategies for four sample periods: P1, the whole sample period from January 1927 to December 2016; P2, the period from January 1965 to December 1989 covered in Jegadeesh and Titman (1993); P3, the market downturn period from August 2007 to December 2012; and P4, the era of turbulence from January 2000 to December 2016. The starting point of the market downturn period of August 2007 represents the onset of financial market turbulence in the equity market and the intervention of the Federal Reserve for the first time since 2001. The end date of the market downturn period reflects the recovery of the DJIA, S&P 500, and Nasdaq to their pre-GFC levels.[9]

[Insert Table 1 about here]

We adopt a practitioner's version of the Sharpe ratio, where we define an appropriate formulation for a long–short portfolio as

$$Sharpe\ ratio = \frac{\mu_L - \mu_S}{sd(LS)} \qquad (1)$$

where μ_L and μ_S are returns of a long portfolio and short portfolio, respectively. $sd(LS)$ is the standard deviation of the long–short portfolio.[10] A justification is presented in Appendix A.1.

Table 1 shows performances of the plain momentum strategy on a WML basis (*11×1*) for the aforementioned four sample periods. During P1 and P2, the *11×1 (WML)* strategy (with an 11-month formation (*J*) period, a 1-month holding (*K*) period, and a 1-month gap between *J* and *K*) can

[8] All analyses are repeated using all-firm breakpoints momentum returns as a robustness check. The results are consistent with those based on our original WML strategy. Our WML returns using NYSE breakpoints are moderately lower than momentum returns on the same basis using all-firm breakpoints.
[9] The GFC period classification by Bekaert, Ehrmann, Fratzscher, and Mehl (2014) covers the period August 2007 to March 2009.
[10] A more conservative formulation for defining the Sharpe ratio for a long–short portfolio is $\mu_L - \mu_S$ minus the riskless rate divided by the long–short portfolio standard deviation, which is shown in Sharpe (1994).

earn 15.15% and 21.60% annualized returns, respectively. We also construct four other plain momentum strategies, *3×3*, *6×6*, *9×9*, and *12×12* following the same construction methods in Subsection 2.2. These four strategies also report positive returns during P1 and P2 as shown in Table B.1 of Appendix B. However, apart from the *WML* strategy, none of the other four plain strategies can generate profits during P3, the market downturn or during P4, the era of turbulence.

Consistent with the findings of Cooper, Gutierrez, and Hameed (2004), we find extreme losses occurring in plain momentum strategies constructed immediately after sharp market declines, such as in March 2009[11]. One exception is the *11×1 (WML)* strategy, which continues to yield positive returns. We observe that the momentum strategy with a long formation period and a short holding period is easier to turn around in the event of a momentum reversal.

In this study, we use the *11×1 (WML)* plain momentum strategy as the benchmark to demonstrate the effectiveness of our partial moments-based strategies, since this strategy yields the highest and most significant return for all four sample periods among those five plain momentum strategies (see Table 1 and Table B.1). Thus, we set our benchmark as high as possible. Also, by using this benchmark, we maintain consistency with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) for our performance comparisons.

3. Partial moment momentum

Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) present evidence that, by scaling the weights of momentum portfolios, this new scaling strategy increases the Sharpe ratio of the plain momentum strategy.

Rather than scaling plain momentum portfolios, we first construct a PMM strategy by switching positions of the winner and loser portfolios during the holding periods.

[11] A counter-trend bear market rally began after the DJIA, S&P 500, and Nasdaq reached their troughs and started to rebound.

3.1 Partial moments and realized semi-variance

We can construct the sample realized volatility using equi-spaced data for *[t-1, t]*, which we define as RV. This is defined for *n+1* prices or their logarithms to define *n* returns, r_i, *i=1,...,n*. We define RV as

$$RV_t = \sum_{i=1}^{n} r_{i,t}^2 \tag{2}$$

We call this the realized variance for *[t-1, t]*, which is known to be a consistent estimator of quadratic variation if we assume the prices are generated by a particular class of processes; see Baruník, Kočenda, and Vácha (2016) for more details. The properties of RV and related measures can be found in Andersen, Bollersley, Diebold, and Ebens (2001), Barndorff-Nielsen (2002) and Barndorff-Nielsen, Kinnebrock, and Shephard (2010). Following the literature, we define two statistics, negative semi-variance RS^- and positive semi-variance RS^+, as

$$RS_t^- = \sum_{i=1}^{n} r_{i,t}^2 \, I(r_{i,t} < 0) \tag{3}$$

and

$$RS_t^+ = \sum_{i=1}^{n} r_{i,t}^2 \, I(r_{i,t} \geq 0) \tag{4}$$

where $I()$ is the indicator function, these being sample lower and upper partial moments of order 2 with truncation at zero in both cases, $r_{i,t}$ is the return of stock *i* in day *t*, and n is the number of valid sample stocks in day *t*. There is an identity,

$$RV_t = RS_t^- + RS_t^+ \tag{5}$$

If the population has a mean of zero, then we might hope to treat these quantities as estimators of the unconditional population variance and the unconditional lower and upper semi-variances of the process over the interval *[t-1, t]*, which we could denote by σ^2, σ^{2-}, and σ^{2+}, respectively. Hopefully, $\sigma^2 = \sigma^{2-} + \sigma^{2+}$. However, the difficulty here is that, for example, $E(RS^+)$ is not equal to σ^{2+} but rather $\sigma^{2+} + E^2[\sum_{i=1}^{n} r_{i,t}\, I(r_{i,t} \geq 0)]$. This means that descriptions in terms of partial moments seem more appropriate. Thus, we refer to these two statistics as upper and lower partial moments (RPM^{+} and RPM^{-}). This follows if we define $E(RV) = E[\sum_{i=1}^{n} r_{i,t}^2]$, $E(RPM^{-}) = E[\sum_{i=1}^{n} r_{i,t}^2\, I(r_{i,t} < 0)]$, and $E(RPM^{+}) = E[\sum_{i=1}^{n} r_{i,t}^2\, I(r_{i,t} \geq 0)]$.

Barroso and Santa-Clara (2015) argue that momentum volatility is strongly forecastable relative to other styles (see in particular Table 2 of p. 114). This suggests that momentum volatility or momentum partial moments might be useful in forecasting momentum returns. The authors use 126 days overlapping momentum returns to estimate volatilities [see Barroso and Santa-Clara (2015) and equations (5) and (6)]. There is also some theoretical support for the notion that momentum returns are volatility dependent; see, for example, He, Li, and Li (2016). We believe that part of the forecastability is artificial. To observe this, we oversimplify and assume that a momentum strategy can be described as a position in an asset where the "weight" of the asset is equal to the last period's return, which is coined as a relative strength portfolio by Lo and Mackinlay (1990). This can be identified as a version of single variable time-series momentum. Thus, R_t, the return of the strategy, is equal to $r_t\, r_{t-1}$.

We first show in Appendix A.3 that momentum is forecastable even when returns are independently and identically distributed (iid). We observe that, when underlying returns are white noise, the forecastability of momentum volatility is connected to underlying kurtosis, and the higher the kurtosis, the lower the forecastability. Thus, we do not use momentum volatility in guiding our strategies, but instead use a market index defined in Subsection 2.1. The lower partial moment RPM^{-} and the higher partial moment RPM^{+} are defined analogously to (3) and (4).

While we continue to report Sharpe ratios, we also provide a performance measure called the adapted Sortino ratio, defined in Appendix A.2.

3.2 Partial moments and reference points

We consider the medians of RPM_t^+ and RPM_t^- as the reference points for the upper and lower market partial moments. This reflects a more general case but our assumption follows from the most straight-forward method of median sorting and a prior belief based on a uniform distribution on [0, 1]. We are agnostic about what our central point (CP^+, CP^-) should be for our strategies. If we held a Bayesian prior consisting of a pair of independent uniform distributions, then we would choose the means of both variables. However, for reasons of robustness, we choose the medians, which in any case are equal to the means.

[Insert Figures 1a and 1b about here]

Figures 1a and 1b show the histograms of upper and lower partial moments (RPM_t^+ and RPM_t^-) with kernel density curves. Values of medians and maximum observations of both RPM_t^+ and RPM_t^- are reported. For the upper partial moment (RPM_t^+), the normal kernel estimate for c=0.7852 has a bandwidth of 0.0002 and an approximate mean integrated square error (AMISE) of 2.1726. For the lower partial moment (RPM_t^-), the normal kernel estimate for c=0.7852 has a bandwidth of 0.0002 and an AMISE of 1.8252.

[Insert Table 2a, 2b, and 2c about here]

Table 2a reports the distributions of monthly realized variance (RV_t) and upper and lower market partial moments (RPM_t^+ and RPM_t^-) throughout the whole sample period from January 1927 to December 2016.

Table 2b presents the joint distribution of upper and lower market partial moments (RPM_t^+ and RPM_t^-) quartiles for the four sample periods. Of particular interest is the fact that the probability that both upper and lower market partial moments are relatively large or small increases dramatically over the two sub-periods of market turbulence. We note that, if the average frequency of a 4×4 table is 6.25%, then in all cases, $P(RPM_t^{+\,(4)}, RPM_t^{-\,(4)})$, where $P(RPM_t^{+\,(i)}, RPM_t^{-\,(j)})$ is the joint frequency of RPM_t^+ and RPM_t^- and in which i and j stand for the four quartiles of the upper and lower partial moments, from the lowest quartile to the highest quartile, is two to three times greater. Likewise, the frequencies in $P(RPM_t^{+\,(1)}, RPM_t^{-\,(1)})$ are obviously higher.

Table 2c shows the correlation between pairs of variables from among the upper and lower market partial moments (RPM_t^+ and RPM_t^-) and the skewness of returns of plain momentum (winner-minus-loser), winner, and loser portfolios ($Skew\_mom_t$, $Skew\_win_t$, and $Skew\_los_t$) for the four sample periods. In all cases, RPM_t^+ and RPM_t^- are positively correlated, especially in P3 and P4 where they exceed 0.9 and are highly significant. Overall, the skewness of momentum, winner, or loser portfolios is correlated much more with RPM_t^+ than RPM_t^-.

The previous approach analyzed in Subsection 3.1 used the past month RPM_t^+ and RPM_t^- based on daily data as our forecast of partial moments. An alternative to this approach would be to assume that the partial moments satisfy a statistical model, such as a vector-autoregressive process of order p ($VAR(p)$), which is discussed in Appendix C.1. In Table C.1, based on a $VAR(1)$ model, we observe that both the past upper and lower partial moments forecast the current partial moments, as all coefficients are significant and positive. We note also that the estimated unconditional means are positive.

3.3 Rules and performance of partial moment momentum strategies

[Insert Table 3 about here]

The idea behind our switching strategies is to change momentum strategies depending upon our current estimates of partial moments (RPM_t^+, RPM_t^-). We refer to it simply as a PMM strategy.

The switching methods are shown in Table 3 and Figure 2a. Panel A of Table 3 shows four conditions of upper and lower partial moments based on their reference points. It presents the corresponding switching method(s) during the holding periods for each of these four conditions. Panel B illustrates the actions and returns for different holding periods for each of the four conditions presented in Panel A of our six PMM strategies, which we call PMM strategies 1 to 6, represented by *PMM_S1* to *PMM_S6*.

[Insert Figures 2a and 2b about here]

Figure 2a illustrates the four PMM strategy conditions in the coordinate plane. The origin point represents the reference points for both the upper partial moment, RPM_t^+, and the lower partial moment, RPM_t^-, in month *t*. Each quadrant represents a PMM condition based on the upper and lower partial moments and their reference points. In particular, the combination of condition 1, in which both RPM_t^+ and RPM_t^- are greater than the reference points, represents an environment of high market volatility, which is not conducive to momentum trading profits. On the other hand, condition 3 contains RPM_t^+ and RPM_t^-, which are less than the reference points. This reflects an environment in which market trends tend to persist in the same direction. In this case, we expect high profitability as the outcome of momentum-based strategies.

Figure 2b reports the actual possibility of occurrence of each of these four PMM strategy conditions when the medians of RPM_t^+ and RPM_t^- are used as reference points for RPM_t^+ and RPM_t^-, respectively. If the two processes are independent, the probability that each of these four conditions occurs is exactly the same, at 25%. However, the actual possibilities reveal that the upper and lower partial moments are not independent. Condition 1 (3), in which both RPM_t^+ and RPM_t^- are higher (lower) than the reference points, shows a higher chance of occurrence at 34.91% than condition 2 or 4. This result is supported by the significant positive correlations between RPM_t^+ and RPM_t^- in Table 2c. The same possibility of condition 1 and 3 as well as condition 2 and 4 reflects their complementary relationship when medians are used as reference points. In the coordinate plane, this relationship appears as the combined possibility of occurrence of any two adjacent quadrants, equal to 50%.

[Insert Figure 3 about here]

Figure 3 demonstrates the timeline of an *11×1* partial moment momentum (PMM) strategy constructed at time *t*. In any month *t*, all sample stocks are ranked and sorted into deciles based on their past 11-month formation period returns from month *t-11* to *t-1* (owing to a 1-month gap, month *t* to *t+1,* between the formation and holding periods for returns). Then, we classify holding strategies into four conditions based on partial moments determined in the period month *t-1* to *t* and their reference points analyzed in Subsection 3.2. Then, during the 1-month holding period from month *t* to *t+1,* we compare monthly upper and lower partial moments (RPM_t^+ and RPM_t^-) during the period [*t-1, t*] with their reference points and switch positions of winners, losers, and cash assets to maintain a zero net position based on Panel A of Table 3. In other words, for any of these six PMM strategies, we keep rebalancing and switching positions of all assets held.

In particular, all six PMM strategies switch to the same positions if condition 2 or 3 occurs but act differently if condition 1 or 4 occurs. Moreover, we hold cash long or short to keep our PMM strategies in net zero positions. $r_{w,t+1}$, $r_{l,t+1}$, and $r_{f,t+1}$ represent the returns of winners, losers, and risk-free assets in month *t*+1, respectively. For example, for PMM strategy 4 (*PMM_S4*), if condition 1 applies during the period [*t-1, t*] in which the upper and lower partial moments are all higher than their reference points, then we close out our positions in both winners and losers. The PMM return for month *t+1* is 0; if condition 2 applies during the period [*t-1, t*] in which the upper partial moment is lower than its reference point and the lower partial moment is higher than its reference point, then we short losers only, liquidating our long positions and holding cash long. The PMM return for month *t+1* is $r_{f,t+1} - r_{l,t+1}$; if condition 3 applies during the period [*t-1, t*] in which upper and lower partial moments are all lower than their reference points, then we carry on the momentum strategy by buying winners and short-selling losers. The PMM return for month *t+1* is $r_{w,t+1} - r_{l,t+1}$; if condition 4 applies during the period [*t-1, t*] in which the upper partial moment is higher than its reference point and the lower partial moment is lower than its reference point, then we buy winners only and short cash. The PMM return for month *t+1* is $r_{w,t+1} - r_{f,t+1}$.

The key characteristic of our PMM strategies is that we change positions of winners, losers, and cash assets based on market partial moments during holding periods. For instance, if we construct PMM strategy 4 (*PMM_S4*) on a $J \times K$ basis with a zero net position in month *t*, we rank all sample stocks and sort into deciles based on their past *J-month* formation period returns from month *t-J* to *t-1*. Then, during the *K-month* holding period from month *t* to *t+K,* we compare the upper and lower partial moments with their reference points in each month and switch positions of winners, losers, and cash assets to maintain a zero net position based on Panel A of Table 3. However, if the current partial moment condition persists over consecutive months, then we hold the same positions based on this condition's method and keep rebalancing until other conditions apply. Assume that in month *t*, this $J \times K$ based PMM strategy 4 (*PMM_S4*) meets condition 4; then, we buy winners only and short cash. If in month *t+1* condition 4 still holds, then we hold current positions and rebalance the winners' portfolio based on the past *J-month* performance from month *t-1* to *t*, which is the same rebalancing method as our plain momentum strategy in Subsection 3.2. If condition 1 holds in month *t+2*, then we close out long positions by selling winners and close out short positions in cash for the month. This process continues repeatedly.

[Insert Table 4a and 4b about here]

Table 4a compares the performances of six PMM strategies on an 11×1 basis and the benchmark strategy for four sample periods. The results show that PMM strategies 1 and 4 (*PMM_S1* and *PMM_S4*) persistently outperform the benchmark for the two sub-periods of market downturn, P3 and P4, by generating higher Sharpe ratios and adapted Sortino ratios as well as yielding significant positive returns. Even during P4, the era of turbulence, PMM strategy 4 (*PMM_S4*) generates the most significant return and the highest Sharpe ratio (0.33) among all seven strategies, including the benchmark.

Table 4b presents decomposed returns of PMM strategies for the four conditions in the whole sample period. As Panel B shows, results support our expectations that momentum-based strategies are highly profitable when condition 3 applies as market trends tend to persist in the same direction but are almost unprofitable when condition 1 applies as high market turbulence is not conducive to momentum trading profits. In particular, condition 1 applies 34.91% of the time but contribute less than 12% (1.87/15.65) to the total return of the plain momentum strategy.

4. An extended partial moment-decomposed momentum strategy

PMM strategies, by switching positions of winner and loser portfolios, do not work well when the market is calm. Thus, we propose a different type of partial moments-based momentum strategy, named the extended partial moment-decomposed momentum strategy.

Recent studies have enhanced momentum strategies by weighting the momentum positions using volatility in various ways [see Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016)]. These methods do not differentiate between upside or downside risk and typically are, by construction, net zero funds. Practitioner methods based on optimization, however, scale momentum mean forecasts by their standard deviations. We extend this class of strategies by tilting our strategy long or short toward favorable/unfavorable volatility signals and holding an offsetting position in cash.

We define a ($\varphi_1(RPM_t^+, RPM_t^-), \varphi_2(RPM_t^+, RPM_t^-)$) strategy as a net zero portfolio long $\varphi_1(RPM_t^+, RPM_t^-)$ in winners and short $\varphi_2(RPM_t^+, RPM_t^-)$ in losers with an offsetting position of $\varphi_1(RPM_t^+, RPM_t^-) - \varphi_2(RPM_t^+, RPM_t^-)$ in cash; we call this portfolio *p*. We denote its return at time *t+1* as

$$r_{p,t+1} = \varphi_1(RPM_t^+, RPM_t^-) r_{w,t+1} - \varphi_2(RPM_t^+, RPM_t^-) r_{l,t+1}$$

$$+(\varphi_2(RPM_t^+, RPM_t^-)) - \varphi_1(RPM_t^+, RPM_t^-)) \ r_{f,t+1} \quad (6)$$

where $r_{w,t+1}$, $r_{l,t+1}$, and $r_{f,t+1}$ are the returns at time *t+1* to the winners, losers, and "cash" portfolios, respectively.

There is no obvious guidance as to the functional form of $\varphi_1(RPM_t^+, RPM_t^-)$ and $\varphi_2(RPM_t^+, RPM_t^-)$, but we might expect $\varphi_1(RPM_t^+, RPM_t^-)$ to be increasing in its first argument and decreasing in its second argument and $\varphi_2(RPM_t^+, RPM_t^-)$ to have the opposite properties. Furthermore, we might want to normalize them, as in Barroso and Santa-Clara (2015), in terms of some target volatility, σ_{tar}. We pick the same target annualized volatility of 12% to maintain consistency. With these considerations in mind, we might require the constraint to be

$$\varphi_1(RPM_t^+, RPM_t^-) + \varphi_2(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}} \tag{7}$$

which is broadly in accord with Barroso and Santa-Clara (2015). We choose $\varphi_1(RPM_t^+, RPM_t^-)$ and $\varphi_2(RPM_t^+, RPM_t^-)$ as follows:

$$\varphi_1(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}}\left(\frac{RPM_t^+}{RPM_t^+ + RPM_t^-}\right) \tag{8}$$

$$\varphi_2(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}}\left(\frac{RPM_t^-}{RPM_t^+ + RPM_t^-}\right) \tag{9}$$

If $RPM_t^+ = RPM_t^-$, then we have a conventional long–short portfolio with scaling $\frac{\sigma_{tar}}{\sqrt{RV_t}}$, as in Barroso and Santa-Clara (2015, p. 115, formula (5)).[12]

There are many alternative ways to specify our strategy; Formulas (7) to (9) could be criticized in that we might want to shrink our exposure on the downside when RPM_t^- is large. Such an approach would be more consistent with the empirical result that momentum profits mainly result from the long side of a portfolio.[13] Furthermore, it might be argued that in choosing functions that are homogeneous of degree zero, some of the partial moment information is lost. The argument in

[12] We note that actual choices of σ_{tar}, RV_t, etc., may differ in our application from those of other authors.
[13] Jegadeesh and Titman (1993) state that "the abnormal performance of the zero-cost (momentum) portfolio is due to the buy side of the transaction rather than the sell side" (p. 77). Moskowitz and Grinblatt (1999) argue that "industry momentum strategies appear to profit mostly on the buy side" (p. 1272). See, for instance, Chan, Jegadeesh, and Lakonishok (1996), Jegadeesh and Titman (2001), and Israel and Moskowitz (2013) for further evidence.

favor of increasing one's exposure on the downside is that, in prospect theory, agents are usually deemed risk-loving on the downside.[14]

In practice, leverage is an issue in long–short portfolios. Many institutional hedge funds have strict restrictions on leverage, with 200% leverage being a typical upper bound. The previous popularity of 130-30 funds provides evidence that leverage is not unconstrained in practice[15]. Leverage is defined as the sum of absolute value of long and short weights (ignoring cash positions) so that our abovementioned strategies have a leverage as in Formula (7).

If the leverage as in Formula (9) exceeds the upper bound, to rescale our weights to obey the 200% leverage condition, we need to change our scaling to

$$\varphi_1(RPM_t^+, RPM_t^-) = 2\left(\frac{RPM_t^+}{RPM_t^+ + RPM_t^-}\right) \tag{10}$$

$$\varphi_2(RPM_t^+, RPM_t^-) = 2\left(\frac{RPM_t^-}{RPM_t^+ + RPM_t^-}\right) \tag{11}$$

with corresponding positions in cash.

[Insert Table 5 about here]

Table 5 shows the performance of two PMD strategies, with and without the 200% leverage constraint, for the four sample periods. Both PMD strategies significantly outperform the plain momentum strategy for all four sample periods by generating higher positive returns and having larger Sharpe ratios and adapted Sortino ratios. Even during financial turbulence (P3 and P4) when holding a plain 11×1 momentum strategy only earns insignificant low returns, the unconstrained

[14] Ang, Chen, and Xing (2006) conclude that "the behavioural framework of Kahneman and Tversky's (1979) loss aversion preferences and the axiomatic approach taken by Gul's (1991) disappointment aversion preferences allow agents to place greater weights on losses relative to gains in their utility functions" (p. 1192). The authors also argue that "agents who place greater weight on downside risk demand additional compensation for holding stocks with high sensitivities to downside market movements" (p. 1191).

[15] "130-30" represents a leveraged long–short mutual fund strategy that allows fund managers to hold short positions up to 30% of the initial investment and to use the corresponding funds to take long positions. Other popular strategies of this type include but are not limited to "150-50" and "120-20."

PMD strategy (*PMD*) persistently yields positive returns. In particular, during P4, this strategy earns an annualized Sharpe ratio of 1.62 and an annualized adapted Sortino ratio of 1.00. The 200% leverage-constrained PMD strategy (*PMD_C*) reveals similar good performance even though it slightly underperforms the unconstrained strategy.

[Insert Table 6 about here]

Table 6 provides a summary of the performance of two PMD strategies, the best and the worst PMM strategy with the plain momentum strategy for the four sample periods. PMM_S4 and PMM_S3 are chosen as the best- and worst-performing PMM strategies based on their performances during the whole sample period and market downturn period (see Table 5). To check the normality of returns of these five momentum strategies, we also conduct the Jarque-Bera normality test[16]. Results show that during the whole sample period, apart from the underperforming PMM Strategy 3, all the other three partial moments-based momentum strategies reduce skewness and kurtosis and increase both the Sharpe ratio and the adapted Sortino ratio compared to the benchmark. In particular, the skewness of PMM_S4 is reduced and turns positive for all four sample periods compared to that of the plain momentum strategy. We also note that, in almost all cases, the relatively small p-values of the Jarque-Bera normality test suggest rejection of the null hypothesis; that is, strategy returns are not normally distributed. This is consistent with the theory where Kwon and Satchell (2017) show that the returns of the cross-sectional momentum strategy are generically non-normally distributed. However, the Jarque-Bera statistic of four partial moments-based momentum strategies are reduced in most cases. This suggests that our partial moments-based momentum strategies help in managing momentum risk.

[16] The Jarque-Bera statistic follows (asymptotically) the chi-squared distribution with two degrees of freedom. Under the null hypothesis of normality, the expected value of the skewness is zero and the excess kurtosis is zero (which is equivalent as a kurtosis of three). The test statistic is defined as $\left[\frac{skeness^2}{6/n} + \frac{(kurtosis-3)^2}{24/n}\right] \sim \chi^2(2)$, where n is the number of observations. See Jarque and Bera (1980, 1987) for further details.

5. Out-of-sample analysis and rolling window PMM

The PMM performance in Section 3 is conditional on the realized semi-variance reference points for our whole sample period from January 1927 to December 2016. There may be concerns that this involves using out-of-sample information and lacks realism.

To strengthen the effectiveness of our PMM model and to provide practical investment insights for momentum investors, we further construct PMM strategies using parameters and reference points computed within an in-sample period from January 1927 to December 1999. We call the out-of-sample period from January 2000 to December 2016 the era of turbulence, since it contained the IT bubble of the early 2000s, the hedge fund crisis of 2006, the GFC since late 2007, and the European debt crisis since late 2009. Therefore, it might be more convincing if our selected PMM and PMD strategies outperform plain momentum during this period.

[Insert Table 7 about here]

Table 7 shows the use of RPM_t^+ and RPM_t^- during in-sample periods for the vector autoregression of upper and lower partial moments, respectively, in order to maintain consistency with previous sections (see Appendix C.1 for further details). In common with Table C.1, we observe that the forecastability of past upper and lower partial moments remains strong, as all coefficients are significant and positive. We note also that the stationarity conditions are satisfied and the unconditional means are positive.

[Insert Table 8 about here]

Table 8 presents the frequencies of the four PMM switching conditions based on in-sample partial moments for the whole out-of-sample period, the era of turbulence, from January 2000 to December 2016. The table shows that in any given month *t* during the out-of-sample period, there is a high probability that condition 1 or 4 occurs. Panel B presents the out-of-sample frequencies of PMM conditions based on in-sample estimates. Compared to the estimated frequencies, the major variation is that the frequency of condition 1 almost doubled during the era of turbulence (actual frequency of 61.08% compared to an expected frequency of 32.53% based on the in-sample periods). A possible explanation is that this period contains a lower proportion of periods of market upturn compared to periods of market downturn, in contrast to the in-sample period from January 1927 to December 1999. Thus, condition 1, in which upper and lower partial moments are greater than the in-sample reference points, is fulfilled more frequently during this period.

[Insert Table 9 about here]

Table 9 presents a comparison of the performances of six PMM strategies and the benchmark strategy over the whole out-of-sample period and the market downturn period. Four out of six PMM strategies outperform the benchmark strategy during the market downturn period. In particular, PMM strategy 4 (*PMM_S4*) earns significant positive returns during the period of market downturn. In addition to Table 4a, these results provide further evidence that PMM strategies outperform plain momentum during financial turbulence. However, even though PMM strategy 4 doubles both the Sharpe ratio and adapted Sortino ratio of the plain momentum strategy during the whole out-of-sample period, it does not generate higher-than-benchmark returns. In this instance, the plain momentum strategy has higher returns. However, *PMM_S4* has much better risk-adjusted returns as evidenced by the higher Sharpe ratio and adapted Sortino ratio. Furthermore, we can improve our PMM strategies by updating our reference points sequentially rather than fixing them at the start of the out-of-sample period.

By using the estimated coefficients for the in-sample periods from Table 7, we generate a dynamic out-of-sample PMD strategy. During the out-of-sample periods, we employ estimated partial moments using formulas (14) and (15) in the models developed in Section 4. The estimated model is

$$RPM_t^+ = 0.00041 + 0.45557 * RPM_{t-1}^+ + 0.17777 * RPM_{t-1}^- \quad (12)$$

$$RPM_t^- = 0.00052 + 0.28695 * RPM_{t-1}^+ + 0.24295 * RPM_{t-1}^- \quad (13)$$

While the results presented in Table 7 show that the partial moments are forecastable, we find the returns of two PMD strategies based on these forecasts do not improve. This is possibly due to estimation errors swamping any forecasting benefits. However, two PMD strategies more than doubled the Sharpe ratio and adapted Sortino ratio of the plain momentum strategy during both sample periods.

[Insert Table 10 about here]

Furthermore, to avoid the look-ahead bias issue, we use the 20-year rolling window medians of RPM_t^+ and RPM_t^- for the period [*t-240, t-1*], rather than the medians for the whole sample period as reference points to construct PMM strategies[17]. Table 10 presents the performances of six PMM strategies using rolling window medians as reference points. Compared to Table 4a, results show robustness as five out of six PMM strategies generate both higher holding returns and risk-adjusted returns during market downturns.

6. Robustness check and performance comparison

Following Jegadeesh and Titman (1993), we also use a *6×6* strategy as a robustness check since this strategy is the best-performing strategy among four plain momentum strategies during all four periods by having the highest returns and Sharpe ratios (see Table B.1).

6.1 Partial moments-based momentum strategies on a *6×6* basis

We follow the switching rules presented in Table 3 and construct six PMM strategies on a *6×6* basis for the whole sample period (January 1927 to December 2016). Results are presented in Table 10.

[17] As robustness checks, all analyses are repeated using rolling window medians of RPM_t^+ and RPM_t^- on two different bases: a 10-year rolling window during the period [*t-120, t-1*] and a whole sample rolling window during the period [*i, t-1*], where *i* represents January 1927, the first month of our whole sample period. Both methods show similar results.

[Insert Table 11 about here]

From Table 11, all six PMM strategies outperform the benchmark strategy (*M66*) for both periods: P3, the market downturn and P4, the era of turbulence. In particular, PMM strategy 4 (*PMM_S4*) on a *6×6* basis earns an annualized return of 9.23%, which is significant at the 1% level during P4, the era of turbulence, while the benchmark strategy causes plain momentum investors to lose approximately 7.63 basis points a month. Interestingly, four out of six PMM strategies on a *6×6* basis outperform the benchmark strategy during the whole sample period, 1927-2016, and show better performances than the PMM strategies on an *11×1* basis (see Table 4a). Nevertheless, the results, especially the risk-adjusted ones, in Table 10 show the effectiveness of our PMM strategies. The mechanism of this profitability, however, requires further investigation.

We then conduct two PMD strategies, with and without the 200% leverage condition, on a *6×6* basis, also shown in Table 11. Like the results in Table 5, both unconstrained and leverage-constrained PMD strategies on a WML basis consistently outperform the plain momentum strategy (*WML*) in all sample periods, including periods of financial turbulence (P3 and P4).

The robustness checks reveal that our partial moments-based momentum strategies are robust across strategy construction and multiple time periods. While we do not claim that investors could earn these returns in practice, we suggest that partial moments-based strategies seem a good way to manage momentum risk.

6.2 Comparison between partial moment-decomposed strategies and Barroso and Santa-Clara (2015) volatility-scaled momentum strategy

[Insert Table 12 about here]

Table 12 presents a comparison of the performance of two PMD strategies and the volatility-scaled momentum strategy constructed by Barroso and Santa-Clara (2015). We construct a 12% constant volatility-scaled momentum portfolio consistent with the risk-managed momentum strategy in Barroso and Santa-Clara (2015). The authors term their volatility-scaled momentum strategy the "risk-managed momentum" [see Barroso and Santa-Clara (2015, p. 115, Section 4)]. In particular, we also construct a Barroso and Santa-Clara (2015) volatility-scaled momentum portfolio with a 200% leverage constraint (*BSC_C*)[18]. The results reveal that our two PMD strategies outperform the Barroso and Santa-Clara (2015) volatility-scaled momentum strategy for all four sample periods, including periods of financial booms (P2) and financial turbulence (P3 and P4). In particular, our unconstrained PMD strategy (*PMD*) earns an annualized return of 13.78%, which is significant at the 1% level, with an annualized adapted Sortino ratio of 1.00 during the era of turbulence, compared to an annualized return of 6.39%, which is significant at the 10% level, with an annualized adapted Sortino ratio of 0.22 for the Barroso and Santa-Clara (2015) volatility-scaled momentum strategy during the same period. This suggests our PMD strategy might be useful as an alternative tool in the management of momentum risk, in particular, the downside risk during market downturns.

In conclusion, we find that our extended PMD momentum strategy tilted long or short towards favorable/unfavorable volatility signals is more efficient and profitable than a constant volatility strategy, during both good and bad times.

7. Conclusion

We have demonstrated good performance for cross-sectional momentum strategies, using information in past partial moments. We investigate two types: first, a portfolio choice based on the region in which the upper and lower partial moments lie; second, a portfolio choice based on partial moment scaling whereby we load up on long positions if upper partial moments are forecast to be relatively large or we load up on short positions when lower partial moments are forecast to be relatively large.

[18] Apart from very few cases, we also observe that the weights of the Barroso and Santa-Clara (2015) volatility-scaled momentum strategy do not exceed a value of 2 over our whole sample period. Thus, the *BSC_C* strategy shows almost identical performances as those of *BSC*. For a detailed distribution of the weights, see Barroso and Santa-Clara (2015, p. 116, Figure 4).

Our approach, relative to what a fund manager might do, is conservative in the sense of using minimal calculation. While we use CRSP data, which involve many stocks too small or illiquid to be considered by a fund of reasonable size, we do not update our models, estimating them once based on in-sample data and using them, with fixed parameters, when we carry out our out-of-sample model evaluation.

We leave open the question of whether partial moments are fully or partially priced by markets. Linking partial moments to expected utility has been examined by numerous authors; see Fishburn (1977) and references therein.

The success of our strategies occurs because of the forecastability of partial moments (see Table 3). This immediately raises the question whether this evidences market inefficiency? We can best answer this by quoting Timmermann and Granger (2004, p. 25):

> "There is now substantial evidence that volatility of asset returns varies over time in a way that can be partially predicted. For this reason, there has been considerable interest in improved volatility forecasting models in the context of option pricing; see, for example, Engle, Hong, Kane, and Noh (1993). Does this violate market efficiency? Clearly the answer is no unless a trading strategy could be designed that would use this information in the options markets to identify under- and over-valued options."

While we note possible subtleties in that options are priced directly off volatility, we also note that partial moments are barely traded at all. Thus, it seems, to us at least, that the presentation of a trading strategy that generates significant returns by forecasting partial moments is an argument against market efficiency. The source of this inefficiency, whether it be behavioral or predictable risk premia or investors' unwillingness to embrace a more challenging measure of risk, such as partial moments, awaits further research.

Overall, we have derived strategies that both complement and extend recent literature that uses volatility measures to enhance a cross-sectional momentum. Extensions of our analysis include applications of our procedures to other momentum strategies, including time-series momentum and relative strength strategies. Further applications such as applying the techniques to other asset classes can also be implemented.

Appendix A. Performance ratios and some theoretical consideration

A.1 Practitioner's version of Sharpe ratio

Suppose that w_i is the weight of stock *i* in the portfolio, $\sum_{1=1}^{n} w_i = 0$ and $\sum_{1=1}^{n} \mu_i w_i = \mu_p$. If the Capital Asset Pricing Model (CAPM) for long-only portfolios $\mu_i - r_f = \beta_i(\mu_m - r_f)$ holds for stock *i*, then the CAPM for the net zero portfolio is $\mu_p = \beta_p(\mu_m - r_f)$, where $\beta_p = \sum_{1=1}^{n} \beta_i w_i$. Thus, $\mu_p - r_f = \beta_p(\mu_m - r_f) - r_f$ does not seem correct.

Alternatively, suppose we view the Sharpe ratio as 100% cash plus a long–short portfolio. Then, the $Sharpe\ ratio = \frac{\mu_L - \mu_S}{sd(LS)}$ as the riskless rates cancel out, which is consistent with the CAPM for long–short portfolios, $\mu_p = \mu_L - \mu_S$.

A.2 Adapted Sortino ratio

We define the adapted Sortino ratio as

$$Adapted\ Sortino\ ratio = \frac{Excess\ Return}{2 * Downside\ SemiDeviation} \tag{14}$$

where

$$Excess\ Return_t = R_t - Desired\ Target\ Return_t \tag{15}$$

Sortino and Price (1994), for example, define sample downside semi-deviation as[19]

$$Downside\ SemiDeviation = \sqrt{\frac{\sum_{i=1}^{N}(Excess\ Return_i - \overline{Excess\ Return})^2}{N}\ I(Excess\ Return_i < 0)} \tag{16}$$

[19] See, in particular Sortino and Price (1994, p. 61, third paragraph). Sortino and Price (1994) refer to this "downside risk" measure as downside deviation, but we consider downside semi-deviation to be more appropriate.

where $Excess\ Return$ is the portfolio's excess return on the $Desired\ Target\ Return$, for which we use a 1-month Treasury bill. $I()$ is the indicator function, this being a sample target excess return lower than zero. Our adaptation differs from the standard Sortino ratio with target return equal to the riskless rate as, in the event that downside and upside standard deviations are equal, we recover the Sharpe ratio. Thus, we place our version of the Sortino ratio into a broadly similar scale to the Sharpe ratio.

A.3 Momentum forecastability

Under the assumption that r_t is $iid\ (\mu, \sigma^2)$, it follows by elementary calculation that

$$Var(R_t) = E(r_t^2 r_{t-1}^2) - E^2(r_t\, r_{t-1}) \tag{17}$$

and

$$Cov(R_t, R_{t-1}) = E(r_t\, r_{t-1}^2 r_{t-2}) - E(r_t\, r_{t-1})E(r_{t-1}\, r_{t-2}) \tag{18}$$

This leads to an autocorrelation coefficient as $\rho = \frac{\mu^2\sigma^2}{\sigma^4+2\mu^2\sigma^2}$. If we interpret the signal-to-noise ratio of the strategy as $SN = \frac{\mu}{\sigma}$, then $\rho = \frac{SN^2}{1+2SN^2}$. Thus, if the strategy has a signal–to-noise ratio of 0.5, then the returns will appear to have an autocorrelation coefficient of 0.17 although the underlying data are pure white noise.

Furthermore, as SN becomes large, we reach an upper bound for ρ of ½. Assuming that $E(r_t) = 0$, we now turn to

$$Var(R_t^2) = E(r_t^4 r_{t-1}^4) - E^2(r_t^2 r_{t-1}^2) \tag{19}$$

and

$$Cov(R_t^2, R_{t-1}^2) = E(r_t^2 r_{t-1}^4 r_{t-2}^2) - E\left((r_t^2 r_{t-1}^2)E\left((r_{t-2}^2 r_{t-1}^2)\right)\right) \tag{20}$$

Let $\mu_j = E(r_t^j)$. Then $\rho = \frac{1}{\frac{\mu_4}{\mu_2^2}+1}$.

This shows that when underlying returns are white noise, the lower the forecastability of momentum volatility, the higher the kurtosis. If we take this to be the kurtosis of semi-annual index returns, we might expect a number near 5, and the (spurious) autocorrelation might thus be of a similar magnitude to that seen before.

Appendix B. Other four plain momentum strategies

[Insert Table B.1 about here]

Table B.1 shows performances of four plain momentum strategies, *3×3*, *6×6*, *9×9*, and *12×12*, during the four sample periods. During P1 and P2, the *6×6* strategy, which represents a momentum strategy, has 6-month formation (*J*) and holding (*K*) periods with a 1-month gap between *J* and *K*, can earn 6.23% and 12.03% annualized returns, respectively. It is the best-performing plain momentum strategy during these two periods by having the highest Sharpe ratios.[20] The other three strategies also report positive returns during P1 and P2. However, none of these four plain strategies can generate profits during P3, the market downturn, or P4, the era of turbulence.

We find that long-term momentum strategies, such as *9×9* and *12×12*, report negative annualized returns of around 30% right after March 2009 when the market reaches its lowest level. This is possibly because the market declined persistently before it reached the bottom, and longer-term momentum strategies with this negative information are more difficult to turn around.

Appendix C. Some further analyses

C.1 Vector-autoregressive process

We assume, for simplicity, that monthly partial moments satisfy a $VAR(1)$ given by regressions (21) and (22)

$$RPM_t^+ = \alpha_1 + \beta_{11} RPM_{t-1}^+ + \beta_{12} RPM_{t-1}^- + \varepsilon_{1t} \tag{21}$$

[20] Previous findings also reveal this phenomenon. See, for example, Jegadeesh and Titman (1993, 2001).

$$RPM_t^- = \alpha_2 + \beta_{21} RPM_{t-1}^+ + \beta_{22} RPM_{t-1}^- + \varepsilon_{2t} \tag{22}$$

Suppose that the conditional properties of μ_{2t}^+ and μ_{2t}^-, the upper and lower partial moments of degree 2, respectively, can be described by the following equations:

$$\mu_{2t}^+ = \alpha_1 + \beta_{11} RPM_{t-1}^+ + \beta_{12} RPM_{t-1}^- \tag{23}$$

$$\mu_{2t}^- = \alpha_2 + \beta_{21} RPM_{t-1}^+ + \beta_{22} RPM_{t-1}^- \tag{24}$$

Formulas (23) and (24) can be interpreted as a matrix analogue of an $ARCH(1)$ model except that it is a model for conditional partial moments. Replacing population moments by sample counterparts together with the errors involved yields regressions (21) and (22).

Such a model has certain features we can exploit for analytic purposes. We can compute the conditional and unconditional means of the partial moments and use potentially better forecasts. Writing regressions (21) and (22) in terms of vectors and matrixes yields

$$RPM_t = \alpha + \beta RPM_{t-1} + \varepsilon_t \tag{25}$$

Then, $E(RPM_t) = (I - \beta)^{-1} \alpha$ is the unconditional mean, and the one-period-ahead forecast for time *t*+1 at time *t* is given by $\alpha + \beta RPM_t$. The stationarity condition is that all the roots of β are less than one in absolute value.[21] This is satisfied in all cases in this study, based on the estimated β.

C.2 Distribution of days of positive and negative market returns per month

[Insert Table C.2 about here]

Monthly upper (lower) partial moment is the sum of daily market positive (negative) realized semi-variances. We compute the joint distribution of days of positive and negative market returns per month for the period 1927-2016. Results in Table C.2 imply that the distribution is close to a bivariate normalized one as, in most of the months, the number of days when the market return is

[21] We run a standard unit root test on our vector autoregression (VAR) process given by regressions (21) and (22). We find that the VAR process is stationary.

positive and negative range between 6 and 13. This confirms that our definitions on the monthly upper and lower partial moments are appropriate.

C.3 Conditional decomposed returns of PMM strategies on an *11×1* basis during the out-of-sample period

[Insert Table C.3 about here]

As shown in Table C.3, we compute the decomposed returns of six PMM strategies for four conditions in the out-of-sample period. Similar to the results in Table 4b, returns generated when condition 1 applies are much lower than expected. This relatively weak performance could result from the choice of sub-sample periods, as markets are more volatile in the out-of-sample period than in the in-sample period.

C.4 Out-of-sample partial moments-based momentum strategies on a *6×6* basis

The partial moments-based momentum strategies are to change positions or to scale weights of plain momentum strategies. In addition to the original benchmark set in Section 2 as an *11×1* plain momentum strategy with a 1-month gap between the formation and holding periods, we repeat all analyses using a *6×6* plain momentum strategy with a 1-month gap as the benchmark.

[Insert Table C.4 about here]

We further test the *6×6*-based PMM and PMD strategies on out-of-sample analyses, as in Section 5 (see Table C.4). These results show that all six *6×6*-based PMM strategies outperform the plain WML strategy during Pb, the market downturn period, and four out of six PMM strategies show better performance during Pa, the whole out-of-sample period. Results also provide further evidence that PMD strategies outperform plain momentum during financial turbulence.

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Figure 1a Histogram of upper partial moments with kernel density curves

Figure 1a shows the histogram of upper partial moments (RPM_t^+) with kernel density curves. The values of median and maximum observations of RPM_t^+, skewness, and kurtosis are reported. The normal kernel estimate for c = 0.7852 has a bandwidth of 0.0002 and an approximate mean integrated square error of 2.1726. Since the distribution of upper partial moments has a fat tail, we combine all upper partial moments with values between the 95th percentile (0.00420) and the maximum observation (0.02765). The vertical line represents the median.

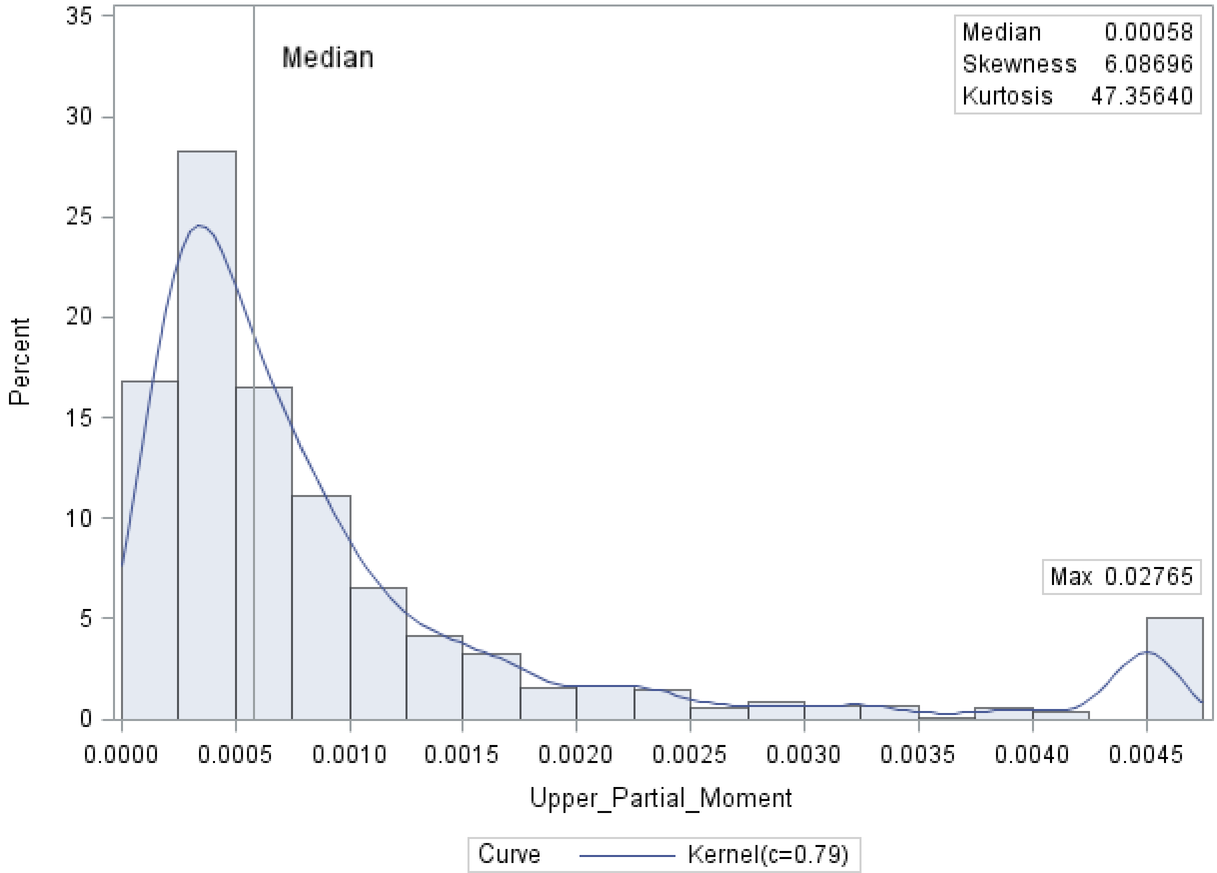

Figure 1b Histogram of lower partial moments with kernel density curves

Figure 1b shows the histogram of lower partial moments (RPM_t^-) with kernel density curves. The values of median and maximum observations of RPM_t^-, skewness, and kurtosis are reported. The normal kernel estimate for c = 0.7852 has a bandwidth of 0.0002 and an approximate mean integrated square error of 1.8252. Since the distribution of lower partial moments has a fat tail, we combine all lower partial moments with values between the 95th percentile (0.00436) and the maximum observation (0.04299). The vertical line represents the median.

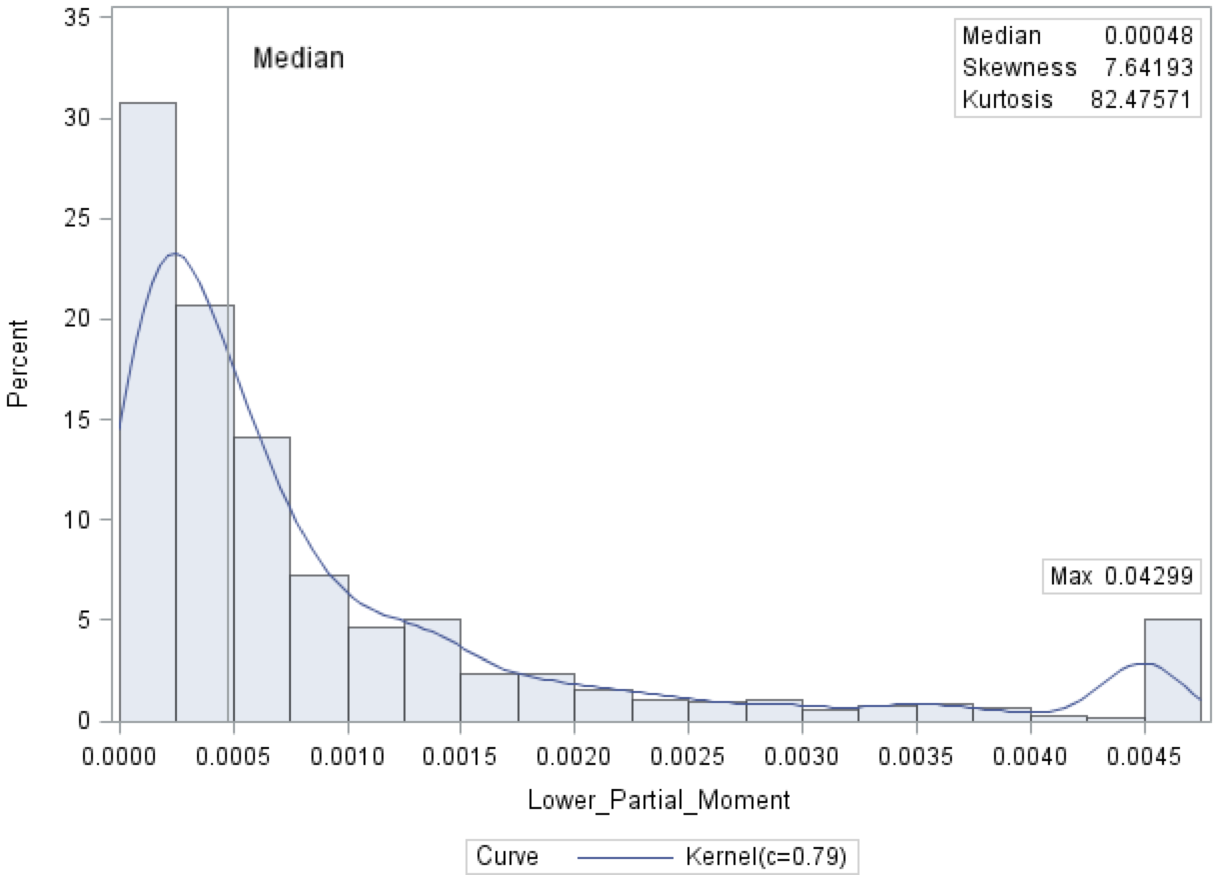

Figure 2a PMM strategy conditions in the coordinate plane

Figure 2a illustrates the four PMM strategy conditions in the coordinate plane. The origin represents the reference points for both the upper partial moment, RPM_t^+, and the lower partial moment, RPM_t^-, in month *t*. Each quadrant represents a PMM condition based on the upper and lower partial moments and their reference points. For example, if in month *t*, both the upper and lower partial moments are higher than their reference points, this combination represents an environment of high market volatility, which is not conducive to momentum trading profits. Then, condition 1 applies in month *t* and the corresponding trading methods for condition 1 are employed as shown on Panel B of Table 3.

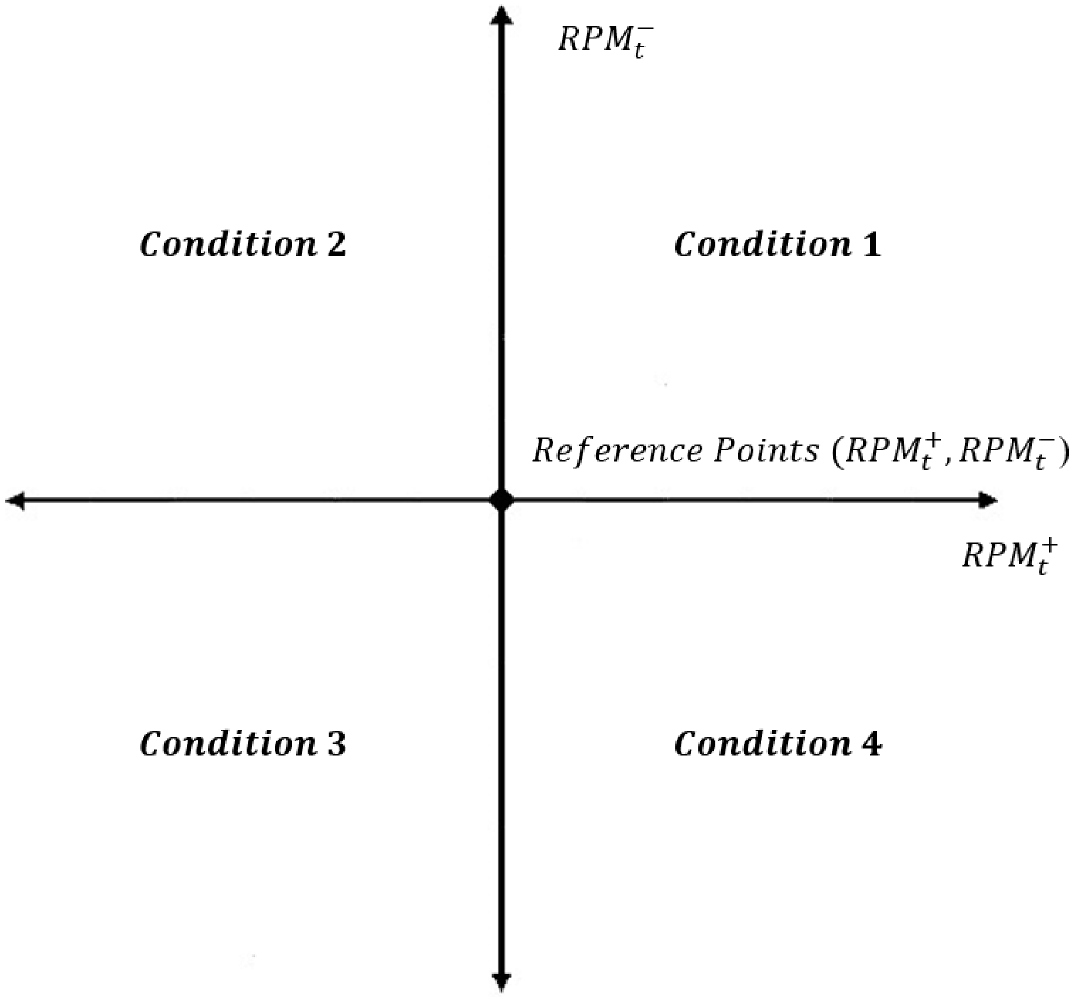

Figure 2b PMM strategy conditions for 1927-2016

Figure 2b reports the numbers and percentages (in parenthesis) of four PMM strategy conditions for the whole sample period when the medians of RPM_t^+ and RPM_t^- are used as reference points for RPM_t^+ and RPM_t^-, respectively. The numbers within parentheses indicate the actual possibility of occurrence of each of these four PMM strategy conditions.

Condition	Possibility of Occurrence
1	377 (34.91)
2	163 (15.09)
3	377 (34.91)
4	163 (15.09)
Total	1080 (100.00)

Figure 3 Timeline of an *11×1* partial moment momentum strategy

Figure 3 demonstrates the timeline of an *11×1* partial moment momentum (PMM) strategy constructed at time *t*. In any month *t*, all sample stocks are ranked and sorted into deciles based on their past 11-month formation period returns from month *t-11* to *t-1* (owing to a 1-month gap, month *t* to *t+1,* between the formation and holding periods for returns). Then, we classify holding strategies into four conditions based on partial moments determined in the period month *t-1* to *t* and their reference points analyzed in Subsection 3.2. Then, during the 1-month holding period from month *t* to *t+1,* we compare monthly upper and lower partial moments (RPM_t^+ and RPM_t^-) during the period [*t-1, t*] with their reference points and switch positions of winners, losers, and the cash assets to keep a zero net position based on Panel A of Table 3. In other words, for any of these six PMM strategies, we keep rebalancing and switching positions of all assets held.

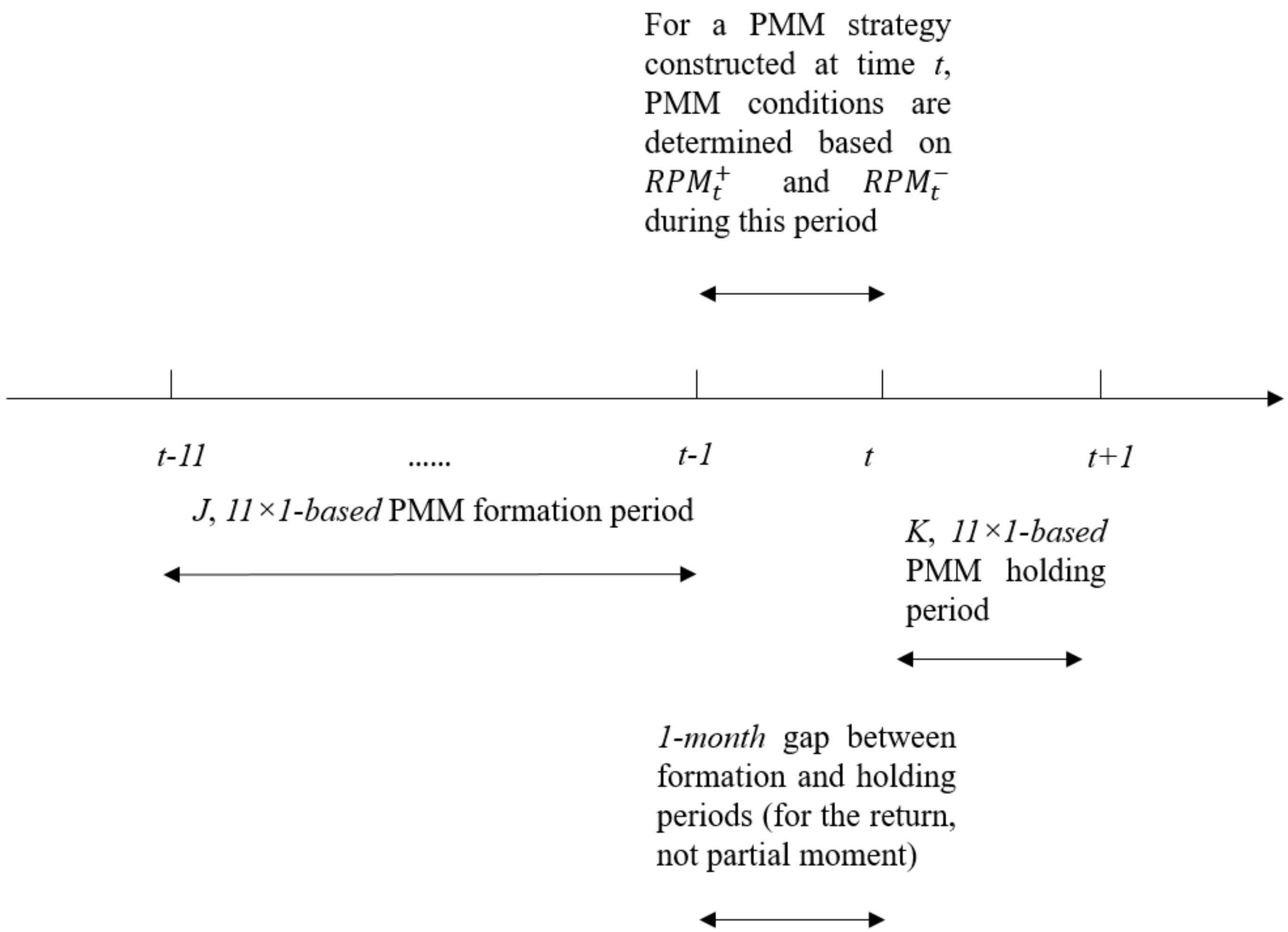

Table 1 Performances of plain momentum strategies in the US equity markets

Table 1 shows the performance of the plain momentum strategies on a WML basis (*11×1*) for the four sample periods. The Return column reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. WML represents the *11×1* plain momentum strategies with 1-month gap between formation and holding periods, respectively. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
Panel A: P1, whole sample period: January 1927 to December 2016				
WML	15.65	4.96 (***)	0.52	0.20
Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989				
WML	21.60	5.12 (***)	1.02	0.40
Panel C: P3, market downturn: August 2007 to December 2012				
WML	5.65	0.33	0.14	0.06
Panel D: P4, era of turbulence: January 2000 to December 2016				
WML	4.83	0.60	0.15	0.05

Table 2a Distributions of partial moments in the US equity market

Table 2a reports the distributions of monthly realized variance (RV_t) and upper and lower partial moments (RPM_t^+ and RPM_t^-) for the whole sample period from January 1927 to December 2016. 10th Percentile, 25th Percentile, 75th Percentile, and 90th Percentile represent the 10th, 25th, 75th, and 90th percentiles of each variable. The median, mean, max, standard deviation, skewness, and kurtosis are also reported. Amounts in brackets represent annual standard deviation equivalent (in percentages) of monthly realized variance/partial moments. For example, RV_t with a mean average of 24.25 corresponds to an annual standard deviation of 17.06%. Values of all percentiles, medians, means, and standard deviations are in 0.0001.

Variable	10th Percentile	25th Percentile	Median	75th Percentile	90th Percentile	Mean	Max	Standard Deviation	Skewness	Kurtosis
RV_t	3.84 (6.79)	6.28 (8.68)	10.46 (11.20)	21.07 (15.90)	51.07 (24.75)	24.25 (17.06)	548.27 (81.11)	47.66	6.19	50.80
RPM_t^+	1.87 (4.73)	3.11 (6.11)	5.80 (8.34)	10.85 (11.41)	22.97 (16.60)	12.12 (12.06)	276.48 (57.60)	24.02	6.09	47.36
RPM_t^-	0.87 (3.22)	2.02 (4.92)	4.75 (7.55)	11.23 (11.61)	25.12 (17.36)	12.13 (12.06)	429.88 (71.82)	27.84	7.64	82.48

Table 2b Joint distribution of partial moments

Table 2b presents the joint distribution of the upper and lower market partial moments (RPM_t^+ and RPM_t^-) quartiles throughout four sample periods. $P(RPM_t^{+\,(i)}, RPM_t^{-\,(j)})$ is the joint probability of upper and lower market partial moments, where $i, j = 1,2,3,4$. i and j stand for the four quartiles of the upper and lower partial moments from the lowest quartile to the highest quartile, respectively. Amounts reported represent the probability for each pair of $P(RPM_t^{+\,(i)}, RPM_t^{-\,(j)})$ during the sample period. All figures are in percentage. For example, during P1, the whole sample period, $P(RPM_t^{+\,(i)}, RPM_t^{-\,(j)})$ with an amount of 7.87 represents a 7.87% probability that in any month *t* during the whole sample period, the upper market partial moment (RPM_t^+) falls in the lowest quartile ($RPM_t^{+\,(1)}$) and the lower market partial moment (RPM_t^-) falls in the second lowest quartile ($RPM_t^{-\,(2)}$), respectively.

Panel A: P1, whole sample period: January 1927 to December 2016					*Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989*				
$P(RPM_t^{+\,(i)}, RPM_t^{-\,(j)})$	$RPM_t^{+\,(1)}$	$RPM_t^{+\,(2)}$	$RPM_t^{+\,(3)}$	$RPM_t^{+\,(4)}$	$P(RPM_t^{+\,(i)}, RPM_t^{-\,(j)})$	$RPM_t^{+\,(1)}$	$RPM_t^{+\,(2)}$	$RPM_t^{+\,(3)}$	$RPM_t^{+\,(4)}$
$RPM_t^{-\,(1)}$	10.28	8.61	5.28	0.83	$RPM_t^{-\,(1)}$	8.33	6.00	7.67	3.00
$RPM_t^{-\,(2)}$	7.87	8.15	6.57	2.41	$RPM_t^{-\,(2)}$	8.33	7.00	6.67	3.00
$RPM_t^{-\,(3)}$	5.93	6.11	7.50	5.46	$RPM_t^{-\,(3)}$	6.33	8.33	4.00	6.33
$RPM_t^{-\,(4)}$	0.93	2.13	5.65	16.30	$RPM_t^{-\,(4)}$	2.00	3.67	6.67	12.67
Panel C: P3, market downturn: August 2007 to December 2012					*Panel D: P4, era of turbulence: January 2000 to December 2016*				
$P(RPM_t^{+\,(i)}, RPM_t^{-\,(j)})$	$RPM_t^{+\,(1)}$	$RPM_t^{+\,(2)}$	$RPM_t^{+\,(3)}$	$RPM_t^{+\,(4)}$	$P(RPM_t^{+\,(i)}, RPM_t^{-\,(j)})$	$RPM_t^{+\,(1)}$	$RPM_t^{+\,(2)}$	$RPM_t^{+\,(3)}$	$RPM_t^{+\,(4)}$
$RPM_t^{-\,(1)}$	14.22	7.84	2.45	0.49	$RPM_t^{-\,(1)}$	15.15	7.58	3.03	0.00
$RPM_t^{-\,(2)}$	8.82	8.33	6.37	1.47	$RPM_t^{-\,(2)}$	6.06	10.61	6.06	1.52
$RPM_t^{-\,(3)}$	1.96	7.35	11.27	4.41	$RPM_t^{-\,(3)}$	4.55	6.06	10.61	4.55
$RPM_t^{-\,(4)}$	0.00	1.47	4.90	18.63	$RPM_t^{-\,(4)}$	0.00	0.00	6.06	18.18

Table 2c Correlation between partial moments and the skewness of plain momentum profits

Table 2c shows the correlation between the upper and lower market partial moments (RPM_t^+ and RPM_t^-) and the skewness of returns of momentum (winner-minus-loser), winner, and loser portfolios ($Skew\_mom_t$, $Skew\_win_t$, and $Skew\_los_t$) throughout four sample periods, respectively. $Skew\_mom_t$, $Skew\_win_t$, and $Skew\_los_t$ report the skewness of plain momentum, winner, and loser portfolios on a WML basis (*11×1*) constructed on each trading day in any month *t*, respectively. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Panel A: P1, whole sample period: January 1927 to December 2016

	RPM_t^+	RPM_t^-	$Skew\_mom_t$	$Skew\_win_t$	$Skew\_los_t$
RPM_t^+	1.00				
RPM_t^-	0.69 (***)	1.00			
$Skew\_mom_t$	-0.05 (*)	-0.03	1.00		
$Skew\_win_t$	0.18 (***)	-0.01	0.24 (***)	1.00	
$Skew\_los_t$	0.17 (***)	-0.02	-0.28 (***)	0.47 (***)	1.00

Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989

	RPM_t^+	RPM_t^-	$Skew\_mom_t$	$Skew\_win_t$	$Skew\_los_t$
RPM_t^+	1.00				
RPM_t^-	0.71 (***)	1.00			
$Skew\_mom_t$	-0.12 (**)	-0.00	1.00		
$Skew\_win_t$	0.13 (**)	-0.09	0.24 (***)	1.00	
$Skew\_los_t$	0.09	-0.10 (*)	-0.24 (***)	0.45 (***)	1.00

Panel C: P3, market downturn: August 2007 to December 2012

	RPM_t^+	RPM_t^-	$Skew\_mom_t$	$Skew\_win_t$	$Skew\_los_t$
RPM_t^+	1.00				
RPM_t^-	0.92 (***)	1.00			
$Skew\_mom_t$	-0.13	-0.12	1.00		
$Skew\_win_t$	0.10	0.01	0.06	1.00	
$Skew\_los_t$	0.10	0.06	-0.29 (**)	0.54 (***)	1.00

Panel D: P4, era of turbulence: January 2000 to December 2016

	RPM_t^+	RPM_t^-	$Skew\_mom_t$	$Skew\_win_t$	$Skew\_los_t$
RPM_t^+	1.00				
RPM_t^-	0.90 (***)	1.00			
$Skew\_mom_t$	-0.01	-0.04	1.00		
$Skew\_win_t$	0.14 (**)	0.07	0.14 (**)	1.00	
$Skew\_los_t$	0.08	0.04	-0.33 (***)	0.43 (***)	1.00

Table 3 Partial moment momentum strategy construction

Table 3 shows how partial moment momentum (PMM) strategies are constructed. Panel A presents four conditions of the upper and lower partial moments based on their reference points: medians of RPM_t^+ and RPM_t^-. In any given month t, we classify holding strategies into four conditions based on partial moments in the period [*t-1, t*] (there is a 1-month gap between the formation and holding periods for returns) and their reference points analyzed in Subsection 3.2. Correspondingly, switching methods during holding periods under each of these four conditions are presented. Panel B illustrates the actions and returns for different holding periods on each of the four conditions presented in Panel A of our six PMM strategies, which we call PMM strategies 1 to 6, represented by *PMM_S1* to *PMM_S6*. In particular, all six PMM strategies switch to the same positions if conditions 2 or 3 occur, but act differently if conditions 1 or 4 occur. Moreover, we hold cash long or short to keep our PMM strategies in net zero positions. $r_{w,t+1}$, $r_{l,t+1}$, $r_{f,t+1}$ represent the returns of winners, losers, and risk-free assets in month *t+1*, respectively. For example, for PMM strategy 4 (*PMM_S4*), if condition 1 applies during the period [*t-1, t*], in which the upper and lower partial moments are all higher than their reference points, then we close out our positions in both winners and losers. The PMM return for month *t+1* is 0. If condition 2 applies during the period [*t-1, t*], in which the upper partial moment is lower than its reference point and the lower partial moment is higher than its reference point, then we short losers only, liquidating our long positions and holding cash long. The PMM return for month *t+1* is $r_{f,t+1} - r_{l,t+1}$. If condition 3 applies during the period [*t-1, t*], in which upper and lower partial moments are all lower than their reference points, then we carry on with the momentum strategy by buying winners and short selling losers. The PMM return for month *t+1* is $r_{w,t+1} - r_{l,t+1}$. If condition 4 applies during the period [*t-1, t*], in which the upper partial moment is higher than its reference point and the lower partial moment is lower than its reference point, then we buy winners only and short cash. The PMM return for month *t+1* is $r_{w,t+1} - r_{f,t+1}$.

Panel A: Four conditions of partial moments based on their reference points

Condition 1	If	$RPM_t^+ \geq Reference\ point\ (RPM_t^+)$	and if	$RPM_t^- \geq Reference\ point\ (RPM_t^-)$	then	Method 1.1: Close out Method 1.2: Go contrarian Method 1.3: Short losers only
Condition 2		$RPM_t^+ < Reference\ point\ (RPM_t^+)$		$RPM_t^- \geq Reference\ point\ (RPM_t^-)$		Method 2: Short losers only
Condition 3		$RPM_t^+ < Reference\ point\ (RPM_t^+)$		$RPM_t^- < Reference\ point\ (RPM_t^-)$		Method 3: Go momentum
Condition 4		$RPM_t^+ \geq Reference\ point\ (RPM_t^+)$		$RPM_t^- < Reference\ point\ (RPM_t^-)$		Method 4.1: Go momentum Method 4.2: Buy winners only

Panel B: Methods and returns of six PMM strategies (in any given month t, conditions are classified based on partial moments in the previous month t-1 due to a 1-month gap between the formation and holding periods; $\boldsymbol{r_{w,t+1}}$, $\boldsymbol{r_{l,t+1}}$, $\boldsymbol{r_{f,t+1}}$ *represent returns of winners, losers, and the risk-free assets in month t+1, respectively. Correspondingly, returns of momentum and contrarian strategies are* $\boldsymbol{r_{w,t+1}}$*-* $\boldsymbol{r_{l,t+1}}$ *and* $\boldsymbol{r_{l,t+1}}$*-* $\boldsymbol{r_{w,t+1}}$*, respectively.)*

PMM Strategies	Condition 1		Condition 2		Condition 3		Condition 4	
	Method	Return	Method	Return	Method	Return	Method	Return
PMM_S1	1.1	0	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.1	$r_{w,t+1} - r_{l,t+1}$
PMM_S2	1.2	$r_{l,t+1} - r_{w,t+1}$	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.1	$r_{w,t+1} - r_{l,t+1}$
PMM_S3	1.3	$r_{f,t+1} - r_{l,t+1}$	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.1	$r_{w,t+1} - r_{l,t+1}$
PMM_S4	1.1	0	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.2	$r_{w,t+1} - r_{f,t+1}$
PMM_S5	1.2	$r_{l,t+1} - r_{w,t+1}$	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.2	$r_{w,t+1} - r_{f,t+1}$
PMM_S6	1.3	$r_{f,t+1} - r_{l,t+1}$	2	$r_{f,t+1} - r_{l,t+1}$	3	$r_{w,t+1} - r_{l,t+1}$	4.2	$r_{w,t+1} - r_{f,t+1}$

Table 4a Performance of partial moment momentum strategies

Table 4a shows the performance of partial moment momentum (PMM) strategies on a WML basis (*11×1*) for four sample periods. Partial moment reference points are computed for the whole sample period P1. WML represents an *11×1* plain momentum strategy with a 1-month gap between the formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies on a WML basis and are constructed according to the switching rules presented in Table 3. Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
Panel A: P1, whole sample period: January 1927 to December 2016				
WML	15.65	4.96 (***)	0.52	0.20
PMM_S1	10.02	6.14 (***)	0.65	0.31
PMM_S2	8.23	2.58 (**)	0.27	0.13
PMM_S3	7.73	2.00 (**)	0.21	0.06
PMM_S4	10.15	6.24 (***)	0.66	0.32
PMM_S5	8.37	2.62 (***)	0.28	0.13
PMM_S6	7.87	2.04 (**)	0.21	0.06
Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989				
WML	21.60	5.12 (***)	1.02	0.40
PMM_S1	14.75	4.46 (***)	0.89	0.33
PMM_S2	10.80	2.49 (**)	0.50	0.13
PMM_S3	13.77	2.94 (***)	0.59	0.18
PMM_S4	13.06	3.92 (***)	0.78	0.26
PMM_S5	9.16	2.12 (**)	0.42	0.07
PMM_S6	12.08	2.58 (**)	0.52	0.14
Panel C: P3, market downturn: August 2007 to December 2012				
WML	5.65	0.33	0.14	0.06
PMM_S1	2.47	0.77	0.33	0.17
PMM_S2	0.37	0.02	0.01	-0.01
PMM_S3	4.68	0.24	0.10	0.05
PMM_S4	4.71	1.34	0.57	0.33
PMM_S5	2.56	0.15	0.06	0.05
PMM_S6	6.96	0.35	0.15	0.07
Panel D: P4, era of turbulence: January 2000 to December 2016				
WML	4.83	0.60	0.15	0.05
PMM_S1	2.66	0.88	0.21	0.06
PMM_S2	2.37	0.30	0.07	0.02
PMM_S3	0.67	0.08	0.02	-0.02
PMM_S4	3.86	1.37	0.33	0.16
PMM_S5	3.57	0.45	0.11	0.05
PMM_S6	1.85	0.21	0.05	0.00

Table 4b Conditional decomposed returns of PMM strategies

Table 4b presents the decomposed returns of six PMM strategies for the four conditions in the whole sample period. Panel A reports the actual possibility of occurrence of each of these four PMM strategy conditions, shown and discussed previously in Figure 2b. Return reports the annualized equivalent return and frequency-weighted return (in parenthesis) per each condition in percentage. The combined return is the sum of the frequency-weighted return of each of the four conditions.

	Condition 1		Condition 2		Condition 3		Condition 4		Combined
Panel A: Possibility of occurrence									
	34.91%		15.09%		34.91%		15.09%		100.00%
Panel B: Returns per condition									
WML	5.36	(1.87)	42.10	(6.36)	14.06	(4.90)	16.65	(2.51)	15.65
PMM_S1	0.00	(0.00)	17.23	(2.60)	14.06	(4.90)	16.65	(2.51)	10.02
PMM_S2	-5.11	(-1.79)	17.23	(2.60)	14.06	(4.90)	16.65	(2.51)	8.23
PMM_S3	-6.53	(-2.28)	17.23	(2.60)	14.06	(4.90)	16.65	(2.51)	7.73
PMM_S4	0.00	(0.00)	17.23	(2.60)	14.06	(4.90)	17.56	(2.65)	10.15
PMM_S5	-5.11	(-1.79)	17.23	(2.60)	14.06	(4.90)	17.56	(2.65)	8.37
PMM_S6	-6.53	(-2.28)	17.23	(2.60)	14.06	(4.90)	17.56	(2.65)	7.87

Table 5 Performance of partial moment-decomposed strategies

Table 5 shows the performance of two partial moment-decomposed (PMD) strategies on a WML basis (11 × 1) for four sample periods. WML represents an 11 × 1 plain momentum strategy with a 1-month gap between formation and holding periods. PMD and PMD_C represent an unconstrained PMD strategy and a 200% leverage-constrained PMD strategy, respectively. Both strategies are on a WML basis and are constructed according to the methods described in Section 4. Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
Panel A: P1, whole sample period: January 1927 to December 2016				
WML	15.65	4.96 (***)	0.52	0.20
PMD	26.13	24.39 (***)	2.57	2.27
PMD_C	19.94	24.69 (***)	2.60	1.93
Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989				
WML	21.60	5.12 (***)	1.02	0.40
PMD	31.66	14.83 (***)	2.97	2.65
PMD_C	23.31	15.84 (***)	3.17	2.00
Panel C: P3, market downturn: August 2007 to December 2012				
WML	5.65	0.33	0.14	0.06
PMD	13.84	4.35 (***)	1.86	1.08
PMD_C	13.15	4.28 (***)	1.82	1.03
Panel D: P4, era of turbulence: January 2000 to December 2016				
WML	4.83	0.60	0.15	0.05
PMD	13.78	6.68 (***)	1.62	1.00
PMD_C	12.25	6.65 (***)	1.61	1.00

Table 6 Comparison of performance of partial moments-based momentum strategies versus the plain momentum strategy

Table 6 compares the performance of two PMD strategies, the best and the worst PMM strategy with the plain momentum strategy for four sample periods. All momentum portfolios are constructed on a WML basis. WML represents an 11×1 plain momentum strategy with a 1-month gap between the formation and holding periods. PMD and PMD_C represent an unconstrained PMD strategy and a 200% leverage-constrained PMD strategy, respectively. Both strategies are constructed according to the methods described in Section 4. PMM_S4 and PMM_S3 represent the best- and worst-performing PMM strategies, respectively, during periods of market downturn (see Table 4a). Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. Jarque-Bera reports the statistics and p-values (in parenthesis; for instance, the reference point when p = 0.05 is 5.991.) of the Jarque-Bera normality test in Jarque and Bera (1987). The null hypothesis states that the variable is normally distributed. Skewness and kurtosis of strategy returns are also reported. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio	Skewness	Kurtosis	Jarque-Bera
P1: whole sample period: January 1927 to December 2016							
WML	15.65	4.96 (***)	0.52	0.20	-2.34	17.46	10394.75 (<0.001)
PMM_S3	7.73	2.00 (**)	0.21	0.06	-2.07	18.19	11159.07 (<0.001)
PMM_S4	10.15	6.24 (***)	0.66	0.32	0.53	3.70	73.24 (<0.001)
PMD	26.13	24.39 (***)	2.57	2.27	0.83	2.27	149.12 (<0.001)
PMD_C	19.94	24.69 (***)	2.60	1.93	0.54	2.02	94.80 (<0.001)
P2: Jegadeesh and Titman (1993): January 1965 to December 1989							
WML	21.60	5.12 (***)	1.02	0.40	-0.88	2.19	46.87 (<0.001)
PMM_S3	13.77	2.94 (***)	0.59	0.18	-0.51	3.09	13.07 (0.001)
PMM_S4	13.06	3.92 (***)	0.78	0.26	0.00	1.29	36.54 (<0.001)
PMD	31.66	14.83 (***)	2.97	2.65	0.99	1.98	61.65 (<0.001)
PMD_C	23.31	15.84 (***)	3.17	2.00	0.41	0.96	60.29 (<0.001)
P3: market downturn: August 2007 to December 2012							
WML	5.65	0.33	0.14	0.06	-1.91	5.69	60.06 (<0.001)
PMM_S3	4.68	0.24	0.10	0.05	-1.12	3.28	13.97 (<0.001)
PMM_S4	4.71	1.34	0.57	0.33	0.01	5.35	15.23 (<0.001)
PMD	13.84	4.35 (***)	1.86	1.08	-0.27	2.01	3.47 (0.176)
PMD_C	13.15	4.28 (***)	1.82	1.03	-0.36	2.18	3.29 (0.193)
P4: era of turbulence: January 2000 to December 2016							
WML	4.83	0.60	0.15	0.05	-1.38	5.76	129.58 (<0.001)
PMM_S3	0.67	0.08	0.02	-0.02	-0.79	3.46	22.82(<0.001)
PMM_S4	3.86	1.37	0.33	0.16	2.38	16.59	1762.93 (<0.001)
PMD	13.78	6.68 (***)	1.62	1.00	-0.11	1.58	17.44 (<0.001)
PMD_C	12.25	6.65 (***)	1.61	1.00	-0.06	1.45	20.52 (<0.001)

Table 7 Vector autoregression results of partial moments from January 1927 to December 1999

Table 7 reports the VAR (1) process results of partial moments throughout the in-sample period from January 1927 to December 1999. For Equation (14), the dependent variable is RPM_t^+, which represents the upper partial moments at month *t*. For Equation (15), the dependent variable is RPM_t^-, which represents the lower partial moments at month *t*. For each regression, α represents the coefficient of the intercept; RPM_{t-1}^+ and RPM_{t-1}^- represent the value of the upper and lower partial moments at month *t-1*, respectively. R_{adj}^2 reports the adjusted R-squared value. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Coefficient	Variable	Estimated coefficients (t-statistics)	
		(12)	(13)
		RPM_t^+	RPM_t^-
α_1	1	0. 00041 (5.89) ***	
β_{11}	RPM_{t-1}^+	0.45557 (13.26) ***	
β_{12}	RPM_{t-1}^-	0.17777 (6.13) ***	
α_2	1		0. 00052 (5.59) ***
β_{21}	RPM_{t-1}^+		0.28695 (6.21) ***
β_{22}	RPM_{t-1}^-		0.24295 (6.23) ***
R_{adj}^2		0. 3704	0. 1893

Table 8 Switching conditions for the out-of-sample periods

Table 8 reports the frequencies of the four PMM switching conditions based on in-sample partial moments for the whole out-of-sample period, the era of turbulence from January 2000 to December 2016. Panel A shows the actual out-of-sample frequencies of PMM conditions based on in-sample estimates. The variable "Sab" represents a situation in which the condition in the current month *t* is *a* and the condition in the next month *t+1* is *b*. N represents the numbers of each situation. Pct represents the percentage of each situation for each of the four conditions. Cond represents the total numbers of months when a condition applies to the whole out-of-sample period; there are 203 months during the 17-year period (there is no PMM in January 2000 owing to the 1-month lag before the formation and holding periods). For example, Cond 1 equals 124, which means there are 124 months during the out-of-sample period when condition 1 applies. The N of S14 is 15, which means there are 15 months in the out-of-sample period when condition 1 applies and, in the meantime, condition 4 applies in the next month. The Pct value of S14 is 7.39, which means that no matter what condition applies in the current month, there is a 7.39% chance that condition 4 will apply in the next month. Panel B compares the estimations of the four PMM strategies with the actual observations. Estimated probability reports the theoretical possibility of occurrence of each of these four PMM strategy conditions, which is consistent with Figure 2b.

Panel A: Frequencies of the four PMM switching conditions

Variable	N	Pct	Variable	N	Pct	Variable	N	Pct	Variable	N	Pct
S11	93	45.81	S21	13	6.40	S31	8	3.94	S41	9	4.43
S12	3	1.48	S22	0	0.00	S32	8	3.94	S42	5	2.46
S13	13	6.40	S23	3	1.48	S33	15	7.39	S43	5	2.46
S14	15	7.39	S24	0	0.00	S34	4	1.97	S44	9	4.43
Cond 1	124	61.08	Cond 2	16	7.88	Cond 3	35	17.24	Cond 4	28	13.79

Panel B: Comparison of estimated and actual frequencies

Condition	Estimated probability	Actual probability
1	32.53	61.08
2	17.47	7.88
3	32.53	17.24
4	17.47	13.79
Total	100.00	100.00

Table 9 Out-of-sample performance of partial moments-based momentum strategies

Table 9 shows the out-of-sample performance of six partial moment momentum (PMM) strategies and two partial moment-decomposed (PMD) strategies on a WML basis (*11×1*) for two sample periods. Partial moment reference points are computed for the in-sample sample period from January 1964 to December 1999. WML represents an *11×1* plain momentum strategy with a 1-month gap between the formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies on a WML basis and are constructed according to the switching rules presented in Table 3. PMD and PMD_C represent an unconstrained PMD strategy and a 200% leverage-constrained PMD strategy, respectively. Both strategies are on a WML basis and are constructed according to the methods described in Section 4. All partial moments-based momentum strategies follow the estimated partial moments. Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
Panel A: Pa, era of turbulence: January 2000 to December 2016				
WML	5.05	0.60	0.15	0.05
PMM_S1	1.41	0.52	0.13	-0.02
PMM_S2	-0.23	-0.04	-0.01	-0.05
PMM_S3	1.49	0.16	0.04	0.00
PMM_S4	3.22	1.36	0.33	0.12
PMM_S5	1.57	0.18	0.04	0.00
PMM_S6	3.30	0.36	0.09	0.03
PMD	2.03	1.29	0.31	0.04
PMD_C	2.03	1.29	0.30	0.04
Panel B: Pb, market downturn: August 2007 to December 2012				
WML	5.65	0.33	0.14	0.06
PMM_S1	3.20	1.23	0.52	0.43
PMM_S2	2.16	0.13	0.06	0.04
PMM_S3	5.96	0.30	0.13	0.06
PMM_S4	5.45	1.83 (*)	0.78	0.61
PMM_S5	4.39	0.26	0.11	0.11
PMM_S6	8.26	0.41	0.18	0.09
PMD	2.97	1.01	0.43	0.19
PMD_C	2.97	1.01	0.43	0.19

Table 10 Performance of rolling window PMM strategies

Table 10 shows the performance of six PMM strategies on a WML basis (*11×1*) for four sample periods. Partial moment reference points are computed on a 20-year rolling window basis. WML represents an *11×1* plain momentum strategy with a 1-month gap between the formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies on a WML basis and are constructed according to the switching rules presented in Table 3. Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
Panel A: P1, whole sample period: January 1927 to December 2016				
WML	15.15	4.96 (***)	0.52	0.20
PMM_S1	8.28	4.59 (***)	0.48	0.18
PMM_S2	5.41	1.78 (*)	0.19	0.05
PMM_S3	7.95	2.20 (**)	0.23	0.07
PMM_S4	8.53	4.77 (***)	0.50	0.20
PMM_S5	5.66	1.87 (*)	0.20	0.06
PMM_S6	10.82	2.61 (***)	0.30	0.10
Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989				
WML	21.60	5.12 (***)	1.02	0.40
PMM_S1	11.09	3.87 (***)	0.77	0.20
PMM_S2	3.92	0.92	0.18	-0.11
PMM_S3	14.05	2.85 (***)	0.57	0.18
PMM_S4	10.75	3.80 (***)	0.76	0.19
PMM_S5	3.59	0.85	0.17	-0.13
PMM_S6	13.26	2.30 (**)	0.52	0.16
Panel C: P3, market downturn: August 2007 to December 2012				
WML	5.65	0.33	0.14	0.06
PMM_S1	8.81	2.08 (**)	0.89	0.70
PMM_S2	11.76	0.67	0.29	0.29
PMM_S3	9.60	0.47	0.20	0.10
PMM_S4	8.49	2.02 (**)	0.86	0.61
PMM_S5	11.43	0.65	0.28	0.29
PMM_S6	6.42	0.29	0.13	0.06
Panel D: P4, era of turbulence: January 2000 to December 2016				
WML	4.83	0.60	0.15	0.05
PMM_S1	6.04	1.69 (*)	0.41	0.22
PMM_S2	7.30	0.88	0.21	0.14
PMM_S3	3.57	0.4	0.10	0.03
PMM_S4	6.83	2.00 (**)	0.49	0.31
PMM_S5	8.10	0.99	0.24	0.17
PMM_S6	3.36	0.35	0.09	0.03

Table 11 Performance of partial moments-based momentum strategies on a *6×6* basis

Table 11 shows the performance of partial moment momentum (PMM) strategies and two partial moment-decomposed (PMD) strategies on a *6×6* basis for four sample periods. Partial moment reference points are computed for the whole sample period, P1. M66 (strategy) represents a *6×6* plain momentum strategy with a 1-month gap between the formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies on a *6×6* basis; these are constructed according to the switching rules presented in Table 3. PMD and PMD_C represent an unconstrained PMD strategy and a 200% leverage-constrained PMD strategy, respectively. Both strategies are on a *6×6* basis and are constructed according to the methods described in Section 4. Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
Panel A: P1, whole sample period: January 1927 to December 2016				
M66	5.62	2.15 (**)	0.23	0.04
PMM_S1	5.67	3.76 (***)	0.40	0.10
PMM_S2	6.79	2.44 (**)	0.26	0.11
PMM_S3	-1.78	-0.46	-0.05	-0.07
PMM_S4	9.31	5.79 (***)	0.61	0.26
PMM_S5	10.47	3.65 (***)	0.39	0.24
PMM_S6	1.63	0.41	0.04	-0.02
PMD	23.93	23.57 (***)	2.48	1.96
PMD_C	21.92	20.06 (***)	2.11	1.37
Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989				
M66	12.03	3.59 (***)	0.72	0.14
PMM_S1	9.55	3.06 (***)	0.61	0.10
PMM_S2	7.02	1.80 (*)	0.36	0.00
PMM_S3	6.38	1.27	0.25	-0.01
PMM_S4	12.10	3.44 (***)	0.69	0.19
PMM_S5	9.51	2.25 (**)	0.45	0.08
PMM_S6	8.87	1.67 (*)	0.33	0.04
PMD	28.06	13.05(***)	2.61	3.44
PMD_C	24.43	13.48(***)	2.70	2.03
Panel C: P3, market downturn: August 2007 to December 2012				
M66	-8.98	-0.83	-0.35	-0.16
PMM_S1	-0.07	-0.03	-0.01	-0.08
PMM_S2	10.50	0.88	0.38	0.64
PMM_S3	4.64	0.28	0.12	0.08
PMM_S4	5.85	1.75 (*)	0.75	0.63
PMM_S5	16.98	1.38	0.59	1.01
PMM_S6	1.03	0.06	0.03	0.00
PMD	12.46	3.74 (***)	1.59	1.19
PMD_C	9.88	3.09 (***)	1.32	0.74
Panel D: P4, era of turbulence: January 2000 to December 2016				
M66	-0.92	-0.12	-0.03	-0.04
PMM_S1	3.30	1.31	0.32	0.12
PMM_S2	8.96	1.12	0.27	0.21
PMM_S3	6.04	0.62	0.15	0.10
PMM_S4	9.23	3.41 (***)	0.83	0.71
PMM_S5	15.19	1.85 (**)	0.45	0.37
PMM_S6	-0.60	-0.06	-0.01	-0.03
PMD	15.58	7.24 (***)	1.76	1.15
PMD_C	14.07	5.66(***)	1.37	0.67

Table 12 Performance comparison of partial moment-decomposed strategies versus the Barroso and Santa-Clara (2015) volatility-scaled momentum strategy

Table 12 compares the performance of two PMD strategies with the volatility-scaled momentum strategy introduced by Barroso and Santa-Clara (2015) for four sample periods. To maintain consistency with Barroso and Santa-Clara (2015), all momentum portfolios are constructed on a WML basis (11×1). WML represents an 11 × 1 plain momentum strategy with a 1-month gap between the formation and holding periods. BSC and BSC_C represent an unconstrained volatility-scaled momentum strategy constructed by Barroso and Santa-Clara (2015) and a similar strategy with 200% leverage constraint, respectively. PMD and PMD_C represent an unconstrained PMD strategy and a 200% leverage-constrained PMD strategy, respectively. Both strategies are constructed according to the methods described in Section 4. Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
P1: whole sample period: January 1927 to December 2016				
WML	15.65	4.96 (***)	0.52	0.20
BSC	20.24	9.30 (***)	0.98	0.58
BSC_C	19.26	8.73 (***)	0.92	0.51
PMD	26.13	24.39 (***)	2.57	2.27
PMD_C	19.94	24.69 (***)	2.60	1.93
P2: Jegadeesh and Titman (1993): January 1965 to December 1989				
WML	21.60	5.12 (***)	1.02	0.40
BSC	31.63	8.03 (***)	1.34	0.75
BSC_C	26.73	7.17 (***)	1.23	0.60
PMD	31.66	14.83 (***)	2.97	2.65
PMD_C	23.31	15.84 (***)	3.17	2.00
P3: market downturn: August 2007 to December 2012				
WML	5.65	0.33	0.14	0.06
BSC	13.84	2.06 (**)	0.88	0.58
BSC_C	11.67	1.85 (*)	0.76	0.48
PMD	13.84	4.35 (***)	1.86	1.08
PMD_C	13.15	4.28 (***)	1.82	1.03
P4: era of turbulence: January 2000 to December 2016				
WML	4.83	0.60	0.15	0.05
BSC	6.39	1.71 (*)	0.41	0.22
BSC_C	5.20	1.60	0.35	0.17
PMD	13.78	6.68 (***)	1.62	1.00
PMD_C	12.25	6.65 (***)	1.61	1.00

Table B.1 Performances of plain momentum strategies in the US equity markets

Table B.1 shows the performance of four plain momentum strategies for the four sample periods. Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated following the method in formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. M33, M66, M99, and M1212 represent the *3×3*, *6×6*, *9×9,* and *12×12* plain momentum strategies with 1-month gap between formation and holding periods, respectively. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
Panel A: P1, whole sample period: January 1927 to December 2016				
M33	2.14	0.91	0.10	-0.02
M66	5.62	2.15(**)	0.23	0.04
M99	2.15	0.83	0.09	-0.02
M1212	1.49	0.59	0.07	-0.03
Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989				
M33	8.96	2.95(***)	0.59	0.06
M66	12.03	3.59(***)	0.72	0.15
M99	9.10	2.68(***)	0.54	0.06
M1212	6.11	1.90(**)	0.38	0.05
Panel C: P3, market downturn: August 2007 to December 2012				
M33	-6.08	-0.66	-0.28	-0.14
M66	-8.98	-0.83	-0.35	-0.16
M99	-10.19	-0.95	-0.40	-0.19
M1212	-10.36	-1.00	-0.43	-0.21
Panel D: P4, era of turbulence: January 2000 to December 2016				
M33	-0.88	-0.13	-0.03	-0.04
M66	-0.92	-0.12	-0.03	-0.04
M99	-6.17	-0.98	-0.24	-0.14
M1212	-9.51	-1.68	-0.41	-0.24

Table C.1 Vector autoregression results of partial moments

Table C.1 reports the $VAR(1)$ process results of partial moments for the whole sample period, from January 1927 to December 2016. For Equation (23), the dependent variable is RPM_t^+, which represents the upper partial moment at month *t*. For Equation (24), the dependent variable is RPM_t^-, which represents the lower partial moment at month *t*. For each regression, α represents the coefficient of the intercept; RPM_{t-1}^+ and RPM_{t-1}^- represent, respectively, the upper and lower partial moment values at month *t-1.* R_{adj}^2 reports the adjusted R-squared value. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Coefficient	Variable	Estimated coefficients (t-statistics)	
		(21)	(22)
		RPM_t^+	RPM_t^-
α_1	1	0.00044 (6.81) ***	
β_{11}	RPM_{t-1}^+	0.40117 (12.20) ***	
β_{12}	RPM_{t-1}^-	0.23641 (8.33) ***	
α_2	1		0.00051 (6.17) ***
β_{21}	RPM_{t-1}^+		0.27294 (6.47) ***
β_{22}	RPM_{t-1}^-		0.30533 (8.38) ***
R_{adj}^2		0.3861	0.2462

Table C.2 Joint distribution of days of positive and negative market returns per month

Table C.2 presents the joint distribution of number of days when the market return is positive and negative in month t for the whole sample period, 1927-2016. (x, y) represents the number of months when there are x days when market return is negative and y days when market return is positive, where x=2, 3, …19 and y=3, 4, …19. The value-weighted index of all listed firms in the CRSP is chosen as the market proxy as defined in Subsection 2.1. The numbers represent the number of months for each (x, y) pair for the 90-year sample period, which is equivalent to a total of 1080 months. For instance, (8, 13) with a value of 56 represents 56 months during the period 1927-2016, when the market generates negative returns in 8 trading days and positive returns in 13 trading days.

Number of days when the market return is positive in month *t*	Number of days when the market return is negative in month *t*																	
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	19	Total
3																	1	1
4															1			1
5									1			1	1		3			6
6									1	1		4	4	6	2	1		19
7									2	7	6	11	11	10	1			48
8								2	3	10	19	20	13	2				69
9								2	21	24	56	25	5					133
10							6	18	26	35	24	7						116
11						2	20	30	44	37	15							148
12					4	14	25	51	45	17								156
13				3	7	22	56	40	18									146
14		1	1	5	22	28	33	17										107
15	2		1	14	20	21	8											66
16		1	3	13	10	5												32
17		6	5	11	4													26
18		2	2	1														5
19		1																1
Total	2	11	12	47	67	92	148	160	161	131	120	68	34	18	7	1	1	1080

Table C.3 Conditional decomposed returns of partial moment momentum strategies for the out-of-sample period

Table C.3 presents the decomposed returns of partial moment momentum (PMM) strategies on an *11×1* basis for four conditions in the out-of-sample period. Panel A reports the actual possibility of occurrence of each of these four PMM strategy conditions, shown and discussed previously in Panel B of Table 8. Return reports the annualized equivalent return and frequency-weighted return (in parenthesis) for each condition, in percentage. The combined return is the sum of the frequency-weighted return for each of the four conditions.

	Condition 1		Condition 2		Condition 3		Condition 4		Combined
Panel A: Possibility of occurrence									
	61.08%		7.88%		17.24%		13.79%		100.00%
Panel B: Returns per condition									
WML	2.76	(1.69)	32.44	(2.56)	8.22	(1.42)	-4.44	(-0.61)	5.05
PMM_S1	0.00	(0.00)	7.72	(0.61)	8.22	(1.42)	-4.44	(-0.61)	1.41
PMM_S2	-2.69	(-1.65)	7.72	(0.61)	8.22	(1.42)	-4.44	(-0.61)	-0.23
PMM_S3	0.13	(0.08)	7.72	(0.61)	8.22	(1.42)	-4.44	(-0.61)	1.49
PMM_S4	0.00	(0.00)	7.72	(0.61)	8.22	(1.42)	8.63	(1.19)	3.22
PMM_S5	-2.69	(-1.65)	7.72	(0.61)	8.22	(1.42)	8.63	(1.19)	1.57
PMM_S6	0.13	(0.08)	7.72	(0.61)	8.22	(1.42)	8.63	(1.19)	3.30

Table C.4 Out-of-sample performance of partial moments-based momentum strategies on a *6×6* basis

Table C.4 shows the out-of-sample performance of six partial moment momentum (PMM) strategies and two partial moment-decomposed (PMD) strategies on a *6×6* basis over two sample periods, respectively. Partial moment reference points are computed for the in-sample period from January 1964 to December 1999. M66 (strategy) represents a *6×6* plain momentum strategy with a 1-month gap between the formation and holding periods. PMM_S1 to PMM_S6 represent six PMM strategies on a *6×6* basis constructed according to the switching rules presented in Table 3. PMD and PMD_C represent an unconstrained PMD strategy and a 200% leverage-constrained PMD strategy on a *6×6* basis, respectively. Both strategies are on a *6×6* basis and are constructed according to the methods described in Section 4. Return reports the annualized return of each strategy in percentage. The Sharpe ratio reports the annualized Sharpe ratio of each strategy. It is calculated according to formula (1), as the long–short portfolio return divided by its standard deviation. The adapted Sortino ratio reports the annualized adapted Sortino ratio of each strategy. It is calculated according to formulas (16) to (18), as the excess return divided by twice its downside semi-deviation. The Newey–West (1987) t-test indicates significance at the 10% (*), 5% (**), and 1% (***) levels.

Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
Panel A: Pa, era of turbulence: January 2000 to December 2016				
M66	-0.92	-0.12	-0.03	-0.04
PMM_S1	2.92	1.43	0.35	0.11
PMM_S2	8.10	1.03	0.25	0.19
PMM_S3	-5.16	-0.53	-0.13	-0.09
PMM_S4	7.09	3.12 (***)	0.76	0.52
PMM_S5	12.46	1.54	0.37	0.30
PMM_S6	-1.29	-0.13	-0.03	-0.04
PMD	1.27	0.94	0.23	-0.04
PMD_C	1.18	0.93	0.21	-0.04
Panel B: Pb, market downturn: August 2007 to December 2012				
M66	-8.98	-0.83	-0.35	-0.16
PMM_S1	0.10	0.05	0.02	-0.08
PMM_S2	11.85	1.00	0.42	0.74
PMM_S3	-4.47	-0.27	-0.11	-0.08
PMM_S4	6.02	2.07 (**)	0.88	1.24
PMM_S5	18.41	1.50	0.64	1.12
PMM_S6	1.20	0.07	0.03	0.01
PMD	-2.59	-0.16	-0.07	-0.13
PMD_C	-2.30	-0.16	-0.06	-0.11