Trading Complex Risks

Felix Fattinger

Brain, Mind and Markets Lab
Department of Finance, University of Melbourne

January, 2018
Where I start from ...

That economic decisions are made without certain knowledge of the consequences is pretty self-evident.

Kenneth J. Arrow
Roadmap

1. What do I mean by ‘complex’ risks?

2. How to derive theoretical predictions?

3. How does the theory hold up against the experimental data?
My Terminology: Simple vs. Complex Risks

- The aim is to study the effects of complexity on the trading and pricing of consumption risk in a *well-defined* environment.

- I therefore rely on the following distinction:
  
  **Simple risks:** Agents possess perfect information about the underlying objective probabilities.
  
  **Complex risks:** Agents only have access to imperfect information about the underlying objective probabilities.

- In the context of complex risks, the quality of agents’ information depends on the cognitive resources at their disposal.
An Example
What is the probability $\pi$ of receiving a dividend $X$ equal to 150?
What is the probability \( \pi \) of receiving a dividend \( X \) equal to 150?
Theory in a Nutshell
(Intuition!)
Agent $i$’s expected utility from consumption depends on $\pi$, $\mu_i$, and $\sigma_i$. 
Agent $i$’s expected utility from consumption depends on $\pi_i$, $\mu_i$, and $\sigma_i$. 

- Dominated ($\mu_i \downarrow$, $\sigma_i \uparrow$)
- Dominated ($\Delta \mu_i = 0$)
- Dominated ($\mu_i \downarrow$, $\sigma_i \uparrow$)

Diagram: Cartesian plane with axes $Q$ and $P$, showing the dominance of different risk scenarios.
Equilibrium for Simple Risks (Benchmark)

In the absence of aggregate risk (if $\exists \hat{Q}$), market completeness implies:

$$P^* = \hat{Q}$$

$$Q^* = \hat{Q}$$

$$P^* = E[X]$$
Trading Complex Risks

If risks are complex, ambiguity-averse agents are more reluctant to bear them.
Trading Complex Risks

If risks are complex, agents likely have different beliefs.
Equilibrium for Complex Risks

If risks are complex, market outcomes are a function of agents' beliefs.
Equilibrium for Complex Risks

If agents are ambiguity-averse, efficient risk sharing prevails under complexity.
RESULTS ON A FIRST GLANCE
The Beauty of Aggregation (for $\hat{Q} = 2$ and $\pi = 1/2$, i.e., $E[X] = 75$)
Aggregate Market Outcomes
Simple vs. Complex Risks (cont’d): Wilcoxon Signed-Rank Test

Supply for $E[X] = 75$

Demand for $E[X] = 75$

Supply for $E[X] = 50$

Demand for $E[X] = 50$
Bootstrapped Equilibrium Distribution (resampling size: 10k)

$E[X] = 75$

$E[X] = 50$

Estimated density

$P^*$

Estimated density

$Q^*$

Estimated density

$Q^*$
I propose the following measure to assess markets’ information aggregation efficiency:

$$Std(P^\star)\text{-Ratio} = \sqrt{\frac{Var(P^\star_c)}{Var(P^\star_s + E^\star_c[X])}}.$$
Individual Behavior
Inconclusive Results

\[ \text{Avg} \left( \Delta Q_i( E_i[X] ) \right) \]

\[ \text{Avg}( |\{Q = \widehat{Q}\}_i| ) \]

- Simple
- Complex

- Simple
- Complex
Reconciling Individual and Aggregate Behavior

- What about complexity induced errors/noise in decision making?

- More severe bounds on rationality than in Biais et al. (2017)?

- Random choices in the spirit of McKelvey and Palfrey (1995, 98)'s quantal response model:

\[
P_i(Q_j | P) = \frac{\psi_i (E_i[U_i(Q_j | P)])}{\Sigma_k \psi_i (E_i[U_i(Q_k | P)])}
\]

- Implications:

1. \( P = E_i[X] \): distribution of \( Q \)s symmetric around \( \hat{Q} \)
2. \( P < E_i[X] \): Distribution of \( Q \)s asymmetric around \( \hat{Q} \) and decreasing above (below) \( \hat{Q} \) for sellers (buyers)
3. \( P > E_i[X] \): Distribution of \( Q \)s asymmetric around \( \hat{Q} \) and decreasing below (above) \( \hat{Q} \) for sellers (buyers)
What about complexity induced errors/noise in decision making?

More severe bounds on rationality than in Biais et al. (2017)?

Random choices in the spirit of McKelvey and Palfrey (1995, 98)'s quantal response model:

\[
P_i(Q_j|P) = \frac{\psi_i(E_i[U_i(Q_j|P)])}{\sum_k \psi_i(E_i[U_i(Q_k|P)])}
\]

Hypotheses:

1. \(\psi_i\) likely to depend on complexity: \(\overline{\psi}_i\) vs. \(\underline{\psi}_i\)

2. \(\overline{\psi}_i(x) > \underline{\psi}_i(x)\) and \(\overline{\psi}_i'(x) > \underline{\psi}_i'(x)\)
Reconciling Individual and Aggregate Behavior: Sellers

\[ Q_i(E_i[X]), \quad E[X] = 75 \]

\[ Q_i(E_i[X]), \quad E[X] = 50 \]
Reconciling Individual and Aggregate Behavior: Sellers (cont’d)
From Unconditional to Conditional Individual Behavior

\[ \text{Avg} \left( |\{Q = \hat{Q}\}_i| \right) \]

\[ \text{Avg} \left( |\{Q = \hat{Q} | Q(E[X]) = \hat{Q}\}_i| \right) \]

Bar charts showing the average values for simple and complex cases.
What do we learn?

- Consistent with decision theory under ambiguity, subjects’ demand and supply curves are less price sensitive for complex relative to simple risks.

- In the presence of complex risks, equilibrium prices are more sensitive whereas risk allocations are less sensitive to subjects’ incorrect beliefs.

- Markets’ effectiveness in aggregating beliefs about complex risks is determined by the trade-off between reduced price sensitivity and reinforced bounded rationality.
Appendix
Now, what is the probability of receiving a dividend equal to 150?

We start with the SDE of the GBM

\[ dS_t = 10\% S_t \, dt + 32\% S_t \, dW_t. \]

Applying Itô to \( f := \ln(S_t) \), we get

\[ S_2 = \exp \left\{ \left( 10\% - \frac{32\%^2}{2} \right) + 32\%(W_2 - W_1) \right\}. \]

Hence,

\[ \mathbb{P}(S_2 \geq 1.05) = \mathbb{P} \left( W_2 - W_1 \leq \left( \ln(1.05) - 10\% + \frac{32\%^2}{2} \right) \frac{1}{32\%} \right). \]

Given the distribution of \( W_2 - W_1 \) (known), we find \( \mathbb{P}(S_2 \geq 1.05) = \frac{1}{2} \).
Agent $i$’s expected utility from consumption is given by

$$E[U_i(C_i(\omega))] = \pi U_i \left( \mu_i + \sqrt{\frac{1-\pi}{\pi}} \sigma_i \right) + (1-\pi) U_i \left( \mu_i - \sqrt{\frac{\pi}{1-\pi}} \sigma_i \right),$$

where $\mu_i \equiv \pi C_i(u) + (1-\pi) C_i(d)$ and $\sigma_i^2 \equiv \pi (1-\pi) (C_i(u) - C_i(d))^2$.

No Aggregate Risk

If there is no aggregate risk, i.e., there exists a tradeable quantity $\hat{Q}$ at which every seller and buyer is perfectly hedged, i.e., $\sigma_i = 0 \ \forall \ i \in I$, then:

For any family of concave utility functions $(U_i)_{i \in I}$, seller $i$’s supply and buyer $j$’s demand curve have the unique intersection point $(E[X], \hat{Q}) \ \forall \ \{i, j\} \subset I$. 

[back]
<table>
<thead>
<tr>
<th>Round</th>
<th>Session 1 (#16)</th>
<th>Session 2 (#18)</th>
<th>Session 3 (#16)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>π Type Pricing</td>
<td>π Type Pricing</td>
<td>π Type Pricing</td>
</tr>
<tr>
<td>1</td>
<td>1 C (P) MC</td>
<td>1 C (P) MC</td>
<td>1 C (P) MC</td>
</tr>
<tr>
<td>2</td>
<td>high C (P) random</td>
<td>high C (P) random</td>
<td>high C (P) random</td>
</tr>
<tr>
<td>3</td>
<td>low C (P) MC</td>
<td>low C (P) MC</td>
<td>low C (P) MC</td>
</tr>
<tr>
<td>4</td>
<td>1/2 C MC</td>
<td>1/3 C random</td>
<td>1/2 C random</td>
</tr>
<tr>
<td>5</td>
<td>1/3 C MC</td>
<td>1/2 C random</td>
<td>1/3 C random</td>
</tr>
<tr>
<td>6</td>
<td>1/2 C random</td>
<td>1/3 C MC</td>
<td>1/2 C random</td>
</tr>
<tr>
<td>7</td>
<td>1/3 C random</td>
<td>1/2 C MC</td>
<td>1/3 C random</td>
</tr>
<tr>
<td>8</td>
<td>1/2 R MC</td>
<td>1/2 R random</td>
<td>1/2 R random</td>
</tr>
<tr>
<td>9</td>
<td>1/3 R random</td>
<td>1/3 R MC</td>
<td>1/3 R random</td>
</tr>
<tr>
<td>10</td>
<td>ambig A MC</td>
<td>ambig A random</td>
<td>ambig A MC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Round</th>
<th>Session 4 (#16)</th>
<th>Session 5 (#16)</th>
<th>Session 6 (#16)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>π Type Pricing</td>
<td>π Type Pricing</td>
<td>π Type Pricing</td>
</tr>
<tr>
<td>1</td>
<td>1/2 R (P) MC</td>
<td>1/2 R (P) MC</td>
<td>1/2 R (P) MC</td>
</tr>
<tr>
<td>2</td>
<td>9/10 R (P) random</td>
<td>9/10 R (P) random</td>
<td>9/10 R (P) random</td>
</tr>
<tr>
<td>3</td>
<td>1/2 R MC</td>
<td>1/2 R random</td>
<td>1/2 R random</td>
</tr>
<tr>
<td>4</td>
<td>1/3 R random</td>
<td>1/3 R MC</td>
<td>1/3 R random</td>
</tr>
<tr>
<td>5</td>
<td>high C (P) MC</td>
<td>high C (P) MC</td>
<td>high C (P) MC</td>
</tr>
<tr>
<td>6</td>
<td>1/2 C MC</td>
<td>1/3 C random</td>
<td>1/3 C random</td>
</tr>
<tr>
<td>7</td>
<td>1/3 C MC</td>
<td>1/2 C random</td>
<td>1/2 C random</td>
</tr>
<tr>
<td>8</td>
<td>1/2 C random</td>
<td>1/3 C MC</td>
<td>1/2 C random</td>
</tr>
<tr>
<td>9</td>
<td>1/3 C random</td>
<td>1/2 C MC</td>
<td>1/2 C random</td>
</tr>
<tr>
<td>10</td>
<td>ambig A MC</td>
<td>ambig A random</td>
<td>ambig A MC</td>
</tr>
</tbody>
</table>
Price-taking Behavior under Complex Risks?

Supply for $E[X] = 75$

Supply for $E[X] = 50$

Demand for $E[X] = 75$

Demand for $E[X] = 50$
Price-taking Behavior (cont’d): Wilcoxon Signed-Rank Test

Supply for $E[X] = 75$

Demand for $E[X] = 75$

Supply for $E[X] = 50$

Demand for $E[X] = 50$