Estimating and Accounting for the Output Gap with Large Bayesian Vector Autoregressions

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The view do not necessarily represent those of the Reserve Bank of New Zealand

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Most T-C methods are univariate (e.g. HP filter, Bandpass filter, Watson (1986) UC model etc)

Beveridge-Nelson (BN) decomposition is a natural way to incorporate multivariate information (e.g. Evans and Reichlin, 1994)

\[
\tau_t = \lim_{j \to \infty} E_t [y_{t+j} - j \cdot E [\Delta y]]
\]
Estimated U.S. Output Gap from Univariate and Multivariate BN Decompositions (% Dev from trend)

2 variable VAR includes output growth and the unemployment rate. 3 variable VAR includes output growth, CPI inflation, and the federal funds rate. 7 variable VAR includes all of the variables in the 2 and 3 variable systems, as well as capacity utilization, the growth of industrial production, and the growth of real personal consumption expenditure.
Punchlines

Contribution

1. Show how to incorporate multivariate information into trend-cycle decomposition
   - Requires only large standard BVARs ala “Minnesota with a twist”
2. Show how to interpret trend-cycle decomposition through the included multivariate information

Main Findings

- BVARs with up to 138 variables produce plausible/intuitive estimates of the U.S. output gap
- Unemployment rate, CPI, housing starts, consumption, stock prices, real M1, and federal funds rate are key informational variables
- Estimates largely robust to including additional variables
- Monetary policy shocks play little role in the output gap, while oil price shocks explain about 10% of variance over different horizons
“Minnesota with a Twist”

Standard BVAR

\[
\mathbb{E}[\beta_{ij}^2] = 0
\]

\[
\nabla[\beta_{ij}^2] = \begin{cases} \frac{\lambda^2}{I^2}, & i = j \\ \frac{\lambda^2}{I^2} \frac{\sigma_i^2}{\sigma_j^2}, & \text{otherwise} \end{cases}
\]

“Twist” (Kamber, Morley & Wong, forthcoming, REStat)

Output is \(s^{th}\) equation

\[
\mathbb{E}\left[ \sum_{l=1}^{p} \beta_{ls}^2 \right] = \rho(\delta)
\]

\[
\nabla\left[ \sum_{l=1}^{p} \beta_{ls}^2 \right] = \left( \frac{\lambda}{10} \right)^2
\]

- One hyperparameter: \(\lambda\)
  - We want \(\lambda \rightarrow 0\) (i.e., more shrinkage) as more series are added in
  - We optimize \(\lambda\) based on out of sample RMSE

Key Advantage

- No need for MCMC simulation of posterior
- Analytical. Trivially implemented using dummy observations
U.S. Output Gap (BN Filter aka Wellington Prior), $\bar{\delta} = 0.25$ (Kamber, Morley & Wong, REStat, forthcoming)
Data

Benchmark model includes output growth (target variable) + 22 variables (taking logs as appropriate and differencing until stationary):

1. Oil Prices
2. CPI inflation
3. Unemployment Rate
4. Hourly Earnings
5. Federal Funds Rate
6. Stock Price Index
7. Yield Spread
8. GDP Deflator
9. Employment
10. Income
11. Real PCE
12. Industrial Production
13. Capacity Utilization
14. Housing Starts
15. PPI (all commodities)
16. PCE Deflator
17. Hours
18. Productivity
19. Total Reserves
20. Non Borrowed Reserves
21. Real M1
22. Real M2
U.S. Output Gap (Benchmark Model, % Dev from trend)
Trend and Cycle can be written as a linear decomposition of all the historical forecast errors

Consider companion form of VAR(p) forecasting model:

\[(\Delta x_t - \mu) = F(\Delta x_{t-1} - \mu) + H \nu_t\]

Let \(\Gamma = F^i(I - F)^{-1}\), BN decomposition implies

\[c_t \approx \sum_{i=0}^{t-1} \Gamma i+1 H \nu_{t-i} - \mu + \Gamma_0 H \nu_t.\]

Two Decompositions

1. Sources of information
   - Which variables contain the most information for estimating trend and cycle?
   - Which variables should be included in forecasting model?

2. Role of Structural Shocks
   - Given forecast errors and identification restrictions, SVAR analysis straightforward
   - What drives the trend and cycle?
Historical Decomposition of Role of Forecast Errors (Benchmark Model)
Historical Decomposition of Role of Forecast Errors (Benchmark Model)
Historical Decomposition of Role of Forecast Errors (Benchmark Model)
Standard Deviations of Informational Contributions

- Oil Price
- GDP growth
- PCE
- Income
- IP
- CAPU
- Employment
- Unemployment

- Hours
- Housing Starts
- PCE inflation
- GDP Deflator
- CPI
- PPI
- Hourly Earnings
- Productivity

- Fed Funds Rate
- Yield Curve
- Real M1
- Real M2
- TR
- NBR
- Stock Price
Varying the Information Set (% Dev from trend)
Omitting Important Information (% Dev from trend)
Out of Sample RMSE (one-step ahead, real GDP growth)
Causal Determinants of Output Gap and Trend Growth

- We identify two shocks using standard timing restrictions
  - An oil price shock
  - A monetary policy shock
- Then we consider a forecast error variance decomposition (FEVD) and a historical decomposition
Variance Shares (%)
Historical Decomposition (% Dev from trend)
Summary

- Bayesian shrinkage makes application of BN decomposition with large information sets feasible and avoids overfitting.
- Movements in trend and cycle can be accounted for based on different sources of information or structural shocks.
- When estimating the U.S. output gap, it is more important to include key variables than to consider a really large information set (i.e., unemployment).
Other Applications and Extensions

Work-in-Progress

- Global Influences of Trend Inflation (Kamber and Wong, 2018, BIS working paper)
- Role of foreign shocks in driving output gap and trend growth for open economies (Morley, Vehbi, and Wong, in progress)

Pipeline

- Mixed frequency modeling
- Multiple target variables—neutral rates
- Financial cycles
Canada Trend Inflation (Kamber and Wong, 2018, BIS WP)
Decompose Trend Inflation and Inflation Gap

Source: Kamber and Wong (2018)
Share of Foreign Shocks (%) (Kamber and Wong, 2018, BIS WP)
Canadian Output Gap (Morley, Vehbi, and Wong)
Historical Decomposition of the Canadian Output Gap (Morley, Vehbi, and Wong)
Historical Decomposition of Canadian Trend Growth (YoY) (Morley, Vehbi, and Wong)
U.S. Output Gap (Benchmark Model)
Additional Slides
Why is estimated output gap deeper in 1982 than in 2009?