## Fake news

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## Introduction

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Fake news

- Deliberate misinformation or hoaxes.
- Traditional print and broadcast news media or online social media.
- Intend to mislead in order to gain financially or politically.
- Effectively influencing the society.

- Communication games (Crawford and Sobel 1982, Kamenica and Gentzkow 2011,...)
  - No/full commitment power vs. Dynamically learned trust
- Media market (Mullainathan and Shleifer 2002, Gentzkow and Shapiro 2006)
  - Deception due to biased belief vs. Deception due to stochasticity
- Deception games (Anderson and Smith 2013)
  - Brownian motion process vs. Point process

- Point Processes: A stochastic process that models the discrete occurrence of events as a series of random points in continuous time or geographic space.
- News traffic:  $dY = Z_0 dN_0 + Z_a dN_a$ 
  - N<sub>0</sub>, N<sub>a</sub>: Point processes capturing the timing of true news and fake news. Denote their intensities as Λ<sub>0</sub>(t) and Λ<sub>a</sub>(t).
  - Z<sub>0</sub>, Z<sub>a</sub>: Random variables capturing the content of true news and fake news. Denote their event space as Ω and pdf's as P<sub>0</sub> and P<sub>a</sub>.
  - Common knowledge for receiver and sender.

# Information structure

- Information asymmetry: The fake news sender knows that he is sending fake news but the receiver does not.
- Formally speaking, there are two states:
  - State 1: The sender is sending fake news and  $dY = Z_0 dN_0 + Z_a dN_a$ .
  - State 2: The sender is not sending fake news and  $dY = Z_0 dN'_0$ , where  $N'_0$  is a point process whose intensity  $\Lambda'_0$  satisfies  $\Lambda'_0 = \Lambda_0 + \Lambda_a$ .
- At each time t, the receiver has a belief q(t) ∈ [0, 1] that the world is in State 1 and a belief 1 − q(t) that the world is in State 2.
- q(t) is Bayesian updated through the observation of Y:

$$dq = rac{q(1-q)(rac{\Lambda_a P_a(dY)+\Lambda_0 P_0(dY)}{\Lambda_0+\Lambda_a}-P_0(dY))}{q\cdot rac{\Lambda_a P_a(dY)+\Lambda_0 P_0(dY)}{\Lambda_0+\Lambda_a}+(1-q)P_0(dY)}$$

 Assume that receiver's initial belief q<sub>0</sub> is common knowledge, then the receiver's belief q(t) is common knowledge for all t.  At each time t, the receiver decides her dependence on the focal sender, p(t) ∈ [0, 1] to maximize her expected payoff:

$$E[\int_0^\infty p((1-q)(\Lambda_0+\Lambda_a)+q(\Lambda_0-L\Lambda_a))dt]$$

- Normalize her payoff from outside options as 0.
- Assume that she obtains 1 positive payoff consuming each piece of true news and suffers L > 0 loss from consuming each piece of fake news.
- At each time t, the sender decides the fake news intensity
   Λ<sub>a</sub>(t) ∈ [0, c] to maximize his expected payoff, which is how much he
   misleads the receiver:

$$E[\int_0^\infty e^{-rt} p \Lambda_a dt]$$

#### Markov equilibrium

- Bayesian Nash equilibrium.
- Both players' strategies are Markovian and the state variable is the receiver's belief *q*(*t*).
- The receiver's strategy will not influence the evolution of the game, therefore, her dynamic optimization problem is equivalent to optimization at each static point.

 $p(q) \in \arg \sup_{p \in [0,1]} p((1-q)(\Lambda_0 + \Lambda_a) + q(\Lambda_0 - L\Lambda_a)) \quad \forall q \in [0,1]$ (1)

# Equilibrium Analysis

- The sender is facing a dynamic programming problem: trade-off between immediate gain from misleading the receiver and a dynamic loss of the trust of the receiver.
- In equilibrium, with Hamilton-Jacobi-Bellman equation, the sender's strategy Λ<sub>a</sub>(q) and value function V(q) follow

$$\Lambda_{a}(q) \in \arg \sup_{\Lambda_{a} \in [0,c]} p\Lambda_{a} + V'(E[g(Z_{0})|q,\Lambda_{a}]\Lambda_{0} + E[g(Z_{a})|q,\Lambda_{a}]\Lambda_{a})$$
(2)

and

$$rV = p\Lambda_a + V'(E[g(Z_0)|q,\Lambda_a]\Lambda_0 + E[g(Z_a)|q,\Lambda_a]\Lambda_a) \quad \forall q \in [0,1]$$
(3)

A Markov equilibrium is a 3-tuple (p, V, Λ<sub>a</sub>), where each entry is a function of q, such that conditions (1),(2),(3) are satisfied.

### Theorem (Existence and uniqueness of equilibrium)

There exists a unique Markov equilibrium.

- Assume that the random variables capturing the contents of the true and fake news,  $Z_0$  and  $Z_a$ , are binary, with frequency  $p_0$  and  $p_a$  in one state and  $1 p_0$ ,  $1 p_a$  in the other state.
  - Likelihood of not passing some fact checking tool.
  - *p<sub>a</sub>* characterizes the sender's technology of producing fake news.

#### Theorem (The game evolution)

When 0 < q < 1, E[dq/dt] > 0. Therefore, there are only two absorbing states: q = 0 and q = 1. If the receiver's initial belief  $q_0 > 0$ ,  $q \to 1$  when  $t \to \infty$ .

• Set L = 3, c = 3, r = 0.1,  $\Lambda_0 = 1$ ,  $p_0 = 0.1$ , and compare strategies and payoffs between cases where  $p_a = 0.3$  and  $p_a = 0.4$ .



Figure: Comparison of sender's strategies



Figure: Comparison of receiver's strategies



Figure: Comparison of sender's payoffs



#### Figure: Receiver's expected payoffs

- In equilibrium, it is assumed that the receiver knows the technology that the fake news sender would be using.
- Could be restrictive especially when the fake news sender is specialized while the receiver is relatively naive.
- With the binary characterization as in previous section, assume that the receiver anticipates the sender's technology to be characterized by  $p'_a$ .

#### Underestimation

We say that the receiver *underestimates* the sender's technology if  $p_0 < p_a < p'_a$  or  $p'_a < p_a < p_0$ . The larger  $(p_0 - p'_a)/(p_0 - p_a)$  is, we say that the sender is *more underestimated*.

- Although the sender's technology is characterized by  $p_a$ , the receiver is using the equilibrium strategy where the technology is characterized by  $p'_a$ .
- The belief is updated based on the wrong anticipation.
- The sender knows that the receiver is misunderstanding and utilizes this by optimally responding to the receiver's suboptimal strategy.

#### Theorem (The off-equilibrium game revolution)

Assume that  $Lc > \Lambda_0$ . If the sender is underestimated, then there exists a unique  $q_e \in (0, 1)$ , such that: i) when  $0 < q < q_e$ , E[dq/dt] > 0ii) when  $q = q_e$ , E[dq/dt] = 0iii) when  $q_e < q < 1$ , E[dq/dt] < 0Therefore, other than q = 0 and q = 1,  $q_e$  is another absorbing state. If  $0 < q(t_0) < 1$ , as  $t \to \infty$ , q will fluctuates around  $q_e$  and the sender's intensity fluctuates around  $\Lambda_a(q_e)$ . Specifically,  $\Lambda_a(q_e)$  satisfies: i)  $\Lambda_a(q_e) = \frac{p_0 - p'_a}{Lp_0 + p'_a - p_a - Lp_a} \Lambda_0$ , if  $\frac{p_0 - p'_a}{p_0 - p_a} < \frac{c}{c + \Lambda_0} (1 + L)$ . ii)  $\Lambda_a(q_e) = c$ , if  $\frac{p_0 - p'_a}{p_0 - p_a} \geq \frac{c}{c + \Lambda_c} (1 + L)$ 

#### Theorem (Effect of underestimation)

If the receiver has initial belief  $0 < q(t_0) < 1$  and underestimates the sender's technology, she will receive more fake news in the long run if the sender is more underestimated. Formally,  $\Lambda_a(q_e)$  is an increasing function with respect to  $(p_0 - p'_a)/(p_0 - p_a)$ .

# Off-equilibrium Analysis

• Set L = 3, c = 3, r = 0.1,  $\Lambda_0 = 1$ ,  $p_0 = 0.1$ , and compare strategies and payoffs between equilibrium cases where  $p_a = 0.3$  and  $p_a = 0.4$ and off-equilibrium case where  $p_a = 0.3$ ,  $p'_a = 0.4$ 



Figure: Comparison of sender's equilibrium strategies and off-equilibrium strategy

# Off-equilibrium Analysis



Figure: Comparison of sender's equilibrium payoffs and off-equilibrium payoff

- Synergies between pieces of news
- The receiver's optimal fake news checking method.
- The sender's optimal fake news generation technology.
  - Trade-off between how deceptive the news is and how much the news can mislead readers.

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# Thank you!

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