Why Have Interest Rates Fallen Far Below the Return on Capital

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The opinions herein do not necessarily represent those of the Banque de France, the Federal Reserve Bank of Chicago, or the Federal Reserve System.
The decrease of real interest rates

Why Have Interest Rates Fallen Far Below the Return on Capital
Which does not reflect the evolution of capital return

Why Have Interest Rates Fallen Far Below the Return on Capital
The usual suspects

- Low rates have been loosely tied to “secular stagnation”
- A number of potential explanations have been cited:
  - productivity slowdown
  - changing demographics (population slowdown, increased longevity)
  - change in the price of investment goods
  - tightening of borrowing constraint
  - shortage of safe assets
  - rising inequality
Our goal

- quantitative assessment of the various factors cited
- embed them in a single, tractable model
- explain both the evolution of capital return and risk-free rate
  - this means having risk, and attitudes toward risk, in the model
Related literature

- Low rates: King and Low (2014); Hamilton et al. (2016); Holston et al. (2016); Del Negro et al. (2017)
- Safe assets: Coeurdacier et al. (2015); Caballero et al. (2008); Caballero and Farhi (2014)
- Deleveraging: Eggertsson and Krugman (2012); Korinek and Simsek (2016); Farhi and Werning (2013)
- Secular stagnation: Bean et al. (2015); Rachel and Smith (2015); Ferrero et al. (2017); Borio et al. (2016, 2017)
- Demographics: Carvalho et al. (2016); Gagnon et al. (2016)
- Risk: Kozlowski et al. (2015); Hall (2016)
- Return on capital: Caballero et al. (2017)
The Model

- add risk to Eggertsson and Mehrotra (2014) and Coeurdacier et al. (2015).
- time is discrete, infinite
- 3-period OLG structure \((y, m, o)\)
  - population \(N_t\), growth rate \(g_L\)
- recursive preferences with Epstein-Zin-Weil utility function
- capital and labor (supplied inelastically), age-specific productivities \((e^y, 1, 0)\)
- output \(Y = K^\alpha (AL)^{1-\alpha}\)
  - productivity \(A\): trend growth \(g_A\) + shock with variance \(\sigma\) (only source of risk)
  - growth in price of investment \(g_I\)
Preferences

Epstein and Zin (1989)–Weil (1990) recursive preferences:

\[ V_t = U(c_t, E_t V_{t+1}) = \left( c_t^{1-\rho} + \beta \left( (E_t V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}} \right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \]

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CES functional form applied to

- **time:** \( (c_t^{1-\rho} + \beta (\cdot_{t+1})^{1-\rho})^{\frac{1}{1-\rho}} \)
  - \( \rho \): inverse of intertemporal elasticity of substitution

- **risk:** \( (E_t V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}} \)
  - \( \gamma \): risk aversion

- **when** \( \rho = \gamma \)
  - standard time-additive preferences
  - tension between
    - high \( \gamma \) required to match asset pricing
    - low \( \rho \) required to match consumption growth with interest rates
Budget constraints

- young borrow from middle-aged up to a fraction $\theta$ of their $t+1$ labor income
  - we focus on equilibria where this binds
  - no other frictions (e.g., price stickiness)
- middle-aged lend to young, buy capital from old, invest
- old collect returns, sell depreciated capital

\[
\begin{align*}
  c_t^y &= b_{t+1}^y + w_t e_t^y \\
  b_{t+1}^y &\leq \theta_t E_t(w_{t+1}/R_{t+1}) \\
  c_{t+1}^m - b_{t+2}^m + p_{t+1}^k k_{t+2}^m &= w_{t+1} - R_{t+1} b_{t+1}^y \\
  c_{t+2}^o &= (p_{t+2}(1-\delta) + r_{t+2}^k) k_{t+2}^m - R_{t+2} b_{t+2}^m
\end{align*}
\]

market-clearing:

\[
g_{L,t} b_{t+1}^y + b_{t+1}^m = 0
\]
Production

\[ Y_t = (N_{t-2}k_t^m)^\alpha [A_t(e_t^yN_t + N_{t-1})]^{1-\alpha} \]

- \(N_{t-2}k_t^m\): capital (chosen by current old in the previous period)
- \(e_t^yN_t + N_{t-1}\): labor (of young and middle-aged)

Competitive factor markets:

\[ w_t = (1 - \alpha)A_t^{1-\alpha}k_t^\alpha \]
\[ r_t^k = \alpha A_t^{1-\alpha}k_t^{\alpha-1} \]

both written in terms of the capital/labor ratio \(k_t\) defined as

\[ k_t \equiv \frac{N_{t-2}k_t^m}{e_t^yN_t + N_{t-1}} = \frac{k_t^m}{g_{L,t-1}(1 + e_t^yg_{L,t})}. \]
only the middle-aged have an intertemporal problem
  ▶ how much to save
  ▶ in what form: bonds or capital
write the middle-aged’s Euler equation and substitute equilibrium quantities
  ▶ quantity of bonds determined by young’s constraint
  ▶ Euler equation also relates risk-free rate $R$ and return to capital $R^k$
we derive a law of motion expressed in terms of $R$ or equivalently $k$
Solution strategy (2)

Middle-aged FOCs:

\[
(c_t^m)^{-\rho} = \beta \left[ E_t(c_{t+1}^o)^{1-\gamma} \right]^{\frac{\gamma-\rho}{1-\gamma}} E_t \left[ (c_{t+1}^o)^{-\gamma} R_{t+1}^k \right] \\
(c_t^m)^{-\rho} = \beta \left[ E_t(c_{t+1}^o)^{1-\gamma} \right]^{\frac{\gamma-\rho}{1-\gamma}} E_t \left[ (c_{t+1}^o)^{-\gamma} R_{t+1}^k \right].
\]

Define \( R_{t+1}^m = \alpha_t R_t^k + (1 - \alpha_t) R_{t+1} \) and express budget constraints as

\[
W_t = Y_t - c_t^m \\
c_{t+1}^o = R_{t+1}^m W_t.
\]

Portfolio choice: set \( \alpha_t \) so that

\[
E_t(R_{t+1}^m)^{1-\gamma}) R_{t+1} = E_t \left( R_{t+1}^m - \gamma R_{t+1}^k \right)
\]

Saving decision:

\[
Y_t = \left( 1 + (\beta \phi_t R_{t+1}^1)^{-\frac{1}{\rho}} \right) W_t
\]

Then use market clearing to express \( Y_t, W_t, R_{t+1}^m \) in term of the aggregate capital stock
Law of motion

\[
\left(1 + (\beta \phi_t)^{-1/\rho} R^{1-1/\rho}_{t+1}\right)^{-1} (1 - \frac{\theta_{t-1}}{\tilde{a}_t})(1 - \alpha \left(\frac{R^k_{t+1}}{g_l} - 1 + \delta\right)) k_t
\]

- **saving rate**
- **income**

\[
g_{L,t} = \alpha (1 + e^y g_{L,t+1}) + \left(\frac{1}{\xi_t}\right) (1 - \alpha) \theta_t (1 - g_l \frac{1 - \delta}{R_{t+1}}) k_{t+1}
\]

- **overlapping generations**
  - saving only done out of labor income
- **borrowing constraint**
  - disappears if $\theta = 0$, $e^y = 0$
- **risk**
  - $\phi_t$: precautionary saving, acts like discount factor distortion ($\leq 1$)
  - $1/\xi_t$: portfolio choice
Risk terms

The factors $\phi_t$ and $\xi_t$ are

\[
\xi_t = \frac{\mathbb{E}_t(u_{t+1}^{1-\gamma}\tilde{a}_{t+1})}{\mathbb{E}_t(u_{t+1}^{1-\gamma})}
\]

\[
\phi_t = \left[\mathbb{E}_t u_{t+1}^{1-\gamma}\right]^{(\gamma-\rho)/(1-\gamma)} \mathbb{E}_t u_{t+1}^{1-\gamma} v_t^{\rho}
\]

with

\[
u_t \equiv \alpha(1 + e^{y} g_{L,t+1})\tilde{a}_{t+1} + (1 - \alpha)\theta_t
\]

\[
\tilde{a}_{t+1} \equiv \frac{A_{t+1}^{1-\alpha}}{\mathbb{E}_t A_{t+1}^{1-\alpha}}.
\]

only functions of (moments of) the exogenous process $A_{t+1}$

- when $\delta \neq 1$, $\phi_t$ involves $R_{t+1}$ as well
Risk steady state

to account for risk in a tractable way, we appeal to the concept of “risky steady state”:
- exogenous trends as in the data
- productivity shock is assumed i.i.d.
- in the law of motion, $\tilde{a}_t$ set at its mean, $\tilde{a}_{t+1}$ is stochastic
- agents take into account the uncertainty
Risk and borrowing constraint

When $\delta = 1$, $\rho < 1$:

$$\phi_t \approx 1 + \frac{1}{2} \gamma (1 - \rho) \frac{\alpha^2 (1 + e^\gamma g_L)^2}{(\alpha(1 + e^\gamma g_L) + (1 - \alpha) \theta)^2} \sigma^2$$

$$\frac{1}{\xi_t} \approx 1 + \gamma \frac{\alpha(1 + e^\gamma g_L)}{\alpha(1 + e^\gamma g_L) + (1 - \alpha) \theta} \sigma^2$$

Risky steady-state:

$$g_{Ag}^{-\frac{1}{1-\alpha}} = (1 + (\phi \beta)^{-\frac{1}{\rho}} R^{1-\frac{1}{\rho}})^{-1} \left[\frac{1 - \alpha}{\alpha g_L \xi} \frac{R}{g_L}\right] \frac{\alpha(1 - \theta)}{\alpha(1 + e^\gamma g_L) \xi + (1 - \alpha) \theta}$$

$$R^k = \frac{R}{\xi}$$
Long run determinants

of the bond interest rate $r$ and the return on capital $r^K$

$\delta = 1, \rho = 1$:

- Observable factors
  - productivity growth $g_A$
  - evolution of working age population $g_L$
  - trend in investment price $g_I$

- Unobservable factors
  - borrowing constraint $\theta$
  - variance of the shock on the trend of productivity $\sigma$. 
Long run determinants
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  - borrowing constraint $\theta$
  - variance of the shock on the trend of productivity $\sigma$.

\[
\begin{align*}
r &= \bar{r} + (g_L - 1) + (g_A - 1) - \frac{\alpha}{1 - \alpha} (g_I - 1) + c\theta + \gamma u(\theta|\sigma, \sigma^2) \\
r^K &= r + \gamma v(\theta|\sigma, \sigma^2)
\end{align*}
\]

The wedge between $r$ and $r^K$ is only affected by $\theta$ and $\sigma$.
Empirical strategy

- our targets are the risk-free rate and the return on capital
- we segregate the usual suspects into
  - the observables: productivity, demographics, price of investment
  - the “less observables”: borrowing constraint, productivity risk

Three steps:
1. input the observables, set $\theta$ and $\sigma$ constant to match the levels of the targets
2. input the observables, compute $\theta$ to match the risk-free rate, keep $\sigma$ constant
3. input the observables, compute $\sigma$ to match the risk-free rate, keep $\theta$ constant
4. input the observables, compute $\theta$ and $\sigma$ to match both targets

- repeat for US and Euro area (and the world)
- then stare at the pictures...
- caveats
  - we interpret the generations loosely (10-year averages)
  - risk-free rates before the 1980s are less meaningful (financial repression etc), so we focus on 1990s to present
## Model calibration and data sources

### Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>length of period (years)</td>
<td>10</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>$0.98^T$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>capital share</td>
<td>0.28</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>risk aversion</td>
<td>100</td>
</tr>
<tr>
<td>$\rho$</td>
<td>inverse of IES</td>
<td>0.8</td>
</tr>
<tr>
<td>$\delta$</td>
<td>capital depreciation rate</td>
<td>$0.1 \times T$</td>
</tr>
<tr>
<td>$e^y$</td>
<td>relative productivity of young</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### Factors

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<thead>
<tr>
<th>Factor</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{L,t}$</td>
<td>growth rate of population 20-64</td>
<td>US, EA (France), China, Japan: OECD</td>
</tr>
<tr>
<td>$g_{I,t}$</td>
<td>investment price growth</td>
<td>DiCecio (2009)</td>
</tr>
<tr>
<td>$g_{A,t}$</td>
<td>productivity growth</td>
<td>US: Fernald (2012), Euro: NAWM model</td>
</tr>
<tr>
<td>$R_t$</td>
<td>real interest rate</td>
<td>US: Hamilton et al. (2016), France</td>
</tr>
<tr>
<td>$R^k_t$</td>
<td>return on capital</td>
<td>US, EA: our calculations à la Gomme et al. (2015)</td>
</tr>
<tr>
<td>$\tilde{a}_t$</td>
<td>productivity shock</td>
<td>$\ln(\tilde{a})$ is a i.i.d. $N(-\sigma^2/2, \sigma^2)$</td>
</tr>
</tbody>
</table>

### Free parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>borrowing constraint on young</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>variance of $\tilde{a}_t$</td>
</tr>
</tbody>
</table>
Why Have Interest Rates Fallen Far Below the Return on Capital
Impact of observable factors, in the US
Observable factors explain about 1.4% from 1992 to 2014

Why Have Interest Rates Fallen Far Below the Return on Capital
Impact of observable factors, in the EA
Observable factors explain about 1.8% from 1992 to 2014

Why Have Interest Rates Fallen Far Below the Return on Capital
Impact of the borrowing constraint, in the US.
A tighter constraint can account for the fall in the risk-free rate and 0.8% increase of the risk premium.
Impact of the borrowing constraint, in the EA.
A tighter constraint can account for the fall in the risk-free rate and 0.7% increase of the risk premium.
Impact of risk, in the US.
A higher risk perception can account for the fall in the risk-free rate and the increase in the risk premium.
Impact of risk, in the EA.

A higher risk perception can account for the fall in the risk-free rate and the increase in the risk premium.

Why Have Interest Rates Fallen Far Below the Return on Capital
Impact of risk and the borrowing constraint, in the US.

With higher risk perception data are consistent with non decreasing debts.
Impact of risk and the borrowing constraint, in the EA.

With higher risk perception data are consistent with non decreasing debts.
Borrowing constraint and risk, in the US.

Why Have Interest Rates Fallen Far Below the Return on Capital
Borrowing constraint and risk, in the EA.

Why Have Interest Rates Fallen Far Below the Return on Capital
Global perspective
Impact of observable factors

**Observed and simulated rates**

**Observed and simulated risk premium**

**Borrowing ratio $\theta$**

**Productivity Risk (std) $\sigma$**

Why Have Interest Rates Fallen Far Below the Return on Capital
Global perspective
Impact of the borrowing constraint

Observed and simulated rates

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Why Have Interest Rates Fallen Far Below the Return on Capital
Global perspective
Impact of risk and the borrowing constraint

Why Have Interest Rates Fallen Far Below the Return on Capital
Conclusion

- usual suspects aren’t enough
  - deleveraging story
- increased (perception of) risk can account for the patterns
  - but it’s a residual
- extensions on
  - longevity
  - increasing capital share
  - inequality (through a bequest motive)
- more work to be done on exogenous supply of safe assets
Sensitivity to $\gamma$ (US)

Why Have Interest Rates Fallen Far Below the Return on Capital
Sensitivity to $\gamma$ (EA)

Why Have Interest Rates Fallen Far Below the Return on Capital
The inputs for the US

**Productivity growth**

**Working Age Population**

**Relative investment price**

**Debt/GDP (%)**

Why Have Interest Rates Fallen Far Below the Return on Capital
The inputs for the EA

Productivity growth

Working Age Population

Relative investment price

Debt/GDP (%)

Why Have Interest Rates Fallen Far Below the Return on Capital
Measures of uncertainty

Source: Bachmann et al. (2012), dispersion index is based on survey expectations data (disagreement and forecast errors).

Borio, C., P. Disyatat, M. Juselius, and P. Rungcharoenkitkul (2017): “Why so long for so long, a long term view of real interest rates,” mimeo, BIS.


References II


References III


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