



# Is the U.S. Phillips Curve Convex? Some Metro Level Evidence\*

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\*The views expressed are my own, and not those of the Federal Reserve Bank of Dallas or of the Federal Reserve System

## Abstract

Interest in the possible convexity of inflation Phillips Curve waxes and wanes as the economy nears or moves away from full employment. If the Phillips Curve is convex then, as unemployment falls below the natural rate of unemployment or NAIRU, the upward pressure on inflation rises increasingly. If the Phillips Curve is convex and the unemployment rate is close to the NAIRU, monetary policy makers should act pre-emptively and raise rates sooner rather than later, ceteris paribus.

It is difficult to identify convexity using aggregate data covering the Great Moderation period, so I exploit the greater variation in inflation at the metro level. I estimate linear and non-linear, inflation expectations augmented Phillips Curves using core CPI inflation and unemployment data from the mid-1980's for a panel of 27 large U.S. metros. Inter alia, heterogeneous dynamic panel data models with multiple unobserved common factors are estimated.

Labor market slack is always significant, even though the fit of linear and non-linear Phillips curves is similar. Simulations of a simple, three equation IS-PC-MR suggest that the degree of non-linearity is modest. For example, a short-term shock that reduces the unemployment rate by one percentage point might boost core CPI inflation by 30 basis points (bps) in a linear model and less than 40 bps in a non-linear mode. If inflation expectations adjust modestly, the effects might be 15 bps higher.

## Introduction

Evidence for the convexity of the Phillips Curve is rather mixed. Most studies find a convex wage inflation Phillips curve, but not a convex price inflation Phillips Curve (Table 1). Aggregate time series data may not be sufficiently informative, especially when the Central Bank successfully targets inflation. Regional or metro data may be more informative.

Table 1 - Recent Studies of U.S. Phillips Curve

Study	Data and Sample	Main Model	Evidence of Convexity
Laxton, Rose & Tambakis (1999)	Quarterly CPI inflation, 1968 Q1 to 1997 Q1	Two equation model; PC with time varying coefficient on convex unemployment gap term and random walk NRU	<b>Weak</b> - convex PC, but fit only marginally better than for linear model
Ball & Mazumder (2011)	Quarterly data, y/y headline and core (median) CPI, 1960Q1	Linear model where slope of PC varies with level and/or variance of inflation	<b>Mixed</b> - prefer model with varying slope to convex PC model; fit of linear and convex PC models similar
Nalewaik (2016)	Annual data, core PCE inflation and growth in non-farm business sector hourly compensation, 1961-2015	Two equation, two regime Markov Switching model with squared low unemployment rate term; one regime is non-stationary	<b>Strong</b> - convex PC
Albuquerque, Baumann (2017)	Quarterly data, y/y PCE inflation, 1992q1-2015q1	Time varying parameter model using unemployment gap and labor market tracking index etc.	<b>Weak</b> - prefer time varying parameter to convex PC model; fit of linear and convex PC models similar.
Detmeister, Babb (2017)	Annual metro data, core CPI inflation, 1984 to 2016	Fixed effects panel model	<b>Weak</b> - some convexity but not economically significant

## Data and Models

Identify effect of labor market slack on inflation using time series and cross-section variation in unemployment rates and core CPI inflation at the metro level.

- $\tilde{\pi}_{m,t}^{core} = \pi_{m,t}^{core} - \pi_{t,10y}^{core}$  = Deviation of core year-on-year CPI inflation in metro  $m$  from long-term expected inflation in the Survey of Professional Forecasters (SPF).
- $ugap_{m,t} = u_{m,t} - u_t^{NRU}$  = Unemployment gap, the deviation from the CBO's natural rate of unemployment (NAIRU).
- $ugap_{m,t}^{neg} = \min(0, ugap_{m,t})$  = Negative unemployment gap (tight labor market)..

The models are formulated in terms of the deviations of inflation from survey based, long run expected inflation and the deviation of the unemployment rate from the NAIRU.

Models with linear, linear spline and convex labor market slack effects estimate. The base linear spline model is:

$$\tilde{\pi}_{m,t}^{core} = \beta_0 + \beta_1 \tilde{\pi}_{m,t-1}^{core} + \beta_2 \tilde{\pi}_{m,t-2}^{core} + \beta_3 ugap_{m,t-2} + \beta_4 ugap_{m,t-2}^{neg} + \beta_5 \Delta u_{m,t-2} + v_{m,t}$$

This is an expectations augmented Phillips Curve, not New Keynesian Phillips Curve, with priors:  $\beta_3 < 0, \beta_4 < 0$  (since  $ugap_{m,t}^{neg} < 0$ ) and  $\beta_5 < 0$ . The data are I(0), and the choice of lags is based on limited pre-searching. Other non-linear specifications for the effect of slack use  $ugap_{m,t}$  and  $ugap_{m,t}^{neg}$  squared as in Nalewaik (2016),  $ugap_{m,t}/u_{m,t}$  as in Debelle and Vickery (1998), or log slack,  $\ln(u_{m,t}/u_t^{NRU})$ .

The Phillips Curves are estimated using pooled OLS, one and two-way fixed effects and dynamic common correlated effects (DCCE) estimators. The DCCE estimator (Chudik and Pesaran, 2015) is the most general one and has many advantages:

$$\tilde{\pi}_{m,t}^{core} = \beta_{m,0} + \beta_{m,1} \tilde{\pi}_{m,t-1}^{core} + \beta_{m,2} \tilde{\pi}_{m,t-2}^{core} + \beta_{m,3} (ugap_{m,t-2}/u_{m,t-2}) + \sum_j \gamma_{mj} f_{j,t} + v_{m,t}$$

It provides consistent estimates of the mean effects in dynamic, heterogeneous panel data models with weakly exogenous variables and cross section dependence. The cross section dependence is modelled in a flexible way (as unobserved factors  $f_{j,t}$ ), which are "partialled out" by adding current and lagged cross section averages of the dependent regressors and other related covariates to the individual equations.

## Results

Some representative regression results are set out in Table 2. Consider the linear spline results initially. The pooled OLS and FE results are very similar - inflation is highly persistent; lagged labor market slack and changes in slack are economically and statistically significant. The linear spline term in lagged slack significant suggesting that the Phillips Curve is convex.

However, the pooled OLS and FE results do not account of any common omitted factors, such as imported core goods inflation, driving metro-level inflation. The DCCE results, which do, are rather different. Inflation is not as persistent and lagged labor market slack, but not the lagged change in slack, is significant. The spline term is insignificant, which suggests that the Phillips Curve is linear. Other convex specifications need to be examined before reaching this conclusion. The fit of the two convex models is about the same as that of the linear / linear spline models.

Similar results are obtained using quarterly data for approx. 13 metros and in sub-samples. Lagged labor market slack is always economically and statistically significant. The linear spline term is insignificant in the DCCE results. The convex Phillips Curve models fit marginally better. The effects of slack are fairly stable in the sub-samples. Changes in lagged slack are also statistically significant in the quarterly models, but are hard to identify in the sub-samples. Results hold up to various robustness checks - breaks in CPS-based unemployment series, threshold effects, alternative measures of expected inflation etc.

Table 2: Linear Spline and Convex Phillips Curve Specifications

Dependent Variable =  $\tilde{\pi}_{m,t}^{core}$ ; Sample: 24 to 27 Metros, 1985 or 1986 H1 to 2016 H2 (Semi-Annual).

Regressors	Linear Spline in $u_{m,t}$			Slack = $ugap_{m,t}/u_{m,t}$			Slack = $\ln(u_{m,t}/u_t^{NRU})$		
	OLS	FE	DCCE	OLS	FE	DCCE	OLS	FE	DCCE
$\tilde{\pi}_{m,t-1}^{core}$	0.867*** (0.027)	0.836*** (0.025)	0.670*** (0.032)	0.866*** (0.027)	0.835*** (0.025)	0.679*** (0.033)	0.872*** (0.028)	0.843*** (0.025)	0.672*** (0.032)
$\tilde{\pi}_{m,t-2}^{core}$	-0.244*** (0.023)	-0.256*** (0.022)	-0.331*** (0.030)	-0.244*** (0.024)	-0.256*** (0.022)	-0.334*** (0.024)	-0.241*** (0.024)	-0.252*** (0.023)	-0.336*** (0.027)
$ugap_{m,t-2}$	-0.017 (0.012)	-0.032** (0.012)	-0.319*** (0.053)	-	-	-	-	-	-
$ugap_{m,t-2}^{neg}$	-0.195*** (0.036)	-0.235*** (0.039)	-	-	-	-	-	-	-
$\Delta u_{m,t-2}$	-0.172*** (0.029)	-0.163*** (0.021)	-	-0.170*** (0.030)	-0.160*** (0.021)	-	-	-	-
$ugap_{m,t-2}/u_{m,t-2}$	-	-	-	-0.471*** (0.069)	-0.633*** (0.064)	-1.587*** (0.275)	-	-	-
$\ln(u_{m,t-2}/u_t^{NRU})$	-	-	-	-	-	-	-0.423*** (0.072)	-0.564*** (0.065)	-1.763*** (0.275)
$\Delta \ln u_{m,t-2}$	-	-	-	-	-	-	-1.093*** (0.206)	-1.027*** (0.159)	-
Metro Fixed Effects	-	Yes	-	-	Yes	-	-	Yes	-
Adjusted R <sup>2</sup>	0.621	0.609	0.620	0.620	0.607	0.607	0.618	0.604	0.615
SE	0.671	0.661	0.607	0.672	0.662	0.612	0.674	0.665	0.608
No of Observations	1679	1679	1599	1679	1679	1599	1679	1679	1599

Notes: Standard errors are shown in parentheses. The superscripts \*, \*\* and \*\*\* denote significance at the 10%, 5% and 1% levels respectively. FE denotes fixed effects estimators. The DCCE estimates use three lags of the cross section averages.

## Discussion – Does Convexity Matter?

Two ways of assessing the importance of convexity of the Phillips Curve are considered. First, are estimated linear and convex Phillips Curves very far apart when slack is negative (the unemployment rate is below the NAIRU)? No, the linear and convex Phillips Curves are close when slack is the historically relevant range, i.e. -0% to -2% in the metro panel, and 0% to -1% at the aggregate level (Figure 1).

Second, do the results of simulating an exogenous fall in lack in a simple, three equation IS-PC-MR model differ significantly when the Phillips Curve is linear vs. when it is convex? The dynamics of the IS curve are based on estimates from before the Great Recession. The Phillips curve are based on the quarterly linear and convex (slack =  $ugap_{m,t}/u_{m,t}$ ) DCCE estimates. An inertial Taylor Rule is used, with inertia coefficient of 0.85 and equal weights on the deviation of inflation from target and the unemployment gap. The NAIRU and inflation target are assumed to be fixed, and inflation expectations are either constant or slowly adjusting.

In the simulations, a short-term shock that reduces the unemployment rate by one percentage point might boost core CPI inflation by 30 basis points (bps) when the Phillips Curve is linear, and less than 40 bps when it is convex (Figure 2). If inflation expectations adjust modestly, the effects might be 15 bps higher. Similar results hold in more elaborate models.

Figure 1: Is the Convexity of the Phillips Curve Important?

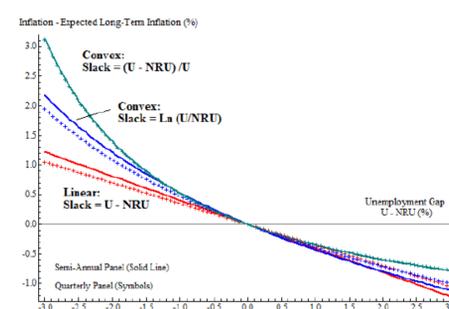
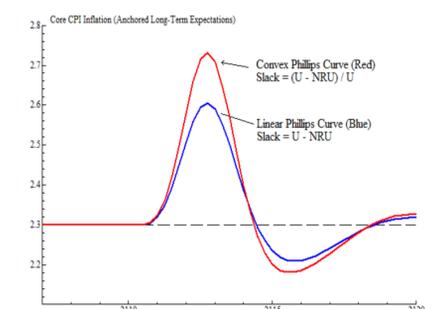


Figure 2: Time Path of Inflation – Linear (Blue) and Convex (Red) Phillips Curves



## Conclusion

The degree of convexity in the price Phillips Curve appears to be relatively small, and not economically significant. Labor market slack is always economically and statistically significant, and there is no compelling evidence that the effect of slack has significantly declined over time in the metro-level dataset used here. Although the fit of convex Phillips Curves is somewhat better than the fit of linear curves, the degree of convexity is modest.