# Interest Rate Dynamics, Variable-Rate Loan Contracts, and the Business Cycle

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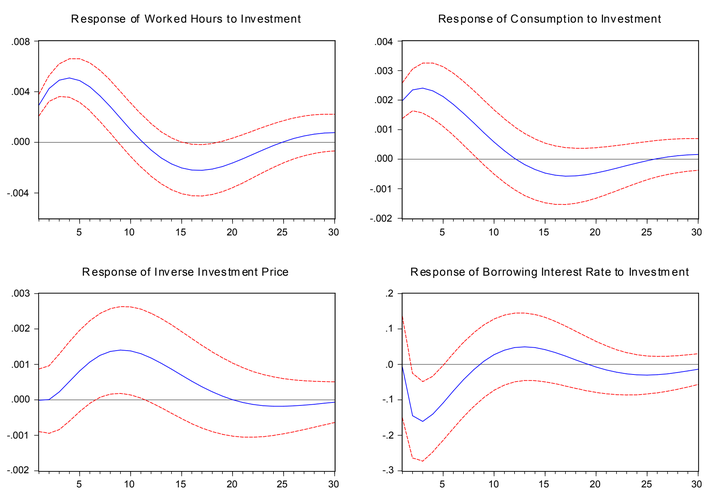
## Borrowing Cost of US firms

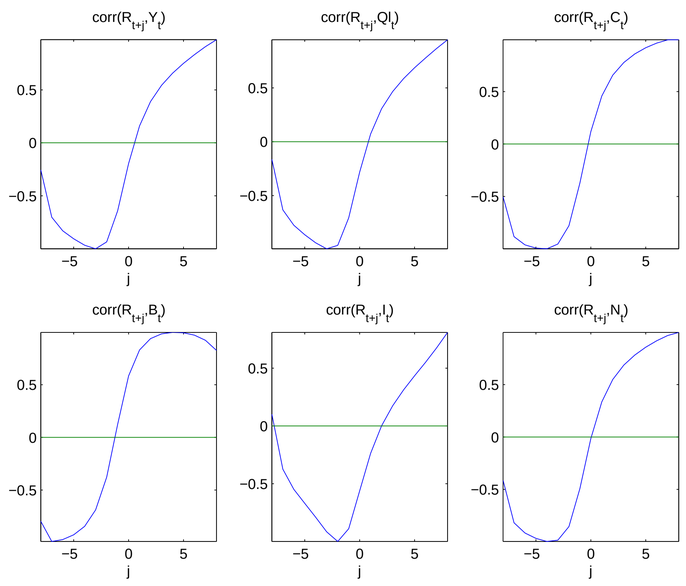
- Interest rate at which US firms borrow has two features: (i) countercyclical; (ii) an inverted leading indicator: low rate forecasts future booms.
- Feature (ii) is a long-standing puzzle (King-Watson, 1996)
  - 1-sector RBC model at odds with (i) and (ii);
  - 2-sector RBC model: Boldrin-Christiano-Fisher (2001).

## Empirical Evidence

- VAR IRFs with investment first:




- Lead-lag correlations:




## What We Do

Show that a Kiyotaki-Moore model accounts for (i) and (ii), with the key assumption: loan contract with **variable interest rate**, which is prevalent in practice (Vickery, 2008).

## A Model with Analytical Solution

- A risk-neutral representative lender consumes non-nondurable goods $\tilde{C}_t$ and durable land $\tilde{L}_t$

$$\max_{\{\tilde{C}_t,\tilde{L}_{t+1},B^l_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \{\tilde{C}_t + \psi \tilde{L}_t\}$$

$$\text{s.t. } \tilde{C}_t + Q_t(\tilde{L}_{t+1} - \tilde{L}_t) + B^l_{t+1} \leq R_t B^l_t$$

- A representative producer faces linear technology $Y_t = L_t$ and borrowing constraint:

$$\max_{\{C_t,L_{t+1},B^l_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log C_t$$

$$\text{s.t. } C_t + Q_t(L_{t+1} - L_t) + R_t B^l_t \leq B^l_{t+1} + L_t$$

$$\mathbb{E}_t R_{t+1} B^l_{t+1} \leq \theta_t \mathbb{E}_t Q_{t+1} L_{t+1}$$

### *Proposition (Analytical Global Sunspot Equilibria)*

*There exist global sunspot equilibria such that the dynamics of the land stock allocated to borrower follows*

$$L_{t+1} = [1 + Q(1 - \tilde{\beta} R_t)] L_t$$

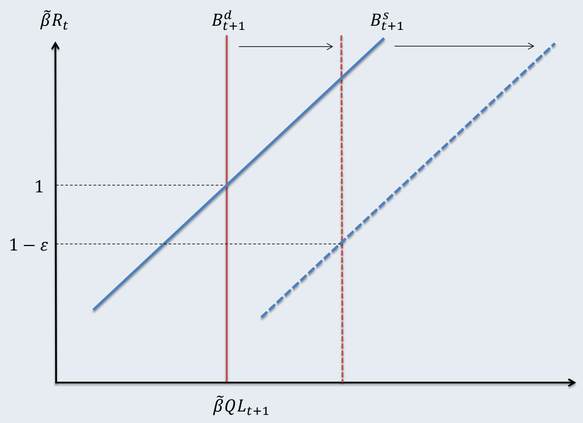
*for all $t \geq 0$, given $L_0 > 0$, where $Q = \beta/(1-\tilde{\beta})$, $R_t = \tilde{\beta}^{-1}(1+\varepsilon_t)$ and sunspot innovation $\varepsilon_t$ is an i.i.d. random variable with zero mean.*

## Variable v.s. Predetermined-rate

- In the variable-rate economy, the real interest $R_{t+1}$ applied to period $t$ loan $B^l_{t+1}$ will be realized in period $t+1$.
- In the predetermined-rate economy the rate applied to $B^l_{t+1}$ is $R_t$, **predetermined** and known in $t$. Such an economy is always in steady state absent fundamental shocks: interest rate fixed at $R_t = \tilde{\beta}^{-1}$.

## Intuition for Indeterminacy with Variable-rate

- Under variable interest rate, credit demand and supply are: $B^d_{t+1} = \tilde{\beta} Q L_{t+1}$, $B^s_{t+1} = Q L_{t+1} - \beta X_t L_t$, where $X_t \equiv 1 + Q(1 - \tilde{\beta} R_t)$.




- Bottom line: sunspot equilibria under variable interest rate because of aggregate credit-demand externality.
- Pecuniary externality does not generate sunspot equilibria.

## A Quantitative Model

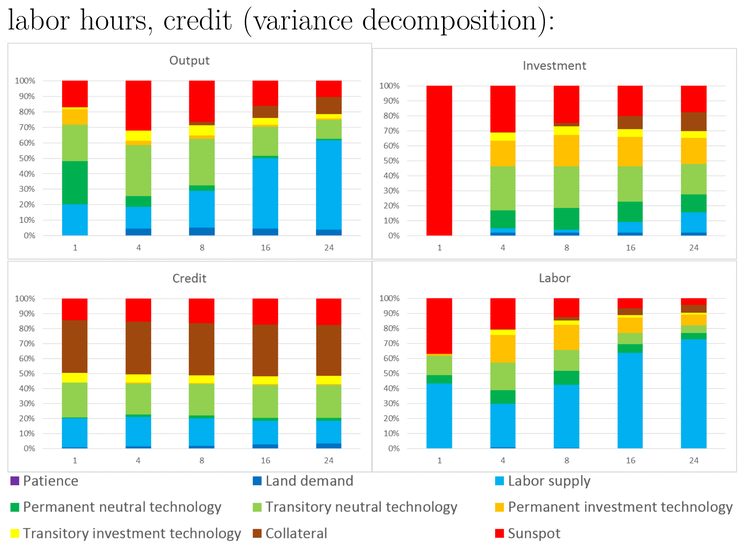
- We introduce variable-rate loans in Liu-Wang-Zha (2013), a model with additional features on top of Pintus-Wen (2013): consumption habits, investment adjustment costs, productivity growth.
- Shocks:
  - discount rate, land demand, labor supply;
  - production technology (transitory and permanent);
  - investment technology (transitory and permanent);
  - collateral (leverage);
- Indeterminacy arises if the fraction of variable-rate loans in the economy $\omega$ is higher than 0.5 (the same rule as in the simple analytical model).

## Bayesian Estimation Strategy

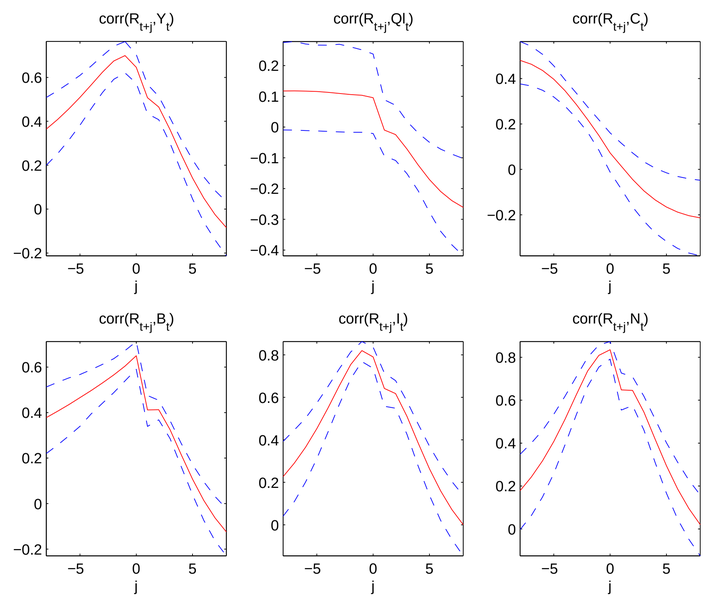
- Estimate the model in both the determinate regime with $\omega \leq 0.5$ and the indeterminate regime with $\omega > 0.5$;
- Use the same US 1975-2010 dataset as LWZ (2013): consumption, investment, land price, hours, debt, (inverse of) investment price.

## Estimation Results

- In the determinate regime, data pushes towards the highest possible value for $\omega$ (that is, 0.5);
- The indeterminate model dominates the determinate model in terms of model fit;
- The indeterminate model with sunspot shocks on investment ("animal spirits")
  - fits the data best (the highest log marg. data density);
  - explains significant share of volatility for output, investment, labor hours, credit (variance decomposition):




- The animal spirits model generates lead-lag correlations (LLCs) consistent with the inverted leading indicator property of the real interest rate, whereas the determinate model does not:
  - The LLCs of the determinate model:




  - The LLCs of the indeterminate (animal spirits) model: