Noise-Ridden Lending Cycles[☆]

PRELIMINARY DRAFT

This Version: December 1, 2017

Elena Afanasyeva[*]

20th Street and Constitution Avenue NW, Washington, D.C. 20551, USA.
Email address: elena.afanasyeva@frb.gov.

Jochen Güntner[**]

Altenberger Straße 69, 4040 Linz, Austria.
Email address: jochen.guentner@jku.at.

Abstract

This paper builds on a neoclassical investment model to study the effect of imperfect information on the lending behavior of financial intermediaries. We first develop the intuition in partial equilibrium, where a risk-neutral competitive lender has limited knowledge of the current state of the economy. Non-fundamental noise shocks lower the equilibrium interest rate on risky credit as the lender is willing to extend more loans, which leads to higher ex-post default. We then estimate a simplified general equilibrium version of the neoclassical investment model and find that noise shocks account for up to one third of the forecast error variance of the credit spread in our theoretical model.

Keywords: Bayesian Maximum Likelihood, Credit Booms, Imperfect Information, Kalman Filter

JEL classification: D83, D84, E13, E44

[☆]We are grateful to Yuriy Gorodnichenko and Mirko Wiederholt for helpful comments and discussions. We thank Luigi Bocola, Johannes Brumm, Andrea Ferrero, Albert Marcet, Johannes Pfeifer, Morten Ravn, and Wouter den Haan as well as workshop and seminar participants at the 2nd Annual MACFINROBODS Conference in Brussels, the 8th RCEA Macro-Money-Finance Workshop in Rimini, the 2017 Annual Meeting of the SED, the 32nd Annual Congress of the European Economic Association, the Midwest Macro Meetings in Pittsburgh, Goethe University Frankfurt, and the Federal Reserve Board. Elena Afanasyeva gratefully acknowledges financial support by the FP7 Research and Innovation Funding Program (grant FP7-SSH-2013-2) and the hospitality of the Hoover Institution. Jochen Güntner gratefully acknowledges funding by the German Academic Exchange Service (DAAD) and the hospitality of the University of California at Berkeley. The views expressed in this paper are those of the authors and should not be attributed to the Board of Governors of the Federal Reserve System.

[*]Elena Afanasyeva is Economist at the Board of Governors of the Federal Reserve System.

[**]Jochen Güntner is Assistant Professor at the Department of Economics, Johannes Kepler University Linz.

1. Introduction

Do expectations about the current and future state of the economy matter for banks' lending decisions? According to the *Senior Loan Officer Opinion Survey* conducted by the Board of Governors of the Federal Reserve, commercial banks in the U.S. respond in the affirmative to this question. Figure 1 illustrates that the category "economic outlook" is an important reason for survey participants' decision to adjust their lending standards. In particular, current and future expected economic conditions seem to play a role when banks tighten standards in a recession as well as when they ease standards in a boom or recovery.

The existing literature on the role of banks' expectations in the emergence of lending cycles focuses on deviations from rational expectations when modeling the expectations formation process of economic agents. For example, Bordalo et al. (2017) build on insights from cognitive psychology and assume that risk-neutral investors form so-called *diagnostic* rather than rational expectations. De Grauwe and Macchiarelli (2015) instead assume that, due to limited cognitive abilities, banks apply simple extrapolative rules when deciding whether and under which conditions to grant a loan. While deviations from rational expectations represent an appealing alternative explanation for the emergence of credit booms and busts, they often

Figure 1: The importance of "economic outlook" for changes in lending standards of U.S. commercial banks

Notes: Number of banks reporting that "economic outlook" was unimportant, somewhat important, or very important for tightening (if positive) or loosening (if negative) lending standards. **Source:** Senior Loan Officer Opinion Survey on Bank Lending Practices dispose of the modeling discipline associated with the former. Moreover, irrational behavior comes in countless varieties, few of which make empirically testable predictions.

For this reason, we maintain the assumption of rational expectations, while departing from the frequently associated assumption of full or perfect information. Under *imperfect information*, rational economic agents are assumed to know the model environment, whereas they must "learn" the value of certain unobservable variables such as total factor productivity, for example. These learning dynamics can alter the endogenous propagation of shocks by affecting the behavior of economic agents.

Given that the profitability of investment is intimately linked to future economic activity, the expectations formation process of financial intermediaries is of particular interest. In this paper, we model the lending decision of a risk-neutral competitive bank subject to imperfect information about the state of the economy, which determines the default probability of a given loan. Hence, a positive signal about the current state of the economy induces the bank to lend to objectively riskier borrowers in expectation of higher future returns. If the signal is pure noise, however, the expansion of credit is not backed by economic fundamentals and would not occur under full information and rational expectations. Building on the partial equilibrium neoclassical investment model of Bordalo et al. (2017), we show that pure noise shocks can generate credit booms, if the bank cannot observe the relevant state of the economy and must infer the riskiness of a loan from other observables and a noisy public signal. This is costly in the sense that ex-post default increases as the bank's expectations turn out to be incorrect. It is important to note that such credit booms are driven only by our assumption of imperfect information and do not rely on any kind of financial friction.

In order to quantify the role of noise shocks in aggregate fluctuations, we embed our neoclassical investment model in a general equilibrium model with only two risk types of borrowers. We close the partial equilibrium model by assuming that bank lending is funded by the bank's accumulated net worth as well as by external funds in the form of risk-free deposits. In order to focus on the propagation of noise shocks through credit supply, we assume that the supply of bank deposits is perfectly elastic. We solve the linearized general equilibrium model under imperfect information and estimate its equivalent full information representation using Bayesian maximum likelihood techniques (see Blanchard et al., 2013).

Calibrating the model's driving processes along the lines of our empirical estimates, we find that noise shocks contribute up to one third to the forecast error variance of the spread between Moody's seasoned Baa corporate bond yield and the yield on 10-year treasury constant maturity at the 5-year horizon, whereas the contribution to the forecast error variance of total asset growth of U.S. corporate banks is negligible.

The rest of the paper is organized as follows. In Section 2, we provide evidence of imperfect information in the financial sector and of a link with credit market conditions. Section 3 uses a neoclassical investment model to illustrate how credit booms can emerge in the presence of imperfect information. Section 4 embeds the former model in a general equilibrium setting, where the equilibrium interest rate on risk-free deposits is determined by the consumption Euler equation of a representative household, and proposes a simplified version with only two risk types of borrowers. We estimate this model using Bayesian maximum likelihood techniques and evaluate the importance of informational frictions based on the recovered parameter estimates of the forcing processes. Section 5 concludes.

2. Empirical Evidence

In this section, we use survey data on banks' expectations from the Blue Chip Economic Indicators to provide evidence of imperfect information in the financial sector and a link between Blue Chip forecast errors and financial market conditions.[1] We focus on the so-called *Blue Chip consensus* forecast, i.e. the simple average of forecasts across all survey participants, of real GDP growth as a key measure of participants' expectations regarding current and future U.S. economic conditions.

Figure 2 plots Blue Chip consensus *forecasts* of year-on-year real GDP growth for forecast horizons from 0 to 4 quarters against a real-time benchmark released by the Federal Reserve Bank of Philadelphia with a lag of four quarters.[2] In general, the Blue Chip consensus forecasts are relatively accurate in real time and track the $t+4$-benchmark closely. Nevertheless, we detect quantitatively large and persistent deviations during selected historical episodes.[3] During 1987-1990 and during 1997-2000, for example, forecasts were persistently more pessimistic about current real GDP growth than suggested by real-time data. In contrast, forecasts were optimistic relative to the $t+4$-benchmark during 1990-1992 and during 2005-2010.

The observed persistence in deviations of Blue Chip consensus forecasts from the real-time benchmark

[1] The Blue Chip Economic Indicators is a monthly survey of more than 50 economists employed by some of America's largest manufacturers, banks, insurance companies, and brokerage firms. It was established in 1976 and collects professional forecasts on macroeconomics aggregates for the U.S. economy. What makes it particularly interesting and suitable for our question is that the majority of the survey participants are U.S. commercial and investment banks.

[2] We use real-time rather than final data on real GDP growth because reclassifications and redefinitions might make final data not directly comparable to Blue Chip forecasts (see Croushore, 2010). We use the $t+4$-benchmark rather than the so-called *advance release*, i.e. the $t+1$-benchmark, of the Federal Reserve Bank of Philadelphia, given that a large share of revisions in real-time data occurs within one year after the advance release.

[3] We plot the Blue Chip consensus nowcasts and forecasts based on the *third* monthly round of a given quarter, i.e. from the March, June, September, and December releases, to measure expectations as of quarter I, II, III, and IV, respectively. Releases from the first and second month of each quarter yield similar results.

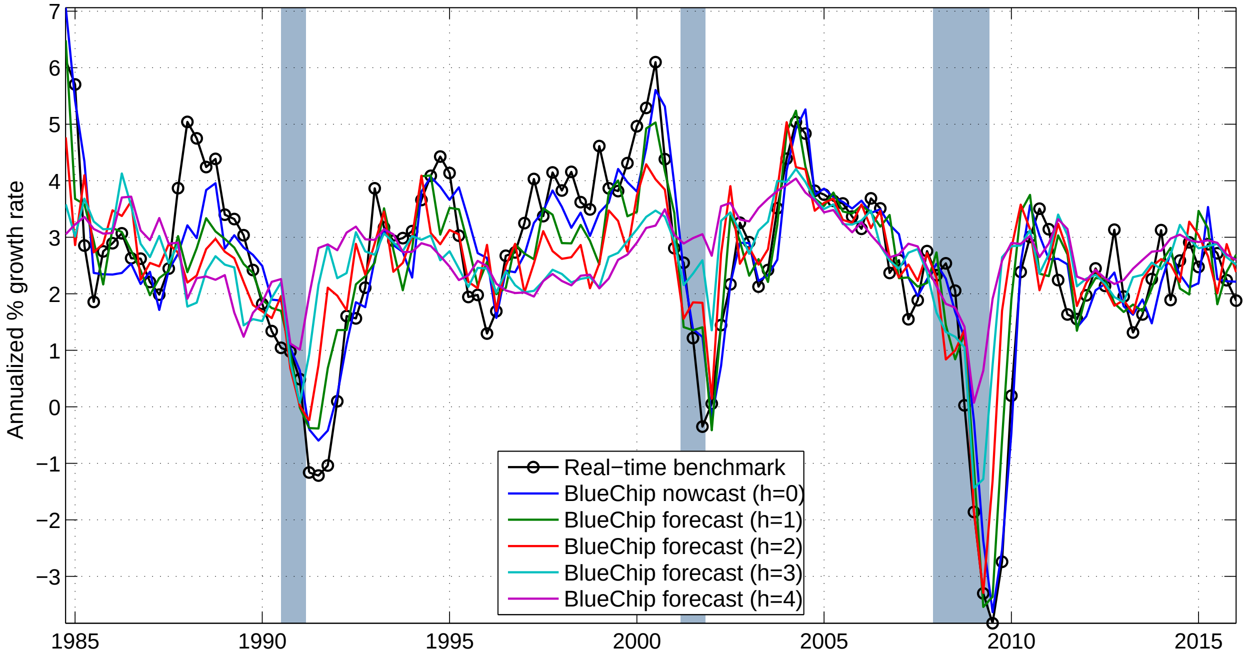

Figure 2: Blue Chip consensus nowcasts and forecasts of year-on-year real GDP growth and $t + 4$ real-time benchmark

is inconsistent with the assumption of full information and rational expectations. The latter implies that forecast errors are *unpredictable* if the underlying disturbances are unpredictable, whereas we find significant positive serial correlation in Blue Chip consensus forecast errors for all forecast horizons. In order to test more formally for the presence of informational frictions in banks' expectations, we draw on Coibion and Gorodnichenko (2015), who show that, if agents observe the variable x_t with normally distributed mean-zero noise that is i.i.d. across time and across agents, then the individual forecasts based on agents' information sets and the Kalman filter imply the following relationship between ex-post mean forecast errors and ex-ante mean forecast revisions:

$$x_{t+h} - F_t x_{t+h} = \frac{1-G}{G}\,(F_t x_{t+h} - F_{t-1} x_{t+h}) + u_{t+h,t}, \tag{1}$$

where F_t denotes the average forecast across agents at time t, G the so-called Kalman gain, and $u_{t+h,t}$ the rational expectations error. The predictability of average forecast errors reflects the gradual adjustment of conditional expectations to new information by the agents, who don't know whether this information reflects a change in fundamentals or pure noise. Note that the specification in (1) holds for any forecast horizon h as well as for forecasts over multiple horizons (see Coibion and Gorodnichenko, 2015).

Table 1: Tests of banks' expectations process for real GDP growth based on the regression equation in (2)

		yoy real GDP growth			qoq real GDP growth			
Horizon	Benchmark	c	β	R^2	c	β	R^2	N
$h = 0$	$t+1$	0.038	0.072*	0.013	0.233*	0.145	0.012	125
	$t+2$	0.048	0.098	0.012	0.288*	0.167	0.010	125
	$t+3$	0.045	0.209**	0.040	0.297**	0.087	0.003	125
	$t+4$	0.042	0.192*	0.026	0.272*	0.105	0.004	125
	$t+5$	0.029	0.190	0.022	0.280*	0.170	0.010	125
$h = 1$	$t+1$	0.089**	0.869***	0.673	0.156	1.011***	0.167	124
	$t+2$	0.100*	0.903***	0.560	0.211	1.044***	0.133	124
	$t+3$	0.102	1.008***	0.550	0.212	0.909***	0.101	124
	$t+4$	0.098	0.987***	0.468	0.197	0.979***	0.114	124
	$t+5$	0.092	0.972***	0.430	0.238	1.078***	0.131	124
$h = 2$	$t+1$	0.077	0.974***	0.480	−0.074	1.548***	0.118	123
	$t+2$	0.092	1.021***	0.442	0.010	1.824***	0.131	123
	$t+3$	0.096	1.121***	0.477	0.019	1.633***	0.106	123
	$t+4$	0.096	1.114***	0.445	−0.026	1.751***	0.117	123
	$t+5$	0.091	1.110***	0.410	0.018	1.984***	0.139	123
$h = 3$	$t+1$	0.101	0.911***	0.135	−0.276	0.928	0.019	122
	$t+2$	0.126	1.004***	0.145	−0.210	1.242	0.027	122
	$t+3$	0.141	1.120***	0.169	−0.177	1.180	0.025	122
	$t+4$	0.142	1.119***	0.158	−0.229	1.386	0.034	122
	$t+5$	0.136	1.140***	0.154	−0.213	1.524	0.037	122

Notes: $***/**/*$ indicates statistical significance at the 1/5/10% level based on Newey-West (HAC-robust) standard errors.

Accordingly, models with informational frictions such as noisy information, for example, imply that ex-ante average forecast revisions have predictive power for ex-post average forecast errors, whereas this is not the case under full information and rational expectations (FIRE). We replicate the regression analysis in Coibion and Gorodnichenko (2015) using Blue Chip consensus forecast errors and revisions as of period t for forecast horizon h:

$$x_{t+h} - F_t x_{t+h} = c + \beta (F_t x_{t+h} - F_{t-1} x_{t+h}) + error_t, \tag{2}$$

where a statistically significant coefficient β indicates a rejection of the null hypothesis of FIRE in favor of the alternative hypothesis of informational frictions in the presence of rational expectations. Table 1 reports our regression results for U.S. banks' expectations on year-on-year and quarter-on-quarter real GDP growth, respectively, for 1984Q4 through 2015Q4.

For year-on-year growth rates, the null of FIRE is strongly rejected in favor of informational frictions. The coefficient estimate on forecast revisions, β, is statistically significant (most frequently at the 1% level),

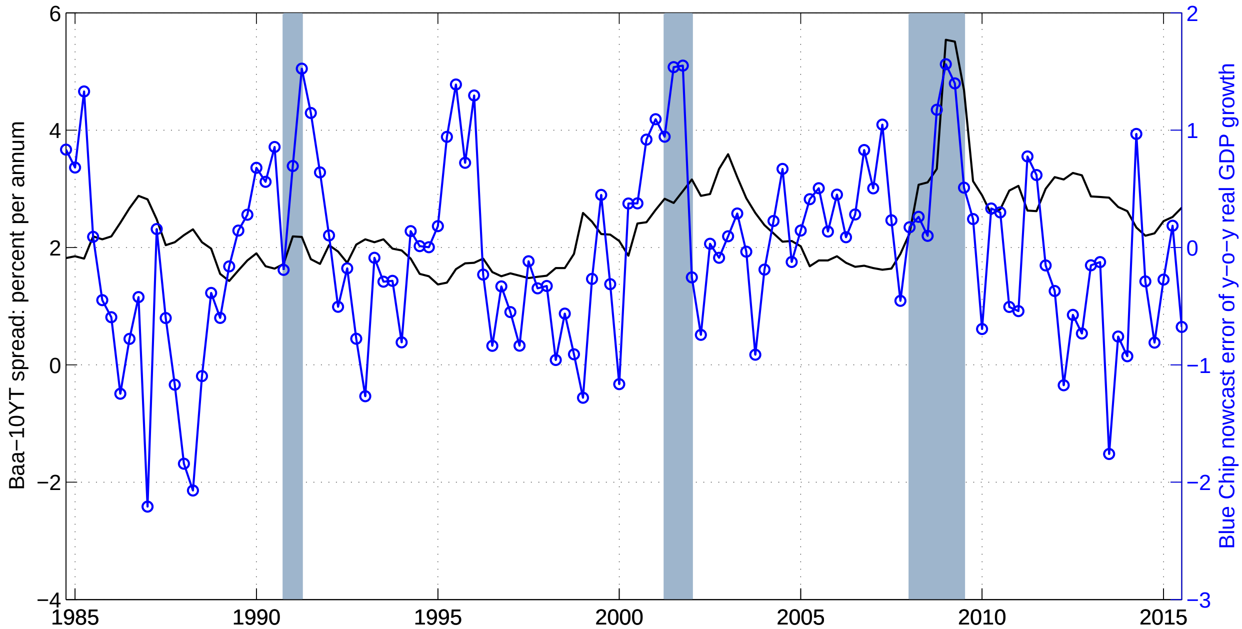

Figure 3: Blue Chip consensus nowcast errors of year-on-year real GDP growth and Baa−10YT spread

whereas the estimated intercept coefficient c is close to zero and not statistically significant. This finding is quantitatively more pronounced for $h = 1, 2, 3$ and also reflected in a higher value of R^2 in these cases. It is important to note that our results are robust to the choice of the real-time benchmark used to evaluate Blue Chip consensus forecast errors, where $t + 1$ corresponds to the so-called *advance release* and $t + l$ to the data on period t made available by the Federal Reserve Bank of Philadelphia with a lag of l quarters.

For quarter-on-quarter growth rates of real GDP growth, the results are qualitatively similar for $h = 1$ and $h = 2$, yet less clear-cut for $h = 0$ and $h = 3$. Given that quarter-on-quarter data displays more unexplained fluctuations, the corresponding R^2 values are lower than their year-on-year counterparts. At the same time, the point estimates of β tend to be larger.

Coibion and Gorodnichenko (2015) show that the point estimate of β translates directly into the degree of informational rigidity in a noisy information model. For $\beta = 1.1$, i.e. the mode of our point estimates, the Kalman gain $G = 1/(1+\beta) = 0.476$ reflects the weight that agents assign to new information relative to their previous forecasts.[4] Based on this evidence, we reject the null hypothesis of FIRE in favor of the alternative hypothesis of informational frictions in Blue Chip consensus forecasts of real GDP growth – a measure of U.S. banks' expectations about the current and future state of the economy.

[4]The Kalman gain also reflects the average reduction in the variance of contemporaneous forecast errors relative to the variance of one-step ahead forecast errors (see Coibion and Gorodnichenko, 2015).

Table 2: Results of Granger non-causality tests based on bivariate VAR models

	Blue Chip consensus nowcast error			
	yoy real GDP growth		qoq real GDP growth	
Financial conditions	error → fin. cond.	fin. cond. → error	error → fin. cond.	fin. cond. → error
Baa−Aaa	*	−	**	−
Baa−10YT	* * *	−	* * *	−
Aaa−10YT	* * *	*	* * *	*
GZ-Spread	**	−	* * *	−
NFCI	−	−	−	−
NFCI credit	* * *	**	* * *	*
NFCI risk	−	−	−	−
NFCI leverage	−	−	−	−
NFCI nonfinleverage	* * *	−	**	−

Notes: * * */ * */* indicates statistical significance at the 1/5/10% level. Baa−Aaa denotes the spread between Moody's seasoned Baa and Aaa corporate bond yields, Baa−10YT the spread between Moody's seasoned Baa corporate bond yield and the yield on 10-year treasury constant maturity, Aaa−10YT the spread between Moody's seasoned Baa corporate bond yield and the yield on 10-year treasury constant maturity, GZ-spread the credit spread index constructed by Gilchrist and Zakrakšek (2012) based on micro-level data, NFCI the *National Financial Conditions Index* published by the Federal Reserve Bank of Chicago, and NFCI x the corresponding credit, risk, financial sector leverage, and nonfinancial sector leverage sub-indices.

Figure 3 plots the Blue Chip consensus *nowcast error* of year-on-year real GDP growth against the difference between Moody's seasoned Baa corporate bond yield and the yield on 10-year treasury constant maturity, illustrating potential comovement between nowcast errors and a measure of financial conditions. To test whether banks' expectations have predictive power for fluctuations in credit conditions, we relate the Blue Chip nowcast errors to alternative measures of financial market conditions including the Baa−10YT spread. Table 2 reports the results of Granger non-causality tests based on bivariate vector-autoregressive (VAR) models in the Blue Chip nowcast error of real GDP growth and various measures of financial market conditions, where the number of asterisks indicates the degree of statistical significance.

The first column of Table 2 illustrates that, using year-on-year real GDP growth, we can reject the null hypothesis of *no* predictive power of Blue Chip nowcast errors for financial conditions in six out of nine cases at the 10%-level or better, whereas Granger non-causality from financial conditions to nowcast errors can be rejected in two out of nine cases at a lower level of statistical significance. The second column illustrates that this finding is robust to the use of quarter-on-quarter rather than year-on-year real GDP growth. Hence, there is suggestive evidence of imperfect information in financial sector forecasts of real economic activity, while the resulting forecast errors have significant predictive power for U.S. credit market conditions.

3. The Partial Equilibrium Model

We start by illustrating the intuition with a neoclassical partial equilibrium investment model similar to the one proposed by Bordalo et al. (2017).

3.1. Partial Equilibrium Model under Full Information

Time $t = 1, 2, \ldots$ is discrete and the economy's state at time t, $\Omega_t \in \mathbb{R}$ with realization ω_t, follows a Markov process with a normal distribution conditional on Ω_{t-1}, as in the AR(1) case

$$\omega_t - \mu_\omega = b\,(\omega_{t-1} - \mu_\omega) + \epsilon_t^\omega, \tag{3}$$

with $\epsilon_t^\omega \sim N\left(0, \sigma^2\right)$ and $\mu_\omega \in \mathbb{R}$, $b \in [0, 1]$.

3.1.1. Credit demand by firms

A unit measure of atomistic firms uses capital to produce output, where productivity at time t depends on ω_t to a different extent for different firms. Each firm is identified by its *risk type*, $\rho \in \mathbb{R}$. Firms with higher ρ are less likely to be productive in any state ω_t and represent thus a riskier investment. The output of a type-ρ firm at time t is given by

$$y\,(k|\omega_t, \rho) = \begin{cases} k^\alpha & \text{if} \quad \omega_t \geq \rho \\ 0 & \text{if} \quad \omega_t < \rho \end{cases},$$

where $\alpha \in (0, 1)$.

The capital used for production at time $t + 1$ must be installed at time t already, before ω_{t+1} is known. For simplicity, we assume that capital depreciates fully in production. Each firm's risk type ρ is *common knowledge* and distributed across firms according to the continuous density function $f(\rho)$.

Each firm's capital investment is fully debt funded. A firm of type ρ borrows funds $l_t(\rho) = k_{t+1}(\rho)$ from a bank, taking the contractual interest rate $r_t(\rho)$ as given. It repays the loan only if the realized state of the economy allows the firm to be productive. Else, it defaults and repays nothing.

Assuming perfect competition in production, each firm type ρ borrows up to the point where the marginal product of capital equals the cost of borrowing from the bank, i.e.

$$l_t(\rho) = k_{t+1}(\rho) = \left[\frac{\alpha}{r_t(\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{4}$$

3.1.2. Credit supply by banks

Suppose that the credit market is perfectly competitive, while the representative bank owns a stock of net worth that can be used to grant loans. In contrast to the representative household in Bordalo et al. (2017), which elastically supplies any amount of capital at the risk-free interest rate, the lenders in our model do *not* make optimal consumption-saving decisions. Instead, we assume that the risk-neutral representative bank invests all of its net worth, n_t, until it exits the credit market (exogenously) and gets to consume its accumulated net worth.

Similar to the behavior of entrepreneurs in Bernanke et al. (1999), the representative bank myopically maximizes its expected net worth in the next period. Given a predetermined amount of bank net worth, n_t, and a risk-free interest rate on deposits, r_t^d, that is assumed to be determined outside the partial equilibrium model (e.g. by monetary policy or household preferences), banks maximize $E_t n_{t+1}$ by optimally choosing the amount of lending $l_t(\rho)$ supplied to each firm type ρ:

$$\begin{aligned}
&\max_{l_t(\rho)} \quad Prob_t\left(\omega_{t+1} \geq \rho\right) r_t(\rho) l_t(\rho) - \left[l_t(\rho) - n_t(\rho)\right] r_t^d \\
&\frac{\partial E_t n_{t+1}}{\partial l_t(\rho)} = Prob_t\left(\omega_{t+1} \geq \rho\right) r_t(\rho) - r_t^d = 0 \\
\Leftrightarrow \quad & r_t^*(\rho) = \frac{r_t^d}{Prob_t\left(\omega_{t+1} \geq \rho\right)},
\end{aligned} \tag{5}$$

where $Prob_t(\bullet)$ denotes the expected repayment probability as of period t, which is assumed to be independent of $r_t(\rho)$. As a result, we have a unique interior solution for the bank's optimal choice of $r_t(\rho)$ for each firm type ρ.[5] Equations (4) and (5) yield a unique interior solution for the firm's optimal demand for credit:

$$l_t^*(\rho) = \left[\frac{\alpha \cdot Prob_t\left(\omega_{t+1} \geq \rho\right)}{r_t^d}\right]^{\frac{1}{1-\alpha}}. \tag{6}$$

From equations (5) and (6), a higher expected probability of repayment translates into a lower interest rate on bank credit, $r_t^*(\rho)$, and thus a higher demand for bank credit and capital, $l_t^*(\rho)$, for each firm type ρ.

Figure 4 plots the equilibrium interest rate in (5) and the demand for capital and bank credit in (6) *by firm type* ρ and illustrates that low-ρ types with a negligible risk of default pay an interest rate on bank credit

[5] Our formulation of the representative bank's profit maximization problem implies that the *marginal* unit of credit is financed by deposits rather than bank net worth. Hence, the marginal cost of extending an additional unit of credit equals the risk-free interest rate r_t^d rather than the opportunity cost of bank net worth.

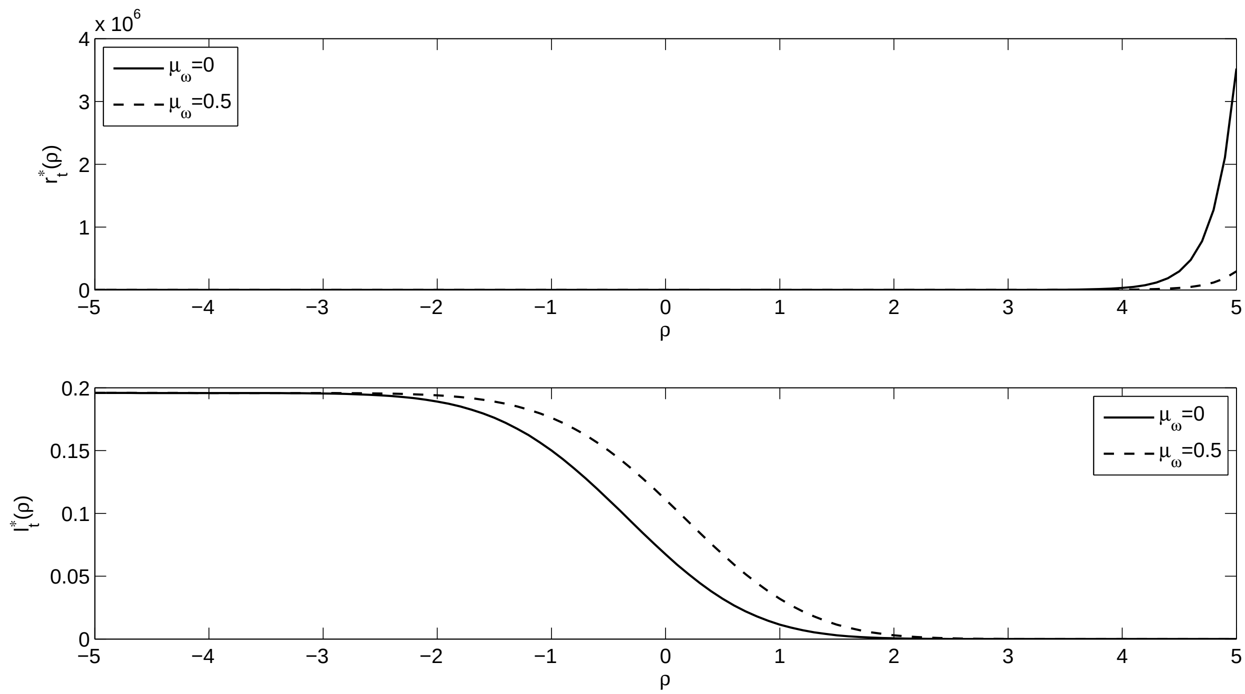

Figure 4: Equilibrium interest rates and demand for bank credit by firm type ρ

that is close to the risk-free rate. The figure represents a snapshot of the economy for a given expected value and variance of ω_{t+1} conditional on the information available in period t. The broken lines depict the effect of an increase in $E_t\omega_{t+1} = \mu_\omega$ without any change in $E_t\sigma_{\omega,t+1}$.

In contrast, Figure 5 plots the equilibrium interest rate in (5) and the demand for capital and bank credit in (6) *as a function of the expected repayment probability*. Changes in $E_t\omega_{t+1}$ affect the position of a given firm type ρ on the x-axis rather than shifting graphs (as in Figure 4). Figure 5 illustrates that symmetric increases and decreases in the expected repayment probability have *asymmetric effects* on the equilibrium interest rate and thus on the demand for bank credit for a given firm type ρ. Due to the presence of $Prob_t\left(\omega_{t+1} \geq \rho\right)$ in the denominator of (5), this asymmetry is more pronounced for low expected repayment probabilities, where a decrease in $Prob_t\left(\omega_{t+1} \geq \rho\right)$ raises the equilibrium interest rate by more than a symmetric increase would lower $r^*\left(\rho\right)$.

3.2. Partial Equilibrium Model under Imperfect Information

Suppose that the exogenous default threshold ω_t summarizes the persistent and transitory components ν_t and η_t, respectively, which are both *unobservable*. As a consequence, the bank must form expectations about ω_{t+1} based on its nowcast of the unobservable components in period t. Using the above notation, the

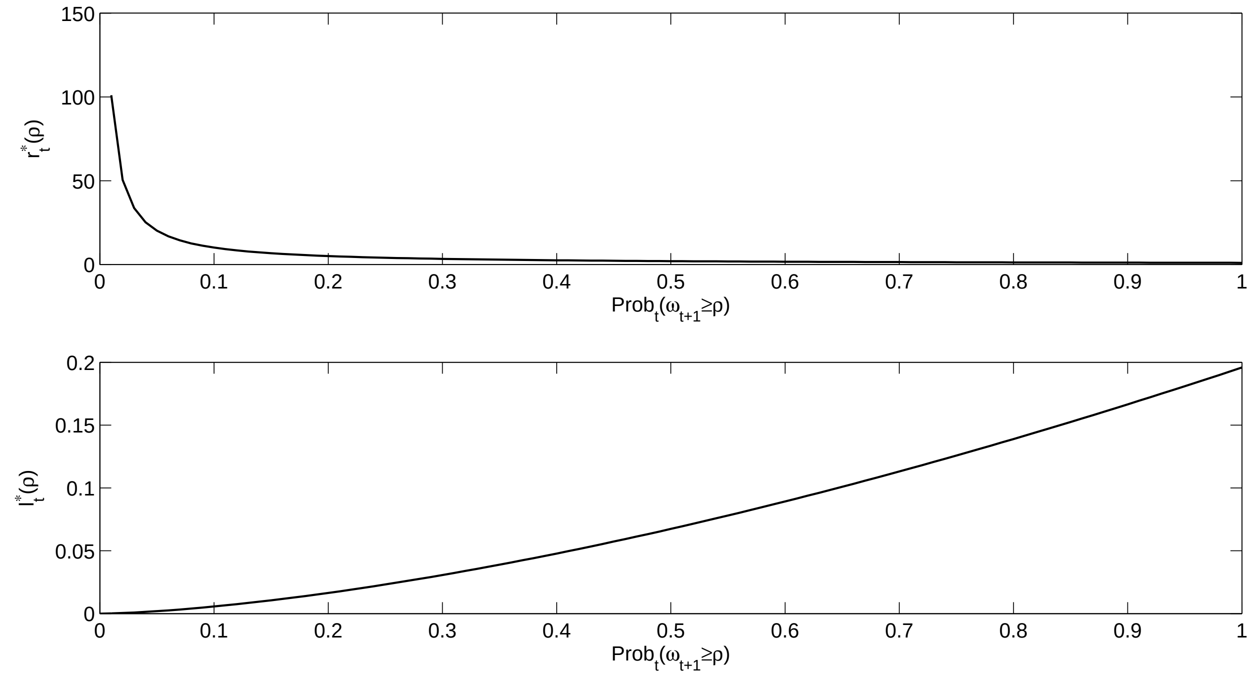

Figure 5: Equilibrium interest rates and demand for bank credit by expected repayment probability

exogenous processes of the bank's information problem can be written as

$$\begin{aligned} \omega_t &= \nu_t + \eta_t, \\ \nu_t &= \rho_\nu \nu_{t-1} + e_t, \qquad e_t \sim N\left(0, \sigma_e^2\right), \\ \eta_t &= \rho_\eta \eta_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N\left(0, \sigma_\epsilon^2\right), \end{aligned} \tag{7}$$

where e_t and ϵ_t are assumed to be *contemporaneously* and *serially uncorrelated*, and $\rho_\nu > \rho_\eta$. Following Lorenzoni (2009), we further assume that the bank receives a noisy public signal of the persistent component ν_t at time t, i.e.

$$\tilde{s}_t = \nu_t + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right), \tag{8}$$

where ε_t is assumed to be contemporaneously and serially uncorrelated with e_t and ϵ_t.

The disturbance term ε_t in (8) plays two roles. First, it prevents the bank from perfectly observing the persistent component of the aggregate state. Second, it generates an independent source of variation in the bank's beliefs about ν_t. Note that the disturbance terms in η_t and $\tilde{s}_t$ have very different interpretations. While ϵ_t is a shock to the transitory component of the aggregate state and affects thus ω_t, ε_t is a non-fundamental "noise shock", which propagates only through the bank's period-t expectations about ω_{t+1}.

In our partial equilibrium model, the market-clearing interest rate of a loan to firm type ρ thus equals

$$r_t^*(\rho) = \frac{r_t^d}{Prob\left(\omega_{t+1} \geq \rho | \omega_t, \omega_{t-1}, \ldots, \tilde{s}_t, \tilde{s}_{t-1}, \ldots\right)} \quad \forall \rho, \tag{9}$$

where the information set of the bank in period t is confined to current and past realizations of the observable variables ω_t and $\tilde{s}_t$.

Note that only shocks to the persistent component contribute to fluctuations in $\nu_t = \rho_\nu \nu_{t-1} + e_t$. However, ν_t is not observable either contemporaneously or with a lag. Assuming rational expectations (RE), agents must therefore infer the current value of ν_t from observable variables. Applying Kalman filtering techniques and assuming that $\rho_\eta = 0$ in the transitory component, it can be shown that the optimal period-t nowcast of the persistent component corresponds to the following projection on ω_t, $\tilde{s}_t$, and the period-$t-1$ nowcast of ν_{t-1}:

$$\nu_{t|t} = \left(\rho_\nu - \kappa_1 - \kappa_2\right) \nu_{t-1|t-1} + \kappa_1 \omega_t + \kappa_2 \tilde{s}_t, \tag{10}$$

where $\kappa_1 \equiv \frac{\sigma_\nu^2 \sigma_\varepsilon^2}{\sigma_\nu^2 \sigma_\epsilon^2 + \sigma_\nu^2 \sigma_\varepsilon^2 + \sigma_\epsilon^2 \sigma_\varepsilon^2}$, $\kappa_2 \equiv \frac{\sigma_\nu^2 \sigma_\epsilon^2}{\sigma_\nu^2 \sigma_\epsilon^2 + \sigma_\nu^2 \sigma_\varepsilon^2 + \sigma_\epsilon^2 \sigma_\varepsilon^2}$, and σ_ν^2 implicitly solves $\sigma_\nu^2 = \rho_\nu^2 \left(\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_\nu^2} \right)^{-1} + \sigma_e^2$.

Ceteris paribus, the weight on either observable is increasing in the variance of the other observable as well as in the variance of the persistent component. Conversely, $\kappa_1 \to 0$ as $\sigma_\varepsilon^2 \to 0$, $\kappa_2 \to 0$ as $\sigma_\epsilon^2 \to 0$, and $\kappa_1, \kappa_2 \to 0$ as $\sigma_\nu^2 \to 0$. Hence, a higher weight is put on the relatively more precise observable, whereas both observables receive a higher weight if ν_t exhibits more volatility. In Appendix A, we derive the state-space representation of the bank's signal extraction problem and the complete set of Kalman filter updating and forecasting expressions in matrix notation.

3.3. Impulse Response Functions under Imperfect Information

Based on the Kalman filter updating and forecasting expressions in equations (A.1) and (A.2), we can simulate the bank's expectation about ω_{t+1} and thus the expected repayment probability for each firm type ρ as of time t, $Prob\left(\omega_{t+1} \geq \rho | \omega_t, \omega_{t-1}, \ldots, \tilde{s}_t, \tilde{s}_{t-1}, \ldots\right)$. Using these probabilities in the partial equilibrium model in equations (6) and (9), we can then derive the bank's profit-maximizing interest rate for a given loan and the corresponding credit demand for each firm type ρ.

To compute the *ex-ante* aggregate demand for credit by firms at time t *before the realization of* ω_t, we integrate over the support of the density function of firm types, $f(\rho)$. To compute the *ex-post* aggregate amount of capital available for production *after the realization of* ω_t, we integrate over $f(\rho)$ from the lower bound of the support up to ω_{t+1}. Accordingly, the aggregate demand for credit is computed for *all* firm types ρ, whereas the aggregate amount of capital at time t is computed only for non-defaulting firm types $\rho < \omega_t$.

Due to the non-linearity of the partial equilibrium model in (6) and (9), we compute *generalized impulse response functions* (see Koop et al., 1996) to a shock in the persistent (a.k.a. "trend"), the transitory (a.k.a. "cycle"), and the noise component, respectively, by simulation as follows:

1. Drawing e_t, ϵ_t, and ε_t from their stochastic distributions, we simulate equations (7) and (8) for $T = 250$ periods and save the resulting time series for ω_t and $\tilde{s}_t$. We then impose a unit shock on the persistent, transitory, or noise component at time t_0, i.e. $e_{t_0} = 1$, $\epsilon_{t_0} = 1$, or $\varepsilon_{t_0} = 1$, and save the resulting time series for ω_t' and $\tilde{s}_t'$.
2. Given initial values for $\mathbf{s}_{0|0}$ and $\Sigma_{0|0}$, we compute $\mathbf{s}_{1|0}$, $\Sigma_{1|0}$, and $\mathbf{y}_{1|0}$ and simulate equations (A.1) and (A.2) recursively for $t = 1, ..., T$. Using the Kalman filter forecast of $Prob_t(\omega_{t+1} \geq \rho)$, we compute the equilibrium interest rate and the corresponding credit demand for each firm type ρ and integrate over all firm types.
3. The path-dependent impulse response function to a shock in the persistent, transitory, or noise component can then be computed as the period-by-period difference between the path of a certain ρ-specific or aggregate variable with and without the respective shock for $t = 1, ..., T$.
4. We repeat steps 1-3 a large number (N) of times and take the period-by-period average across the path-dependent impulse response functions for all N replications.

In all simulations, we use $N = 500$ replications and allow for a burn-in of 100 periods to be discarded when computing impulse response functions.

The exogenous processes in (7) and (8) are calibrated in line with our estimates of a general equilibrium version of this model discussed below, i.e. $\rho_\nu = 0.7445$, $\sigma_e = 0.0645$, $\sigma_\epsilon = 1.5432$, and $\sigma_\varepsilon = 0.6415$, while we set $\rho_\eta = 0$ for illustrative purposes. The remaining parameters of our partial equilibrium model are set to conventional values. In particular, we set the elasticity of output with respect to capital to $\alpha = 0.35$ and the gross risk-free interest rate on bank deposits to $r_t^d = r^d = 1.01$. We further assume that firm types ρ are normally distributed with zero mean and unit variance, i.e. $\rho \sim N(0, 1)$.[6]

[6] The assumption on the distribution of ρ is not crucial for the results. We get qualitatively similar results under the assumption of a uniform distribution.

The impulse responses to a *positive unit shock* in the persistent, transitory, and noise component, respectively, are plotted in Figures 6, 7, and 8. In each case, we simulate and plot impulse response functions under imperfect information ("Kalman") as well as under full information and rational expectations ("FIRE"), where the information set in period t contains the current and past realizations of ω_t, $\tilde{s}_t$, ν_t, and η_t, i.e. the bank separately observes the transitory and persistent components of ω_t under FIRE.

3.3.1. "Trend" shocks

The upper left panel in Figure 6 illustrates that, in response to a unit shock in the persistent component, ω increases by the amount of the disturbance *on impact*.[7] Under imperfect information and rational expectations, the source of this increase is unobservable, whereas ν_t and η_t can be observed separately under full information and rational expectations. Accordingly, the imperfectly informed bank initially attributes a substantial probability to the possibility that the observed increase in ω_t indicates a transitory (i.e. cycle) or noise shock rather than a shock to the persistent component. As the observable remains positive afterwards, the bank revises its nowcast of the persistent component and thus its forecast of ω_{t+1} upwards, while the uncertainty about the origin of the observed change in ω_t remains. Only as ν_t and ω_t converge to their long-run means of zero, the bank stops making systematic forecast errors. In contrast, the bank immediately knows that the increase in the observable variable is due to its persistent component under FIRE.

For illustrative purposes, we report the responses of two ρ-specific equilibrium lending rates in line with equation (5). We choose $\rho_1 = -3$ and $\rho_2 = -1.5$ in order to represent a relatively safer and a relatively riskier firm type from $\rho \sim N(0, 1)$. In response to a positive shock to the trend component, the bank optimally lowers the lending rate for both ρ_1 and ρ_2. Given the necessity of learning about the actual state of the economy, however, the decrease in $r_t^* (\rho)$ is strongly muted and deferred under imperfect information (Kalman), whereas the equilibrium interest rates under FIRE replicate qualitatively the impulse response function of the persistent component.

The lower left panels in Figure 6 illustrate the effects of a trend shock on ex-ante aggregate credit and ex-post aggregate capital. The reduction in equilibrium lending rates induces all firm types ρ to purchase more capital financed by bank credit. This increase in the demand for credit is driven exclusively by the increase in the bank's expected probability of repayment in the lower right panel, which follows the same qualitative pattern as $\nu_{t|t}$.

[7]Note that the term "trend" shocks in the title is a slight misnomer here, given that the persistent component ν_t is stationary.

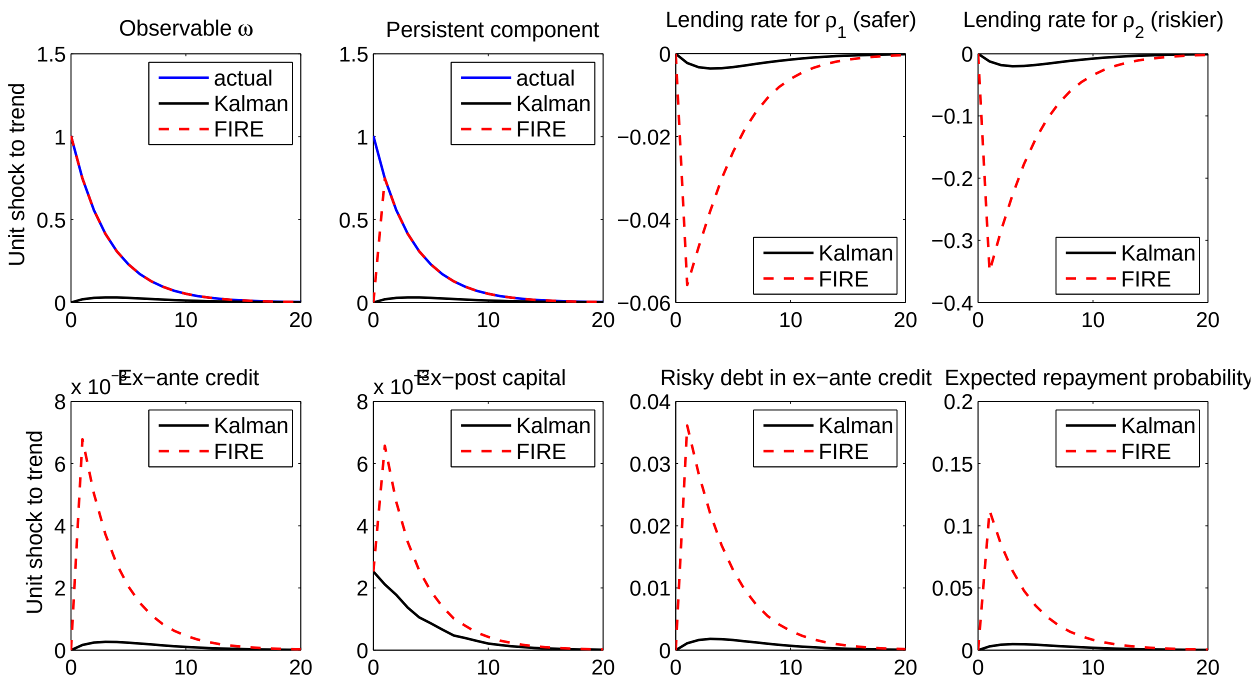

Figure 6: Impulse response functions to a unit shock in the persistent component based on the partial equilibrium investment model

Finally, the lower next-to-right panel in Figure 6 illustrates that the share of loans to objectively riskier firm types in the bank's portfolio *increases* in response to a positive innovation in the persistent component. Due to the fact that a relatively riskier firm type, such as $\rho_2 = -1.5$, is generally more likely to default than a firm type $\rho_1 = -3$, a change in $\omega_{t+1|t}$ has a relatively larger effect on $Prob\left(\omega_{t+1} \geq \rho_2 | \omega_t, \omega_{t-1}, \ldots, \tilde{s}_t, \tilde{s}_{t-1}, \ldots\right)$ than on $Prob\left(\omega_{t+1} \geq \rho_1 | \omega_t, \omega_{t-1}, \ldots, \tilde{s}_t, \tilde{s}_{t-1}, \ldots\right)$. Note that this result is not sensitive to our assumptions about $f(\rho)$.

3.3.2. "Cycle" shocks

The upper left panels of Figure 7 illustrate that, although the effect of a shock in the "cycle" component is purely transitory, the bank still attributes part of the observed increase in ω to the persistent component. Accordingly, the bank expects a higher probability of repayment and expands its supply of credit to both the relatively safer and the relatively riskier firm type. From the upper right panels, the decrease in $r_t^*(\rho_2)$ is quantitatively more pronounced than the decrease in $r_t^*(\rho_1)$. As a consequence, firms' aggregate demand for credit increases, replicating the pattern in ρ-specific equilibrium lending rates with an opposite sign.

The slow learning process under imperfect information implies that the bank's perceived probability of repayment for each firm type ρ remains elevated for an extended period before converging back to the long-

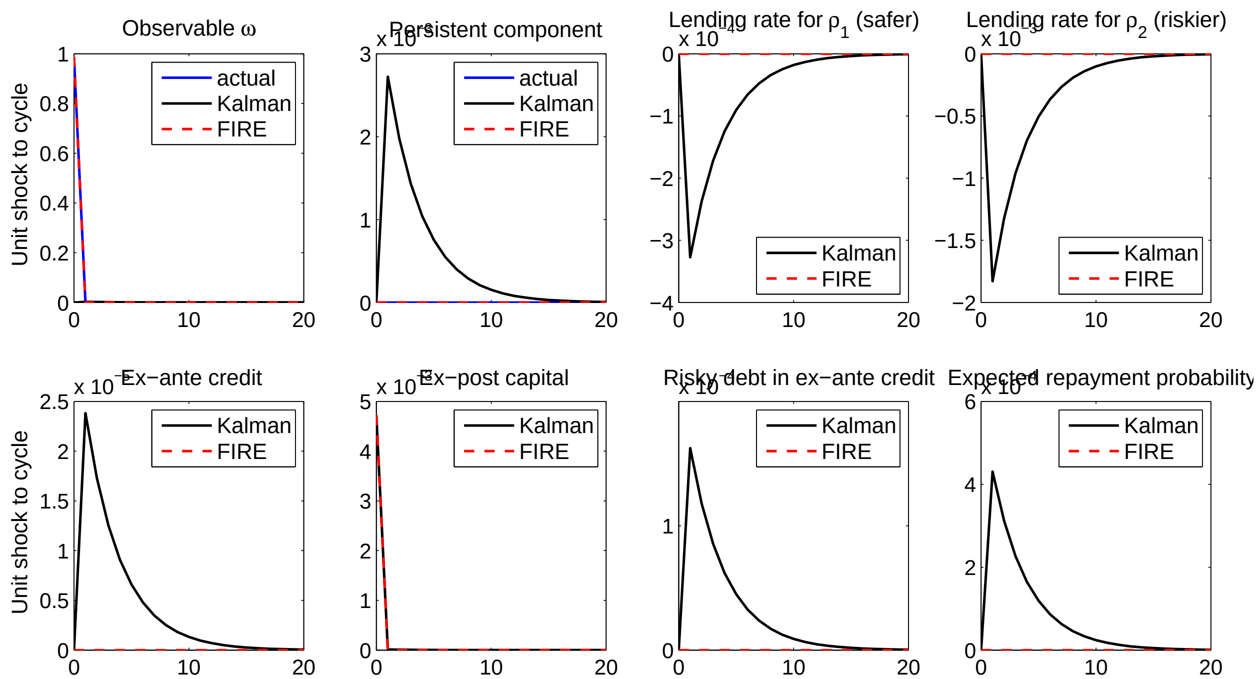

Figure 7: Impulse response functions to a unit shock in the transitory component based on the partial equilibrium investment model

run equilibrium. Accordingly, equilibrium lending rates and ex-ante aggregate credit slowly converge back to the long-run equilibrium. In contrast, neither the lending rates nor ex-ante aggregate credit respond to a cycle shock under FIRE at all. Once the agent observes the shock, its transitory nature ($\rho_\eta = 0$) implies that it is too late to expand the supply of credit by lowering lending rates. Under FIRE, it is therefore optimal to "let bygones be bygones", if a shock to fundamentals is purely transitory.

Both under imperfect information and under FIRE, ex-post aggregate capital increases on impact due to the transitory increase in the observable ω_t and the corresponding decrease in ex-post firm default, while its impulse response function remains quantitatively below that of ex-ante aggregate credit from period 2 onwards. The vertical distance between the response of ex-ante credit and the response of ex-post capital corresponds to ex-post aggregate default of credit due to imperfect information.

From the lower right panels of Figure 7, a positive shock to the cycle component results in a persistent increase in the share of loans to *objectively* riskier (i.e. higher-ρ) firm types and in the *subjectively* expected probability of repayment of the bank's loan portfolio at the same time.

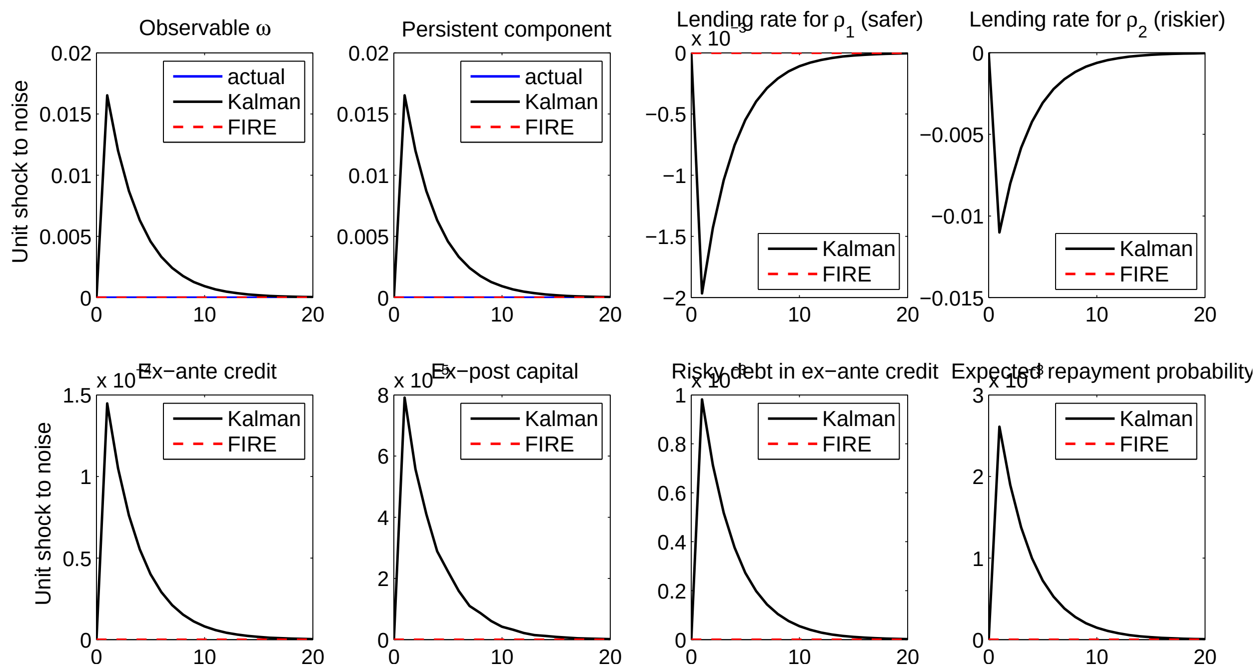

Figure 8: Impulse response functions to a unit shock in the noise component based on the partial equilibrium investment model

3.3.3. Noise shocks

Figure 8 plots the impulse responses to a unit shock in the noise component of the public signal in (8). From the upper left panel, a pure noise shock does not have any impact on the observable ω_t. Nevertheless, the positive signal induces the bank to attribute a nonzero probability to the possibility of an increase in the persistent component and thus in the Kalman-filter forecast of ω_t under imperfect information.

Given its expectation of a higher value for ω_{t+1}, the bank revises its expected probability of repayment upwards and expands its supply of credit to relatively safer firm types, such as $\rho_1 = -3$, and even more so to relatively riskier firm types, such as $\rho = -1.5$. As a consequence, the corresponding equilibrium lending rates decrease, while firms' demand for credit increases, replicating the pattern in ρ-specific lending rates with an opposite sign. In contrast, neither the lending rates nor aggregate credit respond to a cycle shock under FIRE, where the bank realizes that the increase in $\tilde{s}_t$ represents pure noise.

In order to illustrate the role of imperfect information as a possible driver of credit booms in our partial equilibrium model, Figure 9 plots the impulse responses of ex-ante credit and ex-post capital to a pure noise shock and indicates ex-post aggregate default as the shaded area between the two responses. Accordingly, the learning process in this simple model implies that the bank might become more optimistic in response to misleading news about the state of the economy, lower its lending rates accordingly to attract borrowers,

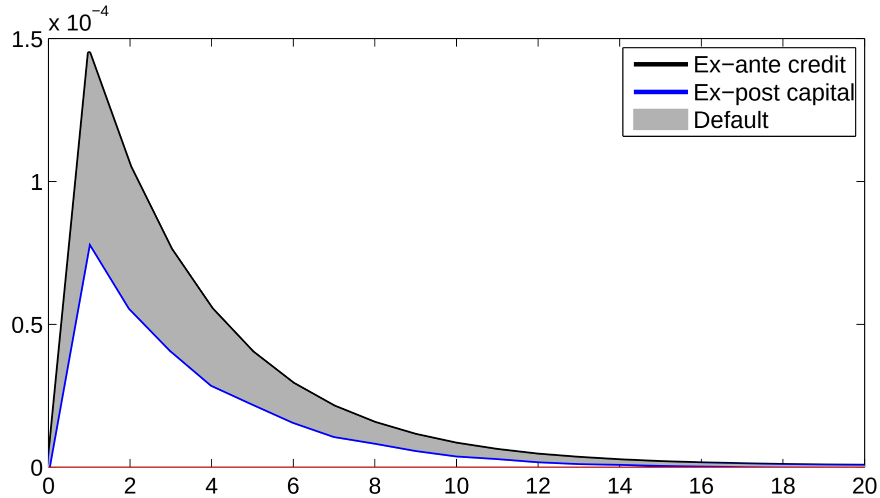

Figure 9: Impulse response function of ex-post credit default to a unit shock in the noise component

and cause thus an aggregate credit expansion accompanied by an increase in firm default. Note that a similar noise-ridden credit boom does not occur under FIRE.

4. The General Equilibrium Model

4.1. A General Version

Suppose that lending to firms is undertaken by a continuum of perfectly competitive financial intermediaries, which are endowed with aggregate bank equity N_t and collect aggregate deposits D_t in each period t. Assuming that firm credit represents the only asset, while bank equity and deposits represent the only liabilities, the financial sector's aggregate balance sheet identity is given by

$$L_t \equiv N_t + D_t, \tag{11}$$

where L_t denotes ex-ante aggregate credit at the *end of period t*. The sole purpose of bank equity is to shield depositors from unexpected fluctuations in the aggregate return on firm credit and guarantee them a risk-free rate of return on deposits in each state of the world, ω_t. To avoid that bank equity N_t grows without bound, each period a constant fraction δ is assumed to be consumed exogenously by the financial sector. Assuming a uniform distribution for firm types $\rho \in \left[\underline{\rho}, \bar{\rho}\right]$, aggregate bank equity evolves according to

$$N_t = (1-\delta)\left[\int_{\underline{\rho}}^{\omega_t} r_{t-1}(\rho)\, k_t^*(\rho)\, d\rho - R_{t-1}^d D_{t-1}\right] = (1-\delta)\left[\int_{\underline{\rho}}^{\omega_t} r_{t-1}(\rho)\, k_t^*(\rho)\, d\rho - R_{t-1}^d (L_{t-1} - N_{t-1})\right],$$

where R^d_{t-1} denotes the *gross* risk-free rate of return on bank deposits between period $t-1$ and period t.

In order to isolate the propagation of noise shocks via the supply of bank credit from potential demand-side effects (e.g. consumption and savings), we assume that bank deposits are supplied perfectly elastically by risk-neutral foreign depositors that demand the exogenous interest rate R^w_t, a.k.a. the "world interest rate". Accordingly, the economy-wide resource constraint must account for the payments of principal and interest on maturing period-$t-1$ deposits as well as for the inflows of new deposits in period t, i.e.

$$Y_t = C_t + N_t - D_t + R^d_{t-1} D_{t-1}, \tag{12}$$

where aggregate consumption C_t captures any residual demand for domestic output. Given that we abstract from the possibility of bank failure, deposits are effectively risk-free and pay the gross rate of return $R^d_t = R^w_t$ *in all states* of the economy. Without loss of generality, we further abstract from exogenous fluctuations in the world interest rate and assume that $R^w_t = R^w$ *in each period* t.

Note that aggregate *ex-ante* credit in period t,

$$L_t = \int_{\underline{\rho}}^{\bar{\rho}} l^*_t(\rho)\, d\rho,$$

is weakly larger than aggregate *ex-post* capital available for production in period $t+1$,

$$K_{t+1} = \int_{\underline{\rho}}^{\bar{\omega}_{t+1}} k^*_{t+1}(\rho)\, d\rho.$$

Making use of the bank's balance sheet identity in (11) and the production function for firm type ρ,

$$y^*_t(\rho) = \begin{cases} k^*_t(\rho)^\alpha & \text{if } \omega_t \geq \rho \\ 0 & \text{if } \omega_t < \rho \end{cases},$$

we can rewrite the resource constraint in (12) as

$$Y_t = \int_{\underline{\rho}}^{\omega_t} k^*_t(\rho)^\alpha\, d\rho = C_t + \int_{\underline{\rho}}^{\bar{\rho}} l^*_t(\rho)\, d\rho - 2D_t + R^d_{t-1} D_{t-1},$$

where $\rho \in [\underline{\rho}, \omega_t]$ contains only firm types that are productive in period t, whereas $\rho \in [\underline{\rho}, \bar{\rho}]$ indicates that *all* types demand bank loans in period t which may or may not turn into productive capital in period $t+1$.

Under these assumptions, we get the following set of equilibrium conditions for period t:

$$R_t^d = R^w, \tag{13}$$

$$Y_t = C_t + L_t - 2D_t + R_{t-1}^d D_{t-1}, \tag{14}$$

$$L_t = N_t + D_t, \tag{15}$$

$$K_t = \int_{\underline{\rho}}^{\omega_t} k_t^* (\rho)\, d\rho, \tag{16}$$

$$Y_t = \int_{\underline{\rho}}^{\omega_t} k_t^* (\rho)^\alpha\, d\rho, \tag{17}$$

$$L_t = \int_{\underline{\rho}}^{\bar{\rho}} l_t^* (\rho)\, d\rho, \tag{18}$$

$$N_t = (1 - \delta) \left[\int_{\underline{\rho}}^{\omega_t} r_{t-1}^* (\rho)\, l_{t-1}^* (\rho)\, d\rho - R_{t-1}^d D_{t-1} \right], \tag{19}$$

where

$$l_t^*(\rho) = \left[\frac{\alpha}{r_t^* (\rho)} \right]^{\frac{1}{1-\alpha}} \qquad \text{and} \qquad r_t^* (\rho) = \frac{R_t^d}{Prob\, (\omega_{t+1} \geq \rho | \omega_t, \omega_{t-1}, \ldots, \tilde{s}_t, \tilde{s}_{t-1}, \ldots)}.$$

Equations (13)-(19) yield a system of seven equilibrium conditions in the seven endogenous variables C_t, Y_t, K_t, L_t, D_t, N_t, and R_t^d. Given exogenous processes for the observable variables ω_t and $\tilde{s}_t$, we can investigate the general-equilibrium implications of imperfect information for credit spreads, lending, and output.

4.2. A Simplified Version with Two Risk Types

Suppose instead that there is a continuum of *safer firms* and a continuum of *riskier firms*, both of mass 1, where the latter have a higher probability of defaulting than the former in each state of the world and the corresponding variables carry superscripts "1" and "2", respectively. As before, $\omega_t = \nu_t + \eta_t$ contains the persistent component $\nu_t = \rho_\nu \nu_{t-1} + e_t$, $e_t \sim N\left(0, \sigma_e^2\right)$, and the transitory component $\eta_t = \rho_\eta \eta_{t-1} + \epsilon_t$, $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$, which are not separately observable, while the noisy signal of the trend component $\tilde{s}_t = \nu_t + \varepsilon_t$, $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$, is publicly observable.

Given that ω_t is defined on the real line, whereas the probability of default is a *percentage*, we map the real line onto the [0, 1]-interval by defining the aggregate repayment probabilities of type-1 and type-2 firms, respectively, as

$$Prob_t^1 \equiv \frac{a}{a + e^{-\omega_t}} \qquad \text{and} \qquad Prob_t^2 \equiv \frac{b}{b + e^{-\omega_t}}. \tag{20}$$

For $\omega_{ss} = 0$, this implies that $Prob_{ss}^1 = a/\,(a + 1)$ and $Prob_{ss}^2 = b/\,(b + 1)$, i.e., $100 \cdot a/\,(a + 1)\%$ of the

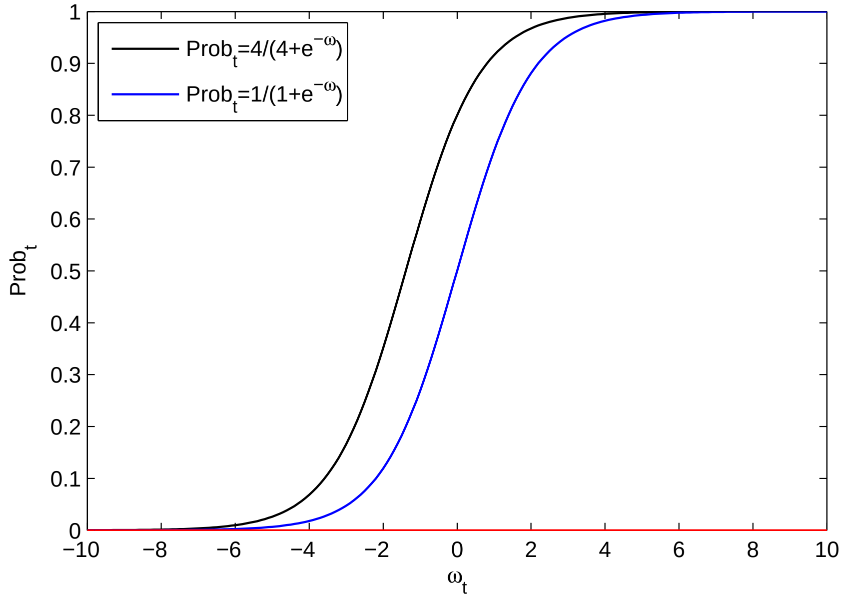

Figure 10: Continuous mappings from $\omega_t \in \mathbb{R}$ onto $Prob_t^1 \in (0, 1)$ and $Prob_t^2 \in (0, 1)$

less risky and $100 \cdot b/ (b + 1)\%$ of the more risky loans are repaid in the steady state. Figure 10 depicts the continuous mapping from $\omega_t \in \mathbb{R}$ onto $Prob_t^1 \in (0, 1)$ and $Prob_t^2 \in (0, 1)$ for $a = 4$ and $b = 1$, respectively.[8] Based on the ex-ante expected probabilities of repayment, the equilibrium lending rates of type-i firms can then be computed as

$$R_t^i = \frac{R_t^d}{E_t Prob_{t+1}^i}, \quad i = 1, 2,$$

which pins down the corresponding demand for *expected* productive capital by either firm type in period t:

$$E_t K_{t+1}^i = E_t Prob_{t+1}^i L_t^i = \left(\frac{\alpha}{R_t^i}\right)^{\frac{1}{1-\alpha}}, \quad i = 1, 2.$$

Note that the aggregate demand for *ex-ante* bank credit by either firm type in period t is thus given by

$$L_t^i = \frac{E_t K_{t+1}^i}{E_t Prob_{t+1}^i}, \quad i = 1, 2,$$

while the actual amount of *ex-post* productive capital in period $t + 1$ depends on the realized probability of

[8] While the mapping in (20) and the parameter values in Figure 10 are chosen for illustrative purposes, their functional forms can be adjusted in order to target arbitrary repayment probabilities for the two risk types.

repayment of a firm of type i, i.e.

$$K^i_{t+1} = Prob^i_{t+1} L^i_t, \quad i = 1, 2.$$

The equilibrium conditions of the simplified model with two risk types can be summarized as follows:

$$R^d_t = R^w, \tag{21}$$

$$R^1_t = \frac{R^d_t}{E_t Prob^1_{t+1}}, \tag{22}$$

$$R^2_t = \frac{R^d_t}{E_t Prob^2_{t+1}}, \tag{23}$$

$$E_t Prob^1_{t+1} L^1_t = \left(\frac{\alpha}{R^1_t}\right)^{\frac{1}{1-\alpha}}, \tag{24}$$

$$E_t Prob^2_{t+1} L^2_t = \left(\frac{\alpha}{R^2_t}\right)^{\frac{1}{1-\alpha}}, \tag{25}$$

$$L_t = L^1_t + L^2_t, \tag{26}$$

$$K^1_t = Prob^1_t L^1_{t-1}, \tag{27}$$

$$K^2_t = Prob^2_t L^2_{t-1}, \tag{28}$$

$$K_t = K^1_t + K^2_t, \tag{29}$$

$$Y_t = \left(K^1_t\right)^\alpha + \left(K^2_t\right)^\alpha, \tag{30}$$

$$Y_t = C_t + L_t - 2D_t + R^d_{t-1} D_{t-1}, \tag{31}$$

$$L_t = N_t + D_t, \tag{32}$$

$$N_t = (1-\delta)\left(R^1_{t-1} K^1_t + R^2_{t-1} K^2_t - R^d_{t-1} D_{t-1}\right). \tag{33}$$

Equations (21)-(33) yield a system of 13 equilibrium conditions in the 13 endogenous variables C_t, Y_t, K_t, K^1_t, K^2_t, L_t, L^1_t, L^2_t, D_t, N_t, R^d_t, R^1_t, and R^2_t, with ex-post default ratios $Prob^1_t$ and $Prob^2_t$ defined in (20). Given exogenous processes for ω_t and $\tilde{s}_t$, we can investigate the general-equilibrium implications of imperfect information for credit spreads, lending, and output.[9] Moreover, equation (34) defines the aggregate lending rate R_t as the average of type-specific interest rates weighted by the respective loan volumes, while equation

[9] Appendix B summarizes the log-linearized equilibrium conditions of the simplified model with two risk types in (21)-(33).

(35) defines the aggregate credit spread as the ratio of R_t to the risk-free rate on deposits:

$$R_t \equiv \frac{R_t^1 L_t^1 + R_t^2 L_t^2}{L_t^1 + L_t^2}, \tag{34}$$

$$spread_t \equiv \frac{R_t}{R_t^d}. \tag{35}$$

4.3. Estimation of the Simplified Model

In order to identify the structural shocks in (7) and (8) and to quantify their importance for aggregate fluctuations, we estimate the simplified version of the general equilibrium model with two risk types while allowing for imperfect information about the underlying state of the economy.[10,11] The aim of this section is to quantify the role of noise shocks in the financial cycle by matching the general equilibrium model to economic data. For this purpose, we draw on the approach to estimating DSGE models with a signal extraction problem proposed by Blanchard et al. (2013).

4.3.1. Equivalent full information representation

Suppose that the simplified general equilibrium model can be expressed in terms of the following system of stochastic difference equations:

$$\mathbf{D}E_t\left[\mathbf{x}_{t+1}\right] + \mathbf{F}\mathbf{x}_t + \mathbf{G}\mathbf{x}_{t-1} + \mathbf{M}\mathbf{y}_t + \mathbf{N}E_t\left[\mathbf{y}_{t+1}\right] = 0,$$

where $\mathbf{x}_t$ denotes a vector of *endogenous* state variables, $\mathbf{D}, \mathbf{F}, \mathbf{G}, \mathbf{M}$, and $\mathbf{N}$ are parameter matrices, and the *unobservable* exogenous state vector $\mathbf{s}_t$ only enters the system through the observable vector $\mathbf{y}_t$.[12] Suppose further that the model has the unique stable solution

$$\mathbf{x}_t = \mathbf{O}\mathbf{x}_{t-1} + \mathbf{P}\mathbf{s}_t + \mathbf{R}\mathbf{s}_{t|t},$$

[10] Note that the financial sector's information problem implies that the corresponding VAR representation is *not* invertible (see Blanchard et al., 2013).

[11] In contrast to the dynamic stochastic general equilibrium (DSGE) exercise in Blanchard et al. (2013), as a starting point, we allow only for the three structural shocks in Section 3 rather than adding additional shocks that are standard in the DSGE literature.

[12] This reflects the assumption that the information set of the representative agent contains past and current values of $\mathbf{y}_t$ and $\mathbf{x}_t$ rather than current and past values of $\mathbf{s}_t$ (see Blanchard et al., 2013).

where, as in Section 3, $\mathbf{s}_{t|t}$ denotes the agents' expectation of the state vector conditional on all information available in period t, i.e. $\mathbf{x}_{t|t} \equiv E[\mathbf{s}_t|\mathbf{y}_t, \mathbf{y}_{t-1}, \ldots]$, and the matrices $\mathbf{O}, \mathbf{P}$, and $\mathbf{R}$ can be found by solving

$$\mathbf{DO}^2 + \mathbf{FO} + \mathbf{G} = 0, \qquad (\mathbf{DO} + \mathbf{F})\,\mathbf{P} + \mathbf{M} = 0, \qquad (\mathbf{DO} + \mathbf{F})\,\mathbf{R} + [\mathbf{D}\,(\mathbf{PZ} + \mathbf{R}) + \mathbf{NZ}]\,\mathbf{T} = 0,$$

where the matrices $\mathbf{Z}$ and $\mathbf{T}$ are defined in Section 3. While the second and third equation are linear in $\mathbf{P}$ and $\mathbf{R}$ and thus straightforward to solve, Uhlig (1995) provides solution techniques for the first equation in $\mathbf{O}$.

Making use of the Kalman filter updating and forecasting expressions for the state vector $\mathbf{s}_t$ in (A.1) and (A.2), respectively, we can express the joint dynamics of $\mathbf{s}_{t|t}$ and the vector of observables $\mathbf{y}_t$ as

$$\begin{aligned}
\mathbf{s}_{t|t} &= \mathbf{T}\mathbf{s}_{t-1|t-1} + \mathbf{K}\left(\mathbf{y}_t - \mathbf{y}_{t|t-1}\right) = \mathbf{T}\mathbf{s}_{t-1|t-1} + \mathbf{K}\left(\mathbf{y}_t - \mathbf{Z}\mathbf{T}\mathbf{s}_{t-1|t-1}\right), \\
\mathbf{y}_t &= \mathbf{Z}\mathbf{T}\mathbf{s}_{t-1|t-1},
\end{aligned}$$

where $\mathbf{K} \equiv \Sigma_{t|t-1}\mathbf{Z}'\left(\mathbf{Z}\Sigma_{t|t-1}\mathbf{Z}' + \mathbf{H}\right)^{-1}$ denotes the Kalman filter gain and $\Sigma_{t|t-1} \equiv \mathrm{Var}_{t-1}(\mathbf{y}_t)$ the variance-covariance matrix of $\mathbf{y}_t$ conditional on the information available in period t.

Suppose that the latter can be factorized as $\Sigma_{t|t-1} = \mathbf{F}\mathbf{F}'$ for some matrix $\mathbf{F}$ and consider the model

$$\begin{aligned}
\hat{\mathbf{s}}_t &= \mathbf{T}\hat{\mathbf{s}}_{t-1} + \mathbf{K}\mathbf{F}\hat{\mathbf{v}}_t, \\
\mathbf{y}_t &= \mathbf{Z}\mathbf{T}\hat{\mathbf{s}}_{t-1} + \mathbf{F}\hat{\mathbf{v}}_t,
\end{aligned} \tag{36}$$

where $\hat{\mathbf{v}}_t$ is an m-dimensional vector of mutually independent i.i.d. standard normal shocks. *Lemma 2* in Blanchard et al. (2013) states that, identifying $\mathbf{s}_t$ with $\mathbf{s}_{t|t}$ and $\mathbf{v}_t$ with $\mathbf{y}_t - \mathbf{Z}\mathbf{T}\mathbf{s}_{t-1|t-1}$, the original signal extraction model is *observationally equivalent* to the model in (36) with the assumption that the agent perfectly observes $\hat{\mathbf{s}}_t$ and $\hat{\mathbf{v}}_t$ for any matrix $\mathbf{F}$. From *Lemma 2*, we can estimate the equivalent full information model subject to a restriction on the shocks' correlation matrix and recover the parameters of the original signal extraction model.

4.3.2. Bayesian maximum likelihood estimation

In the absence of additional shocks, the equivalent full information representation of the simplified general equilibrium model with imperfect information in (36) allows for $m = 2$ mutually independent shock series. Hence, we estimate the parameters of the exogenous processes in (7) and (8) by matching aggregate output, Y_t, and the aggregate spread between lending rates and the risk-free rate on deposits, $spread_t$, to their empirical counterparts. For the log-difference of aggregate output, we use the seasonally adjusted quarter-on-quarter growth rate of U.S. real GDP growth. For $spread_t$, we employ the difference between Moody's seasoned Baa corporate bond yield and the yield on 10-year treasury constant maturity. The variables are transformed as described in Pfeifer (2017).

We use Dynare to solve and estimate the model by maximizing the likelihood function at the posterior mode for the vector of parameter values. Following Blanchard et al. (2013), the impulse response functions of the economic variables to the $n = 3$ shocks in the original signal extraction model can then be backed out from the impulse responses to the $m = 2$ mutually orthogonal shocks in the vector $\hat{\mathbf{v}}_t$ in the equivalent full information representation in (36).

4.3.3. Bayesian priors and parameter estimates

Table 3 reports the posterior mode of the estimates of the parameters in equations (7) and (8) as well as the type and mean of the corresponding Bayesian priors, which are standard in the DSGE literature. We assume a beta-distributed prior with a mean of 0.6 and a standard deviation of 0.2 for both ρ_ν and ρ_η, while we set the means of the inverse gamma-distributed priors for σ_e, σ_ϵ, and σ_ε to 0.5, 1.0, and 1.0, respectively, and the corresponding standard deviations to unity.

We find substantial autocorrelation in the estimated persistent component ν_t and virtually zero serial correlation in the estimated transitory component η_t. While the posterior mode of the coefficient ρ_ν is equal to 0.7445, the posterior mode of ρ_η hits the imposed lower bound of 0.01 from above. As a consequence, the effects on the observable ω_t of shocks to the persistent component are qualitatively different from shocks to the transitory component, as illustrated by the impulse response functions in Figures 6 and 7.

Lines 3-5 in Table 3 indicate that non-fundamental "noise shocks" to the publicly observable signal $\tilde{s}_t$ are more volatile than shocks to the persistent component ν_t, yet less volatile than shocks to the transitory component η_t. Note that a larger estimate of σ_ϵ relative to σ_ε implies that $\tilde{s}_t$ is a relatively more precise

Table 3: Bayesian priors and maximum likelihood posterior estimates of the parameters in equations (7) and (8)

Parameter	Prior type	Prior mean	Prior s.d.	Posterior mode	Posterior s.d.
ρ_ν	beta	0.600	0.200	0.7445	0.0353
ρ_η	beta	0.600	0.200	0.0100	0.0004
σ_e	inverse gamma	0.500	1.000	0.0645	0.0015
σ_ϵ	inverse gamma	1.000	1.000	1.5432	0.1169
σ_ε	inverse gamma	1.000	1.000	0.6415	0.1335

indicator of the unobservable persistent component ν_t than ω_t, which is "polluted" by shocks to its transitory component. As a result, rational agents put some trust in the public signal, which slows down their speed of learning and amplifies the effect of noise shocks on the bank's expectations about the state of the economy in period $t + 1$ and thus on its lending behavior.

4.4. Impulse Response Functions

It is straightforward to solve the simplified general equilibrium model and compute impulse response functions of the variables of interest. In what follows, we set $\alpha = 0.35$ and $R^w = \beta_w^{-1} = 1.01$, as in our partial equilibrium model, while 10% of the financial sector's net worth is consumed each period, i.e. $\delta = 0.1$. The volatility of shocks to the persistent component, shocks to the transitory component, and shocks to the public signal are taken from our parameter estimates in Table 3, i.e. $\sigma_e = 0.0645$, $\sigma_\epsilon = 1.5432$, and $\sigma_\varepsilon = 0.6415$. In the exogenous AR(1) processes for ν_t and η_t, we set $\rho_\nu = 0.7445$ and $\rho_\eta = 0.01$.

4.4.1. "Trend" shocks

Figure 11 plots the impulse response functions based on the simplified general equilibrium model with two risk types of borrowers for a positive one-standard-deviation innovation in the persistent component. In period 1, ν_t and thus both ω_t and $\tilde{s}_t$ increase by the full amount of the "trend" shock e_t, while the Kalman filter nowcast of the persistent component, $\nu_{t|t}$, increases only by a fraction of e_t, as economic agents assign a substantial probability to the possibility that the observed changes in ω_t and $\tilde{s}_t$ are driven by a shock in the transitory or noise component rather than by a "trend" shock.[13]

The increase in $\nu_{t|t}$ translates into an increase in the expected repayment probabilities for both risk types, $E_t Prob^i_{t+1}$, $i = 1, 2$, and a decrease in the corresponding credit spreads defined in equations (24) and (25). Hence, the demand for capital by both types and thus the ex-ante aggregate demand for credit increase. Note that the shock to the persistent component is *fundamental* in the sense that it affects the state of the world ω_t that is relevant for the default probability of each firm type. In response to a "trend" shock, the ex-post aggregate capital stock increases on impact, as a smaller fraction of firms of both types defaults.

The surprise increase in the stock of productive capital directly translates into higher output, while the increased ex-post repayment of credit raises bank net worth on impact. Note that the subsequent hump-shaped response of *bank net worth* is reminiscent of the hump-shaped response of *entrepreneurial net worth*

[13] Recall that the Kalman filter nowcast of the persistent component follows directly from equation (10) as a projection on the period-t realizations of the observable variables ω_t and $\tilde{s}_t$.

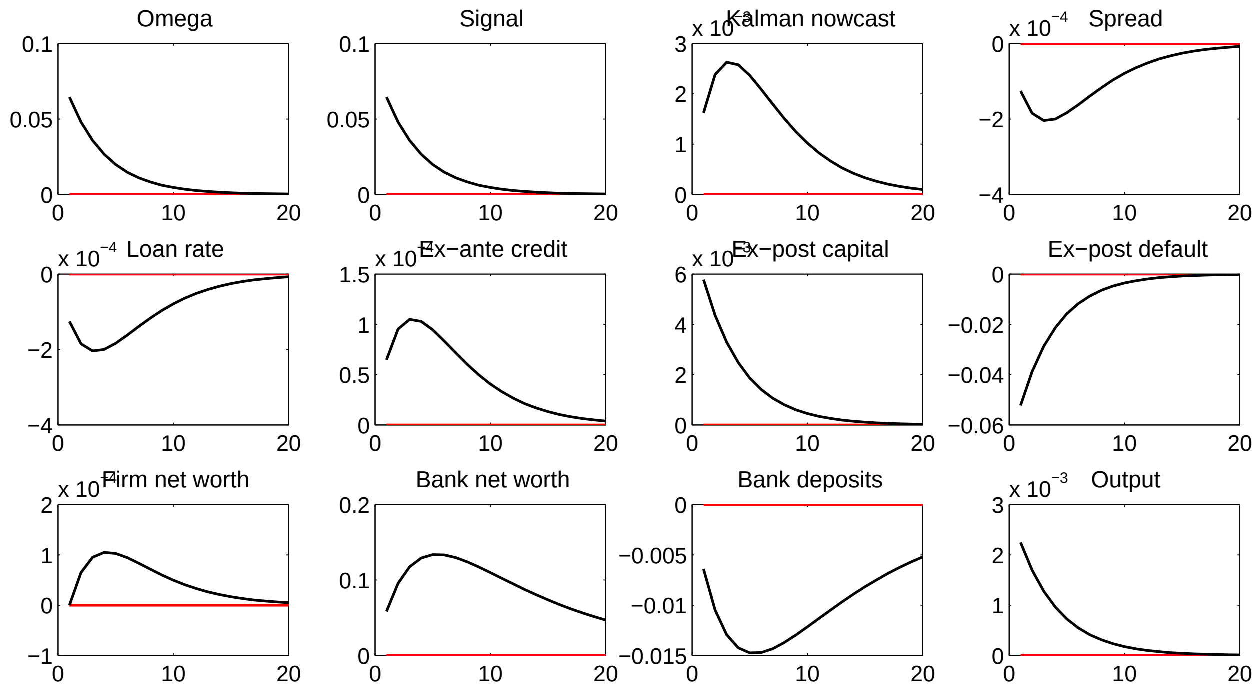

Figure 11: Impulse responses to a shock in the persistent component based on the simplified general equilibrium model in (21)-(33)

to fundamental shocks in Bernanke et al. (1999). As a consequence, banks can rely on a higher stock of net worth rather than on external funding in order to satisfy the increased demand for credit, and bank deposits fall short of their steady-state value.

4.4.2. "Cycle" shocks

Consider now the impulse response functions to a positive one-standard-deviation shock in the transitory component in Figure 12. Given our assumption that $\rho_\eta = 0.01$ in (7), ω_t increases in period 1 by the full amount of the shock and virtually falls back to zero in period 2. In contrast, the unobservable persistent component v_t and the observable signal $\tilde{s}_t$ remain equal to their steady-state values throughout.

Under imperfect information, the uncertainty about the origin of the observed increase in ω_t implies that the Kalman filter nowcast of the persistent component, $v_{t|t}$, increases nevertheless, as agents attribute part of the observed increase in ω_t to an increase in the persistent component. As a result, the equilibrium credit spread decreases — reflecting higher expected probabilities of repayment — and the ex-ante aggregate demand for credit increases despite the fact that the fundamental source of the disturbance has all but vanished in period 2.

Due to the fundamental nature of the shock, reduced default of firms implies a higher ex-post aggregate

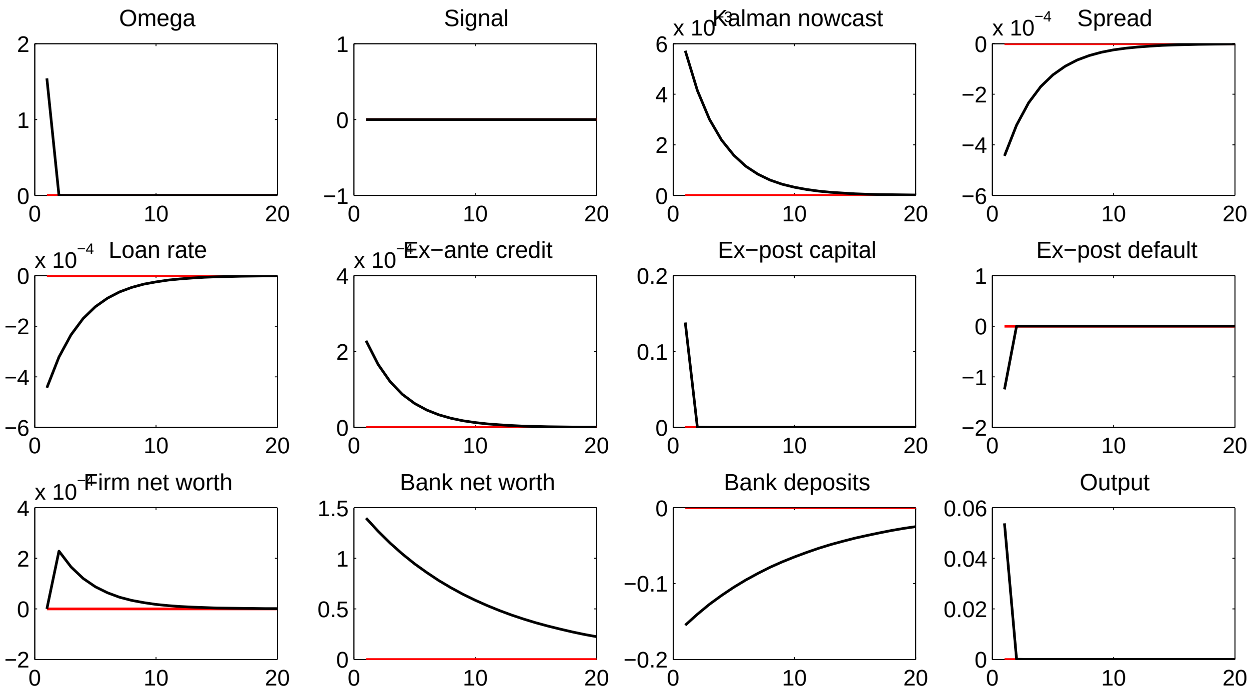

Figure 12: Impulse responses to a shock in the transitory component based on the simplified general equilibrium model in (21)-(33)

stock of productive capital and thus higher output in the economy. Note that ex-post default decreases only in the period of the shock, thus raising bank net worth on impact, while a larger fraction of loans defaults from period 2 onwards. The reason is that, in response to a shock in the transitory component, banks lend disproportionately more to the riskier firm type, for which the equilibrium credit spread decreases by more. After the initial increase, bank net worth monotonically converges to its steady-state value from above. Due to the ample supply of internal funds, the bank relies less on external funding in order to satisfy the increased demand for ex-ante credit, and bank deposits decrease.

4.4.3. Noise shocks

While the impulse responses in Figures 11 and 12 arise from shocks to the fundamental determinants of credit risk, pure noise shocks can also affect the bank's lending behavior and thus real economic activity in the presence of imperfect information. Figure 13 plots the impulse response functions for a positive one-standard-deviation shock to the public signal $\tilde{s}_t$ *without any change in fundamentals.*

Although the observable ω_t and its latent persistent component ν_t are unaffected, the positive signal induces rational economic agents to revise their Kalman filter nowcast of the "trend" component, $\nu_{t|t}$, upwards. While the "objective" repayment probabilities of both risk types are therefore unchanged, the equilibrium

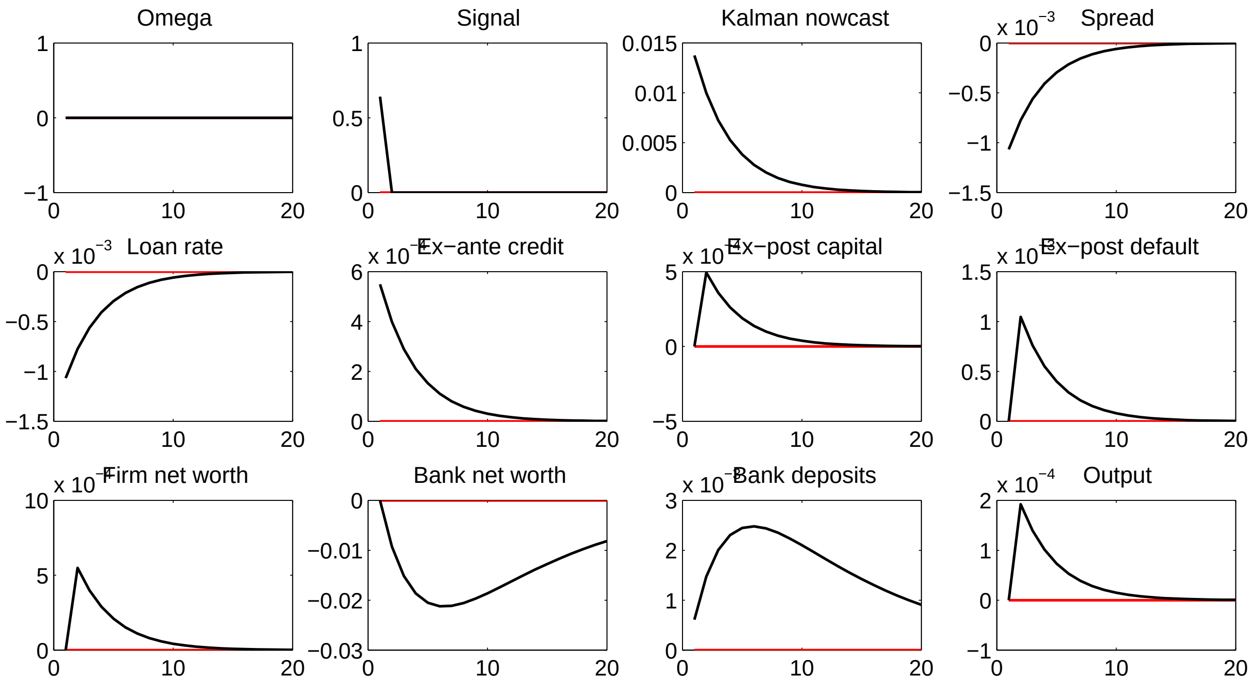

Figure 13: Impulse responses to a shock in the noise component based on the simplified general equilibrium model in (21)-(33)

credit spread decreases on impact before monotonically converging back to its steady-state value from below, while the impulse response function of ex-ante aggregate credit follows a similar but inverted pattern. Note that, in response to a non-fundamental noise shock, ex-post default, the aggregate stock of productive capital, and thus output remain unaffected on impact. Only after the increase in ex-ante aggregate credit in period 1 translates into a increase in ex-post aggregate capital in period 2, both capital and output increase slightly. Importantly, the increase in ex-post aggregate capital is less pronounced than the expansion of ex-ante aggregate bank lending, leading to an increase in ex-post aggregate default in line with Figure 9.

This increase in ex-post aggregate default cuts into the bank's net worth, which follows U-shaped pattern and converges back to its steady-state value from below. The loss of internal funds induces the bank to draw on additional external funding in order to satisfy the increased demand for ex-ante aggregate credit. Hence, bank deposits increase in Figure 13. This is in stark contrast with the findings in Blanchard et al. (2013), where consumption *increases* in response to a pure noise shock. The reason is that, in our model, the signal extraction problem has to be solved by financial intermediaries rather than the household sector, implying a qualitatively different propagation of noise shocks.

4.5. Forecast Error Variance Decomposition

Table 4 reports the forecast error variance (FEV) contribution of the three structural shocks for selected variables from the simplified general equilibrium model with two risk types in (21)-(35). Consistent with our findings for the partial equilibrium investment model, the first two panels illustrate that noise shocks do *not* contribute to the FEV of ω_t, while shocks to the transitory component do *not* affect the signal $\tilde{s}_t$. At the same time, both cycle and noise shocks contribute to the FEV of the Kalman filter nowcast of the persistent component, $\nu_{t|t}$, where the intuition follows from equation (10). Due to the large estimated variance of cycle shocks in Table 3, ω_t represents a comparatively noisy measure of the persistent component that is largely disregarded by economic agents, while a higher weight is placed on the relatively more precise signal $\tilde{s}_t$. As a consequence, non-fundamental noise shocks to the public signal enter the Kalman filter nowcast of ν_t with a large weight and contribute thus more to its FEV.

Turning to selected economic variables in the second line of Table 4, we find that the large contribution of noise shocks to the Kalman filter nowcast of the persistent component translates into an increasing contribution of 21-33% in the FEV of the aggregate credit spread defined in equation (35). Note also that the contribution of shocks to the persistent component is monotonically increasing, while that of shocks to the transitory component is monotonically decreasing with the forecast horizon. The relative importance of the two fundamental shocks is largely determined by our estimates of σ_e and σ_ϵ.

Due to the assumption of a constant interest rate on deposits, the FEV of the credit spread is inherited by the interest rates on loans to either bank type and thus by the corresponding equilibrium credit volumes.

Table 4: Forecast error variance decomposition of selected variables based on the model in (21)-(35)

	Omega			Signal			Kalman nowcast		
Quarter	Trend	Cycle	Noise	Trend	Cycle	Noise	Trend	Cycle	Noise
1	0.002	0.998	0.000	0.010	0.000	0.990	0.012	0.144	0.844
4	0.004	0.996	0.000	0.020	0.000	0.980	0.048	0.139	0.813
8	0.004	0.996	0.000	0.022	0.000	0.978	0.074	0.135	0.791
20	0.004	0.996	0.000	0.022	0.000	0.978	0.082	0.134	0.785
	Spread			Ex-ante credit			Ex-post default		
Quarter	Trend	Cycle	Noise	Trend	Cycle	Noise	Trend	Cycle	Noise
1	0.007	0.785	0.208	0.007	0.785	0.208	0.002	0.998	0.000
4	0.029	0.649	0.322	0.029	0.649	0.322	0.004	0.996	0.000
8	0.044	0.625	0.331	0.044	0.625	0.331	0.004	0.996	0.000
20	0.044	0.621	0.331	0.047	0.621	0.331	0.004	0.996	0.000

As a result, noise shocks display a large contribution to the FEV of ex-ante aggregate credit, whereas they have no explanatory power for fluctuations in ex-post default, which is determined almost exclusively by shocks to the transitory component. This is due to the comparatively larger variance of the latter, which implies that economic variables such as ex-post default and ex-post capital, for example, respond strongly to the fundamental cycle shocks on impact (also see Figure 12).

5. Concluding Remarks

In this paper, we investigate whether imperfect information of financial intermediaries about the state of the economy can be a source of lending cycles. We start by analyzing a partial equilibrium neoclassical investment model, where a competitive bank has to solve a signal extraction problem in order to distinguish between fundamental components that determine the underlying state of the economy and non-fundamental "noise" shocks. The corresponding impulse response functions suggest that credit booms can arise from informational frictions and that such credit booms are associated with higher ex-post default relative to a full-information and rational expectations benchmark.

In order to quantify the role of noise shocks in aggregate fluctuations, we embed our neoclassical investment model in a general equilibrium model with only two risk types of borrowers. We close the partial equilibrium model by assuming that bank lending is funded by the bank's accumulated net worth as well as by external funds in the form of risk-free deposits by a risk-neutral foreign depositor at the "world interest rate". Accordingly, the equilibrium deposit rate is assumed to be exogenous and constant. We solve and estimate the model using Bayesian techniques and an equivalent full information representation of the general equilibrium model with imperfect information (see Blanchard et al., 2013).

Calibrating the model's driving processes along the lines of our empirical estimates, we find that noise shocks contribute up to one third to the forecast error variance of the spread between Moody's seasoned Baa corporate bond yield and the yield on 10-year treasury constant maturity at the 5-year horizon, whereas the contribution to the forecast error variance of total asset growth of U.S. corporate banks is negligible.

6. References

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework." in: Taylor, J., Woodford, M. (Eds.), *Handbook of Macroeconomics*.

Blanchard, Olivier J., Jean-Paul L'Huillier, and Guido Lorenzoni (2013). "News, Noise, and Fluctuations: An Empirical Exploration." *American Economic Review* 103(7), 3045-3070.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer (2017). "Diagnostic Expectations and Credit Cycles." forthcoming in the *Journal of Finance*.

Coibion, Olivier and Yuriy Gorodnichenko (2015). "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts." *American Economic Review* 105(8), 2644-2678.

Croushore, Dean (2010). "An Evaluation of Inflation Forecasts from Surveys Using Real-Time Data." *B.E. Journal of Macroeconomics* 10(1).

Gilchrist, Simon and Masashi Saito (2008). "Expectations, Asset Prices, and Monetary Policy: The Role of Learning." NBER Chapters, in: Asset Prices and Monetary Policy, pages 45-102. National Bureau of Economic Research, Inc.

Gilchrist, Simon and Egon Zakrakšek (2012). "Credit Spreads and Business Cycle Fluctuations." *American Economic Review* 102(4), 1692-1720.

De Grauwe, Paul and Corrado Macchiarelli (2015). "Animal Spirits and Credit Cycles." *Journal of Economic Dynamics and Control* 59, 95-117.

Gennaioli, Nicola, Yueran Ma, and Andrei Shleifer (2016). "Expectations and Investment." *NBER Macroeconomics Annual*, Vol. 30 (2016), 379-442.

Koop, Gary, Hashem M. Pesaran, and Simon M. Potter (1996). "Impulse Response Analysis in Nonlinear Multivariate Models." *Journal of Econometrics* 74(1), 119-147.

Ilut, Cosmin and Hikaru Saijo (2016). "Learning, Confidence, and Business Cycles." working paper.

Lorenzoni, Guido (2009). "A Theory of Demand Shocks." *American Economic Review* 99(5), 2050-2084.

Pfeifer, Johannes (2017). "A Guide to Specifying Observation Equations for the Estimation of DSGE Models." mimeo, University of Cologne.

Uhlig, Harald (1995). "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily." *Center for Economic Research Discussion Paper* 1995-97, Tilburg University.

Appendix A. Kalman Filter

Solving equation (7) for the transitory component η_t and substituting into $\eta_t = \rho_\eta \eta_{t-1} + \epsilon_t$, we obtain

$$\begin{aligned}
\omega_t &= \nu_t + \rho_\eta (\omega_{t-1} - \nu_{t-1}) + \epsilon_t, \\
\tilde{s}_t &= \nu_t + \varepsilon_t, \\
\nu_t &= \rho_\nu \nu_{t-1} + e_t,
\end{aligned}$$

where the first two represent measurement equations, while the third represents the transition equation of ν_t. Defining the vector of states $\mathbf{s}_t \equiv [\nu_t, \nu_{t-1}]'$, we can write the above system in matrix notation as

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{Z}\mathbf{s}_t + \mathbf{B}\mathbf{y}_{t-1} + \mathbf{u}_t, \\
\mathbf{s}_t &= \mathbf{T}\mathbf{s}_{t-1} + \mathbf{v}_t,
\end{aligned}$$

where $\mathbf{y}_t \equiv \begin{bmatrix} \omega_t \\ \tilde{s}_t \end{bmatrix}$, $\mathbf{Z} \equiv \begin{bmatrix} 1 & -\rho_\eta \\ 1 & 0 \end{bmatrix}$, $\mathbf{B} \equiv \begin{bmatrix} \rho_\eta & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{u}_t \equiv \begin{bmatrix} \epsilon_t \\ \varepsilon_t \end{bmatrix}$, $\mathbf{T} \equiv \begin{bmatrix} \rho_\nu & 0 \\ 1 & 0 \end{bmatrix}$, and $\mathbf{v}_t \equiv \begin{bmatrix} e_t \\ 0 \end{bmatrix}$.

From the above system of equations, it is straightforward to derive Kalman filter *updating* expressions for the state vector and its variance conditional on information available at time t, $\mathbf{s}_{t|t}$ respectively $\Sigma_{t|t}$:

$$\begin{aligned}
\mathbf{s}_{t|t} &= \mathbf{s}_{t|t-1} + \Sigma_{t|t-1}\mathbf{Z}' \left(\mathbf{Z}\Sigma_{t|t-1}\mathbf{Z}' + \mathbf{H}\right)^{-1} \left(\mathbf{y}_t - \mathbf{y}_{t|t-1}\right), \\
\Sigma_{t|t} &= \Sigma_{t|t-1} - \Sigma_{t|t-1}\mathbf{Z}' \left(\mathbf{Z}\Sigma_{t|t-1}\mathbf{Z}' + \mathbf{H}\right)^{-1} \mathbf{Z}\Sigma_{t|t-1},
\end{aligned} \tag{A.1}$$

where the corresponding Kalman filter *forecasting* expressions are given by

$$\begin{aligned}
\mathbf{s}_{t+1|t} &= \mathbf{T}\mathbf{s}_{t|t}, \\
\Sigma_{t+1|t} &= \mathbf{T}\Sigma_{t|t}\mathbf{T}' + \mathbf{Q}, \\
\mathbf{y}_{t+1|t} &= \mathbf{Z}\mathbf{s}_{t+1|t} + \mathbf{B}\mathbf{y}_t,
\end{aligned} \tag{A.2}$$

and the covariance matrices of the disturbance term vectors $\mathbf{u}_t$ and $\mathbf{v}_t$, respectively, are given by

$$\mathbf{H} = \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} \sigma_e^2 & 0 \\ 0 & 0 \end{pmatrix}.$$

Note that the Kalman filter gain, $\mathbf{K} \equiv \Sigma_{t|t-1}\mathbf{Z}' \left(\mathbf{Z}\Sigma_{t|t-1}\mathbf{Z}' + \mathbf{H}\right)^{-1}$, is increasing in σ_e while it is decreasing in σ_ϵ and σ_ε, i.e., the extent to which the bank optimally updates its *nowcast* of $\mathbf{s}_{t|t}$ depends on the so-called "signal-to-noise ratio".

Appendix B. Log-Linearized General Equilibrium Model with Two Risk Types

Assuming that $R_t^d = R^w \; \forall t$, it is straightforward to log-linearize the equilibrium conditions in equations (21)-(35) around the non-stochastic steady state:

$$\hat{R}_t^d = 0, \tag{B.1}$$

$$\hat{R}_t^1 = \hat{R}_t^d - E_t\widehat{Prob}_{t+1}^1, \tag{B.2}$$

$$\hat{R}_t^2 = \hat{R}_t^d - E_t\widehat{Prob}_{t+1}^2, \tag{B.3}$$

$$E_t\widehat{Prob}_{t+1}^1 + \hat{L}_t^1 = -\frac{1}{1-\alpha}\hat{R}_t^1, \tag{B.4}$$

$$E_t\widehat{Prob}_{t+1}^2 + \hat{L}_t^2 = -\frac{1}{1-\alpha}\hat{R}_t^2, \tag{B.5}$$

$$L_{ss}\hat{L}_t = L_{ss}^1\hat{L}_t^1 + L_{ss}^2\hat{L}_t^2, \tag{B.6}$$

$$\hat{K}_t^1 = \widehat{Prob}_t^1 + \hat{L}_{t-1}^1, \tag{B.7}$$

$$\hat{K}_t^2 = \widehat{Prob}_t^2 + \hat{L}_{t-1}^2, \tag{B.8}$$

$$K_{ss}\hat{K}_t = K_{ss}^1\hat{K}_t^1 + K_{ss}^2\hat{K}_t^2, \tag{B.9}$$

$$Y_{ss}\hat{Y}_t = \alpha\left(Y_{ss}^1\hat{K}_t^1 + Y_{ss}^2\hat{K}_t^2\right), \tag{B.10}$$

$$Y_{ss}\hat{Y}_t = C_{ss}\hat{C}_t + N_{ss}\hat{N}_t - D_{ss}\hat{D}_t + R_{ss}^d D_{ss}\left(R_{t-1}^d + D_{t-1}\right), \tag{B.11}$$

$$L_{ss}\hat{L}_t = D_{ss}\hat{D}_t + N_{ss}\hat{N}_t, \tag{B.12}$$

$$N_{ss}\hat{N}_t = (1-\delta)\left[R_{ss}^1 K_{ss}^1\left(R_{t-1}^1 + K_t^1\right) + R_{ss}^2 K_{ss}^2\left(R_{t-1}^2 + K_t^2\right) - R_{ss}^d D_{ss}\left(R_{t-1}^d + D_{t-1}\right)\right], \tag{B.13}$$

$$R_{ss}\left(L_{ss}^1\hat{L}_t^1 + L_{ss}^2\hat{L}_t^2\right) = R_{ss}^1 L_{ss}^1\left(\hat{R}_t^1 + \hat{L}_t^1\right) + R_{ss}^2 L_{ss}^2\left(\hat{R}_t^2 + \hat{L}_t^2\right) - R_{ss}\left(L_{ss}^1 + L_{ss}^2\right)\hat{R}_t, \tag{B.14}$$

$$\widehat{spread}_t = \hat{R}_t - \hat{R}_t^d. \tag{B.15}$$

where X_{ss} denotes the steady-state value of variable X and $\hat{X}_t$ the percentage deviation of X in period t from its steady-state value, i.e. $\hat{X}_t \equiv \left(X_t - X_{ss}\right)/X_{ss}$.