Multi-Dimensional Pass-Through, Incidence, and the Welfare Burden of Taxation and Other External Changes in Oligopoly

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September 27, 2017
Firms are faced with a complex set of policy interventions and other external changes. Naturally, it is **multi-dimensional**.

Example: Consumers pay unit and value-added taxes, $t \geq 0$ and $v \in [0, 1)$. Firm’s profit is:

$$\pi = [(1 - v)p - t]q - c(q) = [pq - c(q)] - [tq + vpq]$$

Now, the government plans to change the value-added part: $s > 0$ fraction of $c(q)$ is tax deductible.

$$\pi = [pq - c(q)] - [tq + v\{pq - s \cdot c(q)\}]$$

$$\equiv \phi(p,q,T)$$: additional cost

“What are the effects of introducing $s$, **when $t > 0$ and $v > 0$ are already implemented**?”

Intervention vector, $T = (t, v, s)$, is **three-dimensional**.
Research Question (1/2)

- Better not to specify a particular type of competition:
  
  Many industries are *oligopolistic*, more or less

  However,

  Quantity- or Price-Setting?

  Collusive to some degree?

- “How can one evaluate the welfare consequences of a change in such multi-dimensional environments, taking into account *imperfect competition*?”
Research Question (2/2)

- We provide **general formulas** for welfare evaluation in consideration of **multi-dimensionality** and **oligopoly**.

  In this way, we generalize Weyl and Fabinger’s (2013) analysis of **single-dimensional pass-through**, to include **multi-dimensional pass-through**.

  Our analysis is flexible in the sense that the degree of competition is captured by a single variable, **conduct index**, $\theta \in [0, 1]$.

  We mainly work on imperfect competition with a fixed number of firms. However, in the paper, we also allow **free entry** to endogenize the number of firms (**monopolistic competition**).
Importance of Pass-Through

- **Pass-Through**: How final prices are affected by exogenous changes to firms, $\frac{dp}{dT}$
  - Tax scheme
  - Emission regulations (additional cost)
  - Change in exchange rate

- “Pass-Through Renaissance,” initiated by Weyl and Fabinger (2013)
  - It has increasingly been recognized as an important measure for welfare evaluation.
  - Clear and tractable both in theory and empirics.
Generality of Our Framework

- Our framework can be used to study policy issues under **imperfect competition** in such fields as (but not limited to):
  - Industrial Organization
  - Public Economics
  - International Economics
  - Agricultural Economics
  - Environmental/Energy Economics
  - Macroeconomics
  
  ....
What We Do (1/2)

- We mainly work on **two-dimensional** taxation under symmetric oligopoly, \( T \in \{ t, v \} \):

  \( \text{1) Unit Tax, } t \geq 0 \quad \text{2) Ad Valorem Tax, } v \in [0, 1], \)

  - Firm \( i \)'s profit: \( \pi_i = [p_i q_i - c(q_i)] - [tq_i + vp_i q_i] \)

  (Per-firm) tax revenue: \( R(q) \equiv t q + v p q \)

  to characterize

  (i) Unit-tax **pass-through**: \( \rho_t \equiv \frac{\partial p}{\partial t} \), where \( T = t \)

  and two welfare measures:

  (ii) Tax **incidence**: \( l_T \equiv \frac{\partial CS}{\partial PS} \frac{\partial PS}{\partial T} \) for \( T = t \)

  (iii) Marginal Cost of Public Funds ("welfare burden"):

  \( MCPF_T \equiv -\frac{\partial W}{\partial R} \frac{\partial R}{\partial T} \) for \( T = t \)

- Results for ad valorem tax \( (T = v) \) are analogous.
We then generalize our two-dimensional results under symmetric oligopoly to include:

Multi-Dimensionality

Asymmetric Firms

Taxation and Other External Changes

Firm-Specific Taxation/Changes
Related Literature
Relation to the Literature: Theory

- This paper is a generalization of

  (1) Weyl and Fabinger (2013, *JPE*)
  (2) Häckner and Herzing (2016, *JET*)

<table>
<thead>
<tr>
<th></th>
<th>Tax Scheme</th>
<th>Initial Tax Level</th>
<th>Model</th>
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<tbody>
<tr>
<td>WF ('13)</td>
<td>Unit Tax: $t$</td>
<td>$t = 0$</td>
<td>General</td>
</tr>
<tr>
<td>HH ('16)</td>
<td>Unit Tax: $t$ Ad Valorem Tax: $v$</td>
<td>$(t, v) = (0, 0)$</td>
<td>Linear Demands Constant MC</td>
</tr>
<tr>
<td>This Paper</td>
<td>Multi-Dimensional</td>
<td>Non-Zero</td>
<td>General</td>
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</table>
Relation to the Literature: Empirics

- Our framework is in line with the “sufficient-statistics” approach (Chetty, 2009).

   (1) **Price elasticities** (own and cross; $\epsilon$)

   (2) **Conduct** ($\theta$)
       Estimable if the mode of competition is specified

   (3) **Pass-through** ($\rho$)
       Directly estimated if variation on the cost side is observed, or
       Indirectly estimated from the estimated $\epsilon$ and the demand **curvatures**, and 1st- and 2nd-order cost characteristics (using our formulas)

For example, Atkin and Donaldson (2016) study the welfare implications of changes in intra-national trade costs. See also Miller, Osborne, and Sheu (2017, *RAND*).
Quick Preview
Perfect Competition

- **(i) Pass-through:** $\rho_t = \frac{1}{1 + \frac{\epsilon^D}{\epsilon^S}}$, where $\epsilon^D \equiv -\frac{D'p}{Q}$ and $\epsilon^S \equiv \frac{S'p}{Q}$ are the **elasticities** of demand and supply.

- **(ii) Incidence:** Tax Burden is devided into:

  \[ \Delta t = \Delta p + (\Delta t - \Delta p) \]

  \[ \Leftrightarrow 1 = \rho_t + (1 - \rho_t) \]

  - for **consumers**
  - for **producers**
Oligopoly

- **Conduct Index** is implicitly defined from FOC:

\[
\theta(q) \equiv \frac{\epsilon^D(q)}{p(q)} \left[ p(q) - \frac{t + mc(q)}{1 - \nu} \right] \in [0, 1],
\]

where \( mc(q) \equiv c'(q) \) is the marginal cost

- Special cases:
  
  Perfect competition: \( \theta = 0 \)
  
  Monopoly: \( \theta = 1 \)
  
  Cournot oligopoly with \( n \) firms: \( \theta = 1/n \)
Comment on Multi-Product Oligopoly

- We mainly work on the case of single-product oligopoly. However, our analysis can be extended to the case of multi-product oligopoly with some more notations (see the paper’s appendix).
Marginal Cost of Public Funds: Monopoly

- Marginal Cost of Public Funds (MCPF):

\[
MCPF_t \equiv \frac{-\Delta W}{\Delta R} = \frac{-p'q \cdot dq}{q \cdot dt} = \frac{p'q \cdot dp/p'}{q \cdot dt} = \frac{dp}{dt} \equiv \rho_t
\]
Marginal Cost of Public Funds: Oligopoly

- MCPF (with initial $t = 0$; HH 2016):

$$MCPF_t \equiv \frac{\Delta W}{\Delta R} = \frac{\theta p' q \cdot dq}{q \cdot dt} = \frac{\theta p' q \cdot dp}{p' q \cdot dt} = \frac{\theta dp}{dt} \equiv \theta \rho_t$$
Comparison in the Two-Dimensional Case

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<tr>
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<th>Perfect, and ( (t, v) = (0, 0) )</th>
<th>Imperfect, and ( (t, v) \geq (0, 0) )</th>
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<tr>
<td>(i) Pass-Through</td>
<td>[ \frac{1}{1 + \frac{\epsilon D}{\epsilon S}} ]</td>
<td>[ \frac{1}{1 - v} \left( 1 + \frac{1 - \tau}{\epsilon D} \right) + \epsilon D q \frac{\partial (\theta/\epsilon D)}{\partial q} ]</td>
</tr>
<tr>
<td>(ii) Incidence</td>
<td>[ \frac{1}{\rho_t - 1} ]</td>
<td>[ \frac{1}{\rho_t - (1 - v)(1 - \theta)} ]</td>
</tr>
<tr>
<td>(iii) MCPF</td>
<td>0</td>
<td>[ \frac{(1 - v)\theta}{\epsilon D} + \tau ]</td>
</tr>
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</table>

where

\[ \tau(q) \equiv \frac{R(q)}{p(q)q} = \frac{t}{p(q)} + v \]

is the fraction of the gov’s (per-firm) revenue to firm’s pre-tax revenue.
(i) Pass-Through
Pass-Through

Proposition

Pass-through, $\rho_t$, is characterized by:

$$\rho_t = \frac{1}{1 - v} \cdot \left[ 1 + \frac{1 - \tau}{1 - v} \frac{\epsilon_D}{\epsilon_S} \right] + \left[ -\left( \frac{1}{\epsilon_D} + \frac{1}{\epsilon_S} \right) \theta \right] + \left[ -\epsilon_D q \frac{\partial(-\theta/\epsilon_D)}{\partial q} \right].$$

Additional two terms from oligopoly:

- **Direct** effect: $\rho_t$ becomes *larger* if the demand becomes *inelastic* (i.e., $\frac{1}{\epsilon_D}$ becomes larger), propagated by $\theta$.

- **Indirect** effect: (i) Suppose $\epsilon_D$ is close to a constant. Then, $\rho_t$ becomes *smaller* if $\left( -\frac{\partial q}{\partial \theta} \right)$ is *larger*. This is the case of *greater distortion*. Similar in the case of $\theta$ being close to a constant.
Relation to Weyl-Fabinger (2013)

- The original single pass-through formula by WF (2013) is:

$$\rho_t = \frac{1}{1 + \frac{\epsilon^D - \theta}{\epsilon^S} + \frac{\theta}{\epsilon^\theta} + \frac{\theta}{\epsilon^{ms}}}$$

where $\epsilon^\theta \equiv \theta / [q \cdot (\theta)']$ and $\epsilon^{ms} \equiv ms / [q \cdot (ms)']$ are the quantity elasticities of the conduct index, and the marginal consumer’s surplus, respectively.

- If two-dimensional $(t, v) \geq (0, 0)$ is considered, their formula is reformulated as:

$$\rho_t = \frac{1}{1 - v} \cdot \frac{1}{1 + \frac{1 - \tau}{1 - v} \frac{\epsilon^D - \theta}{\epsilon^S} + \frac{\theta}{\epsilon^\theta} + \frac{\theta}{\epsilon^{ms}}}$$

which is equivalent to our formula.
(ii) Incidence
Proposition

*Incidence of unit tax,*

\[
\frac{1}{l_t} = \frac{\partial PS}{\partial t} = \frac{\partial CS}{\partial t}.
\]

is characterized by:

\[
\frac{1}{l_t} = \frac{1}{\rho_t} - (1 - \nu)(1 - \theta).
\]
The effects of an increase in unit tax \( dt \) on the producer surplus can be decomposed into the following five parts:

\[
dPS = [(-qdt) + (1 - v)pdq] + [(1 - v)qdp + (-mcdq) + (-tdq)]
\]

(1) (Direct) **loss** from an increase in unit tax; the tax increase multiplied by output \( q \)

(2) (Indirect) **loss** from a reduction in production; multiplied by the ad valorem tax adjusted unit price \((1 - v)p\)

(3) (Direct) **gain** from the associated price increase, mitigated by \((1 - v)\) due to the ad valorem tax, multiplied by \( q \)

(4) (Indirect) **gain** from cost saving by output reduction \( dq \)

(5) (Indirect) **gain** from unit tax saving by \( dq \)
Proof (2/3)

• By rewriting:

\[ dPS = \left[ -qdt + (1 - v)qdp \right] + \left[ (1 - v)p - \left( mc + t \right) \right] dq \]

(1) \(<0 \quad (3) >0 \quad \text{Marginal Cost} \]

• Now, in symmetric equilibrium, the marginal cost \( mc + t \) is equal to the marginal revenue \( (1 - v)p(1 - \frac{\theta}{\epsilon D}) \), which implies

\[ dPS = \left[ -qdt + (1 - v)qdp \right] + \left[ (1 - v)p \right] \left( \frac{\theta}{\epsilon D} \right) dq. \]

(1) \(<0 \quad (3) >0 \]

• Under perfect competition: \((2) = (4)+(5)\), and only (1) and (3) survive.
Proof (3/3)

- However, under imperfect competition, the marginal cost is less than \((1 - \nu) p\): \((2) > (4) + (5)\). The third term expresses this difference \((2) - [(4) + (5)]\).

- Now recall: \(dp = \rho_t dt\) and \(\left(\frac{P}{\epsilon_D}\right) dq = -qdp = -q\rho_t dt\). Thus,

\[
dPS = \left[ -qdt + (1 - \nu)qdp \right] + \left[ (1 - \nu)p \right] \left( \frac{\theta}{\epsilon_D} \right) dq
\]

\[
= \left[ -qdt + (1 - \nu)q\rho_t dt \right] - (1 - \nu)q\theta\rho_t dt
\]

\[
= \left[ -1 + (1 - \nu)\rho_t - (1 - \nu)\theta\rho_t \right] qdt
\]

\[
= \left[ -1 + (1 - \nu)(1 - \theta)\rho_t \right] qdt
\]

\[
(1) < 0 \quad (3) \geq 0
\]
Comment on $dPS$

- On the other hand, $dCS = -\rho_t(qdt)$. Thus, while it is always the case that $dCS < 0$, it is possible that $dPS > 0$.

- Finally,

$$\frac{1}{l_t} \equiv \frac{dPS}{dCS} = \frac{-1 + (1 - \nu)(1 - \theta)\rho_t}{-\rho_t} = \frac{1}{\rho_t} - (1 - \nu)(1 - \theta)$$
(iii) Marginal Cost of Public Funds (MCPF)
Marginal Cost of Public Funds

Proposition

Define the marginal welfare cost of raising the government’s revenue by unit tax \( t \), \( \text{MCPF}_t \), by:

\[
\text{MCPF}_t \equiv -\frac{\partial W}{\partial t} \frac{\partial R}{\partial t}.
\]

Then, it is characterized by:

\[
\text{MCPF}_t = \frac{(1-v)\theta}{e^D} + \tau + \tau,
\]

where \( \tau(q) \equiv R(q)/[p(q)q] = t/p(q) + v \) is the fraction of the government’s per-firm revenue to the firm’s pre-tax revenue.
Proof (1/4)

- Under oligopoly, the effects of an increase in unit tax $dt$ on social welfare is written as: $dW = (p - mc) dq$.

- Thus, $(p - mc)$ serves as a measure for welfare change.

- It is decomposed into two parts:

  (1) Surplus from imperfect competition: $\frac{(1-v)p\theta}{\epsilon D}$

  (2) Tax payment: $t + vp$

- Thus,

  $$MCPF_t = \frac{-dW}{dR} = \frac{-p[\frac{(1-v)\theta}{\epsilon D} + \left(\frac{t}{p} + v\right)] dq}{dR} \equiv \tau$$
Next, the effects of an increase in unit tax $dt$ on the tax revenue are:

$$dR = qdt + vqdp + (t + vp)dq$$

(1) (Direct) **gain**, multiplied by the output $q$
(2) (Indirect) **gain**, due to the associated price increase, multiplied by $vq$
(3) (Indirect) **loss** from the output reduction for both unit tax revenue and ad valorem tax revenue
Proof (3/4)

Now recall again: \( dp = \rho_t dt \) and \( \left( \frac{p}{\epsilon_D} \right) dq = -qdp \). Thus,

1. \( qdt = \frac{q}{\rho_t} dp = -\frac{p}{\epsilon_D \rho_t} dq \)

2. \( vqdp = -\left( \frac{vq_p}{q \epsilon_D} \right) dq = -\left( \frac{vp}{\epsilon_D} \right) dq \), which implies that

\[
dR = - \left( \frac{p}{\epsilon_D \rho_t} \right) dq + \left[ - \left( \frac{vp}{\epsilon_D} \right) dq \right] + (t + vp) dq
\]

\[
= p \left[ \left( -\frac{1}{\epsilon_D \rho} \right) + \left( -\frac{v}{\epsilon_D} \right) + \left( \frac{t}{p} + v \right) \right] dq
\]
Proof (4/4)

• Now, in the per-price term,

\[ MCPF_t = -\frac{dW}{dR} = -p \left[ \frac{(1-v)\theta}{\epsilon^D} + \tau \right] dq \]

\[ = p \left[ \frac{(1-v)\theta}{\epsilon^D} + \tau \right] dq \]

\[ = \frac{1}{\rho_t + \frac{v}{\epsilon^D} - \tau} \left[ \frac{(1-v)\theta}{\epsilon^D} + \tau \right] dq \]

\[ = \text{Welfare Loss expressed by the Profit Margin} \]

\[ \frac{1}{\rho_t} + \frac{v}{\epsilon^D} + \left( -\tau \right) \]

\[ \text{Gain in Revenue} \quad \text{Loss in Revenue} \]
Recap

- Under symmetric oligopoly, we have provided concise yet general formulas for (i) pass-through, (ii) tax incidence, and (iii) the marginal cost of public funds:

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• What if the mode of competition (price or quantity) is specified?

→ Conduct Index is now expressed by the first-order demand characteristics.

Pass-through is characterized by up to the second-order demand (and supply) characteristics.

• Generalization to

Multi-Dimensionality

Firm Heterogeneity
Pass-Through Expressions under Price and Quantity Competition
Price Elasticities

- Recall that \( \epsilon^D(p) \equiv -\frac{pq'(p)}{q(p)} > 0 \) is the price elasticity of the industry demand.

- Additionally, we define the own price elasticity of the firm’s demand by

  \[
  \epsilon_F(p) \equiv - \left( \frac{p}{q(p)} \right) \frac{\partial q_j(p)}{\partial p_j} \bigg|_{p=(p,...,p)},
  \]

  and the cross price elasticity by

  \[
  \epsilon_C(p) \equiv (n - 1) \left( \frac{p}{q(p)} \right) \frac{\partial q_{j'}(p)}{\partial p_j} \bigg|_{p=(p,...,p)},
  \]

  for any distinct pair of indices \( j \) and \( j' \).

- These are related by \( \epsilon_F = \epsilon^D + \epsilon_C \).
Demand Curvature

- We also define the **curvature of the industry’s direct demand** by \( \alpha(p) \equiv -pq''(p)/q'(p) \).

- \( \alpha \) is **positive** (**negative**) if and only if the industry demand is **convex** (**concave**).
Pass-Through under Price Competition

Proposition

Under price competition, the unit-tax and the ad valorem tax pass-through rates are characterized by:

\[ \rho_t = \frac{1}{1-v} \cdot \frac{1}{1 + \left( \frac{1-\alpha/\epsilon_F}{\epsilon_F} \right) \frac{\epsilon D}{\epsilon F} + \left( \frac{1-\tau}{1-v} \frac{\epsilon F}{\epsilon F-1} - \frac{1}{\epsilon F-1} \right) \left( \frac{\epsilon D}{\epsilon S} \right) }, \]

and

\[ \rho_v = \frac{1}{1-v} \cdot \frac{1}{1 - \frac{1/\epsilon_F}{\epsilon_F-1} + \left( \frac{1-\alpha/\epsilon_F}{\epsilon_F-1} \right) \frac{\epsilon D}{\epsilon F-1} + \left( \frac{1-\tau}{1-v} \frac{\epsilon F}{\epsilon F-1} - \frac{1}{\epsilon F-1} \right) \left( \frac{\epsilon D}{\epsilon S} \right) }, \]

respectively.
Proof for $\rho_t$

- First, recall that

$$\rho_t = \frac{1}{1 - v} \cdot \frac{1}{\left[ \left(1 - \frac{\theta}{\epsilon^D}\right) + \left(\frac{\theta}{\epsilon^D}\right)' \epsilon^D q \right] + \left[ \frac{1 - \tau}{1 - v} \frac{\epsilon^D}{\epsilon} - \theta \right] \frac{1}{\epsilon S}}.$$ 

  - revenue increase
  - cost savings

- Then, with $\theta = \epsilon^D / \epsilon_F$, $1 - \theta / \epsilon^D = 1 - 1 / \epsilon_F$, $(\theta / \epsilon^D)' \epsilon^D q = (1 + \epsilon^D - \alpha \epsilon^D / \epsilon_F) / \epsilon_F$, it is rewritten as:

$$\rho_t = \frac{1}{1 - v} \cdot \frac{1}{\left[ \left(1 - \frac{1}{\epsilon_F}\right) + \frac{1 + \epsilon^D - \alpha \epsilon^D / \epsilon_F}{\epsilon_F} \right] + \left[ \frac{1 - \tau}{1 - v} - \frac{1}{\epsilon_F} \right] \left(\frac{\epsilon^D}{\epsilon S}\right)}.$$ 

  - revenue increase
  - cost savings
Quantity Elasticities

• Define $\eta^D(q) = 1/\epsilon^D(p)|_{q(p)=q}$.

• We also define the own quantity elasticity of the firm’s inverse demand by

$$\eta_F(q) \equiv - \left( \frac{q}{p(q)} \right) \frac{\partial p_j(q)}{\partial q_j} |_{q=(q,...,q)} ,$$

and the the cross quantity elasticity by

$$\eta_C(q) \equiv (n - 1) \left( \frac{q}{p(q)} \right) \frac{\partial p_{j'}(q)}{\partial q_j} |_{q=(q,...,q)} ,$$

for any distinct pair of indices $j$ and $j'$.

• These are related by $\eta_F = \eta^D + \eta_C$. 
We also define the curvature of the industry’s inverse demand \( \sigma(q) \equiv -q \frac{p''(q)}{p'(q)} \).

\( \sigma \) is positive (negative) if and only if the industry’s inverse demand is convex (concave),
Pass-Through under Quantity Competition

**Proposition**

*Under quantity competition, the unit-tax and the ad valorem tax pass-through rates are characterized by:*

\[
\rho_t = \frac{1}{1-v} \cdot \frac{\frac{1}{\eta F}}{1 + \frac{\eta F}{\eta D} - \sigma + \left( \frac{1-\tau}{\eta D} - \eta F \right) \left( \frac{1}{\eta D} S \right)},
\]

*and*

\[
\rho_v = \frac{1}{1-v} \cdot \frac{1-\eta F}{1 + \frac{\eta F}{\eta D} - \sigma + \left( \frac{1-\tau}{\eta D} - \eta F \right) \left( \frac{1}{\eta D} S \right)},
\]

*respectively.*
Proof for $\rho_t$

- Recall again that

$$\rho_t = \frac{1}{1 - \nu} \cdot \frac{1}{[(1 - \theta/\epsilon^D) + \left(\frac{\theta}{\epsilon^D}\right)' \epsilon^D q] + \left[\frac{1 - \tau}{1 - \nu} \epsilon^D - \theta\right] \frac{1}{\epsilon^S}}.$$ 

revenue increase

cost savings

- Then, $\theta = \eta_F/\eta^D$ implies $(1/\epsilon^S - \eta^D) \cdot \theta = [(1/\epsilon^D \eta^D) - 1] \eta_F$ and $(\theta \eta^D)' (q/\eta^D) = (1 + \eta^D - \sigma \eta^D/\eta_F) (\eta_F/\eta^D)$. Thus, it is rewritten as

$$\rho_t = \frac{1}{1 - \nu} \cdot \left[\left(1 - \eta_F\right) + \frac{1 + \eta^D - \sigma \eta^D/\eta_F}{\eta^D} \eta_F\right] + \left[\frac{1 - \tau}{1 - \nu} \frac{1}{\epsilon^S \eta^D} - \frac{\eta_F}{\epsilon^S \eta^D}\right].$$ 

revenue increase

cost savings
Parametric Example: Linear Demands

- Functional form:

\[ q_j(p_1, \ldots, p_n) = b - \lambda p_j + \mu \sum_{j' \neq j} p_{j'}, \]

where \( b > mc \) and \( \mu \in [0, \lambda/(n - 1)) \) measures the degree of substitutability.

- To focus on \( n \) and \( \mu \), we set: \( b = 1, \ mc = 0, \) and \( \lambda = 1. \)

- When we change \( n \), we set \( \mu = 0.1. \) When we change \( \mu \), we set \( n = 5. \)
Pass-Through
Incidence \((dCS/dPS)\)
Marginal Cost of Public Funds
Generalization
Generalization

- Up to now, the firm’s profit is:

\[ \pi = [(1 - \nu)p - t]q - c(q) = [pq - c(q)] - [tq + \nu pq] \equiv \phi(p, q, T) \]

- How to proceed:

1. First, consider **Multi-Dimensionality**, maintaining the symmetry assumption:

\[ \phi(p, q, T) \leftarrow tq + \nu pq \]

\[ \pi = [pq - c(q)] - \phi(p, q, T) \]

2. Then, incorporate **Firm Heterogeneity**:

\[ \pi_i = [p_i q_i - c_i(q_i)] - \phi_i(p_i, q_i, T) \]
Multi-Dimensionality
• Now, the additional cost the firm has to pay is written as:

$$\phi(p, q, T)$$

where $T \equiv (T_1, ..., T_\ell, ..., T_L)$ is an $L$-dimensional vector of policy/shock parameters.

• Then, the firm’s profit is written as:

$$\pi = [pq - c(q)] - \phi(p, q, T).$$
Motivating Example (again)

- The government considers a new tax scheme which makes \( s > 0 \) fraction of the cost \( c(q) \) tax deductible.

"What are the effects of introducing \( s \), when \( t > 0 \) and \( \nu > 0 \) are already implemented?"

- The firm’s profit is written as:

\[
\pi = (p - t)q - c(q) - \nu[pq - s \cdot c(q)]
\]

\[
= [pq - c(q)] - [tq + \nu\{pq - s \cdot c(q)\}]
\]

\[\equiv \phi(p, q, T): \text{additional cost}\]

- Intervention vector, \( T = (t, \nu, s) \), is three-dimensional.
Again, the conduct index $\theta$ is implicitly defined by:

$$
\left[(1 - \tau) - (1 - \nu) \left( \frac{\theta}{\epsilon D} \right) \right] p = mc,
$$

where

$$
\tau(p, q, T) \equiv \frac{1}{p} \frac{\partial \phi}{\partial q}(p, q, T)
$$

is the (first-order) quantity sensitivity of the (per-firm) tax revenue, and

$$
\nu(p, q, T) \equiv \frac{1}{q} \frac{\partial \phi}{\partial p}(p, q, T)
$$

is its (first-order) price sensitivity.
• Note that

\[
\left[ (1 - \tau) - (1 - \nu) \left( \frac{\theta}{\epsilon D} \right) \right] p = mc
\]

is the generalization of

\[
\left[ 1 - \left( \nu + \frac{t}{p} \right) - (1 - \nu) \left( \frac{\theta}{\epsilon D} \right) \right] p = mc
\]

in our two-dimensional case of taxation above.
Now, we define the pass-through rate vector by:

\[ \tilde{\rho} \equiv \left( \frac{\partial p(T)}{\partial T_1}, \ldots, \frac{\partial p(T)}{\partial T_\ell}, \ldots, \frac{\partial p(T)}{\partial T_L} \right) \]

and the pass-through quasi-elasticity vector by:

\[ \rho \equiv (\rho_1, \ldots, \rho_\ell, \ldots, \rho_L), \quad \rho_\ell \equiv \frac{q}{\frac{\partial \phi}{\partial T_\ell}(p, q, T)} \tilde{\rho}_\ell \]
Multi-Dimensional Pass-Through (2/2)

Proposition

Each $\ell$-th element of $\rho$ is characterized by

$$\rho_\ell = \frac{pq}{\phi_\ell} \left[ \tau_\ell - \left( \frac{\theta}{\epsilon D} \right) \nu_\ell \right] \rho(0),$$

where

$$\frac{1}{\rho(0)} = \left[ (1 - \kappa) + \epsilon^D \tau(2) + (1 - \tau) \left( \frac{\epsilon^D}{\epsilon S} \right) \right]$$

$$+ \left[ \left( \nu - \kappa + \frac{\nu(2)}{\epsilon^D} \right) - (1 - \nu) \left( \frac{1}{\epsilon^D} + \frac{1}{\epsilon S} \right) \right] \theta + (1 - \nu) \epsilon^D q \frac{\partial (\theta / \epsilon^D)}{\partial q},$$

with $\kappa \equiv \frac{\partial^2 \phi}{\partial p \partial q}$, $\tau(2) \equiv \frac{q}{p} \frac{\partial^2 \phi}{\partial q^2}$, and $\nu(2) \equiv \frac{p}{q} \frac{\partial^2 \phi}{\partial p^2}$. 
For \( \ell = t \) (Unit Tax Pass-Through: 1/2)

- If \( \phi(p, q, T) = tq + vpq \), then
  
  \[
  \rho_t = \frac{pq}{\partial \phi/\partial t} \left[ \tau_t - \left( \frac{\theta}{\epsilon D} \right) \nu_t \right] \rho(0) = \rho(0)
  \]

  because \( \partial \phi/\partial t = q, \tau_t = 1/p \), and \( \nu_t = 0 \). Then,

  \[
  \frac{1}{\rho(0)} = \left[ (1 - \kappa) + \epsilon^D \tau(2) + (1 - \tau) \left( \frac{\epsilon^D}{\epsilon S} \right) \right]
  \]

  \[
  + \left[ \nu - \kappa + \eta \nu(2) - (1 - \nu) \left( \frac{1}{\epsilon D} + \frac{1}{\epsilon S} \right) \right] \theta + (1 - \nu) \epsilon^D q \frac{\partial (\theta/\epsilon^D)}{\partial q}
  \]

  \[
  = (1 - \nu) \left\{ \left[ 1 + \frac{1 - \tau}{1 - \nu} \left( \frac{\epsilon^D}{\epsilon S} \right) \right] - \left( \frac{1}{\epsilon D} + \frac{1}{\epsilon S} \right) \theta + \epsilon^D q \frac{\partial (\theta/\epsilon^D)}{\partial q} \right\}
  \]

  because \( \kappa \equiv \frac{\partial^2 \phi}{\partial p \partial q} = \nu \), and \( \tau(2) \equiv \frac{q}{p} \frac{\partial^2 \phi}{\partial q^2} = 0 = \nu(2) \equiv \frac{p}{q} \frac{\partial^2 \phi}{\partial p^2} \).
For $\ell = t$ (Unit Tax Pass-Through: 2/2)

- Thus, it coincides with:

\[
\rho_t = \frac{1}{1 - \frac{\tau}{1 - \nu} \frac{\epsilon_D}{\epsilon_S}} + \left[ - \left( \frac{1}{\epsilon_D} + \frac{1}{\epsilon_S} \right) \theta \right] + \left[ -\epsilon_D q \frac{\partial(-\theta/\epsilon_D)}{\partial q} \right],
\]

as we saw above.
Proposition: Welfare Measures

<table>
<thead>
<tr>
<th></th>
<th>Two-Dimensional, ((t, \nu))</th>
<th>Multi-Dimensional</th>
</tr>
</thead>
</table>
| Incidence           | \[
\frac{1}{\rho_t} - (1 - \nu)(1 - \theta)
\]                     | \[
\frac{1}{\rho_\ell} - (1 - \nu)(1 - \theta)
\]                     |
| MCPF                | \[
\frac{(1 - \nu)\theta}{\epsilon D} + \tau
\]                     | \[
\frac{(1 - \nu)\theta}{\epsilon D} + \tau
\]                     |

where \(\nu \equiv \frac{1}{q} \frac{\partial \phi}{\partial p} (p, q, T)\) and \(\tau \equiv \frac{1}{p} \frac{\partial \phi}{\partial q} (p, q, T)\).
Including Other Changes Than Taxes
Exchange Rate and Technology

• Up to now, the additional cost consists of taxation only:

\[ \pi = [pq - c(q)] - \phi(p, q, T), \]

which means that the firm’s additional cost contributes to welfare as the government’s revenue:

\[ dW = dCS + dPS + dR. \]

• However, our \( \phi(p, q, T) \) can be extended to include changes in exchange rates and production costs:

\[ \phi(p, q, T) = \tilde{\phi}(p, q, T) + \left[ \phi(p, q, T) - \tilde{\phi}(p, q, T) \right], \]

where 
\[ \tilde{\phi}(p, q, T) \text{ tax} \] and 
\[ \phi(p, q, T) - \tilde{\phi}(p, q, T) \text{ others} \]
Example

- Firm uses some imported inputs for production. Then,

\[ \pi = [(1 - \nu)p - t]q - [(1 - a) + a \cdot e]c(q) \]

\[ = [pq - c(q)] - [tq + vpq + a \cdot (1 - e)c(q)], \]

\[ \equiv \phi(p, q, T) \]

where \( a \) measures the ratio of imported inputs and \( e > 0 \) is the exchange rate.

- Intervention vector, \( T = (t, \nu, e) \), is **three-dimensional**.
Welfare Measures

- Define

\[ g_\ell \equiv \frac{1}{q} \cdot \frac{\partial \tilde{\phi}}{\partial T_\ell} (p, q, T) \]

\[ \frac{\partial \phi}{\partial T_\ell} (p, q, T) \]

as the fraction of an increase in additional cost (\( \phi \)) to the firm that is collected by the government in the form of taxes (\( \tilde{\phi} \)).

Then,

<table>
<thead>
<tr>
<th>Incidence</th>
<th>Two-Dimensional, with Taxation Only</th>
<th>Multi-Dimensional, also with Other Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCPF</td>
<td>[ \frac{(1-\nu)\theta}{\epsilon D} + \tau ]</td>
<td>[ (1-\nu)\theta + \frac{1-g_\ell}{\rho_\ell} ]</td>
</tr>
<tr>
<td></td>
<td>[ \frac{1}{\rho t} + \nu ]</td>
<td>[ \frac{g_\ell}{\rho_\ell} + \nu ]</td>
</tr>
<tr>
<td></td>
<td>[ \frac{1}{\rho t} - (1-\nu)(1-\theta) ]</td>
<td>[ \frac{1}{\rho t} - (1-\nu)(1-\theta) ]</td>
</tr>
</tbody>
</table>
Incorporating Firm Heterogeneity
Definitions (1/2)

- We allow for the tax function

\[ \phi_i (p_i, q_i, T) \]

to depend explicitly on the identity of the firm. Similar for the sensitivities \( \tau_i (p_i, q_i, T) \), \( \nu_i (p_i, q_i, T) \), etc.

- The marginal cost \( mc_i (q_i) \) of firm \( i \) is also allowed to depend on the identity of the firm. We denote its elasticity by

\[ \epsilon_i^S (q_i) \equiv \frac{mc_i (q_i)}{q_i mc_i' (q_i)}. \]
Definitions (2/2)

• Instead of the conduct index, we define the **pricing strength index**

$$\psi_i(q),$$

implicitly from FOC:

$$[1 - \tau_i - \psi_i(q)(1 - \nu_i)] p_i(q) = m c_i(q_i),$$

where $\tau_i = \tau_i(p_i(q), q_i, T)$ and $\nu_i = \nu_i(p_i(q), q_i, T)$.

• In the case of symmetric firms, this definition reduces to:

$$\psi_i = \frac{\theta}{\epsilon D}. $$
Multi-Dimensional Pass-Through (1/3)

- First, we define the \((n \times d)\) pass-through matrix \(\tilde{\rho}\) with columns \(\tilde{\rho}_\ell \equiv \partial p / \partial T_\ell\) and whose \((i, \ell)\) element is:

\[
\tilde{\rho}_{i\ell} = \frac{\partial p_i}{\partial T_\ell}
\]

- Notice here that our framework can easily be extended to include firm-specific taxation/shocks:

\[
\tilde{\rho}_{i\ell_j} = \frac{\partial p_i}{\partial T_{\ell_j}}
\]

is the effect of firm \(j\)'s specific shock in the \(\ell\)-th instrument on firm \(i\)'s price.
Multi-Dimensional Pass-Through (2/3)

Proposition

The pass-through matrix \( \tilde{\rho} = (\tilde{\rho}_1, \ldots, \tilde{\rho}_\ell, \ldots, \tilde{\rho}_L) \) is characterized by:

\[
\tilde{\rho}_\ell = b^{-1} \cdot \iota_\ell,
\]

where matrix \( b \) is an \((n \times n)\) matrix whose \((i,j)\) element is

\[
b_{ij} = 
\begin{bmatrix}
(1 - \kappa_i)\delta_{ij} + \epsilon_{ij}^D \tau(2)i + (1 - \tau_i) \left( \frac{\epsilon_{ij}^D}{\epsilon_i^S} \right)
\end{bmatrix}
\]

\[
+ \left\{ \left[ (\nu_i - \kappa_i)\epsilon_{ij}^D + \nu(2)i\delta_{ij} \right] - (1 - \nu_i) \left( \delta_{ij} + \frac{\epsilon_{ij}^D}{\epsilon_i^S} \right) \right\} \psi_i - (1 - \nu_i)\psi_i \Psi_{ij},
\]

where \( \delta_{ij} \) is the Kronecker delta, and

\[
\psi_{ij} = \frac{p_i}{\psi_i} \frac{\partial \psi_i}{\partial p_j} (q(p)), \quad \epsilon_{ij}^D = -\frac{p_i}{q_i} \frac{\partial q_i}{\partial p_j}, \quad (contd)
\]
Multi-Dimensional Pass-Through (3/3)

Proposition

(contd) and \( \nu_\ell \) is an \( n \)-dimensional vector defined for each \( T_\ell \), whose \( (i,1) \) element is:

\[
\nu_{i\ell} \equiv p_i \cdot \left[ \frac{\partial \tau_i}{\partial T_\ell} - \psi_i \frac{\partial \nu_i}{\partial T_\ell} \right].
\]

• Then, we can define the \( (n \times d) \) pass-through quasi-elasticity matrix \( \rho \) whose \( (i,\ell) \) element is:

\[
\rho_{i\ell} = \frac{q_i}{\frac{\partial}{\partial T_\ell} \phi_i} \tilde{\rho}_{i\ell}.
\]
Welfare Measures: Takeaway

- We have developed **general formulas** for welfare evaluation under **imperfect competition** in consideration of the **multi-dimensionality** of taxation and other external changes.

<table>
<thead>
<tr>
<th></th>
<th>Perfect comp</th>
<th>Two-Dimensional, Symmetric Oligopoly</th>
<th>Multi-Dimensional, Firm Heterogeneity, Also with Other Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incidence</td>
<td>$\frac{1}{\rho_t - 1}$</td>
<td>$\frac{1}{\rho_t} - (1-v)(1-\theta)$</td>
<td>$\frac{1}{\rho_i \ell} - (1-\nu_i)(1-\psi_i \epsilon_i^p \ell)$</td>
</tr>
<tr>
<td>MCPF</td>
<td>0</td>
<td>$\frac{(1-v)\theta}{\epsilon^D} + \tau$</td>
<td>$\frac{(1-\nu_i)\psi_i + \frac{1-g_i \ell}{\rho_i \ell}}{\epsilon_i^p \ell} + \tau_i$</td>
</tr>
</tbody>
</table>

where $\epsilon_i^p \ell = \epsilon_i^D \frac{\tilde{\rho}_\ell}{\tilde{\rho}_i \ell} = \epsilon_i^D \cdot \frac{\rho_\ell}{\rho_i \ell}$.
Possible Extensions

- **Vertical Relationships / Two-Sided Platforms**

  Tremblay (2017): *Taxation on a Two-Sided Platform*

- **Macroeconomics?**

  General-Equilibrium Effects

  Dynamics (incorporating *adjustment/menu costs*)

  “How does imperfect competition matter to the determination of the aggregate price level?”