Social Discounting and Intergenerational Pareto

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Which Social Discount Rate Should We Use?

Many economic decisions are dynamic and affect multiple individuals

- Corporate/household long-term investments
- Durable public good investments
- Intertemporal tax transfers
- Environmental projects

These decisions depend on one number, the social discount rate

- The society’s trade-off between current benefit and future benefit
- No consensus on which social discount rate should be used
"...if we don’t act, the overall costs and risks of climate change will be equivalent to losing at least 5% of global GDP each year, now and forever."

— The Stern Review on the Economics of Climate Change

"...(the Stern Review) depends decisively on the assumption of a near-zero time discount rate..."

— William Nordhaus

"...(using discount rates ranging from 3-5%) is ethically indefensible."

— Lord Nicholas Stern
Questions

1. In what sense is a social discount rate reasonable?

2. What are the reasonable social discount rates?
Social Discounting Depends on Individual Discounting

Social discounting should be more patient than individual discounting (Caplin & Leahy 2004, Farhi & Werning 2007)

- Pure time-preference discounting, rather than consumption discounting
- Social discounting should take into account how future generations value their consumption
- Future generations value future more than the current generation value future
- Thus, social discounting also values future more than the current generation does
- However, these theories only have one individual (representative agent)
A Negative Result

Common in these situations...

- A benevolent planner chooses for multiple generations
- Uncertainty about payoffs

Widely used assumptions in economics:

1. Planner has an exponential discounting expected utility function
2. Some Pareto property

Gollier & Zeckhauser (2005), Zuber (2011), Jackson & Yariv (2014, 2015): even when individuals also discounts exponentially

\[ 1 + 2 \Rightarrow \text{Dictatorship} \]
Preferences
Model Setup

- $2 < T \leq +\infty$ generations/periods

- $N < \infty$ individuals in each generation who live for one period

- One risky public consumption $p_t \in \Delta(X)$ in each period $t$

- Consumption sequence: $\mathbf{p} = (p_1, \ldots, p_T) \in \Delta(X)^T$
Individual Preferences

- Generation-\(t\) individual \(i\)’s preference over \(p\)’s: \(\succeq_{i,t}\)
- Generation-\(t\) individual \(i\)’s discounting utility function:

\[
U_{i,t}(p) = \sum_{\tau=t}^{T} \delta_i(\tau - t) u_i(p_\tau)
\]

- Discount function \(\delta_i(\cdot)\): \(\delta_i(0) = 1, \delta_i > 0\); if \(T = +\infty\), \(\delta_i \in \ell^1\)
- Instantaneous (expected) utility function \(u_i : \Delta(X) \rightarrow \mathbb{R}\)

1. \(U_{i,t}\) only depends on current and future consumption
   - can be relaxed when \(\delta_i\)’s are exponential
2. The offspring inherits the parent’s \(\delta_i\)
   - They rank \(p\)’s differently (\(\delta_i(\cdot)\) is shifted forward)
   - can be relaxed
3. Instantaneous utility does not depend on time
   - can be relaxed
The Planner’s Preference

As in the negative results, we first focus on exponential discounting

- In each period $t$, the planner’s preference over $p$’s: $\succcurlyeq_t$
- In each period $t$, the planner’s utility function:

$$U_t(p) = \sum_{\tau = t}^{T} \delta^{\tau - t} u(p_\tau)$$

- Social discount factor $\delta > 0$; $0 < \delta < 1$ if $T = +\infty$
- Instantaneous utility function $u : \Delta(X) \rightarrow \mathbb{R}$

1. $U_t$ only depends on current and future consumption
2. The discount factor and instantaneous utility do not depend on time
3. Normalization of expected utility functions: for some $x_*$ and $x^*$, $u_i(x_*) = u(x_*) = 0$ and $u_i(x^*) = u(x^*) = 1$
Intergenerational Pareto
A Variant of the Negative Result

- In a dynamic setting, there are different ways to define Pareto

The planner is current-generation Pareto if for each \( t \),
\[ p \succeq_{i,t} q \] for all \( i \) implies \( p \succeq_t q \),
and \( p \succ_{i,t} q \) for all \( i \) implies \( p \succ_t q \).

- An generation-\( t \) individual \( i \) has an exponential discounting utility (EDU) function if

\[
U_{i,t}(p) = \sum_{\tau=t}^{T} \delta_{\tau-t} u_i(p_{\tau})
\]
A Variant of the Negative Result

Proposition Suppose each generation-\(t\) individual \(i\) has an EDU function with \((\delta_i, u_i)\). For a generic \(N\)-tuple of discount factors \((\delta_i)_{i \in N}\), the planner is current-generation Pareto if and only if for each \(t\), there exists a unique \(i\) such that \(U_t = U_{i,t}\).

Sketch of the proof:

- Example: \(N = 2\) and \(u_1 = u_2 = u\)
- Harsanyi 1955: Pareto \(\Leftrightarrow\) Utilitarian, i.e., \(U = \omega U_1 + (1 - \omega) U_2\)
  
  \[
  \begin{align*}
  \omega \delta_1 u_1 + (1 - \omega) \delta_2 u_2 &= \delta u, \\
  \omega \delta_1^2 u_1 + (1 - \omega) \delta_2^2 u_2 &= \delta^2 u.
  \end{align*}
  \]
  
  \(\Rightarrow\) \(\omega = 0, 1\)
Intergenerational Pareto

OUR SURVIVAL PLAN IS TO SACRIFICE OUR CHILDREN.

History Repeats Itself

ARE WE AT FARCE YET?
The planner is intergenerationally Pareto if for each \( t \in T \),
\[
\mathbf{p} \succeq_{i,s} \mathbf{q} \text{ for all } i \text{ and all } s \geq t \text{ implies } \mathbf{p} \succeq_t \mathbf{q},
\]
and \( \mathbf{p} \succ_{i,s} \mathbf{q} \text{ for all } i \text{ and all } s \geq t \text{ implies } \mathbf{p} \succ_t \mathbf{q} \).

- The planner can disagree with a selfish current generation
- The planner ignores past generations whose utility can no longer be changed

Intergenerational Pareto allows the planner to make rather discretionary decisions?
Lemma Suppose $U_{i,t}(p) = \sum_{\tau=t}^{T} \delta_{i,t}(\tau - t)u_i(p_\tau, \tau)$, and $U_t(p) = \sum_{\tau=t}^{T} \delta_t(\tau - t)u_t(p_\tau, \tau)$. Suppose $T < +\infty$. The planner is intergenerationally Pareto if and only if for each $t$, there exists a finite sequence of nonnegative numbers $(\omega_{t}(i,s))_{i\in N, s\geq t}$ such that

$$\sum_{i=1}^{N} \sum_{s=t}^{T} \omega_{i,t}(s) > 0$$

and

$$U_t = \sum_{i=1}^{N} \sum_{s=t}^{T} \omega_{i,t}(s)U_{i,s}.$$
Social Discounting

and Individual Long-Run Discounting:

The Benchmark Case
Strongly Non-Dictatorial

The planner is strongly non-dictatorial if for each $t$,

$$U_t(p) = f_t(U_{1,t}(p), \ldots, U_{1,T}(p), U_{2,t}(p), \ldots, U_{2,T}(p), \ldots, U_{N,T}(p))$$

for some strictly increasing function $f_t$.

- Negative results: The only way for a time-consistent planner to be current-generation Pareto is dictatorship
- Non-dictatorial: The planner cares about more than one individual
Individual Average and Relative Discounting

- $\delta_i(\cdot)$ is defined on $\mathbb{N}$; $T$ may vary

**Average discounting:** $\tau \sqrt{\delta_i(\tau)}$  
**Relative discounting:** $\frac{\delta_i(\tau+1)}{\delta_i(\tau)}$

- **A1:** $\lim_{\tau \to \infty} \tau \sqrt{\delta_i(\tau)}$ exists
- **A2:** $\frac{\delta_i(\tau+1)}{\delta_i(\tau)}$ is bounded
- **A3:** $\frac{\delta_i(\tau+1)}{\delta_i(\tau)}$ is increasing

- $\tau \sqrt{\delta_i(\tau)} = \sqrt{\frac{\delta_i(\tau)}{\delta_i(\tau-1)}} \times \frac{\delta_i(\tau-1)}{\delta_i(\tau-2)} \times \cdots \times \frac{\delta_i(1)}{\delta_i(0)}$
The benchmark case assumes that $T < +\infty$ and $u_i = u$

The main results will highlight how individual instantaneous utility affects the range of “reasonable” social discount rates
Benchmark Case

**Theorem** Suppose $T < +\infty$, and each generation-$t$ individual $i$’s discounting utility function satisfies $A1$, $A2$, and $u_i = u$. Then,

1. if $\delta > \min_i \max_{\tau \in \{0, \ldots, T-1\}} \frac{\delta_i(\tau+1)}{\delta_i(\tau)}$, the planner is intergenerationally Pareto and strongly non-dictatorial;

2. For each $\delta < \min_i \lim_{\tau \to \infty} \sqrt[\tau]{\delta_i(\tau)}$, there exists some $T^* > 0$ such that if $T \geq T^*$, the planner is not intergenerationally Pareto.

- The first part fixes the negative result, and can be used to check whether a planner satisfies intergenerational Pareto.

- The second part: if $\delta$ is too low, there exist $p$ and $q$ such that all individuals from all generations prefer $p$ to $q$, but the planner disagrees.

- In many examples, the two cutoffs are identical.
Individual Long-Run Discounting

- In both examples, two cutoffs coincide

\[ A1: \lim_{\tau \to \infty} \sqrt{\delta_i(\tau)} \text{ exists} \quad \quad A2: \frac{\delta_i(\tau+1)}{\delta_i(\tau)} \text{ is bounded} \]

\[ A3 \text{ (present bias)}: \frac{\delta_i(\tau+1)}{\delta_i(\tau)} \text{ is increasing} \]

- \[ A2 \text{ and } A3 \Rightarrow \lim_{\tau \to \infty} \frac{\delta_i(\tau+1)}{\delta_i(\tau)} = \lim_{\tau \to \infty} \sqrt{\delta_i(\tau)} \]

Define \[ \delta_i^* := \lim_{\tau \to \infty} \frac{\delta_i(\tau+1)}{\delta_i(\tau)} = \lim_{\tau \to \infty} \sqrt{\delta_i(\tau)} \]

as individual \( i \)'s long-run discount factor
**Individual Long-Run Discounting**

**Corollary** Suppose $T < +\infty$ and each generation-$t$ individual $i$’s discounting utility function satisfies $A2$, $A3$, and $u_i = u$. Then,

1. if $\delta > \min_i \delta_i^*$, the planner is intergenerationally Pareto and strongly non-dictatorial;

2. For each $\delta < \min_i \delta_i^*$, there exists some $T^* > 0$ such that if $T \geq T^*$, the planner is not intergenerationally Pareto.

- Social discounting literature: social discounting should be more patient than individual discounting, but which individual and what individual discount factor?

- Benchmark case: the individual with the least patient long-run discount factor

- However, this does not contribute much to the debate on social discounting, because $\min_i \delta_i^*$ can be quite low
Social Discounting and Individual Instantaneous Utility Functions
Instantaneous Utility Functions

\((u_i)_{i \in N}\) is said to be linearly independent if there are no constants \((\alpha_i)_{i \in N}\) such that they are not all zero and \(\sum_i \alpha_i u_i(p) = 0\) for all \(p \in \Delta(X)\).

- Generically, \((u_i)_{i \in N}\) is linearly independent
Theorem Suppose $T < +\infty$, each generation-$t$ individual $i$’s discounting utility function satisfies A2 and A3, and $(u_i)_{i \in N}$ is linearly independent. Let the planner’s $u$ be any strict convex combination of $(u_i)_{i \in N}$. Then,

1. For each $\delta > \max_i \delta_i^*$, the planner is intergenerationally Pareto and strongly non-dictatorial;

2. For each $\delta < \max_i \delta_i^*$, there exists some $T^* > 0$ such that if $T \geq T^*$, the planner is not intergenerationally Pareto.
Remarks

- If A1 and A2 are assumed, rather than A2 and A3, we again have two cutoffs defined analogously.

- The benchmark case is not robust: a small perturbation of $u_i = u$ moves the cutoff from $\min_i \delta_i^*$ to $\max_i \delta_i^*$.

- The choice of $\delta$ is independent of the choice of $u$.

- This result provides support for the use of near-zero discount rate.

- Robustness: (i) $T$ can be $+\infty$; (ii) the offspring does not have to inherit the parent’s preference parameters; (iii) intergenerational Pareto can be strengthened...
Consider a special case where individuals have exponential discounting. In period 1,

\[ U = \sum_{s=1}^{T} \sum_{i=1}^{N} \omega(i, s) U_{i,s} \]

\[ \sum_{\tau=1}^{T} \delta^{\tau-1} u(p_{\tau}) = \sum_{s=1}^{T} \sum_{i=1}^{N} \omega(i, s) \sum_{\tau=s}^{T} \delta_{i}^{\tau-s} u_{i}(p_{\tau}) \]

There is a unique way to write \( u \) as a convex combination of \( u_i \)'s:

\[ \sum_{i} \lambda_i u_i = u \]

First period: \( u = \sum_{i} \omega(i, 1) u_i \Rightarrow \lambda_i = \omega(i, 1) \)

Second period: \( \delta u = \sum_{i} \omega(i, 1) \delta_i u_i + \sum_{i} \omega(i, 2) u_i \Rightarrow \lambda_i \delta = \omega(i, 1) \delta_i + \omega(i, 2) \)

\( \omega(i, 1) \delta = \omega(i, 1) \delta_i + \omega(i, 2) \)
Gradual Transition of the Cutoff

- An individual's instantaneous utility function describes his risk attitude

- \((u^\theta)_{\theta=1}^\Theta\) is a linearly independent \(\Theta\)-tuple of instantaneous utility functions—\(\Theta\) generic types of risk attitude

  - \(\Theta = 1\): \(u_i = u\); \(\Theta = N\): \((u_i)_{i \in N}\) is linearly independent

- Define

  \[
  \delta_{\text{maxmin}}^* := \max_{\theta} \min_{k \in \{i \in N : u_i = u^\theta\}} \delta_k^*.
  \]
Gradual Transition of the Cutoff

Theorem Suppose $T < +\infty$ and each generation-$t$ individual $i$’s discounting utility function has an instantaneous utility function $u_i \in \{u^\theta\}_{\theta=1}^\Theta$ for some linearly independent $\Theta$-tuple of instantaneous utility functions $(u^\theta)_{\theta=1}^\Theta$ such that $\{u_i\}_{i\in N} = \{u^\theta\}_{\theta=1}^\Theta$, and has a discount function $\delta_i$ that satisfies A2 and A3. Let the planner’s $u$ be an arbitrary strict convex combination of $(u_i)_{i\in N}$. Then,

1. If $\delta > \delta^*_{\text{maxmin}}$, the planner is are intergenerationally Pareto and strongly non-dictatorial;

2. For each $\delta < \delta^*_{\text{maxmin}}$, there exists some $T^* > 0$ such that if $T \geq T^*$, the planner is not intergenerationally Pareto.