Fake news

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Overview

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Fake news

- Deliberate misinformation or hoaxes.
- Traditional print and broadcast news media or online social media.
- Intend to mislead in order to gain financially or politically.
- Effectively influencing the society.
Literature Review

- Communication games (Crawford and Sobel 1982, Kamenica and Gentzkow 2011,...)
  - No/full commitment power vs. Dynamically learned trust
- Media market (Mullainathan and Shleifer 2002, Gentzkow and Shapiro 2006)
  - Deception due to biased belief vs. Deception due to stochasticity
- Deception games (Anderson and Smith 2013)
  - Brownian motion process vs. Point process
**Point Processes:** A stochastic process that models the discrete occurrence of events as a series of random points in continuous time or geographic space.

**News traffic:** \( dY = Z_0 dN_0 + Z_a dN_a \)
- \( N_0, N_a \): Point processes capturing the timing of true news and fake news. Denote their intensities as \( \Lambda_0(t) \) and \( \Lambda_a(t) \).
- \( Z_0, Z_a \): Random variables capturing the content of true news and fake news. Denote their event space as \( \Omega \) and pdf’s as \( P_0 \) and \( P_a \).
- Common knowledge for receiver and sender.
Information asymmetry: The fake news sender knows that he is sending fake news but the receiver does not.

Formally speaking, there are two states:

- State 1: The sender is sending fake news and \( dY = Z_0 dN_0 + Z_a dN_a \).
- State 2: The sender is not sending fake news and \( dY = Z_0 dN'_0 \), where \( N'_0 \) is a point process whose intensity \( \Lambda'_0 \) satisfies \( \Lambda'_0 = \Lambda_0 + \Lambda_a \).

At each time \( t \), the receiver has a belief \( q(t) \in [0,1] \) that the world is in State 1 and a belief \( 1 - q(t) \) that the world is in State 2.

\( q(t) \) is Bayesian updated through the observation of \( Y \):

\[
 dq = q(1 - q)(\frac{\Lambda_a P_a(dY) + \Lambda_0 P_0(dY)}{\Lambda_0 + \Lambda_a} - P_0(dY))
\
\frac{q \cdot \frac{\Lambda_a P_a(dY) + \Lambda_0 P_0(dY)}{\Lambda_0 + \Lambda_a} + (1 - q) P_0(dY)}
\]

Assume that receiver’s initial belief \( q_0 \) is common knowledge, then the receiver’s belief \( q(t) \) is common knowledge for all \( t \).
Payoff structure

- At each time $t$, the receiver decides her dependence on the focal sender, $p(t) \in [0, 1]$ to maximize her expected payoff:

$$E \left[ \int_0^\infty p((1 - q)(\Lambda_0 + \Lambda_a) + q(\Lambda_0 - L\Lambda_a)) dt \right]$$

- Normalize her payoff from outside options as 0.
- Assume that she obtains 1 positive payoff consuming each piece of true news and suffers $L > 0$ loss from consuming each piece of fake news.

- At each time $t$, the sender decides the fake news intensity $\Lambda_a(t) \in [0, c]$ to maximize his expected payoff, which is how much he misleads the receiver:

$$E \left[ \int_0^\infty e^{-rt} p\Lambda_a dt \right]$$
Equilibrium Analysis

- Markov equilibrium
  - Bayesian Nash equilibrium.
  - Both players' strategies are Markovian and the state variable is the receiver’s belief $q(t)$.
- The receiver’s strategy will not influence the evolution of the game, therefore, her dynamic optimization problem is equivalent to optimization at each static point.

\[
p(q) \in \arg \sup_{p \in [0,1]} p((1 - q)(\Lambda_0 + \Lambda_a) + q(\Lambda_0 - L\Lambda_a)) \quad \forall q \in [0,1] \quad (1)
\]
The sender is facing a dynamic programming problem: trade-off between immediate gain from misleading the receiver and a dynamic loss of the trust of the receiver.

In equilibrium, with Hamilton-Jacobi-Bellman equation, the sender’s strategy $\Lambda_a(q)$ and value function $V(q)$ follow

$$
\Lambda_a(q) \in \arg \sup_{\Lambda_a \in [0,c]} p\Lambda_a + V'(E[g(Z_0)\big| q, \Lambda_a]\Lambda_0 + E[g(Z_a)\big| q, \Lambda_a]\Lambda_a)
$$

and

$$
rV = p\Lambda_a + V'(E[g(Z_0)\big| q, \Lambda_a]\Lambda_0 + E[g(Z_a)\big| q, \Lambda_a]\Lambda_a) \quad \forall q \in [0,1]
$$

A Markov equilibrium is a 3-tuple $(p, V, \Lambda_a)$, where each entry is a function of $q$, such that conditions (1),(2),(3) are satisfied.
Theorem (Existence and uniqueness of equilibrium)

There exists a unique Markov equilibrium.
Assume that the random variables capturing the contents of the true and fake news, $Z_0$ and $Z_a$, are binary, with frequency $p_0$ and $p_a$ in one state and $1-p_0$, $1-p_a$ in the other state.

- Likelihood of not passing some fact checking tool.
- $p_a$ characterizes the sender’s technology of producing fake news.

**Theorem (The game evolution)**

When $0 < q < 1$, $E[ dq / dt ] > 0$. Therefore, there are only two absorbing states: $q = 0$ and $q = 1$. 

*If the receiver’s initial belief $q_0 > 0$, $q \rightarrow 1$ when $t \rightarrow \infty$.***
An Illustrative Example

Set \( L = 3, c = 3, r = 0.1, \Lambda_0 = 1, p_0 = 0.1 \), and compare strategies and payoffs between cases where \( p_a = 0.3 \) and \( p_a = 0.4 \).

Figure: Comparison of sender’s strategies
Figure: *Comparison of receiver’s strategies*
An Illustrative Example

Figure: Comparison of sender's payoffs
An Illustrative Example

**Figure:** Receiver’s expected payoffs

**Figure:** Receiver’s expected payoffs
In equilibrium, it is assumed that the receiver knows the technology that the fake news sender would be using.

Could be restrictive especially when the fake news sender is specialized while the receiver is relatively naive.

With the binary characterization as in previous section, assume that the receiver anticipates the sender’s technology to be characterized by $p'_a$.

**Underestimation**

We say that the receiver *underestimates* the sender’s technology if $p_0 < p_a < p'_a$ or $p'_a < p_a < p_0$. The larger $(p_0 - p'_a)/(p_0 - p_a)$ is, we say that the sender is *more underestimated*.
Although the sender’s technology is characterized by \( p_a \), the receiver is using the equilibrium strategy where the technology is characterized by \( p'_a \).

The belief is updated based on the wrong anticipation.

The sender knows that the receiver is misunderstanding and utilizes this by optimally responding to the receiver’s suboptimal strategy.
Off-equilibrium Analysis

Theorem (The off-equilibrium game revolution)

Assume that $Lc > \Lambda_0$. If the sender is underestimated, then there exists a unique $q_e \in (0, 1)$, such that:

i) when $0 < q < q_e$, $E[\frac{dq}{dt}] > 0$

ii) when $q = q_e$, $E[\frac{dq}{dt}] = 0$

iii) when $q_e < q < 1$, $E[\frac{dq}{dt}] < 0$

Therefore, other than $q = 0$ and $q = 1$, $q_e$ is another absorbing state. If $0 < q(t_0) < 1$, as $t \to \infty$, $q$ will fluctuates around $q_e$ and the sender’s intensity fluctuates around $\Lambda_a(q_e)$. Specifically, $\Lambda_a(q_e)$ satisfies:

i) $\Lambda_a(q_e) = \frac{p_0 - p'_a}{Lp_0 + p'_a - p_a - Lp_a} \Lambda_0$, if $\frac{p_0 - p'a}{p_0 - p_a} < \frac{c}{c + \Lambda_0} (1 + L)$.

ii) $\Lambda_a(q_e) = c$, if $\frac{p_0 - p'_a}{p_0 - p_a} \geq \frac{c}{c + \Lambda_0} (1 + L)$.
Theorem (Effect of underestimation)

If the receiver has initial belief $0 < q(t_0) < 1$ and underestimates the sender’s technology, she will receive more fake news in the long run if the sender is more underestimated. Formally, $\Lambda_a(q_e)$ is an increasing function with respect to $(p_0 - p'_a)/(p_0 - p_a)$. 
Off-equilibrium Analysis

- Set $L = 3$, $c = 3$, $r = 0.1$, $\Lambda_0 = 1$, $p_0 = 0.1$, and compare strategies and payoffs between equilibrium cases where $p_a = 0.3$ and $p_a = 0.4$ and off-equilibrium case where $p_a = 0.3$, $p'_a = 0.4$

Figure: Comparison of sender’s equilibrium strategies and off-equilibrium strategy
Figure: Comparison of sender’s equilibrium payoffs and off-equilibrium payoff
Future directions

- Synergies between pieces of news
- The receiver’s optimal fake news checking method.
- The sender’s optimal fake news generation technology.
  - Trade-off between how deceptive the news is and how much the news can mislead readers.
- ...

Thank you!