Malas Notches

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Important new development in public economics - the sufficient statistic approach, which "derives formulas for the welfare consequences of policies that are functions of high-level elasticities rather than deep primitives" (Chetty (2009), p 451).

Feldstein (1999): for a proportional income tax $t$, marginal excess burden (MEB) only depends on behavioral responses via the elasticity of taxable income (ETI), $e$.

- $e$ measures the intensive margin response to a change in $t$ i.e. the change in the taxable income of a given individual to $t$

Now a large literature on empirical estimates of $e$ (e.g. Gruber and Saez (2002), Saez, Slemrod, and Giertz (2012), Kleven and Schultz (2014), Weber (2014)).
Saez (2001) showed that the Feldstein formula could be extended to a proportional income tax with an allowance (single-bracket tax), with one more sufficient statistic \( a \) of the income distribution, constant if the top tail of the income distribution is Pareto.

Also, he showed that revenue-maximising tax rate depends only on \( e, a \) and the welfare maximising tax rate depends on \( e, a \) and a welfare weight \( \bar{g} \).
The “sufficient statistics” approach fails with notches

Specifically, \( MEB = \frac{te+C}{1-t-te-C} \) where \( C > 0 \) is a correction factor

This is the formula for the MEB of a proportional tax (Feldstein (1999)) plus a correction factor \( C \)

But, correction factor is complex (does not depend on simple sufficient statistics) and is quantitatively important for a calibrated version of the model

At baseline values, ignoring \( C \) underestimates the MEB by about 86%, and the revenue-maximising tax is overestimated by around 100%

Application to VAT: MEB is underestimated by about 50%
Some notches in income taxes:

- PIT; Pakistan has notches of up to 5% (Kleven and Waseem (2013)), Ireland, an emergency income levy with a notch of up to 4% (Hargaden (2015)), small notches in the federal PIT in the US (Slemrod (2013)).
- notches in the CIT in Costa Rica (Bachas and Mauricio (2015)).

Notches in housing transactions taxes in the UK and the US (Best and Kleven (2014), Kopczuk and Munroe (2014)).

Slemrod (2013): many examples of commodity tax notches

- a marginal change in some characteristic can change the product classification so as to produce a discrete change in the tax liability e.g. the US Gas Guzzler Tax

Most important case: a VAT threshold can be a tax notch (Liu & Lockwood (2015))
Already known that due the sufficient statistic approach is limited due to externalities

Saez, Slemrod and Giertz (2012); positive externalities if “socially valuable” activities can be deducted from income tax e.g. charitable giving/mortgage interest payments

Chetty (2010): possible positive fiscal externalities with income tax evasion if (part of) the cost of evasion is a transfer payment (e.g. a fine to the government)

By contrast, our results nothing to do with externalities—rather, difference between intensive margin and total ETI.
The Set-Up

- Individual taxpayers indexed by a skill or taste parameter $n \in [n, \bar{n}]$, distributed with density $h(n)$.
- A type $n$ individual has preferences over consumption $c$ and taxable income $z$ of $u(c, z; n) = c - d(z; n)$
- Assume $d_z, d_{zz} > 0, d_n, d_{nz} < 0$
- Iso-elastic case: $d(z; n) = \frac{n}{1+1/e}(z/n)^{1+1/e}$
- The budget constraint is $c = z - T(z)$, where $T(.)$ is the tax function.
- Household $n$'s utility over $z$ is $u(z; n) = z - T(z) - d(z; n)$.
- For any marginal rate $t$, $z(1-t, n)$ is household $n$'s optimal taxable income
  - In iso-elastic case, $z(1-t, n) = (1-t)^e n$
For simplicity, we focus on a two-bracket tax; results extend straightforwardly to the highest tax in a piecewise-linear tax system with any number of brackets.

So, kinked and notched two-bracket taxes are:

\[
T_K(z) = \begin{cases} 
  t_Lz, & z \leq z_0 \\
  t_Lz_0 + t_H(z - z_0), & z > z_0 
\end{cases}
\]

\[
T_N(z) = \begin{cases} 
  t_Lz, & z \leq z_0 \\
  t_Hz, & z > z_0 
\end{cases}
\]
With either a kink or a notch, all types in an interval \( n \in [n_L, n_H] \) will bunch at taxable income \( z_0 \).

With both a kink and a notch: \( z(1 - t_L; n_L) = z_0 \)

With a kink, \( n_H \) is defined by \( z(1 - t_H; n_H) = z_0 \)

With a notch, \( n_H \) is defined by

\[
(1 - t_L)z_0 - d(z_0; n_H) = v(t_H; n_H)
\]

where \( v(t; n) \equiv \max_z (1 - t)z - d(z; n) \)
Tax revenue $R$ depends on $t_H$ both directly, and indirectly, via its effect on bunching i.e. $R(t_H, n_H(t_H))$

So:

$$\frac{dR}{dt_H} = \frac{\partial R}{\partial t_H} \left| \begin{array}{c} \text{intensive} \\ \text{bunching} \end{array} \right| + \frac{\partial R}{\partial n_H} \frac{\partial n_H}{\partial t_H}$$

In the kink case, the bunching effect is zero, because $\frac{\partial R}{\partial n_H} = 0$

In the notch case, $\frac{\partial R}{\partial n_H} = (t_Lz_0 - t_Hz(1 - t_H; n_H))h(n_H) < 0$
The Bunching Effect on Tax Revenue with A Kink

Assume iso-elastic disutility so $z(1 - t; n) = (1 - t)^e n$

Tax revenue

![Graph showing the bunching effect on tax revenue with a kink at $n_L$ and $n_H$.]
Assume iso-elastic disutility so $z(1 - t; n) = (1 - t)^{e} \cdot n$
Generally, $\textit{MEB} = -\frac{dW/dt_H}{dR/dt_H}$, where welfare $W$ is calculated assuming that tax revenue is redistributed as a lump-sum back to households.

With iso-elastic utility and a Pareto upper tail of the income distribution:

$$\textit{MEB} = \frac{t_H e + C}{1 - t_H (1 + e) - C},$$

$$C = \frac{(1 - t_H)(t_H(1 - t_H)^e - t_L z_0/n_H)(1 - a)(1 + e)}{(1 - t_H)^{1+e} - (z_0/n_H)^{1+1/e}} > 0$$

C cannot be written in terms of sufficient statistics $e, a$ (depends also on tax parameters $t_H, t_L, z_0$, and on $n_H$, which is endogenous)
The Welfare-Maximizing Top Rate of Tax with a Notch

- Government’s objective is $W = \int G(v(n)) h(n) dn$

- $G$ is strictly concave, so government has a redistribution objective, $G' = g$ are the welfare weights

- Also, government budget constraint is that $R$ must exceed some exogenous amount

- Then, the welfare-maximising level of $t_H$ is $t^* = \frac{1 - \bar{g} - C}{1 + e}$, where $\bar{g}$ is the average welfare weight on all top-rate taxpayers

- Special case of revenue-maximising $t_H$ is $\bar{g} = 0$ i.e. $t^* = \frac{1 - C}{1 + e}$
### Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline Value</th>
<th>Range</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>0.25</td>
<td>0.1-0.4</td>
<td>SSG (2012), Kleven and Schultz (2014)</td>
</tr>
<tr>
<td>$a$</td>
<td>1.5</td>
<td>1.01-2.0</td>
<td>Piketty and Saez (2003)</td>
</tr>
<tr>
<td>$t_H - t_L$</td>
<td>0.03</td>
<td>0.0-0.05</td>
<td>Kleven and Waseem (2013)</td>
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<tr>
<td>$t_L$</td>
<td>0.2</td>
<td></td>
<td></td>
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<tr>
<td>$z_0$</td>
<td>2.168</td>
<td></td>
<td>20% of population have $z \geq z_0$</td>
</tr>
</tbody>
</table>
The Marginal Excess Burden

Figure: MEB as $e, a$ vary
The Welfare-Maximizing Top Rate of Tax

Figure: $t^*$ as $e, a$ vary ($\bar{g} = 0.25$)
A Simple Model of VAT Registration

- Simplified version of Liu and Lockwood (2015): a single industry with a large number of small traders producing a homogeneous good.
- Every small trader combines his own labor input with an intermediate input to produce output via a fixed coefficients technology.
- Buyers have perfectly elastic demand for the good (like the assumption made implicitly in the taxable income literature that labor demand is perfectly elastic at a fixed wage.)
- This is formally equivalent to the notched income tax model.
- But MEB is the marginal excess burden on *producers* (demand is perfectly elastic).
A Simple Model of VAT Registration

- Key variable is $s$, units of input required per unit of output.

- If $s = 0$, then the MEB of the VAT is mathematically identical to income tax case: $MEB = \frac{e^{\frac{t}{1+t}} + C}{1 - \frac{t}{1+t} (1+e) - C}$.

- If $s > 0$, then MEB is similar, but details are more complex, because a change in the statutory rate of VAT also changes the effective tax on non-registered firms via unrecovered input VAT.

- Important to consider $s > 0$, as empirically relevant (for the UK, $s = 0.45$)
## Calibration

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<td></td>
<td>Luettmer (1995)</td>
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<td>(t_R - t_N)</td>
<td>0.17/0.14</td>
<td></td>
<td>LL: (t = 0.2, s = 0.0/0.45)</td>
</tr>
<tr>
<td>(t_N)</td>
<td>0.0/0.16</td>
<td></td>
<td>LL: (t = 0.2, s = 0.0/0.45)</td>
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<tr>
<td>(z_0)</td>
<td>2.168</td>
<td></td>
<td>LL: 37.5% of firms have (z \geq z_0)</td>
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LL=Liu and Lockwood (2016)
The Marginal Excess Burden of VAT

Figure: MEB as $a$ varies ($s = 0.0, 0.45$)
Conclusions

- We show that sufficient statistic approach does not apply to notched tax systems due to the fact that bunching response has a first-order effect on tax revenue.
- Formulae for MEB and revenue-maximising top rate of tax can be written as proportional tax formulae plus a correction factor.
- But, correction factor is complex (does not depend on simple sufficient statistics) and is quantitatively important.
- For example, at baseline values, the MEB is underestimated by about 86%, and the revenue-maximising tax is overestimated by around 30%, and the errors can be much larger for some parameter values.
- Analysis can be applied to VAT; treating VAT as a simple proportional tax underestimates the MEB of the VAT by about 50%.