Security Design, Informed Intermediation, and the Resolution of Borrowers' Financial Distress*

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Abstract

Banks as informed intermediaries have information about their borrowers to make efficient liquidation versus restructuring decisions for distressed loans, but their information also creates an adverse selection problem when they seek financing from uninformed investors. We demonstrate that a bank with high-quality loans faces incentives to distort its resolution policy in order to improve allocative efficiency and to signal information about loan quality, with the direction of the distortion depending on whether the security issued to uninformed investors is concave or convex. We find that the bank's equilibrium resolution policy is biased towards liquidation when it optimally designs and sells a debt (concave) security to raise financing. Regulations aimed at promoting ex post efficient liquidation may increase banks' financing costs and discourage their screening effort, thereby reducing welfare. (*JEL:* D8, G21, G23, G24)

Keywords. Security design, asset-backed securities, resolution of financial distress, liquidation, restructuring, asymmetric information

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1 Introduction

The recent financial crisis has put the resolution of borrowers' financial distress back to the spotlight. For example, the amount of public company assets entering Chapter 11 bankruptcy protection during the two-year period of 2008-09 were almost 20 times more than during the previous two year, with over $3.5 trillion of corporate debt in distress or in default at one point.[1] Households were severely impacted too, with over 14 million U.S. properties with foreclosure filings from 2008 to 2014.[2] Anecdotal reports and recent empirical research, in particular on securitised mortgages, have argued that the resolution of financial distress might be inefficient.[3]

Inefficient resolution of borrowers' financial distress can arise if there are information frictions between lenders and borrowers.[4] Theories of financial intermediation have proposed that banks emerge as delegated information producers, who can achieve efficient resolution of borrowers' financial distress thanks to their ability and incentives to acquire borrower-specific, 'soft' information (e.g. Berlin and Loeys (1988), Rajan (1992), Chemmanur and Fulghieri (1994) and Bolton and Freixas (2000)).[5] Yet, banks' information about their loans creates an adverse selection problem when they need to raise funds from uninformed investors (e.g. Winton (2003) and DeMarzo (2005)). Indeed, banks routinely need funds to finance borrowers and to comply with regulatory requirements. In this paper we investigate banks' incentives to efficiently resolve borrowers' financial distress in the presence of adverse selection friction in banks' funding.

We present a model in which an informed bank determines the resolution policy for its loans in case of borrowers' financial distress, and raises funds from uninformed investors by selling a security backed by the loans.[6] We show that, depending on the security design, a bank may face incentives to distort its resolution policy in order to improve allocative efficiency and to signal information about its loan quality. In equilibrium, a bank with high-quality loans adopts a resolution policy biased towards liquidation (as opposed to restructuring), when the bank endogenously designs and sells a debt security. Our results suggest that policies aimed at

[1]Source: Gilson (2012)

[2]Source: RealtyTrac (2015).

[3]Related empirics are discussed in the empirical implications part in the introduction.

[4]The information frictions between lenders and borrowers and the associated inefficiency in the resolution of borrower financial distress can be found in Haugen and Senbet (1978), Giammarino (1989), Gertner and Scharfstein (1991), Repullo and Suarez (1998) and Wang et al. (2002).

[5]Leland and Pyle (1977) and Diamond (1984) first point out that a bank would emerge endogenously as delegated information producer to economise on investment in information.

[6]Begley and Purnanandam (2017) and Balasubramanyan et al. (2017) provide direct evidence that banks use their private information about loan quality when designing the securities in mortgage securitisation and in syndicated loan market respectively.

promoting efficient liquidation of distressed loans ex post may ex ante hinder banks' ability to obtain funding and disincentivise banks' screening effort, hence reducing welfare.

Model preview. The bank has a continuum of ex ante identical assets (a "pool" of loans). Upon realisation of an aggregate shock (the bad state), some borrowers enter into financial distress – "default". The bank's resolution policy determines whether a distressed loan is liquidated or restructured. Liquidation of a distressed loan delivers a safe cash flow, whereas restructuring a loan may result in a higher (lower) cash flow if the borrower recovers (re-defaults), depending on the realisation of a future aggregate shock. The efficient resolution policy is to liquidate all delinquent loans that have a higher liquidation value than the expected recovery value, and restructure the rest.

To meet its funding needs, the bank designs and sells a security backed by the cash flow from the loan pool to uninformed investors (as in DeMarzo (2005)), and chooses the resolution policy for the loans. A key friction in our model is that investors do not have access to the information the bank has on its loans, particularly regarding the default probability of its borrowers. Clearly, if information is symmetric, the bank should choose the efficient resolution policy and reap the full value of the assets immediately by simply selling a pass-through security to investors. In other words, if the market for funds is frictionless, financing is irrelevant for the bank's resolution policy.

Results preview. We show that adverse selection problem at the bank's financing stage could cause the bank with high-quality, less distress-prone loans (the high type) to optimally biase its resolution policy for its borrowers towards liquidation, when it endogenously designs and sells a debt security to uninformed investors. In line with the literature on security design, debt is the optimal security to raise financing from uninformed investors because it is least sensitive to the private information held by the bank.[7,8]

[7] Equivalently, the bank optimally signals the quality of its assets to investors via costly retention of the residual equity claim. The notion of debt as the optimal security due to its information insensitivity dates back to the Pecking Order Theory in Myers and Majluf (1984). DeMarzo et al. (2015) shows that in an ex-post liquidity-based security design game like ours, standard debt is the least information sensitive and thus the optimal monotone security when the cash flow satisfies Hazard Rate Ordering (HRO) property. See also Chemla and Hennessy (2014) and Vanasco (2016) for recent theoretical works with costly retention of the equity tranche as signals. Empirically, Begley and Purnanandam (2017) find that conditional on observable characteristics, RMBS deals with larger equity tranche have lower delinquency rate and command higher prices, suggesting that information asymmetry is relevant and the signalling mechanism is at play.

[8] As also discussed in Begley and Purnanandam (2017), even if the bank sells off the equity tranche at a later date, the initial retention of the equity tranche could still be a costly signal because i) the opportunity cost of the locked-up capital could still be significant in a high-growth market and ii) the equity tranches in practice are often sold to sophisticated and informed investors like hedge funds and mutual fund managers, who are likely to have stronger bargaining power and/or scarcer capital than uninformed senior tranche investors.

The bank distorts its resolution policy towards liquidation for two reasons, namely to improve allocative efficiency and to signal the quality of its loans. In equilibrium, the bank's payoff is comprised of the proceeds from selling the optimal debt security and the expected value of the retained claim. A key observation is that biasing the resolution policy towards liquidation reduces the riskiness of the cash flow from the loan pool by reducing the loan pool's exposure to borrowers' re-default risks. On the one hand, this increases the expected value of the optimal debt, a concave security, by Jensen's inequality, resulting in greater gains from trade between the bank and the investors and thus greater allocative efficiency. On the other hand, a resolution policy with a liquidation bias could serve as a costly signal because it reduces the value of the bank's retained , convex, claim and crucially, more so for the bank with lower-quality, more-distress-prone loans. We find that the liquidation bias in the equilibrium resolution policy is greater when information asymmetry is more severe and is increasing in the quality of the loans. Interestingly, different from the results of canonical security design models, banks with higher-quality loans may not retain more of the loans' cash flow in equilibrium, because the resolution policy could emerge as the more efficient signal.

To highlight the relationship between security design and the resolution policy in our model, we can generalise the above intuition to any security. A distortion in the bank's resolution policy towards liquidation (restructuring) is optimal if the security issued is concave (convex) in the cash flow in the bad state in which the loans become distressed, through both the allocative efficiency channel and the signalling channel. Our analysis thus reveals that, the *direction* of the distortion in the resolution policy depends on the security issued in equilibrium. We believe the insights of the model is general and can be applied to other economic settings.

Extension and Policy. We extend the model to allow the bank to ex ante exert screening effort at loan origination to increase the likelihood of creating a high-quality loan pool. We find that adverse selection in the bank's funding discourages the bank from screening diligently because part of the gains from having a high-quality pool is lost to costly retention. Nonetheless, a resolution policy biased towards liquidation mitigates such inefficiency and in turn restores some of the weakened screening incentives.

The main policy implication from our results is a cautionary one in the Lucas' critique fashion: policies aimed at promoting ex post efficient resolution of distressed loans, such as the Home Affordable Modification Program (HAMP) for mortgages, could inadvertently *reduce* the bank's incentive to screen loans diligently, leading to lower average asset quality and overall welfare.[9] The analysis suggests that, while policies mitigating liquidations might be warranted due to some negative externalities not considered in our model, policy makers should take into account their potential effects on loan origination and on banks' ability to obtain financing.

Empirical implications. Our model generates novel empirical predictions. Our main result predicts an average liquidation bias in banks' resolution policy, consistent with Maturana (2017).[10] To the extent that banks rely on mortgage servicers to carry out the resolution of borrowers' financial distress, our model is consistent with the evidence that mortgage servicers have biased incentives towards liquidation (Thompson (2009), Kruger (2016)). Moreover, after controlling for observable loan pool characteristics, the liquidation bias in banks' resolution policy and the associated loan losses should be larger for banks with higher-quality loans.[11] Finally, because banks signal quality through a combination of security design and a liquidation bias in the resolution policy, the size of retention may be non-monotonic in the quality of the loans. This can reconcile the empirical literature that tests retention as a signal of quality and finds mixed results.[12]

Contribution to the literature. To our best knowledge, this paper is the first to study the role played by banks' resolution policy of their borrowers in mitigating the adverse selection friction in banks' financing. Our results suggest that while information asymmetry regarding asset quality increases banks' funding costs and undermines their screening incentives, a distorted resolution policy could alleviate some of these inefficiencies. Our paper therefore complements the analysis of one of the fundamental roles of banks as delegated information producers to achieve efficient monitoring (Diamond (1984)) and resolution of borrowers' financial distress (Berlin and Loeys (1988), Chemmanur and Fulghieri (1994) and Bolton and Freixas (2000)), by acknowledging their limits due to adverse selection frictions in banks' financing.[13]

[9] In the aftermath of the subprime mortgage crisis, the U.S. experienced a large number of delinquencies and foreclosure filings. In response, the U.S. government developed the HAMP to incentivise mortgage modification instead of foreclosure, by providing direct one-off and annual monetary incentives to mortgage servicers for each successfully modified delinquent mortgage. For a detailed description and an empirical evaluation of HAMP, see Agarwal et al. (2012a).

[10] Maturana (2017) shows that in the context of securitised mortgages, the marginal returns of liquidation exceeds that of restructuring, suggesting that the associated loan loss could have been smaller if there were fewer liquidations.

[11] Testing signalling models is challenging due to the unobservable nature of private information. Recent studies like Begley and Purnanandam (2017) has proxied the ex ante unobservable pool quality with ex post *abnormal* default rate. Our cross-sectional prediction about the resolution policy can be tested using similar methodology.

[12] Garmaise and Moskowitz (2004) and Agarwal et al. (2012b) fail to find strong evidence of retention as a signal of quality, while Begley and Purnanandam (2017) find consistent evidence in the residential mortgage-backed securities market.

[13] Rajan (1992) highlights another dark side of having an informed lender that the lender cannot commit not to extract rent from the borrower with its private information by threatening to liquidate the firm.

Some important elements of our paper can be found in Winton (2003). Winton also observes that a bank's private information regarding its loans enables efficient resolution but also increase its funding costs due to adverse selection. The focus of Winton, however, is different from ours. We study the bank's optimal funding structure and show that the bank adopts a distorted resolution policy in order to reduce its funding cost, whereas Winton assumes that the bank issues equity and instead emphasises that the bank should hold the borrowers' debt to reduce its funding cost.

Our paper extends the canonical liquidity-based security design models, such as DeMarzo (2005) and Biais and Mariotti (2005), by allowing the informed issuer to take actions that affect the distribution of the underlying asset's cash flow, specifically by choosing the resolution policy of borrowers' financial distress. While we also have debt as the optimal security thanks to its relative information insensitivity, we contribute to this literature by showing that a distortion in the bank's resolution policy i) can further reduce the optimal security's information sensitivity and ii) may substitute retention as a signal of quality.

In terms of application, our paper contributes to the burgeoning literature on the interaction between the financing problem of banks and banks' roles as informed intermediaries. Chemla and Hennessy (2014) and Vanasco (2016) explore the trade-off between secondary market liquidity and the incentive to originate good assets. While the result that the adverse selection problem in banks' financing reduces banks' screening incentive is also present in our paper, in addition, we show that banks' optimal resolution policy is distorted from the efficient benchmark to mitigate such adverse selection problem.

Roadmap. The rest of the paper is organised as follows. Section 2 describes the model setup. Section 3 carries out the main analysis of the equilibrium with an endogenous resolution policy. Section 4 extends the model to consider ex ante screening incentives of the bank. Section 5 shows that our results are robust in a general model with multiple types. Section 6 concludes.[14]

2 Model setup

There are three dates in the baseline model: $t = 1$, 2 and 3.[15] The model's participants consist of a bank who owns a continuum of loans and competitive outside investors. All agents are risk neutral. The bank has a discount factor $\delta < 1$ between $t = 1$ and $t = 3$. Outside investors are

[14]The Internet Appendix associated with the paper can be found at https://goo.gl/AgSTGy

[15]We extend the model to a loan-origination stage $t = 0$ in Section 4.

deep-pocketed and have a discount factor equal to 1. Hence, there are gains from trade between the bank and the investors. This follows the assumption of DeMarzo and Duffie (1999) and can be interpreted as the bank's funding needs.[16]

Loan pool and borrowers' financial distress

The bank owns a loan pool containing a continuum of ex ante identical loans that pay off at $t = 3$. We model the loan pool as a well-diversified portfolio of loans. The loan pool is thus only exposed to aggregate risks, which affect the ability for all borrowers to repay.[17] Specifically, with probability π, the loan pool is in the good state (G) at $t = 2$ and no borrowers default. In the good state, each loan returns a riskless cash flow $Z > 0$ at $t = 3$. With probability $1 - \pi$, the loan pool is in the bad state (B) at $t = 2$ and each borrower defaults with some i.i.d. probability d. Thanks to the diversification benefit, the proportion of the loans that become distressed at $t = 2$ is also d. The remaining performing loans continue to return a riskless cash flow $(1 - d)Z$ at $t = 3$.

When a fraction d of the loans become distressed in the bad state at $t = 2$, the bank can choose to liquidate a fraction λ of the distressed loans and restructure the remaining fraction $1 - \lambda$. We will henceforth refer to $\lambda \in [0, 1]$ as the bank's resolution policy. If a distressed loan is liquidated, the loan is terminated and the collateral asset is sold to outside investors. Let $\mathcal{L}(\lambda)$ denote the total liquidation proceeds. Alternatively, if a distressed loan is restructured, the restructured loan pays off a cash flow $X > 0$ with probability θ at $t = 3$ (recovery) or zero otherwise (re-default). For simplicity, we assume that the recovery (and re-default) of restructured loans in a given pool are perfectly correlated. This is also in line with the assumption of a well-diversified loan pool so that only aggregate risks affect the repayment of the borrowers. Finally, we assume that $X \leq Z$, so that the payoff of a loan in the good state is at least as high as in a bad state, even if the loan is restructured and resumes payment. Intuitively, the difference may account for the value of the temporary missing payments and the reduced repayments after restructuring.

The exact functional form of the total liquidation proceeds $\mathcal{L}(\lambda)$ depends on characteristics of the loans as well as the direct and indirect costs associated with liquidation. We abstract

[16]Modelling gains from trade as a discount factor $\delta < 1$ is standard in the literature to capture liquidity needs stemming from, e.g., capital constraints, new investment opportunities, risk-sharing, etc. (see Holmström and Tirole (2011)).

[17]Such aggregate risks can be aggregate property prices or employment opportunities for the borrowers.

from these considerations to keep the analysis general and make the following assumption on the liquidation technology.

Assumption 1. *For* $\lambda \in [0,1]$, *(i)* $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} > 0 > \frac{\partial^2 \mathcal{L}(\lambda)}{\partial \lambda^2}$ *and (ii)* $\lim_{\lambda \to 0^-} \frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} > \theta X > \lim_{\lambda \to 1^+} \frac{\partial \mathcal{L}(\lambda)}{\partial \lambda}$.

Part *(i)* of Assumption 1 states that the total liquidation proceeds increases at a decreasing rate in the fraction of loans liquidated. Part *(ii)* of Assumption 1 implies that the efficient resolution policy that maximises the expected value of distressed loans involves some liquidations and some restructuring. Intuitively, Assumption 1 is satisfied if the distressed loans have heterogeneous liquidation values (with those with highest liquidation values liquidated first), which in turn could be motivated by heterogeneity in collateral values of the distressed loans or in the costs associated liquidation.[18] Heterogeneity in restructuring proceeds could be introduced but does not bring any additional insights.

The loan pool's exposure to aggregate risks is characterised by the probability of entering state G. This probability $\pi \in \{\pi_H, \pi_L\}$, where $\pi_H > \pi_L$, is loan-pool specific and is the source of information asymmetry between the bank and outside investors, as detailed in the next section. We interpret π_i as the "quality" of the loan pool and thus the "type" of the bank (subscript "H" stands for "High" and "L" for "Low").[19] The assumption that the delinquency rate of a loan pool being the bank's private information is in line with empirical studies such as Begley and Purnanandam (2017).[20] A high-quality pool is less exposed or more resilient to aggregate risks and hence is more likely to have no distressed loan (be in the good state G). At $t = 1$, all model participants have the prior belief that $\pi = \pi_H$ with probability γ.[21] One interpretation of γ is a publicly observable signal about the quality of the loans in the pool, e.g. the average FICO scores of the borrowers in a loan pool. Therefore, a pool with higher γ is observably better because it is more likely to be a high-quality pool.

To summarise, for a given resolution policy λ, the overall cash flow c from a type i loan pool at $t = 3$ is given by $c_1 \equiv Z$ with probability π_i (the "Good" state), $c_2(\lambda) \equiv (1-d)Z + d[\mathcal{L}(\lambda) +$

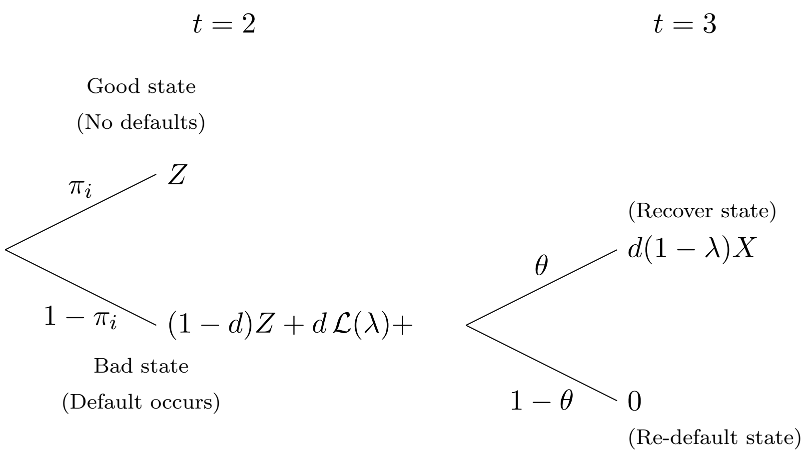
[18] A microfoundation of such a liquidation technology based on heterogeneous liquidation values is provided in the Internet Appendix, where we show that it is indeed optimal for the bank to liquidate the distressed loans with higher liquidation values first.

[19] We extend the baseline two-type model to a multiple-type one in Section 5.

[20] In addition, we could also allow the bank to have some private information on the recovery probability θ. It complicates the analysis without much additional insights as in that case the first-best benchmark will depend on types as well. For simplicity, we assume information about θ is symmetric.

[21] In Section 4 we endogenise this probability γ in the loan-origination stage $t = 0$ through the bank's screening effort choice.

Figure 1: Loan pool cash flow for a given resolution policy λ



$(1-\lambda)X]$ with probability $(1-\pi_i)\theta$ (the "Recovery" state), and $c_3(\lambda) \equiv (1-d)Z + d\,\mathcal{L}(\lambda)$ with probability $(1-\pi_i)(1-\theta)$ (the "Re-default" state), as illustrated in Figure 1.

Financing and security design

Because of the liquidity discount δ, at $t = 1$, the bank would like to raise cash today by selling a security backed by the cash flow of the loan pool to outside investors. The bank receives the cash proceeds from selling the security at $t = 1$, and retains any residual cash flow from the loan pool after paying off the investors at $t = 3$.

We mentioned earlier that there is asymmetric information between the bank and the investors. This creates a financing friction for the bank akin to the classical lemon's problem in Akerlof (1970). Specifically, at the beginning of $t = 1$, the bank receives private information regarding the quality of the mortage pool $\pi_i \in \{\pi_H, \pi_L\}$. This reflects the bank's soft information about its borrowers.

We model the financing stage as follows, similar to the ex-post security design problem in DeMarzo (2005).[22] After observing the private information π_i, the bank chooses to offer outside investors a security $\mathcal{F}$ and promises a resolution policy λ. The security $\mathcal{F}_i$ is contracted upon the cash flow of the loan pool at $t = 3$, specifying a payment $f(c)$ to investors for each realisation of the cash flow $c \in \{c_1, c_2(\lambda), c_3(\lambda)\}$. The security can be expressed as $\mathcal{F} = (f_1, f_2(c_2), f_3(c_2))$, specifying the payments to investors given the cash flow realisation in the good, recovery and

[22] DeMarzo and Duffie (1999) solves the ex ante security design problem, whereas we solve for the ex post security design problem after the banks learn about their private information. As shown by DeMarzo (2005) and DeMarzo et al. (2015), similar intuition carries through in the ex post problem, although the problem becomes more complicated as the design itself becomes a signal.

Table 1: Payoffs of a generic security backed by the loan pool cash flow

Realisation of cash flow	Security payoff $\mathcal{F}$
$c_1 \equiv Z$	f_1
$c_2(\lambda) \equiv (1-d)Z + d\,\mathcal{L}(\lambda) + d(1-\lambda)X$	$f_2(c_2(\lambda))$
$c_3(\lambda) \equiv (1-d)Z + d\,\mathcal{L}(\lambda)$	$f_3(c_3(\lambda))$

re-default state respectively. Table 1 summarises the possible realisations of the cash flow and the corresponding payments specified by a security. We restrict our attention to monotone securities subject to limited liability.[23] The value of the security $\mathcal{F}$ backed by a loan pool of quality i, given a resolution policy λ, is thus given by

$$p_i(\mathcal{F}, \lambda) = \pi_i f_1 + (1-\pi_i)[\theta f_2(c_2(\lambda)) + (1-\theta) f_3(c_3(\lambda))] \tag{1}$$

After observing the offer $(\mathcal{F}, \lambda)$, the competitive investors form a posterior belief $\hat{\pi}$ regarding the private information of the bank, and bid the price of the security p up to its fair value given the belief, $p_{\hat{\pi}}(\mathcal{F}, \lambda)$, defined analogously to Eq. 1. At $t = 3$, after paying investors according to $\mathcal{F}$ from the loan pool cash flow, the bank consumes any residual cash flow.

Timeline and the equilibrium concept

The timeline of the model is summarised in Table 2. The main analysis of the baseline model concerns only $t = 1$, 2 and 3. We extend the model to a loan-origination stage $t = 0$ in Section 4.

The equilibrium concept in this model is the perfect Bayesian equilibrium (PBE). Formally, a PBE consists of a security $\mathcal{F}_i$ issued by the bank of each type $i \in \{H, L\}$, the resolution policy λ_i of the bank of each type, and a system of beliefs such that i) the bank chooses the security and the resolution policy at $t = 1$ to maximise its expected payoff, given the equilibrium choices of the other agents and the equilibrium beliefs, and ii) the beliefs are rational given the equilibrium choices of the agents and are formed using Bayes' rule (whenever applicable). As there can be multiple equilibria in games of asymmetric information, we invoke the Intuitive

[23] A monotone security satisfies that, a higher realisation of the loan pool cash flow should leave both the outside investors and the bank a (weakly) higher payoff. Although this implies some loss of generality, it is not uncommon in the security design literature, e.g. Innes (1990) and Nachman and Noe (1994). One potential justification provided by DeMarzo and Duffie (1999) is that, the issuer has the incentive to contribute additional funds to the assets if the security payoff is not increasing in the cash flow. Similarly, the issuers has the incentive to abscond from the loan pool if the security leaves the issuer a payoff that is not increasing in the cash flow. The full characterisation of a monotone security subject to limited liability is given in Appendix A.

Table 2: Model timeline

$t = 0$	Bank exerts screening effort (Section 4 only)
$t = 1$	Bank observes π_i and offers $(\mathcal{F}_i, \lambda_i)$ Investors purchase the security $\mathcal{F}_i$ at price p
$t = 2$	Loan defaults occur in state B Distressed loans are liquidated or restructured according to the resolution policy λ_i
$t = 3$	Loan pool cash flow is realised Bank and investors are paid off

Criterion of Cho and Kreps (1987) to eliminate equilibria with unreasonable out-of-equilibrium beliefs. This allows us to eliminate all but the least cost separating equilibrium (as shown in Lemma 4).

Discussion of the framework

We adopt and extend the liquidity-based security design framework developed by DeMarzo (2005) to study the joint optimisation problem of security design and the resolution of borrowers' financial distress under asymmetric information. As we shall later illustrate, our mechanism involves the retention of the residual equity claim as a signal of quality, the observability of the bank's resolution policy, and the bank's ability to commit to an ex post inefficient resolution policy. We believe our framework captures some realistic aspects of the securitisation market and here we will discuss some of the crucial features.

On the evidence on equity tranche as a signal

As in DeMarzo (2005), the optimal security in our model is risky debt. In other words, banks with high-quality loan pools signal information to investors through the retention of the residual equity claim. The signalling mechanism is supported by empirical evidence in the residential mortgage market.[24] For example, Begley and Purnanandam (2017) find that, conditional on observable characteristics, residential mortgage-backed securities (RMBS) deals with larger equity tranches have lower delinquency rates and command higher prices, suggesting that investors could and do learn from the equity tranche size.[25]

[24]See the discussion in footnote 7 for recent theoretical works featuring the same mechanism and for supporting evidence in the RMBS market.

[25]While it is possible that in practice the retained tranche might be subsequently sold off in the secondary market, the *initial* retention of the equity tranche could still signal information as long as there are substantial (opportunity) costs associated with it. First, the delayed sale of the equity tranche could be costly to the banks because it implies that some capital is locked-up and thus the banks have to forgo some profitable lending in the

On the observability of resolution policy and the bank's ability to commit

In the model, the bank can commit to a resolution policy, which is observed by investors as part of the signal. We argue that this is realistic. In the mortgage context for instance, banks often delegate the resolution decisions of delinquent mortgages to third-party servicers. Banks can effectively commit to a resolution policy through either the fine-tuning of incentives in the servicers' compensation contract or the choice of servicers with different liquidation capacity. Empirical studies have shown that incentives and identity of servicers matter. Thompson (2009) and Kruger (2016) have argued and documented that the compensation of servicers overall is biased towards liquidation (foreclosure), whereas Agarwal et al. (2011), among others, have shown that the identity of servicers has explanatary power for the liquidation probability of delinquent mortgages. In practice, Moody's, a rating agency, produces "Servicer Quality" (SQ) rating which assesses RMBS and ABS servicers' loss mitigation ability in case of delinquency in securitisation (Moody's (2016)). Therefore, by observing the servicers' identity and compensation structure listed in the prospectus of the deals, investors can infer the resolution policy indirectly chosen by the issuing banks. In the Internet Appendix, we discuss these mechanisms in details and formalise them as extensions of the model.

3 Security design and resolution policy

In this section we analyse our model of security design with endogenous resolution. We first present the benchmark case under symmetric information, then establish the key properties of the model, and finally proceed to characterise the equilibrium under asymmetric information.

3.1 First-best and the symmetric-information benchmark

In this section we first characterise the first-best resolution policy. We then analyse the benchmark equilibrium under symmetric information and show that the first best is achieved in the symmetric-information equilibrium.

loan market or investments in general. Second, due to the segmentation of markets for different ABS tranches, the banks are also likely to be in a less advantageous position in the sale of equity tranche than those of the senior tranche. In practice, the typical equity tranche investors are sophisticated investors like hedge fund with whom the banks are likely to have a weaker bargaining position. For a detailed discussion, see Begley and Purnanandam (2017).

The first-best, efficient, resolution policy maximises the value of the loan pool $V_i(\lambda)$.

$$\lambda_i^{FB} = \arg\max_{\lambda} \; V_i(\lambda) \tag{2}$$

$$\text{where} \quad V_i(\lambda) \equiv \pi_i Z + (1-\pi_i)[(1-d)Z + d\,\mathcal{L}(\lambda) + d(1-\lambda)\theta X] \tag{3}$$

The solution is characterised by the first order condition, $\frac{\partial \mathcal{L}(\lambda_i^{FB})}{\partial \lambda} = \theta X$. That is, since the marginal value obtained from liquidation is decreasing in the fraction of liquidated loans, the first-best level of liquidation is determined such that the the margin value from liquidation is equal to the expected recovery value given restructuring, conditional on the bad state (B). Furthermore, as the H and L type loan pools only differ in the probability of entering state (B), the first-best level of liquidation is identical across types. We therefore drop the type subscript and denote the first-best resolution policy by $\lambda^{FB} \in (0,1)$.[26]

We now characterise the equilibrium under symmetric information. First consider the optimal security issued at $t = 1$. Since any retention of the cash flow by the bank incurs a discount, it is optimal for the bank to sell the entire cash flow of the loan pool to investors, given that all securities are fairly priced under symmetric information. Second, given that the entire cash flow is sold, the bank optimally adopts the first-best resolution policy λ^{FB} to maximise the value of the loan pool and hence its payoff.

The following proposition summarises the symmetric-information benchmark results. All proofs are in the Appendix.

Lemma 1. *In the symmetric-information benchmark, the bank of both types sells the entire cash flow of the loan pool, and chooses the first-best resolution policy λ^{FB}.*

Denote henceforth the expected payoff to a type i bank in the symmetric-information benchmark by $U_i^{FB} \equiv V_i(\lambda^{FB})$. We would like to stress the fact that the first-best resolution policy is achieved in the symmetric-information benchmark equilibrium. Therefore, any distortion in the equilibrium resolution policy in this paper is driven by information asymmetry between the bank and the investors.

3.2 Security design and resolution policy: key properties

The main analysis of the paper focuses on how the bank's private information affects its optimal security design and resolution policy. Before turning to characterise the equilibrium under

[26] That λ^{FB} is strictly interior follows from Part (ii) of Assumption 1.

asymmetric information, here we first highlight two key properties of the model that underlie the relationship between the bank's security design and resolution policy.

It is useful to write down the expected payoff of the bank for a given security design and a given resolution policy. At $t = 1$, a bank with private information π_i issues a security $\mathcal{F}$ backed by the cash flow of the loan pool, and promises a resolution policy λ. Upon observing the offer from the bank $(\mathcal{F}, \lambda)$, investors form a belief about the quality $\hat{\pi}$ of the loan pool. Let $U_i(\mathcal{F}, \lambda; \hat{\pi})$ denote the expected payoff to the bank, where

$$U_i(\mathcal{F}, \lambda; \hat{\pi}) = p_{\hat{\pi}}(\mathcal{F}, \lambda) + \delta \left[V_i(\lambda) - p_i(\mathcal{F}, \lambda)\right] \tag{4}$$

For a given $(\mathcal{F}, \lambda)$, the bank's expected payoff consists of two parts. The first term of Eq. 4 is the proceeds from issuing the security $p_{\hat{\pi}}(\mathcal{F}, \lambda)$, i.e. investors' valuation of the security given belief $\hat{\pi}$. The second term of Eq. 4 is the bank's own valuation of the retained cash flow $\delta \left[V_i(\lambda) - p_i(\mathcal{F}, \lambda)\right]$, which is equal to the expected value of the loan pool less the expected value of the security sold, given the bank's private information π_i and its liquidity discount δ. $p_i(\mathcal{F}, \lambda)$ is given by Eq. 1 and $p_{\hat{\pi}}(\mathcal{F}, \lambda)$ is defined analogously.

The first key property of the model, presented in the following lemma, shows that security design directly affects the bank's choice of resolution policy, even when information is symmetric, that is when $\hat{\pi} = \pi_i$. Denote by $\hat{\lambda}_i(\mathcal{F}) = \arg\max_{\lambda} U_i(\mathcal{F}, \lambda; \pi_i)$ the optimal resolution policy for the type i bank for a given monotone security $\mathcal{F}$.

Lemma 2 (Directional distortion to improve allocative efficiency)**.**

$$\hat{\lambda}_i(\mathcal{F}) = \hat{\lambda}(\mathcal{F}) \begin{cases} > \lambda^{FB}, & \textit{if } f_3'(c_3(\lambda)) \geq f_2'(c_2(\lambda)) \textit{ for all } \lambda \textit{ and } f_3'(c_3(\lambda^{FB})) > f_2'(c_2(\lambda^{FB})) \\ < \lambda^{FB}, & \textit{if } f_3'(c_3(\lambda)) \leq f_2'(c_2(\lambda)) \textit{ for all } \lambda \textit{ and } f_3'(c_3(\lambda^{FB})) < f_2'(c_2(\lambda^{FB})) \\ = \lambda^{FB}, & \textit{if } f_3'(c_3(\lambda)) = f_2'(c_2(\lambda)) \textit{ for all } \lambda. \end{cases}$$

Note the distortion only depends on the security $\mathcal{F}$ but not bank type i.

When a security's cash flow is more sensitive to the loan pool's cash flow in the re-default state than that in the recovery state, i.e. $f_3'(c_3(\lambda)) \geq f_2'(c_2(\lambda))$ for all λ, we loosely refer to it as "concave" in the underlying loan pool's cash flow in the bad state. Similarly, a security is "convex" if $f_3'(c_3(\lambda)) \leq f_2'(c_2(\lambda))$ for all λ. This lemma implies that even in the absence of information frictions, the bank prefers to distort its resolution policy towards liquidation (restructuring) if it issues a security that is concave (convex) in the bad state. To see this, when information is symmetric, $p_{\hat{\pi}}(\mathcal{F}, \lambda) = p_i(\mathcal{F}, \lambda)$ and the bank's expected payoff becomes

$$U_i(\mathcal{F}, \lambda; \pi_i) = \delta V_i(\lambda) + (1 - \delta) p_i(\mathcal{F}, \lambda) \tag{5}$$

The bank's expected payoff consists of the discounted value of the pool $V_i(\lambda)$, and the gains from trade by selling a security $\mathcal{F}$ fairly priced at $p_i(\mathcal{F}, \lambda)$. Suppose the security is concave in the underlying cash flow in the bad state and consider a marginal distortion in the resolution policy towards liquidation from the efficient level λ^{FB}. Such distortion does not affect the expected value of the pool but reduces the underlying cash flow's riskiness, as it reduces the loan pool's exposure to borrowers' re-default risks. By Jensen's inequality, it increases the expected value of the concave security, thereby increasing the total gains from trade. Conversely, given a security that is convex in the cash flow in the bad state, distorting the resolution policy towards restructuring increases the bank's payoff. Lemma 2 thus suggests that, for a given security design, distortion in the resolution policy can lead to greater allocative efficiency.

The second key property underlies the signalling function of the resolution policy. It is a single-crossing property of the informed bank's indifference curves, which is stated in the following lemma.

Lemma 3 (Directional single-crossing). $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ *is*

- *constant in i for all λ if $f_2(c_2(\lambda)) = c_2(\lambda)$ and $f_3(c_3(\lambda)) = c_3(\lambda)$; otherwise, it is*
- *strictly decreasing in i for $\lambda \geq \lambda^{FB}$ if $f_3'(c_3(\lambda)) \geq f_2'(c_2(\lambda))$ and $f_3'(c_3(\lambda^{FB})) > f_2'(c_2(\lambda^{FB}))$,*
- *strictly increasing in i for $\lambda \leq \lambda^{FB}$ if $f_3'(c_3(\lambda)) \leq f_2'(c_2(\lambda))$ and $f_3'(c_3(\lambda^{FB})) < f_2'(c_2(\lambda^{FB}))$,*
- *strictly decreasing in i for $\lambda > \lambda^{FB}$ and strictly increasing in i for $\lambda < \lambda^{FB}$ if $f_3'(c_3(\lambda)) = f_2'(c_2(\lambda))$.*

The single-crossing property states that, the ratio of marginal cost (in the form of a lower retained payoff due to a distortion in the resolution policy) to marginal benefit (in the form of a higher issuance price due to an improvement in the bank's perceived quality) is monotonic in the bank's type. Since the latter $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \hat{\pi}} = \frac{\partial p_{\hat{\pi}}(\mathcal{F},\lambda)}{\partial \hat{\pi}}$ is the same for all types of banks, the single-crossing property is equivalent to the monotonicity of the marginal cost $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \lambda}$ in type, where

$$\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \lambda} = \frac{\partial p_{\hat{\pi}}(\mathcal{F},\lambda)}{\partial \lambda} + \delta \frac{\partial}{\partial \lambda}\left[V_i(\lambda) - p_i(\mathcal{F},\lambda)\right] \tag{6}$$

We show that $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \lambda}$ can be increasing or decreasing in type i, depending on the security design. Notice that the first term of equation 6 is about the issuance proceeds and only depends on investors' belief $\hat{\pi}$ but not the bank's true type i. The second term of Eq. 6 represents the effect of increasing liquidation on the bank's valuation of the retained cash flow, which drives the directional single-crossing property for two reasons. First, the effect of a distortion in the resolution policy depends on security design. When the security issued is concave and hence the retained claim is convex in the underlying cash flow in the bad state, for any $\lambda \geq \lambda^{FB}$, biasing the resolution policy towards liquidation (increasing λ) decreases the value of the retained security. This is because both the expected value and the riskiness of the underlying cash flow are decreased. Second, any effect from the change in resolution policy is stronger for lower-quality banks, as their loans default more often. Therefore, when the issued security is concave, biasing the resolution policy towards liquidation is *more costly* for lower-quality banks than for higher-quality ones. Conversely, if the issued security is convex and hence the retained security is concave, for all $\lambda \leq \lambda^{FB}$, biasing towards restructuring (reducing λ from λ^{FB}) is more costly for lower-quality banks than for higher-quality banks.

The single-crossing property is a maintained assumption of the signaling literature and a key step to establishing the uniqueness of the refined equilibrium under the Intuitive Criterion (e.g. Cho and Kreps (1987), Mailath (1987) and DeMarzo (2005)). While the single-crossing property is satisfied in our model with respect to the resolution policy, Lemma 3 emphasises that the *direction* of the single-crossing condition depends on the security design. It suggests that if the bank with high-quality loans issues a concave (convex) security, it can signal its quality by adopting a resolution policy with a liquidation (restructuring) bias.

To sum up, we show in this section that security design could affect a bank's resolution policy through an allocative efficiency channel and a signalling channel. Importantly, both channels bias the bank's resolution policy in the same direction: if the issued security is concave (convex), a liquidation (restructuring) bias in resolution policy can improve allocative efficiency and may serve as a costly signal of information.

3.3 Equilibrium security design and resolution policy under asymmetric information

The bank's private information about the quality of the loan pool creates an adverse selection problem when the bank raises financing from uninformed investors. In this section, we show how the bank optimises its security design and resolution policy to mitigate the adverse selection problem. The analysis leads to the main result of the paper: biasing the resolution policy towards liquidation is optimal because the optimal security is debt.

Our analysis focuses on the least cost separating equilibrium, which is the unique equilibrium that satisfies the Intuitive Criterion in our model. We state this uniqueness result in the following lemma and formally prove it in the Appendix.

Lemma 4. *The unique equilibrium that satisfies the Intuitive Criterion is the least cost separating equilibrium.*

Let's start the analysis with the bank who owns a low-quality, i.e. a more distress-prone loan pool. In the least cost separating equilibrium, the low-type bank achieves the first best outcome because the high-type bank has no incentive to mimic the low type (verified in equilibrium). Denote by U_i^*, λ_i^* and $\mathcal{F}_i^*$ the expected payoff, the resolution policy, and security design of a type i bank in equilibrium respectively. Therefore, $U_L^* = U_L^{FB}$, $\lambda_L^* = \lambda^{FB}$, and $\mathcal{F}_L^* = (c_1, c_2(\lambda^{FB}), c_3(\lambda^{FB}))$.

Meanwhile, in order to deter the low type's mimicry, the high-type bank has to take costly actions such as retention of the loan pool's cash flow and distortion in the resolution policy. More precisely, in the least cost separating equilibrium, the high-type bank's expected payoff is maximised by the choices of the monotone security $\mathcal{F}_H$ under limited liability and the resolution policy λ_H, subject to the incentive compatibility constraint (IC) that the low type does not mimic. The maximisation problem is stated below.[27]

$$U_H^* = max_{(\mathcal{F}_H,\lambda_H)}\, U_H(\mathcal{F}_H, \lambda_H; \pi_H) \quad s.t. \quad (IC) \quad U_L^{FB} \geq U_L(\mathcal{F}_H, \lambda_H; \pi_H) \qquad (7)$$

where $U_i(\mathcal{F}, \lambda; \hat{\pi})$ is defined by Eq. 4. The optimisation problem in Eq. 7 involves the simultaneous optimisation along two dimensions: the security design and the resolution policy. We solve the joint optimisation problem sequentially and first characterise the optimal security

[27] The characterisation of the monotonicity and limited liability constraints on the security design is in Appendix A.

for any given resolution policy.

Lemma 5 (Optimal security). *For any resolution policy λ_H, risky debt with a promised repayment $F_H \in (c_3(\lambda_H), Z)$ implements the optimal security for the high-type bank.*

This result is consistent with the classic literature on the pecking order of external financing under asymmetric information (e.g. Myers (1984)).[28] The optimal monotone security issued by the high type is a debt security, because it is least information sensitive. The high type exhausts its capacity to issue risk-free debt ($F_H > c_3(\lambda)$), which is free from any information asymmetry. The retention of future cash flow being a costly signal is a well-established result in the security design literature such as Leland and Pyle (1977) and DeMarzo and Duffie (1999).

We now proceed to characterise the joint determination of the optimal resolution policy and the optimal security. To solve the problem stated in Eq. 7, by Lemma 5, we can restrict the security $\mathcal{F}_H$ to be risky debt, which is characterised by the promised repayment F_H. Hence, the value of the security $p_i(\mathcal{F}, \lambda)$ can be expressed as $p_i(F, \lambda)$ for a given promised repayment F and a resolution policy λ, backed by a loan pool of quality i, where

$$p_i(F, \lambda) = \pi_i F + (1 - \pi_i) \left[\theta \min\{c_2(\lambda), F\} + (1 - \theta)c_3(\lambda)\right] \tag{8}$$

The following proposition states the main result of the paper.

Proposition 1 (Equilibrium resolution policy and security design under asymmetric information). *In the least cost separating equilibrium, the low-type bank sells the entire cash flow and adopts the first-best resolution policy $\lambda_L^* = \lambda^{FB}$, whereas the high-type bank issues risky debt with a promised repayment $F_H^* \in (c_3(\lambda_H^*), Z)$ and adopts a resolution policy with a liquidation bias $\lambda_H^* \geq \lambda^{FB}$. The inequality is strict if and only if $G(\lambda^{FB}) > 0$, where*

$$G(\lambda) \equiv \delta\pi_L Z + (1 - \delta\pi_L)c_2(\lambda) - (1 - \pi_H)(1 - \theta)d(1 - \lambda)X - U_L^{FB} \tag{9}$$

The result that the high-type bank adopts a resolution policy with a liquidation bias follows from the two channels – the allocative efficiency channel and the signalling channel – through which security design affects the bank's resolution policy (Lemmas 2 and 3), and from the

[28] Technically, the cash flow distribution in our model satisfies the Hazard Rate Ordering (HRO) property, which is weaker than the Monotone Likelihood Ratio Property (MLRP) commonly assumed in signalling environments. DeMarzo et al. (2015) show that the (HRO) is a sufficient condition to ensure that debt is the optimal monotone security in a signalling framework with liquidity needs.

optimality of debt (Lemma 5). Importantly, the bias is strict if only if $G(\lambda^{FB}) > 0$. We discuss this condition below.

There is no distortion in equilibrium resolution policy when $G(\lambda^{FB}) \leq 0$. This condition, as shown in the Appendix, implies that $F_H^* \geq c_2(\lambda^{FB})$ and thus the optimal debt defaults whenever the bad state occurs. In this case, both channels imply no distortion. For the allocative efficiency channel, when the bad state realises, the debt becomes a claim of total cash flow from the loan pool. The debt is thus linear in the underlying cash flow and by Lemma 2, the value of the debt security is maximised at λ^{FB}. For the signalling channel, the strict single-crossing property is not satisfied because the retained claim has a value of zero for all types of banks. Therefore, the equilibrium resolution policy is the efficient one.

On the other hand, the equilibrium resolution policy is distorted towards liquidation when $G(\lambda^{FB}) > 0$. In this case, the optimal debt only defaults in the re-default state, but not in the recovery state. The security is thus concave in the underlying cash flow of the loan pool in the bad state. Thus, both channels imply a distortion in the resolution policy towards liquidation. First, the allocatively efficient resolution policy is given by $\hat{\lambda}(F_H^*) > \lambda^{FB}$, following immediately from Lemma 2. Second, the signalling channel further biases the high-type bank's equilibrium resolution policy towards liquidation *beyond* the allocatively efficient level, i.e. $\lambda_H^* \geq \hat{\lambda}(F_H^*)$.[29] To see this, suppose $\hat{\lambda}(F_H^*)$ is given by the first order condition. Then, at $\lambda = \hat{\lambda}(F_H^*)$,

$$\frac{\partial U_L(F_H^*, \lambda_H; \pi_H)}{\partial \lambda_H}|_{\lambda_H = \hat{\lambda}(F_H^*)} < \frac{\partial U_H(F_H^*, \lambda_H; \pi_H)}{\partial \lambda_H}|_{\lambda_H = \hat{\lambda}(F_H^*)} = 0$$

The inequality follows immediately from the single-crossing condition for $\lambda_H > \lambda^{FB}$ shown in Lemma 3. That is, a marginal increase in λ_H strictly decreases the low type's mimicking payoff in Eq. 7 without affecting the high-type bank's payoff in equilibrium. Hence, it is strictly optimal for the high type to liquidate more than the allocatively efficient level $\lambda_H^* > \hat{\lambda}(F_H^*)$ in this case. In general, given the optimal security F_H^*, increasing λ beyond $\hat{\lambda}(F_H^*)$ is costly and crucially, more costly for the low-type bank than for the high-type bank. Therefore, biasing the resolution policy beyond $\hat{\lambda}(F_H^*)$ can serve as a signal of information in equilibrium.

To summarise, Proposition 1 highlights that a resolution policy with a liquidation bias can arise in the equilibrium under asymmetric information to mitigate adverse selection. In particular, it arises when $G(\lambda^{FB}) > 0$, which is the case if i) the adverse selection problem

[29] The claim that $\lambda_H^* \geq \hat{\lambda}(F_H^*)$ is formally shown in the proof of Proposition 1.

is severe, i.e. π_H is high or π_L is low, and/or ii) the cost of retention $(1-\delta)$ is low, so that retention is ineffective in signalling information.

Finally, the following comparative statics further illustrates the effect of adverse selection on the high-type bank's resolution policy. An increase in the high-type bank's loan quality, π_H, exacerbates the information asymmetry because it creates greater mimicking incentives. This then leads to a larger liquidation bias in the high-type bank's resolution policy in equilibrium.

Corollary 1. *The liquidation bias in the high-type bank's resolution policy is increasing in the quality of its loans. That is,* $\frac{\partial \lambda_H^*}{\partial \pi_H} \geq 0$*, where the inequality is strict if and only if* $G(\lambda^{FB}) > 0$*.*

4 Screening and resolution policy

In this section, we study the implication of the bank's distortion in the resolution policy for the bank's screening incentives. We extend the model to incorporate a loan-origination stage $t = 0$, at which point the bank can exert non-observable costly screening effort to increase the probability γ of receiving a high-quality loan pool at $t = 1$. The main finding is that while information asymmetry leads to underinvestment in screening effort, adopting the optimal resolution policy featuring a liquidation bias mitigates this underinvestment problem and the associated inefficiency.

At $t = 0$, the bank is endowed with 1 unit of funds and can invest in a loan pool. When investing, the bank can exert non-observable effort to affect $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, the probability that the loan pool is of high quality at $t = 1$, where $0 \leq \underline{\gamma} < \bar{\gamma} \leq 1$. Such effort can be interpreted as, for example, time and resources spent to assess the quality of the borrowers' investment projects and to screen out borrowers who have less valuable projects. Effort incurs a quadratic cost of $\dfrac{1}{2}k(\gamma - \underline{\gamma})^2$. We assume $k \geq \frac{U_H^{FB} - U_L^{FB}}{\bar{\gamma} - \underline{\gamma}}$ to guarantee an interior optimal level of effort, and $U_L^{FB} \geq 1$ so that investing in the loan pool is always efficient.

4.1 Equilibrium screening effort

In this section we solve for the optimal screening effort exerted by the bank in equilibrium. The bank is willing to exert costly effort because the expected payoff to the high type U_H is higher than that to the low type U_L. Since U_H and U_L are potentially affected by the information environment, the security design, and the resolution policy in the subsequent stages of the model, so is the bank's optimal screening effort.

Notice that since the subsequent equilibrium in the funding stage is separating, the equilibrium payoffs $\{U_H, U_L\}$ do not depend on γ. We can therefore consider any generic pair of $\{U_H, U_L\}$ that represents the expected payoffs to the bank in the separating equilibrium in the subgame starting at $t = 1$. At $t = 0$, the bank chooses the optimal level of effort to maximise its ex ante expected payoff

$$\max_{\gamma} \quad \gamma U_H + (1-\gamma)U_L - \frac{1}{2}k(\gamma - \underline{\gamma})^2 \tag{10}$$

The optimal effort is thus

$$\gamma^*(U_H, U_L) = \underline{\gamma} + \frac{U_H - U_L}{k} \tag{11}$$

The optimal effort chosen by the bank is increasing in the difference in the expected payoff $(U_H - U_L)$ between having a high-quality and a low-quality pool. We will look at how this difference changes under symmetric and asymmetric information, and given different resolution policy.

Under symmetric information, both banks with high- and low-quality pools adopt the efficient resolution policy λ^{FB} and achieve payoff (U_H^{FB}, U_L^{FB}) respectively. Under asymmetric information, the low-type bank attains the same payoff as under symmetric information, i.e. $U_L^* = U_L^{FB}$, because it suffers no information friction and hence optimally chooses the efficient resolution policy λ^{FB} and sells a full pass-through security. On the other hand, the high type is strictly worse off under asymmetric information because of the signalling cost $U_H^* < U_H^{FB}$. As a result, the bank exerts *strictly less* effort.

Lemma 6. *Compared to the symmetric information case, the bank under-expends screening effort under asymmetric information. That is* $\gamma^*(U_H^{FB}, U_L^{FB}) > \gamma^*(U_H^*, U_L^*)$.

4.2 Inefficiency of intervention in resolution policy

Next we turn to the question of how regulatory interventions in banks' resolution policies can affect banks' screening effort. As shown in our main result, adopting a resolution policy with a liquidation bias allows the high-type bank to alleviate adverse selection problem in equilibrium. Our extension reveals that this also creates stronger incentives for the bank to screen borrowers in order to create a high-quality loan pool, further increasing welfare. The following proposition summarises the effect of a regulatory intervention in the resolution policy on banks' ex ante screening effort and on welfare.

Proposition 2 (Policy implication)**.** *If the government imposes a resolution policy λ_H different from the equilibrium policy λ_H^*, including the ex post efficient policy λ^{FB}, the bank exerts less screening effort at $t = 0$, hence reducing the total welfare.*

Proposition 2 suggests that there is an unintended consequence of government policy like Home Affordable Modification Program (HAMP) which aims to restore efficiency in the resolution decision of delinquent mortgages. When banks have financing needs, imposing any resolution policy different from λ_H^* on the bank *reduces* its payoff in the case of receiving a high-quality loan pool due to adverse selection. This in turn lowers its incentive to exert screening effort to obtain a high-quality pool. This under-provision of value-enhancing screening effort decreases social welfare.

5 Extension: multiple types

The goal of this section is to show that our result is robust to an extension to multiple types. In line with the baseline two-type model, we find that the liquidation bias is (weakly) larger for banks with higher-quality loan pools.

We extend the baseline model with two types to n types. That is, the probability that the loan pool enters the good (G) state is given by $\pi_i \in \{\pi_1, \pi_2, ..., \pi_n\}$, where $1 > \pi_i > \pi_{i-1} > 0$ for all $i \in \{2, ..., n\}$. As before, we focus on the least cost separating equilibrium.[30] Analogous to Eq. 7, the least cost separating equilibrium with n types is given by

$$
\begin{aligned}
U_1^n &= \max_{(\mathcal{F}_1,\lambda_1)} \quad p_1(\mathcal{F}_1,\lambda_1) + \delta\left[V_1(\lambda_1) - p_1(\mathcal{F}_1,\lambda_1)\right] \\
U_i^n &= \max_{(\mathcal{F}_i,\lambda_i)} \quad p_i(\mathcal{F}_i,\lambda_i) + \delta\left[V_i(\lambda_i) - p_i(\mathcal{F}_i,\lambda_i)\right] \quad s.t. \quad (IC_i) \quad \forall\, i \geq 2
\end{aligned} \tag{12}
$$

where the incentive compatibility constraint (IC_i) that type $i-1$ will not mimic type i is given by

$$
(IC_i) \quad U_{i-1}^n \geq p_i(\mathcal{F}_i,\lambda_i) + \delta\left[V_{i-1}(\lambda_i) - p_{i-1}(\mathcal{F}_i,\lambda_i)\right] \tag{13}
$$

[30]The prior distribution of types is thus irrelevant.

and the value of the security $p_i(\mathcal{F}, \lambda)$ is given by Eq. 8. We show in the Appendix that the set of local incentive compatibility constraints (IC_i) implies no mimicking by all other types, and thus characterises the equilibrium. We summarise the result in the following proposition and discuss the intuition below. We denote by superscript n all equilibrium quantities in a model with n types.

Proposition 3. *In a model with n types, the least cost separating equilibrium exists, in which the bank of type i issues risky debt with a promised repayment F_i^n. There exists a unique type $j > 1$ such that the equilibrium resolution policies are*

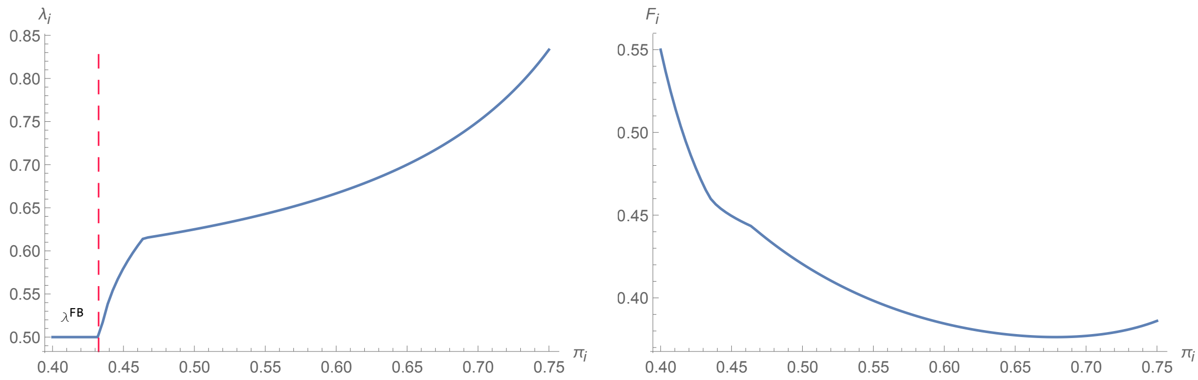
$$\lambda_i^n \begin{cases} = \lambda^{FB} & \textit{for} \quad i < j \\ > \lambda^{FB} & \textit{for} \quad i \geq j \end{cases}$$

Moreover, $\lambda_i^n \geq \lambda_{i-1}^n$ for all $i \geq j$, where the inequality is strict whenever $\lambda_i^n < 1$.

Proposition 3 shows that the insights from our baseline two-type model (Proposition 1) can be extended to a model with multiple types. That is, banks with higher-quality loans adopt resolution policies with (weakly) larger liquidation bias. The intuition behind this result is similar to the two-type case, and is a consequence of the allocative efficiency channel and the signalling channel given by Lemmas 2 and 3. In particular, banks with higher-quality loans $(i \geq j)$ who face severe adverse selection retain larger amounts of the loan pool's cash flow $F_i^n < c_2(\lambda^{FB})$. These banks issue debt securities that are concave in the cash flow of the loan pool in the bad state, as discussed in Section 3.3, and therefore distort their resolution policies in equilibrium towards liquidation. Banks with lower-quality loans $(i < j)$ adopt the first-best resolution policy and separate by retaining more cash flow of the loan pool.

This extension sheds additional light on the interaction between the bank's security design and the choice of resolution policy. Banks with higher-quality loans $(i \geq j)$ signal their quality through a combination of cash flow retention and distortion in the resolution policy. In contrast to DeMarzo (2005) and DeMarzo et al. (2015) who only allow retention as a signal, we find that cash flow retention by banks in equilibrium may be non-monotonic in loan quality. This is because a liquidation bias in the resolution policy can substitute retention as a signal of quality in our model. While a bank with higher-quality loans may issue debt with a higher promised repayment, separation is still achieved as long as the distortion in the resolution policy is sufficiently costly to deter mimicry. This result is formally presented in Corollary 2

Figure 2: The equilibrium resolution policy λ_i^n (left panel) and promised repayment F_i^n (right panel). The parameter values used in this plot are $Z = 0.55$, $(1-d)Z = 0.2$, $dX = 0.3$, $\theta = 0.5$, $\delta = 0.9$, $\pi_1 = 0.4$, $\pi_{100} = 0.75$, $\pi_i - \pi_{i-1} = \frac{0.35}{99}$ for all $i \in \{2, ..., 100\}$, and $\mathcal{L}(\lambda) = \frac{1-(1-\lambda)^2}{2} X$. The red dashed line in the left panel marks the type j.



and illustrated in Figure 2.

Corollary 2. *In equilibrium, F_i^n is strictly decreasing in loan quality i for all $i < j$. F_i^n may be non-monotonic in loan quality i for $i \geq j$.*

6 Conclusion

This paper studies a bank's resolution policy for its borrowers in case the borrowers enter financial distress, when the bank must raise financing from uninformed investors. We show that the bank may distort its resolution policy to i) improve allocative efficiency and ii) to signal information about the quality of its loans to investors. Our analysis highlights the importance of security design for the resolution policy. For a normative perspective, our results caution that policies attempting to restore ex post resolution efficiency can have the unintended consequence of reducing the banks' ex ante screening effort, thereby worsening the average quality of the loan pools and reducing social welfare.

We conclude with some conjectures for directions for future work and extensions. First, this framework can be extended to a setting with multiple banks to study the spillover effects of distressed loan liquidation due to fire-sale or information externality. It could also be fruitful to analyse, in a general equilibrium, the potential impact of banks' resolution policy on the quantity, quality, and the prices of loans originated. Finally, a dynamic framework could shed light on how banks' incentive to liquidate loans vary across business cycles.

References

Agarwal, S., Amromin, G., Ben-David, I., Chomsisengphet, S., and Evanoff, D. D. (2011). The role of securitization in mortgage renegotiation. *Journal of Financial Economics*, 102(3):559–578.

Agarwal, S., Amromin, G., Ben-David, I., Chomsisengphet, S., Piskorski, T., and Seru, A. (2012a). Policy intervention in debt renegotiation: Evidence from the Home Affordable Modification Program. NBER Working Paper No. 18311.

Agarwal, S., Chang, Y., and Yavas, A. (2012b). Adverse selection in mortgage securitization. *Journal of Financial Economics*, 105(3):640–660.

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics*, 84(3):488–500.

Balasubramanyan, L., Berger, A. N., Koepke, M. M., et al. (2017). How do lead banks use their private information about loan quality in the syndicated loan market?

Begley, T. and Purnanandam, A. (2017). Design of financial securities: Empirical evidence from private-label RMBS deals. *Review of Financial Studies*, 30(1):120–161.

Berlin, M. and Loeys, J. (1988). Bond covenants and delegated monitoring. *The Journal of Finance*, 43(2):397–412.

Biais, B. and Mariotti, T. (2005). Strategic liquidity supply and security design. *Review of Economic Studies*, 72(3):615–649.

Bolton, P. and Freixas, X. (2000). Equity, bonds, and bank debt: Capital structure and financial market equilibrium under asymmetric information. *Journal of Political Economy*, 108(2):324–351.

Chemla, G. and Hennessy, C. A. (2014). Skin in the game and moral hazard. *The Journal of Finance*, 69(4):1597–1641.

Chemmanur, T. J. and Fulghieri, P. (1994). Reputation, renegotiation, and the choice between bank loans and publicly traded debt. *Review of Financial Studies*, 7(3):475–506.

Cho, I.-K. and Kreps, D. M. (1987). Signaling games and stable equilibria. *The Quarterly Journal of Eonomics*, 102(2):179–222.

DeMarzo, P. and Duffie, D. (1999). A liquidity-based model of security design. *Econometrica*, 67(1):65–99.

DeMarzo, P. M. (2005). The pooling and tranching of securities: A model of informed intermediation. *Review of Financial Studies*, 18(1):1–35.

DeMarzo, P. M., Frankel, D. M., and Jin, Y. (2015). Portfolio liquidation and security design with private information. Stanford University Graduate School of Business Research Paper No. 15–24.

Diamond, D. W. (1984). Financial intermediation and delegated monitoring. *Review of Economic Studies*, 51:393–414.

Garmaise, M. J. and Moskowitz, T. J. (2004). Confronting information asymmetries: Evidence from real estate markets. *Review of Financial Studies*, pages 405–437.

Gertner, R. and Scharfstein, D. (1991). A theory of workouts and the effects of reorganization law. *The Journal of Finance*, 46(4):1189–1222.

Giammarino, R. M. (1989). The resolution of financial distress. *Review of Financial studies*, 2(1):25–47.

Gilson, S. (2012). Coming through in a crisis: How chapter 11 and the debt restructuring industry are helping to revive the us economy. *Journal of Applied Corporate Finance*, 24(4):23–35.

Haugen, R. A. and Senbet, L. W. (1978). The insignificance of bankruptcy costs to the theory of optimal capital structure. *Journal of Finance*, 33(2):383–393.

Holmström, B. and Tirole, J. (2011). *Inside and Outside Liquidity*. MIT Press.

Innes, R. D. (1990). Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory*, 52(1):45–67.

Kruger, S. A. (2016). The effect of mortgage securitization on foreclosure and modification. Available at SSRN: http://ssrn.com/abstract=2794122.

Leland, H. and Pyle, D. (1977). Informational asymmetries, financial structure and financial intermediation. *Journal of Finance*, 32:371–87.

Mailath, G. J. (1987). Incentive compatibility in signaling games with a continuum of types. *Econometrica*, pages 1349–1365.

Maturana, G. (2017). When are modifications of securitized loans beneficial to investors? *The Review of Financial Studies*, 30(11):3824–3857.

Moody's (2016). *Rating Symbols and Definitions.*

Myers, S. C. (1984). The capital structure puzzle. *Journal of Finance*, 39(3):575–592.

Myers, S. C. and Majluf, N. S. (1984). Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics*, 13(2):187–221.

Nachman, D. C. and Noe, T. H. (1994). Optimal design of securities under asymmetric information. *Review of Financial Studies*, 7(1):1–44.

Rajan, R. G. (1992). Insiders and outsiders: The choice between informed and arm's-length debt. *The Journal of Finance*, 47(4):1367–1400.

RealtyTrac (2015). Year-end 2014 U.S. foreclosure market report. Retreived on Jan 30, 2015.

Repullo, R. and Suarez, J. (1998). Monitoring, liquidation, and security design. *The review of financial studies*, 11(1):163–187.

Thompson, D. E. (2009). *Why Servicers Foreclose When They Should Modify and Other Puzzles of Servicer Behavior: Servicer Compensation and its Consequences.* National Consumer Law Center.

Vanasco, V. (2016). The downside of asset screening for market liquidity. *Working paper.*

Wang, K., Young, L., and Zhou, Y. (2002). Nondiscriminating foreclosure and voluntary liquidating costs. *The Review of Financial Studies*, 15(3):959–985.

Winton, A. (2003). Institutional liquidity needs and the structure of monitored finance. *The Review of Financial Studies*, 16(4):1273–1313.

Appendices

A Monotone security under limited liability

We restrict to monotone securities under limited liability. In this section we formalise these constraints as follows

$$\begin{aligned}
(MNO) \quad & f_1 \geq f_2(c_2) \geq f_3(c_3) \geq 0 \; \forall c_2 \in [c_2(1), c_2(0)] \text{ and } c_3 \in [c_3(0), c_3(1)] \text{ and} \\
& \text{and } \frac{\partial f_j(c_j)}{\partial c_j} \geq 0 \; \forall j \in \{2,3\} \\
(MNI) \quad & c_1 - f_1 \geq c_2 - f_2(c_2) \geq c_3 - f_3(c_3) \; \forall c_2 \in [c_2(1), c_2(0)] \text{ and } c_3 \in [c_3(0), c_3(1)] \\
& \text{and } \frac{\partial}{\partial c_j}(c_j - f_j(c_j)) \geq 0 \; \forall j \in \{2,3\} \qquad (14)
\end{aligned}$$

where (MNO) and (MNI) stand for the monotonicity constraints of the outside investors and the insider respectively. These constraints state that, respectively, the payoff of the security and the residual payoff to the bank are weakly increasing in the realisation of the cash flow. Note by restricting the payoff of the security and the residual payoff to the bank to be positive, the limited liability constraint is satisfied.

B Proofs

B.1 Proof of Lemma 1

This result follows immediately from the discussion.

B.2 Proof of Lemma 2

Following the discussion immediately following the lemma in Section 3.2, $U_i(\mathcal{F}, \lambda; \pi_i)$ can be expressed as Eq. 5. Therefore

$$\begin{aligned}
\frac{\partial U_i(\mathcal{F}, \lambda; \pi_i)}{\partial \lambda} &= \delta \frac{\partial V_i(\lambda)}{\partial \lambda} + (1-\delta) \frac{\partial p_i(\mathcal{F}, \lambda)}{\partial \lambda} \\
&= (1-\delta)(1-\pi_i) \left[f_2'(c_2(\lambda)) \theta c_2'(\lambda) + f_3'(c_3(\lambda))(1-\theta) c_3'(\lambda) \right]
\end{aligned}$$

where the second line follows by substituting $p_i(\mathcal{F}, \lambda)$ given by Eq. 1 into the first line.

If $f_3'(c_3(\lambda)) \geq f_2'(c_2(\lambda))$ for all λ and $f_3'(c_3(\lambda^{FB})) > f_2'(c_2(\lambda^{FB}))$, then

$$\frac{\partial U_i(\mathcal{F}, \lambda^{FB}; \pi_i)}{\partial \lambda} = (1-\delta)(1-\pi_i) f_3'(c_3(\lambda)) \left[\underbrace{\frac{f_2'(c_2(\lambda))}{f_3'(c_3(\lambda))}}_{\leq 1} \theta c_2'(\lambda) + (1-\theta) c_3'(\lambda) \right]$$

In this case, $\frac{\partial U_i(\mathcal{F},\lambda^{FB};\pi_i)}{\partial \lambda} > 0$ for all $\lambda \leq \lambda^{FB}$, because and $\frac{\partial U_i(\mathcal{F},\lambda^{FB};\pi_i)}{\partial \lambda} \geq (1-\delta)(1-\pi_i) f_3'(c_3(\lambda))[\theta c_2'(\lambda) + (1-\theta)c_3'(\lambda)] \geq 0$, where the first inequality is strictly for λ^{FB} and the second inequality is strictly for all $\lambda > \lambda^{FB}$. This implies that $\hat{\lambda}(\mathcal{F}) > \lambda^{FB}$.

If $f_3'(c_3(\lambda)) \leq f_2'(c_2(\lambda))$ for all λ and $f_3'(c_3(\lambda^{FB})) < f_2'(c_2(\lambda^{FB}))$, then

$$\frac{\partial U_i(\mathcal{F}, \lambda^{FB}; \pi_i)}{\partial \lambda} = (1-\delta)(1-\pi_i) f_2'(c_2(\lambda)) \left[\theta c_2'(\lambda) + \underbrace{\frac{f_3'(c_3(\lambda))}{f_2'(c_2(\lambda))}}_{\leq 1} (1-\theta) c_3'(\lambda) \right]$$

In this case, $\frac{\partial U_i(\mathcal{F},\lambda^{FB};\pi_i)}{\partial \lambda} < 0$ for all $\lambda \geq \lambda^{FB}$, because and $\frac{\partial U_i(\mathcal{F},\lambda^{FB};\pi_i)}{\partial \lambda} \leq (1-\delta)(1-\pi_i) f_3'(c_3(\lambda))[\theta c_2'(\lambda) + (1-\theta)c_3'(\lambda)] \leq 0$. where the first inequality is strict for λ^{FB} and the second inequality is strictly for all $\lambda < \lambda^{FB}$. This implies that $\hat{\lambda}(\mathcal{F}) < \lambda^{FB}$.

B.3 Proof of Lemma 3

Using Eq. 1, $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \hat{\pi}} = f_1 - [\theta f_2(c_2(\lambda)) + (1-\theta) f_3(c_3(\lambda))]$, which is strictly greater than 0 by (MNO) given by Eq. 14. Following Eq. 6, the ratio $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ is increasing in i if and only if $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \lambda}$ is increasing in i, which is the case if and only if $\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda}$ is increasing in i, where

$$\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F}, \lambda)}{\partial \lambda} = (1-\pi_i) \left[\theta(1 - f_2'(c_2(\lambda))) c_2'(\lambda) + (1-\theta)(1 - f_3'(c_3(\lambda))) c_3'(\lambda) \right]$$

Notice that $1 - f_2'(c_2), 1 - f_3'(c_3) \in [0, 1]$ by (MNI) given by Eq. 14.

If $f_3(c_3(\lambda)) = c_3(\lambda)$ and $f_2(c_2(\lambda)) = c_2(\lambda)$, then $\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda} = 0$ and $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ is constant in i.

If $f_3'(c_3(\lambda)) \geq f_2'(c_2(\lambda))$ for all λ and $f_3'(c_3(\lambda^{FB})) > f_2'(c_2(\lambda^{FB}))$, then

$$\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda} = (1-\pi_i)(1-f_2'(c_2(\lambda)))\left[\theta c_2'(\lambda) + (1-\theta)\underbrace{\frac{1-f_3'(c_3(\lambda))}{1-f_2'(c_2(\lambda))}}_{\leq 1} c_3'(\lambda)\right]$$

In this case, $\theta c_2'(\lambda) + (1-\theta)\frac{1-f_3'(c_3(\lambda))}{1-f_2'(c_2(\lambda))}c_3'(\lambda) \leq \theta c_2'(\lambda) + (1-\theta)c_3'(\lambda) \leq 0$ for all $\lambda \geq \lambda^{FB}$, where the first inequality is strict for λ^{FB} and the second inequality is strict for all $\lambda > \lambda^{FB}$. Therefore $\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda}$ and thus $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ are strictly decreasing in i for all $\lambda \geq \lambda^{FB}$.

If $f_3'(c_3(\lambda)) \leq f_2'(c_2(\lambda))$ for all λ and $f_3'(c_3(\lambda^{FB})) < f_2'(c_2(\lambda^{FB}))$, then

$$\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda} = (1-\pi_i)(1-f_3'(c_3(\lambda)))\left[\theta \underbrace{\frac{1-f_2'(c_2(\lambda))}{1-f_3'(c_3(\lambda))}}_{\leq 1} c_2'(\lambda) + (1-\theta)c_3'(\lambda)\right]$$

In this case, $\theta\frac{1-f_2'(c_2(\lambda))}{1-f_3'(c_3(\lambda))}c_2'(\lambda) + (1-\theta)c_3'(\lambda) > \theta c_2'(\lambda) + (1-\theta)c_3'(\lambda) \geq 0$ for all $\lambda \leq \lambda^{FB}$, where the first inequality is strict for λ^{FB} and the second inequality is strict for all $\lambda < \lambda^{FB}$. Therefore $\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda}$ and thus $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ are strictly increasing in i for all $\lambda \leq \lambda^{FB}$.

Finally, the above analysis implies that, if $f_3'(c_3(\lambda)) = f_2'(c_2(\lambda))$, then $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ is strictly decreasing in i for $\lambda > \lambda^{FB}$ and strictly increasing in i for $\lambda < \lambda^{FB}$.

B.4 Proof of Lemma 4, Lemma 5, and Proposition 1

We establish the proofs of these related results in two steps. We first solve for the least cost separating (LCS) equilibrium which is characterised by the maximisation problem stated in Eq. 7 (Lemma 5 and Proposition 1). Then we show that only the LCS equilibrium survives the Intuitive Criterion (Lemma 4).

In the least cost separating equilibrium is characterised by the opitimisation problem stated in Eq. 7, as discussed in the main text. Here we re-write the problem as follows, with explicit constraint for a monotone security under limited liability.

$$\begin{aligned} & max_{(\mathcal{F}_H,\lambda_H)} \quad p_H(\mathcal{F}_H,\lambda_H) + \delta\left[V_H(\lambda_H) - p_H(\mathcal{F}_H,\lambda_H)\right] \\ s.t. \quad & (IC) \quad U_L^{FB} \geq p_H(\mathcal{F},\lambda_H) + \delta\left[V_L(\lambda_H) - p_L(\mathcal{F}_H,\lambda_H)\right] \text{ and} \\ & (MNO) \text{ and } (MNI) \text{ given by Eq. 14} \end{aligned} \tag{15}$$

In what follows, since the resolution policy λ_H is pre-committed, only three cash flow occur in equilibrium, namely c_1, $c_2(\lambda_H)$ and $c_3(\lambda_H)$.[31] For brevity we suppress the dependency of $f_j(c_j)$ on c_j, $j \in \{2, 3\}$, whenever it is clear.

Lemma 5: The optimality of risky debt

We can now begin to prove Lemma 5. The proof is constructed by establishing several claims in succession. For any given λ_H, an optimal security maximises the high-type bank's expected payoff

$$\delta V_H(\lambda_H) + (1 - \delta) p_H(\mathcal{F}_H, \lambda_H)$$

subject to the constrains (IC), (MNO) and (MNI). Since $V_H(\lambda_H)$ is not affected by the security design, the security maximises the sales proceeds $p_H(\mathcal{F}_H, \lambda_H) = \pi_H f_1 + (1 - \pi_H)[\theta f_2 + (1 - \theta) f_3]$. Since λ_H plays no role in this proof, we subsequently denote proceeds from the sale of the security by $p_H(\mathcal{F}_H)$ for the ease of notation.

Given a committed resolution policy λ_H, there can only be three cash flow realisations c_1, $c_2(\lambda_H)$, and $c_3(\lambda_H)$ in equilibrium. Denote by f_1^*, f_2^*, and f_3^* the payoffs of the optimal security for these equilibrium cash flow realisations respectively. Claim 1–4 below aim to establish the properties that the equilibrium payoffs of the optimal security must satisfy. We finally characterise the properties of the full security and show that a risky debt as described in Lemma 5 is indeed an optimal security.

Claim 1. *For an optimal security $\mathcal{F}_H^*$, $f_1^* < c_1$.*

Proof. If $f_1^* = c_1$, by (MCI), $f_2^* = c_2(\lambda_H)$ and $f_3^* = c_3(\lambda_H)$. This security (full equity) violates (IC). □

Claim 2. *For any optimal security $\mathcal{F}_H^*$, the (IC) must bind.*

Proof. Suppose instead the (IC) is slack for some optimal security with payoffs $\{f_1^*, f_2^*, f_3^*\}$. By Claim 1, $f_1^* < c_1$. Unless $c_1 - f_1^* = c_2(\lambda_H) - f_2^*$, there exists a security $\hat{\mathcal{F}}$ with payoffs $\{\hat{f}_1, f_2^*, f_3^*\}$ with $\hat{f}_1 > f_1^*$ that satisfies the (IC). As $p_H(\cdot)$ strictly increases with f_1, $p_H(\hat{\mathcal{F}}) > p_H(\mathcal{F}_H^*)$, contradicting the supposition that the security is optimal.

[31] The equilibrium security is uniquely defined for these cash flow that occur in equilibrium. Although the payoff of the optimal security may not be uniquely pinned down for the cash flow associated with off-equilibrium resolution policies, this is inconsequential for solving the optimal resolution policy.

If $f_1^* < c_1$ and $c_1 - f_1^* = c_2(\lambda_H) - f_2^*$, one can increase the objective function $p_H(\cdot)$ by increasing both f_1^* and f_2^* by some $\epsilon > 0$ without violating the (IC), unless $f_2^* = c_2(\lambda_H)$ or $c_2(\lambda_H) - f_2^* = c_3(\lambda_H) - f_3^*$. Note that $f_2^* = c_2(\lambda_H)$ implies $f_1^* = c_1$ hence violates Claim 1.

Suppose now $f_1^* < c_1$ and $c_1 - f_1^* = c_2(\lambda_H) - f_2^* = c_3(\lambda_H) - f_3^*$, similarly one can increase all f_1^*, f_2^*, f_3^* without violating the (IC) to strictly increase $p_H(\cdot)$, unless $f_3^* = c_3(\lambda_H)$. And $f_3^* = c_3(\lambda_H)$ implies $f_1^* = c_1$ hence violates Claim 1.

Since we have shown that any security with a slack (IC) can be improved upon, the (IC) must be binding at any optimal security. □

Claim 3. *For any optimal security $\mathcal{F}_H^*$, $f_1^* > c_3(\lambda_H)$.*

Proof. Suppose instead that $f_1^* \leq c_3(\lambda_H)$. By (MNO), $c_3(\lambda_H) \geq f_1^* \geq f_2^* \geq f_3^*$. This implies that the (IC) is slack because the mimicking payoff

$$\delta V_L(\lambda_H) + p_H(\mathcal{F}_H^*) - \delta p_L(\mathcal{F}_H^*) \leq \delta V_L(\lambda_H) + (1-\delta)c_3(\lambda_H) < V_L(\lambda_H) \leq V_L(\lambda^{FB}) = U_L^*$$

By Claim 2, a slack (IC) contradicts the optimality of $\mathcal{F}_H^*$. □

Claim 4. *Any optimal security $\mathcal{F}_H^*$ has either*

1. $f_1^* = f_2^* > f_3^* = c_3(\lambda_H)$ *or*

2. $f_1^* > f_2^* = c_2(\lambda_H) > f_3^* = c_3(\lambda_H)$

Proof. Consider a security that pays off $\hat{f}_1$, $\hat{f}_2$, and $\hat{f}_3$ for cash flow c_1, $c_2(\lambda_H)$ and $c_3(\lambda_H)$ respectively, such that with the (IC) binds. Using the (IC), write $\hat{f}_1$ as a function of $\hat{f}_2$ and $\hat{f}_3$

$$\hat{f}_1(\hat{f}_2, \hat{f}_3) = \frac{(1-\delta)U_L^* - [(1-\pi_H) - \delta(1-\pi_L)](\theta \hat{f}_2 + (1-\theta)\hat{f}_3)}{\pi_H - \delta\pi_L} \tag{16}$$

Substitute this $\hat{f}_1$ into the objective function. After some algebraic manipulation, the objective function becomes

$$\delta V_H + (1-\delta)\left[\frac{\pi_H}{\pi_H - \delta\pi_L}(1-\delta)U_L^* + \delta\frac{\pi_H - \pi_L}{\pi_H - \delta\pi_L}(\theta\hat{f}_2 + (1-\theta)\hat{f}_3)\right] \tag{17}$$

which is strictly increasing in $\hat{f}_2$ and $\hat{f}_3$. Since $\hat{f}_2$ is bounded above by either $c_2(\lambda_H)$ or $\hat{f}_1$, and $\hat{f}_3$ only by $c_3(\lambda_H)$, any optimal security $\mathcal{F}_H^*$ must have $f_3^* = c_3(\lambda_H)$ and $f_2^* = \min\{f_1^*, c_2(\lambda_H)\}$. Finally, by Claim 3, $f_1^* > c_3(\lambda_H)$ and hence $f_2^* > c_3(\lambda_H)$.

$\square$

Having now analysed the properties of an optimal security's equilibrium payoffs $\{f_1^*, f_2^*, f_3^*\}$, we now consider the security's payoffs associated with the off-equilibrium cash flow realisations, i.e. $f_2(c_2)$ and $f_3(c_3)\ \forall\lambda \in (0,1)$.

Claim 5. *For any optimal security $\mathcal{F}_H^*$, $f_3(c_3) = c_3(\lambda)\ \forall\lambda \leq \lambda_H$, and either*

1. $f_1^* = f_2^* = f_2(c_2(\lambda))\ \forall\lambda \leq \lambda_H$, *or*
2. $f_1^* > f_2(c_2) = c_2(\lambda)$ *and* $f_3(c_3) = c_3(\lambda)\ \forall\lambda \geq \lambda_H$

Proof. Notice that these payoffs do not affect either the objective function or the (IC). Therefore they are only restricted by the (MNO) and the (MNI). By Claim 4, $f_3^* = c_3(\lambda_H)$. The (MNI) thus implies that $f_3(c_3) = c_3(\lambda)\ \forall\lambda \leq \lambda_H$, because $c_3(\lambda)$ is increasing in λ.

By Claim 4, there are two cases. In the first case, $f_1^* = f_2^*$. The (MNO) then implies that $f_1^* = f_2^* = f_2(c_2(\lambda))\ \forall\lambda \leq \lambda_H$, because $c_2(\lambda)$ is decreasing in λ. In the second case, $f_2^* = c_2(\lambda_H) > f_3^* = c_3(\lambda_H)$. The (MNI) then implies that $f_2(c_2) = c_2(\lambda)$ and $f_3(c_3) = c_3(\lambda)\ \forall\lambda \geq \lambda_H$. $\square$

Finally we can now verify that a risky debt with face value $F_H \in (c_3(\lambda_H), c_1)$, as defined in Lemma 5, indeed is an optimal security as it satisfies Claim 1–5.

Proposition 1: the LCS equilibrium

We prove Proposition 1 by solving the optimisation programme in Eq. 7, which characterises the LCS equilibrium. Our goal is to highlight the properties of the equilibrium security design and resolution policy.

Firstly, we establish that any optimiser of the programme must bind the (IC). We prove this by contradiction. Suppose there exists (F_H, λ_H) that is an optimiser of the programme such that the (IC) is slack. Then there exists $F_H' > F_H$ such that the (IC) is still satisfied at (F_H', λ_H). However, the objective function is strictly greater at (F_H', λ_H) than at (F_H, λ_H). This contradicts with the supposition that (F_H, λ_H) is an optimiser of the programme. Therefore any optimiser of the programme must bind the (IC).

We then substitute the binding (IC) into the objective function to eliminate F_H, and solve the resulting univariate optimisation problem. Let $\hat{F}_H(\lambda_H)$ denote the F_H implied by a binding

(IC). Let $u(\lambda_H)$ denote the objective function of the resulting univariate optimisation problem. The solution to the problem characterised by Eq. 7 is equal to $\lambda_H^* = \arg\max_{\lambda_H} u(\lambda_H)$, where

$$u(\lambda_H) = (1-\delta)p_H(\hat{F}_H(\lambda_H), \lambda_H) + \delta V_H(\lambda_H) \tag{18}$$

There can be two cases:

(i) $\hat{F}_H(\lambda_H) \in [(1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X, Z)$ if and only if $G(\lambda_H) \leq 0$, or

(ii) $\hat{F}_H(\lambda_H) \in ((1-d)Z + d\,\mathcal{L}(\lambda_H), (1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X)$ if and only if $G(\lambda_H) > 0$,

where $G(\lambda)$ is given by Eq. 9

Case (i): $G(\lambda_H) \leq 0$ **and** $\hat{F}_H(\lambda_H) \in [(1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X, Z)$

In this case, the market value of the high type's security is given by

$$p_H(F_H, \lambda_H) = \pi_H F_H + (1-\pi_H)[(1-d)Z + d\,\mathcal{L}(\lambda_H) + d\theta(1-\lambda_H)X] \tag{19}$$

A binding (IC) implies that

$$\hat{F}_H(\lambda_H) = \frac{U_L^* - \delta\pi_L Z - (1-\pi_H)[(1-d)Z + d\,\mathcal{L}(\lambda_H) + \theta d(1-\lambda_H)X]}{\pi_H - \delta\pi_L} \tag{20}$$

We now show that, the objective function of the resulting univariate optimisation programme, $u(\lambda_H)$, is increasing in λ_H if and only if $\lambda_H \leq \lambda^{FB}$. To see this, we differentiate $u(\lambda_H)$ w.r.t. λ_H:

$$\frac{\partial u(\lambda_H)}{\partial \lambda_H} = (1-\delta)\left[\frac{\partial p_H(\hat{F}_H(\lambda_H), \lambda_H)}{\partial \lambda_H} + \frac{\partial p_H(\hat{F}_H(\lambda_H), \lambda_H)}{\partial F_H}\frac{\partial \hat{F}_H(\lambda_H)}{\partial \lambda_H}\right] + \delta\frac{\partial V_H(\lambda_H)}{\partial \lambda_H} \tag{21}$$

Notice that $\frac{\partial u(\lambda_H^{FB})}{\partial \lambda_H} = 0$ because $\frac{\partial p(F_H, \lambda^{FB})}{\partial \lambda_H} = 0$ and $\frac{\partial \hat{F}_H(\lambda^{FB})}{\partial \lambda_H} = 0$. Moreover, $u(\lambda_H)$ is strictly concave in λ_H. After some algebraic manipulation, we have

$$\frac{\partial^2 u(\lambda_H)}{\partial \lambda_H^2} = \frac{\delta(1-\pi_H)(\pi_H - \pi_L)}{\pi_H - \delta\pi_L} d\,\mathcal{L}''(\lambda_H) < 0 \tag{22}$$

Therefore, for all λ_H such that $G(\lambda_H) \leq 0$, $u(\lambda_H)$ is increasing in λ_H if and only if $\lambda_H \leq \lambda^{FB}$.

Case (ii): $G(\lambda_H) > 0$ **and** $\hat{F}_H(\lambda_H) \in ((1-d)Z + d\,\mathcal{L}(\lambda_H), (1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X)$

In this case, the market value of the high type's security is given by

$$p_H(F_H, \lambda_H) = [\pi_H + (1-\pi_H)\theta]F_H + (1-\pi_H)(1-\theta)[(1-d)Z + d\,\mathcal{L}(\lambda_H)] \tag{23}$$

A binding (IC) implies that

$$\hat{F}_H(\lambda_H) = \frac{\begin{array}{r} U_L^* - \delta\pi_L Z - \delta(1-\pi_L)\theta[(1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X] \\ -\,(1-\pi_H)(1-\theta)[(1-d)Z + d\,\mathcal{L}(\lambda_H)] \end{array}}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} \tag{24}$$

We now show that, there exists $\tilde{\lambda}_H \in (\lambda^{FB}, 1]$, such that $u(\lambda_H)$ is increasing in λ_H if and only if $\lambda_H \leq \tilde{\lambda}_H$, which is equivalent to $u(\lambda_H)$ being quasi-concave in λ_H. To see this, we evaluate the first derivative of $u(\lambda_H)$ w.r.t. λ_H, given by Eq. 21 using Eq. 23–24:

$$\begin{aligned} \frac{\partial u(\lambda_H)}{\partial \lambda_H} &= (1-\delta)\frac{[\pi_H + (1-\pi_H)\theta]\,[-\delta(1-\pi_L)\theta d(\mathcal{L}'(\lambda_H) - X) - (1-\pi_H)(1-\theta)d\,\mathcal{L}'(\lambda_H)]}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} \\ &\quad + (1-\delta)(1-\pi_H)(1-\theta)d\,\mathcal{L}'(\lambda_H) + \delta(1-\pi_H)d(\mathcal{L}'(\lambda_H) - \theta X) \end{aligned}$$

And the second derivative is given by

$$\begin{aligned} \frac{\partial^2 u(\lambda_H)}{\partial \lambda_H^2} &= [(1-\pi_H)(1-\theta) - \theta(1-\delta)] \\ &\qquad \times \frac{\delta(\pi_H - \pi_L)}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} d\,\mathcal{L}''(\lambda_H) \end{aligned} \tag{25}$$

Notice that, depending on the sign of $[(1-\pi_H)(1-\theta) - \theta(1-\delta)]$, $u(\lambda_H)$ can be either concave or convex.

Suppose $[(1-\pi_H)(1-\theta) - \theta(1-\delta)] \leq 0$, then $u(\lambda_H)$ is convex in λ_H. This implies that $u(\lambda_H)$ is increasing in λ_H for all λ_H, as

$$\begin{aligned} \frac{\partial u(\lambda_H)}{\partial \lambda_H} &> -(1-\delta)\frac{[\pi_H + (1-\pi_H)\theta](1-\pi_H)d(1-\theta)X}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} + (1-\pi_H)(1-\theta)X \\ &= \delta\frac{[\pi_H + (1-\pi_H)\theta] - [\pi_L + (1-\pi_L)\theta]}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]}(1-\pi_H)d(1-\theta)X > 0 \end{aligned}$$

where we have used the fact that $\mathcal{L}'(\lambda_H) < X$ for all λ_H (Assumption 1) when deriving the first line.

Suppose $[(1-\pi_H)(1-\theta) - \theta(1-\delta)] > 0$, then $u(\lambda_H)$ is concave in λ_H. At $\lambda_H = \lambda^{FB}$, after some algebraic manipulation using the fact that $\mathcal{L}'(\lambda^{FB}) = \theta X$, we have

$$\frac{\partial u(\lambda^{FB})}{\partial \lambda_H} = \frac{(1-\delta)\delta(\pi_H - \pi_L)\theta d(1-\theta)X}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} > 0 \tag{26}$$

Therefore there exists $\tilde{\lambda}_H \in (\lambda^{FB}, 1]$, such that $u(\lambda_H)$ is increasing in λ_H if and only if $\lambda \leq \tilde{\lambda}_H$, where $\tilde{\lambda}_H$ is given by $\frac{\partial u(\tilde{\lambda}_H)}{\partial \lambda_H} = 0$ if $\frac{\partial u(1)}{\partial \lambda_H} \leq 0$, and $\tilde{\lambda}_H = 1$ otherwise.

To summarise, there exists $\tilde{\lambda}_H \in (\lambda^{FB}, 1]$, such that for all λ_H such that $G(\lambda_H) > 0$, $u(\lambda_H)$ is increasing in λ_H if and only if $\lambda_H \leq \tilde{\lambda}_H$. After some algebraic manipulation, $\tilde{\lambda}_H$ can be defined by

$$\tilde{\lambda}_H \begin{cases} \text{is defined by} & \text{if } [(1-\pi_H)(1-\theta) - \theta(1-\delta)] > 0 \\ \qquad \mathcal{L}'(\tilde{\lambda}_H) = \frac{\delta - [\pi_H + (1-\pi_H)\theta]}{(1-\pi_H)(1-\theta) - \theta(1-\delta)}\theta X, & \qquad \text{and } \mathcal{L}'(1) < \frac{\delta - [\pi_H + (1-\pi_H)\theta]}{(1-\pi_H)(1-\theta)-\theta(1-\delta)}\theta X \\ = 1, & \text{otherwise} \end{cases} \tag{27}$$

We can now describe the equilibrium resolution policy λ_H^*. Notice that $G(\tilde{\lambda}_H) < G(\lambda^{FB})$, where $G(\lambda)$ is given by Eq. 9. This follows because $G(\lambda)$ is decreasing in λ for $\lambda \geq \lambda^{FB}$. To see this,

$$\frac{\partial G(\lambda)}{\partial \lambda} = (1-\delta\pi_L)d[\mathcal{L}'(\lambda) - X] + (1-\pi_H)d(1-\theta)X \tag{28}$$

For $\lambda \geq \lambda^{FB}$, $\mathcal{L}'(\lambda) \leq \theta X$, and the above expression is smaller than

$$-(1-\delta\pi_L)d(1-\theta)X + (1-\pi_H)d(1-\theta)X = (\delta\pi_L - \pi_H)d(1-\theta)X < 0 \tag{29}$$

The equilibrium resolution policy is thus characterised as follows

1. If $G(\tilde{\lambda}_H) < G(\lambda^{FB}) \leq 0$, then $\lambda_H^* = \lambda^{FB}$.

   This follows because, for $\lambda_H \leq \lambda^{FB}$, $u(\lambda_H)$ is increasing in λ_H in either case; for $\lambda_H \geq \lambda^{FB}$, $G(\lambda_H) < \lambda^{FB} \leq 0$, and $u(\lambda^{FB})$ is decreasing as described by Case (i). Therefore, $u(\lambda_H)$ is maximised at $\lambda_H^* = \lambda^{FB}$.

2. If $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{FB})$, then $\lambda_H^* = \breve{\lambda}_H$, where $\breve{\lambda}_H > \lambda^{FB}$ is given by $G(\breve{\lambda}_H) = 0$.

   To see this, notice that $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{FB})$ and that $G(\lambda)$ is decreasing in λ for $\lambda \geq \lambda^{FB}$ imply that there exists $\breve{\lambda}_H \in (\lambda^{FB}, \tilde{\lambda}_H]$ such that $G(\breve{\lambda}_H) = 0$. Moreover, $G(\lambda_H) > 0$ for

$\lambda_H \in (\lambda^{FB}, \breve{\lambda}_H)$ and $G(\lambda_H) < 0$ for $\lambda_H > \breve{\lambda}_H$. The result thus follows because, for $\lambda_H \leq \lambda^{FB}$, $u(\lambda_H)$ is increasing in λ_H in either case; for $\lambda_H \in (\lambda^{FB}, \breve{\lambda}_H)$, $G(\lambda_H) > 0$ and $u(\lambda_H)$ is increasing as described by Case (ii); for $\lambda \geq \breve{\lambda}_H > \lambda^{FB}$, $G(\lambda_H) \leq 0$ and $u(\lambda_H)$ is decreasing as described by Case (i). Therefore, $u(\lambda_H)$ is maximised at $\breve{\lambda}_H$.

3. If $0 < G(\tilde{\lambda}_H) < G(\lambda^{FB})$, then $\lambda_H^* = \tilde{\lambda}_H > \lambda^{FB}$, where $\tilde{\lambda}_H$ is given by Eq. 27.

   This follows because, for $\lambda \leq \lambda^{FB}$, $u(\lambda_H)$ is increasing in λ_H in either case; for $\lambda_H \in (\lambda^{FB}, \tilde{\lambda}_H]$, $G(\lambda_H) > 0$ and $u(\lambda_H)$ is increasing in λ_H as described by Case (ii); for $\lambda_H > \tilde{\lambda}_H$, $u(\lambda_H)$ is decreasing in λ_H in either case. Therefore, $u(\lambda_H)$ is maximised at $\tilde{\lambda}_H$.

To summarise, the equilibrium resolution policy is $\lambda_H^* \geq \lambda^{FB}$, where the inequality is strict if and only if $G(\lambda^{FB}) > 0$.

Lemma 4: only the LCS equilibrium survives the Intuitive Criterion

We prove Lemma 4 in two steps. We will first show no pooling PBE satisfy the Intuitive Criterion. And then we show the same for any separating PBE other than the least cost separating PBE.

The logic of the proof is as follows: for any candidate pooling PBE (U_H^P, U_L^P) with an offer $\{\mathcal{F}^P, \lambda^P\}$, we construct an off-equilibrium pooling offer $\{\mathcal{F}', \lambda^P\}$ that prunes the candidate PBE with Intuitive Criterion. Since we do not involve changing λ^P in the following analysis, for the ease of notation we will simply denote an offer with $\mathcal{F}$ whenever it does not create confusion.

We begin by applying the Intuitive Criterion to our two-type model as follows: a PBE fails to satisfy the Intuitive Criterion if there exists an unsent offer $\mathcal{F}'$, such that the type H is strictly better off than at the posited PBE by proposing $\mathcal{F}'$ for all best responses with beliefs focused on H, and the type L is strictly better at the posited PBE than at $\mathcal{F}'$ for all best responses for all beliefs in response to $\mathcal{F}'$.

Define $J_H(\mathcal{F}')$ and $J_L(\mathcal{F}')$ as the payoff of the H and L type when they deviate to the off-equilibrium offer $\mathcal{F}'$ under a belief focused on H

$$\begin{aligned} J_H(\mathcal{F}') &\equiv p_H(\mathcal{F}') + \delta[V_H - p_H(\mathcal{F}')] \\ J_L(\mathcal{F}') &\equiv p_H(\mathcal{F}') + \delta[V_L - p_L(\mathcal{F}')] \end{aligned} \tag{30}$$

Therefore a pooling PBE (U_H^P, U_L^P) does not satisfy the intuitive criterion if there exists an

$\mathcal{F}'$ such that $J_H(\mathcal{F}') > U_H^P$ and $J_L(\mathcal{F}') < U_L^P$.

We begin the proof with establishing some useful properties of any pooling PBE (U_H^P, U_L^P). First, the payoffs can be computed as follows:

$$
\begin{aligned}
U_H^P &\equiv \bar{p}(\mathcal{F}^P) + \delta[V_H - p_H(\mathcal{F}^P)] \\
U_L^P &\equiv \bar{p}(\mathcal{F}^P) + \delta[V_L - p_L(\mathcal{F}^P)]
\end{aligned} \tag{31}
$$

where $\bar{p}(\mathcal{F}) = \bar{\pi} f_1 + (1-\bar{\pi})[\theta f_2 + (1-\theta) f_3)]$ and $\bar{\pi} \equiv \gamma \pi_H + (1-\gamma)\pi_L$.

Second, in any pooling PBE that satisfies Intuitive Criterion, both types must attain weakly higher payoffs than the least cost separating (LCS) payoffs (U_H^*, U_L^*). The following claim establishes this property formally.

Claim 6. *For any pooling PBE (U_H^P, U_L^P) that satisfies the Intuitive Criterion, $U_H^P \geq U_H^*$ and $U_L^P \geq U_L^*$.*

Proof. This claim is proved by contradiction. First of all, $U_L^P < U_L^*$ cannot be a PBE because the low type can always attain at least the LCS payoffs U_L^* by deviating to the first-best offer of the low type.

Suppose now $U_H^P < U_H^*$ and $U_L^P \geq U_L^*$. To invoke the Intuitive Criterion, consider a set of beliefs that all deviations are done by the high type. Then by deviating to (F_H^*, λ_H^*), the high type achieves its LCS payoff $U_H^* > U_H^P$ whereas the low type's payoff $p_H(F_H^*, \lambda_H^*) + \delta[V_L(\lambda_H^*) - p_L(F_H^*, \lambda_H^*)]$, is also equal to its LCS payoff U_L^* because (F_H^*, λ_H^*) is the solution of the LCS problem in Eq. 7 and the (IC) therein is binding at the solution. Now consider another offer $\{F', \lambda_H^*\}$ with $F' = F_H^* - \epsilon$ for some arbitrarily small and positive ϵ such that the high type's payoff with this off-equilibrium offer is $U'_H \in (U_H^P, U_H^*)$. Such an F' exists because $U_H^P < U_H^*$ and $F_H^* > c_3$ (Lemma 5). Finally the low type's payoff with the offer $\{F', \lambda_H^*\}$ is $U'_L < U_L^* \leq U_L^P$. □

The third property is shown in the following claim

Claim 7. *In any pooling PBE with offer $\{\mathcal{F}^P, \lambda^P\}$, $f_1^p > c_3(\lambda^P)$.*

Proof. Suppose instead $f_1^P \leq c_3(\lambda^P)$. Because of (MNO), $c_3 \geq f_1^P \geq f_2^P \geq f_3^P$

$$
\begin{aligned}
U_L^P &= \delta V_L(\lambda^P) + (\bar{\pi} - \delta\pi_L) f_1^P + [(1-\bar{\pi}) - \delta(1-\pi_L)][\theta f_2^P + (1-\theta) f_3^P] \\
&\leq \delta V_L(\lambda^P) + (\bar{\pi} - \delta\pi_L)[\theta f_2^P + (1-\theta) f_3^P] + [(1-\bar{\pi}) - \delta(1-\pi_L)][\theta f_2^P + (1-\theta) f_3^P] \\
&\leq \delta V_L(\lambda^P) + (1-\delta) c_3(\lambda^P) < V_L(\lambda^P) \leq V_L(\lambda^{FB}) \equiv U_L^*
\end{aligned}
$$

which contradicts the fact that $U_L^P \geq U_L^*$. $\square$

We are now equipped to construct the PBE pruning offer $\mathcal{F}'$ for any pooling PBE with offer $\mathcal{F}^P$. First, we parametrise a series of offers with y such that

$$\mathcal{F}(y) = \{f_1^P - y, f_2^P - \max\{y - (f_1^P - f_2^P), 0\}, f_3^P\} \tag{32}$$

for $y \in [0, f_1^P - f_3^P]$. Note that $\mathcal{F}(0) = \mathcal{F}^P$ and the domain of y is non-empty thanks to Claim 7 and $f_3^P \leq c_3(\lambda^P)$ due to limited liability. The rest of the proof involves two claims with the parametrised offer $\mathcal{F}(y)$.

Claim 8. *There exists a unique $\tilde{y} \in (0, f_1^P - f_3^P)$ that satisfies $J_L(\mathcal{F}(\tilde{y})) = U_L^P$*

Proof. The proof is based on the Intermediate Value Theorem. First, $J_L(\mathcal{F}(\epsilon)) > U_L^P$ with $\epsilon \to 0$ because

$$\begin{aligned} J_L(\mathcal{F}(\epsilon)) - U_L^P &= p_H(\mathcal{F}(\epsilon)) - \bar{p}(\mathcal{F}^P) - \delta[p_L(\mathcal{F}(\epsilon)) - p_L(\mathcal{F}^P)] \\ &= p_H(\mathcal{F}^P) - \bar{p}(\mathcal{F}^P) > 0 \quad \text{as } \epsilon \to 0 \end{aligned}$$

Second, $J_L(\mathcal{F}(f_1^P - f_3^P)) < U_L^P$ as $\mathcal{F}(f_1^P - f_3^P) = \{f_3^P, f_3^P, f_3^P\}$, $f_3^P \leq c_3(\lambda^P)$ due to (LL), and following the same argument as in Claim 7,

$$J_L(\mathcal{F}(f_1^P - f_3^P)) \leq \delta V_L + (1-\delta)c_3(\lambda^P) < V_L(\lambda^P) \leq V_L(\lambda^{FB}) = U_L^* \leq U_L^P$$

Finally, $J_L(\mathcal{F}(y))$ is strictly decreasing and continuous in y

$$\frac{\partial J_L(\mathcal{F}(y))}{\partial y} = \begin{cases} -\pi_H + \delta\pi_L < 0 & \text{for } y \in [0, f_1^P - f_2^P) \\ (1-\theta)(\delta\pi_L - \pi_H) - \theta(1-\delta) < 0 & \text{for } y \in [f_1^P - f_2^P, f_1^P - f_3^P) \end{cases} \tag{33}$$

Therefore, the Intermediate Value Theorem applies. $\square$

Claim 9. $J_H(\mathcal{F}(\tilde{y})) > U_H^P$

Proof. This result relies on two properties:

(i) $J_H(\mathcal{F}(\epsilon)) - U_H^P = J_L(\mathcal{F}(\epsilon)) - U_L^P = p_H(\mathcal{F}^P) - \bar{p}(\mathcal{F}^P) > 0$ as $\epsilon \to 0$;

(ii) $0 > \dfrac{\partial J_H(\mathcal{F}(y))}{\partial y} > \dfrac{\partial J_L(\mathcal{F}(y))}{\partial y}$ for $y \in [0, f_1^P - f_3^P]$

(i) is immediate from the definition of J_H while (ii) from the direct comparison between Eq. 33 and

$$\frac{\partial J_H(\mathcal{F}(y))}{\partial y} = \begin{cases} -\pi_H + \delta\pi_H < 0 & \text{for } y \in [0, f_1^P - f_2^P) \\ (1-\theta)(\delta\pi_H - \pi_H) - \theta(1-\delta) < 0 & \text{for } y \in [f_1^P - f_2^P, f_1^P - f_3^P) \end{cases} \tag{34}$$

These two properties imply that the wedges $J_H - U_H^P$ and $J_L - U_L^P$ are the same when y is arbitrarily close to zero. As y increases, J_L decreases strictly faster then J_H . Therefore, at $\tilde{y}$, the wedge of $J_L - U_L^P$ is zero while the wedge $J_H - U_H^P$ is strictly positive. □

The last step of constructing the PBE pruning $\mathcal{F}'$ is to set $\mathcal{F}' = \mathcal{F}(\tilde{y}+\epsilon_y)$ with an arbitrarily small but positive ϵ_y such that $J_H(\mathcal{F}') > U_H^P$. This ϵ_y exists because $J_H(\mathcal{F}(\tilde{y})) > U_H^P$ as in Claim 9. And by the properties of $\tilde{y}$ in Claim 8 and J_L, $J_L(\mathcal{F}') < J_L(\mathcal{F}(\tilde{y})) = U_L^P$. As a result, the posited pooling PBE (U_H^P, U_L^P) cannot satisfy the Intuitive Criterion.

The proof for showing that no separating PBE other than the LCS PBE can satisfy Intuitive Criterion is very similar to Claim 6. Consider a separating PBE (U_H, U_L), by definition of LCS, $U_H \leq U_H^*$ and $U_L \leq U_L^*$ with at least one strict inequality. First U_L cannot be strictly less than U_L^* because the low type can always achieve at least U_L^* by giving the first-best offer. The relevant class of separating PBE is thus with $U_H < U_H^*$ and $U_L = U_L^*$. The remaining argument of the proof follows exactly the same as the one in Claim 6 and therefore is omitted.

Finally, we show that in equilibrium, $\lambda_H^* \geq \hat{\lambda}(F_H^*)$, implying that the signalling channel always biases the equilibrium resolution policy further towards liquidation.

Corollary 3. $\lambda_H^* \geq \hat{\lambda}(F_H^*)$.

Proof. We first characterise $\hat{\lambda}(F) = \arg\max_\lambda U_i(F, \lambda; \pi_i)$ for $F = F_H^*$. We then show that $\lambda_H^* \geq \hat{\lambda}(F_H^*)$.

To characterise $\hat{\lambda}(F)$ for $F = F_H^*$, let us consider the following two cases.

1. $F \in (c_2(1), c_2(\lambda^{FB})]$. In this case, there exists $\lambda'(F) > \lambda^{FB}$, given by $F = c_2(\lambda'(F))$, such that $p_i(F, \lambda; \pi_i)$ is strictly increasing in λ for $\lambda < \lambda'(F)$ and independent of λ for $\lambda \geq \lambda'(F)$. Therefore $U_i(F, \lambda; \pi_i)$ is strictly increasing in λ for all $\lambda \leq \lambda^{FB}$ and strictly decreasing for all $\lambda > \lambda'(F)$. For $\lambda \leq \lambda'(F)$, $c_2(\lambda) > F$ and

$$U_i(F, \lambda; \pi_i) = \delta V_i(\lambda) + (1-\delta)\left(\pi_i f_1 + (1-\pi_i)\left[\theta F + (1-\theta)c_3(\lambda)\right]\right) \tag{35}$$

It follows that $\hat{\lambda}$ is given by the first order condition to Eq. 35 $\mathcal{L}'(\lambda) = \frac{\delta}{\delta\theta+1-\theta}\theta X$ if the solution is smaller than $\lambda'(F)$, otherwise $\hat{\lambda}(F) = \lambda'(F)$. Therefore in this case, $\hat{\lambda}(F) \in (\lambda^{FB}, \lambda'(F)]$.

2. $F \in (c_2(\lambda^{FB}), Z)$. In this case, both $p_i(F, \lambda)$ and $V_i(\lambda)$ are strictly increasing in λ if and only if $\lambda < \lambda^{FB}$. Therefore in this case, $\hat{\lambda}(F) = \lambda^{FB}$.

We can now show that $\lambda_H^* \geq \hat{\lambda}(F_H^*)$. If $G(\lambda^{FB}) \leq 0$, then $F_H^* > c_2(\lambda^{FB})$. In this case, $\hat{\lambda}(F_H^*)$ is as described by Case 2 above, and $\lambda_H^* = \hat{\lambda}(F_H^*) = \lambda^{FB}$.

If $G(\lambda^{FB}) > 0$, $F_H^* < c_2(\lambda^{FB})$ and $\lambda_H^* > \lambda^{FB}$. In this case, $\hat{\lambda}(F_H^*)$ is as described by Case 1 above, and there can be two cases. (i) If $\lambda_H^* = \breve{\lambda}_H$, where $\breve{\lambda}_H$ is defined in the proof of Proposition 1 and $\breve{\lambda}_H = \lambda'(F_H^*)$. Therefore $\lambda_H^* = \lambda'(F_H^*) \geq \hat{\lambda}(F_H^*)$. (ii) If $\lambda_H^* = \tilde{\lambda}_H$, then it is characterised by the first order conditions (following from the proof of Proposition 1) given by

$$\begin{aligned}\frac{\partial u(\lambda_H)}{\partial \lambda_H} &= \frac{\partial U_H(\hat{F}_H(\lambda_H), \lambda_H; \pi_H)}{\partial \lambda} + \frac{\partial U_H(\hat{F}_H(\lambda_H), \lambda_H; \pi_H)}{\partial F}\frac{\partial \hat{F}_H(\lambda_H)}{\partial \lambda_H} \\ &= \frac{\partial U_H(\hat{F}_H(\lambda_H), \lambda_H; \pi_H)}{\partial \lambda} + \frac{\partial U_L(\hat{F}_H(\lambda_H), \lambda_H; \pi_H)}{\partial \lambda}\frac{\partial U_H(\hat{F}_H(\lambda_H), \lambda_H; \pi_H)/\partial F}{\partial U_L(\hat{F}_H(\lambda_H), \lambda_H; \pi_H)/\partial F}\end{aligned}$$

When evaluated at (F_H^*, λ_H^*), we have

$$\frac{\partial U_H(F_H^*, \lambda_H^*; \pi_H)}{\partial \lambda} + \frac{\partial U_L(F_H^*, \lambda_H^*; \pi_H)}{\partial \lambda}\frac{\partial U_H(F_H^*, \lambda_H^*; \pi_H)/\partial F}{\partial U_L(F_H^*, \lambda_H^*; \pi_H)/\partial F} = 0$$

Notice that,

$$\underbrace{\frac{\partial U_H(F_H^*, \hat{\lambda}(F_H^*); \pi_H)}{\partial \lambda}}_{\leq 0} + \underbrace{\frac{\partial U_L(F_H^*, \hat{\lambda}(F_H^*); \pi_H)}{\partial \lambda}}_{<0}\underbrace{\frac{\partial U_H(F_H^*, \hat{\lambda}(F_H^*); \pi_H)/\partial F}{\partial U_L(F_H^*, \hat{\lambda}(F_H^*); \pi_H)/\partial F}}_{>0} < 0$$

where the $\frac{\partial U_H(F_H^*, \hat{\lambda}(F_H^*); \pi_H)}{\partial \lambda} \leq 0$ because $\hat{\lambda}(F_H^*)$ is as described in Case 1 above. Moreover, $\frac{\partial U_L(F_H^*, \hat{\lambda}(F_H^*); \pi_H)}{\partial \lambda} < \frac{\partial U_H(F_H^*, \hat{\lambda}(F_H^*); \pi_H)}{\partial \lambda}$, because for $\lambda > \lambda^{FB}$, $\frac{\partial U_i(F, \lambda; \pi_H)}{\partial \lambda}$ is strictly increasing in i, as

$$\begin{aligned}\frac{\partial U_i(F, \lambda; \pi_H)}{\partial \lambda} &= \frac{\partial p_H(F, \lambda)}{\partial \lambda} + \delta\left[\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(F, \lambda)}{\partial \lambda}\right] \\ &= \frac{\partial p_H(F, \lambda)}{\partial \lambda} + \delta(1 - \pi_i)\theta \underbrace{c_2'(\lambda)}_{<0}\end{aligned}$$

following the proof of Lemma 3. Therefore in this case $\lambda_H^* = \tilde{\lambda}_H > \hat{\lambda}(F_H^*)$.

$\square$

B.5 Proof of Corollary 1

We characterise the comparative statics for the three cases discussed in Appendix B.4.

1. If $G(\tilde{\lambda}_H) < G(\lambda^{FB}) \leq 0$, $\lambda_H^* = \lambda^{FB}$.

2. If $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{FB})$, $\lambda_H^* = \check{\lambda}_H$, where $\check{\lambda}_H$ is given by $G(\check{\lambda}_H) = 0$. In this case, $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$, because $\frac{\partial G(\lambda_H^*)}{\partial \lambda_H} < 0$ and

$$\frac{\partial G(\cdot)}{\partial \pi_H} = (1-\theta)(1-\lambda)X > 0 \tag{36}$$

3. If $0 < G(\tilde{\lambda}_H) < G(\lambda^{FB})$, $\lambda_H^* = \tilde{\lambda}_H$, where $\tilde{\lambda}_H$ is given by Eq. 27. In this case, $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$, because the LHS of Eq. 27 is decreasing in λ_H, and the RHS of Eq. 27 is decreasing in π_H. The derivative of the RHS of Eq. 27 w.r.t π_H is equal to

$$-\frac{1-[\delta+(1-\delta)\theta]}{[(1-\pi_H)(1-\theta)-(1-\delta)\theta]^2} < 0 \tag{37}$$

To summarise, $\frac{\partial \lambda_H^*}{\partial \pi_H} \geq 0$, where the inequality is strict if and only if $G(\lambda^{FB}) > 0$.

B.6 Proof of Lemma 6

This proposition follows immediately from Eq. 11 and the preceding discussion.

B.7 Proof of Proposition 2

Denote with $U_i(\lambda)$ the expected payoff obtained by the high-type bank in the least cost separating equilibrium, for a given resolution policy. In this equilibrium, the high-type bank chooses a security to offer at $t = 1$ to maximise its expected payoff, while preventing mimicking from the low type. Formally, $U_i(\lambda)$ is equal to the value of the optimisation programme Eq. 7, given $\lambda_H = \lambda$.

By definition of λ_H^* as the optimiser of Eq. 7 , $U_H(\lambda_H^*) = U_H^* > U_H(\lambda_H)$ for any $\lambda_H \neq \lambda_H^*$. Thus the screening effort γ^* decreases as

$$\gamma^*(U_H(\lambda_H), U_L^{FB}) < \gamma^*(U_H(\lambda_H^*), U_L^{FB}) \quad \forall \lambda_H \neq \lambda_H^*$$

For efficiency, we only need to look at the bank's expected payoff as the investors are always indifferent. The expected payoff is lower when λ_H^* is replaced with λ_H, i.e.

$$
\begin{aligned}
&\gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H) + [1-\gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H), U_L^{FB}) \\
&< \gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H^*) + [1-\gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H), U_L^{FB}) \\
&\leq \gamma^*(U_H(\lambda_H^*), U_L^{FB})U_H(\lambda_H^*) + [1-\gamma^*(U_H(\lambda_H^*), U_L^{FB})]U_L^{FB} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H^*), U_L^{FB})
\end{aligned}
$$

The first inequality comes from $U_H(\lambda_H) < U_H(\lambda_H^*)$ and the second weak inequality follows from the definition of optimal γ^*. Finally, λ_H^{FB} is one of the possible $\lambda_H \neq \lambda_H^*$ if and only if $G(\lambda^{FB}) > 0$, where $G(\lambda)$ is given by Eq. 9.

B.8 Proof of Proposition 3

We first characterise the solution to Eq. 12 and 13. We then show that this solution subject to a set of local incentive compatibility constraints (Eq. 13) indeed is the equilibrium by showing that it satisfies global incentive compatibility (Claim 10).

It is immediate that for the lowest type $i = 1$, $\mathcal{F}_1^n$ is a pass-through security (or debt with face value $F_1 = Z$), and $\lambda_1^n = \lambda^{FB}$. It then follows from the proof of Lemma 5 that the optimal security for all types $i \geq 2$ is a risky debt with face value $F_i^n \in ((1-d)Z + d\,\mathcal{L}(\lambda_i), Z)$.

We next characterise the equilibrium security F_i^n and resolution policy λ_i^n for $i \geq 2$. Following the proof of Proposition 1, the (IC_i) binds in equilibrium for all $i \geq 2$. We can then substitute the binding (IC_i) into the objective function of type i to eliminate F_i^n, and solve the resulting univariate optimisation problem. Let $\hat{F}_i(\lambda_i, U_{i-1}^n)$ denote the F_i implied by a binding (IC_i), given that the equilibrium expected payoff to type $i-1$ is equal to U_{i-1}^n. Let $u_i(\lambda_i, U_{i-1}^n)$ denote the objective function of the resulting univariate optimisation problem. The solution to the problem characterised by Eq. 7 is equal to $\lambda_i^n = \arg\max_{\lambda_i} u_i(\lambda_i)$ for all $i \geq 2$, where analogous to Eq. 18,

$$
u_i(\lambda_i, U_{i-1}^n) = (1-\delta)p_i(\hat{F}_i(\lambda_i, U_{i-1}^n), \lambda_i) + \delta V_i(\lambda_i) \tag{38}
$$

There can be two cases for $i \geq 2$:

(i) $\hat{F}_i(\lambda_i, U_{i-1}^n) \in [c_2(\lambda_i), Z)$ if and only if $G_i(\lambda_i, U_{i-1}^n) \leq 0$, or

(ii) $\hat{F}_i(\lambda_i, U_{i-1}^n) \in [c_3(\lambda_i), c_2(\lambda_i))$ if and only if $G_i(\lambda_i, U_{i-1}^n) > 0$, where $G_i(\lambda, U)$ is given by

$$G_i(\lambda, U) = c_2(\lambda) - \frac{U - \delta\pi_{i-1}Z - (1-\pi_i)\left[(1-d)Z + d\,\mathcal{L}(\lambda) + d\theta(1-\lambda)X\right]}{\pi_i - \delta\pi_{i-1}} \tag{39}$$

and $c_2(\lambda)$ and $c_3(\lambda)$ are defined in Table 1

Case (i): $G_i(\lambda_i, U_{i-1}^n) \leq 0$ **and** $\hat{F}_i(\lambda_i, U_{i-1}^n) \in [c_2(\lambda_i), Z)$

In this case, the market value of the type i's security is given by

$$p_i(F_i, \lambda_i) = \pi_i F_i + (1-\pi_i)\left[(1-d)Z + d\,\mathcal{L}(\lambda_i) + d\theta(1-\lambda_i)X\right] \tag{40}$$

A binding (IC_i) implies that

$$\hat{F}_i(\lambda_i, U_{i-1}^n) = \frac{U_{i-1}^n - \delta\pi_{i-1}Z - (1-\pi_i)\left[(1-d)Z + d\,\mathcal{L}(\lambda_i) + d\theta(1-\lambda_i)X\right]}{\pi_i - \delta\pi_{i-1}} \tag{41}$$

After some algebraic manipulation similar to those in the proof of Proposition 1, we have $\frac{\partial u(\lambda^{FB}, U_{i-1}^n)}{\partial \lambda_i} = 0$ and $\frac{\partial^2 u(\lambda_i, U_{i-1}^n)}{\partial \lambda_i^2} < 0$. Therefore, for all λ_i and U_{i-1}^n such that $G_i(\lambda_i, U_{i-1}^n) \leq 0$, $u_i(\lambda_i, U_{i-1}^n)$ is increasing in λ_i if and only if $\lambda_i \leq \lambda^{FB}$.

Case (ii): $\hat{F}_i(\lambda_i, U_{i-1}^n) \in [c_3(\lambda_i), c_2(\lambda_i))$

In this case, the market value of the type i's security is given by

$$p_i(F_i, \lambda_i) = [\pi_i + (1-\pi_i)\theta]F_i + (1-\pi_i)(1-\theta)[(1-d)Z + d\,\mathcal{L}(\lambda_i)] \tag{42}$$

A binding (IC) implies that

$$\hat{F}_i(\lambda_i, U_{i-1}^n) = \frac{\begin{array}{r} U_{i-1}^n - \delta\pi_{i-1}Z - \delta(1-\pi_{i-1})\theta[(1-d)Z + d\,\mathcal{L}(\lambda_i) + d(1-\lambda_i)X] \\ - (1-\pi_i)(1-\theta)[(1-d)Z + d\,\mathcal{L}(\lambda_i)] \end{array}}{[\pi_i + (1-\pi_i)\theta] - \delta[\pi_{i-1} + (1-\pi_{i-1})\theta]} \tag{43}$$

After some derivation similar to those in the proof of Proposition 1, we can show that there exists $\tilde{\lambda}_i \in (\lambda^{FB}, 1]$, such that for all λ_i and U_{i-1}^n such that $G_i(\lambda_i, U_{i-1}^n) > 0$, $u_i(\lambda_i, U_{i-1}^n)$ is increasing in λ_i if and only if $\lambda_i \leq \tilde{\lambda}_i$, where $\tilde{\lambda}_i$ is given by

$$\tilde{\lambda}_i \begin{cases} \text{is defined by} & \text{if } [(1-\pi_i)(1-\theta)-\theta(1-\delta)] > 0 \\ \quad \mathcal{L}'(\tilde{\lambda}_i) = \frac{\delta-[\pi_i+(1-\pi_i)\theta]}{(1-\pi_i)(1-\theta)-\theta(1-\delta)}\theta X, & \quad \text{and } \mathcal{L}'(1) < \frac{\delta-[\pi_i+(1-\pi_i)\theta]}{(1-\pi_i)(1-\theta)-\theta(1-\delta)}\theta X \\ = 1, & \text{otherwise} \end{cases} \tag{44}$$

Notice that $G_i(\lambda_i, U_{i-1}^n)$ (Eq. 39) is decreasing in λ_i for all λ_i. Following similar reasoning as those in the proof of Proposition 1, the equilibrium resolution policy for type $i \geq 2$ thus satisfies the following conditions:

1. If $G_i(\tilde{\lambda}_i, U_{i-1}^n) < G_i(\lambda^{FB}, U_{i-1}^n) \leq 0$, then $\lambda_i^n = \lambda^{FB}$.
2. If $G_i(\tilde{\lambda}_i, U_{i-1}^n) \leq 0 < G_i(\lambda^{FB}, U_{i-1}^n)$, then $\lambda_i^n = \breve{\lambda}_i(U_{i-1}^n)$, where $\breve{\lambda}_i(U_{i-1}^n) > \lambda^{FB}$ is given by $G_i(\breve{\lambda}_i(U_{i-1}^n), U_{i-1}^n) = 0$.
3. If $0 < G_i(\tilde{\lambda}_i, U_{i-1}^n) < G_i(\lambda^{FB}, U_{i-1}^n)$, then $\lambda_i^n = \tilde{\lambda}_i$, where $\tilde{\lambda}_i > \lambda^{FB}$ is given by Eq. 44.

Notice that Points 2 and 3 imply that, if $G_i(\lambda^{FB}, U_{i-1}^n) > 0$, then $\lambda_i^n = \max\{\breve{\lambda}_i(U_{i-1}^n), \tilde{\lambda}_i\}$.

We now prove the first part of the proposition, that there exists a unique type $j > 1$, such that $\lambda_i^n = \lambda^{FB}$ for all $i \leq j$ and $\lambda_i^n > \lambda^{FB}$ for all $i > j$. Recall that $\lambda_1^n = \lambda^{FB}$ and $F_1^n = Z$. If $G_2(\lambda^{FB}, U_1^n) > 0$, then $j = 1$.

For $G_2(\lambda^{FB}, U_1^n) \leq 0$, notice that for any type $i-1 \geq 2$ such that $G_{i-1}(\lambda^{FB}, U_{i-2}^n) \leq 0$, $\lambda_{i-1} = \lambda^{FB}$, and $U_{i-1}^n = \delta\pi_{i-1}Z + (1-\delta)\pi_{i-1}F_{i-1}^n + (1-\pi_{i-1})[(1-d)Z + d\,\mathcal{L}(\lambda^{FB}) + \theta d(1-\lambda^{FB})X]$. This implies that

$$\begin{aligned} G_i(\lambda^{FB}, U_{i-1}^n) &= (1-d)Z + d\,\mathcal{L}(\lambda^{FB}) + d(1-\lambda^{FB})X \\ &\qquad - \frac{(1-\delta)\pi_{i-1}F_{i-1}^n + (\pi_i - \pi_{i-1})\left[(1-d)Z + \mathcal{L}(\lambda^{FB}) + d\theta(1-\lambda^{FB})X\right]}{\pi_i - \delta\pi_{i-1}} \\ &> (1-d)Z + d\,\mathcal{L}(\lambda^{FB}) + d(1-\lambda^{FB})X - F_{i-1}^n = G_{i-1}(\lambda^{FB}, U_{i-2}^n) \end{aligned} \tag{45}$$

where the last equality follows because $G_{i-1}(\lambda^{FB}, U_{i-2}^n) \leq 0$ implies that $F_{i-1}^n = \hat{F}_{i-1}(\lambda^{FB}, U_{i-2}^n)$ as given by Eq. 41 (Case i). The inequality follows because $F_{i-1}^n > (1-d)Z + d\,\mathcal{L}(\lambda^{FB}) + d\theta(1-\lambda^{FB})X$. This implies that there exists j, such that for all types $i \leq j$, $G_i(\lambda^{FB}, U_{i-1}^n) \leq 0$ and $\lambda_i^n = \lambda^{FB}$, and $G_{j+1}(\lambda^{FB}, U_j^n) > 0$.

Having shown that $G_i(\lambda^{FB}, U_{i-1}^n) \leq 0$ for all $i \leq j$, we now show by contradiction that $G_i(\lambda^{FB}, U_{i-1}^n) > 0$ for all $i > j$. Notice that Eq. 45 implies that $G_{j+1}(\lambda^{FB}, U_j^n) > G_j(\lambda^{FB}, U_{j-1}^n) =$

0. Suppose that there exists $k > j+1$ such that $G_k(\lambda^{FB}, U^n_{k-1}) \leq 0$. This implies that $\lambda^n_{j+1} > \lambda^{FB} = \lambda^n_k$, and $F^n_{j+1} < c_2(\lambda^n_{j+1}) < c_2(\lambda^n_k) \leq F^n_k$, where $c_2(\lambda)$ is defined in Table 1. This violates incentive compatibility as the type $j+1$ can profitably deviate to (F^n_k, λ^n_k):

$$\begin{aligned} U^n_{j+1} &= p_{j+1}(F^n_{j+1}, \lambda^n_{j+1}) + \delta\left[V_{j+1}(\lambda^n_{j+1}) - p_{j+1}(F^n_{j+1}, \lambda^n_{j+1})\right] \\ &= (1-\delta)[\pi_{j+1} + (1-\pi_{j+1})\theta]F^n_{j+1} + \delta\pi_{j+1}Z + \delta(1-\pi_{j+1})\theta c_2(\lambda^n_{j+1}) \\ &\qquad\qquad\qquad\qquad\qquad\qquad + (1-\pi_{j+1})(1-\theta)c_3(\lambda^n_{j+1}) \\ &< (1-\delta)\pi_{j+1}F^n_k + \delta\pi_{j+1}Z + (1-\pi_{j+1})\left[\theta c_2(\lambda^n_{j+1}) + (1-\theta)c_3(\lambda^n_{j+1})\right] \\ &< (1-\delta)\pi_{j+1}F^n_k + \delta\pi_{j+1}Z + (1-\pi_{j+1})\left[\theta c_2(\lambda^n_k) + (1-\theta)c_3(\lambda^n_k)\right] \\ &< p_k(F^n_k, \lambda^n_k) + \delta\left[V_{j+1}(\lambda^n_k) - p_{j+1}(F^n_k, \lambda^n_k)\right] \end{aligned}$$

Finally, we show that λ^n_i is increasing in i for all $i > j$. Recall that, for $i > j$, $\lambda^n_i = \max\{\breve{\lambda}_i(U^n_{i-1}), \tilde{\lambda}_i\}$. If $\lambda^n_{i-1} = \tilde{\lambda}_{i-1}$, then it is immediate that $\lambda^n_i \geq \lambda^n_{i-1}$, since $\tilde{\lambda}_i$ (Eq. 44) is increasing in π_i, where the inequality is strict whenever $\tilde{\lambda}_{i-1} < 1$. If $\lambda^n_{i-1} = \breve{\lambda}_{i-1}(U^n_{i-2})$, then $U^n_{i-1} = \delta\pi_{i-1}Z + (1-\delta)\pi_{i-1}F^n_{i-1} + (1-\pi_{i-1})[(1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d\theta(1-\breve{\lambda}_{i-1}(\cdot))X]$. This implies that

$$\begin{aligned} G_i(\breve{\lambda}_{i-1}(\cdot), U^n_{i-1}) &= (1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d(1-\breve{\lambda}_{i-1}(\cdot))X \\ &\quad - \frac{(1-\delta)\pi_{i-1}F^n_{i-1} + (\pi_i - \pi_{i-1})[(1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d\theta(1-\breve{\lambda}_{i-1}(\cdot))X]}{\pi_i - \delta\pi_{i-1}} \\ &> (1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d(1-\breve{\lambda}_{i-1}(\cdot))X - F^n_{i-1} = G_{i-1}(\breve{\lambda}_{i-1}(\cdot), U^n_{i-2}) = 0 \end{aligned}$$

where the inequality follows because $F^n_{i-1} = (1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d(1-\breve{\lambda}_{i-1}(\cdot))X > (1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d\theta(1-\breve{\lambda}_{i-1}(\cdot))X$. This then implies that $\lambda^n_i \geq \breve{\lambda}_i(U^n_{i-1}) > \breve{\lambda}_{i-1}(U^n_{i-2}) = \lambda^n_{i-1}$. To summarise, for all $i > j$, $\lambda^n_i \geq \lambda^n_{i-1}$, where the inequality is strict whenever $\lambda^n_{i-1} < 1$.

Having now characterize the solution to Eq. 12 and 13, we now show that this solution indeed characterises the least cost separating equilibrium.

Claim 10. *The optimisation programme subject to global incentive compatibility, defined as*

$$U^n_i \geq p_k(F^n_k, \lambda^n_k) + \delta\left[V_i(\lambda^n_k) - p_i(F^n_k, \lambda^n_k)\right] \quad \forall i, k \in \{1, 2, ..., n\}$$

is equivalent to the optimisation programme subject to local incentive compatibility (IC_i) defined *by Eq. 13.*

Proof. It is immediate that global incentive compatibility implies local incentive compatibility.

It remains to show that the solution to Eq. 12 and 13 indeed satisfies the global incentive compatability constraint. Following the proof of Proposition 1, the (IC_i) binds in equilibrium for all $i \geq 2$. Consider any $k \geq i$. The binding local (IC_i) implies

$$\begin{aligned}
& p_i(F_i^n, \lambda_i^n) \\
\text{by } (IC_i) \quad &= p_{i+1}(F_{i+1}^n, \lambda_{i+1}^n) + \delta\left(\left[V_i(\lambda_{i+1}^n) - p_i(F_{i+1}^n, \lambda_{i+1}^n)\right] - \left[V_i(\lambda_i^n) - p_i(F_i^n, \lambda_i^n)\right]\right) \\
\text{by } (IC_{i+1}) \quad &= p_{i+2}(F_{i+2}^n, \lambda_{i+2}^n) + \delta\left(\left[V_{i+1}(\lambda_{i+2}^n) - p_{i+1}(F_{i+2}^n, \lambda_{i+2}^n)\right] - \left[V_{i+1}(\lambda_{i+1}^n) - p_{i+1}(F_{i+1}^n, \lambda_{i+1}^n)\right]\right) \\
&\quad + \delta\left(\left[V_i(\lambda_{i+1}^n) - p_i(F_{i+1}^n, \lambda_{i+1}^n)\right] - \left[V_i(\lambda_i^n) - p_i(F_i^n, \lambda_i^n)\right]\right) \\
\text{by } (IC_s) \quad &= p_k(F_k^n, \lambda_k^n) + \delta \sum_{s=i}^{k-1} \left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right] \qquad (46)
\end{aligned}$$

This implies $U_i^n = p_i(F_i^n, \lambda_i^n) + \delta\left[V_i(\lambda_i^n) - p_i(F_i^n, \lambda_i^n)\right] \geq p_k(F_k^n, \lambda_k^n) + \delta\left[V_i(\lambda_k^n) - p_i(F_k^n, \lambda_k^n)\right]$ for all $k \geq i$ if

$$\begin{aligned}
&\sum_{s=i}^{k-1} \left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right] \\
&\qquad \geq \left[V_i(\lambda_k^n) - p_i(F_k^n, \lambda_k^n)\right] - \left[V_i(\lambda_i^n) - p_i(F_i^n, \lambda_i^n)\right] \\
&\qquad = \sum_{s=i}^{k-1} \left[V_i(\lambda_{s+1}^n) - p_i(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_i(\lambda_s^n) - p_i(F_s^n, \lambda_s^n)\right]
\end{aligned}$$

which is implied by

$$\begin{aligned}
&\left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right] \\
&\qquad \geq \left[V_i(\lambda_{s+1}^n) - p_i(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_i(\lambda_s^n) - p_i(F_s^n, \lambda_s^n)\right] \quad \forall s \geq i \\
\Leftrightarrow \quad & -F_{s+1}^n + F_s^n - \left[c_2(\lambda_{s+1}^n) - F_{s+1}^n\right]^+ + \left[c_2(\lambda_s^n) - F_s^n\right]^+ \geq 0 \qquad (47)
\end{aligned}$$

Similarly, Eq. 46 implies $U_k^n = p_k(F_k^n, \lambda_k^n) + \delta\left[V_k(\lambda_k^n) - p_k(F_k^n, \lambda_k^n)\right] \geq p_i(F_i^n, \lambda_i^n) + \delta\left[V_k(\lambda_i^n) - p_k(F_i^n, \lambda_i^n)\right]$

for all $k \geq i$ if

$$\sum_{s=i}^{k-1} \left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right]$$

$$\leq \left[V_k(\lambda_k^n) - p_k(F_k^n, \lambda_k^n)\right] - \left[V_k(\lambda_i^n) - p_k(F_i^n, \lambda_i^n)\right]$$
$$= \sum_{s=i}^{k-1} \left[V_k(\lambda_{s+1}^n) - p_k(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_k(\lambda_s^n) - p_k(F_s^n, \lambda_s^n)\right]$$

which is implied by

$$\left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right]$$
$$\leq \left[V_k(\lambda_{s+1}^n) - p_k(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_k(\lambda_s^n) - p_k(F_s^n, \lambda_s^n)\right] \quad \forall k \geq s$$
$$\Leftrightarrow \quad -F_{s+1}^n + F_s^n - \left[c_2(\lambda_{s+1}^n) - F_{s+1}^n\right]^+ + \left[c_2(\lambda_s^n) - F_s^n\right]^+ \geq 0 \qquad \Leftrightarrow \text{Eq. } 47$$

Therefore it suffices to show that Eq. 47 is true for all s. Following the proof of Proposition 3, there can be three cases.

(i) $s + 1 \leq j$. In this case, $\lambda_{s+1}^n = \lambda_s^n$, and Eq. 47 $\Leftrightarrow F_{s+1}^n < F_s^n$.

(ii) $s \leq j < s + 1$. In this case, $\lambda_{s+1}^n > \lambda_s^n$, and Eq. 47 $\Leftrightarrow -c_2(\lambda_{s+1}^n) + F_s^n \geq 0$, which is true as $-c_2(\lambda_{s+1}^n) + F_s^n > -c_2(\lambda_s^n) + F_s^n \geq 0$.

(iii) $s = 1 > j$. In this case, $\lambda_{s+1}^n > \lambda_s^n$, and Eq. 47 $\Leftrightarrow -c_2(\lambda_{s+1}^n) + c_2(\lambda_s^n) \geq 0$.

□