Developers, Wall Street, and the Taxmen: A Theory of Real Estate Development*

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Abstract

We build a general equilibrium model of commercial real estate (CRE) development (aka, CRE asset creation), for an economy with several jurisdictions, and segmented commercial good and equity markets. The CRE assets' capital structures and cash flows, the prices of equity, non-recourse mortgage debt and commercial goods, and the jurisdictions' property taxes and land use policies are all endogenous. We show that an equilibrium exists for this economy. Our equilibrium characterization shows that CRE construction booms in jurisdictions where developers hold a larger share of debt and equity (possibly, due to lower mortgage default risk), and contracts otherwise. Regional negative shocks to developers' funding debt capacity result in leverage shifting toward other jurisdictions. Leverage also increases in jurisdictions that pursue a high property tax fiscal policy. Finally, we highlight how global real estate markets can influence local jurisdictions' fiscal policies.

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1 Introduction

Commercial real estate (CRE) development plays an integral role in the economy. It creates spaces for jobs and provides a permanent source of revenue to investors. In the U.S. alone, there is about $5 trillion worth of commercial real estate. The construction of these assets contributes approximately 5% percent of the U.S. Gross Domestic Product (GDP), which in 2016 terms amounts to a contribution of $861 billion to the U.S.'s economic output. In terms of employment, commercial real estate development supports 6.25 million American jobs. But not all cities experience the same patterns in construction. Some cities grow and others shrink. Developers' access to debt and equity capital is crucial for a city to succeed.

In the last decades, regional and local commerical real estate markets have rapidly changed in part because of the dominant presence of Wall Street financial intermediaries in real estate development deals. The most common types of real estate investors are Real Estate Investment Trusts (REITs), pension funds, insurance companies, and, to some extent, foreign investors. Only the REIT sector owns more than $3 trillion in gross assets, supports an estimated 1.8 million full-time workers, and contributes more than $56 billion in new construction and capital expenditures.[1]

Since the seminal contributions of Mills (1993) and Dinsmore (1998) on the prevalence of REITs and other types of real estate investors in the development and acquisition of commercial real estate, there has been a spirited debate on the ability of cities to attract capital investments. Economic strains such as Brexit and the cooling of the Chinese economy are now driving international capital into commercial real estate in the U.S. in search for higher returns and stability. Not only traditional gateway cities, such as New York, Los Angeles and Miami, are the focus of investors' attention, but also other secondary markets such as Denver, Phoenix and Nashville. Local authorities are aware of the importance for developers to raise debt and equity, and compete accordingly to make their cities attractive destinations for investors.

However, the presence of Wall Street – rather than Main Street investors – puts in question

[1]REITs are corporate entities that can be privately or publicly held. In the United States, there are more than 200 stock exchange-listed REITs with a total equity market capitalization of approximately $1 trillion.

the role that global real estate investors have in urban and metropolitan economic development. Real estate investors have a clear self-interest in reducing some of the costs decided by local municipalities, including local property taxes, zoning and land use controls, and utility rates. Depending on the maginitude of the real estate investors' portfolios, stakeholders maybe be able to influence whether and how a local community deals with a particular issue. An alarming example is the case of Texas, where REITs own more than 29,600 properties, including office buildings, luxury hotels, urban apartments, regional and strip malls, storage, and warehouse.

These important practical questions deserve a careful economic analysis and a general equilibrium model happens to be the natural way to analyze these issues. The financing aspects of a real estate development project cannot be analyzed without taking into consideration the local fiscal and land use policies, nor can they be analyzed without a proper evaluation of the city's economic fundamentals. Local households are the final buyers of the goods produced and sold by the commercial real estate assets and their prices determine the cash flows used to evaluate the equity price of a commercial real estate asset. Neither we can ignore a developer's access to funding in the debt market because debt collateral constraints may determine a developer's need for equity. Also, a developer's choice of capital structure has general equilibrium implications in an economy with segmented equity markets. A more leveraged development in a jurisdiction may be the result of real estate equity funds flowing to other more attractive jurisdictions. Moreover, because interests on debt are tax exempt but not the equity dividends, a local jurisdiction fiscal policy (in the form of property taxation) may influence not only the capital structure of a local development project, but also the capital structures of other projects outside the jurisdiction (a general equilibrium effect).

We propose a model of real estate development that captures these considerations. We focus on equity investments and commercial (income-producing) real estate properties, such as office buildings, industrial space (e.g., heavy manufacturing, light assembly, and warehouses), retail space (e.g., strip centers and regional malls), multifamily buildings (e.g., student housing and mid-rise apartments), land (e.g., greenfield land), and hotels. We consider an economy with multiple jurisdictions and pay special attention to the financing aspects of the real estate development deals, as well as their interaction with the local governments' fiscal and land use policies. For each development project in each jurisdiction, the capital structure, composed by the developer's debt, the developer's (or entrepreneurial partner's) equity, and the investors' (or capital partners') equity, is endogenously determined in our model and driven by factors such as the developer's funding capacity, the jurisdiction's property taxes, and the performance of the income-producing real estate asset. Other important elements such as real estate equity cash flows and development construction costs are also endogenous in our model.

We build our economy in a two-period setting. Uncertainty enters into the model because we consider several states of nature in the second period. Our notion of equilibrium is in the tradition of general competitive equilibrium models with incomplete financial markets and restricted participation, where agents do not choose their location (jurisdiction), but are exogenously assigned to jurisdictions. There are multiple jurisdictions and each jurisdiction has both local households and a representative local developer. Global real estate equity investors can invest in all jurisdictions, an important feature to model interdependent segmented commercial equity markets.

Because CRE development projects are capital intensive, developers need to obtain financing by issuing non-recourse mortgage debt in a global market and selling equity to investors in the first period. Developers then use raised the capital to buy construction inputs, including land. Given the jurisdiction's choice of the type of a CRE that a developer is allowed to construct, inputs are transformed into a CRE asset using a Cobb-Douglas production technology. The CRE asset produces (multiple) commercial goods in the second period, possibly in different amounts across states of nature. Local households purchase these commercial goods. This interpretation of commercial goods produced by CRE assets and sold to local households is in the spirit of Debreu (1959)'s classical book *Theory of Value*. For example, for the case of hotels, the consumption of a "hotel good" should be interpreted as the purchase of a certain number of nights in a hotel room with specific characteristics in a given location. Similar interpretations apply for the consumption of office space or student housing. Because households may have different preferences for the consumption of commercial goods, the price of similar commercial goods can differ across jurisdictions. This in turn implies that similar CRE assets in different jurisdictions may generate different cash flows in equilibrium.

Given the jurisdictions' fiscal and land use policies, we identify mild conditions under which a competitive equilibrium exists for our economy. This result is not trivial because of the additional economic structure that our model requires in order to accommodate the development (aka creation) of CRE assets and the segmentation of commercial good and CRE equity markets. One particular difficulty is to guarantee that a developer's budget set correspondence takes convex values. This is particularly challenging in a setting where the market value of a CRE asset depends on the developer's choice of materials and the equity return in the second period depends on the (endogenous) type of CRE asset chosen in the first period. Another subtlety of our existence proof is the property of lower semicontinuity of the budget correspondence (the standard approach of embeding all prices in the simplex does not guarantee the existence of an interior point with segmented real estate equity and commercial good markets). We circumvent this problem by finding an endogenous upper bound for real estate equity prices (previous results in the literature of market segmentation with a fixed point theory approach are not useful in our setting).

Having established this general result, we provide a characterization of equilibrium in a model with only two jurisdictions and two states of nature in the second period. This setting allows us to understand the impact of several economic shocks and public policies on the feasibility of CRE development projects and capital structures.

First, we analyze the determinants of default risk on the valuation of CRE equity and mortgage debt. We show that a higher risk of mortgage debt default decreases the equilibrium debt market price. The expected default risk is also the main driver of the shadow value of the equity collateral constraint that a developer must satisfy in order to get a non-recourse loan. This shadow value in turn increases the equity price for a CRE asset facing a higher mortgage default risk. Roughly speaking, mortgage default increases market pressure on equity through the corresponding equity price.

Second, we investigate the determinants of differences in construction patters between jurisdictions. We find that a higher mortgage default loss in a jurisdiction induces real estate investors to rebalance their equity holdings toward the high default risk jurisdiction, crowding out developer's equity in the high default risk jurisdiction and expanding the developer's equity in the low default risk jurisdiction. As a result, construction booms in the low default risk jurisdiction and contracts in the high default risk jurisdiction.

Third, we show that global real estate equity investors increase their equity exposure in those CRE projects whose developers become more debt constrained. In addition, we find an equity-debt substitution effect not only within the capital structure of a real esatate development project, but also across jurisdictions: levarage in the real estate construction sector shifts from one jurisdiction to another when access to the debt market changes for developers in one of the two jurisdictions.

Fourth, we consider a situation where a jurisdiction decides to increase its property tax rate to improve the provision of local public goods. Given the differences in tax treatment between equity and debt, the developer in the higher property tax jurisdiction increases leverage. Because the price of equity becomes relatively less expensive than debt, the developer in the low tax jurisdiction rebalances its portfolio toward equity, so leverage decreases in that jurisdiction.

This last result ignores the possibility that jurisdictions strategically choose property taxes in a non-cooperative way to increase profits. To understand the implications of dropping this assumption, we extend our two-period model to incorporate a Nash game in a pre-stage of the economy where strategic jurisdiction authorities choose their property taxes and the types of CRE developments allowed in their jurisdictions. These decisions influence the type of competitive equilibrium (e.g., the composition of the different capital structures and the commercial good and equity prices). High property taxes are not always optimal for jurisdictions that seek to maximize profits. Low taxes may generate higher profits by drawing more real estate equity investments to the jurisdictions, increasing equity tax revenues as a result. We provide an example that illustrates this, which resembles the "race to the bottom" concept in the literature of financial competition among political jurisdictions (Cary 1974, Drezner 2001, and Carruthers and Lamoreaux 2016). Our result extends to commercial real estate equity investments and highlights the impact that global real estate markets have on local jurisdictions. Rather than having fiscal policy driving financial markets, our economy captures the dominant role of Wall Street on the fiscal policies of "small" municipalities.

Further contribution to the literature

The question of what determines the capital structure of a firm has been extensively researched in the last decades since the seminal work of Modigliani and Miller (1958, 1963). Significant contributions are Miller (1977) on the impact of taxes, Jensen and Meckling (1976) on agency costs, Kraus and Litzenberger (1973) on bankruptcy costs, and DeAngelo and Masulis (1980) on non-debt tax shields. Titman and Wessels (1988) analyze the explanatory power of these and other related theories of optimal capital structure. The literature has expanded and recently the focus has been on financial intermediation and the implications of certain regulations on the banks' optimal choice of capital structure (Gale 2004, DeAngelo and Stulz 2015, Gornall and Strebulaev 2015, Allen, Carletti and Marquez 2015, Gale and Gottardi 2015, and Amaral, Corbae, and Quintin 2017). However, capital structure theory is not as well developed for real estate development and investments.

While there is some work that has highlighted the importance of capital constraints, taxes on equity dividends, and default risk on the capital structure of a real estate asset (Gau and Wang 1990, Giambona, Mello, and Riddiough 2016), it is fair to say that most research is empirical. Existing theoretical papers rely either on a "real options" pricing partial equilibrium model (see e.g. Titman 1985, Capozza and Helsley 1990, Capozza and Sick 1990, Williams 1991, Childs, Riddiough, and Triantis 1996, and Grenadier 1995a,b) or on a general equilibrium model with entrepreneurial developers and a housing/land market (see e.g. Henderson 1974, Helsley and Strange 1997, Konishi 2013, and Anglin, Dale-Johnson, Gao and Zhu 2014).[2] Our paper pro-

[2]Henderson (1974) studies the evolution of cities when intracity Marshallian externalities in production are present and discusses the role of entrepreneurship. Helsley and Strange (1997) consider the case of (city) devel-

poses a different approach than these two literatures by considering a general equilibrium model that focuses on the financing aspects of commercial real estate development when jurisdictions compete to attract real estate equity investments.

We incorporate and endogenize for the first time and in a consistent way several important financial variables of real estate development and investment decisions, such as the developer's capital structure – composed by debt, common equity of the capital partners, and common equity of the developer partner –, the property taxes chosen by jurisdiction managers, the cash flows of the real estate asset, and the types of development projects chosen by developers and jurisdiction managers. Our results that illustrate an equity-debt substitution effect not only within the capital structure of a real esatate development project, but also across jurisdictions contributes to the recent theoretical literature on risk shifting and asset substitution. See the seminar paper of Stiglitz and Weiss (1981), and Martinez-Miera and Repullo (2010) for a recent contribution.

In addition to having developers choosing their optimal capital structure, our paper also departs from the literature in that it captures how global capital markets influence local fiscal policy. Our model exploits the trade-off between equity taxes, default risk on debt and leverage, but leaves aside other considerations such as agency costs.

Our equilibrium model is also related to the literature of competitive market economies with incomplete markets, developed by Diamond (1967), Radner (1974), and Grossman and Hart (1979), among others. The main departure from this literature is our focus on income-producing real estate assets. This particular type of asset generates cash flows that are dependent on the market price of the goods sold in local segmented markets. Thus, real estate cash flows are endogenous because commercial good prices are determined by market clearing given the demand of households at the jurisdiction level and the construction of CRE assets that produce and sell these commercial goods. Even when the size and type of real estate assets is the same in two jurisdictions, cash flows may differ if the fundamentals of their respective local economies are

opers that provide local public goods with limited power and an explicit geographical structure of the city. Konishi (2013) considers an economy with a large number of atomless land developers who can enter the market freely in an idealized version of Tiebout (1956).

different.

In our setting, developers are subject to collateralized debt constraints and markets can be incomplete. In addition, we depart from the standard one good economy because we allow for multiple construction inputs in the development phase and also for multiple consumption goods that are sold in the jurisdiction where the commercial real estate asset was constructed. Our setting comes at a cost because with multiple goods and incomplete markets, we are not able to establish the efficiency property of a decentralized competitive economy (Geanakoplos and Polemarchakis 1986). In this sense, our work departs from Gale and Gottardi (2017), who follow a similar approach than Makowski (1983) and Hart (1979) by considering an alternative general equilibrium model of financial intermediation with one good and complete markets where the efficiency property holds.

Our work is also closely related to the the literature on financial innovation – also referred to as the security design literature – pioneered by Allen and Gale (1991) (see Allen and Gale 1994 and Duffie and Rahi 1995 for surveys). This literature has focused on the design of financial securities, such as bonds, stocks, mortgages, and mortgage-backed securities, and the relationship with economic development.[3] Up to our knowledge, we are the first paper to uncover the specific case of real estate development (aka creation) in a model with segmented equity and commercial good markets.

Our paper also relates to Rahi and Zigrand (2009), who study financial innovation and welfare in a two-stage equilibrium model with segmented markets. These markets ("jurisdictions" in our terminology) are characterized by differential marginal valuations and can list the same assets. Investors cannot trade assets in more than one market (jurisdiction). The ability to trade across markets is left for arbitrageurs, who turn out to be the issuers of assets in the first stage of their model. However, as Rahi and Zigrand (2009) recognize, this interpretation of issuance and implied listing is rather specific (when a company lists its shares in an jurisdiction, arbitrage possibilities are not the main reason to go public). In our theory of real estate development

[3] See, for example, Amaral, Corbae, and Quintin (2017) for a recent study of the relationship between financial engineering (repackaging) and development.

and investments, we depart from these assumptions and allow global investors to buy real estate equity in multiple jurisdictions. In addition, besides the obvious difference in motivations, our model also departs from Rahi and Zigrand (2009) in that financial innovation is not driven by arbitrageurs, but by the sequential actions of local jurisdiction authorities and developers. The former choose the type of commercial real estate asset that developers are allowed to construct in their respective jurisdictions (e.g., hotel, shopping mall, etc). The latter choose, for a given type of real estate asset, the combination of construction inputs that determines the size of the development.

The remainder of this paper is structured as follows. In Section 2, we present the model, the equilibrium concept, and the result of equilibrium existence. Section 3 proposes an extension to the model in which the jurisdiction's fiscal and land use policies become endogenous variables. Section 4 builds a simplified economy with two jurisdictions and provides an equilibrium characterization analysis. Section 5 concludes. The Appendix is devoted to the proofs.

2 The model

Consider a multi-jurisdictional economy with $\mathcal{K} > 0$ jurisdictions. In each jurisdiction k, there is a representative local developer d_k that finances new developments by issuing debt and equity. In addition, there are $\mathcal{H}_k \geq 1$ households, whose consumption of commercial goods is restricted to jurisdiction k.[4] We denote the set of households in jurisdiction k by $\mathbf{H}_k = \{1, ..., h_k, ..., \mathcal{H}_k\}$. The set of developers in the economy is $\mathbf{D} = \{d_1, ..., d_k, ..., d_{\mathcal{K}}\}$. For simplicity, we ignore banks as financial intermediaries and assume that households buy debt directly from developers. In addition, in this economy there are $\mathcal{I} \geq 1$ global real estate investors that can buy equity in all new developments across jurisdictions. The set of investors is $\mathbf{I} = \{1, ..., i, ..., \mathcal{I}\}$. We write

[4]Extending the model to allow for multiple developers in each jurisdiction is feasible; all that is needed is to rewrite the market clearing equations of debt and equity to accommodate the additional CRE assets of the same type constructed by the developers in a jurisdiction. However, this extension would complicate the analysis without adding much additional economic insight, as it would introduce sharing rules for investors' equity investments among the different CRE assets within the same jurisdiction. Another extension could be to consider global developers, in which case risk sharing across projects in different jurisdictions would be a possibility.

$a \in \mathbf{A}$ to denote an agent independently of its type, where $\mathbf{A} = \mathbf{I} \cup \mathbf{D} \cup \{\mathbf{H}_k\}_{k=1}^{\mathcal{K}}$. Finally, we write $\mathbf{K} = \{1, ..., \mathcal{K}\}$ to denote the set of all jurisdictions.

Our focus is not on where agents choose to live, but on the implications of allowing global real estate investors to do business in multiple jurisdictions. In this sense, our approach departs from the Tiebout literature where consumers "vote with their feet" where to live and work (see e.g. Tiebout 1956, Konishi 2008, and Luque 2013). Thus, we assume that the distribution of agents into jurisdictions is exogenously given and satisfies the following conditions.[5]

In this economy, there are two periods, $t = 1, 2$, and S states of nature in the second period. $\mathbf{S}$ denotes the set of states of the second period. Because we allow for the construction of CRE assets, the space of commodities changes between periods 1 and 2. There is a consumption good, indexed by $l = 0$ and called the "*numeraire*". This good can be any good that facilitates trade, e.g., cash. We assume that all agents can trade the numeraire good in the global market in both periods. In addition, in period 1, there are also L_1 inputs used for construction. We denote the set of construction inputs by $\mathbf{L}_1 = \{1, ..., L_1\}$. We denote an agent a's endowments of the numeraire good and construction inputs in the first period by $\omega_1^a \in \mathbb{R}_{++}^{1+L_1}$. The set of construction inputs $\mathbf{L}_1$ includes the land available for new development in each jurisdiction.[6] For example, if input $l \in \mathbf{L}_1$ is land in jurisdiction k, then the total amount of land available for development in jurisdiction k is $\sum_{a \in \mathbf{A}} \omega_{l1}^a$. Because $\omega_{l1}^a < \infty$ for all $a \in \mathbf{A}$ and all $l \in \mathbf{L}_1$, this upper bound limits the supply of land in each jurisdiction.

In the second period, agents are endowed with the numeraire good and each CRE asset produces L_2 commercial goods, with corresponding set $\mathbf{L}_2 = \{1, 2, ..., L_2\}$. We denote agent a's endowment of the numeraire good at state s of the second period by $\omega_0^a(s) \in \mathbb{R}_{++}$.

We refer to the numeraire, construction inputs, and commercial goods as "commodities". The vector of commodities purchased by an agent a is $x^a = (x_1^a, (x^a(s), s = 1, ..., S)) \in \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{S(1+\mathcal{K}L_2)}$. Notice that the consumption possibilities in the second period at a state $s \in \mathbf{S}$

[5] See Berliant and ten Raa (1994) for a discussion of the different methodologies in the fields of regional science, regional economics, and urban economics.

[6] For simplicity, we do not explicitly model the production of construction materials (e.g., the process of cutting trees and using timber pieces to construct the frames of large structures) and take their supply as given.

are $1 + \mathcal{K}L_2$ goods (the numeraire good and $\mathcal{K}L_2$ commercial goods). This is because there are $\mathcal{K}$ jurisdictions in the economy and we assume that a jurisdiction's CRE asset produces L_2 commercial goods. We introduce, however, restrictions on the consumption of some of these goods. In particular, when an agent is a developer or a household in a jurisdiction k, the consumption of a commercial good produced in another jurisdiction is zero, i.e., $x^a_{lk'}(s) = 0$ if $a \in \{d_k\} \cup \mathbf{H}_k$ and $l \in \mathbf{L}_2(k')$ with $k' \neq k$. Investors belong to all jurisdictions, so they are not restricted to consume commercial goods in different jurisdictions.

We find convenient to denote the vector of all individual commodity purchases in the economy by $x^A = (x^a : a \in \mathbf{A})$. The vector of commodity prices is $p = (p_1, (p(s), s = 1, ..., S)) \in \mathbb{R}^{1+L_1}_+ \times \mathbb{R}^{S(1+\mathcal{K}L_2)}_+$. We refer to the price of the numeraire good in the first period by p_{01}. For the second period, we denote the price of a commercial good $l \in \mathbf{L}_2$ sold to households of jurisdiction k at state s by $p_{lk}(s)$. We write the price of the numeraire good at state s as $p_0(s)$.

Agents derive utility from the consumption of the numeraire and the commercial goods available in the juridictions where they belong to. The consumption of construction inputs does not provide agents any (direct) utility. We denote an agent a's utility function by $u^a : \mathbb{R}^{(1+L_1)+S(1+\mathcal{K}L_2)}_+ \rightarrow \mathbb{R}$.

Assumption 1: **(i)** *For any agent $a \in \mathbf{A}$, the utility function u^a is continuous and strongly quasi-concave;*[7] **(ii)** *For all $k \in \mathbf{K}$ and for any agent $a \in \{d_k\} \cup \mathbf{H}_k$, $u^a(x_1, (x(s), s = 1, ..., S)) = u^a(x_{01}, (x_k(s), s = 1, ..., S))$, where $x_k(s) \in \mathbb{R}^{1+L_2}$ is the bundle of commercial goods available at jurisdiction k, and, for any investor $i \in \mathbf{I}$, $u^i(x_1, (x(s), s = 1, ..., S)) = u^i(x_{01}, (x(s), s = 1, ..., S))$; that is, developers, households, and investors do not assign any utility to the construction inputs; moreover, developers and households do not assign any utility to commercial goods in jurisdiction where they do not belong to. (iii) For any agent $a \in \mathbf{A}$, u^a is strictly increasing in all commodities to which agent a assigns utility.*

[7]Given a convex set $X \subset \mathbb{R}^n$, a function $f : X \rightarrow \mathbb{R}$ is strongly quasi-concave if $f(\lambda x + (1-\lambda)y) > \min\{f(x), f(y)\}$, for any $(x, y) \in X \times X$ such that $f(x) \neq f(y)$. This property is weaker than strict quasi-concavity, which requires $f(\lambda x + (1-\lambda)y) > \min\{f(x), f(y)\}$, for any $(x, y) \in X \times X$ such that $x \neq y$.

2.1 CRE development projects

In the first period, developer d_k can buy construction inputs to develop one (and only one) CRE asset j_k in jurisdiction k. These construction inputs are evaluated by the following Cobb-Douglas production function:

$$y_k = TFP_k \cdot \Pi_{l \in \mathbf{L}_1} (x_{l1}^{d_k})^{\alpha_{lk}} \tag{1}$$

where

- TFP_k is a parameter that stands for the "total factor productivity" specific to jurisdiction k. TFP_k may differ across jurisdictions because their natural resources, their agglomeration economies, or the intensity of competition.[8] TFP_k may also capture the infrastructure and the amenities of a jurisdiction.[9]

- parameter $\alpha_{lk} \in [0, 1]$ denotes the weight assigned to construction input $l \neq 1$ by jurisdiction manager k.

- $x_{l1}^{d_k}$ is the amount of construction input $l \in \mathbf{L}_1$ that developer d_k buys in period 1. Materials, once employed for the construction of the CRE asset, cannot be further traded.

- The type of CRE asset is determined by the vector of production weights $\alpha_k = (\alpha_{lk})_{l \in \mathbf{L}_1}$, i.e., different vectors α_k leads to different types of CRE assets. For example, in a two-jurisdiction economy with two construction inputs, we can associate the vector $\alpha_k = (\alpha_{1k}, \alpha_{2k}) = (1/2, 1/2)$ with a hotel, and the vector $\alpha_{k'} = (\alpha_{1k'}, \alpha_{2k'}) = (1/3, 2/3)$ with a shopping mall. If the jurisdiction manager k chooses $\alpha_k = (\alpha_{1k}, \alpha_{2k}) = (1/2, 1/2)$ and if the representative developer d_k chooses $x_1^{d_k} = (2, 3)$, then the CRE development project j_k has a size equal to $y_k = 2.45$. Different combinations of $x_1^{d_k}$ result in different type-specific CRE asset sizes.

[8] The fact that firms in large jurisdictions are more productive is a well-established fact in the empirical literature; see, e.g., Rosenthal and Strange (2004) and Duraton et al. (2012).

[9] In addition, we could make TFP_k a function of the jurisdiction's fixed and variable costs, which in turn determine its public investments and depend on parameter α_k (see Section 2.3). Here, for simplicity, we just take TFP_k as a parameter specific of a jurisdiction k.

If developer d_k constructs a positive amount of CRE asset $y_k > 0$, it is initially entitled to all CRE equity in that asset and we write $\bar{e}_k^{d_k} = 1$. If $y_k = 0$, then $\bar{e}_k^{d_k} = 0$. And because only developers can engage in construction activities, we set $\bar{e}_k^a = 1$ if $a \neq d_k$. We find convenient to write $\bar{e}^A = (\bar{e}^a : a \in \mathbf{A})$ to denote the initial CRE equity ownership bundles of all agents in the economy.

The production of commercial goods only depends on the type and size of the CRE asset. In particular, at state s of the second period, the supply of consumption goods by CRE asset j_k in jurisdiction k is given by the following function:

$$f_k(y_k)(s) : [0, +\infty) \to \mathbb{R}_+^{L_2}$$

with $f_k(0)(s) = 0$.[10]

For a CRE asset j_k with size y_k, the amount of commercial good $l \in \mathbf{L}_2$ supplied to local households at state s is $f_{lk}(y_k)(s)$. The price of this commercial good is denoted by $p_{lk}(s)$. Thus, a CRE asset with size y_k generates an endogenous price-dependent cash flow equal to

$$c_k(y_k; p)(s) = \sum_{l \in \mathbf{L}_2} p_{lk}(s) f_{lk}(y_k)(s) \text{ at } s \in \mathbf{S}.$$

We impose the following assumptions on function f_k:

Assumption 2: **(i)** *$f_k(\cdot)(s)$ is additively separable and homogeneous of degree 1;* **(ii)** *for every $k \in \mathbf{K}$ and $l \in \mathbf{L}_2$, f_{lk} is increasing and concave;* ***(iii)*** *and for all $k \in \mathbf{K}$ and $s \in \mathbf{S}$, $f_k(y_k)(s) \neq 0$ when $y_k \neq 0$.*

Assumption 2.i implies that, if $y_k = \varkappa\theta_1 + \theta_2$ and $\theta_1, \theta_2 > 0$, then $f_k(y_k)(s) = \varkappa f_k(\theta_1)(s) +$

[10]For CRE assets such as hotels, office space, and student housing, consumption of those commercial goods should be interpreted as the purchase of a particular type of space for a given period in a given location with specific surrounding amenities. For CRE assets that involve the production and sale of physical goods, we differentiate between industry and retail. The former produces some goods that are sold to retail owners. The latter buys those goods from the industry and sells them to the individuals in the jurisdiction. For the sake of simplicity, our model does not differentiate between the two. Instead, we assume that households purchase commercial goods directly from the local CRE property. An extension of our model that differentiates between industry and retail real assets could be done by considering an additional period ($t = 3$), where the goods purchased in $t = 2$ by retail owners are sold to local households.

$f_k(\theta_1)(s)$. This allows us to split the cash flows of a CRE asset among different equity owners. We impose Assumption 2.ii to guarantee that the developer's budget constraints are convex.[11] We will use Assumption 2.iii to prove that there is no excess of supply in the equity market.

2.2 Equity and debt

In this economy, only equity is subject to taxes (this model of taxation is consistent with the so-called "pass-through taxation", see discussion in Section 2.4). Issuing debt also requires a developer to keep with some equity in the property that can be pledge to the lender in case of default.

A developer d_k can sell all, part, or none of its initial CRE equity $\bar{e}_k^{d_k}$ to investors. Investors can buy equity in one or several CRE development projects and thus can be thought of as equity REITs.[12] We assume that households are not financially sophisticated in the sense that they do not have access to the real estate equity market.[13]

With these considerations at hand, we proceed to introduce the following notation. Let E_k^a denote an agent a's equity positions on CRE project j_k, where $a = d_k, i$. We write $E^A \equiv (E^a : a \in \mathbf{D} \cup \mathbf{I}) \in \mathbb{R}_+^{\mathcal{K}+\mathcal{IK}}$ to denote the vector of equity positions corresponding to all developers and investors of the economy, respectively (there are $\mathcal{I}$ investors and $\mathcal{K}$ developers – one per jurisdiction – and we do not allow developers to buy CRE equity in jurisdictions other than their own). Since households are just consumers and do not have access to the CRE equity market, we set $E_{k'}^{h_k} = 0$ for all $k, k' \in \mathbf{K}$.

An equity stake on a CRE asset is just a claim to the future payoffs generated by this asset.

[11]Later, in section 4, we shall provide an example of a simplified economy, where an equilibrium can also exist in a context where f_{lj_k} is a linear increasing function.

[12]*Equity REITs* are real estate companies that acquire commercial properties – such as office buildings, shopping centers and apartment buildings – and lease the space in the structures to tenants. See "Guide to REITs": https://www.reit.com/investing/reit-basics/guide-equity-reits

[13]This modelling choice requires an impatience assumption on the utility function (see below)

The market clearing condition for equity shares corresponding to CRE asset j_k is as follows:

$$E_k^{d_k} + \sum_{i \in \mathbf{I}} E_k^i = \bar{e}_k^{d_k}. \tag{2}$$

When $E_k^{d_k} = 0$, the developer sells all the initial equity and does not keep any equity for himself. If instead $E_k^{d_k} \in (0, \bar{e}_k^{d_k})$, the developer keeps part but not all of the ownership on the property. When $E_k^{d_k} = \bar{e}_k^{d_k}$, the developer owns all equity of the project, so he is entitled to all of the property's cash flows.

We normalize $\bar{e}_k^{d_k} = 1$ to have E_k^a denoting both the agent a's *face value* of equity on CRE property j_k in jurisdiction k, and the agent a's *equity share* on j_k. Thus, an agent a with E_k^a is entitled to the CRE asset j_k's cash flow $E_k^a c_k(y_k; p)(s)$. By market clearing equation (2), we can write

$$c_k(y_k; p)(s) = E_k^{d_k} c_k(y_k; p)(s) + \sum_{i \in \mathbf{I}} E_k^i c_k(y_k; p)(s), \text{ for all } s \in \mathbf{S}.$$

Roughly speaking, the endogenous price-dependent cash flow that a CRE asset with size y_k generates at state s equals the sum of all payments received by the equity owners of this asset.

We denote the (endogenous) price of one unit of CRE equity in asset j_k in period 1 by q_k. Then, an agent a that purchases an equity stake $E_k^a > 0$ on asset j_k pays $q_k E_k^a$ in the first period and receives $E_k^a c_k(y_k; p)(s)$ units of the numeraire good at state s of the second period.

We assume that debt is a nominal asset (we need this assumption for our proof of Theorem 1 below in order to guarantee the existence of an interior point in an agent's budget constraint). An agent buying (lender) a *face value* of debt equal to $D_+^a \geq 0$ pays τD_+^a in the first period, where τ is the (endogenous) discount price of debt.[14] Similarly, an agent selling (borrower) a face value of debt equal to $D_-^a \geq 0$ receives τD_-^a in the first period. We allow households and developers to trade debt in the global market. Howerver, global investors are not allowed to borrow, i.e., $D_-^i = 0$ (we rule out modelling globar investors as mortgage REITs).[15]

[14]Debt prices, which reflect the discounted cashflow of future debt payments, are endogenous in our model.
[15]See Campello and Giambona (2013) and Cvijanović (2014) for insights on this possibility.

CRE debt is non-recourse and must be collateralized with CRE equity. The collateral debt constraint in period 1 for an agent $a = d_k, h_k$ in jurisdiction k is as follows:

$$E_k^a \geq \sigma_k D_-^a \equiv \upsilon_k(D_-^a) \tag{3}$$

where $\sigma_k > 0$ stands for the *equity collateral requirement* for each unit of debt issued by an agent a in jurisdiction k.[16] See Giambona, Mello, and Riddiough (2016) for a discusison of covenants imposed on secured (non-recourse) mortgage contracts.

We denote the interest rate at state s especified in the mortgage contract by $r(s) - 1$ and for simplicity we set $r(s) = r$, for all $s \in \mathbf{S}$. Because the commercial mortgage is assumed to be non-recourse, the borrower's delivery rate at state s per unit of debt purchased will not be the face value r, but rather the minimum between the debt face value and the cash flow corresponding to the equity collateral. Formally, a borrower a's effective mortgage payoff at state s for CRE project in jurisdiction k is

$$Q_k(s) = \min\{r, \sigma_k c_k(y_k; p)(s)\}$$

Thus, if $r > \sigma_k c_k(y_k; p)(s)$, the borrower declares default and transfers the CRE asset payoff $\upsilon_k(D_-^a)c_k(y_k; p)(s)$ to the lender.

As in Geanakoplos and Zame (2014), we consider a "perfectly competitive world in which lenders and borrowers meet in a large market, and not a world with a single lender and borrower negotiating with each other". There is an agency (or clearing house) that manages the mortgage payments and determines the mortgage rate of return $\Phi_1 \in [0, 1]^S$ that is paid to the lenders at the different states of nature of period 2. In particular, given the borrowers' effective mortgage

[16]Notice that collateral constraint (3) differs from the standard one in general equilibrium where collateral is a durable consumption good (see e.g. Geanakoplos and Zame 2014). We instead require the borrower to keep a minimum amount of CRE equity as collateral in case of default. Also notice that constraint (3) substitutes the standard exogeneous short sale constraint of type $D^a \geq -\bar{D}^a$ with $\bar{D}^a > 0$. This is because, once we take into account the equity market clearing equation (2), we have $E_k^a \leq \bar{e}_k^{d_k}$ and therefore $D^a \geq -\bar{D}^a$ where $\bar{D}^a = \bar{e}_k^{d_k}/\sigma_k$ for our economy. Finally, notice also that in equilibrium we expect developers to be in the long side of the debt market, and thus the short sale constraint would become $D^{d_k} \geq -\bar{e}_k^{d_k}/\sigma_k$. This constraint rules out the possibility of indeterminacy in agents' portfolios choices in our setting with an endogenous financial structure and possibly redundant assets.

payments across jurisdictions, the agency delivers the following return at state s of period 2 for each unit of mortgage purchased in the first period:

$$\phi_s = \begin{cases} \frac{\sum_{k\in\mathbf{K}}\sum_{a\in\{\mathbf{D}_k\cup\mathbf{H}_k\}} Q_k(s)D^a_-}{r\sum_{k\in\mathbf{K}}\sum_{a\in\{\mathbf{D}_k\cup\mathbf{H}_k\}} D^a_-} & \text{if } \sum_{k\in\mathbf{K}}\sum_{a\in\{\mathbf{D}_k\cup\mathbf{H}_k\}} D^a_- > 0 \\ \\ 1 & \text{if } \sum_{k\in\mathbf{K}}\sum_{a\in\{\mathbf{D}_k\cup\mathbf{H}_k\}} D^a_- = 0 \end{cases}$$

2.3 The jurisdiction's profit function

Jurisdictions incur in some costs when providing public goods. Examples of these costs are roads, sewerage, fire protection, police, legal advisors, supervision, and accounting. Here we split the costs for local authority k between fixed costs, $\lambda(\alpha_k) > 0$, and a variable cost $\varepsilon(\alpha_k) > 0$. Both cost functions are defined in terms of the numeraire good. We assume that $\varepsilon(0) = 0$ and $\lambda(0) = 0$, and also that $\varepsilon(\alpha_k)$ and $\lambda(\alpha_k)$ are, respectively, homogenous of degree 1 and 0 in α_k. The latter assumption guarantees that, without loss of generality, we can normalize the return vector of a CRE asset. In terms of economic interpretation, making fixed and variable costs a function of α_k implies that the type of CRE asset dictates the associated cost of public infrastructure.

To finance these costs, the local authority imposes a tax on CRE property that is proportional to the agent's equity holdings. In particular, an agent holding E^a_k equity shares in asset j_k must pay $\gamma^a_k E^a_k$ units of the numeraire good to the jurisdiction, where $\gamma^a_k > 0$ is the agent type specific property tax rate. This model of taxation is consistent with the so-called "pass-through taxation", in which the owners of the CRE property are personally responsible for paying taxes and expenses according to some pari-passu rule, which normally takes the form of a proportional sharing rule with respect to the owners' equity holdings. The "pass-through taxation" model includes Limited Liability Companies (LLC), which are one of the most prevalent business forms in the United States.

We take tax rate γ_k as given in Sections 2 and 3. Later, in Section 4, we provide an extension to the model that makes this variable endogenous.

Assumption 3: *For all* $k \in \mathbf{K}$ *and all* $a \in \mathbf{A}$, γ_k^a *belongs to the compact set* $\Gamma_k^a = \{\gamma_k^a \in \mathbb{R}_+ : \varepsilon(\alpha_k) \leq \gamma_k^a \leq \bar{\gamma}_k\}$ *where* $\bar{\gamma}_k > \varepsilon(\alpha_k)$.

Let us now define a jurisdiction by a triplet $(\mathbf{k}, \alpha_k, \gamma_k)$ that specifies the set of players $\mathbf{k}$ in jurisdiction k, the type of CRE asset that developers can construct (determined by vector α_k), and the vector of property taxes $\gamma_k = (\gamma_k^{h_k}, \gamma_k^{d_k}, \gamma_k^i)$, respectively. Hereafter, we refer to $\mathbf{K}$ as the *jurisdiction structure,* i.e., $\mathbf{K} \equiv \{(\mathbf{k}, \alpha_k, \gamma_k),\ k = 1, ..., \mathcal{K}\}$. By defining the set of all jurisdictions in $\mathbf{K}$ that contain agent a by $\mathbf{K}^a \equiv \{(\mathbf{k}, \alpha_k, \gamma_k) \in \mathbf{K} : a \in \mathbf{k}\}$, we can restrict agents' choices. By abuse of notation, k designates de jurisdiction and also stands for the triplet $(\mathbf{k}, \alpha_k, \gamma_k)$. Later, in Section 3 we introduce a pre-stage where we endogenize (α_k, γ_k).

Given vector E^A, the profits of a jurisdiction k in the first period are given by

$$\pi_k \equiv \sum_{a \in \mathbf{k}} \gamma_k^a E_k^a - (\lambda(\alpha_k) + \varepsilon(\alpha_k) y_k)$$

Profits are redistributed among jurisdiction members. Consider an agent $a \in \mathbf{k}$ and let its share of jurisdiction k's profit be $\delta_k^a \in [0, 1]$. By choosing a vector $(\delta_k^a)_{a \in \mathbf{k}}$, such that $\sum_{a \in \mathbf{k}} \delta_k^a = 1$, the jurisdiction manager is effectively redistributing resources among agents in the jurisdiction. Thus, CRE property taxes have a redistributive effect in our model because jurisdiction profits revert to the agents that live and do business in the jurisdiction according to some weights.[17] A theory of political economy could be elaborated by endogenizing these shares (we leave this possibility for future research).

[17] This assumption is standard in competitive financial economies with intermediation costs. See e.g. Markeprand (2008) and Prechac (1996).

2.4 Optimization and equilibrium

The budget constraints of an agent $a \in \mathbf{A}$ in period 1 and state $s \in \mathbf{S}$ of period 2 are, respectively,

$$\sum_{l \in \{0\} \cup \mathbf{L}_1} p_{l1}(x^a_{l1} - \omega^a_{l1}) - p_{01} \sum_{k \in \mathbf{K}^a} \delta^a_k \pi_k + \tau D^a_+ - \tau D^a_- + \sum_{k \in \mathbf{K}^a} q_k(E^a_k - \bar{e}^a_k) + p_{01}\gamma_k E^a_k \leq 0 \tag{4}$$

$$p_0(s)(x^a_0(s) - \omega^a_0(s)) + \sum_{k \in \mathbf{K}^a} \sum_{l \in \mathbf{L}_2} p_{lk}(s) x^a_{lk}(s) \leq \phi_s D^a_+ - Q_k(s) D^a_- + \sum_{k \in \mathbf{K}^a} E^a_k c_k(y_k; p)(s) \tag{5}$$

Given equity prices $q \in \mathbb{R}^K_+$, commodity prices $p \in \mathbb{R}^{1+L'_1}_+ \times \mathbb{R}^{S(1+L_2K)}_+$, and the price of debt $\tau \in \mathbb{R}_+$, an agent a's *optimization problem* consists of choosing a vector

$$(x^a_1, x^a(1),\ \ldots,\ x^a(S), D^a_-, D^a_+, E^a) \in \mathbb{R}^{1+L'_1}_+ \times \mathbb{R}^{S(1+L_2K)}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}^K_+$$

that maximizes his utility function u^a, subject to his budget constraints of periods 1 and 2, the collateralized debt constraint (3), and the sign constraints $D^i_- = 0$, $E^{h_k}_{k'} = 0$ for all $k, k' \in \mathbf{K}$, and $x^a_{lk'}(s) = 0$ if $a \in \{d_k\} \cup \mathbf{H}_k$ and $l \in \mathbf{L}_2(k')$ with $k' \neq k$. W

The formal definition of an equilibrium is as follows:

Definition 1: *Given a jurisdiction structure $\mathbf{K}$, a competitive equilibrium for this economy consists of a system $(x^A, D^A_-, D^A_+, E^A, p, q, \tau, (\pi_k)_{k \in \mathbf{K}})$, such that:*

(i) each agent a solves its optimization problem;

(ii) for each jurisdiction k, the profit function π_k is

$$\pi_k = \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_k E^a_k - (\lambda(\alpha_k) + \varepsilon(\alpha_k) y_k)$$

(iii) the return of mortgage debt at state s is

$$\phi_s = \begin{cases} \dfrac{\sum_{k \in \mathbf{K}} \sum_{a \in \{\mathbf{D}_k \cup \mathbf{H}_k\}} Q_k(s) D^a_-}{r \sum_{k \in \mathbf{K}} \sum_{a \in \{\mathbf{D}_k \cup \mathbf{H}_k\}} D^a_-} & \text{if } \sum_{k \in \mathbf{K}} \sum_{a \in \{\mathbf{D}_k \cup \mathbf{H}_k\}} D^a_- > 0 \\ \\ 1 & \text{if } \sum_{k \in \mathbf{K}} \sum_{a \in \{\mathbf{D}_k \cup \mathbf{H}_k\}} D^a_- = 0 \end{cases}$$

(iv) the following market clearing conditions hold:

- *global market clearing for the numeraire consumption good in period 1:*

  $\sum_{a\in\mathbf{A}}(x_{01}^a - \omega_{01}^a + \sum_{k\in\mathbf{K}}(\lambda(\alpha_k) + \varepsilon(\alpha_k)y_k)) = 0$

- *global market clearing for the construction inputs in period 1:*

  $\sum_{a\in\mathbf{A}}(x_{l1}^a - \omega_{l1}^a) = 0, \forall l \in \mathbf{L}_1$

- *global market clearing for the numeraire consumption good at state $s \in \mathbf{S}$ of period 2:*

  $\sum_{a\in\mathbf{A}}(x_0^a(s) - \omega_0^a(s)) = 0,$

- *local market clearing for commercial goods:*

  $\forall k \in \mathbf{K}, \forall l \in \mathbf{L}_2, \sum_{a\in\mathbf{k}} x_{lk}^a(s) - f_{lk}(y_k)(s) = 0$

- *local market clearing for equity:*

  $\forall k \in \mathbf{K}, E_k^{d_k} + \sum_{i\in\mathbf{I}} E_k^i = \bar{e}_k^{d_k}$

- *global market clearing for debt:*

  $\sum_{a\in\mathbf{A}}(D_+^a - D_-^a) = 0$

In equilibrium we expect developers need to raise equity and issue non-recourse mortgage debt to pay for construction materials in the first period. For this equilibirum configuration, the capital structure of a CRE asset has the following structure:

Remark 1: *The capital structure of a CRE asset at jurisdiction k is endogenous and composed by the developer's debt $D_k^{d_k}$, the common equity of the entrepreneurial (developer) partner $E_k^{d_k}$, and the common equity of the capital (investor) partners $(E_k^i)_{i\in\mathbf{I}}$.*

2.4.1 Additional key assumptions and the existence result

A subtle condition in general equilibrium models is the lower semicontinuity property of the agent's budget constraint in the first period. The usual approach to guarantee this property is assuming that all agents of the economy have positive endowments of all goods. In our model, we cannot impose this assumption for the commercial goods because these goods are endogenously produced. To guarantee the existence of an interior point in the budget constraint set for each profile of prices, we consider the following assumptions on the endowments of construction inputs ($l \in \mathbf{L}_1$) and the numeraire good (indexed by "0"):[18]

Assumption 4: *For all agents* $a \in \mathbf{A}$, $\omega_{01}^a > (\lambda(\alpha_k)+\varepsilon(\alpha_k)(1+\mathcal{K}))\mathcal{K}$, $\omega_{l1}^a > 0, \forall\, l \in \mathbf{L}_1$, *and* $\omega_0^a(s) > 0$. *Moreover, for all developers* $d_k \in \mathbf{D}$, $\omega_{01}^{d_k} > \gamma_k \hat{y}_k$, *where* $\hat{y}_k \equiv TPF_k \cdot \Pi_{l\in\mathbf{L}_1}(\omega_{l1}^{d_k})^{\alpha_{lk}}$.

Another subtlety to guarantee the property of lower semicontinuity of the budget set correspondence has to do with the presence of portfolio constraints, which prevent the usual normalization of commodities and assets prices in the first period. Since we cannot impose additional restrictions on the agent's budget constraints and portfolio sets – as these are written to capture our particular setting – we consider the following mild impatience assumption for an economy with restricted participation:

Assumption 5: *For any agent* $a \in \mathbf{A}$ *and any* $x \in X^a$, *there exists a bundle* $\varrho(\theta, x) \in \mathbb{R}_+^{1+\mathbf{L}_1}$, *given* $\theta \in (0,1)$, *such that* $u^a(x_1 + \varrho(\theta, x), (\theta(x(s))_{s=1,\ldots,S}) > u^a(x_1, (x(s))_{s=1,\ldots,S})$.

Assumption 5 says that we can always find a large consumption for an agent in period 1 such that this agent is better off with this extra consumption in period 1 but less consumption in every state of period 2.[19] This assumption is satisfied by many different types of utility functions that are unbounded on the first period consumption, such as von-Neumann utility functions with quasi-linear, Cobb-Douglas, or Leontieff kernels, e.g., Cobb-Douglas, CES, and CARA (see Seghir and Torres-Martinez 2011). Also, notice that this type of utility function does not depend on the representation of individuals' preferences and does not require further assumptions on the portfolio sets.

[18]We leave for future research the relaxation of some of the elements in Assumption 4, namely, getting rid of the interiority assumptions of agents' endowments (see Rincon-Zapatero and Santos 2009 for similar issues). In Section 4, we provide a simplified version of our economy in which an equilibrium exists without requiring agents having positive endowments of all commodities in the first period.

[19]This way to reframe the standard impatience assumption of an agent preferring to consume today rather than tomorrow is convenient to state in terms of the "primitives" of the model (see).

Theorem 1: *Let Assumptions 1.i-iii, 2.i, 2.ii, 3, 4, and 5 hold. Then, given a jurisdiction structure* $\mathbf{K} \equiv \{(\mathbf{k}, \alpha_k, \gamma_k),\ k = 1, ..., \mathcal{K}\}$*, there exists a competitive equilibrium.*

The proof of Theorem 1 is left for the Appendix. This proof relies on fixed point theory.

2.4.2 Subtleties of our existence proof

In the rest of this section, we discuss the implications of imposing Assumptions 2, 4 and 5, and the technical subtleties of our existence proof in view of existing results in the literature of general equilibrium with segmented markets.

First, it is not trivial that a developer's budget set correspondence takes convex values. To see this, notice that the market value of a CRE asset depends on the developer's choice of materials. Also notice that the equity return in the second period depends on the CRE asset throught functions $(f_k)_{k \in \mathbf{K}}$, and so is endogenous. Then, to guarantee the convexity of a developer's budget set, we have to impose Assumption 2.ii, namely, we require that f_k is a concave and increasing function for every $k \in \mathbf{K}$.[20]

The main difficulty of our equilibrium existence proof has to do with the lower semicontinuity property of the agents' budget constraints. Since the CRE equity markets are segmented (CRE equity not available to all agents in the economy), we cannot take the usual approach where an auctioneer chooses both the commodity and security prices in the simplex. For if the auctioneer chooses the price of one type of CRE equity equal to 1, the remaining commodity, debt, and equity prices would be zero. But then, there would be jurisdictions with commodity and security prices equal to zero and the budget constraints of agents with single jurisdiction memberships would hold with equality. Lower semicontinuity of the budget correspondence would fail as a result since we could not guarantee the existence of an interior point. To circumvent this problem, we let the price auctioneer for the first period choose commodity prices in the simplex (a compact set). For asset prices, we have to find an endogenous upper bound.

[20]The convexity of a developer's budget set is not compromised by the budget constraint in the first period because the production function for the CRE asset is Cobb-Douglas and this function is assumed to be concave.

Another issue related to the lower semi-continuity of the budget set correspondence has to do with the existence of an interior point in the budget constraint. Even if agents have strictly positive endowments of the numeraire good in the second period, the value of endowments may not be strictly positive for some agents because we embed commodity prices into the simplex. To overcome this difficulty, we rely on two assumptions: 1) endowments of the numeraire and construction imputs are stritly positive for all agents in the first period, and 2) the nominal return of risk-free debt is positive. With prices chosen in the simplex and strictly positive endowments of all commodities in the first period, the value of the endowment in the first period is strictly positive. The following bundle is an interior point of the budget constraints for all agents: all commodity purchases equal to zero, CRE equity positions also equal to zero, and a small position in risk-free debt.

Next, we argue that previous results in the literature of market segmentation with a fixed point theory approach are not useful for finding endogenous upper bounds for CRE equity prices.

In one strand of this literature, authors consider exogenous trading constraints, but impose financial survival assumptions or spanning conditions on the set of admissible portfolios (see for instance Balasko, Cass, and Siconolfi 1990 for a seminal contribution, and, more recently, Angeloni and Cornet 2006, Aouani and Cornet 2009, and Cornet and Gopalan 2010). These assumptions are not suitable for our particular setting. For instance, households do not satisfy the survival assumption because they do not trade equity. Also, the property tax on equity prevents us from considering a spanning condition on the set of admissible portfolios.

Another strand of the literature applies fixed point theory but in a setting with endogenous portfolio constraints. For instance, the paper by Cea-Echenique and Torres-Martinez (2016) imposes a super-replication condition that allows payments associated with segmented contracts to be super-replicated by durable goods and/or contracts that agents can short sell. This assumption does not fit into our framework because of the following reasons: 1) we do not have durable goods, 2) there is a collateral constraint on debt, and 3) CRE equity returns are endogenous as they depend on the developers' choices. In a similar context, Faias and Torres-Martinez

(2017) consider instead assumptions on the utility function – precisely, indiference curves through individuals' endowments do not intersect the consumption set boundary. However, the "essentiality of commodities" assumption requires endowments to be strictly positive, which is not true in our setting for the case of commercial goods in the second period. Finally, the paper by Seghir and Torres Martinez (2011) considers an impatience assumption in the utility function, which requires that small reductions in the consumption of the second period can be compensated by an increase in the consumption of the first period. We adapt this assumption to our framework by imposing Assumption 5. A key difference from Seghir and Torres Martinez (2011) is that they assume that second period endowments are strictly positive, which is not true in our case. We overcome this difficult by imposing Assumption 4, which says that developers have enough endowment of the numeraire in the first period to pay the property tax associated with the equity of the CRE asset that they could produce with their endowment of construction inputs. With this trick, we make sure that developers can always transfer wealth from the first period to the second period.

2.4.3 Financial innovation with segmented markets: the case of real estate development

We conclude this section with a remark that relates our equilibrium setting with Rahi and Zigrand (2009) model of financial innovation with segmented markets.

Remark 2: *Our Walrasian economy has segmented good and equity markets, and financial markets can be incomplete. This setting is similar to Section 3 of Rahi and Zigrand (2009) in the sense that we do not consider "arbitrageurs". The main difference from our models is that we allow for global investors that can buy and sell CRE equity in multiple jurisdictions. An equilibrium exists for our economy and, therefore, we can rule out arbitrage opportunities even when global investors can operate in multiple jurisdictions.*

3 Optimal property tax and selection of CRE assets

The presence of global investors makes public policy an important tool for local and national governments as they seek to attract capital for commercial real estate development. Recurrent policies used by local governments are fiscal and land use policies. The most important fiscal instrument for local governments is property taxes (e.g., property taxes contribute more than 60 percent of the City of Madison's revenues in Wisconsin). There is also a wide spectrum of land use policies, but one of the most important and effective one for urban design is the restriction that a jurisdiction may impose on the type of a real estate development, e.g., size and asset class. Local governments' public policies are crucial in attracting CRE investments. If well executed, they create value for the jurisdiction, not only in terms of tax revenues, but also in terms of job creation and amenities for its citizens.

So far, we have taken the property tax γ_k and the Cobb-Douglas exponents α_k for all jurisdictions $k \in \mathbf{K}$ as parameters and proved that the equilibrium set is non-empty.[21] We denote by $\mathcal{E}(\alpha, \gamma)$ the set of competitive equilibria for a given profile of types of CRE assets and corresponding taxes, $(\alpha, \gamma) = ((\alpha_1, \gamma_1), ..., (\alpha_{\mathcal{K}}, \gamma_{\mathcal{K}}))$. In this section, we propose an extension to our model of Section 2 in which strategic jurisdiction managers choose (α, γ) in a non-cooperative game. More precisely, we consider a strategic game in which jurisdiction authorities decide their respective types of CRE projects and property taxes. For this, each of these jurisdiction authorities evaluates a profile of strategies by replacing the equilibrium (or combination of equilibria) associated with the profile of policy variables into their corresponding profit function.

Let each jurisdiction authority consider a discrete set Λ_k of possible types of CRE development projects (that is, $\alpha_k \in \Lambda_k$ for all $k \in \mathbf{K}$) and a discrete set $\Gamma_k = \Gamma_k^{h_k} \times \Gamma_k^{d_k} \times \Gamma_k^i$ of possible fiscal policies for all $k \in \mathbf{K}$.

Assumption 7: *The spaces Λ_k and Γ_k are discrete for all $k \in \mathbf{K}$.*

[21] Recall that developers in a jurisdiction can only construct one type of CRE asset, which is chosen by the jurisdiction manager. However, it is possible that developers in the same jurisdiciton would choose a different combination of materials and thus develop the same CRE asset but with different sizes. Notice that allowing for the construction of multiple types of CRE assets would complicate our analysis because that would require having the jurisdiction manager choose multiple vectors of Cobb-Douglas production weights.

Examples of available CRE development projects in Λ_k are shopping centers, retail spaces, offices, and hotels. Examples of property taxes in Γ_k are a non-invasive fiscal policy with low property taxes and a redistributive fiscal policy with high property taxes. The jurisdiction manager chooses pairs $(\alpha_k, \gamma_k) \in \Lambda_k \times \Gamma_k$ that satisfy certain criteria. An example of such criteria could be to consider only those CRE development projects that are compatible with a low tax scheme, i.e., those CRE development projects with low fixed and variable costs, i.e., low $\lambda(\alpha_k)$ and $\varepsilon(\alpha_k)$, respectively.

The aim of each jurisdiction manager is to maximize its profits, which consist of the difference between the property taxes the jurisdiction receives minus the costs it incurs to provide a CRE project. In particular, given a profile (α, γ) of CRE projects and taxes, let $(x^A, D^A, E^A, p, q, \tau, (\pi_k)_{k\in\mathbf{K}})$ be an equilibrium vector of the economy; then, for this equilibrium, the profit of a jurisdiction k is

$$\pi_k = \sum_{a\in\{d_k\}\cup\mathbf{I}} \gamma_k E^a_k - (\lambda(\alpha_k) + \varepsilon(\alpha_k) y_k)\,.$$

As discussed by Allen and Gale (1989), it would be difficult to provide general conditions that guarantee uniqueness in this environment with endogenous security design. Here, we face the same difficulty because, for each profile (α, γ), the set of equilibria may not be single. We circumvent this problem by assuming that jurisdiction k's manager evaluates its payoff using an index $\Psi^k(\mathcal{E}(\alpha, \gamma))$ that depends on the equilibrium set of profits that emerge given the profile of parameters (α, γ).

Let $\Pi_k(\alpha, \gamma) = \{\pi_k : (x^A, D^A_-, D^A_+, E^A, p, q, \tau, (\pi_k)_{k\in\mathbf{K}}) \in \mathcal{E}(\alpha, \gamma)\}$, then, $\Psi^k(\mathcal{E}(\alpha, \gamma)) = \Psi^k(\Pi_k(\alpha, \gamma))$. When the equilibrium set $\mathcal{E}(\alpha, \gamma)$ is unique, then for each profile $(\alpha, \gamma) \in \Lambda_k \times \Gamma_k$, $\Pi_k(\alpha, \gamma) = \{\pi_k\}$ is also unique and thus $\Psi^k(\mathcal{E}(\alpha, \gamma)) = \pi_k$. When the equilibrium set for a profile (α, γ) is not single, we define index $\Psi^k(\mathcal{E}(\alpha, \gamma))$ using a measurable selector of the equilibrium correspondence $\mathcal{E}(\alpha, \gamma)$. In particular, let $(\tilde{x}^A, \tilde{D}^A_-, \tilde{D}^A_+, \tilde{E}^A, \tilde{p}, \tilde{q}, \tilde{\tau}, (\tilde{\pi}_k)_{k\in\mathbf{K}})(\alpha, \gamma)$ be the measurable selector of correspondence $\mathcal{E}(\alpha, \gamma)$. Then, $\Psi^k(\mathcal{E}(\alpha, \gamma)) = \tilde{\pi}_k$. This index is well defined if such measurable selector exists.[22]

[22]The notion of an equilibrium selector is well-known and has been used in different strands of the literature; see,

Lemma 7: *There exists a measurable selector* $(\tilde{x}^A, \tilde{D}_-^A, \tilde{D}_+^A, \tilde{E}^A, \tilde{p}, \tilde{q}, \tilde{\tau}, (\tilde{\pi}_k)_{k\in\mathbf{K}})$ *for the equilibrium correspondence* $\mathcal{E}$.

The proof of this lemma follows by applying the Kuratowski-Ryll-Nardzewski measurable selection theorem (see Aliprantis and Border 2006, p. 600). To see this, notice that in our model, the set of available CRE development projects and tax profiles is finite and, therefore, the equilibrium correspondence $\mathcal{E}(\alpha, \lambda)$ is trivially a weak measurable correspondence.

Considering equilibrium selections has implications for economic policy. To see this, notice that the index function $\Psi^k(\mathcal{E}(\alpha, \gamma))$ can be defined in many different ways. For example, if the jurisdiction manager is risk averse and has prudent behavior, then it would be reasonable to consider an index that corresponds to the minimal profit generated among the set of possible equilibria, i.e.,

$$\Psi^k(\alpha, \gamma) = \min\{\pi_k : \pi_k \in \Pi_k(\alpha, \gamma)\}.$$

Because for each profile (α, γ) the set of equilibrium profits belongs to a compact set, the proof of existence of this minimum is trivial.

Local authorities play a strategic game $\mathbb{G} = \{(\Lambda_k \times \Upsilon_k, \Psi^k)_{k\in\mathbf{K}}\}$, where $\Lambda_k \times \Upsilon^k$ and Ψ^k are, respectively, the strategy set and the payoff function of local authority $k \in \mathbf{K}$. A Nash equilibrium in mixed strategies for this game consists of a probability measure over the set of property taxes and CRE projects.

The game $\mathbb{G}$ pins down the local authorities' equilibrium strategies.

Definition 2: *An equilibrium for the economy is a profile* $(\alpha, \gamma) \in (\Lambda_1 \times \Upsilon_1) \times \cdots (\Lambda_{\mathcal{K}} \times \Upsilon_{\mathcal{K}})$ *and a Walrasian equilibrium system* $(x^A, D_-^A, D_+^A, E^A, p, q, \tau, (\pi_k)_{k\in\mathbf{K}}) \in E(\alpha, \gamma)$, *such that*

(i) (α, γ) *is a Nash equilibrium for the game* $G = \{(\Lambda_k \times \Upsilon_k, \Psi^k(\alpha, \gamma))_{k\in\mathbf{K}}\}$, *and*

for example, Miao (2006) in recursive macroeconomics, Berliant and Page (2001) in public economics, Simon and Zame (1990) in game theory, Faias, Moreno, and Pascoa (2002) and Luque and Faias (2017) in financial economics, and Stahn (1999) for a general equilibrium model with monopolistic behavior. In these models, in general, a profile of actions gives rise to a set of equilibrim outcomes. Then, to obtain an equilibrium existence result with well-defined payoff functions, which themselves depend on these profiles, authors use equilibrium selections. For example, this is the case of Cournot-Walras models with a continuous space of actions, where continuous random selections are used.

(ii) *for each* $k \in K$, $\pi_k = \Psi^k(\varepsilon(\alpha, \gamma))$.

Theorem 2: *Let Assumption 7 hold. Then, there exists an equilibrium for the first stage of the economy.*

Theorem 2 guarantees the existence of an equilibrium, possibly in mixed strategies (due to the discreteness of sets Λ and Γ).

4 Equilibrium characterization

The seminar paper by Modigliani and Miller (1958) shows that the capital structure of a financial asset is irrelevant. However, this result ignores important institutional features, such as limits on debt issuance, different tax treatment between equity and debt, and default risk – see Admati and Hellwig (2013) for a recent paper that discusses these possibilities in the rather different context of the banking sector, Gau and Wang (1990) for a discussion of usual restrictions in real estate investments and their relationship with the capital structure of an income-producing property, and Sun, Titman, and Twite (2015) for evidence of the impact of the recent financial crisis on the limits to debt capacity for commercial real estate assets. In our model, the capital structure of a real estate development is relevant because we consider property taxes, default on non-recourse collateralized mortgage debt, and potentially incomplete markets. Moreover, capital structures of CRE development projects in different jurisdictions are interdependent in our setting with segmented markets because investors can buy equity in multiple jurisdictions.

In this section, we perform an equilibrium analysis of these possibilities. For simplicity, we focus on an economy with two jurisdictions ($k = 1, 2$), two states of nature in the second period ($s = s_1, s_2$), and only one construction material ($l = 1$) used for development in both jurisdictions. We normalize to 1 the prices of the numeraire good ($l = 0$) in period 1 and also at states s_1 and s_2 in period 2. Function $f_k(y_k)(s) : y_k \to x_k(s) \in \mathbb{R}_+$, which maps the CRE asset size into an amount of a single commercial good at state s, is assumed to be linear. In particular, $f_k(y_k)(s) = Y_k(s)y_k$, where $Y_k(s) \geq 0$ for $s = s_1, s_2$ and $k = k_1, k_2$. For simplicity, we consider only one type of CRE asset and accordingly we equate the coefficient α_k in production function 1 between jurisdictions, i.e., $\alpha_1 = \alpha_2$. The promised mortgage payoff r is set equal to 1.

Each jurisdiction k has two households and one developer, denoted by h_k, H_k, and d_k, respectively. The real estate equity investor i belongs to both jurisdictions. The utility functional form of an agent $a \in \mathbf{A}$ is

$$u^a(x^a) = \sum_{l=\{0,1\}} \theta_l^a \ln x_l^a + \sum_{s=\{1,2\}} (\theta_0^a(s) \ln x_0^a(s) + \theta_1^a(s) \ln x_1^a(s) + \theta_2^a(s) \ln x_2^a(s))$$

where the θ-parameters denote agent a's weights assigned to the different available goods.[23] We set all θ-parameters are zero, except for $\theta_0^{H_1} > 0$, $\theta_1^{h_1}(s) > 0$, $\theta_2^{h_2}(s) > 0$, $\theta_0^{d_1}(s) > 0$, $\theta_0^{d_2}(s) > 0$, $\theta_0^i(s) > 0$, for $s = s_1, s_2$. Households h_k and H_k differ by when they prefer to consume. We think of h_k as a young household who enjoys the consumption of his respective commercial good at both states of the second period, while H_k is an old household who enjoys the consumption of the numeraire good in the first period. Both developers and the investor only enjoy the consumption of the numeraire good in the second period.

Endowments are as follows. Old household H_k owns $\omega_1^{H_k} > 0$ units of the construction material. Young household h_k is endowed with $\omega_0^{h_k} > 0$ units of the numeraire good (we assume $\omega_0^{h_1} = \omega_0^{h_2}$) and $\omega_0^{h_k}(s) > 0$ units of the numeraire good at state $s = 1, 2$. The real estate equity investor i is endowed with $\omega_0^i(s) > 0$ units of the numeraire good at state $s = 1, 2$ of the second period. Developers have no commodity endowments whatsoever, so debt and equity are the only means to transfer wealth from the first to the second period.

We think of the real estate investor as an "equity REIT", whose business only consists of buying CRE equity; thus, we exclude this investor from borrowing in the debt market; thus, $D_-^i = 0$. In addition, we restrict households from investing in CRE equity and leave this possibility only to the real estate equity investor and the developers.

[23]The consumption of commercial goods is restricted to only those households that belong to the jurisdiction in question. For example, the consumption of the commercial good produced in the second jurisdiction, denoted by $x_2^a(s)$, can only be positive in this simplified economy if $a \in \mathbf{k}_2$.

Finally, to avoid dealing with the sharing of jurisdiction profits, we set a policy that assigns jurisdiction profits to the corresponding old household, i.e., $\delta^{H_k} = 1$ for $k = k_1, k_2$.

4.1 Determinants of mortgage debt and equity market prices

For this economy, an equilibrium as specified by Definition 1 satisfies the following properties.

Proposition 1 (Mortgage recovery rate): *ϕ_s, the equilibrium mortgage recovery rate at state s, can be expressed as a weighted sum of effective delivery rates $Q_1(s)$ and $Q_2(s)$, where weights are the corresponding developer's share of debt with respect to total debt in the economy. Delivery rate ϕ_s also increases with the young households endowments of the numeraire good at state s. Formally,*

$$\phi_s = \sum_k \omega_0^{h_k}(s) + \frac{D_-^{d_1}}{D_-^{d_1} + D_-^{d_2}} Q_1(s) + \frac{D_-^{d_2}}{D_-^{d_1} + D_-^{d_2}} Q_2(s) \tag{6}$$

We define *mortgage yield* at state s as ϕ_s/τ, *mortgage default loss* of the CRE project in jurisdiction k as $1 - Q_k(s)$, and *high-risk default jurisdiction* as the jurisdiction facing a negative shock to its effective mortgage delivery rate $Q_k(s)$. With this definitions in mind, we work out expression (6) to assert the following:

Corollary 1 (Mortgage yield): *The mortgage yield at a given state s decreases with mortgage default losses at that state, the developer's debt face value in the high-risk default jurisdiction, and the scarcity of young households' resources at state s with respect to their resources in the first period. Formally,*

$$\frac{\phi_s}{\tau} = \frac{\sum_k \omega_0^{h_k}(s)}{\sum_k \omega_0^{h_k}} + \frac{D_-^{d_1}}{\sum_k \omega_0^{h_k}} Q_1(s) + \frac{D_-^{d_2}}{\sum_k \omega_0^{h_k}} Q_2(s) \tag{7}$$

Next, let us define $\hat{\phi}_s = (\lambda_s^{h_k}/\lambda_1^{h_k})\phi_s$ as the household h_k's discounted personalized recovery rate using household h_k's shadow values $\lambda_s^{h_k}$ and $\lambda_1^{h_k}$ for his budget constraints in period 1 and state s, respectively. Similarly, we define $\hat{Q}_k(s) = (\lambda_s^{d_k}/\lambda_1^{d_k})Q_k(s)$ as the developer d_k's discounted personalized effective delivery rate using developer d_k's shadow values $\lambda_s^{d_k}$ and $\lambda_1^{d_k}$ for his budget constraints at state s and period 1, respectively.

Proposition 2 (Binding collateral constraint and expected default): *The shadow value of collateral constraint* (3) *corresponding to mortgage debt $D_-^{d_k}$ and denoted by ξ^{d_k} is a function of the expected default losses of the CRE project in jurisdiction k. Formally,*

$$\sum_s \left(\hat{\phi}_s - \hat{Q}_k(s)\right) = \frac{\xi^{d_k}\sigma_k}{\lambda_1^{d_k}} \tag{8}$$

Corollary 2 (Mortgage debt valuation): *The negative effect of a higher default loss $1 - Q_k(s)$ on the mortgage debt equilibrium price τ more than offsets the positive impact of a higher default loss $1 - Q_k(s)$ on τ through the developer d_k's collateral constraint shadow price ξ^{d_k}.*

Proposition 3 (Equity valuation): *The equity equilibrium price q_k accrued of the CRE tax γ_k increases with the developer d_k's discounted value of CRE cash flows and the expected default losses of mortgage debt in jurisdiction k (through shadow value ξ^{d_k}). Formally,*

$$q_k + \gamma_k = \sum_s \frac{\lambda_s^{d_k}}{\lambda_1^{d_k}} c_k(y_k; p)(s) + \xi^{d_k} \tag{9}$$

where $c_k(y_k; p)(s) = p_k(s)Y_k(s)y_k$.

Equilibrium condition (8) captures how a higher risk of mortgage debt default in jurisdiction k's CRE project increases market pressure on equity through price q_k. Equilibrium condition (8) also shows how fiscal policy can be used to offset the positive impact of higher default risk (through higher ξ^{d_k}) on q_k; namely, a decrease in γ_k may (partially) offset an increase in ξ^{d_k}.

4.2 CRE construction patters and capital markets

We now turn our attention to differences in construction patterns between jurisdictions. We assume that both jurisdictions have the same space available for construction, but may differ in the amount of construction material employed for CRE development, here denoted by $x_1^{d_k}$ for developer d_k in jurisdiction k. We define the *height* of commercial real estate in jurisdiction k by $Height_k = x_1^{d_k}$. For our next result, we also find convenient to define the developer d_k's expected market price of a CRE space unit (square foot) in his development project j_k, net of the productivity factor TFP_k, by

$$m_k \equiv \sum_s \frac{\lambda_s^{d_k}}{\lambda_1^{d_k}} p_k(s) Y_k(s)$$

Proposition 4 (Differences in construction patterns between jurisdictions): *CRE height in jurisdiction k_1 increases with respect to jurisdiction k_2 if:*

1. *Developer d_1's "equity capital" $E_1^{d_1}$ in CRE project j_1 increases with respect to developer d_2's "equity capital" $E_2^{d_2}$.*
2. *The CRE project j_1's productivity parameter TFP_1 increases with respect to TFP_2;*
3. *m_1, the expected market price of a CRE space unit (square foot) net of productivity gains increases in CRE project j_1, increases with respect to m_2.*

Formally, the following equilibrium equation captures items 1, 2 and 3 of Proposition 4:

$$\frac{Height_1}{Height_2} = \left(\frac{E_1^{d_1}}{E_2^{d_2}} \frac{TFP_1}{TFP_2} \frac{m_1}{m_2} \right)^{1/1-\alpha} \tag{10}$$

The next corollary uses the collateral constraint (3) on mortgage debt $D_-^{d_k}$ to provide an alternative way of looking at item 1 in Proposition 4 in terms of collateral requirements σ_1 and σ_2.

Corollary 3: *When developers' collateral constraints bind in both jurisdictions, a laxer collateral requirement in jurisdiction k_1 with respect to jurisdiction k_2 (a higher σ_1 with respect to σ_2) increases CRE height in k_1 with respect to CRE height in k_2. Formally, we can rewrite expression (10) as follows:*

$$\frac{Height_1}{Height_2} = \left(\frac{\sigma_1 D_-^{d_1}}{\sigma_2 D_-^{d_2}} \frac{TFP_1}{TFP_2} \frac{m_1}{m_2} \right)^{1/1-\alpha} \tag{11}$$

Now we discuss the general equilibrium effects of a higher expected default risk on equity and debt prices and the capital structures of CRE development projects in the two jurisdictions. Without loss of generality, let jurisdiction k_2 experience a higher mortgage default loss at state s_2, i.e., $1 - Q_2(2)$ increases. This shock may be due to a decrease in production capacity $Y_2(2)$ or a lower productivity parameter TFP_2.

Proposition 5: *A higher mortgage default loss at state s_2 in jurisdiction k_2 induces the real estate investor i to rebalance its equity investments toward the high-risk jurisdiction, crowding out developer d_2's equity $E_2^{d_2}$ and debt $D_-^{d_2}$, while increasing developer d_1's equity $E_1^{d_1}$ and debt $D_-^{d_1}$. As a result, construction booms in the low-risk jurisdiction k_1 and contracts in the high-risk jurisdiction k_2, in turn increasing the CRE height in jurisdiction k_1 with respect to jurisdiction k_2.*

4.3 Interdependent capital structures without mortgage default

Mortgage default and competition for construction materials are not necessary conditions for the real estate investor to rebalance its equity portfolio between jurisdictions. To illustrate this, we slightly modify our previous economy by assuming recourse mortgage debt where payoff is always $r = 1$ (debt is risk-free). Also, there are now two construction materials. CRE development uses a different material in each jurisdiction; in particular, $y_1^{d_1} = TFP_1(x_{11}^{d_1})^{\alpha_1}$ and $y_2^{d_2} = TFP_2(x_{21}^{d_2})^{\alpha_2}$, where material $l = 1$ is used for CRE development in jurisdiction k_1 and material $l = 2$ is used for CRE development in jurisdiction k_2. We ignore default and the collateral constraint (3) on mortgage debt, but impose the following exogenous short sale constraint on recourse debt to guarantee equilibrium existence (see Hart 1979): $D_-^a \leq \bar{D}^a$, where $\bar{D}^a > 0$. We refeer to $\bar{D}^a$ as the agent a's debt capacity. Finally, we let the old household H_k be the sole owner of material $l = k$, i.e., $\omega_1^{H_1} > 0$, $\omega_2^{H_1} = 0$, $\omega_2^{H_2} > 0$, and $\omega_1^{H_2} = 0$. The rest of specifications are similar to our previous economy. The following proposition highlights the particular equilibrium properties for this alternative economy.

Proposition 6: *For this economy with risk-free debt and no developers' competition for con-*

struction materials, an equilibrium satisfies the following properties:

1. *Commercial good price differentials across states of nature in the second period are driven by the CRE asset's commercial good production capacity (i.e., $f_k(y_k)(s_1)$ versus $f_k(y_k)(s_2)$);*

2. *The CRE equity price accrued of the property tax in a jurisdiction is driven by the CRE asset's cash flows (the default term is absent in this specification);*

3. *Construction input price differentials across jurisdictions are driven by the difference in marginal productivity of the jurisdiction-specific CRE assets.*

We finish this section with some numerical examples that illustrate how the CRE capital structures in different jurisdictions may change after a shock to a developer's funding capacity or a different local fiscal policy.

Example 1 (benchmark): Let us consider the following parameter values: $\omega_1^{H_1} = \omega_2^{H_2} = \omega_0^{h_1} = \omega_0^{h_2} = \omega_0^i = 1$, $\omega_0^{H_1}(1) + \omega_0^{H_2}(1) = \omega_0^{H_1}(2) + \omega_0^{H_2}(2) = 1$, $r = 1$, $\bar{D}^{d_1} = \bar{D}^{d_2} = 1$, $TFP_1 = TFP_2 = 1$, $\alpha_1 = \alpha_2 = 0.6$, $Y_1(1) = Y_2(1) = 1$, $Y_1(2) = Y_2(2) = 0.5$, $\varepsilon(\alpha_1) = \alpha_1/6$, $\varepsilon(\alpha_2) = \alpha_2/6$, $\lambda(\alpha_1) = \hat{\lambda}_1\alpha_1$ with $\hat{\lambda}_1 = 0.333$, $\lambda(\alpha_2) = \hat{\lambda}_2\alpha_2$ with $\hat{\lambda}_2 = 0.333$, $\delta_1^{H_1} = 0$, $\delta_1^{h_1} = \delta_2^{h_2} = 0.7$, $\delta_1^{d_1} = \delta_2^{d_2} = 0.2$, $\delta_1^i = \delta_2^i = 0.1$, and $\gamma_1^{a_1} = \gamma_1 = 0.5$ and $\gamma_2^{a_2} = \gamma_2 = 0.5$ for $a_1 = d_1, i$ and $a_2 = d_2, i$. All θ-parameters are equal to zero, except for $\theta_0^{H_1} = \theta_0^{H_2} = 3$, $\theta_1^{h_1}(1) = \theta_2^{h_2}(1) = 1$, $\theta_1^{h_1}(2) = \theta_2^{h_2}(2) = 0.5$, $\theta_0^{d_1}(s) = \theta_0^{d_2}(s) = 0.24$, and $\theta_0^i(s) = 0.52$. For these parameter values, we obtain a unique equilibrium solution where $q_1 = q_2 = 2.500$, $\tau = 1.440$, $p_1 = p_2 = 1.500$, $p_1(1) = p_2(1) = 1.500$, $p_1(2) = p_2(2) = 3.000$, and $E_1^i = E_2^i = 0.173$.

For the above parameters, Figure 1 illustrates the equilibrium capital structures of CRE assets j_1 and j_2 in an economy where the developer's debt, the developer's equity, and the investor's equity are the same in both jurisdictions (for the sake of brevity, we leave the details of parameter and equilibrium values for the Appendix).

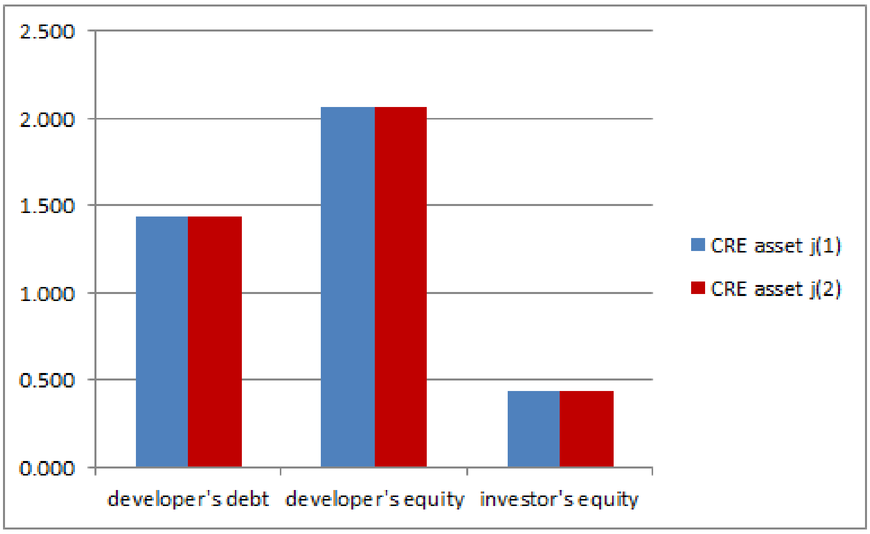

Figure 1: This figure illustrates the different capital structure components of CRE assets j_1 and j_2, given the parameter values of Example 1 in the Appendix. Quantities are expressed in real terms, i.e., nominal amount times the corresponding price.

Example 2 (Shocks to developers' funding capacities):[24] Let us modify our benchmark example by increasing the developer d_1's debt capacity to $\bar{D}^{d_1} = 1.1$, while decreasing the developer d_2's debt capacity to $\bar{D}^{d_2} = 0.9$. Roughly speaking, the access to funding for the developer in the second jurisdiction (k_2) worsens with respect to developer in the first jurisdiction (k_1).

Figure 2 illustrates the equilibrium values of the different capital structure components of CRE assets j_1 and j_2 for this case. There we see that, compared to our benchmark example in Figure 1, the developer with worse funding capacity (d_2) decreases its absolute real exposure to CRE debt and equity, while the investor increases its equity exposure. The opposite happens in the CRE development project of the developer with better funding capacity. Thus, when a developer has poorer access to debt financing, the equity investor finds optimal to fill the gap. Leverage (debt-to-equity ratio) decreases (increases) in the jurisdiction where the developer has worse (better) funding capacity. Thus, leverage shifts from the jurisdiction with shortage of debt to the jurisdiction with better debt funding capacity.

[24]See Sun, Titman, and Twite (2015) for evidence of the impact of the recent financial crisis on the limits to debt capacity for commercial real estate assets.

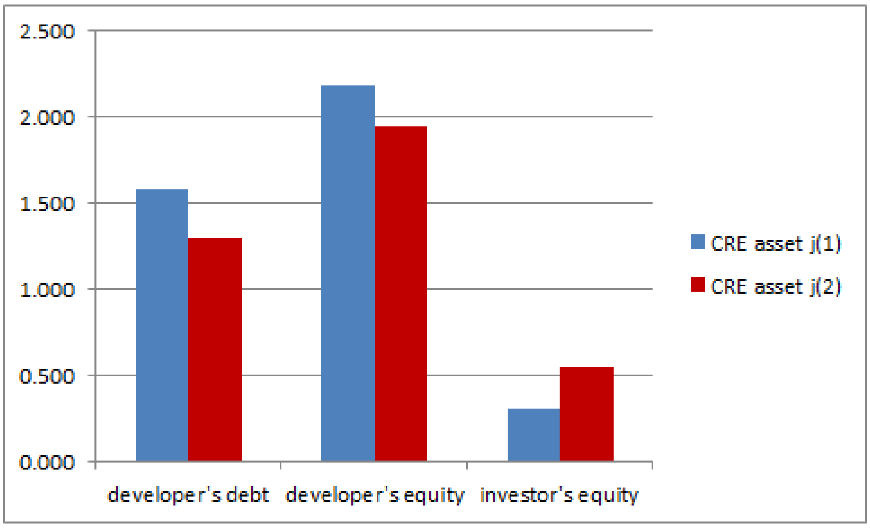

Figure 2: This figure illustrates the different capital structure components of CRE assets j_1 and j_2, given the parameter values of Example 2 in the Appendix. Quantities are expressed in real terms.

Example 3 (The property tax rate increases in jurisdiction k_2 to finance an increase in public spending): We next consider a situation where jurisdiction k_2's fixed cost $\hat{\lambda}_2$ increases from $0.2.$ to 0.5 (public spending increases) and finances this by increasing its tax rate γ_2 from 0.5 to 0.8. Figure 3 illustrates how the CRE capital structures in both jurisdictions change as a result. The increase in γ_2 decreases the developer d_2's equity compared to the equilibrium value in Figure 1 due to the developer's substitution of equity for tax free debt. The developer d_2's debt and the investor's equity contributions increase as a result. Roughly speaking, we find that a jurisdiction that increases local government spending on public infrastructure and finances these additional spending with higher taxes ends up with more levered developers, while jurisdictions with more conservative fiscal policies can effectively reduce developers' leverage.

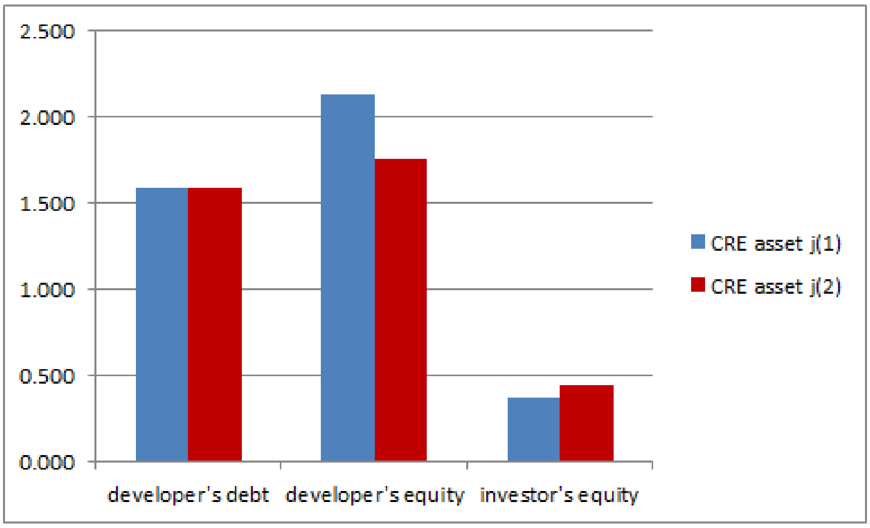

Figure 3: This figure illustrates the different capital structure components of CRE assets j_1 and j_2, given the parameter values of Example 3 in the Appendix. Quantities are expressed in real terms.

Example 4 (Competition between jurisdiction managers): Finally, we illustrate by means of a numerical example how jurisdiction competition in property taxation may result in a "race to the bottom". For this, we consider a setting where the two jurisdictions compete through taxation and land regulation to attract equity investments. For this, we consider again the parameter values used to construct our benchmark example in Figure 1, with the exception that now jurisdiction k's set of strategies is $\Lambda_k \times \Gamma_k = \{(\alpha_k^{high}, \gamma_k^{high}), (\alpha_k^{low}, \gamma_k^{low})\} = \{(0.80, 0.53), (0.60, 0.50)\}$, for both $k = k_1, k_2$. The economic interpretation is that a high Cobb-Douglas exponential parameter α is associated with a high variable cost for the jurisdiction, which in turn requires a high property tax. Jurisdiction manager $k = 1, 2$ chooses (α_k, γ_k) in $\Lambda_k \times \Gamma_k$ in order to maximize the following profit function:[25]

$$\pi_k = \left(1 - \frac{\alpha_k}{6\gamma_k}\right)\left(\tau(\alpha,\gamma)\bar{D}^{d_k} + (q_k(\alpha,\gamma) + \gamma_1)E_k^i(\alpha,\gamma) - p_k(\alpha,\gamma)\omega_k^{H_k}\right) - \hat{\lambda}_k\alpha_k \qquad (12)$$

where $(\alpha, \gamma) = (\alpha_1, \alpha_2, \gamma_1, \gamma_2)$. For simplicity, we assume that $\delta_k^{d_k} = 0$ and $\delta_k^i = 0$, for $k = 1, 2$.

[25]We obtain function (12) by first writing the jurisdiction k's profit function as $\pi_k = (\gamma_k - \alpha_k/6)TFP_k\left(x_{1k}^{d_k}\right)^{\alpha_k} - \hat{\lambda}_1\alpha_k$, and then replacing $TFP_k\left(x_{1k}^{d_k}\right)^{\alpha_k}$ with $\left(\tau(\alpha,\gamma)\bar{D}^{d_k} + (q_k(\alpha,\gamma)+\gamma_1)E_k^i(\alpha,\gamma) - p_{1k}(\alpha,\gamma)\omega_{1k}^I\right)/\gamma_k$ using (27a) and (27b), and taking $\delta_k^{d_k} = 0$.

After computing the equilibrium associated with each pair of strategies and the corresponding profit functions for each jurisdiction manager, we obtain the following payoffs:

Jurisdiction $k = 2$ Jurisdiction $k = 1$	$(\alpha_2^{high}, \gamma_2^{high})$	$(\alpha_2^{low}, \gamma_2^{low})$
$(\alpha_1^{high}, \gamma_1^{high})$	$(0.13, 0.13)$	$(0.13, 0.20)$
$(\alpha_1^{low}, \gamma_1^{low})$	$(0.20, 0.13)$	$(0.20, 0.20)$

$(\alpha_k^{low}, \gamma_k^{low})$ is a dominant strategy for both $k = 1, 2$ in this game. Thus, the Nash equilibrium consists of the pair of strategies $((\alpha_1^{low}, \gamma_1^{low}), (\alpha_2^{low}, \gamma_2^{low}))$. This result resembles the "race to the bottom" concept in literature of competition among political jurisdictions (Cary 1974, Drezner 2001, and Carruthers and Lamoreaux 2016). In particular, we find that low property taxes are optimal for profit maximizing jurisdictions that compete to attract global real estate equity investments.

5 Conclusions

In this paper we build a general equilibrium model of CRE development. The cash flows of a CRE asset depend on the amount of commercial goods sold to households in the jurisdiction where the asset is located. Global real estate investors compete to buy equity stakes on these assets. Because investors can buy CRE equity in different jurisdictions, the economy does not consist of isolated markets. Market interdependence means that the investors' decisions to buy more CRE equity in a jurisdiction affect the capital structure of CRE assets located in other jurisdictions. We identify mild conditions that guarantee the existence of equilibrium for this economy. This result contributes to the literatures of optimal security design and market segmentation in general equilibrium (Allen and Gale's 1991, Allen and Gale 1994, Duffie and Rahi 1995, and Rahi and Zigrand 2009) and also to the literature pioneered by Modigliani and Miller (1958) and Bradley, Jarrel, and Kim (1894) on the existence of an optimal capital structure in equilibrium by considering the case of commercial real estate assets.

Our model endogenizes many important variables, such as the capital structure of a CRE asset, the CRE cash flows, the construction costs, the prices of commercial goods, the property taxes, and the type of CRE assets that a jurisdiction selects. To motivate this aspect of our theory, we propose a simplified version of our general equilibrium model that illustrates the market mechanism of different shocks on the economy. In addition, we provide several numerical examples. For instance, we see that a negative shock to a developer's funding capacity increases the equity-to-total-equity ratio of the capital partner and decreases the debt-to-equity and the developer's equity-to-total-equity ratios. Because the capital structures of CRE assets in different jurisdictions are interconnected, this shock increases the leverage ratio in those CRE assets belonging to jurisdictions that are not hit by the shock. Moreover, developers in those jurisdictions must increase their equity contributions in order to offset the investors' decrease of CRE equity purchases in their jurisdictions. We also explore other shocks to the economy, such as a decrease in the production capacity of a CRE asset and an increase in the property tax in one jurisdiction. These examples also offer interesting insights regarding the inflation of commercial goods and the changes in the capital structures of CRE assets.

There are many other issues that can be explored in future research under the lens of our model. For example, our economy could be extended to accommodate the difference between industrial real estate and retail properties. This extension is briefly discussed in Section 2.

General equilibrium models with collateral constraints seem to be the correct approach for this extension (see e.g. Geanakoplos and Zame 2014, Gale and Gottardi 2015, and Fostel and Geanakoplos 2016). An interesting question that could be addressed following this approach would be to quantify the importance of collateral versus taxes for the capital structures of different CRE assets (see Li, Whited, and Wu 2016 for a similar question in the context of corporations). Also it would be interesting to characterize the relationship between the CRE asset collateral and the developer's funding capacity (see Campello and Giambona 2013 and Cvijanović 2014 for empirical work on this issue).

Another avenue for future research is to understand the role that transfers of property tax revenue to the agents that live and do business in a jurisdiction have in the economy. When the jurisdiction's profits revert to local households, taxes can be seen as a standard redistributive device from CRE developers and investors to households. When the jurisdiction profits revert to developers, transfers can be seen as Tax Incremental Financing. And if the jurisdiction transfers tax revenues to investors, the subsidies can be seen as tax credits (see Minnassian 2016). As in the classical theory of general equilibrium, all that we would require is that the profit sharing weights sum up to one across the agents of the jurisdiction (Luque 2018).

Our model can also guide empirical evaluations of the role that CRE property taxes and debt collateral requirements have on attracting real estate equity investments in a globalized economy in which jurisdictions compete to attract global equity investors.

References

[1] A. Admati and M. Hellwig (2013), *The bankers' new clothes: What's wrong with banking and what to do about it.* Princeton University Press: Princeton and Oxford.

[2] C. D. Aliprantis and Border, K. C. (2006), *Infinite Dimensional Analysis*, 3rd Ed., Springer, Berlin.

[3] F. Allen, E. Carletti, and R. Marquez (2015), "Deposits and Bank Capital Structure", *J Finan Econ* 118, 601-619.

[4] F. Allen and D. Gale (1991): "Arbitrage, Short Sales and Financial Innovation," *Econometrica* 59, 1041–1068.

[5] F. Allen and D. Gale (1994): Financial Innovation and Risk Sharing. Cambridge, MA: MIT Press.

[6] P. Amaral, D. Corbae, and E. Quintin (2017), "Financial Engineering and Economic Development", mimeo.

[7] L. Angeloni and B. Cornet (2006), "Existence of Financial Equilibria in a Multi-period Stochastic Economy", *Advances in Math Econ* 8, 933-955.

[8] Anglin, P. M., D. Dale-Johnson, Y. Gao and G. Zhu (2014), "Patterns of Growth in Chinese Cities: Implications of the Land Lease", *J Urban Econ* 83, 87-107.

[9] Z. Aouani and B. Cornet (2009), "Existence of Financial Equilibria with Restricted Participation", *J Math Econ* 45, 772-786.

[10] Y. Balasko, D. Cass, and P. Siconolfi (1990), "The Structure of Financial Equilibrium with Exogenous Yields: The Case of Restricted Participation", *J Math Econ* 19, 195-216.

[11] M. Berliant and T. ten Raa (1994), "Regional Science: The State of the Art", *Reg Sci and Urban Econ* 24, 631-647.

[12] M. Berliant and F. Page (2001), "Income Taxes and the Provision of Public Goods: Existence of an Optimum", *Econometrica* 69, 771-784.

[13] M. Bradley, G. Jarrel, and E. Kim (1894), "On the Existence of an Optimal Capital Structure: Theory and Evidece", *J Finan* 39, 857-878.

[14] M. Campello and E. Giambona (2013), "Real Assets and Capital Stucture", *J Finan Quant Analysis* 48, 1333-1370.

[15] D. R. Capozza and R.W. Helsley (1990), "The Stochastic City", *J Urban Econ* 28, 187-203.

[16] D. R. Capozza and G. A. Sick (1990), "Risk and Return in Land Markets", mimeo, University of Michigan.

[17] W. Cary (1974), "Federalism and Corporate Law: Reflections Upon Delaware", *Yale Law Journal* 83, 663-705.

[18] B. G. Carruthers and N. R. Lamoreaux (2016), "Regulatory Races: The Effects of Jurisdictional Competition on Regulatory Standards", *J Econ Lit*, 54 (1): 52-97.

[19] S. Cea-Echenique and J.P. Torres-Martinez (2016), "Credit Segmentation in General Equilibrium", *J Math Econ* 62, 19-27

[20] P. D. Childs, T. Riddiough, and A. J. Triantis (1996), "Mixed-Uses and the Redevelopment Option", *Real Estate Econ* 24, 317-339.

[21] D. Cvijanović (2014), "Real Estate Prices and Firm Capital Structure", *Rev Fin Stud* 27, 2690-2735.

[22] B. Cornet and R. Gopalan (2010), "Arbitrage and equilibrium with portfolio constraints", *Econ. Theory* 45, 227-252.

[23] H. DeAngelo and R. Masulis (1980), "Optimal Capital Structure Under Corporate and Personal Taxation", *J Finan Econ* 8, 3-29.

[24] H. DeAngelo and R. Stulz (2015), "Liquidity-claim Production, Risk Management, and Bank Capital Structure: Why High Leverage is Optimal for Banks", *J Finan Econ* 116, 219-236.

[25] G. Debreu (1959), "Theory of Value: An Axiomatic Analysis of Economic Equilibrium", Cowles Foundation Monographs Series, Yale University Press.

[26] D. Diamond (1967), "The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty", *American Economic Review* 57, 759-776.

[27] C. Dismore (1998), "The Impact of Public Capital Markets on Urban Real Estate", discussion paper prepared for the Brookings Institution Center on Urban and Metropolitan Policy.

[28] D. W. Drezner (2001), "Globalization and Policy Convergence", *Int Stud Rev* 3, 53-78.

[29] D. Duffie and R. Rahi (1995): "Financial Market Innovation and Security Design: An Introduction," *J Econ Theory*, 65 (1), 1-42.

[30] G. Duranton, P. Philippe-Combes, L. Gobillon, D. Puga, and S. Roux (2012), “The Productivity Advantages of Large Cities: Distinguishing Agglomeration from Firm Selection”, *Econometrica* 80, 2543-2594.

[31] M. Faias (2008), “Approximate Equilibrium in Pure Strategies for a Two-Stage Game of Asset Creation”, *Decisions in Economics and Finance* 31, 117-136.

[32] M. Faias, E. Moreno, and M. Pascoa (2002), “Real Indeterminacy of Equilibria and Manipulability”, *J Math Econ* 37, 325-340.

[33] M. Faias and J. Luque (2017), “Endogenous Formation of Security Exchanges”, *Econ Theory,* forthcoming. DOI: 10.1007/s00199-016-0989-9.

[34] M. Faias and J.P. Torres-Martinez (2017), “Credit Market Segmentation, Essentiality of Commodities, and Supermodularity”, *J Math Econ* 70, 115-122.

[35] A. Fostel and J. Geanakoplos (2016), “Financial Innovation, Collateral and Investment”, *Amer Econ J Macro* 8, 242-284.

[36] D. Gale (2004), “Notes on Optimal Capital Regulation”, in: St-Amant, P., C. Wilkins (Eds.), *The Evolving Financial System and Public Policy.* Bank of Canada, Ottawa, Ontario, Canada, pp. 225-253.

[37] D. Gale and P. Gottardi (2015), “A General Equilibrium Theory of Capital Structure”, mimeo.

[38] D. Gale and P. Gottardi (2015), “Capital Structure, Investment and Fire Sales”, *Rev Finan Stud* 28, 2502-2533.

[39] G. W. Gau and K. Wang (1990), “Capital Structure Decisions in Real Estate Investment”, *AREUEA Journal* 18, 501-521.

[40] J. Geanakoplos and W.R. Zame (2014), “Collateral Equilibrium, I: A Basic Framework”, *Econ Theory* 56, 443-492.

[41] J. Geanakoplos and H. Polemarchakis (1986), “Existence, Regularity, and Constrained Sub-optimality of Competitive Allocations When the Asset Market Is Incomplete”, Cowles Foundation Working Paper 652.

[42] E. Giambona, A. Mello, and T. Riddiough (2016), “Real Assets, Collateral, and the Limits of Debt Capacity”, *Real Estate Econ*, forthcoming.

[43] W. Gornal and I. A. Strebulaev (2016), “Financing as a Supply Chain: The Capital Structure of Banks and Borrowers”, *J Finan Econ*, forthcoming.

[44] S. R. Grenadier (1995a), “Valuing Lease Contracts: A Real-Options Approach”, *J Finan Econ* 38, 297-331.

[45] S. Grenadier (1995b), “The Persistence of Real Estate Cycles”, *Real Estate Finan and Econ* 10, 95-119.

[46] S. J. Grossman and O. Hart (1979), “A Theory of Competitive Equilibrium in Stock Market Economies”, *Econometrica* 47, 293-329.

[47] O. Hart (1979), “On Shareholder Unanimity in Stock Market Economies”, *Econometrica* 47, 1057-1083.

[48] R. W. Helsley and W. C. Strange (1997), “Limited Developers”, *Canadian J Econ* 30, 329-348.

[49] J.V. Henderson (1974), “The Sizes and Types of Cities’, *Amer Econ Rev* 64, 640-656.

[50] M. C. Jensen and W. Meckling (1976), “Theory of the Firm: Managerial Behavior, Agency Cost and Capital Structure”, *J. Finan Econ* 3, 11-25.

[51] A. Krausand and R. Litzenberger (1973), “A State Preference Model of Optimal Financial Leverage”, *J Finan* 28, 911-922.

[52] H. Konishi (2013), "Entrepreneurial Land Developers: Joint Production, Local Externalities and Mixed Housing Developments", *J Urban Econ* 75, 68-79.

[53] H. Konishi (2008), "Tiebout's Tale in Spatial Economies: Entrepreneurship, Self-Selection, and Efficiency", *Reg Sci and Urban Econ* 38, 461-471.

[54] S. Li, T. M. Whited, and Y. Wu (2016), "Collateral, Taxes, and Leverage", *Rev Finan Studi* 29, 1453-1500.

[55] J. Luque (2018), "Assessing the Role of TIF and LIHTC in an Equilibrium Model of Affordable Housing Development", *Reg Sci and Urban Econ*, forthcoming DOI: 10.1016/j.regsciurbeco.2018.06.005

[56] J. Luque (2013), "Heterogeneous Tiebout Communities with Private Production and Anonymous Crowding", *Reg Sci and Urban Econ* 43, 117-123.

[57] L. Makowski (1983), "Competitive Stock Markets", *Rev Econ Stud* 50, 305-330.

[58] T. Markeprand (2008), "On Financial Equilibrium with Intermediation Costs", *J Math Econ* 44, 148 - 156.

[59] D. Martinez-Miera and R. Repullo (2010), "Does Competition Reduce the Risk of Bank Failure?", *Rev Finan Stud* 23, 3638-3663.

[60] A. Mas-Collel and J. Nachbar (1991), "On the Finiteness of the Number of Critical Equilibria, with Application to the Random Selections", *J Math Econ* 20, 397 - 409.

[61] J. Miao (2006), "Competitive Equilibria of Economies with a Continuum of Consumers and Aggregate Shocks", *J Econ Theory* 128, 274-298.

[62] E. S. Mills (1993), "The Future of Office Markets", Working Paper, Kellogg Graduate School of Business, Northwestern University.

[63] M. H. Miller (1977), "Debt and Taxes" *J Finan* 32, 261-275.

[64] M. Minassian (2016), “Tax Deductions and Credits for Owners of Commercial Real Estate in Massachusetts”, Minassian CPA. http://www.minassiancpa.com/articles/realestate/masstaxincentives.html

[65] F. F. Modigliani and M. H. Miller (1958), “The Cost of Capital, Corporation Finance, and the Theory of Investment”, *Amer Econ Rev* 48, 261-297.

[66] F. F. Modigliani and M. H. Miller (1963), “Corporate Income Taxes and the Cost of Capital”, *Amer Econ Rev* 48, 261-297.

[67] C. Prechac (1996), “Existence of Equilibrium in Incomplete Markets with Intermediation Costs”, *J Math Econ*, 25 (3), 373-380.

[68] R. Radner (1974), “A Note on the Unanimity of Stockholders Preferences Among Alternative Production Plans: A Reformulation of the Ekren-Wilson Model”, *Bell Journal of Economics and Management Science* 5, 181-184.

[69] R. Rahi and J.P. Zigrand (2009), “Strategic Financial Innovation in Segmented Markets”, *Rev Finan Stud* 22, 2941-2971.

[70] J. P. Rincon-Zapatero and M. Santos (2009), “Differentiability of the Value Function Without Interiority Assumptions”, *J Econ Theory* 144, 1948-1964.

[71] S. Rosenthal and W. Strange (2004), “Evidence on the Nature and Sources of Agglomeration Economies,” in Handbook of Regional and Urban Economics, Vol. 4, ed. by V. Henderson and J.-F. Thisse. Amsterdam: North-Holland, 2119–2171.

[72] A. Seghir and J.P. Torres-Martinez (2011), “On Equilibrium Existence with Endogenous Restricted Financial Participation”, *J Math Econ* 47, 37-42.

[73] L. Simon and W. R. Zame (1990), “Discontinuous Games and Endogenous Sharing Rules”, *Econometrica* 58, 861-872 .

[74] H. Stahn (1999), “Monopolistic Behaviors and General Equilibrium: A Generalization of Nikaido’s Work”, *J Math Econ* 32, 87-112.

[75] L. Sun, S. D. Titman, and G. J. Twite (2015), “REIT and Commercial Real Estate Returns: A Postmortem of the Financial Crisis”, *Real Estate Econ* 43, 8-36.

[76] C. M. Tiebout (1956), “A Pure Theory of Local Expenditures”, *J Polit Economy* 64, 416-424.

[77] S. Titman (1985), “Urban Land Prices Under Uncertainty”, *Amer Econ Rev* 75, 505-514.

[78] S. Titman and R. Wessels (1988), “The Determinants of Capital Structure Choice”, *J Finan* 43, 1-19.

[79] S. Van den Heuvel (2008), “The Welfare Cost of Bank Capital Requirements”, *J Mon Econ* 55, 298-320.

[80] J. T. Williams (1991), “Real Estate Development as an Option”, *J Real Estate Finance Econ* 4, 191-208.

A Appendix

In this Appendix we provide the proof of our equilibrium existence theorem 1 for the case where mortgage contracts are recourse (no default is allowed). In addition, we report the proofs of the characterization results for our simplified economy of Section 4 and the computation details of Examples 1 to 3.

A.1 Proof of Theorem 1

Proof of Theorem 1: Our approach is to first construct a generalized game of the economy introduced in Section 2, then prove that the set of equilibria for our generalized game is non-empty, and then show that an equilibrium of the generalized game is in fact a competitive equilibrium that satisfies all conditions in Definition 1.

To this aim, let us first define the following thresholds:

- $W_{01} \equiv \sum_{a\in\mathbf{A}} \omega_{01}^a$ denotes the aggregate endowment of the numeraire good in the first period. We shall use this threshold to bound consumption of the numeraire in the first period.

- We shall use threshold $\hat{E} \equiv \max_{k=1,\ldots,\mathcal{K}}(1/\gamma_k)(W_{01})$ to find an upper bound on equity purchases. Notice that we chose threshold $\hat{E}$ in such a way that the property tax payment, denominated in terms of the numeraire good, cannot exceed the total amount of the numeraire good in the economy.

- We shall use threshold

$$W \equiv \max\{W_{01}, \max_{l\in\mathbf{L}_1} \sum_{a\in A} \omega_{l1}^a, \max_{l\in\mathbf{S}} \sum_{a\in\mathbf{A}} \omega_0^a(s), \max_{(s,l,k)\in\mathbf{S}\times\mathbf{L}_2\times\mathbf{K}} f_{lk}(s)(\overline{Y})\}$$

  to bound consumption in the second period, where $\overline{Y} = \max_{k=1,\ldots,\mathcal{K}} \bar{y}_k$, where $\bar{y}_k = TFP_k \prod_{l\in\mathbf{L}_1} (\omega_l^a)^{\alpha_{lk}}$.

- $n \in \mathbb{N}_+$ is a parameter that we shall use for bounding consumption of commodities in the first period. The subtlety is that in this economy the prices of debt and equity stakes can be very large and, if this happens, the consumption bundles of commodities in the first period can also be very large. However, we shall find an upper bound for these variables that depend only on the primitives of the economy and thus we shall conclude that this constant n will not be binding.

The set of players in this generalized game consists of the set of agents $\mathbf{A} = \mathbf{I}\cup\{d_k\}_{k=1}^{\mathcal{K}}\cup\{\mathbf{H}_k\}_{k=1}^{\mathcal{K}}$ and the following auctioneers: an auctioneer that chooses period 1 prices, an auctioneer that chooses period 2 prices, and an auctioneer per jurisdiction that chooses profits.

Next, let us truncate the set of admissible consumption bundles and financial positions corresponding to households, developers, and investors.

- Each household $h \in \{\mathbf{H}_k\}_{k=1}^{\mathcal{K}}$ chooses a vector $(x^h, D_+^h, D_-^h,)$ in the compact set $\Omega^h(n) = [0,n]^{1+L_1} \times [0, 2W]^{(1+L_2)S} \times [0, 2(\#\mathbf{A})D]^2 \subset \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{(1+L_2)S} \times \mathbb{R}$.

- Each developer $d \in \mathbf{D} \equiv \{d_k\}_{k=1}^{\mathcal{K}}$ chooses a vector (x^d, D_+^d, D_-^d, E^d) in the compact set $\Omega^d(n) = [0,n]^{1+L_1} \times [0, 2W]^{(1+L_2)S} \times [0, 2(\#\mathbf{A})D]^2 \times [0, 2\hat{E}] \subset \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{(1+L_2)S} \times \mathbb{R} \times \mathbb{R}_+$, where $\#\mathbf{A}$ denotes the number of agents in the economy.

- Each investor $i \in \mathbf{I}$ chooses a plan (x^i, D_+^i, E^i) in the compact set $\Omega^i(n) = [0,n]^{1+L_1} \times [0, 2W]^{S(1+L_2\mathcal{K})} \times [0, 2(\#\mathbf{A})D] \times [0, 2\hat{E}]^{\mathcal{K}} \subset \mathbb{R}_+^{1+L_1} \times \mathbb{R}_+^{S(1+L_2\mathcal{K})} \times \mathbb{R} \times \mathbb{R}_+^{\mathcal{K}}$.

The goal of households, developers, and investors is to maximize their utility function by choosing a bundle in their respective compact sets

$$\big((x^h, D_+^h, D_-^h)_{h\in\mathbf{H}}, (x^d, D_+^d, D_-^d, E^d)_{d\in\mathbf{D}}, (x^i, D_+^i, D_-^i, E^i)_{i\in\mathbf{I}}\big) \in (\Omega^h(n))^{\mathcal{H}\mathcal{K}} \times (\Omega^d(n))^{\mathcal{K}} \times (\Omega^i(n))^{\mathcal{I}}$$

which satisfies their respective budget constraints. The consumption sets are $X^a = \mathbb{R}_+^{1+L_1+S(1+L_2)}$ if $a \in \mathbf{D} \cup \mathbf{H}$, and $X^i = \mathbb{R}_+^{1+L_1+S(1+\mathcal{K}L_2)}$ if $i \in \mathbf{I}$.

We now define the auctioneers' feasible sets.

- The price-auctioneers in periods 1 and 2 choose prices in the following simplexes: $\Delta^{L_1} = \{p \in \mathbb{R}_+^{1+L_1} : \sum_{l=1}^{1+L_1} p_l = 1\}$ and $\Delta^{(1+L_2\mathcal{K})S-1} = \{p \in \mathbb{R}_+^{(1+L_2\mathcal{K})S} : \sum_{l=1}^{(1+L_2\mathcal{K})S} p_l = 1\}$, respectively. These sets are non-empty, convex, and compact.

- The profit-auctioneer chooses profits π_k in the compact set $\left[-\lambda(\alpha_k) - \varepsilon(\alpha_k)(1+\mathcal{I}), \sum_{a\in\mathbf{A}} \omega_{01}^a\right]$. The lower bound $-\lambda(\alpha_k) - \varepsilon(\alpha_k)(1+\mathcal{I})$ on the feasible set for profit π_k follows from the fact that parameter γ_k must be such that $\varepsilon(\alpha_k) \leq \gamma_k$, for all $k = 1, ..., \mathcal{K}$. The upper bound on π_k, $\sum_{a\in\mathbf{A}} \omega_{01}^a$, is given by the aggregate of the numeraire good in the first period since profits are denominated in units of the numeraire. Notice that we fix an exogenous upper bound for the sake of simplicity; in fact, we could obtain the upper bound on profits endogenously.

Auctioneers solve the following optimization problems:

- The price auctioneer in period 1 chooses $(p_1, \tau, q) \in \Delta^{L_1} \times [0, m]^{1+\mathcal{K}}$, with $m \in \mathbb{N}_+$, in order to maximize the following function:

$$p_{01} \sum_{a \in \mathbf{A}} (x^a_{01} - \omega^a_{01}) - p_{01} \sum_{a \in \mathbf{A}} \sum_{k \in \mathbf{K}} \delta^a_k \pi_k + \sum_{l \in \mathbf{L}_1} p_{l1} \sum_{a \in \mathbf{A}} (x^a_{l1} - \omega^a_{l1}) + \tau \sum_{a \in \mathbf{A}} (D^a_+ - D^a_-) + \\ + p_{01} \sum_{k \in \mathbf{K}} \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_k E^a_k + \sum_{k \in \mathbf{K}} q_k \left(\sum_{a \in \{d_k\} \cup \mathbf{I}} E^a_k - \bar{e}^a_k \right)$$

- The price auctioneer in period 2 chooses $(p(s), s = 1, ..., S) \in \Delta^{(1+L_2\mathcal{K})S-1}$ in order to maximize the following function:

$$\sum_{s \in \mathbf{S}} p_0(s) \sum_{a \in \mathbf{A}} (x^a_0(s) - \omega^a_0(s)) + \sum_{(s,l,k) \in \mathbf{S} \times \mathbf{L}_2 \times \mathbf{K}} p_{lk}(s) \left(\sum_{a \in \mathbf{H}_k \cup \mathbf{I} \cup \{d_k\}} x^a_{lk}(s) - f_{lk}(y_k)(s) \right).$$

- For all $k = 1, ..., \mathcal{K}$, the profit-auctioneer in jurisdiction k chooses π_k in the closed set $\left[-\lambda(\alpha_k) - \varepsilon(\alpha_k)(1 + \mathcal{I}), \sum_{a \in \mathbf{A}} \omega^a_{01}\right]$ in order to minimize the following function:

$$\left(\pi_k - \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_k E^a_k + \lambda(\alpha_k) + \varepsilon(\alpha_k) y_k \right)^2$$

- For all $s \in \mathbf{S}$, a house clearing auctioneer chooses the delivery rate $\phi_s \in [0, 1]$ to minimize

$$\left(\phi_s - \frac{\sum_{k \in \mathbf{K}} \sum_{a \in \{\mathbf{D}_k \cup \mathbf{H}_k\}} Q_k(s) D^a_-}{r \sum_{k \in \mathbf{K}} \sum_{a \in \{\mathbf{D}_k \cup \mathbf{H}_k\}} D^a_-} \right)^2$$

  if $\sum_{k \in \mathbf{K}} \sum_{a \in \{\mathbf{D}_k \cup \mathbf{H}_k\}} D^a_- > 0$ and minimize $(\phi_s - 1)^2$ if $\sum_{k \in \mathbf{K}} \sum_{a \in \{\mathbf{D}_k \cup \mathbf{H}_k\}} D^a_- = 0$.

We refer to the above generalized game as $\mathcal{G}(n, m)$. Next, we verify that the player's best response correspondences for this game satisfy the conditions of Kakutani's fixed point theorem.

First, the objective functions of households, developers, and investors are continuous and strongly quasi-concave as stated in Assumptions 1.i-iii, and their choice sets are non-empty, convex, and compact.

Second, for each vector $(p_1, (p(s), s = 1, ..., S), \tau, q, (\pi_k)_{k\in\mathbf{K}}, (\phi_s)_{s\in\mathbf{S}})$ of prices, profits and delivery rates, the choice set of each agent $a \in \mathbf{A}$ has an interior point. According to Assumptions 4 and 5, the endowments of the numeraire good and construction inputs are strictly positive for every agent. Moreover, for every agent $a \in \mathbf{A}$, $p_1\omega_1^a + p_{01}\sum_{k\in\mathbf{K}}\delta_k^a\pi_k > 0$ (this follows because $\omega_{01}^a > (\lambda(\alpha_k)+\varepsilon(\alpha_k)(1+\mathcal{K}))\mathcal{K}$ and $p_1 \in \Delta^{1+L_1}$). Thus, $x^a = 0$ and $E^a = 0$, together with a D^a satisfying inequalities $\tau D_+^a < p_1 \cdot \omega_1^a + p_{01}\sum_{k=1}^{\mathcal{K}}\delta_k^a\pi_k$ and $\phi_s D_+^a > 0$ for all $s \in \mathbf{S}$, is an interior point of the budget correspondence. This guarantees the lower hemicontinuity property of the agent's budget set correspondence. Since upper hemicontinuity also holds in our setting, we can use Berge's Maximum Theorem to claim that, for these players, the best response correspondence is upper-hemicontinuous with non-empty and compact values. The best response correspondence also takes convex values – this follows from the convexity of the budget set correspondence and strongly quasi-convavity of the objective function. For developers, this is also true, but the convexity property is not immediate. That property follows from the fact that the production function, which transforms construction inputs into a CRE asset, is concave, and also the fact that production functions $(f_{lk})_{l\in\mathbf{L}_1,k\in\mathbf{K}}$ that assign CRE assets into commercial goods are increasing and concave by Assumption 2.ii.

Third, the objective function of the profit-auctioneer in each jurisdiction is continuous and convex and its choice set is non-empty, convex, and compact. In addition, the price-auctioneers' objective functions are linear and, therefore, continuous and strictly quasi-concave in their choice variable. Moreover, their choice sets are non-empty, convex, and compact. Thus, for each of these auctioneers, its best response correspondence is also upper-hemicontinous an takes non-empty, compact and convex values. Kakutani's fixed point theorem guarantees that the generalized game

$\mathcal{G}(n, m)$ has a Nash equilibrium, which is the fixed point of the product of the best response correspondences.

Lemma A.1: *Suppose that Assumptions 4 and 5 hold. Then, if* $\bar{x}_{01}^a < W_{01}$ *for all* $a \in \mathbf{A}$, *there exists a threshold* $\bar{m} \in \mathbb{N}$ *for an equilibrium* $(\bar{x}^A, \bar{D}^A, \bar{E}^A, \bar{p}, \bar{q}, \bar{\tau}, (\bar{\pi})_{k \in \mathbf{K}})$ *of the generalized game* $\mathcal{G}(n, m)$, *such that* $\max\{\bar{q}_{k_1}, \bar{q}_{k_2}, ..., \bar{q}_{k_K}, \bar{\tau}\} < \bar{m}$.

Proof of Lemma A.1: Let $\tilde{y}_k \equiv TPF_k \cdot \Pi_{l \in L_1} (\omega_{l1}^{d_k})^{\alpha_{lk}}$ and $(\tilde{f}_k(s), s = 1, ..., S) \equiv (f_k(\tilde{y}_k)(s), s = 1, ..., S)$ (here $\tilde{f}_k(s)$ is the bundle of commercial goods that can be produced in period 2 with the CRE assets developed by developer d_k using his endowment of construction inputs). According to Assumption 4, a developer d_k has enough endowment $\omega_{01}^{d_k}$ of the numeraire good in the first period to buy equity of the CRE asset that he develops using his own endowment of construction inputs. Thus, a developer can always transfer at least an amount of wealth $p(s)\tilde{f}_k(s)$ from period 1 to each state of period 2. We conclude that the bundle $(\omega_{01}^{d_k} - g_{j_k}(\tilde{y}_k), 0, (\omega_0(s), \tilde{f}_k(s)), s = 1, ..., S))$ is always feasible for developer d_k.

According to Assumption 5, given $\theta \in (0, 1)$, there exists $\varrho \in \mathbb{R}_+^{1+L_1}$ such that

$$u^{d_k}(W_{01}, (2W(1, ..., 1), s \in \mathbf{S})) < u^{d_k}(W_{01} + \varrho, (2\theta W(1, ..., 1), s \in \mathbf{S})),$$

where $\varrho = \varrho(W_{01}, (2W(1, ..., 1), s = 1, ..., S))$. If we take $\theta \in (0, 1)$, such that $2W(1, ..., 1)\theta < 0.7\tilde{f}_k(s)$ and $2W\theta < 0.7\omega_0^{d_k}(s)$, for all $s \in \mathbf{S}$, then

$$u^{d_k}(W_{01}, (2W(1, ..., 1), s \in \mathbf{S})) < u^{d_k}(W_{01} + \varrho, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_k(s)), \ s \in \mathbf{S}))$$

or

$$u^{d_k}(W_{01}, (2W(1, ..., 1), s \in \mathbf{S})) < u^{d_k}(\omega_{01}^{d_k} - 0.7\gamma_k\tilde{y}_k + \tilde{\varrho}, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_k(s)), \ s \in \mathbf{S})),$$

where $\tilde{\varrho} = W_{01} - \omega_{01}^{d_k} + 0.7\gamma_k\tilde{y}_k + \varrho$ (notice that parameter $\tilde{\varrho}$ only depends on the fundamentals of the economy). Since $\bar{x}_{01}^a < W_{01}$, for all $a \in \mathbf{A}$, by monotonicity it is also true that

$u^{d_k}(\bar{x}_{0,1}^{d_k}, (\bar{x}^{d_k}(s),\, s \in \mathbf{S})) < u^{d_k}(W_{01}, (2W(1,...,1),\, s \in \mathbf{S}))$, which by transitivity implies that

$$u^{d_k}(\bar{x}_{01}^{d_k}, (\bar{x}^{d_k}(s), s = 1,...,S)) < u^{d_k}(\omega_{01}^{d_k} - 0.7\gamma_k \tilde{y}_k + \tilde{\varrho}, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_k(s)),\ s \in \mathbf{S})).$$

This means that developer d_k cannot buy $\tilde{\varrho}$ units of numeraire in period 1 with the resources that he saves when buying only part of the CRE equity, which is $p_{0,1}g_k(0.3y_k) + q_k 0.3y_k$. That is,

$$p_{01}0.3\gamma_k\tilde{y}_k + q_k 0.3\tilde{y}_k < \bar{p}_{0,1}\tilde{\varrho} \Leftrightarrow q_k < \bar{m}_m = \frac{\bar{p}_{0,1}\tilde{\varrho} - 0.3p_{0,1}\gamma_k\tilde{y}_k}{0.3\tilde{y}_k}.$$

Finally, set $\bar{m}_E = \max_{k\in\mathbf{K}} \bar{m}_k$ (observe that, for every $k \in \mathbf{K}$, threshold $\bar{m}_m$ depends only on the primitive parameters of the economy). Inequality $u^{d_k}(W_{01}, (2W(1,...,1),\ s \in \mathbf{S})) < u^{d_k}(W_{01} + \varrho, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_k(s)), s \in \mathbf{S}))$ also implies

$$u^{d_k}(W_{01}, (2W(1,...,1), s = 1,...,S)) < u^{d_k}(\omega_{01}^{d_k} - \gamma_k\tilde{y}_k + \hat{\varrho}, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_k(s)),\ s \in \mathbf{S})),$$

where $\hat{\varrho} = W_{01} - \omega_{01}^{d_k} + \gamma_k\tilde{y}_k + \varrho$ (notice that $\hat{\varrho}$ only depends on the fundamentals of the economy). Then, developer d_k cannot afford the bundle $(\omega_{01}^{d_k} - \gamma_k\tilde{y}_k + \hat{\varrho}, (0.7\omega_0^{d_k}(s), 0.7\tilde{f}_k(s)), s = 1,...,S))$; in particular, d_k cannot buy the bundle $\hat{\varrho} \in \mathbb{R}$ in period 1 using the debt payment that he would receive by selling the bundle $(0.3\omega_0^{d_k}(s), 0.3\tilde{f}_k(s)), s = 1,...,S)$.

Finally, let D^{d_k} be such that $rD^{d_k} < \min\{0.3\omega_0^{d_k}(s), 0.3\tilde{f}_{k,1}(s), ..., 0.3\tilde{f}_{k,1}(s)\}$. Then, $-\tau D^h < \bar{p}_{01}\hat{\rho}$, that is, $\tau < \bar{m}_D = \hat{\rho}/(-D^{d_k}))$. It just remains to set $\bar{m} \equiv \max\{\bar{m}_E, \bar{m}_D\}$ and $\bar{n} = 2W + \bar{m}$. This concludes the proof of Lemma A.1. ■

Lemma A.2: *An equilibrium $(\bar{x}^A, \bar{D}^A, \bar{E}^A, \bar{p}, \bar{q}, \bar{\tau}, (\bar{\pi})_{k\in\mathbf{K}})$ of the generalized game $\mathcal{G}(n,m)$ for $(n,m) > (\bar{n},\bar{m})$ is a competitive equilibrium as defined in Definition 1.*

Proof of Lemma A.2: By adding the first period budget constraints of all agents in the economy, we obtain

$$p_{01}\sum_{a\in\mathbf{A}}(\bar{x}^a_{01}-\omega^a_{01})+\sum_{l\in\mathbf{L}_1}p_{l1}\sum_{a\in\mathbf{A}}(\bar{x}^a_{l1}-\omega^a_{l1})+\tau\sum_{a\in\mathbf{A}}\bar{D}^a+$$
$$p_{01}\sum_{k\in\mathbf{K}}\sum_{a\in\{d_k\}\cup\mathbf{I}}\gamma_k\bar{E}^a_k+\sum_{k\in\mathbf{K}}q_k\left(\sum_{a\in\{d_k\}\cup\mathbf{I}}\bar{E}^a_k-\bar{e}^a_k\right)\le 0.$$

Then, taking into account the problem of the price-auctioneer in period 1, we conclude that there is no excess of demand for commodities and assets; that is,

1. $\sum_{a\in\mathbf{A}}(\bar{x}^a_{01}-\omega^a_{01}-\sum_{k\in\mathbf{K}}\delta^a_k\pi_k)+\sum_{k\in\mathbf{K}}\sum_{a\in\{d_k\}\cup\mathbf{I}}\gamma_k\bar{E}^a_k\le 0$
2. $\sum_{a\in\mathbf{A}}(\bar{x}^a_{l1}-\omega^a_{l1})\le 0$, for all $l\in\mathbf{L}_1$
3. $\sum_{a\in\mathbf{A}}\bar{D}^a\le 0$
4. $\sum_{a\in\{d_k\}\cup\mathbf{I}}\bar{E}^a_k\le\bar{e}^{d_k}_k$, for all $k\in\mathbf{K}\le 0$

If $\sum_{a\in\mathbf{A}}(\bar{x}^a_{01}-\omega^a_{01}-\sum_{k\in\mathbf{K}}\delta^a_k\pi_k)+\sum_{k\in\mathbf{K}}\sum_{a\in\{d_k\}\cup\mathbf{I}}\gamma_k\bar{E}^a_k>0$, then the price auctioneer would choose $p_{01}=1$ and a price equal to zero for the other commodities and assets. This allows this auctioneer to obtain a positive value for its objective function. However, this is in contradiction with the aggregation of the budget constraints. The same argument allows us to conclude that aggregate debt and equity holdings is less than or equal to zero. Finally, if $\sum_{a\in\{d_k\}\cup\mathbf{I}}\bar{E}^a_k-\bar{e}^{d_k}_k>0$, for some asset j_k, then the auctioneer would choose $\bar{q}_k=\bar{m}$ (notice that $\bar{x}^a_{01}<W_{01}$), which would contradict Lemma A.1 for $n>\bar{n}$. The same argument applies for debt.

The inequality in the first numbered list item is equivalent to

$$\sum_{a\in\mathbf{A}}\bar{x}^a_{01}\le\sum_{a\in\mathbf{A}}\omega^a_{01}+\sum_{k\in\mathbf{K}}\sum_{a\in\mathbf{A}}\delta^a_k\pi_k-\sum_{k\in\mathbf{K}}\sum_{a\in\{d_k\}\cup\mathbf{I}}\gamma_k\bar{E}^a_k,$$

which in turn is equivalent to

$$\sum_{a\in\mathbf{A}}\bar{x}^a_{01}\leq\sum_{a\in\mathbf{A}}\omega^a_{01}+\sum_{k\in\mathbf{K}}\pi_k-\sum_{k\in\mathbf{K}}\sum_{a\in\{d_k\}\cup\mathbf{I}}\gamma_k\bar{E}^a_k$$

because $\sum_{a\in\mathbf{A}}\delta^a_k=1$. Given that $\pi_k=\sum_{a\in\{d_k\}\cup\mathbf{I}}\gamma_k\bar{E}^a_k-\lambda(\alpha_k)+\varepsilon(\alpha_k)\bar{y}_k$, we get

$$\sum_{a\in\mathbf{A}}\bar{x}^a_{01}\leq\sum_{a\in\mathbf{A}}\omega^a_{01}-\sum_{k\in\mathbf{K}}(\lambda(\alpha_k)+\varepsilon(\alpha_k)\bar{y}_k).$$

By adding the budget constraints of all agents over all states of nature in period 2, we obtain

$$\begin{aligned}\sum_{s\in\mathbf{S}}\bar{p}_0(s)\sum_{a\in\mathbf{A}}(\bar{x}^a_0(s)-\omega^a_0(s))+\sum_{(s,l,k)\in\mathbf{S}\times\mathbf{L}_2\times\mathbf{K}}\bar{p}_{lk}(s)\left(\sum_{a\in\{d_k\}\cup\mathbf{H}_k\cup\mathbf{I}}\bar{x}^a_l(s)\right)\leq\\ \leq Sr\sum_{a\in\mathbf{A}}\bar{D}^a+\sum_{s\in\mathbf{S}}\sum_{k\in\mathbf{K}}\sum_{a\in\{d_k\}\cup\mathbf{I}}\bar{E}^a_k c_k(\bar{y}_k;\bar{p}(s))\\ \leq Sr\sum_{a\in\mathbf{A}}\bar{D}^a+\sum_{s\in\mathbf{S}}\sum_{k\in\mathbf{K}}c_k(\bar{y}_k)\end{aligned}$$

Using the definition of $c_k(y_k)$ and the fact that $Sr\sum_{a\in\mathbf{A}}\bar{D}^a\leq 0$, we can rewrite the above inequality as follows:

$$\sum_{s\in\mathbf{S}}\bar{p}_0(s)\sum_{a\in\mathbf{A}}(\bar{x}^a_0(s)-\omega^a_0(s))+\sum_{(s,l,k)\in\mathbf{S}\times\mathbf{L}_2\times\mathbf{K}}\bar{p}_{lk}(s)\left(\sum_{a\in\{d_k\}\cup\mathbf{H}_k\cup\mathbf{I}}\bar{x}^a_l(s)-f_{lk}(\bar{y}_k)(s)\right)\leq 0.$$

Then, given the problem of the auctioneer of period 2, we conclude that there is no excess of demand for commodities at date 2, i.e.,

5. $\sum_{a\in\mathbf{A}}(\bar{x}^a_0(s)-\omega^a_0(s))\leq 0$, for all $s\in\mathbf{S}$;

6. $\sum_{a\in\{d_k\}\cup\mathbf{H}_k\cup\mathbf{I}}\bar{x}^a(s)-f_k(y_k)(s)\leq 0$ for all $s\in\mathbf{S}$ and $k\in\mathbf{K}$.

In equilibrium, we also have that, for every jurisdiction $k \in \mathbf{K}$, profits are

$$\pi_k = \sum_{a \in \{d_k\} \cup \mathbf{I}} \gamma_k \bar{E}_k^a - (\lambda(\alpha_k) + \varepsilon(\alpha_k)\bar{y}_k) \,.$$

Observe that this value for the profit function belongs to the profit-auctioneer's strategy set; moreover, this is the value that minimizes that auctioneer's objective function.

Next, we prove that there is no excess of supply of commodities other than construction inputs. For if there were excess supply of one of the commodities in either period 1 or period 2, then the respective price-auctioneer would choose the price of that commodity equal to zero. However, this would be in contradiction with the existence of an optimal plan for the agents of this economy, since utility functions are increasing.

If there is excess supply of a construction input, then the price-auctioneer would choose its price equal to zero and this would be in contradiction with the existence of an optimal plan for developers. To see this, notice that by increasing the purchased amount of construction inputs, the developer could increase his income in period 1 to spend it on the consumption of the numeraire good 01 (recall that the utility function is increasing in the consumption of the numeraire good).

In addition, there is no excess supply of debt or equity. Again, we prove this by contradiction. If there were excess supply of debt or equity, the price-auctioneer would choose the price of that particular asset equal to zero and this would contradict the existence of an optimal plan because, by the monotonicity of the preferences, debt pays strictly positive returns in every state of nature and equity also pays strictly positive returns in all states of nature if $p_k(s) \gg 0$ and Assumption 2.iii hold. Observe that, in a Nash equilibrium of the generalized game, $p_k(s) \gg 0$ for all $s \in \mathbf{S}$, by monotonicity of preferences in period 2.

Finally, we establish the optimality of consumption plans. For this, first notice that, for each agent $a \in \mathbf{A}$, $(\bar{x}^a, \bar{D}^a, \bar{E}^a)$ satisfies the budget constraint given prices and profits $(\bar{p}, \bar{q}, \bar{\tau}, (\bar{\pi})_{k \in \mathbf{K}})$. Also, $(\bar{x}^a, \bar{D}^a, \bar{E}^a)$ belongs to the interior of $\Omega^a(n)$. Therefore, by the convexity of the budget sets and the strongly quasi-concavity of the utility functions, we have that the bundle $(\bar{x}^a, \bar{D}^a, \bar{E}^a)$ is optimal in the budget set with prices and profits $(\bar{p}, \bar{q}, \bar{\tau}, (\bar{\pi})_{k \in \mathbf{K}})$.■

A.2 Equilibrium characterization proofs

Proof of Proposition 1: Proposition 1 follows from the market clearing equation for the numeraire good at state s ($\sum_k p_k(s) Y_k(s) y_k = \omega_0^{h_k}(s) + Q_1(s) D_-^{d_1} + Q_2(s) D_-^{d_2}$) and equation $\sum_k p_k(s) Y_k(s) y_k = \phi_s (D_+^{h_1} + D_+^{h_2})$ (using the young households budget constraints and the market clearing equations for the numeraire good, the commercial goods at state s, and mortgage debt). ■

Proof of Corollary 1: Corollary 1 follows from expression (6), the young households budget constraints, and the market clearing equations for mortgage debt. ■

Proof of Proposition 2: Proposition 2 follows from young household h_k's optimality condition with respect to $D_+^{h_k}$ ($\tau = \sum_s (\lambda_s^{h_k} / \lambda_1^{h_k}) \phi_s$) and the developer d_k's optimality condition with respect to $D_-^{d_k}$ ($\tau = \sum_s (\lambda_s^{d_k} / \lambda_1^{d_k}) Q_k(s) + \xi^{d_k} \sigma_k / \lambda_1^{d_k}$). ■

Proof of Corollary 2: A higher $1 - Q_k(s)$ decreases ϕ_s. Necessary optimality condition $\tau = \sum_s (\lambda_s^{h_k} / \lambda_1^{h_k}) \phi_s$ for $D_+^{h_k} > 0$ then implies that τ must decrease when default loss $1 - Q_k(s)$ increases. Statement in Corollary 2 then follows by this fact and the necessary optimality condition $\tau = \sum_s (\lambda_s^{d_k} / \lambda_1^{d_k}) Q_k(s) + \xi^{d_k} \sigma_k / \lambda_1^{d_k}$ for $D_-^{d_k} > 0$. ■

Proof of Proposition 3: Proposition 3 follows from developer d_k's necessary optimality condition for $E_k^{d_k} > 0$ and Proposition 2's equilibrium condition (8), where $\xi^{d_k} = (\lambda_1^{d_k} / \sigma_k) \sum_s \left(\hat{\phi}_s - \hat{Q}_k(s) \right)$.
■

Proof of Proposition 4: Proposition 4 follows from the developer d_k's necessary optimality condition with respect to $x_1^{d_k} > 0$,

$$p_1 = \sum_s (\lambda_s^{d_k} / \lambda_1^{d_k}) \left(E_k^{d_k} p_k(s) Y_k(s) TFP_k \alpha_k (x_1^{d_k})^{\alpha_k - 1} \right) \tag{13}$$

Then, equating (13) for developers d_1 and d_2 and using $\alpha_1 = \alpha_2$, we obtain after some algebra the equilibrium property (10). ■

Proof of Corollary 3: Equilibrium property (11) follows from expression (10) and the assumption of binding collateral constraints ($\xi^{d_1} > 0$ and $\xi^{d_2} > 0$). ■

Proof of Proposition 5: A higher $1 - Q_2(2)$ translates into a lower mortgage delivery rate at state s_2 (ϕ_2), in turn decreasing the mortgage discount market price τ ($\tau = \sum_s (\lambda_s^{h_k}/\lambda_1^{h_k})\phi_s$, young household h_k's optimality condition with respect to $D_+^{h_k} > 0$) and the developer d_2's amount of debt $D_-^{d_2}$ (market price auctioneer and debt market clearing).

The initial shock also increases expected default risk $\sum_s \left(\hat{\phi}_s - \hat{Q}_2(s)\right)$ in jurisdiction $k = 2$, making the developer d_2's collateral constraint more binding (ξ^{d_2} increases due to equilibrium condition $\sum_s \left(\hat{\phi}_s - \hat{Q}_k(s)\right) = \xi^{d_k}\sigma_k/\lambda_1^{d_k}$; see proof of Proposition 2), in turn increasing the CRE asset j_2's equity market price q_2 ($q_k + \gamma_k = \sum_s (\lambda_s^{d_k}/\lambda_1^{d_k})c_k(y_k;p)(s) + \xi^{d_k}$, see proof of Proposition 3) and the investor i's equity E_2^i in project j_2 (market price auctioneer and equity market clearing). This crowds out developer d_2's equity share in project j_2 (i.e., $E_2^{d_2}$ decreases since $E_2^{d_2} + E_2^i = 1$).

The investor's equity porfolio rebalancing toward jurisdiction k_2 decreases investor i's equity E_1^i and the equity market price q_1 of CRE asset j_1 (following the investor's budget constraint $\omega_0^i = (q_1 + \gamma_1)E_1^i + (q_2 + \gamma_2)E_2^i$). As a result, developer d_1 holds the remaining equity in CRE project j_1 (i.e., $E_1^{d_1}$ increases since $E_1^{d_1} + E_1^i = 1$). This additional constituted equity collateral increases developer d_1's debt when his collateral constraint is binding (i.e., $D_-^{d_1} = \sigma_1 E_1^{d_1}$ increases).

Finally, notice that expression (10) in Proposition 4 then implies that the CRE height in jurisdiction k_1 with respect to jurisdiction k_2. ■

Proof of Proposition 6: The statements in Proposition 6 corresponding to Lemmas 2, 5, and 7 below.

Households h_1 and h_2 are excluded from the CRE equity market and, therefore, the only financial instrument that they can use to transfer wealth from the first to the second period is risk-free debt.[26] Because they prefer to consume tomorrow, they will sell their numeraire good

[26]Household's debt could be seen as a deposit of this household in a bank account. For the sake of simplicity, we

endowment and purchase as much risk-free debt as possible (risk-free debt pays 1 unit of the numeraire good in both states of the second period). Thus, in equilibrium we expect[27]

$$D^{h_1} = \frac{1}{\tau}\omega_0^{h_1} > 0 \tag{14}$$
$$D^{h_2} = \frac{1}{\tau}\omega_0^{h_2} > 0 \tag{15}$$

Households purchase the commercial good produced in their respective jurisdictions and, therefore, the market clearing of the commercial good occurs at the jurisdictional (local) level. This implies that $x_1^{h_1}(s) = Y_1(s)TFP_1(x_1^{d_1})^{\alpha_1}$ and $x_2^{h_2}(s) = Y_2(s)TFP_2(x_2^{d_2})^{\alpha_2}$, for $s = s_1, s_2$.

Because investor i prefers consumption of the numeraire good in the second period, we expect him to sell his endowment of good 0 and buy CRE equity in one or both jurisdictions. The following condition follows from the investor's budget constraint in the first period:

$$(q_1 + \gamma_1)E_1^i + (q_2 + \gamma_2)E_2^i = \omega_{01}^i \tag{16}$$

Next, we prove Proposition 1 with a series of lemmas.

Lemma 1: *The aggregate costs of private development and local public good provision equal the total amount of numeraire good available in the economy in the first period,* i.e.,

$$\underbrace{p_1\omega_1^{H_1} + p_2\omega_2^{H_2}}_{\text{cost of construction inputs}} + \underbrace{\sum_{k\in\mathbf{K}}(\lambda(\alpha_k) + \varepsilon(\alpha_k)TFP_1\left(x_{11}^{d_k}\right)^{\alpha_1})}_{\text{cost of local public good provision}} = \omega_{01}^{h_1} + \omega_{01}^{h_2} + \omega_{01}^{i} \tag{17}$$

Proof: Equation (17) follows from the old households' budget constraint in period 1 and the market clearing equation for the numeraire good. ■

Lemma 2: *Scarcity of a commercial good drives the price differential of this good between states of nature. In particular, the price of a commercial good becomes more expensive at state*

ignore banks as potential financial intermediaries in this economy.

[27] Conditions (14) and (15) are useful to obtain the following Lemmas 2 to 5 in the proof of Proposition 1.

s relative to state $s' \neq s$ if the amount of the commercial good produced and sold at s is smaller than at state s'. In particular,

$$p_1(1)/p_1(2) = Y_1(2)/Y_1(1) \tag{18}$$

$$p_2(1)/p_2(2) = Y_2(2)/Y_2(1) \tag{19}$$

Moreover, if the value of one unit of the commercial good produced in a jurisdiction is the same at both states, consumption of the numeraire good is the same at both states for the two developers and the investor, i.e., $x_0^{d1}(1) = x_0^{d1}(2)$, $x_0^{d2}(1) = x_0^{d2}(2)$, and $x_0^i(1) = x_0^i(2)$.

Proof: The budget constraint of households h_1 and h_2 at states s_1 and s_2 are such that

$$rD^{h_1} = p_1(s)Y_1(s)TFP_1\left(x_1^{d_1}\right)^{\alpha_1}, \text{for } s = s_1, s_2 \tag{20}$$

$$rD^{h_2} = p_2(s)Y_2(s)TFP_2\left(x_2^{d_2}\right)^{\alpha_2}, \text{for } s = s_1, s_2 \tag{21}$$

Conditions (18) and (19) follow from conditions (20) and (21), respectively (i.e., dividing the corresponding expression for state s_1 by the corresponding expression for state s_2). Moreover, conditions (18) and (19), and the developers' budget constraints in the second period imply that $x_0^{d_1}(1) = x_0^{d_1}(2)$ and $x_0^{d_2}(1) = x_0^{d_2}(2)$. These equalities, together with the market clearing conditions of the numeraire good at states s_1 and s_2, and assumption $\omega_0^{H_1}(1) = \omega_0^{H_2}(2)$, imply that $x_0^i(1) = x_0^i(2)$. ■

Lemma 3: *At each state of the second period, the sum of CRE cash flows and debt promises equals the total amount of the numeraire good in the economy, i.e.,*

$$\sum_{k\in\{1,2\}} p_k(s)Y_k(s)TFP_k\left(x_k^{d_k}\right)^{\alpha_k} + r(D^{d_1} + D^{d_2}) = \omega_0^{H_1}(s), \text{for } s = s_1, s_2 \tag{22}$$

Proof: We obtain equation (22) using equalities $x_0^{d_1}(1) = x_0^{d_1}(2)$ and $x_0^{d_2}(1) = x_0^{d_2}(2)$, together with the investor and developers' budget constraints at states s_1 and s_2, and the market clearing equation for the numeraire good in the second period. ■

Lemma 4: *The relative value of commercial goods produced in jurisdictions k_1 and k_2 is driven by the relative amounts of the numeraire good endowments that households h_1 and h_2 have in the first period. In particular,*

$$\frac{p_{11}(s)Y_1(s)TFP_1\left(x_1^{d_1}\right)^{\alpha_1}}{p_{21}(s)Y_2(s)TFP_2\left(x_2^{d_2}\right)^{\alpha_2}} = \frac{\omega_0^{h_1}}{\omega_0^{h_2}} \tag{23}$$

Proof: Condition (23) follows by equating the price of debt (τ) that results from households h_1 and h_2' budget constraints of period 0 and state s_1. ■

Roughly speaking, differences in wealth between households in different jurisdictions determine the difference in credit that these households extend to developers. Because households use their loan payments in the second period to purchase the commercial good of their respective jurisdiction, differences in loan amounts determine the differences in commercial goods valuation and also the differences in (endogenous) cash flows generated by the different CRE assets.

Lemma 5: *The CRE equity price accrued of the property tax in a jurisdiction is driven by the value of the commercial good produced and sold in that jurisdiction. In particular,*

$$r\left(q_1 + \gamma_1\right) = 2(p_1(1)Y_1(1)) \tag{24}$$

$$r\left(q_2 + \gamma_2\right) = 2(p_2(1)Y_2(1)) \tag{25}$$

Proof: Conditions

$$\lambda_1^i\left(q_1 + \gamma_1\right) = 2(\lambda^i(s_1) + \lambda^i(s_2))(p_1(1)Y_1(1))$$

$$\lambda_1^i\left(q_2 + \gamma_2\right) = 2(\lambda^i(s_1) + \lambda^i(s_2))(p_2(1)Y_2(1))$$

follow from investor i's first order conditions with respect to CRE equity E_1^i and E_2^i, respectively, where λ_1^i and $\lambda^i(s)$ are the shadow values of investor's budget constraints in period 1 and state

s of period 2, respectively. These equations take into account conditions (18) and (19).[28] With risk-free debt, we have that $\lambda_1^i/(\lambda^i(s_1) + \lambda^i(s_2)) = r$ and, therefore, conditions (24) and (25) follow accordingly. ■

Lemma 6: *When developers borrow at their maximum capacity, the total amount of debt in the economy equals the resources that households have in the first period, i.e.,*

$$\tau(\bar{D}^{d_1} + \bar{D}^{d_2}) = \sum_{k=\{k_1,k_2\}} \omega_0^{h_k} \tag{26}$$

Moreover, the equilibrium price of debt τ must satisfy the following equations:

$$\tau = \left(p_{11}\omega_{11}^{H_1} - (q_1 + \gamma_1)E_1^i + \gamma_1 TFP_1 \left(x_1^{d_1}\right)^{\alpha_1}\right)/\bar{D}^{d_1} \tag{27a}$$

$$\tau = \left(p_{21}\omega_{21}^{H_1} - (q_2 + \gamma_2)E_2^i + \gamma_2 TFP_2 \left(x_2^{d_2}\right)^{\alpha_2}\right)/\bar{D}^{d_2} \tag{27b}$$

Proof: Equations (26), (27a), and (27b) follow from conditions (14) and (15), together with the market clearing equation for debt, the market clearing condition for equity (at the jurisdiction level), and the developers' budget constraint in the first period. ■

Developers are the only agents in this economy who can create CRE assets. For that, they need to buy construction inputs. The respective market clearing conditions imply that these purchases must be such that $x_1^{d_1} = \omega_1^{H_1}$ and $x_2^{d_2} = \omega_2^{H_2}$. In addition, the equilibrium must satisfy the following condition:

Lemma 7: *The difference in prices of construction inputs in different jurisdictions is driven by the difference in marginal productivity of the CRE assets. In particular,*

$$p_1 - p_2 = \alpha_2 TFP_2(\omega_2^{H_2})^{\alpha_2 - 1} p_2(1) Y_2(1) - \alpha_1 TFP_1(\omega_1^{H_1})^{\alpha_1 - 1} p_{11}(1) Y_{11}(1) \tag{28}$$

Proof: (28) follows from the first order optimality conditions of developers d_1 and d_2 with

[28]Notice that (24) and (25) assume that the shadow values of sign constraints $E_{j_k}^{d_k} \geq 0$ and $E_{j_k}^{d_k} \leq y_{j_k}^{d_k}$ are zero (below, we will verify that our equilibrium is such that these constraints are non-binding for both $k = k_1, k_2$).

respect to construction inputs 11 and 21, respectively, and conditions (18) and (19). ■

A.3 Numerical examples

In this subsection, we provide all details regarding the computation of examples corresponding to Figures 1, 2, and 3 (Examples 1, 2, and 3, respectively). In addition, here we also discuss how poor CRE asset performance may affect price inflation of commercial goods in a jurisdiction (Example 4).

Example 1 (benchmark): For the parameter values in Example 1, we obtain a unique equilibrium solution where $q_1 = q_2 = 2.500$, $\tau = 1.440$, $p_{11} = p_{21} = 1.500$, $p_{11}(1) = p_{21}(1) = 1.500$, $p_{11}(2) = p_{21}(2) = 3.000$, and $E_1^i = E_2^i = 0.173$.[29] Figure 1 illustrates the equilibrium values of the different capital structure components of CRE assets j_1 and j_2, namely, the developer's debt, the developer's equity, and the investor's equity. These quantities are expressed in real terms, i.e., nominal amount times the corresponding price.

When analyzing the capital structure of a CRE development project, analysts look at financial ratios, such as the debt-to-equity ratio or the ratio of an investor's equity-to-total-equity. A debt-to-equity ratio higher than 0.500 indicates that the CRE capital structure has a greater proportion of its capital funding from lenders rather than equity investors. An "investor's equity-to-total-equity" ratio higher than 0.500 indicates that the investor owns more than 50 percent of the equity of a CRE asset. Our theory obtains these ratios endogenously determined in equilibrium. For example, the ratios corresponding to our benchmark example in Figure 1 are 0.576 for the "developer's debt-to-total-equity" ratio, 0.827 for the "developer's equity-to-total-equity" ratio, and 0.173 for the "investor's equity-to-total-equity" ratio. The respective ratios are the same in both jurisdictions. ■

Example 2 (funding capacity): We modify our benchmark example by increasing the debt

[29] We first obtain the value of equilibrium variables q_1, q_2, τ, p_{11}, p_{12}, $p_{11}(1)$, $p_{12}(1)$, $p_{11}(2)$, $p_{12}(2)$, E_1^i, and E_2^i by solving the following system of equations: (17), (18), (19), (22), (23), (16), (24), (25), (26), (27a), (27b), and (**??**). These equilibrium values, in turn, allow us to solve for the rest of the equilibrium variables by using the agents' budget constraints and market clearing equations.

limit for developer d_1 to $\bar{D}^{d_1} = 1.1$, while decreasing the debt limit for developer d_2 to $\bar{D}^{d_2} = 0.9$.[30] Roughly speaking, access to funding for developers in the first jurisdiction improves, while it worsens for those in the second jurisdiction. The new equilibrium is such that $q_1 = q_2 = 2.500$, $\tau = 1.440$, $p_{11} = p_{21} = 1.500$, $p_{11}(1) = p_{21}(1) = 1.500$, $p_{11}(2) = p_{21}(2) = 3.000$, $E_1^i = 0.125$, and $E_2^i = 0.221$. Figure 2 illustrates the equilibrium values of the different capital structure components of CRE assets j_1 and j_2 for the new parameter values. As mentioned in Section 3, we see that, compared to our benchmark example, the developer with worse funding capacity (d_2) decreases his absolute real exposure to CRE debt and equity, while the investor increases his equity exposure. The opposite happens in the CRE development project of the developer with better funding capacity.

We can get further insights into the composition of the two capital structures by looking at and comparing financial ratios. For the parameter values of example 2, we find a "developer's debt-to-total-equity" ratio equal to 0.634 in CRE asset j_1 and 0.518 in CRE asset j_2; a "developer's equity-to-total-equity" ratio equal to 0.875 in CRE asset j_1 and 0.779 in CRE asset j_2; and an "investor's equity-to-total-equity" ratio equal to 0.125 in CRE asset $j_1$and 0.221 in CRE asset j_2. We summarize the equilibrium values of the financial ratios under consideration for Examples 1 and 2 in table 1.

	Example 1 (benchmark example)		Example 2 (funding capacity)	
	k_1	k_2	k_1	k_2
developer's debt to total CRE equity	0.576	0.576	0.634	0.518
developer's equity to total CRE equity	0.827	0.827	0.875	0.779
investor's equity to total CRE equity	0.173	0.173	0.125	0.221

Table 1: This table reports the financial ratios of the equilibria corresponding to Examples 1 and 2.

Compared to our benchmark example, we conclude that both the developer's debt-to-equity ratio and equity-to-total-equity ratio decrease (increase) for the CRE capital structure correspond-

[30]By offsetting the decreasing in $\bar{D}^{d_2}$ with an increase in $\bar{D}^{d_1}$, we are able to keep the remaining components of equations (22) and (26) with the same equilibrium value as in the benchmark example.

ing to the developer with worse (better) funding capacity, while the investor's equity-to-total-equity ratio increases (decreases, respectively). ■

Example 3: Now suppose that jurisdiction 2 experiences an increase in its fixed costs $\hat{\lambda}_2$ from $0.2.$ to 0.5, and that the jurisdiction's manager responds by increasing its tax rate γ_2 from 0.5 to 0.8.[31] The other parameter values are as in Example 1. In this case, $q_1 = 2.500$, $q_2 = 2.200$, $\tau = 1.590$, $p_1 = 1.572$, $p_2 = 1.428$, $p_1(1) = p_2(1) = 1.500$, $p_1(2) = p_2(2) = 3.000$, $E_1^i = 0.147$, and $E_2^i = 0.199$.

Because the property tax γ_2 is paid by both the developer d_2 and the investor i, we expect that this tax increment has an impact on how these two agents allocate their resources. Figure 3 illustrates this. The increase in γ_2 decreases the developer d_2's equity compared to the equilibrium value of our benchmark Example 1. As explained in Section 3, this change is due to the developer's substitution of equity for tax free debt. In particular, developer's debt increases because τ jumped from 1.440 (in Example 1) to 1.590 (in Example 3), while D^{d_2} remained equal to $\bar{D}^{d_2} = -1$. The developer d_2's debt and the investor's equity contributions increase as a result.

In the other jurisdiction, the investor's equity contribution decreases. Interestingly, the increment in E_2^i and the decrease in E_1^i respond to the change in the equity price q_2. Compared to Example 1, q_2 has decreased from 2.500 to 2.200, while q_1 has remained the same (2.500). Roughly speaking, equity in CRE asset j_2 has become relatively cheaper. In CRE asset j_1, we also see that the developer's debt (τD^{d_1}) and equity contributions ($q_1 E_1^{d_1}$) increase.

	Example 1 (benchmark)		Example 3 (Tax policy γ_2)	
	k_1	k_2	k_1	k_2
developer's debt to total CRE equity	0.576	0.576	0.636	0.723
developer's equity to total CRE equity	0.827	0.827	0.853	0.801
investor's equity to total CRE equity	0.173	0.173	0.147	0.199

Table 2: This table reports the financial ratios of the equilibria corresponding to Examples 1 and 4.

[31]Notice that these changes are such that neither jurisdiction k_2's profits nor the profit components of equilibrium equations described in the above simplified economy change.

Table 2 reports the equilibrium values for the three financial ratios under consideration. It provides a complementary perspective on the comparison between the capital structures (in real absolute amounts) in the two CRE assets. There we see that the debt-to-equity ratio increases for j_2, while the developer's equity-to-total-equity ratio $E_2^{d_2}/(E_2^i + E_2^{d_2})$ decreases. This responds to the developer d_2's substitution effect between equity and tax free debt. The investor's equity-to-total-equity ratio $E_2^i/(E_2^i + E_2^{d_2})$ makes up the difference. ■

Example 5: Let us consider again the same parameter values as in the benchmark example, except that now we modify the amount of the commercial good produced at state s_2 by CRE asset j_2; in particular, let $Y_2(2)$ decrease from 0.5 to 0.1. Possible reasons are a natural disaster that negatively impacts the CRE asset's production capacity, a reduction in the supply of intermediate inputs captured by $f_2(y_2^{d_2})(2)$, or even political reasons, such as a policy of expropriation of resources. In this new equilibrium, only the the price of the commercial good 21 at state s_2 changes ($p_2(2) = 15.000$). The values of other equilibrium variables, including the financial ratios under consideration, do not change with respect to the benchmark example.[32] ■

[32] In particular, $q_1 = q_2 = 2.500$, $\tau = 1.440$, $p_1 = p_2 = 1.500$, $p_1(1) = p_2(1) = 1.500$, $p_1(2) = 3.000$, $p_2(2) = 15.000$, and $E_1^i = E_2^i = 0.171$.