Expectation and Duration at the Effective Lower Bound

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1The views expressed here do not represent those of the Chicago Fed or the Federal Reserve System.
I study the relative effects of duration exposures and short-rate expectations in a structural model of the yield curve.

- Important for understanding unconventional monetary policy - forward guidance and QE
- Previous models of this type ignore the ELB
  - Vayanos and Vila, 2009; Greenwood and Vayanos, 2014
- I incorporate the ELB using a shadow-rate structure.
  - Kim and Singleton, 2012; Krippner, 2012; Wu and Xia, 2015

Qualitatively:
- Effects of changes in bond supply on term premia are attenuated at ELB.
- Forward guidance at the ELB has effects on term premia that it does not have during normal times.

Quantitatively:
- The model matches the yield data well, including event-studies on unconventional policy.
- The Fed’s unconventional policy mostly operated by changing the anticipated short-rate path, not by reducing duration exposures.
Following Vayanos-Vila and others, arbitrageurs solve

\[
\max_{x_t(\tau) \forall \tau} E_t [dW_t] - \frac{a}{2} \text{var}_t [dW_t] \tag{1}
\]

subject to

\[
dW_t = \int_0^T x_t(\tau) \frac{dP_t(\tau)}{P_t(\tau)} d\tau + r_t \left( W_t - \int_0^T x_t(\tau) d\tau \right) \tag{2}
\]

where \( W_t \) is wealth, \( x_t(\tau) \) is bond holdings at maturity \( \tau \), \( P_t(\tau) \) is the bond price at maturity \( \tau \), and \( r_t \) is the short rate.
FOC:

\[ E_t \left[ dp_t^{(\tau)} \right] = r_t + a \int_{0}^{T} x_t (\tau') \text{cov}_t \left[ dp_t^{(\tau)}, dp_t^{(\tau')} \right] d\tau' + J_t^{(\tau)} \]  \hspace{1cm} (3)

Can also solve for yields through the usual relationship.

The government supplies bonds \( s_t (\tau) \). Equilibrium is determined by

\[ s_t (\tau) = x_t (\tau) \]  \hspace{1cm} (4)

Levels of \( s_t (\tau) \) that increase the portfolio variance raise required returns (and therefore yields).
The short rate follows

\[ r_t = \max \left[ \hat{r}_t, b \right] \] (5)

where \( b \) is the ELB and

\[ \hat{r}_t = \mu_{\hat{r}} (1 - \phi_{\hat{r}}) + \phi_{\hat{r}} \hat{r}_{t-1} + e_t \]
\[ e_t \sim \text{Niid} \left( 0, \sigma_{\hat{r}} \right) \] (6)

ELB dampens interest-rate uncertainty:
Following Greenwood, Hanson, and Stein (2015), reduce bond supply to a single factor:

\[
s_t(\tau) = \zeta + \left( 1 - \frac{2\tau}{T} \right) \beta_t \tag{7}
\]

\[
\beta_t = \phi \beta_{t-1} + e_t^\beta \quad e_t^\beta \sim Niid \left( 0, \sigma_\beta \right) \tag{8}
\]

Maturity distribution moves in a see-saw pattern in response to shocks to \( \beta_t \).

(The shape of the distribution is not of major importance.)
The WAM of outstanding debt is

\[ WAM_t \equiv v \frac{\int_0^T \tau s_t(\tau) \, d\tau}{\int_0^T s_t(\tau) \, d\tau} = vT \left( \frac{1}{2} - \frac{1}{6\zeta} \beta_t \right) \]

(9)

where \( v \) is the length of one period, in years.

Outstanding 10y equivalents are

\[ \%\Delta 10\text{YE}_t \equiv \frac{\frac{\nu}{10} \int_0^T \tau s_t(\tau) \, d\tau}{\frac{\nu}{10} \int_0^T s_{t-1}(\tau) \, d\tau} = -\frac{\Delta \beta_{t+h}}{3\zeta - \beta_t} \]

(10)
Using data since 1971, I match:

- the annual autocorrelation of Treasury WAM
- the unconditional mean and std. dev. of the 3M and 10Y yield
- the unconditional correlation between the 3M and 10Y yield
- the mean 3M yield during the ELB period

Model is solved numerically.
Evidence on the model’s fit

- The model matches the basic features of yields observed at the ELB:
  - Matches the 10Y slope average to within 0.1%.
  - Matches the 10Y slope std. dev. to within 0.3%.
- Affine model predicts negative short rates, very steep slopes, and excessive volatility.
- Away from the ELB, shadow-rate and affine models perform similarly.

- Model matches regression results on the effects of bond supply (extending Greenwood-Vayanos, 2014).
  - E.g., using 10Y yield as dependent variable:

<table>
<thead>
<tr>
<th></th>
<th>Coef. on WAM above ELB</th>
<th>Coef. on WAM at ELB</th>
<th>Coef. on 2Y yield above ELB</th>
<th>Coef. on 2Y yield at ELB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.19</td>
<td>0.06</td>
<td>0.8</td>
<td>2.3</td>
</tr>
<tr>
<td>Model</td>
<td>~0.12</td>
<td>~0.08</td>
<td>~0.7</td>
<td>&gt;2.0</td>
</tr>
</tbody>
</table>

(Model results are generally within 1 s.e. of regressions.)
Factor loadings in the shadow-rate model

For arbitrary state values, we have

\[ y_t^{(\tau)} \approx C_t^{(\tau)} + A_{r,t}^{(\tau)} \hat{r} + A_{\beta,t}^{(\tau)} \beta \]  

(11)

- In an affine model, \( A_{r,t}^{(\tau)} \) and \( A_{\beta,t}^{(\tau)} \) are constant (and the equation is exact).
- In the nonlinear model, they are state-dependent.

The sensitivity to both factors is quantitatively attenuated by the ELB.

The \( A_{\hat{r}}^{(\tau)} \) loadings change qualitatively, reversing their order across maturities.
Effects of shadow-rate shock on yield curve components

Impact of a one-standard-deviation shock to $\hat{r}_t$ from different initial values:

- **At the ELB:**
  - Overall effects are smaller.
  - Effects are increasing, not decreasing, across maturities.
  - Effects on the term premium are important.
To study the effects of actual Fed policy in this model, I calculate shocks that correspond to what the Fed actually did:

- **Shadow rate shocks** - kept $r_t$ at the ELB for 7 years.
- **Fed balance sheet shocks** - removed 18% of government-backed duration.
  
  These are assumed to be less persistent than the $\beta_t$ shocks above ($\phi = 0.96$), but this makes little difference.

Consider a set of trajectories that are consistent with these observations:
Cumulative yield-curve responses in model sims

Adding up the yield-curve surprises (pseudo event study):

- Magnitude is roughly consistent with the cumulative effects of unconventional policy implied by event studies.
- Model captures the "hump shaped" forward-curve response noted by Rogers et al. (2014) and others.
Decomposition of yields w/r/t unconventional policy shocks

Shadow-rate shocks account for over 80% of the effects of unconventional policy on long-term yields.

About 1/3 of this effect comes from the effects on term premia through reduced volatility.
Simple no-arbitrage model of bond portfolio choice w/ shadow rate.
Captures both forward guidance/signaling and duration channel of QE.
At the ELB, things change dramatically:
  - Effects of both types of shocks are attenuated by the ELB.
  - Forward guidance has effects on term premia at the ELB that don’t exist elsewhere.
Consequently, the effects of unconventional monetary policy at the ELB may not be well described by
  - Empirical estimates from pre-ELB data
  - Theoretical models that assume linearity
Simulations suggest that communications about future short rates were far more important for yields than was duration removal during the ELB period.