

Budget Rules and Political Turnover

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Abstract

This paper studies the welfare implications of a class of budget rules, the ones that determine mandatory spending. I analyze a model with two parties that allocate a fixed budget to private transfers and a public good. Each period a party is chosen to propose an allocation while the other party can accept or reject the proposal. Mandatory spending is modeled in the spirit of dynamic legislative bargaining with an endogenous status quo. The relative taste for public goods defines the level of political conflict in society. I show three results. First, the type of good under mandatory spending is not without loss of generality, as mandatory spending over private goods deliver under-provision of the public good, vis-a-vis the first-best allocation, while the opposite is true for mandatory spending on public goods. This happens because this institution creates a positive intertemporal wedge on the good that is mandatory. Moreover, when political conflict is relatively high, it is welfare superior to have mandatory spending on public goods. However, as the level of political conflict decreases, mandatory spending on private goods may not only become welfare superior, but it may also bring society to the first-best. In fact, net welfare gains are increasing with political conflict when mandatory spending is on private goods. This result highlights that political conflict is not necessary harmful to welfare, specially when there are institutions that prevent expropriation. Finally, I show that net welfare gains have a positive relationship with political turnover, indicating that the *common wisdom* that more political turnover is better holds true when there is such an institution as mandatory spending.

Keywords: Dynamic legislative bargaining, endogenous status quo, political economy, mandatory spending, budget rules, fiscal rules, institutions, entitlements.

JEL Classification: C72, C73, C78, D61, D78, E62, H41, H61

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1 Introduction

Traditional welfare economics assumes that government decisions are made by a social planner with the all means to maximize social welfare. Her decisions consider costs and benefits of all possible alternatives in such a way that allows her to select the level and composition of allocations such that society as a whole is better off. This “first-best” approach is certainly an useful benchmark, but it is far from the how government decisions are made in the public arena.

In fact, government decisions are done via negotiations between groups, each one with their own motivations and incentives, and therefore that potentially disagree on their preferred outcome. The strategic interaction of different interested groups, which from now on I refer as parties, is affected by institutions that govern those negotiations. One important set of institutions is the one that defines how much power politicians that are not currently the main decision makers have in order to prevent reallocations of resources without their consent. The study of reallocation of resources through taxation (Acemoglu et al. (2011), Piguillem and Riboni (2011)) or direct expropriation (Bouton et al. (2016), Diermeier et al. (2017)) have been the core of the political economy debate since its early stage, as in the work of Hobbes (1651) and Marx (1867).

Mandatory expenditures are modern examples of such institutions, the ones that rule political negotiations over the government budget.¹² They give rights for specific groups in society to have a claim over the government budget, despite which group is the active ruler. This suggests that rights to claim the government budget should be thought as an equilibrium result of how much society wants to delegate to their political leaders, not as a constraint on choices that affect the collective. Formally this means that while politicians are not constrained over choices, they will have to respect minimum levels of welfare of other groups in society.

If those budgetary institutions are equilibrium outcomes, they must be the result of the strategic behavior of rational politicians. Therefore, such institutions have an intrinsic dynamic nature, as the group who is the active ruler fluctuates. Following an increasing strand of the literature, I model this budgetary institution as a status quo allocation that remains in place unless some political group proposes another allocation that is individually rational for other groups. This mechanism creates a dynamic strategic link between the two groups by impacting the trade-off current politicians have when choosing an allocation. The models of legislative bargaining following Baron (1996) and Kalandrakis (2004) and many others are a natural start for the study of political economy dynamics of mandatory spending.

Moreover, most allocative decisions are done in a multi-dimensional good space with goods that differ in their basic qualities. In fact, governments usually provide both private and public goods. Example of the former are transfer such as social security and mean-tested transfers. As Samuelson writes in his 1954 paper, a public good is such that “which all enjoy in common in the sense that each individual’s consumption of such a good leads to no subtractions from any other individual’s consumption of that good”. In general, a good is public if citizens cannot be

¹An entitlement is “a provision of law that establishes a legal right to public funds for a class of citizens”, as in Schick (2009). Therefore entitlements can be thought as a claim over a good that is provided by the government. The good can be a public good or a publicly provided private good. My interpretation is that most of entitlement programs, such as social security and social benefit programs are rival and excludable and therefore must be thought as claims of citizens over private goods that are publicly provided. In the case of mandatory expenditures, not all of them are an entitlement. An entitlement program creates the legal right to an eligible class of citizens. A mandatory expenditure not necessarily does the same. For a more detailed discussion on this, see White (1999).

²A different focus on budgetary institutions has been given on papers that study fiscal rules and fiscal constitutions, as in Persson and Tabellini (1996), Halac and Yared (2014) and Azzimonti et al. (2016).

excluded from its use (non-excludable) and if the marginal cost of producing such good to an additional individual is zero (non-rival). Examples of pure public goods would be public defense, clean air and official statistics. Quasi-public goods, such that excludability is a possibility, such as law enforcement, streets, sewage, libraries, museums and education and part of public health care such as hospitals (not without controversy this last two) could also be useful examples for the discussion I will present here.

Most of the political economy literature that studies the welfare impacts of rules over the budgetary allocations focus on unidimensional good's space (Baron and Ferejohn (1989), Baron and Kalai (1993), Baron (1996), Kalandrakis (2004), Dziuda and Loeper (2012), Diermeier et al. (2017), Richter (2014), Piguillem and Riboni (2011) and Bowen et al. (2017)). A basic example is that there is a dollar to be split in every period and parties would like to fully expropriate the others and consume the whole dollar. In a two-dimensional good space with the presence of public goods the focus has been on rules that guarantee the provision of public goods under the status quo. In fact, Bowen et al. (2014a) studies the welfare implications of mandatory spending programs over public goods. They find that mandatory spending over public goods is welfare improving in societies where the level of disagreement over public good provision is low.³ Their findings indicate that welfare improves as mandatory programs over public goods reduce underspending of public good that may happen because of political fluctuation. This result lies on the fact that public goods can always be claimed by citizens if negotiations fail. Other examples are Battaglini et al. (2012), Duggan and Kalandrakis (2012) and Karakas (2017).

The fact that mandatory spending is over goods that are non-excludable and non-rival is not without loss of generality. In their paper, Bowen et al. (2014a) mention that they see as an example of public good entitlement programs such as social security, Medicare and others. The fact that entitlement programs are pure public goods is debatable. In the US, for example, entitlements such as social security and Medicare, are actually private transfers and so rival and excludable. It is relevant to understand if the modeling choice of focusing on entitlement programs as public goods is not without loss of generality.

I then focus on comparing different mandatory spending rules when there is a multidimensional good space and I show that mandatory spending over public goods is not without loss of generality in this setting. In fact, the political dynamics of mandatory spending depend on the type of good that is provided under the status quo and its interplay with other relevant political variables. I show three results. First, the type of good under mandatory spending is not without loss of generality, as mandatory spending over private goods deliver under-provision of the public good, vis-a-vis the first-best allocation, while the opposite is true for mandatory spending on public goods. This happens because this institution creates a positive intertemporal wedge on the good that is mandatory. Moreover, when political conflict is relatively high, it is welfare superior to have mandatory spending on public goods, as its over-provision is less harmful to welfare than its under-provision. However, as the level of political conflict decreases, mandatory spending on private goods may not only become welfare superior, but it may also bring society to the first-best. In fact, net welfare gains are increasing with political conflict when mandatory spending is on private goods. This result highlights that political conflict is not necessary harmful to welfare, specially when there are institutions that prevent expropriation. Finally, I show that net welfare gains have a positive relationship with political turnover, indicating that the *common wisdom* that more political turnover is better holds true when there is such an institution as mandatory spending.

³The authors refer to the level of disagreement over public good provision as *polarization* in their paper.

2 Environment

Economics

Society is formed by a continuum of individuals that have preferences over private goods c_i and public good g and that belong to different groups, organized by how much they value the public good. Time is discrete and every period of time, an amount Y of resources is available to be divided among private transfers for both groups, $c_{L,t}$ and $c_{H,t}$, and public good consumption g_t . I normalize $Y = 1$, fixed across time and this is without any loss of generality. Neither goods can be stored. Denote by $\mathbf{b}_t = (c_{L,t}, c_{H,t}, g_t)$ a feasible allocation. Define the feasible set of allocations by:

$$\mathcal{B} = \{\mathbf{b}_t \geq 0 \mid \sum_{k=1}^3 b_{k,t} \leq Y = 1\}$$

The static payoff of individuals is represented by a separable utility function:

$$u_i(\mathbf{b}_t) = (1 - \alpha_i)f(h_i(c_{i,t})) + \alpha_i f(v(g_t))$$

where α_i is the weight of the public good g with respect to the private transfer c for group $i \in \{L, H\}$. I assume $0 < \alpha_L \leq \alpha_H < 1$. These conditions guarantee that both groups have some benefit from the public good and that is never optimal to spend all resources in public good provision or all in private transfers. This assumption rules out uninteresting cases in which the space of conflict among the agents is unidimensional and they care only about one good.

Definition 1. Political conflict *The level of political conflict is given by how much parties like the public good. In the case*

Moreover, I assume that $f'(x) > 0$, $f''(x) < 0$, $f'(0) > -\infty$. Note that while the initial four assumptions are standard and guarantee that the utility functions are monotonically increasing and concave, the last two are less general. They latter guarantee that the marginal utility of individuals is well defined at zero. This means that Inada conditions don't hold. In fact, h_i and v ($h'_i(x) = v'(x) = 1$, $h''_i(x) = v''(x) = 0$) are level-shift functions, that allow for the well-defined derivatives at zero. An example of such function is $h_i(x) = \bar{x}_i + x$, with $\bar{x}_i \in [0, 1]$. My examples will focus on an utility function that satisfies the conditions above and takes the following form:

$$u_i(c_i, g) = (1 - \alpha_i) \ln(\bar{x}_i + c_i) + \alpha_i \ln(\bar{x} + g)$$

The greater the shift vis-a-vis the amount of resources that are in the hands of the government the less relevant the political bargaining is for individual's well-being. One could interpret this as individuals having additional resources that are external to politics of the model and guarantee them welfare despite the political game played different groups in society. The fact that INADA conditions is crucial to the analysis of budgetary rules that guarantee a subset of the set of allocations implemented in the past and, of course, it is not without loss of generality. In the appendix, I show the consequences of taking this outside consumption to the limit ($\bar{x} \rightarrow 0$) and how this impacts the results presented here.

The Political Economy

I restrict my analysis to $n = 2$ players that have equal measure and therefore I consider a political system with unanimity rule. Allocations are chosen by representatives of one of two groups, like

in citizen candidates model (Osborne and Slivinski (1996), Besley and Coate (1997)). The identity of the individual that is the active ruler is immaterial as individuals are homogenous within a group. Every period of time someone from the groups will be chosen to be the active ruler that can propose the allocation of one dollar the government has ($Y = 1$). The probability of a group to be the proposer is Markovian and defined by p . Once a proposer is drawn, she proposes allocation denoted by $x_t^i = (c_{Lt}, c_{Ht}, g_t)$. Respondent $j \neq i$ gets to accept or reject the proposal. If the proposal is accepted, the proposed allocation is implemented. Otherwise, a status quo allocation defined as $s_t \in \mathcal{S} \subset \mathcal{B}$ is implemented. Denote the vector of allocations implemented at time t as $\mathbf{b}_t \in \mathcal{B}$. Note that under indifference, the respondent favors acceptance of the proposed allocation.

The status quo is endogenous, meaning that it will remain in place until another proposal is accepted. In order to clarify the importance of the status quo rule, I will follow with an example from Bowen et al. (2014a). Denote by Ψ a status quo rule. This rule can be exemplified as the following. Suppose that the implemented allocation at time t is given by $\mathbf{b}_t = (c_{L,t}, c_{H,t}, g_r) = (0.3, 0.3, 0.4)$. Then, if the institution in this society is such that groups that are not the current rulers have right to no claim, then $s_{t+1} = \Psi(\mathbf{b}_t) = (0, 0, 0) \forall \mathbf{b}_t \in \mathcal{B}$. If institutions are such that groups that are not the current rulers have the right to claim both private and public good, then $s_{t+1} = \Psi(\mathbf{b}_t) = (0.3, 0.3, 0.4) \forall \mathbf{b}_t \in \mathcal{B}$. I want to focus here a budgetary institution that guarantees the right to claims over private goods or public goods. In the case rights are over private goods, $s_{t+1} = \Psi(\mathbf{b}_t) = (0.3, 0.3, 0) \forall \mathbf{b}_t \in \mathcal{B}$. In the case rights are over public goods, $s_{t+1} = \Psi(\mathbf{b}_t) = (0, 0, 0.4) \forall \mathbf{b}_t \in \mathcal{B}$. More formally, for any $\Psi : \mathcal{B} \rightarrow \mathcal{S} \subset \mathcal{B}$.

It is not by chance I focus on the two rules mentioned above. In most democratic countries nowadays, political institutions were designed such that citizens that are not the current rulers in society have some right over a minimum amount of welfare that is provided via private transfers or access to a public good. In some countries, the focus is on rights over goods that are rival and excludable, like cash transfers (social security, low income transfers, etc), while in others, the focus is on goods that are non-rival and nonexcludable, like a pure public good, like public education, an universal public health system and so on. Of course, the public good can also be thought in a less material way, like board control, security of the citizens of a country and so on. Moreover, one could think that without the guarantee of such rights as the ones just mentioned, a government has no legitimacy and cannot claim the control of social resources. While I abstract from those issues in terms of the theoretical focus of this paper, this discussion can be captured by the complementarity between private and public goods. This can be done with a generalization of the current environment for a non-separable utility in c_i and g . With that, this same model can be used for the analysis. I will focus on the case in which private goods and public goods that are provided by the government are not complements and, moreover, they are not essential to citizens. Mathematically this just means that INADA conditions don't hold (the limit of the partial derivatives evaluated at 0 are well defined).

Political equilibrium

As it is customary in the dynamic bargaining literature, I focus on Stationary Markov Perfect Equilibria, referred here as MPE. A MPE is a Subgame Perfect Equilibria (SPE) In which strategies were restricted to be stationary Markovian. A strategy profile is stationary Markovian if for any two ex post histories that terminate in the same *state*, the strategies that follow are the same.⁴ Of course this restriction may not be without loss of generality in an infinite horizon game. After

⁴For a more precise definition of stationary Markovian strategies I refer to Mailath and Samuelson (2006).

all, stationary Markovian strategies ignore all details of a history of plays. Much of the applied work of dynamic games has focused on the use of stationary Markovian strategies not only for their simplicity, but also for their prediction power as Folk-like results can be prevented in some cases. Moreover, it seems reasonable in a variety of setting to require strategies to rely only on *relevant* information at the time of the play. In fact, if the dynamic interaction of politicians is seen as a infinite game played by a sequence of legislators who face uncertainty on their reelection, the restrictions of Markovian strategies is justifiable. See Bhaskar et al. (2013) for a theoretical justification of the use of Markovian strategies.

A pure strategy for party i can be defined as a pair of functions $\Pi^i = (X^i, r^i)$ where $X^i : \mathcal{S} \rightarrow \mathcal{B}$ is a proposal strategy for party i and $r^i : \mathcal{S} \times \mathcal{B} \rightarrow \{0, 1\}$ is an acceptance strategy for party i , where 1 indicates acceptance and 0 rejection of the proposed allocation. More explicitly, $X^i = (\chi_H^i, \chi_L^i, \gamma^i)$ is a collection of functions that define the best-reply functions of proposer i . This proposal strategy maps each status quo s into a private transfer to H group, χ_H^i , to group L , χ_L^i and to public good provision, γ^i . The tie-breaking rule favors any proposed allocation, i.e., in case the respondent is indifferent between the status quo and a new proposed policy, the respondent accepts. I focus on MPE that follow the mentioned tie-breaking rule.

Denote a strategy profile $\Pi = (\Pi^L, \Pi^H)$. For each $i \in \{L, H\}$ I denote a $V_i(s; \Pi)$ the dynamic payoff when i is the active proposer in a current period and $W_i(s; \Pi)$ the dynamic payoff of i when $j \neq i \in \{L, H\}$ is the responder and Π is played in the current period. The payoff corresponding to the strategy profile Π is given by (V_L, V_H, W_L, W_H) . For the sake of notation, I suppress the fact that payoffs are functions of strategies and write them just as function of the states.

A strategy profile is a stationary MPE if and only if:

- [E1] Given (V_L, V_H, W_L, W_H) , for any proposal $x \in \mathcal{B}$ and status quo $s \in \mathcal{S}$, the acceptance strategy $r_i(s, x) = 1$ if and only if:

$$u_i(x) + \beta [(1 - p)V_i(\Psi(x)) + pW_i(\Psi(x))] \geq K_i(s)$$

where $K_i(s) = u_i(s) + \beta [(1 - p)V_i(\Psi(s)) + pW_i(\Psi(s))]$ denote the payoff of i from the current status quo s .

- [E2] Given (V_L, V_H, W_L, W_H) and r^j , for any status quo $s \in \mathcal{S}$, the proposal strategy $\Pi^i(s)$ of party $i \neq j$ satisfies:

$$\begin{aligned} \Pi^i(s) &= \max_{x \in \mathcal{B}} u_i(x) + \beta [pV_i(\Psi(x)) + (1 - p)W_i(\Psi(x))] \\ \text{s.t. } &u_j(x) + \beta [(1 - p)V_j(\Psi(x)) + pW_j(\Psi(x))] \geq K_j(s) \end{aligned}$$

- [E3] Given $\Pi = (\Pi^L, \Pi^H)$, the payoff set (V_L, V_H, W_L, W_H) satisfies the following functional equations for any $s \in \mathcal{S}$, $i, j \in \{L, H\}$ with $j \neq i$:

$$\begin{aligned} V_i(s) &= u_i(\chi_i^i(s), \gamma^i(s)) + \beta [pV_i(\Psi(\Pi^i(x))) + (1 - p)W_i(\Psi(\Pi^i(x)))] \\ W_i(s) &= u_i(\chi_i^j(s), \gamma^k(s)) + \beta [pV_i(\Psi(\Pi^j(x))) + (1 - p)W_i(\Psi(\Pi^j(x)))] \end{aligned}$$

3 Optimal Budgetary Rules in a Two-Period Model

3.1 Homogenous parties and goods: analytical characterization

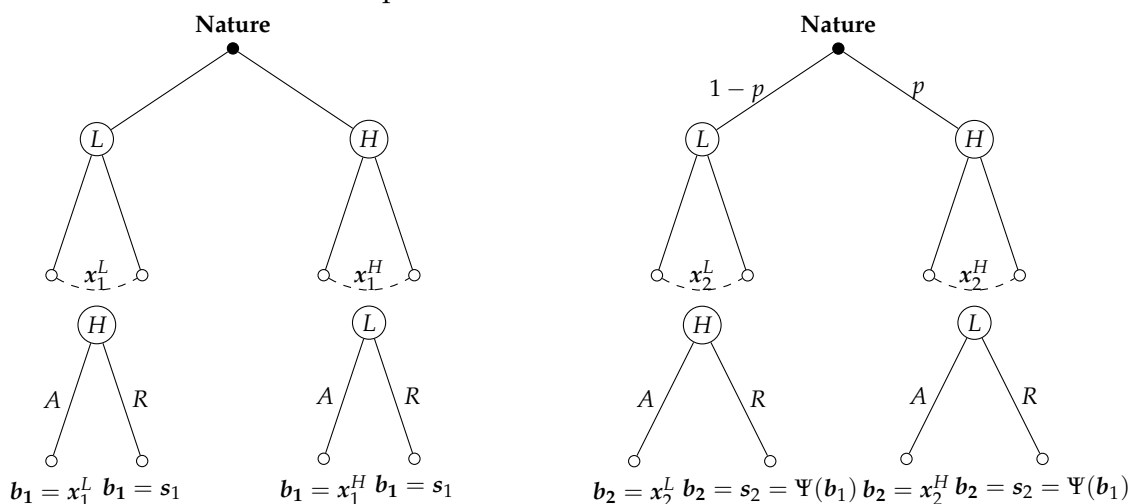
I start my analysis with a specific parameterization of the model that allows for analytical characterization that is helpful in providing intuition on why mandatory spending on private goods differ from mandatory spending on public goods. In this section I focus on a two-period model in which players value public and private goods in the same way ($\alpha = \frac{1}{2}$) and their outside consumption is the same across parties and goods ($\bar{x}_i = \bar{x}_j = \bar{x}$). A simple monotonic transformation allows me to write the static payoff of the players as $u_i(c_i, g) = u(c_i, g) = f(h(c_i)) + f(h(g_t))$, which sets $\alpha = 1$ instead of $\alpha = 0.5$. For the case of the example below, I focus on the following utility function:

$$u_i(c_i, g) = \ln(\bar{x} + c_i) + \ln(\bar{x} + g) \quad (1)$$

with $\bar{x} \in (0, 0.5]$. Since parties value both goods in the same way ($\alpha = 0.5$), the same level of welfare can be guaranteed when using one rule or the other. Therefore, any difference in terms of the optimal strategies from parties will be from how these rules change the trade-offs faced by the politician in power, not because parties value goods in different ways.

Timeline of events.— Given this example is a two-person, two-period, complete information extensive form game, I can focus on its unique subgame perfect equilibrium (SPE). The second-period strategies don't depend on histories except by the status quo. Therefore, I don't have to write strategies as a function of history, but only as a function of the status quo. This would not be true even for a finite horizon problem where $T > 2$. For a finite problem where $T > 2$ or an infinite horizon problem, in order to be independent of histories, I would have to restrain strategies to be Markovian, as discussed above. More of this discussion on Extensions and Conclusions.

The timeline of events can be depicted below. On the left side, the subgame that corresponds to the first period is shown and on the right side the subgame that corresponds to the second period, given that H was chosen in the first period:



In the first period, Nature tosses a coin (fair or not) and chooses the first proposer. The proposer proposes an allocation that can be reject or accepted by the respondent. In the case of rejection, the status quo, which is exogenous, is implemented. In the case of acceptance, the proposed allocation

is implemented. In the second period, Nature draws a Markovian coin that defines the conditional probability that the first proposer remains in power. The second proposer then makes a proposal which can be rejected or accepted. In the case of acceptance, it follows as in the first period. The difference now happens in the case of a rejection, in which the status quo allocation defined by the Ψ mapping is implemented. An example of the second period time-line is depicted above for the case in which H was chosen in the first period.

This problem can be solved backwards. I start with the full characterization of the second-period optimum strategies of party i and show they are statically optimum as they are the same as the first-best allocations. I also show that it doesn't matter if mandatory spending is on private or public goods, the optimum strategy of party i is the same. Moreover, the optimal amount of private consumption given to the respondent is weakly increasing (weakly because for some status quo the individual rationality constraint is not binding and therefore the private consumption to the respondent is 0) in the amount of goods promised under the status quo, being private or public good.

Second-period characterization.— Since the second-period politician takes the status quo s_2 as given and there is no continuation value as the economy ends, the second-period problem is equivalent to one of party i seeking to maximize its static payoff $u(c_{i,2}, g_2) = \ln(\bar{x} + c_{i,2}) + \ln(\bar{x} + g_2)$ given a status quo $s_2 = \Psi(\mathbf{b}_1) \in \mathcal{S} \forall \mathbf{b}_1 \in \mathcal{B}$ and unanimity rule. The problem is given by

$$\begin{aligned} \max_{x_2=(c_{i,2}, c_{j,2}, g_2) \geq 0 \in \mathcal{B}} \quad & \ln(\bar{x} + c_{i,2}) + \ln(\bar{x} + g_2) \\ \text{s.t.} \quad & \ln(\bar{x} + c_{i,2}) + \ln(\bar{x} + g_2) \geq K_{j,2}(s_2), \end{aligned}$$

where $K_{j,2}(s_2) = \ln(\bar{x} + c_{j,1}) + \ln(\bar{x})$ if budgetary rules are on private goods or $K_{j,2}(s_2) = \ln(\bar{x}) + \ln(\bar{x} + g_1)$ if budgetary rules are on public goods. Since in this example we have homogeneity across goods, mandatory spending on private goods or public goods from the perspective of the second-period status quo mean the same in terms of welfare promised to the respondent under the status quo. Denote $s_2 \in \mathcal{S}_2$ the relevant status quo for party i . In the case of rules over private goods, $s_2 = c_{j,1}$ and in the case of rules over public goods, $s_2 = g_1$. I start by showing the characterization of the optimal strategy of proposer $i \in \{H, L\}$ in the second period:

Proposition 1 *In the second-period model with budgetary rules that are over private or public goods the unique proposal strategy for party $i \in \{H, L\}$ is:*

$$\begin{aligned} \gamma_2^i(s_2) &= \begin{cases} \frac{1}{2}, & \text{if } s_2 < \frac{1}{2} \\ s_2, & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \frac{1+\bar{x}}{2}, & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases} \\ \chi_{j,2}^i(s_2) &= \begin{cases} 0, & \text{if } s_2 < \frac{1}{2} \\ 0, & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \frac{\bar{x}(2s_2-1)-\bar{x}^2}{1+3\bar{x}}, & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases} \end{aligned}$$

and $\chi_{i,2}^i(s_2) = 1 - \gamma_2^i(s_2) - \chi_{j,2}^i(s_2)$. Moreover, this allocations are statically first-best optimal.

I leave the proof of Proposition 1 for the Appendix. As we can see above, the party's best-replies depend on the level of outside consumption the parties have \bar{x} . In Figure 1 I illustrate the best-replies of party i for different levels of \bar{x} :

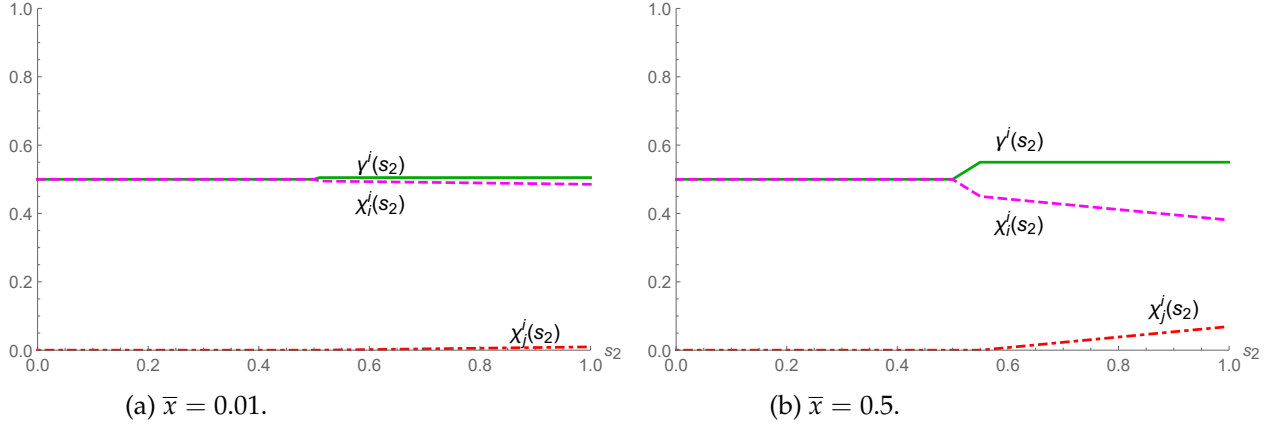


Figure 1: $\chi_{i,2}^i(s_2), \chi_{j,2}^i(s_2), \gamma_2^i(s_2)$.

Figure 1 shows that if $s_2 < \frac{1}{2}$, party $i \in \{H, L\}$ is solving its unconstrained optimal and simply equates her marginal utility of private consumption with her (not the social) marginal utility of public consumption. If $s_2 \geq \frac{1}{2}$, the unconstrained optimal is not enough anymore to guarantee party j 's minimum welfare under the status quo, activating the individual rationality constraint. When this happens, party i initially distorts the marginal utility of private and public consumption, compensating party j 's by increasing the provision of public good, since this is a good party i also enjoys it. However, party i will like to unbalance the marginal utilities across goods up to some point, defined here by $\frac{1+\bar{x}}{2}$. From that point on, party i leaves the public level of consumption the same and start providing private goods to party j to accept the proposal.

Moreover, figure 1 also shows the importance of the outside levels of consumption to party's i best-replies. When \bar{x} is "small", small increases in party's j private consumption will mean a high return in utility terms, since small \bar{x} means we are at a concave part of j 's utility. As we increase \bar{x} , small changes in party's j private consumption will mean less returns in utility terms, as we will be at a less concave part of j 's utility. Therefore, to compensate make j to accept a proposal, i has to provide more private consumption. It is clear in this example that outside consumption in the context of this model represents the party's bargaining powers: the more the allocations chosen in the political layer of society are essential to the party's welfare, the more susceptible they will be to the offers of whoever is the current decision maker for society. The higher their outside consumption, the more they have to be compensated to accept a proposal.

I now characterize first-period allocations. The characterization is only possible when the individual rationality constraint is not binding, as the constraint brings high non-linearity to the problem. For the case in which the constraint is not binding in the first-period, a simple analytical solution can be derived and it is useful to provide good intuition on the difference between rules over private goods and rules over public goods.

First-period partial characterization.— Since this problem is finite, the first period status quo s_1 is exogenously given. Also, since the first-period politician is farsighted, she will take into account the effect of her choices in the current period in the status quo of the next-period. Party i is seeking to maximize its ex-ante dynamic payoff defined as $U(c_{i,2}, g_2) = \ln(\bar{x} + c_{i,1}) + \ln(\bar{x} + g_1) +$

$\beta [pV_i(s_2) + (1 - p)W_i(s_2)]$ given a status quo rule $s_2 = \Psi(b_1) \in \mathcal{S} \forall b_1 \in \mathcal{B}$ and unanimity rule. The exogenous status quo is given by $s_1 = (c_{i,0}, c_{j,0}, 0)$ when mandatory spending is on private goods and $s_1 = (0, 0, g_0)$ otherwise. Moreover, as expressed in the equilibrium definition above, $V_i(\cdot)$ is the value for the current proposer in period one if she is the proposer again in the future, while W_i is the out-of-power value for proposer i . I define again the relevant status quo for the proposer in the first period as $s_1 \in \mathcal{S}$, with $s_1 = c_{j,0}$ when rules are over private goods and $s_1 = g_0$ when rules are over public goods. The problem of party i is given by

$$\begin{aligned} \max_{x_1=(c_{i,1}, c_{j,1}, g_1) \geq 0 \in \mathcal{B}} \quad & \ln(\bar{x} + c_{i,1}) + \ln(\bar{x} + g_1) + \beta [pV_i(s_2) + (1 - p)W_i(s_2)] \\ \text{s.t.} \quad & \ln(\bar{x} + c_{j,1}) + \ln(\bar{x} + g_1) + \beta [pW_j(s_2) + (1 - p)V_j(s_2)] \geq K_{j,1}(s_1), \end{aligned}$$

where $K_{j,1}(s_1) = \ln(\bar{x} + c_{j,0}) + \ln(\bar{x}) + \beta [pW_j(s_1) + (1 - p)V_j(s_1)]$ if budgetary rules are on private goods or $K_{j,1}(s_1) = \ln(\bar{x}) + \ln(\bar{x} + g_0) + \beta [pW_j(s_1) + (1 - p)V_j(s_1)]$ if budgetary rules are on public goods.

The main results of this session hold $\forall \beta \in [0, 1]$ and $\forall p \in [0, 1]$ and that is why I show the characterization for any level of those parameters. The exception will be on the characterization of the boundary conditions for existence of the case in which the first-period constraint is not binding. In order to solve this condition analytically I will require $\beta = 1$ and $p = 0$. I show that the condition that guarantees the individual rationality constraint is not binding in the first-period is increasing in p for the case of mandatory spending on private goods and decreasing in p for mandatory spending on public goods. This means that while for the case of mandatory spending on private goods the bulk of this session's analysis holds for a larger subset of the state space as p increases, this is not true for the case of mandatory spending on g . Therefore, the analysis here has to be taken carefully considering which is the range of the initial status quo that is being considered.

Since in this example the parties value private and public goods in the same way, it is clear that the same level of welfare can be guaranteed by either goods. The difference here between the two types of mandatory spending lies on the impact on the party's trade-off, since rules over private good will distort first-order-conditions of private goods $c_{i,1}$ and $c_{j,1}$ and rules over public goods will distort first-order-conditions of the public goods g_1 . A detailed discussion of the first-order conditions is given in the Appendix, but in the main text I give part of the intuition of why mandatory spending over private goods is different than over public goods in terms of the trade-off faced by the first-period proposer.

Proposition 2 *For any s_1 such that the individual rationality constraint is not binding in the first-period, with $s_1 = c_{j,0}$ for the case of rules over private goods and $s_1 = g_0$ for rules over private goods, the unique proposal strategy for party $i \in \{H, L\}$ when mandatory spending is over private goods is:*

$$\gamma_1^i(s_1) = \frac{1 - \beta(1 - p)\bar{x}}{2 + \beta(1 - p)}$$

an when mandatory spending is over public goods:

$$\gamma_1^i(\mathbf{s}_1) = \begin{cases} \frac{1+\beta(1+\bar{x}(1-p))}{2+\beta(1+p)}, & \text{if } p \geq \frac{1-\bar{x}}{1+3\bar{x}} \\ \frac{B-\sqrt{A}}{8(2+\beta)\bar{x}}, & \text{otherwise} \end{cases}$$

with $\chi_{j,1}^i(\mathbf{s}_1) = 0$, $\chi_{i,1}^i(\mathbf{s}_1) = 1 - \chi_{j,1}^i(\mathbf{s}_1) - \gamma_1^i(\mathbf{s}_1)$ and A and B constants defined in the Appendix.

An immediate result of this characterization is that mandatory spending over private goods induces under-provision of public goods vis-a-vis the first best allocation. The opposite is true for mandatory spending over public goods, where there is over-provision of public goods. Moreover, the mis-allocation created by the mandatory spending is decreasing in probability that the proposer remain in power.

Corollary 1 *For any s_1 such that the individual rationality constraint is not binding in the first-period, mandatory spending over private goods induces over-provision of the public goods, vis-a-vis the first-best allocation. The opposite is true when rules are over public goods, meaning there will be under-provision of public good vis-a-vis the first-best. Moreover, the level of mis-allocation, determined by the amount of over or under-provision of the public good, is decreasing in p , the probability that the proposer remains in office.*

All details are in the Appendix. The main intuition of this proposition is that although we can guarantee the same level of welfare under rules over private goods than over rules under public goods, trade-off politicians face will differ under the two rules, which will mean different allocative distortions vis-a-vis the first-best allocation. I show below a sketch for the proof that provides intuition for the results. When the individual rationality constraint is not binding in the first period, party's i first-order conditions boil down to:

$$\underbrace{\frac{1}{\bar{x} + 1 - g_1}}_{MU_c} + \underbrace{\beta(1-p) \frac{\partial W_i(\mathbf{s}_2)}{\partial c_{i,1}}}_{\text{wedge}_c > 0} = \underbrace{\frac{1}{\bar{x} + g_1}}_{MU_g}$$

for mandatory spending over private goods and

$$\underbrace{\frac{1}{\bar{x} + 1 - g_1}}_{MU_c} = \underbrace{\frac{1}{\bar{x} + g_1}}_{MU_g} + \underbrace{\beta \left(p \frac{\partial V_i(\mathbf{s}_2)}{\partial g_1} + (1-p) \frac{\partial W_i(\mathbf{s}_2)}{\partial g_1} \right)}_{\text{wedge}_g > 0}$$

for mandatory spending over public goods. In the first case, when mandatory spending is over private goods, the current decision of private allocation will determine the value for the proposer i when she is out of power in the following period, creating a positive wedge on the marginal utility of private consumption, which will lead to an optimal value that is above the unconstrained optimal. The opposite happens in the case of mandatory spending over public goods, since the wedge there increases the marginal utility of the public good. This means that in the case of mandatory spending over private goods, there will be under-provision of the public good and in the case of mandatory spending over public goods, there will be over-provision of the public good. The whole proof is left for the Appendix, but the best-reply for specific parameterizations is depicted below.



(a) $\beta = 1$ and $\bar{x} = 0.01$.



(b) $\beta = 1$ and $\bar{x} = 0.5$.

Figure 2: $\beta = 1$ and $\bar{x} = 0.01$. There is over-provision of public good when mandatory spending is on public goods g_1 vis-a-vis the first-best while there is under-provision when mandatory spending is on private goods.

The results above hold for the case in which the first-period individual rationality constraint is not binding. A remaining question is what are the sufficient conditions for this constraint to be slack. I now show sufficient conditions such that the first-period constraint is slack:

Proposition 3 *There is $s_1^* \in s_1$ such that $\forall s_1 < s_1^*$ the individual rationality constraint in the first-period is not binding. Let's take $\beta = 1$ and $p = 0$. In this case, s_1^* can be characterized as following:*

$$c_{j,0} < \underline{c}_{j,0} = \frac{1 - \bar{x}}{3}$$

for when mandatory spending is over private goods and

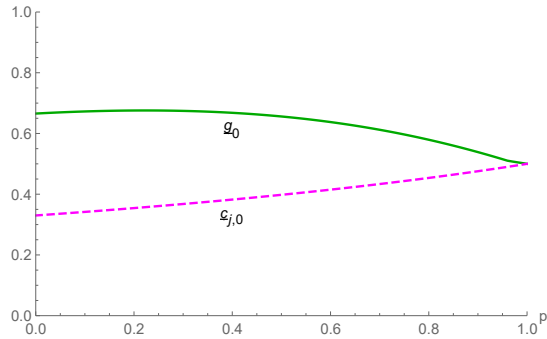
$$g_0 < \underline{g}_0 = \frac{4\bar{x}^2 + 11\bar{x} + 6}{18\bar{x} + 9}$$

for when mandatory spending is over public goods.

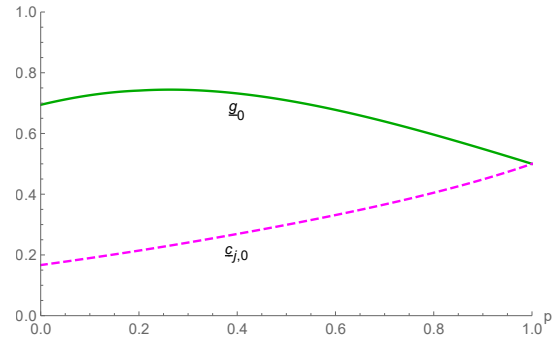
The bulk of the analysis of this session will take into account a comparative statics of the optimal strategies of party i with respect to p , the probability that this party remains in power in period two. Therefore, it is crucial to understand that when p increases, if the constraint gets tighter or looser, which leads to the following corollary:

Corollary 2 *There is $\underline{s}_1 \in s_1$ such that $\forall s_1 < \underline{s}_1$ the individual rationality constraint in the first-period is not binding for $p = 0$. Moreover, \underline{s}_1 is increasing in p when mandatory spending is over private goods and decreasing when mandatory spending is over public goods.*

The proof of this corollary is also on the Appendix. There, I show with the use of the Implicit Function Theorem that the maximum exogenous status quo such that the individual rationality constraint is not binding goes in opposite directions with respect to p depending on which good is mandatory under the status quo. Since \underline{s}_1 is decreasing in p for mandatory spending on private goods but the opposite is true for mandatory spending on public goods, the analysis above has to consider a \underline{s}_1 such that the first-period individual rationality constraint is not binding for both cases. Below the picture depicts the intuition from the above proposition:



(a) $\beta = 1$ and $\bar{x} = 0.01$.



(b) $\beta = 1$ and $\bar{x} = 0.5$.

Figure 3: $\beta = 1$ and $\bar{x} = 0.01$. The maximum exogenous status quo such that the first period's constraint is not binding is increasing in p for mandatory spending on private goods and decreasing for mandatory spending on public goods.

3.2 Numerical solution

The past section highlighted the difference between having mandatory spending over private goods or public goods. Even for the case in which the two goods (as $\alpha = \frac{1}{2}$) and the individual rationality constraint is not binding, mandatory spending over private goods implies different wedges than mandatory spending over public goods: mandatory spending on private goods means under-provision of the public good while mandatory spending on the private public good means over-provision of the public good. It remains to be answered if this misallocation holds in cases in which the constraint is binding in the first period and, moreover, what this misallocation means welfare-wise. In this section, I move to the numerical solution of the problem so we can have the optimal strategies of the players on the entire state space, not just when the first-period constraint is slack. In this way, I can analyze welfare of society under this rules compare to a benchmark in which there is no mandatory spending, defined by the dictator's problem, and with the first-best allocation, defined by the solution of the status planner's problem.

Result 1. (Optimal strategies) *The optimal strategies for parties are given by:*

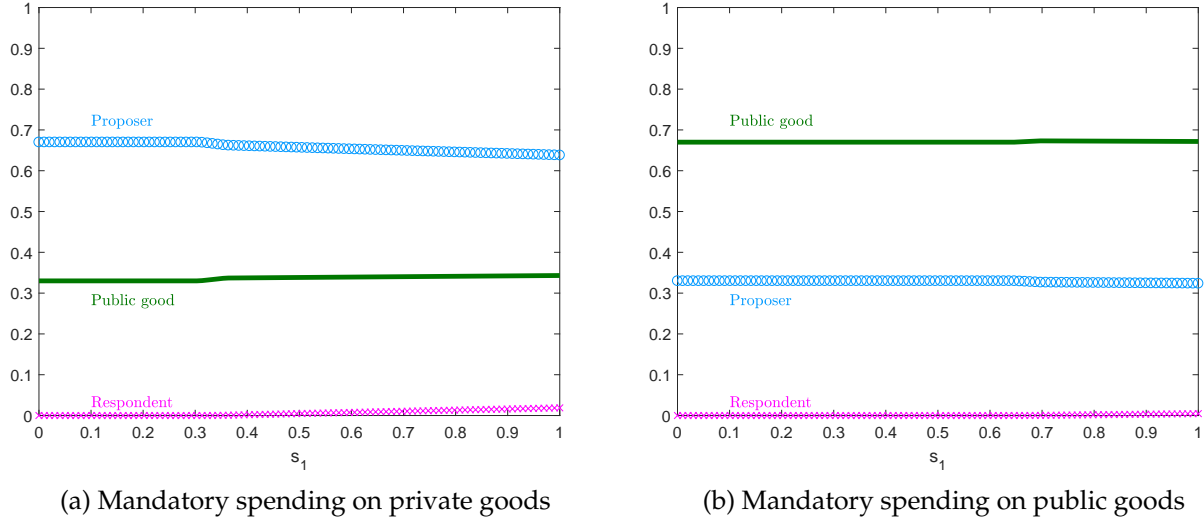


Figure 4: $\chi_{i,1}^i(s_1), \chi_{j,1}^i(s_1), \gamma_1^i(s_2)$ for both mandatory spending rules and $\beta = 1, \bar{x}$ and $p = 0.5$.

The result above shows the optimal strategies of party i with respect to the relevant exogenous status quo s_1 , which is equal to $c_{j,0}$ when mandatory spending is on private goods and equal to g_0 when mandatory spending is on public goods. The intuition from the analytical characterization still holds for the whole state space of the relevant status quo: there will be under-provision of public goods when mandatory spending is on private goods and over-provision of public goods when mandatory spending is on public goods. The wedge created by the rules is not exactly anti-symmetric, as it is clear in the plots above: when the constraint is slack, for low levels of s_1 , private transfers for the respondent under mandatory spending on private goods is the same as public good provision under mandatory spending on public goods. But when the constraint starts to bind, for higher levels of the initial relevant status quo s_1 , the changes in the optimal strategies under mandatory spending on public goods is really small, almost imperceivable in the right chart above. This is not true for the case in which mandatory spending is on private goods, showed in the left plot. There, as the relevant status quo increases the proposer reduces transfer to herself to first increase public good provision up to the point in which it has to start transferring resources as private transfers to the respondent.

Transfers to the respondent, however, are always very small, even to high levels of the initial status quo. There are two factors behind this result. First is the fact that exogenous consumption of parties, measured by $\bar{x} = 0.01$, is small. This means that allocations done by the proposer are valuable and therefore small increases in the scarce good, the private transfer to the respondent, generate significantly high marginal utility for the respondent. The second factor is the fact that $\alpha = 0.5$. This means that for the same levels of both goods being provided, a marginal increase in the public good generates the same utility increase than the private good, which allows for the proposer to compensate the respondent with public goods to a great extent.

There are two immediate questions that still require an answer. The first is what the mis-allocations showed above mean in terms of welfare. Although over or under provision of the goods vis-a-vis the planner will incur in Pareto inefficiencies, society may be better off on net terms, considering winners and losers, with some type of mis-allocation. The second is what happens to optimal strategies and welfare when the level of political disagreement, measured by α is smaller than the one presented in this section. I present the welfare analysis for the case in

which $\alpha = 0.5$ in the last subsection of this chapter, to then move to different levels of political disagreement.

4 Welfare analysis

Benchmarks

I begin by considering two benchmark models: the static Pareto efficient allocations and a model in which all spending is discretionary, which is equivalent to having the proposer will be solving an unconstrained model. Then I show two metrics of welfare, one that is Pareto and another that shows net welfare gains from introducing the mandatory spending rule for both private and public goods. **Pareto efficiency.**—An useful benchmark is the first-best allocation, characterized by the static planner's problem. A Pareto efficient allocation solves the following problem for $\bar{u} \in R$:⁵

$$\begin{aligned} \max_{x_i \geq 0 \in \mathcal{B}} u_H(x_i) \\ \text{s.t. } u_L(x_i) \geq \bar{u}_L \end{aligned} \quad (2)$$

Let $u_i(c_i, g) = \ln(\bar{x}_i + c_i) + \ln(\bar{x} + g)$. Moreover, let's write the minimum level of consumption of party L in terms of goods instead of utility, i.e., $\bar{u}_L = \ln(\bar{x}) + \ln(\bar{x} + s)$, where $s \in \mathcal{S}$ is a relevant status quo allocation. I can characterize the first-best allocations as:

Proposition 4 *The Pareto efficient allocations are characterized by $x^{fb} = (c_H^{fb}, c_L^{fb}, g^{fb})$:*

$$\begin{aligned} g^{fb} &= \begin{cases} \frac{1}{2}, & \text{if } s < \frac{1}{2} \\ s, & \text{if } \frac{1}{2} \leq s < \frac{1+\bar{x}}{2} \\ \frac{1}{2}(1+\bar{x}), & \text{if } \frac{1+\bar{x}}{2} \leq s \end{cases} \\ c_L^{fb} &= \begin{cases} 0, & \text{if } s < \frac{1}{2} \\ 0, & \text{if } \frac{1}{2} \leq s < \frac{1+\bar{x}}{2} \\ \frac{\bar{x}(2s-1)-\bar{x}^2}{1+3\bar{x}}, & \text{if } \frac{1+\bar{x}}{2} \leq s \end{cases} \\ c_H^{fb} &= \begin{cases} \frac{1}{2}, & \text{if } s < \frac{1}{2} \\ 1-s, & \text{if } \frac{1}{2} \leq s < \frac{1+\bar{x}}{2} \\ \frac{1+4\bar{x}(1-s)-\bar{x}^2}{2(1+3\bar{x})}, & \text{if } \frac{1+\bar{x}}{2} \leq s \end{cases} \end{aligned}$$

The characterization, although simple, is extensive as it requires many cases to be checked, which is left for the Appendix.

⁵I show in the Appendix that the planner's problem in which the planner's maximizes one party's utility subject to a minimum level of utility to the other party \bar{u} can be mapped into the more traditional planner's problem with exogenous weights. The representation I use here is more common in the legislative bargaining legislature literature and it is more convenient as first-best allocations will be expressed in terms of status quo allocations instead of weights, which are irrelevant to the analysis carried on this paper. In the Appendix, I show that both problems are equivalent as we potentially, but not without a lot of algebra, can write the exogenous planner's weights as a function of the status quo allocation that would deliver such minimum welfare in the problem above. Moreover, although I show the static efficient allocation, since there is dynamic without politics, the dynamic and static optimal coincide (or time consistent and time inconsistent problems coincide). Hence, we can focus on first-best from the perspective of a time consistent planner.

Dictator levels.—Another useful benchmark is party's i ideal allocation in any period, which is the solution of the unconstrained problem for party i :

$$\max_{\mathbf{x}_t \geq 0 \in \mathcal{B}} u_i(\mathbf{x}_t) \quad (3)$$

Proposition 5 *At any period of time is given by, the unique ideal allocation for party i is the solution to an unconstrained maximization problem, which equates the marginal utilities of consumption of public good with private good, which delivers $c_{i,t} = g_t = \frac{1}{2}$.*

Any party, when solving an unconstrained problem, would like to equate the marginal utilities across goods, unless a corner solution would make them better-off, which is not possible unless $\bar{x} < 0$. Therefore, party's prefer mixed bundles then corner bundles when solving an unconstrained problem.

Welfare measures

In order to make an objective comparison across mandatory spending rules I define below a metric of inefficiency that takes into account the average Pareto improvement that could be made for a given allocation for a fixed rule.

Definition 2. Ex-ante lifetime utility: *The ex-ante lifetime utility of party i can be given as:*

$$U_i(\mathbf{x}_1; \mathbf{X}_2(\Psi(\mathbf{x}_1))) = u_i(\mathbf{x}_1) + \beta [pV_i(\Psi(\mathbf{x}_1)) + (1-p)W_i(\Psi(\mathbf{x}_1))] \quad (4)$$

where for the sake of notation I denote $U_i(\mathbf{x}_1; \mathbf{X}_2(\Psi(\mathbf{x}_1))) = U_i(\mathbf{x}_1)$ and $\mathbf{x}_1 = (c_{H,1}, c_{L,1}, g_1)$ is a vector of first-period allocations.

I also define the ex-ante dynamic payoff of parties and I characterize another useful benchmark, defined as the dictator level, the unconstrained optimum of party's i .

Definition 3. (Pareto inefficiency) *Fix one of the rules that define property rights as defined above. For a given $p \in [0, 1]$, define the Pareto inefficiency level as:*

$$\Delta^{fb} = \underbrace{U_L(\mathbf{x}^*) - U_L(\mathbf{x}^{fb})}_{\Delta_L^{fb}} + \underbrace{U_H(\mathbf{x}^*) - U_H(\mathbf{x}^{fb})}_{\Delta_H^{fb}} \quad (5)$$

where Δ^{fb} measures how much of Pareto inefficiency there is from the first-best. \mathbf{x}^* is the optimal strategy that solves the political game presented above and \mathbf{x}^d is the solution for the dictator's problem presented above.

I define a measure of the net welfare gain of mandatory spending that is given by the difference in the utility pair for both parties with respect to the dictator world, as defined above.

Definition 4. (Net welfare improvement from no mandatory spending) *Fix one of the rules that define budget rules as defined above.*

$$\Delta^d = \underbrace{U_L(\mathbf{x}^*) - U_L(\mathbf{x}^d)}_{\Delta_L^d} + \underbrace{U_H(\mathbf{x}^*) - U_H(\mathbf{x}^d)}_{\Delta_H^d} \quad (6)$$

where \mathbf{x}^* is the optimal strategy that solves the political game presented above and \mathbf{x}^d is the solution for the dictator's problem presented above. Then, Δ^d measures the net welfare gain society had from introducing mandatory spending.

This first measure can also be thought as a certainty equivalent measure, or how much welfare individuals are willing to give up in order to be at the first-best allocation. It shows the opportunity cost society faces for not having a planner making decisions for society. It is a useful measure, but it has one set back: since the comparison is with a benevolent planner that is always in power, the impact of political turnover is two folded: the first is the direct impact of power alternation that creates bumpy consumption and the second is the indirect impact of political turnover on the optimal strategies of the parties. In order to separate the impact of political turnover into these two things, the simple political turnover *per se* and the impact of political turnover on the optimal strategies, I create the second measure of welfare presented above. This second measure is not Pareto, because it considers the net welfare impact of introducing mandatory spending. The plots below show both measures:

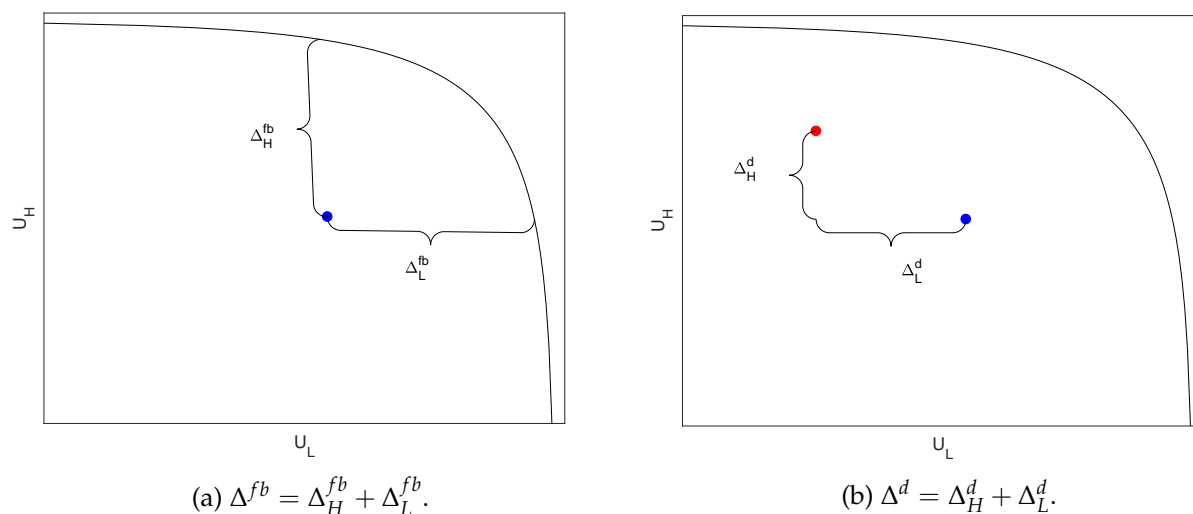


Figure 5: The red dot is the pair of utilities at x^d for a given β, p and relevant initial status quo s_1 , where $s_1 = c_{j,0}$ for mandatory spending on private goods and $s_1 = g_0$. The red dot is the pair of utilities at x^* for some rule on mandatory spending.

With these metrics in mind, I can explore welfare gains for the two mandatory spending rules defined above and its relationship to the initial status quo, the level of political disagreement society and outside consumption of parties.

4.1 Initial status quo

Result 2. (Initial status quo) *When the relevant initial status quo s_1 is low ($s_1 = c_{j,0}$ for mandatory spending on private goods and $s_1 = g_0$ for mandatory spending on public goods), mandatory spending on private goods makes society worse off when compared to when there is no mandatory spending. Mandatory spending on public goods makes society better off.*

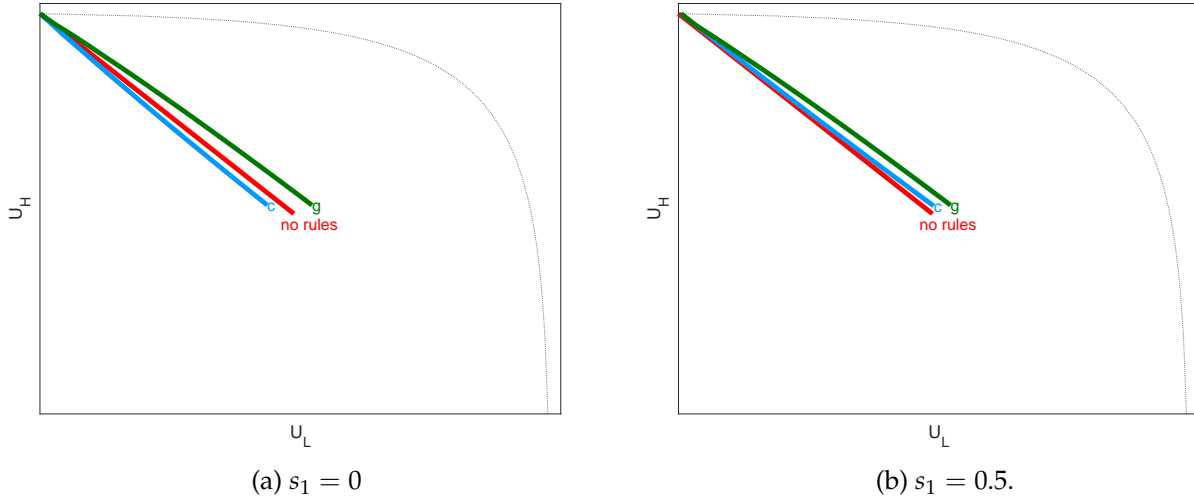


Figure 6: Pareto frontier and welfare for: no mandatory spending (red), mandatory spending over private goods (blue) and mandatory spending over public goods (green) for all $p \in [0, 1]$, for $\beta = 1$ and selected initial status quo s_1 , where $s_1 = c_{j,0}$ for mandatory spending on private goods and $s_1 = g_0$.

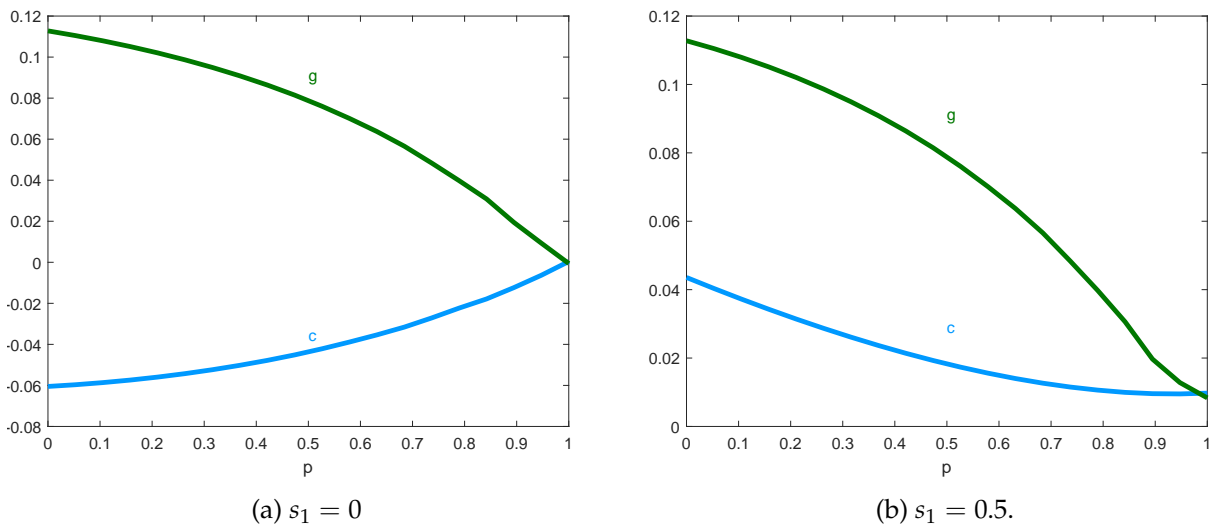


Figure 7: Net welfare improvement for mandatory spending on public goods (green) and private goods (blue) for $\beta = 1$ and selected initial status quo s_1 .

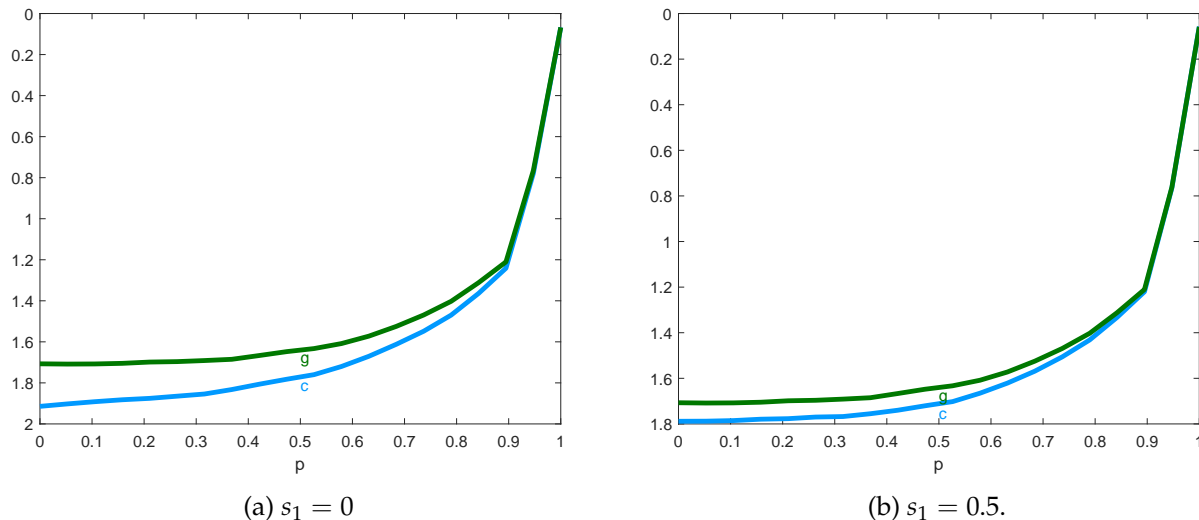


Figure 8: Pareto efficiency for mandatory spending on public goods (green) and private goods (blue) for $\beta = 1$ and selected initial status quo s_1 .

The plots above show that mandatory spending on public goods is always better in terms of net welfare gains. The intuition behind this result is the following. The respondent faces bumpy private consumption, but not bumpy public good consumption. This is because the proposer always want to provide some public good, which will be consumed by the respondent as well. The introduction of mandatory spending on public goods generates over provision of public good, as we've seen in the last section, and this over-provision increases the utilities of both the proposer and the respondent. Mandatory spending on private goods induces under-provision of the public good, making the respondent worse off, as private transfers to the respondent are not high enough to compensate for the loss of public good. This happens because since parties have a "high" value for the public good, any provision of the public good, which is preferred by the proposer, will be enough to make the respondent to accept the proposal. In other words, when the marginal value of the public good is high, the respondent prefers to live under the mandatory provision of public good, as in any of the mandatory spending rules she will be able to smother out her consumption. The plots below illustrate this discussion:

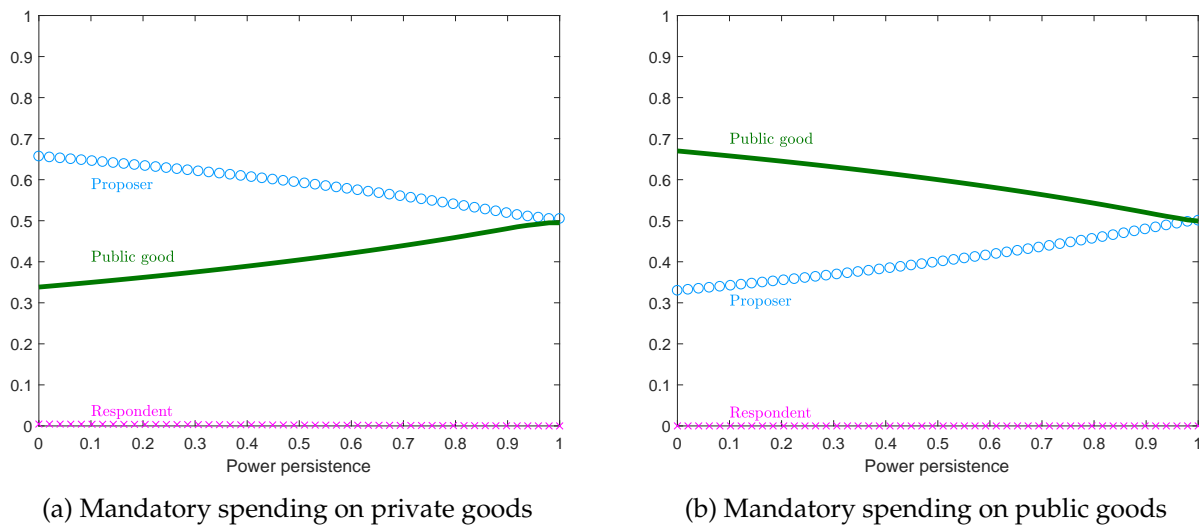


Figure 9: $\chi_{i,1}^i(s_1), \chi_{j,1}^i(s_1), \gamma_1^i(s_2)$ for both mandatory spending rules and $\beta = 1, \bar{x}$ and $s_1 = 0.5$.

It is clear that the marginal value of the public good, defined in this paper as the level of political conflict in society, has a crucial role in the welfare implications of mandatory spending rules on different goods. In the next section I move from the simple case in which $\alpha = 0.5$ and show the impacts of different political conflict levels on welfare.

4.2 Political conflict

The higher the level of the parameter α , the lower is the level of disagreement. In fact, if $\alpha = 1$, then parties enjoy only the public good, the dollar will be spent on the provision of this good and there will be no conflict among parties. Conflict arises when parties value their private consumption, as it is rival and excludable. In the former section, political conflict was relatively high, as $\alpha = 0.5$, indicating groups value marginally both goods to the same extent.

Result 3. (Political conflict) *When political conflict increases, i.e., α decreases, mandatory spending on public goods dominates mandatory spending on private goods. Moreover, the higher the political conflict, the closer society will be to the first-best if mandatory spending is on private goods.*

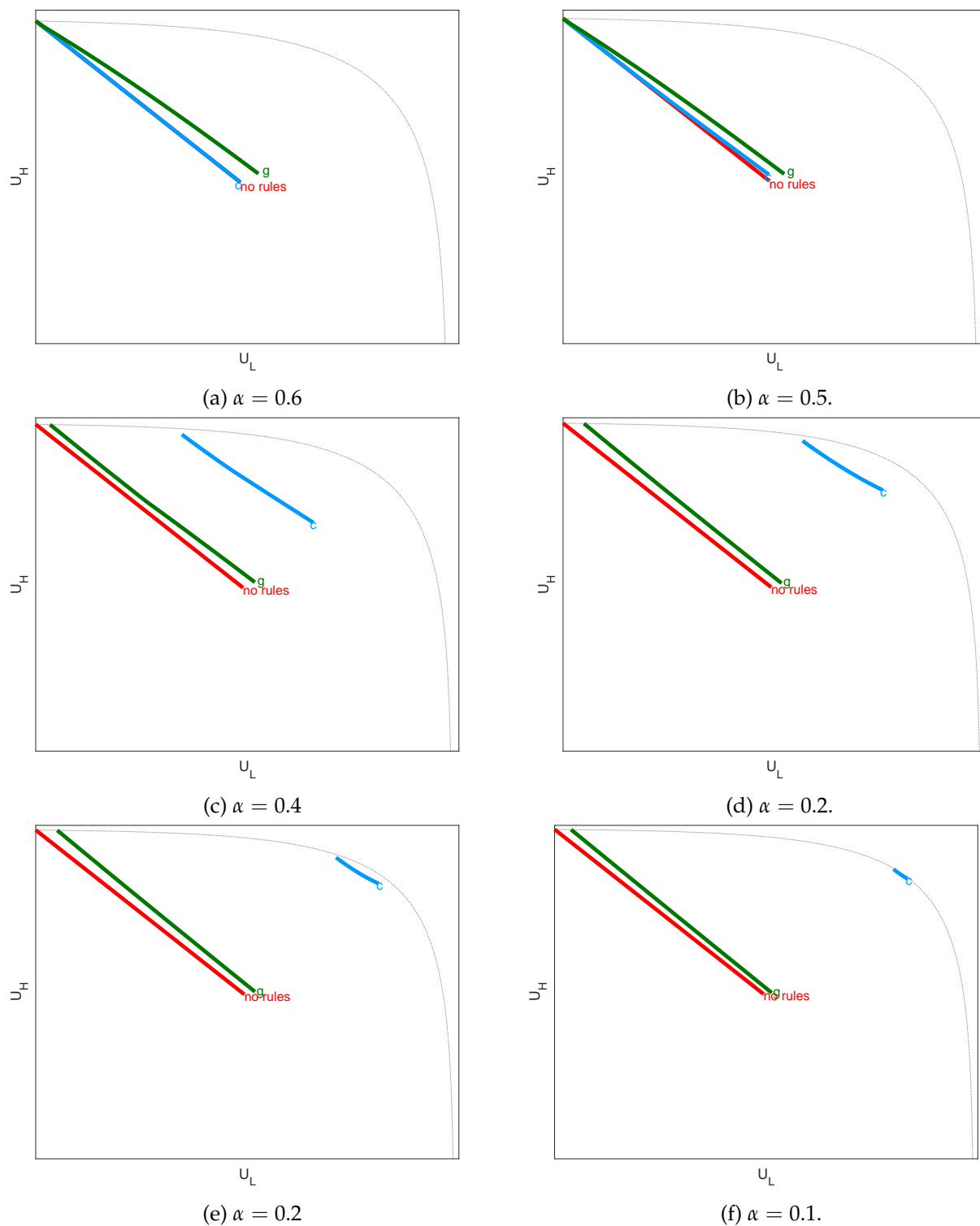


Figure 10: Pareto frontier and welfare for: no mandatory spending (red), mandatory spending over private goods (blue) and mandatory spending over public goods (green) for all $p \in [0, 1]$, for $\beta = 1$ and selected initial status quo $s_1 = 0.5$.

The collection of plots above show that as we increase political conflict, mandatory spending

on private goods is not only superior do mandatory spending on public goods but it can also reach the first-best. A summary of the collection of pictures above can be given by the plot below which shows the net welfare gains, as defined in the former session, with respect to α . As we can see, the α parameter doesn't really change the net welfare gains when mandatory spending is on public goods (green line). When mandatory spending is on private goods (blue line), however, the lower the α parameter, i.e., the higher the political conflict, the higher will be net welfare gains for society.

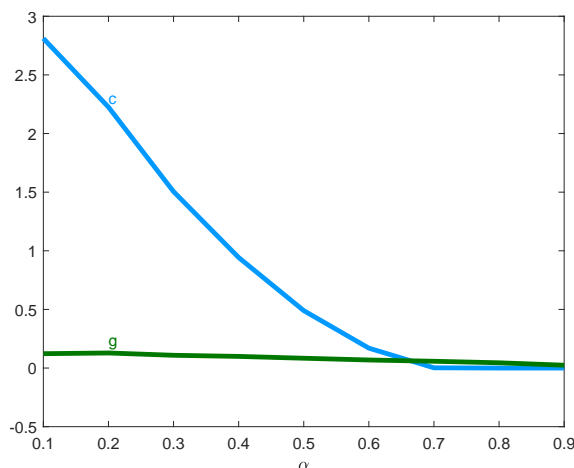
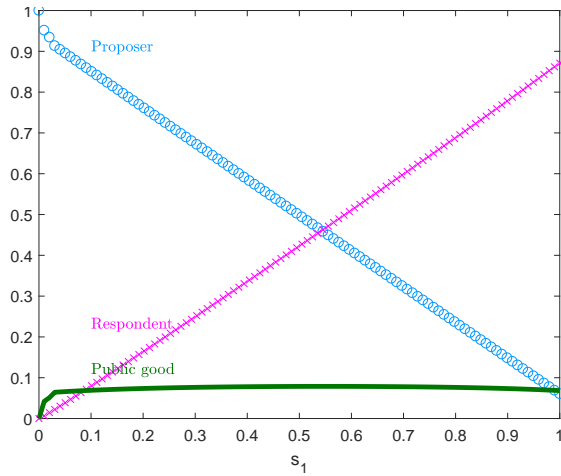


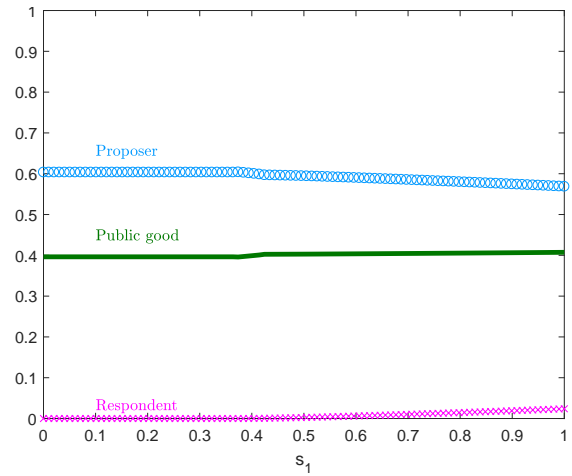
Figure 11: Net welfare gains for $\beta = 1, p = 0.5, s_1 = 0.5$ and $\alpha \in [0.1, 0.9]$

The intuition for the result is the following. If there was no institution, the proposer would fully expropriate the respondent from any private transfers and would choose private transfer to herself and public good provision such that the marginal utility of her private consumption and her private public good consumption are equated. This leaves the respondent with no private good. It is then the private good that is not smoothed across time when there is political fluctuation. Therefore, this is the good that politicians have the greatest demand to insure against expropriation. When the political system is of unanimity, the proposer has to get the approval of the proposer to implement an allocation. Moreover, since there allocations in the first period will be projected into second period's reservation value for whoever gets to be the proposer, a distortion appears in the first-order condition of the proposer. As we saw it above, the type of good that is under the mandatory spending rule, and how much individuals like this good, define where this distortion will appear in the first-order condition of the proposer. Moreover, since mandatory spending on public goods creates a positive wedge on public good provision, this institution cannot satisfy the demand for insurance on the good that suffers the most from expropriation, which is the private good.

It is left to answer now why high political conflict (low α) guarantees more insurance for the respondent and, therefore, is more welfare improving. The easy answer to it is because it is cheap: when α is low, it is relatively cheap for the proposer to guarantee for the proposer private transfers. Moreover, the proposer himself has less demand for that good and it has more marginal utility on the public good. Therefore, the lower the α , the better will do mandatory spending on private goods in terms of smoothing consumption to the respondent.

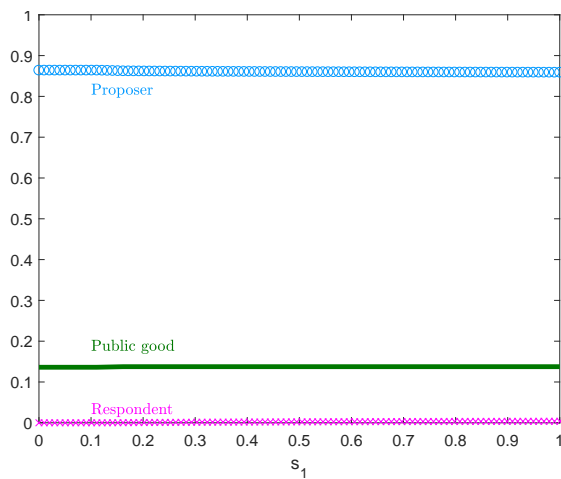


(a) $\alpha = 0.1$

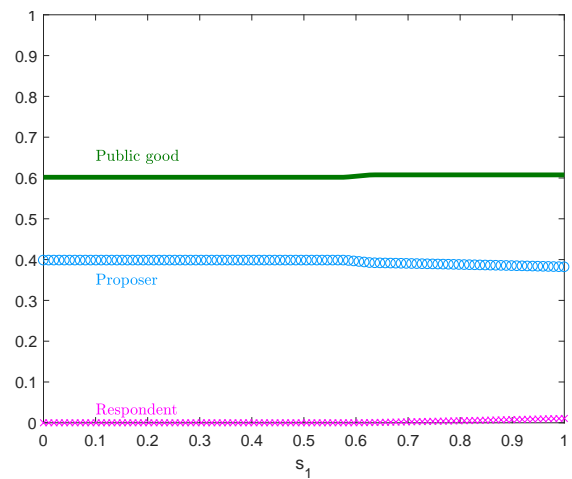


(b) $\alpha = 0.5$

Figure 12: Mandatory spending on private goods c : optimal strategies for incumbent party for $\beta = 1, p = 0.5$ and selected α 's.



(a) $\alpha = 0.1$



(b) $\alpha = 0.5$

Figure 13: Mandatory spending on public good g : optimal strategies for incumbent party for $\beta = 1, p = 0.5$ and selected α 's.

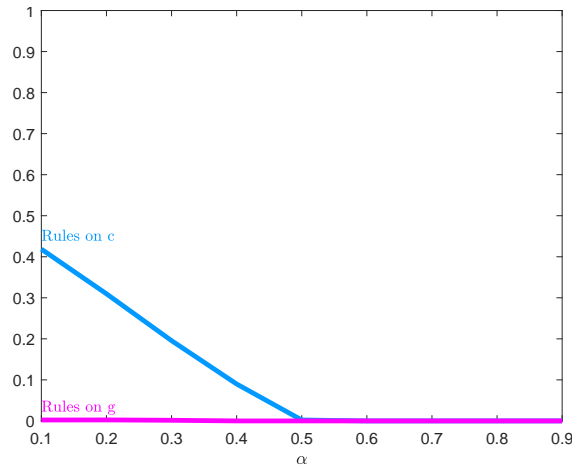


Figure 14: Optimal strategy for incumbent party on respondent's transfer for $\beta = 1, p = 0.5, s_1 = 0.5$.

4.3 Outside consumption

The parameter \bar{x} measures consumption parties receive is from the outside of government decisions. The greater is this outside consumption, the less relevant for the total welfare of parties the government decisions will be, which means that for a marginal increase in a partie's utility, a great

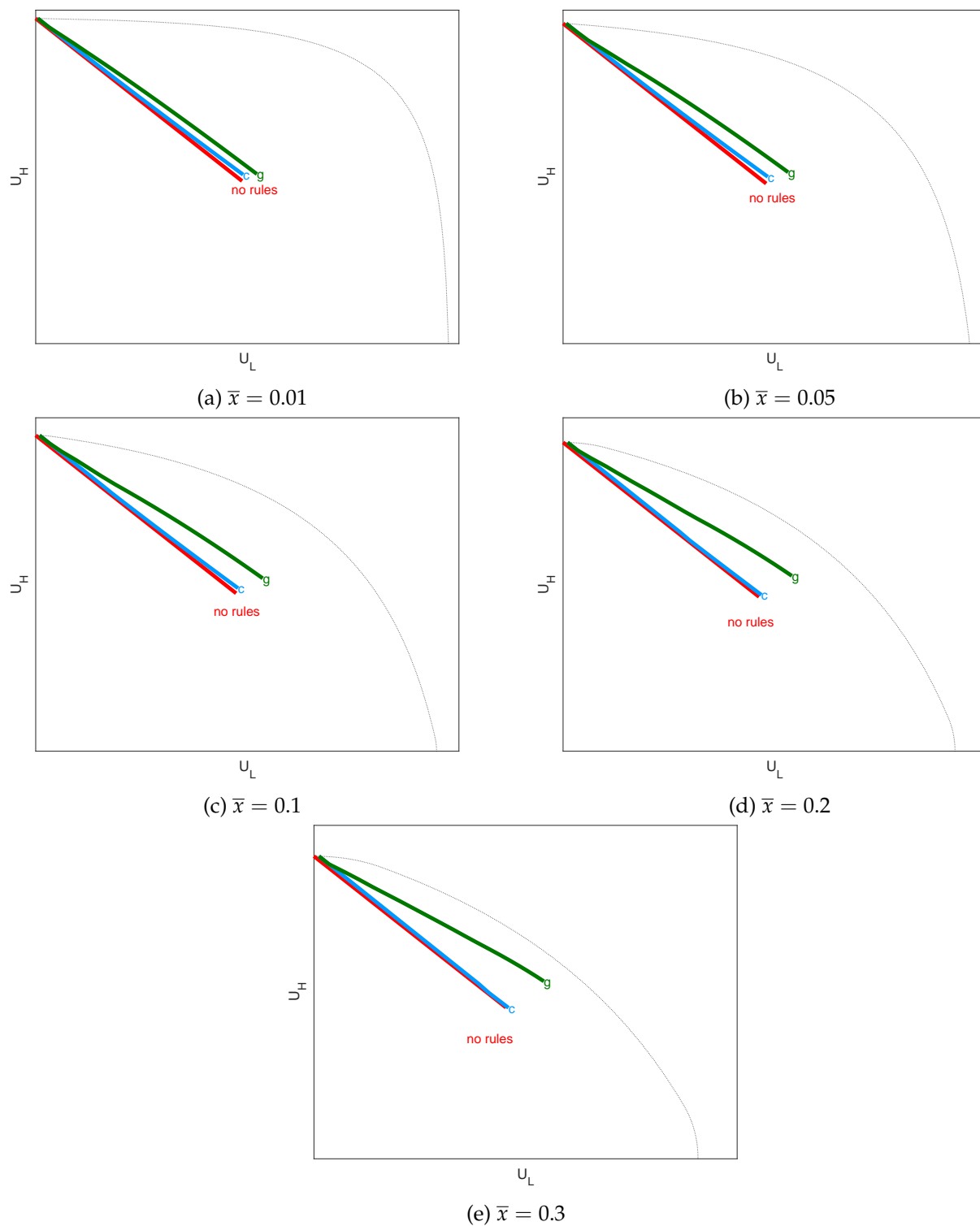


Figure 15: Pareto frontier and welfare for: no mandatory spending (red), mandatory spending over private goods (blue) and mandatory spending over public goods (green) for $\beta = 1, \alpha = 0.5, p = 0.5, s_1 = 0.5$ and $\bar{x} \in [0.01, 0.3]$.

The collection of plots above show that as we increase political conflict, mandatory spending

on private goods is not only superior do mandatory spending on public goods but it can also reach the first-best.

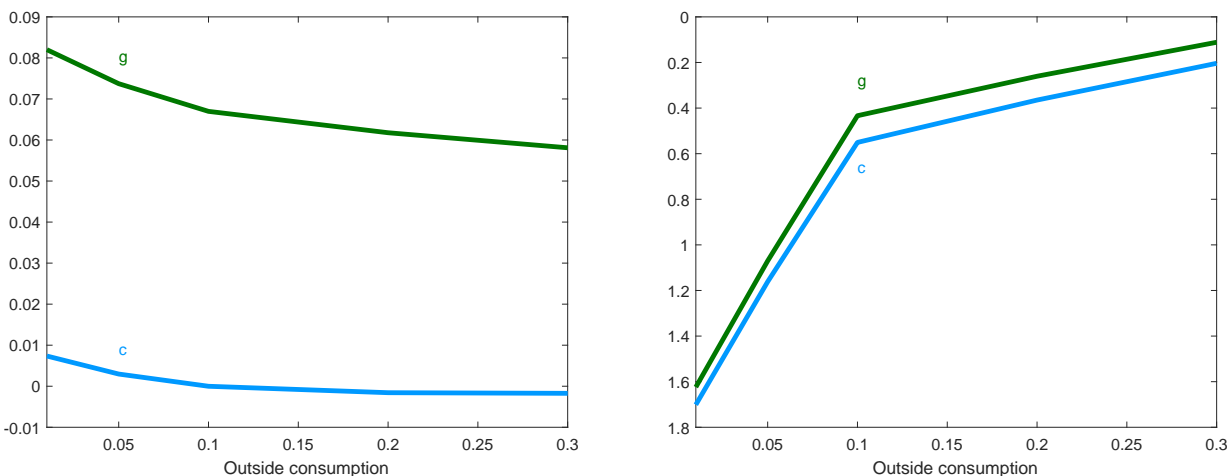


Figure 16: Net welfare gains for $\beta = 1, \alpha = 0.5, p = 0.5, s_1 = 0.5$ and $\bar{x} \in [0.01, 0.3]$

5 Infinite-Horizon Model

To be completed.

6 Conclusions and extensions

This paper studies the welfare implications of a class of budget rules, the ones that determine mandatory spending. I analyze a model with two parties that allocate a fixed budget to private transfers and a public good. Each period a party is chosen to propose an allocation while the other party can accept or reject the proposal. Mandatory spending is modeled in the spirit of dynamic legislative bargaining with an endogenous status quo. The relative taste for public goods define the level of political conflict in society. I show three results. First, the type of good under mandatory spending is not without loss of generality, as mandatory spending over private goods deliver under-provision of the public good, vis-a-vis the first-best allocation, while the opposite is true for mandatory spending on public goods. This happens because this institution creates a positive intertemporal wedge on the good that is mandatory. Moreover, when political conflict is relatively high, it is welfare superior to have mandatory spending on public goods, as its over-provision is less harmful to welfare than its under-provision. However, as the level of political conflict decreases, mandatory spending on private goods may not only become welfare superior, but it may also bring society to the first-best. In fact, net welfare gains are increasing with political conflict when mandatory spending is on private goods. This result highlights that political conflict is not necessary harmful to welfare, specially when there are institutions that prevent expropriation. Finally, I show that net welfare gains have a positive relationship with political turnover, indicating that the *common wisdom* that more political turnover is better holds true when there is such an institution as mandatory spending.

A natural extension is to work with the dynamic interaction of politicians in an infinite horizon, which is work in progress. This is useful to avoid the advantage of the leader in the political game,

which is the group H in my examples. It also helps to see how robust are the results if the strategic interaction of politicians is repeated *ad infinitum*. As discussed above, as in most dynamic games, there is a problem of multiplicity which cannot be addressed by simply restricting strategies of parties to be Markovian. For a theoretical detailed discussion on this subject, please refer to Anesi and Duggan (2015a).

Appendix

Party's i unconstrained optimum

Party's i Lagrangian problem $\forall t$ is given by:

$$\mathcal{L} = \ln(\bar{x} + c_{i,t}) + \ln(\bar{x} + g_t) + \lambda_t [1 - c_{i,t} - c_{j,t} - g_t]$$

Since the status quo enters in the problem via individual rationality constraint, it is irrelevant for this problem. The first order and Kuhn-Tucker conditions for this problem are $c_{i,t}, c_{j,t}, g_t, \lambda_t \geq 0$ and

$$\begin{aligned} [c_{i,t}] \quad & \frac{1}{\bar{x} + c_{i,t}} - \lambda_t \leq 0 \\ & \left[\frac{1}{\bar{x} + c_{i,t}} - \lambda_t \right] c_{i,t} = 0 \end{aligned} \tag{U1}$$

$$\begin{aligned} [c_{j,t}] \quad & -\lambda_t \leq 0 \\ & [-\lambda_t] c_{j,t} \leq 0 \end{aligned} \tag{U2}$$

$$\begin{aligned} [g] \quad & \frac{1}{\bar{x} + g_t} - \lambda_t \leq 0 \\ & \left[\frac{1}{\bar{x} + g_t} - \lambda_t \right] g_t = 0 \end{aligned} \tag{U3}$$

$$\begin{aligned} [RC] \quad & 1 - c_{i,t} - c_{j,t} - g_t \geq 0 \\ & [1 - c_{i,t} - c_{j,t} - g_t] \lambda_t = 0 \end{aligned} \tag{U4}$$

First assume $\lambda_t = 0$. By (U1) we have that $c_{i,t} < \bar{x}$, which contradicts the fact that we require $c_{i,t} \geq 0$. We conclude that $\lambda_t > 0$ which implies that $1 - c_{i,t} - c_{j,t} - g_t = 0$. Since $\lambda_t > 0$, equation (U2) implies that $c_{j,t} = 0$. Also since $\lambda_t > 0$, equation (U1) implies that $c_{i,t} > 0$ and equation (U3) implies that $g_t > 0$. This means that the optimal solution is interior for $c_{i,t}$ and g_t and it is the one that equates the marginal utility across private and public goods:

$$\frac{1}{\bar{x} + c_{i,t}} = \frac{1}{\bar{x} + g_t}$$

Since $c_{i,t} = 1 - g_t$, we have that $c_{i,t} = g_t = \frac{1}{2}$.

Pareto Efficiency: equivalence between static planner with endogenous and exogenous weights

The static planner with exogenous weights solves the following problem, for $\pi \in [0, 1]$:

$$\max_{x_t \geq 0 \in \mathcal{B}} \pi u_H(x_t) + (1 - \pi) u_L(x_t)$$

The Lagrangian for this problem can be written as:

$$\mathcal{L}_1 = \pi [\ln(\bar{x} + c_H) + \ln(\bar{x} + g)] + (1 - \pi) [\ln(\bar{x} + c_L) + \ln(\bar{x} + g)] + \lambda [1 - c_H - c_L - g]$$

The static planner with endogenous weights solves the following problem, for $\underline{u}_L \in \mathbb{R}$:

$$\begin{aligned} \max_{x_t \geq 0 \in \mathcal{B}} & u_H(x_t) \\ \text{s.t.} & u_H(x_t) \geq \bar{u}_L \end{aligned}$$

The Lagrangian problem of this planner can be written as:

$$\mathcal{L}_2 = \ln(\bar{x} + c_H) + \ln(\bar{x} + g) + \lambda_2 [\ln(\bar{x} + c_L) + \ln(\bar{x} + g)] + \lambda_1 [1 - c_H - c_L - g]$$

Let's divide \mathcal{L}_1 by π . Guaranteeing that $\lambda_2 = \frac{1-\pi}{\pi}$ and $\lambda_1 = \frac{\lambda}{\pi}$, I show that both problems are the same.

Pareto Efficiency: static egalitarian planner

The static planner that gives the same weight for both individuals solves the following problem:

$$\max_{x_t \geq 0 \in \mathcal{B}} 0.5u_H(x_t) + 0.5u_L(x_t)$$

The Lagrangian problem of this planner can be written as:

$$\mathcal{L}_e = 0.5 \ln(\bar{x} + c_H) + 0.5 \ln(\bar{x} + c_L) + \ln(\bar{x} + g) + \lambda [1 - c_H - c_L]$$

The first order and Kuhn-Tucker conditions for this problem are $c_H, c_L, g, \lambda \geq 0$ and

$$[c_H] \quad \frac{0.5}{\bar{x} + c_H} - \lambda \leq 0$$

$$\left[\frac{0.5}{\bar{x} + c_H} - \lambda \right] c_H = 0 \quad (\text{P1})$$

$$[c_L] \quad \frac{0.5}{\bar{x} + c_L} - \lambda \leq 0$$

$$\left[\frac{0.5}{\bar{x} + c_L} - \lambda \right] c_L = 0 \quad (\text{P2})$$

$$[g] \quad \frac{1}{\bar{x} + g} - \lambda \leq 0$$

$$\left[\frac{1}{\bar{x} + g} - \lambda \right] g = 0 \quad (\text{P3})$$

$$[\text{RC}] \quad 1 - c_H - c_L - g \geq 0 \quad (\text{P4})$$

$$[1 - c_H - c_L - g] \lambda_1 = 0$$

First assume $\lambda = 0$. By (P1) we have that $c_H < \bar{x}$, which contradicts the fact that we require $c_H \geq 0$. We conclude that $\lambda > 0$ which implies that $1 - c_H - c_L - g = 0$. Next, we have a couple of cases to consider:

- $c_L = 0, c_H, g > 0$: Since $c_L = 0$, (P4) implies that $c_H = 1 - g$. Moreover, (P1) and (P3) imply that $g = \frac{\bar{x}+2}{3}$. In (P4) this implies $c_H = \frac{1-\bar{x}}{3}$. In (P3), this implies $\lambda = \frac{3}{4\bar{x}+2}$. For inequality (P2) to hold, we require $\frac{1}{2\bar{x}} - \frac{3}{4\bar{x}+2} < 0$, which implies $\bar{x} > 1$, a contradiction. Therefore, $c_L > 0$.
- $c_H = 0, c_L, g > 0$: Since the two parties are symmetric, the case goes as the former and we find the same contradiction. We conclude that $c_H > 0$.
- $c_L, c_H, g > 0$: By (P1) and (P2), we have that $c_H = c_L$. Moreover, by (P1) and (P3) we have that $g = \bar{x} + 2c_H$. In (P4) we have that $g = \frac{1+\bar{x}}{2}$. This is the only case that holds.

Pareto Efficiency: static planner with endogenous weights

The static planner with endogenous weights solves the following problem, for $\underline{u}_L \in R$:

$$\begin{aligned} \max_{x_t \geq 0 \in \mathcal{B}} \quad & u_H(x_t) \\ \text{s.t.} \quad & u_H(x_t) \geq \underline{u}_L \end{aligned}$$

The Lagrangian problem of this planner can be written as:

$$\mathcal{L}_2 = \ln(\bar{x} + c_H) + \ln(\bar{x} + g) + \lambda_2 [\ln(\bar{x} + c_L) + \ln(\bar{x} + g) - \underline{u}_L] + \lambda_1 [1 - c_H - c_L - g]$$

where \underline{u} can be re-written in terms allocations as $\underline{u} = \ln(\bar{x} + s) + \ln(\bar{x})$ where $s \in \mathbf{s}$ is the relevant status quo. The first order and Kuhn-Tucker conditions for this problem are $c_H, c_L, g, \lambda_1, \lambda_2 \geq 0$ and

$$\begin{aligned} [c_H] \quad & \frac{1}{\bar{x} + c_H} - \lambda_1 \leq 0 \\ & \left[\frac{1}{\bar{x} + c_H} - \lambda_1 \right] c_H = 0 \end{aligned} \tag{P1}$$

$$\begin{aligned} [c_L] \quad & \lambda_2 \frac{1}{\bar{x} + c_L} - \lambda_1 \leq 0 \\ & \left[\lambda_2 \frac{1}{\bar{x} + c_L} - \lambda_1 \right] c_L \leq 0 \end{aligned} \tag{P2}$$

$$\begin{aligned} [g] \quad & \frac{(1 + \lambda_2)}{\bar{x} + g} - \lambda_1 \leq 0 \\ & \left[\frac{(1 + \lambda_2)}{\bar{x} + g} - \lambda_1 \right] g = 0 \end{aligned} \tag{P3}$$

$$\begin{aligned} [RC] \quad & 1 - c_H - c_L - g \geq 0 \\ & [1 - c_H - c_L - g] \lambda_1 = 0 \end{aligned} \tag{P4}$$

$$\begin{aligned} [IRC] \quad & \ln(\bar{x} + c_L) + \ln(\bar{x} + g) - [\ln(\bar{x} + s) + \ln(\bar{x})] \geq 0 \\ & [\ln(\bar{x} + c_L) + \ln(\bar{x} + g) - [\ln(\bar{x} + s) + \ln(\bar{x})]] \lambda_2 = 0 \end{aligned} \tag{P5}$$

First assume $\lambda_1 = 0$. By (P1) we have that $c_H < \bar{x}$, which contradicts the fact that we require $c_H \geq 0$. We conclude that $\lambda_1 > 0$ which implies that $1 - c_H - c_L - g = 0$. Next, we have a couple of cases to consider:

- $\lambda_2 = 0$: Since $\lambda_1 > 0$, (P2) implies that $c_L = 0$. (P1) and (P3) imply that $\bar{x} + c_H = \bar{x} + g$. Combined with (P4), this implies that $c_H = g = \frac{1}{2}$. For the inequality (P5) to hold, $s < \frac{1}{2}$.
- $\lambda_2 > 0, c_L = 0, c_H, g > 0$: Since $c_L = 0$ (P5) directly implies that $g = s$. (P4) implies that $c_H = 1 - s$. For inequality (P2) to hold, we require $s < \frac{1 + \bar{x}}{2}$.
- $\lambda_2 > 0, g = 0, c_H, c_L > 0$: Since $g = 0$ (P5) directly implies that $c_L = s$ and (P4) implies that $c_H = 1 - s$. By (P1), $\lambda_1 = \frac{1}{2\bar{x} + 1 - s}$. Combining this with (P2) implies $\lambda_2 = \frac{\bar{x} + 1 - s}{\bar{x} + s}$. For the inequality (P3) to hold, we require $\bar{x} < 1$, a contradiction to the assumption that $\bar{x} \leq 1$. This case can be disregarded.
- $\lambda_2 > 0, c_H = 0, c_L, g > 0$: Since $c_H = 0$ (P4) implies that $c_L = 1 - g$. Combining this in (P5) implies that $g = -\frac{1}{2} \left[1 + \sqrt{(1 + 4\bar{x}(1 - s_2))} \right] < 0$ and violates the condition that $g_2 \geq 0$. Therefore, this case can also be disregarded.

- $\lambda_2 > 0, c_H, c_L, g > 0$: By (P1), $\lambda_1 = \frac{1}{2} \frac{1}{\bar{x} + c_H}$. Combining this with (P2) implies $\lambda_2 = \frac{\bar{x} + c_L}{\bar{x} + c_H}$. In (P3) this implies that $\bar{x} + c_H = \bar{x} + g - (\bar{x} + c_L)$. In (P4), this implies that $g = \frac{1}{2}(1 + \bar{x})$. Combining this in (P5) it implies that $c_L = \frac{\bar{x}[\bar{x}(2s-1)-1]}{1+3\bar{x}}$. Going back to (P3), this implies that $c_H = \frac{1+4\bar{x}(1-s)-\bar{x}^2}{2(1+3\bar{x})}$. Since $0 \leq c_H \leq 1, 0 \leq c_L \leq 1$ and $0 \leq g \leq 1$, we require that $0 \leq \frac{1}{2} - \frac{2\bar{x}(\bar{x}+s)}{1} \leq 1$ which implies $\frac{1}{2}(1 + \bar{x}) \leq s \leq \frac{1+\bar{x}(4+\bar{x})}{2\bar{x}}$. The right-hand side of this inequality is never binding, as $\frac{1+\bar{x}(4+\bar{x})}{2\bar{x}} > 1, \forall \bar{x}$. The left-hand-side is the relevant one for this case to hold.

After this extensive analysis, the endogenous weights planner's problem is left with 3 relevant cases: the constraint is not binding, the constraint binds but the status quo is low enough so that $c_L = 0$, the constraint binds and all solutions are interior. Solutions can be summarized as:

$$g = \begin{cases} \frac{1}{2}, & \text{if } s < \frac{1}{2} \\ s, & \text{if } \frac{1}{2} \leq s < \frac{1+\bar{x}}{2} \\ \frac{1}{2}(1 + \bar{x}), & \text{if } s \geq \frac{1+\bar{x}}{2} \end{cases}$$

$$c_L = \begin{cases} 0, & \text{if } s < \frac{1}{2} \\ 0, & \text{if } \frac{1}{2} \leq s < \frac{1+\bar{x}}{2} \\ \frac{\bar{x}(2s-1)-\bar{x}^2}{1+3\bar{x}}, & \text{if } s \geq \frac{1+\bar{x}}{2} \end{cases}$$

$$c_H = \begin{cases} \frac{1}{2}, & \text{if } s < \frac{1}{2} \\ 1 - s, & \text{if } \frac{1}{2} \leq s < \frac{1+\bar{x}}{2} \\ \frac{1+4\bar{x}(1-s)-\bar{x}^2}{2(1+3\bar{x})}, & \text{if } s \geq \frac{1+\bar{x}}{2} \end{cases}$$

Two-period model with homogenous players and homogenous taste across goods

I derive here the solution for the two-period game in which players are homogenous in all dimensions: how much they value the public good ($\alpha_i = \alpha = \frac{1}{2}, \forall i \in \{H, L\}$), and on how much outside consumption available they have. Let's take $u(c_i, g) = \alpha \ln(\bar{x} + c_i) + \alpha \ln(\bar{x} + g)$, where \bar{x} represents the amount of consumption individuals have available for them outside the political game. Also, since I am focusing on a two-period, two-players, complete information game, we can solve it by backward induction.

Second period policies. Party's i Lagrangian for this problem at $t = 2$ is given by:

$$\mathcal{L} = \ln(\bar{x} + c_{i,2}) + \ln(\bar{x} + g_2) + \lambda_{1,2} [1 - c_{i,2} - c_{j,2} - g_2] + \lambda_{2,2} [\ln(\bar{x} + c_{j,2}) + \ln(\bar{x} + g_2) - K_{j,2}(s_2)]$$

where $K_{j,2}(s_2) = \ln(\bar{x} + c_{j,1}) + \ln(\bar{x})$ when budgetary rule are over private goods and $K_{j,2}(s_2) = \ln(\bar{x}) + \ln(\bar{x} + g_1)$ when budgetary rules are over public goods. I define $s_2 \in s$ as the relevant status quo allocation to simplify the notation below. For example, when budgetary rule are over private goods, $s_2 = c_{j,1}$ and when budgetary rule are over public goods, $s_2 = g_1$. The first order and Kuhn-Tucker conditions for this problem are $c_{i,2}, c_{j,2}, g_2, \lambda_{1,2}, \lambda_{2,2} \geq 0$ and

$$\begin{aligned} [c_{i,2}] \quad & \frac{1}{\bar{x} + c_{i,2}} - \lambda_{1,2} \leq 0 \\ & \left[\frac{1}{\bar{x} + c_{i,2}} - \lambda_{1,2} \right] c_{i,2} = 0 \end{aligned} \tag{A1}$$

$$\begin{aligned} [c_{j,1}] \quad & \lambda_{2,2} \frac{1}{\bar{x} + c_{j,2}} - \lambda_{1,2} \leq 0 \\ & \left[\lambda_{2,2} \frac{1}{\bar{x} + c_{j,2}} - \lambda_{1,2} \right] c_{j,2} \leq 0 \end{aligned} \tag{A2}$$

$$\begin{aligned} [g] \quad & \frac{(1 + \lambda_{2,2})1}{\bar{x} + g_2} - \lambda_{1,2} \leq 0 \\ & \left[\frac{(1 + \lambda_{2,2})1}{\bar{x} + g_2} - \lambda_{1,2} \right] g_2 = 0 \end{aligned} \tag{A3}$$

$$\begin{aligned} [RC] \quad & 1 - c_{i,2} - c_{j,2} - g_2 \geq 0 \\ & [1 - c_{i,2} - c_{j,2} - g_2] \lambda_{1,2} = 0 \end{aligned} \tag{A4}$$

$$\begin{aligned} [IRC] \quad & \ln(\bar{x} + c_{j,2}) + \ln(\bar{x} + g_2) - K_j(s) \geq 0 \\ & [\ln(\bar{x} + c_{j,2}) + \ln(\bar{x} + g_2) - K_j(s)] \lambda_{2,2} = 0 \end{aligned} \tag{A5}$$

First assume $\lambda_{1,2} = 0$. By (A1) we have that $c_{i,2} < \bar{x}$, which contradicts the fact that we require $c_{i,2} \geq 0$. We conclude that $\lambda_{1,2} > 0$ which implies that $1 - c_{i,2} - c_{j,2} - g_2 = 0$. Next, we have a couple of cases to consider:

- $\lambda_{2,2} = 0$: Since $\lambda_{1,2} > 0$, (A2) implies that $c_{j,2} = 0$. (A1) and (A3) imply that $\bar{x} + c_{i,2} = \bar{x} + g_2$, since $\alpha = 0.5$. Combined with (A4), this implies that $c_{i,2} = g_2 = \frac{1}{2}$. For the inequality (A5) to hold, $s_2 < \frac{1}{2}$ for which $s_2 = c_{j,1}$ in the case of rules over private goods and $s_2 = g_1$ when rules are over public goods. In the case that (A5) isn't binding, party i can equate the marginal utilities of private and her individual public consumption.

- $\lambda_{2,2} > 0, c_{j,2} = 0, c_{i,2}, g_2 > 0$: Since $c_{j,2} = 0$ (A5) directly implies that $g = s_2$. (A4) implies that $c_{i,2} = Y - s_2$, recalling that $s_2 = c_{j,1}$ in the case of rules over private goods and $s_2 = g_1$ when rules are over public goods. For inequality (A2) to hold, we require $s_2 < \frac{1+\bar{x}}{2}$.
- $\lambda_{2,2} > 0, g_2 = 0, c_{i,2}, c_{j,2} > 0$: Since $g_2 = 0$ (A5) directly implies that $c_{j,2} = s_2$ and (A4) implies that $c_{i,2} = 1 - s_2$. By (A1), $\lambda_{1,2} = \frac{1}{2} \frac{1}{\bar{x}+1-s_2}$. Combining this with (A2) implies $\lambda_{2,2} = \frac{\bar{x}+1-s_2}{\bar{x}+s_2}$. For the inequality (A3) to hold, we require $\bar{x} < 1$, a contradiction to the assumption that $3\bar{x} \leq 1$. This case can be disregarded.
- $\lambda_{2,2} > 0, c_{i,2} = 0, c_{j,2}, g_2 > 0$: Since $c_{i,2} = 0$ (A4) implies that $c_{j,2} = 1 - g_2$. Combining this in (A5) implies that $g_2 = -\frac{1}{2} \left[1 + \sqrt{(1+4\bar{x}(1-s_2))} \right] < 0$ and violates the condition that $g_2 \geq 0$. Therefore, this case can also be disregarded.
- $\lambda_{2,2} > 0, c_{i,2}, c_{j,2}, g_2 > 0$: By (A1), $\lambda_{1,2} = \frac{1}{2} \frac{1}{\bar{x}+c_{i,2}}$. Combining this with (A2) implies $\lambda_{2,2} = \frac{\bar{x}+c_{j,2}}{\bar{x}+c_{i,2}}$. In (A3) this implies that $\bar{x} + c_{i,2} = \bar{x} + g_2 - (\bar{x} + c_{j,2})$. In (A4), this implies that $g_2 = \frac{1}{2} (1 + \bar{x})$. Combining this in (A5) it implies that $c_{j,2} = \frac{\bar{x}[\bar{x}(2s_2-1)-1]}{1+3\bar{x}}$. Going back to (A3), this implies that $c_{i,2} = \frac{1+4\bar{x}(1-s_2)-\bar{x}^2}{2(1+3\bar{x})}$. Since $0 \leq c_{i,2} \leq 1, 0 \leq c_{j,2} \leq 1$ and $0 \leq g_2 \leq 1$, we require that $0 \leq \frac{1}{2} - \frac{2\bar{x}(\bar{x}+s_2)}{Y} \leq 1$ which implies $\frac{1}{2}(1 + \bar{x}) \leq s_2 \leq \frac{1+\bar{x}(4+\bar{x})}{2\bar{x}}$. The right-hand side of this inequality is never binding, as $\frac{1+\bar{x}(4+\bar{x})}{2\bar{x}} > 1, \forall \bar{x}$. The left-hand-side is the relevant one for this case to hold.

After this extensive analysis, the special case of the second-period problem is left with 3 relevant cases: the constraint is not binding, the constraint binds but the status quo is low enough so that $c_{j,2} = 0$, the constraint binds and all solutions are interior. Solutions can be summarized as:

$$\begin{aligned}
g_2 &= \begin{cases} \frac{1}{2}, & \text{if } s_2 < \frac{1}{2} \\ s_2, & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \frac{1}{2} (1 + \bar{x}), & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases} \\
c_{j,2} &= \begin{cases} 0, & \text{if } s_2 < \frac{1}{2} \\ 0, & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \frac{\bar{x}(2s_2-1)-\bar{x}^2}{1+3\bar{x}}, & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases} \\
c_{i,2} &= \begin{cases} \frac{1}{2}, & \text{if } s_2 < \frac{1}{2} \\ 1 - s_2, & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \frac{1+4\bar{x}(1-s_2)-\bar{x}^2}{2(1+3\bar{x})}, & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases}
\end{aligned}$$

Continuation values. Before moving to the first-period problem of party i , let's understand how the continuation value of individuals will change given first-period policies. Since $p = 0$, this means that the proposer of first period will for sure become the respondent in the second period, and vice-versa. Recall that $V_i(s)$ defines the value when party i is the proposer and $W_i(s)$ defines the value when party i is the respondent.

$$\begin{aligned}
W_i(s_2) &= \begin{cases} \ln(\bar{x}) + \ln\left(\frac{2\bar{x}+1}{2}\right), & \text{if } s_2 < \frac{1}{2} \\ \ln(\bar{x}(\bar{x} + s_2)), & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \ln(\bar{x}(\bar{x} + s_2)), & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases} \\
V_j(s_2) &= \begin{cases} 2 \ln\left(\frac{1+2\bar{x}}{2}\right), & \text{if } s_2 < \frac{1}{2} \\ \ln(\bar{x} + 1 - s_2) + \ln(\bar{x} + s_2), & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \ln\left(\frac{1+\bar{x}(6-4s_2+5\bar{x})}{2(1+3\bar{x})}\right) + \ln\left(\frac{3\bar{x}+1}{2}\right), & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases}
\end{aligned} \tag{S1}$$

Moreover,

$$\begin{aligned}
\frac{\partial W_i(s_2)}{\partial s_2} &= \begin{cases} 0, & \text{if } s_2 < \frac{1}{2} \\ \frac{1}{\bar{x}+s_2}, & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \frac{1}{\bar{x}+s_2}, & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases} \\
\frac{\partial V_j(s_2)}{\partial s_2} &= \begin{cases} 0, & \text{if } s_2 < \frac{1}{2} \\ \frac{1-2s_2}{(\bar{x}+s_2)(1+\bar{x}-s_2)}, & \text{if } \frac{1}{2} \leq s_2 < \frac{1+\bar{x}}{2} \\ \frac{-4\bar{x}}{1+\bar{x}(6-4s_2+5\bar{x})}, & \text{if } \frac{1+\bar{x}}{2} \leq s_2 \end{cases}
\end{aligned}$$

Note that although the continuation values for party i are not the same in case 2 (individual rationality constraint binds but there is no private transfer for party i) or in case 3 (individual rationality constraint binds and private transfer for i are strictly positive), the derivative with respect to s_2 are the same.

First period policies. Party's i Lagrangian for this problem at $t = 1$ is given by:

$$\begin{aligned}
\mathcal{L} &= \ln(\bar{x} + c_{i,1}) + \ln(\bar{x} + g_1) + \beta (pV_i(s_2) + (1-p)W_i(s_2)) + \lambda_{1,1} [1 - c_{i,1} - c_{j,1} - g_1] + \\
&\quad \lambda_{2,1} [\ln(\bar{x} + c_{j,1}) + \ln(\bar{x} + g_1) + \beta (pW_j(s_2) + (1-p)V_j(s_2)) - K_{j,1}(s_1)]
\end{aligned}$$

where $s_2 = \Psi(\mathbf{b}_1), \forall \mathbf{b} \in \mathcal{B}$, $K_{j,1}(s_1) = \ln(\bar{x} + c_{j,0}) + \ln(\bar{x}) + \beta (pW_j(s_1) + (1-p)V_j(s_1))$ when budgetary rules are over private goods and $K_{j,1}(s_1) = \ln(\bar{x}) + \ln(\bar{x} + g_0) + \beta (pW_j(s_1) + (1-p)V_j(s_1))$ when budgetary rules are over public goods. Note that because this is a finite problem the initial status quo is exogenous and it is equal to $s_1 \in \mathcal{S}$ with $s_1 = (c_{i,0}, c_{j,0}, 0)$ when rules are over private goods and $s_1 \in \mathcal{S}$ with $s_1 = (0, 0, g_0)$ when rules are over public goods. Also note that while my analytical results presented in the main text will be for the special case in which $\beta = 1$ and there is certain political turnover $p = 0$, these assumptions are only needed when analyzing conditions for the initial exogenous status quo such that the first period's constraint will not be binding, and not to characterize allocations. Therefore, in this Appendix, I leave the parameters β and p explicitly for a richer characterization.

Policies chosen in the first period will determine s_2 the status quo on the second period since $s_2 = \Psi(\mathbf{b}_1), \forall \mathbf{b}_1 \in \mathcal{B}$. Given politicians are farsighted, they will take into account the impact of those choices in the determination of their continuation values. This means that the type of institution that determines the status quo will be relevant for the first-order-conditions of party i , determining what allocations will be distorted vis-a-vis the static efficient problem. Because of

this, I will present first-order-conditions for both when budgetary rules are over private goods and when they are over public goods.

Rules on private transfers.— First note that when rules are over private goods, $\mathbf{s}_2 = \Psi(\mathbf{b}_1) = (c_{i,1}, c_{j,1}, 0)$. In the case of certain political turnover ($p = 0$), proposer $i \in \{H, L\}$ in the first period will be certainly replaced by $j \neq i$. The only relevant state in the second period is the private constraint given to the respondent, which will be i . Therefore, the only relevant state in the second period will be $\mathbf{s}_2 \in \mathbf{s}_2 = c_{i,1}$.

The first-order and Kuhn-Tucker conditions party i are $c_{i,1}, c_{j,1}, g_1, \lambda_{1,1}, \lambda_{2,1} \geq 0$ and

$$[c_{i,1}] \quad \frac{1}{\bar{x} + c_{i,1}} - \lambda_{1,1} + \beta(1-p) \left(\frac{\partial W_i(s_2)}{\partial c_{i,1}} + \lambda_2 \frac{\partial V_j(s_2)}{\partial c_{i,1}} \right) \leq 0$$

$$\left[\frac{1}{\bar{x} + c_{i,1}} - \lambda_{1,1} + \beta(1-p) \left(\frac{\partial W_i(s_2)}{\partial c_{i,1}} + \lambda_2 \frac{\partial V_j(s_2)}{\partial c_{i,1}} \right) \right] c_{i,1} = 0 \quad (\text{A6})$$

$$[c_{j,1}] \quad \lambda_{2,1} \frac{1}{\bar{x} + c_{j,1}} - \lambda_{1,1} + \beta p \left(\frac{\partial W_i(s_2)}{\partial c_{j,1}} + \lambda_2 \frac{\partial W_j(s_2)}{\partial c_{j,1}} \right) \leq 0$$

$$\left[\lambda_{2,1} \frac{1}{\bar{x} + c_{j,1}} - \lambda_{1,1} + \beta p \left(\frac{\partial W_i(s_2)}{\partial c_{j,1}} + \lambda_2 \frac{\partial W_j(s_2)}{\partial c_{j,1}} \right) \right] c_{j,1} = 0 \quad (\text{A7})$$

$$[g_1] \quad \frac{(1 + \lambda_{2,1})}{\bar{x} + g_1} - \lambda_{1,1} \leq 0$$

$$\left[\frac{(1 + \lambda_{2,1})}{\bar{x} + g_1} - \lambda_{1,1} \right] g_1 = 0 \quad (\text{A8})$$

$$[RC] \quad 1 - c_{i,1} - c_{j,1} - g_1 \geq 0 \quad (\text{A9})$$

$$[1 - c_{i,1} - c_{j,1} - g_1] \lambda_{1,2} = 0$$

$$[IRC] \quad \ln(\bar{x} + c_{j,1}) + \ln(\bar{x} + g_1) + \beta (pW_j(s_2) + (1-p)V_j(s_2)) - K_{j,1}(s_1) \geq 0 \quad (\text{A10})$$

$$[\ln(\bar{x} + c_{j,1}) + \ln(\bar{x} + g_1) + \beta (pW_j(s_2) + (1-p)V_j(s_2)) - K_{j,1}(s_1)] \lambda_{2,1} = 0$$

First assume $\lambda_{1,1} = 0$. By (A8) we have that $g_1 < \bar{x}$, since $\alpha > 0$ and $\lambda_{2,1} \geq 0$, which contradicts the fact that we require $g_1 \geq 0$. We conclude that $\lambda_{1,1} > 0$ which implies that $1 - c_{i,1} - c_{j,1} - g_1 = 0$. Next, we have a couple of cases to consider:

- $\lambda_{2,2} = 0$. The initial status quo $\mathbf{s}_1 = (c_{i,0}, c_{j,0}, 0)$ only enters in the problem through the constraint. Therefore, the solution in this case is independent of \mathbf{s}_1 . Since $\lambda_{1,1} > 0$, (A8) implies that $c_{j,1} = 0$. By (A9), $c_{i,1} = 1 - g_1$. Combining (A6), (A8), we have:

$$\frac{1}{\bar{x} + 1 - g_1} + \beta(1-p) \frac{\partial W_i(s_2)}{\partial c_{i,1}} = \frac{1}{\bar{x} + g_1} \quad (7)$$

First note that since $\lambda_{2,2} = 0$, the solution doesn't depend on $V_j(s_2)$, but only on $W_i(s_2)$. As we saw in the characterization of the second period optimal strategies for the parties, they only depend on the *relevant status quo*, which is how much the respondent claims under the status quo, not the proposer. Therefore, when choosing $c_{i,1}$ in the first period, for example, the proposer in the first-period will evaluate how much $c_{i,1}$ will impact her continuation value when she is the respondent in the second period, that is when $c_{i,1}$ will enter as a relevant status quo. Let's assume $c_{i,1} < \frac{1}{2}$. This implies that $\frac{\partial W_i(s_2)}{\partial c_{i,1}} = 0$. By equation (7) we have that $g_1 = c_{i,1} = \frac{1}{2}$, a contradiction. This implies that if the initial constraint is

not binding, then the proposer will set her private consumption to levels that will make the constraint binding in the second-period. Now let's assume $c_{i,1} \geq \frac{1}{2}$. By (S1), $\frac{\partial W_i(s_2)}{\partial c_{i,1}} = \frac{1}{\bar{x} + c_{i,1}} > 0$, if $\frac{1}{2} \leq c_{i,1} \leq \frac{1+\bar{x}}{2}$ or if $\frac{1+\bar{x}}{2} \leq c_{i,1}$. By equation (7), we have that $g_1 = \frac{1-\beta(1-p)\bar{x}}{2+\beta(1-p)}$ and $c_{i,1} = \frac{1+\beta(1-p)(1+\bar{x})}{2+\beta(1-p)}$. Note that $\frac{1}{2} \leq c_{i,1}$ always, but $c_{i,1} \leq \frac{1+\bar{x}}{2}$ if and only if $\beta \leq \frac{2\bar{x}}{(1-p)(1+\bar{x})}$, which implies $p \leq \frac{1-\bar{x}}{1+\bar{x}}$, otherwise, $c_{i,1} \geq \frac{1+\bar{x}}{2}$. This condition is illustrated in the plots below for $\beta = 1$ and for two levels of \bar{x} :

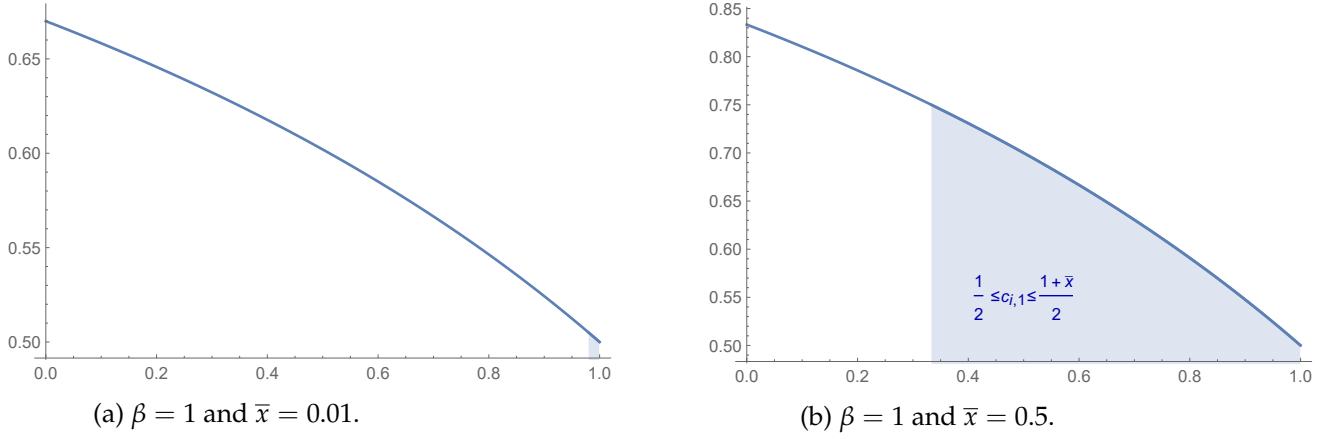


Figure 17: The outside consumption available for parties increases the region such that $\frac{1}{2} \leq c_{i,1} \leq \frac{1+\bar{x}}{2}$. However, for $p = 0$, which is the case of the example in the main text, this case will never hold.

Therefore:

$$g_1^* = \frac{1 - \beta(1-p)\bar{x}}{2 + \beta(1-p)}$$

$$c_{i,1}^* = \frac{1 + \beta(1-p)(1+\bar{x})}{2 + \beta(1-p)}$$

We need to check (A10) to see when this case holds in terms of the initial status quo $s_1 = (c_{i,0}, c_{j,0}, 0)$. Since I am studying the case in which the first period constraint is not binding, it is likely to assume that $c_{j,0}$ is "low". But, in general, we have three relevant cases $c_{j,0} < \frac{1}{2}$, $\frac{1}{2} \leq c_{j,0} < \frac{1+\bar{x}}{2}$ and $\frac{1+\bar{x}}{2} \leq c_{j,0}$.

★ $c_{j,0} < \frac{1}{2}$, $c_{i,0} \geq \frac{1+\bar{x}}{2}$, $p = 0$ and $\beta = 1$. By (A10) we have:

$$\ln(\bar{x}) + \ln\left(\frac{1+2\bar{x}}{3}\right) + V_j(s_2) > \ln(\bar{x}) + \ln(\bar{x} + c_{j,0}) + V_j(s_1)$$

Since $p = 0$, as demonstrated above, $c_{i,1} \geq \frac{1+\bar{x}}{2} \forall \bar{x} \in (0, 0.5]$. By S1, $V_j(s_2) = \ln\left(\frac{1+\bar{x}(6-4c_{i,1}+5\bar{x})}{4}\right)$ and $V_j(s_1) = \ln\left(\frac{1+\bar{x}(6-4c_{i,0}+5\bar{x})}{4}\right)$. The second condition holds because $c_{j,0} < \frac{1}{2}$ and moreover I assume that $c_{i,0}$ is not only greater than $\frac{1}{2}$, since $c_{j,0} + c_{i,0} = 1$ but I also assume that $c_{i,0} \geq \frac{1+\bar{x}}{2}$. After some algebra, the condition for this case to hold (the

first-period individual rationality constraint to not be bind) can be re-written as:

$$c_{j,0} < c_{j,0} = \frac{1 - \bar{x}}{3}$$

★ $c_{j,0} < \frac{1}{2}$, $c_{i,0} \geq \frac{1+\bar{x}}{2}$, $p \in (0, 1]$ and $\beta \in [0, 1)$. By (A10) we have:

$$\begin{aligned} \ln(\bar{x}) + \ln(\bar{x} + g_1^*) + \beta(pW_j(\mathbf{s}_2) + (1-p)V_j(\mathbf{s}_2)) &> \\ \ln(\bar{x}) + \ln(\bar{x} + c_{j,0}) + \beta(pW_j(\mathbf{s}_1) + (1-p)V_j(\mathbf{s}_1)) & \end{aligned}$$

which can be re-written as:

$$EQ_c(c_{i,0}, p) = \ln(\bar{x} + g_1^*) + \beta(1-p)V_j(c_{i,1}^*) - \ln(\bar{x} + c_{j,0}) - \beta(1-p)V_j(c_{i,0}) > 0 \quad (8)$$

this because when j is in power in the second period, $V_j(\mathbf{s}_2) = V_j(c_{i,1})$, since the only relevant state for j as a proposer in the second-period is the mandatory spending on private consumption given to the respondent, i . Also, $W_j(\mathbf{s}_2) = W_j(c_{j,1})$, since the only relevant state for j as the respondent in the second-period is the mandatory spending on private consumption given to herself under the status quo. Since $c_{j,1} = 0$, we have that $W_j(c_{j,1}) = \ln\left(\bar{x}\frac{(1+2\bar{x})}{2}\right)$. Moreover, since I assume $c_{j,0} < \frac{1}{2}$ for this case, $W_j(c_{j,0}) = \ln\left(\bar{x}\frac{(1+2\bar{x})}{2}\right)$. This terms cancel out and deliver the inequality (8). First, note that:

$$\begin{aligned} \frac{\partial EQ_c(c_{j,0}, p)}{\partial c_{j,0}} &= -\frac{1}{\bar{x} + c_{j,0}} - \beta(1-p)V_j'(c_{i,0})\frac{\partial c_{i,0}}{\partial c_{j,0}} \\ &= -\frac{1}{\bar{x} + c_{j,0}} + \beta(1-p)V_j'(c_{i,0}) < 0 \end{aligned}$$

since $c_{i,0} = 1 - c_{j,0}$ and $V_j'(c_{i,0}) < 0$, by S1. Moreover,

$$\begin{aligned} \frac{\partial EQ_c(c_{j,0}, p)}{\partial p} &= \frac{1}{\bar{x} + g_1^*} \frac{\partial g_1^*}{\partial p} - \beta V_j(c_{i,1}^*) - \beta p V_j'(c_{i,1}^*) \frac{\partial c_{i,1}^*}{\partial p} + \beta p V_j(c_{i,0}) \\ &= \underbrace{\frac{1}{\bar{x} + g_1^*} \frac{\partial g_1^*}{\partial p}}_+ - \underbrace{\beta p V_j'(c_{i,1}^*) \frac{\partial c_{i,1}^*}{\partial p}}_+ - \underbrace{\beta (V_j(c_{i,1}^*) - V_j(c_{i,0}))}_{\geq 0} \end{aligned}$$

where $\frac{\partial g_1^*}{\partial p} = \frac{\beta(1+2\bar{x})}{(2+\beta(1-p))^2}$ and $\frac{\partial c_{i,1}^*}{\partial p} = -\frac{\beta(1-\bar{x})}{(2+\beta(1-p))^2}$. Moreover, the last term of the above equation is non-negative. Since $V_j(\cdot)$ is decreasing in c_i , the condition holds if $c_{i,1}^* - c_{i,0} = c_{i,1}^* - (1 - c_{j,0}) < 0$. When $p = 0$ and $\beta = 1$, this is equivalent to have $c_{j,0} < \frac{1-\bar{x}}{3}$, the same condition that guarantees that the individual rationality constraint is not binding in the first-period. For $p \in [0, 1]$ and $\beta \in [0, 1]$, the same rational is valid. By the implicit function theorem:

$$\frac{\partial c_{j,0}}{\partial p} = -\frac{\frac{\partial EQ_c(c_{j,0}, p)}{\partial p}}{\frac{\partial EQ_c(c_{j,0}, p)}{\partial c_{j,0}}} > 0$$

The condition can be illustrated in the picture below:

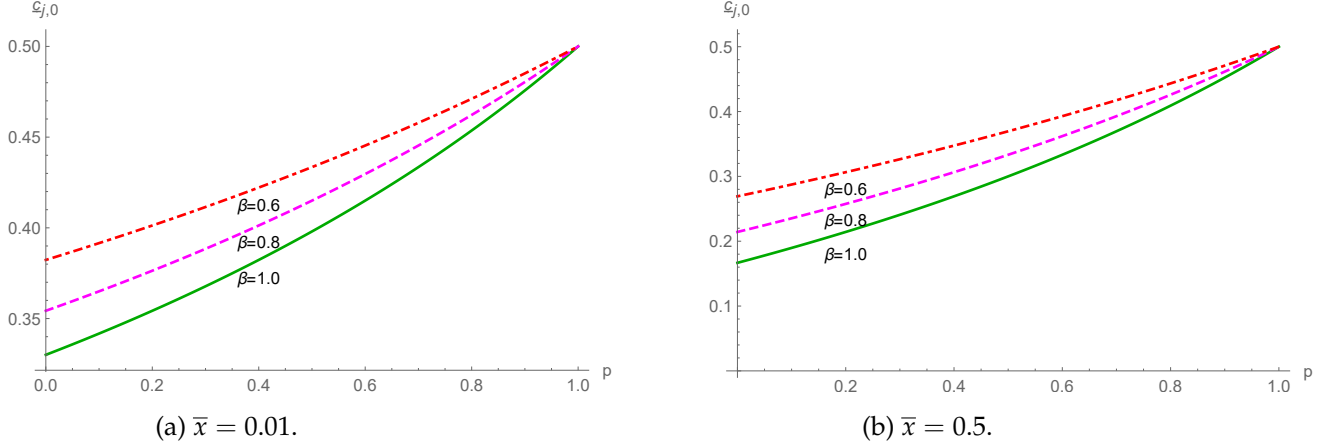


Figure 18: The maximum level of the exogenous status quo $c_{j,0}$ such that the individual rationality constraint is not binding in the first-period is increasing in p and β , but decreasing in \bar{x} .

Rules on public good.— When $s_2 = \Psi(\mathbf{b}_1) = (0, 0, g_1)$, the first-order and Kuhn-Tucker conditions party i are $c_{i,1}, c_{j,1}, g_1, \lambda_{1,1}, \lambda_{2,1} \geq 0$ and

$$[c_{i,1}] \quad \frac{1}{\bar{x} + c_{i,1}} - \lambda_{1,1} \leq 0$$

$$\left[\frac{1}{\bar{x} + c_{i,1}} - \lambda_{1,1} \right] c_{i,1} = 0 \quad (\text{A11})$$

$$[c_{j,1}] \quad \lambda_{2,1} \frac{1}{\bar{x} + c_{j,1}} - \lambda_{1,1} \leq 0$$

$$\left[\lambda_{2,1} \frac{1}{\bar{x} + c_{j,1}} - \lambda_{1,1} \right] c_{j,1} = 0 \quad (\text{A12})$$

$$[g_1] \quad \frac{(1 + \lambda_{2,1})}{\bar{x} + g_1} - \lambda_{1,1} + \beta \left(p \frac{\partial V_i(s_2)}{\partial g_1} + (1-p) \frac{\partial W_i(s_2)}{\partial g_1} + \lambda_2 \left(p \frac{\partial W_j(s_2)}{\partial g_1} + (1-p) \frac{\partial V_j(s_2)}{\partial g_1} \right) \right) \leq 0$$

$$\left[\frac{(1 + \lambda_{2,1})}{\bar{x} + g_1} - \lambda_{1,1} + \beta \left(p \frac{\partial V_i(s_2)}{\partial g_1} + (1-p) \frac{\partial W_i(s_2)}{\partial g_1} + \lambda_2 \left(p \frac{\partial W_j(s_2)}{\partial g_1} + (1-p) \frac{\partial V_j(s_2)}{\partial g_1} \right) \right) \right] g_1 = 0 \quad (\text{A13})$$

$$[RC] \quad 1 - c_{i,1} - c_{j,1} - g_1 \geq 0 \quad (\text{A14})$$

$$[1 - c_{i,1} - c_{j,1} - g_1] \lambda_{1,2} = 0$$

$$[IRC] \quad \ln(\bar{x} + c_{j,1}) + \ln(\bar{x} + g_1) + \beta (pW_j(s_2) + (1-p)V_j(s_2)) - K_{j,1}(s_1) \geq 0 \quad (\text{A15})$$

$$[\ln(\bar{x} + c_{j,1}) + \ln(\bar{x} + g_1) + \beta (pW_j(s_2) + (1-p)V_j(s_2)) - K_{j,1}(s_1)] \lambda_{2,1} = 0$$

where $K_j(s_1) = \ln(\bar{x}) + \ln(\bar{x} + g_0) + \beta (pW_j(s_1) + (1-p)V_j(s_1))$. First assume $\lambda_{1,1} = 0$. By (A11) we have that $c_{i,1} < 0$, a contradiction. We conclude that $\lambda_{1,1} > 0$ which implies that $1 - c_{i,1} - c_{j,1} - g_1 = 0$. Next, we have a couple of cases to consider:

- $\lambda_{2,2} = 0$. The initial status quo $s_1 = (0, 0, g_0)$ only enters in the problem through the constraint. Therefore, the solution in this case is independent of s_1 .

Since $\lambda_{1,1} > 0$, (A12) implies that $c_{j,1} = 0$. By (A9), $c_{i,1} = 1 - g_1$. Combining (A11) and (A13), we have:

$$\frac{1}{\bar{x} + 1 - g_1} = \frac{1}{\bar{x} + g_1} + \beta \left(p \frac{\partial V_i(s_2)}{\partial g_1} + (1 - p) \frac{\partial W_i(s_2)}{\partial g_1} \right) \quad (9)$$

Differently from mandatory spending over private goods, the solution now depends on both $V_i(\cdot)$ and $W_i(\cdot)$. The solution depends on $W_i(s_2)$ and $V_i(s_2)$, which are piecewise functions. Let's assume $g_1 < \frac{1}{2}$. This implies that $\frac{\partial V_i(s_2)}{\partial g_1} = \frac{\partial W_i(s_2)}{\partial g_1} = 0$ and so, $g_1 = \frac{1}{2}$. By (9), we have a contradiction. This implies that if the initial constraint is not binding, then the proposer will set public consumption to levels that will make the constraint binding in the second-period. Now let's assume $\frac{1}{2} \leq g_1 \leq \frac{1+\bar{x}}{2}$. This implies that $\frac{\partial V_i(s_2)}{\partial g_1} = \frac{1-2s_2}{(\bar{x}+1-s_2)(\bar{x}+s_2)}$ and $\frac{\partial W_i(s_2)}{\partial g_1} = \frac{1}{\bar{x}+s_2}$. By (9), we have that $g_1 = \frac{1+\beta(1+\bar{x}(1-p))}{2+\beta(1+p)}$ and $c_{i,1} = \frac{1+\beta(p-\bar{x}(1-p))}{2+\beta(1+p)}$. For this case to hold, we require that $\frac{1}{2} \leq g_1 \leq \frac{1+\bar{x}}{2}$, which implies that $0 \leq \beta \leq \frac{2\bar{x}}{1+\bar{x}-p(1+3\bar{x})}$. Moreover, since we require $0 \leq \beta \leq 1$, this implies that for every p such that $p \geq \frac{1-\bar{x}}{1+3\bar{x}}$, this case to hold. The plot below illustrates the region of p such that this case holds for a given β and \bar{x} .

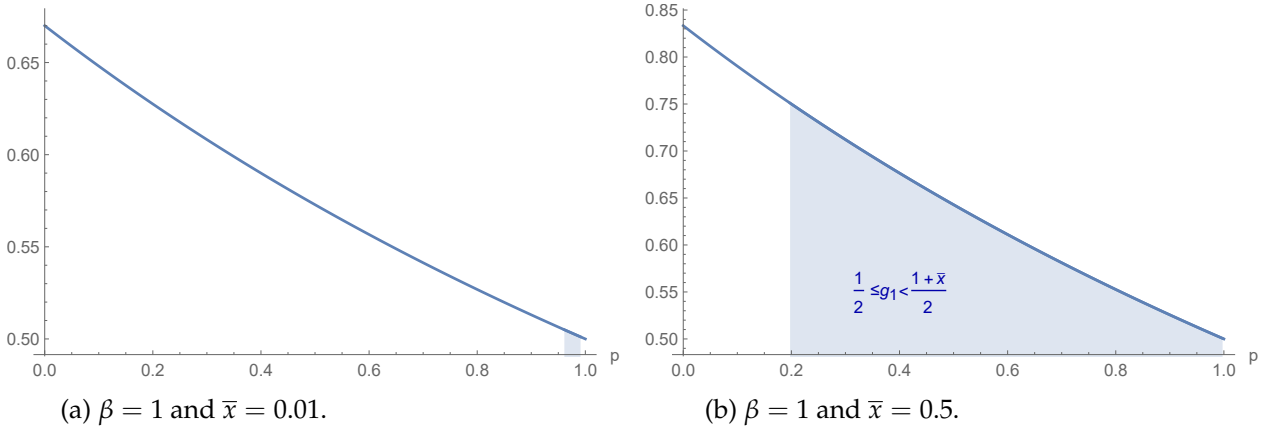


Figure 19: The outside consumption available for parties increases the region such that $\frac{1}{2} \leq g_1 \leq \frac{1+\bar{x}}{2}$.

Finally, let's assume $g_1 \geq \frac{1+\bar{x}}{2}$. This implies that $\frac{\partial V_i(s_2)}{\partial g_1} = \frac{-4s_2}{1+\bar{x}(6-4s_2+5\bar{x})}$ and $\frac{\partial W_i(s_2)}{\partial g_1} = \frac{1}{\bar{x}+s_2}$.

By (9), we have two possible solutions: $g_1 = \frac{B \pm \sqrt{A}}{8(\beta+2)\bar{x}}$ where:

$$A = \beta^2 ((\bar{x} + 1)^2 - p(3\bar{x} + 1)^2)^2 + 4\beta (p(3\bar{x} + 1)^2 (3\bar{x}^2 - 1) + (\bar{x}(5\bar{x} + 4) + 1)(\bar{x} + 1)^2) + 4(\bar{x}(5\bar{x} + 4) + 1)^2$$

$$B = -\beta p(3\bar{x} + 1)^2 + \beta(\bar{x} + 1)(9\bar{x} + 1) + 2\bar{x}(5\bar{x} + 8) + 2$$

For this case to hold, we require that $g_1 \geq \frac{1+\bar{x}}{2}$, which implies that $0 \leq \beta \leq \frac{2\bar{x}}{1+\bar{x}-p(3p\bar{x}+1)}$.

Moreover, since we require $0 \leq \beta \leq 1$, for every p such that $p < \frac{1-\bar{x}}{1+3\bar{x}}$ this case will hold. The plot below illustrates the region of p such that this case holds for a given β and \bar{x} .

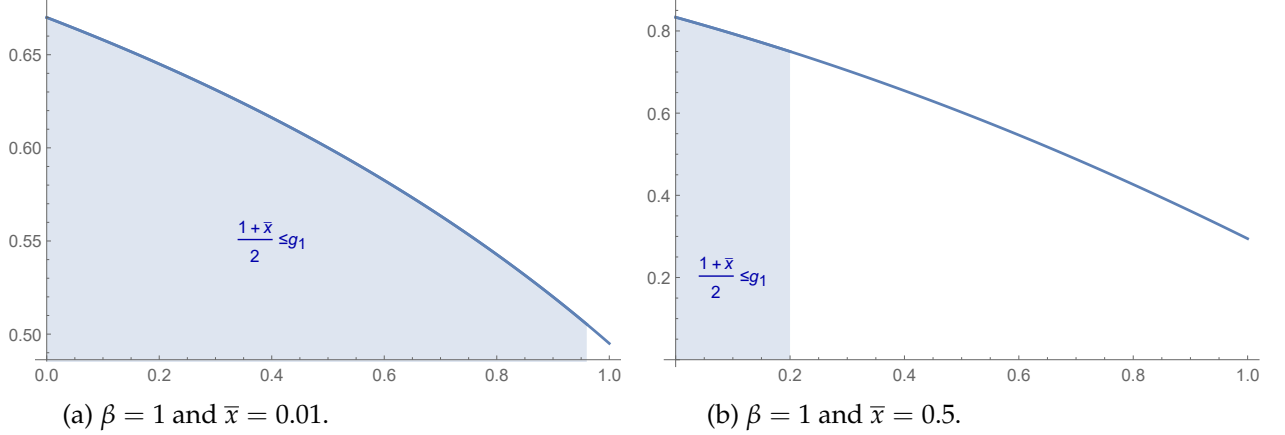


Figure 20: The outside consumption available for parties decreases the region such that $g_1 \geq \frac{1+\bar{x}}{2}$.

Therefore:

$$g_1^* = \begin{cases} \frac{1+\beta(1+\bar{x}(1-p))}{2+\beta(1+p)}, & \text{if } p \geq \frac{1-\bar{x}}{1+3\bar{x}} \\ \frac{B-\sqrt{A}}{8(2+\beta)\bar{x}}, & \text{otherwise} \end{cases} \quad (10)$$

with A and B defined as above. By (A14), $c_{i,1} = 1 - g_1$. Note that for $\beta = 1$ and $p = 0$, the above strategy simplifies to:

$$g_1^* = \frac{2+\bar{x}}{3}$$

since the first case is not possible anymore as it requires $p \geq \frac{1-\bar{x}}{1+3\bar{x}}$.

I now need to check (A15) to see when this case holds in terms of the initial status quo $s_1 = (0, 0, g_0)$. The relevant status quo for this initial analysis is $s_1 \in s_1 = g_0$. Also, since I am studying the case in which the first period constraint is not binding, it is likely to assume that g_0 is "low". But, in general, we have three relevant cases $g_0 < \frac{1}{2}$, $\frac{1}{2} \leq g_0 < \frac{1+\bar{x}}{2}$ and $\frac{1+\bar{x}}{2} \leq g_0$.

★ $g_0 < \frac{1}{2}$ and $p = 0$. In this case, $g_1 \geq \frac{1+\bar{x}}{2}$, since $p = 0$. By (A15) we have:

$$\ln(\bar{x} + g_1^*) - \ln(\bar{x} + g_0) + V_j(s_2) - V_j(s_1) > 0$$

By S1, $V_j(s_2) = \ln\left(\frac{1+\bar{x}(6-4g_1+5\bar{x})}{4}\right)$ and $V_j(s_1) = \ln\left(\frac{(1+\bar{x})^2}{4}\right)$. The second condition holds because I am assuming $g_0 < \frac{1}{2}$. After some more algebra, the condition for this case to hold can be re-written as:

$$g_0 < \underline{g}_0 = \frac{4\bar{x}^2 + 11\bar{x} + 6}{18\bar{x} + 9}$$

★ $g_0 < \frac{1}{2}$, $p \in (0, 1]$ and $\beta \in [0, 1)$. By (A15) we have:

$$\begin{aligned} \ln(\bar{x}) + \ln(\bar{x} + g_1^*) + \beta(pW_j(s_2) + (1-p)V_j(s_2)) > \\ \ln(\bar{x}) + \ln(\bar{x} + g_0) + \beta(pW_j(s_1) + (1-p)V_j(s_1)) \end{aligned}$$

which can be re-written as:

$$EQ_g(g_0, p) = \ln(\bar{x} + g_1^*) - \ln(\bar{x} + g_0) + \beta p W_j(g_1^*) + \beta(1-p)V_j(g_1^*) - \beta p W_j(g_0) - \beta(1-p)V_j(g_0) > 0 \quad (11)$$

this because when j is in power in the second period, $V_j(s_2) = V_j(g_1)$, since the only relevant state for j as a proposer in the second-period is the mandatory spending on public goods. Also, $W_j(s_2) = W_j(g_1)$, since the only relevant state for j as the respondent in the second-period is again the mandatory spending on private goods. Since $g_1 \geq \frac{1}{2}$ but we don't know if $\frac{1}{2} \leq g_1^* < \frac{1+\bar{x}}{2}$ or $g_1^* \geq \frac{1+\bar{x}}{2}$, we have to consider g_1^* as a piecewise function as defines in (10).

Moreover, since the individual rationality constraint is not binding, $g_0 < \frac{1}{2}$, which implies $V_j(g_0) = \ln\left(\frac{1+2\bar{x}}{2}\right)$ and $W_j(g_0) = \ln\left(\frac{\bar{x}(1+2\bar{x})}{2}\right)$.

Note that:

$$\frac{\partial EQ_g(g_0, p)}{\partial g_0} = -\frac{1}{\bar{x} + g_0} - \beta p W_j'(g_0) + \beta(1-p)V_j'(g_0) < 0$$

since $W_j'(g_0) = V_j'(g_0) = 0$ for $g_0 < \frac{1}{2}$ by S1. Moreover,

$$\begin{aligned} \frac{\partial EQ_g(g_0, p)}{\partial g_0} &= \underbrace{\frac{1}{\bar{x} + g_1^*}}_{+} \underbrace{\frac{\partial g_1^*}{\partial p}}_{+} + \underbrace{\beta(pW_j(g_1^*) - (1-p)V_j(g_1^*))}_{+|-} \underbrace{\frac{\partial g_1^*}{\partial p}}_{+} + \\ &\quad \underbrace{\beta(W_j'(g_1^*) - V_j'(g_1^*))}_{+} + \underbrace{\beta(V_j(g_0) - W_j(g_0))}_{+} < 0 \end{aligned}$$

where $\frac{\partial g_1^*}{\partial p} = \frac{\beta(1+2\bar{x})}{(2+\beta(1-p))^2} > 0$. The last term of the above equation is positive since $V_j(g_0) = \ln\left(\frac{(2\bar{x}+1)^2}{4}\right) > W_j(g_0) = \ln\left(\frac{\bar{x}(2\bar{x}+1)}{4}\right)$ if and only if $\frac{2\bar{x}+1}{\bar{x}} > 1$, which implies $\bar{x} < -1$, a contradiction since $\bar{x} \in (0, 0.5]$. Moreover, the third term of the equation is also positive, since $W_j'(g_1^*) > 0$ and $V_j'(g_1^*) \leq 0$ for $\frac{1}{2} \geq g_1^* < 1$ by (S1). The question mark remains for the second term of the above equality:

$$pW_j(g_1^*) - (1-p)V_j(g_1^*) = \begin{cases} p \ln(\bar{x}(\bar{x} + g_1^*)) - (1-p) \ln((\bar{x} + 1 - g_1^*)(\bar{x} + g_1^*)), & \text{if } p \geq \frac{1-\bar{x}}{1+3\bar{x}} \\ p \ln(\bar{x}(\bar{x} + g_1^*)) - (1-p) \ln\left(\frac{1+\bar{x}(6-4g_1^*+5\bar{x})}{4}\right), & \text{if } p < \frac{1-\bar{x}}{1+3\bar{x}} \end{cases}$$

Note that $\ln(\bar{x}(\bar{x} + g_1^*)) < 0$ if and only if $\bar{x}(\bar{x} + g_1^*) < 1$, which implies $g_1^* < \frac{1-\bar{x}^2}{\bar{x}}$. This is always true for $\bar{x} \in \left[\frac{2-\sqrt{20}}{4}, \frac{-1+\sqrt{17}}{4}\right]$, which always holds since $\bar{x} \in (0, 0.5]$. Moreover, $\ln((\bar{x} + 1 - g_1^*)(\bar{x} + g_1^*)) < 0$ following the same rationale. The sign of $\ln\left(\frac{1+\bar{x}(6-4g_1^*+5\bar{x})}{4}\right)$ depends on p since for $\ln\left(\frac{1+\bar{x}(6-4g_1^*+5\bar{x})}{4}\right) < 0$ we require that $\frac{1+\bar{x}(6-4g_1^*+5\bar{x})}{4} \leq 1$, which implies $p < \frac{4(11\bar{x}^2+10\bar{x}-9)}{(3\bar{x}+1)^2(\bar{x}^2+2\bar{x}-3)} \ll \frac{1-\bar{x}}{1+3\bar{x}}$. Therefore, $\ln\left(\frac{1+\bar{x}(6-4g_1^*+5\bar{x})}{4}\right) < 0$ if and only if $p < \frac{4(11\bar{x}^2+10\bar{x}-9)}{(3\bar{x}+1)^2(\bar{x}^2+2\bar{x}-3)}$, otherwise the expression is positive. The condition can be illustrated in the picture below:

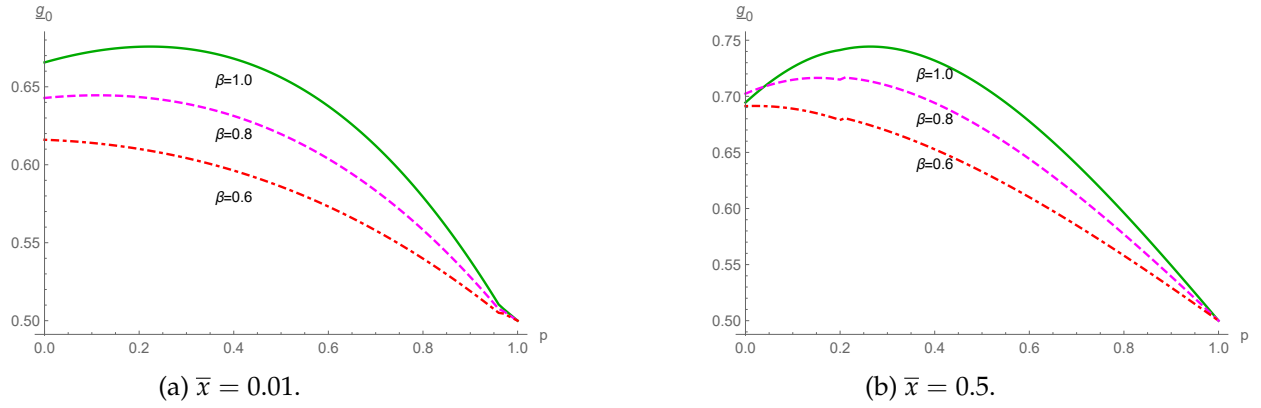


Figure 21: The maximum level of the exogenous status quo g_0 such that the individual rationality constraint is not binding in the first-period is almost always decreasing in p , it is increasing in β and \bar{x} .

In the main text, I depict a picture that shows the relationship between \underline{g}_0 and $\underline{c}_{j,0}$ with respect to \bar{x} .

Net welfare measure for homogenous parties and taste across goods

For the case of homogenous parties and taste across goods, we have:

$$\Delta^d = \underbrace{U_H(\mathbf{x}^*) - U_H(\mathbf{x}^d)}_{\Delta_H^d} + \underbrace{U_L(\mathbf{x}^*) - U_L(\mathbf{x}^d)}_{\Delta_L^d}$$

First recall that the dictator problem for H is given by $c_H = g = 0.5$ and $c_L = 0$. The result is symmetric for L since parties are homogenous. Therefore:

$$U_H(\mathbf{x}^d) = \ln(\bar{x} + 0.5) + \ln(\bar{x} + 0.5) + \beta [p (\ln(\bar{x} + 0.5) + \ln(\bar{x} + 0.5)) + (1 - p) (\ln(\bar{x}) + \ln(\bar{x} + 0.5))]$$

$$U_L(\mathbf{x}^d) = \ln(\bar{x}) + \ln(\bar{x} + 0.5) + \beta [p (\ln(\bar{x} + 0.5) + \ln(\bar{x} + 0.5)) + (1 - p) (\ln(\bar{x}) + \ln(\bar{x} + 0.5))]$$

It is easy to see that:

$$U_H(\mathbf{x}^d) + U_L(\mathbf{x}^d) = (1 + \beta) [2\ln(\bar{x} + 0.5) + \ln(\bar{x}(\bar{x} + 0.5))] \leq 0$$

which doesn't depend on p and it is ≤ 0 since $\bar{x} \leq 0.5$. We want to compute the following partial derivative:

$$\frac{\partial \Delta^d}{\partial p} = \frac{\partial \Delta_L^d}{\partial p} + \frac{\partial \Delta_H^d}{\partial p}$$

as we saw above, $-(U_H(\mathbf{x}^d) + U_L(\mathbf{x}^d))$ doesn't depend on p . Therefore, we can focus on:

$$\frac{\partial \Delta^d}{\partial p} = \frac{\partial U_H(\mathbf{x}^*)}{\partial p} + \frac{\partial U_L(\mathbf{x}^*)}{\partial p}$$

it has to be the case that $\frac{\partial U_H(\mathbf{x}^*)}{\partial p} \geq 0$ and $\frac{\partial U_L(\mathbf{x}^*)}{\partial p} \leq 0$, since the current proposer can only benefit from increasing the chances of remaining in power and the opposite is true for the respondent. Therefore, for $\frac{\partial \Delta^d}{\partial p} < 0$ it has to be that $|\frac{\partial U_H(\mathbf{x}^*)}{\partial p}| < |\frac{\partial U_L(\mathbf{x}^*)}{\partial p}|$, i.e., the marginal impact of increasing political turnover hurts more the respondent than helps the proposer. Why is that the case? Since this is a finite problem, the proposer has leader's advantage. This leader's advantage will materialize in the fact that the proposer will have higher private consumption than the respondent.

where $U_i(\mathbf{x}^*) = \ln(\bar{x} + c_{i,1}^*) + \ln(\bar{x} + g_1^*) + \beta [pV_i(\Psi(\mathbf{x}_1^*)) + (1 - p)W_i(\Psi(\mathbf{x}_1^*))]$ for $i \in \{H, L\}$

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