

# *A Theory of Crime and Vigilance\**

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## **Abstract**

This paper develops a novel equilibrium theory of crime. A population of heterogeneous potential victims elects how much costly vigilance to exert to guard their property, while a population of heterogeneous potential criminals chooses whether to attempt a crime. Attempted crimes arise from random encounters of criminals and potential victims. The deterrence rate is the failure chance of an attempted crime. It rises in vigilance and balances agents' incentives in the unique equilibrium.

Our model predicts how changes in the values of goods to criminal or owner, expected legal punishment, or the vigilance technology affect the crime rate, attempted crime rate, deterrence rate, and vigilance expenses. Most predictions are new and some of them are counterintuitive because of equilibrium effects. For instance, the crime rate is hump-shaped in legal penalties and in property values, resembling a Laffer curve.

Aside from positive predictions, we show that the equilibrium levels of crime and vigilance may be above or below their efficient levels, depending on the optimization criterion, and derive Pigouvian policies to restore efficiency. Also, we offer a graphical analysis of the social costs of crime and find that crime is a classic case of the “Tullock Paradox” — total social costs are strictly less than the potential transfers. Finally, we provide an analysis of the positive spillovers of unobservable vigilance, found in Ayres and Levitt (1998), and discuss some extensions of the model.

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# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>The Model</b>	<b>6</b>
<b>3</b>	<b>A Supply and Demand Equilibrium Analysis</b>	<b>9</b>
<b>4</b>	<b>Comparative Statics</b>	<b>11</b>
4.1	Changing Supply, and the Criminal Laffer Curve . . . . .	11
4.2	Changing Demand . . . . .	15
4.3	Changing Supply and Demand . . . . .	16
<b>5</b>	<b>The Social Costs of Crime</b>	<b>17</b>
<b>6</b>	<b>Optimal Levels of Crime and Vigilance</b>	<b>20</b>
6.1	Potential Victim Losses Minimization: Neighborhood Watch . . . . .	21
6.2	Social Costs Minimization . . . . .	23
<b>7</b>	<b>Vigilance Spillover Effects: Lojack</b>	<b>24</b>
<b>8</b>	<b>Extensions (in progress)</b>	<b>27</b>
<b>9</b>	<b>Concluding Remarks</b>	<b>27</b>
<b>A</b>	<b>Omitted Proofs</b>	<b>28</b>
A.1	Existence and Uniqueness: Proof of Theorem 1 . . . . .	28
A.2	Greater Punishment: Proof of Proposition 1 Finished . . . . .	28
A.3	Greater Gains and Losses: Proof of Proposition 4 Finished . . . . .	30

# 1 Introduction

Motivated by its massive social costs, the path-breaking paper by Becker (1968) gave the first economic analysis of crime. Accounting for the optimal criminal response to changes in punishment and the probability of capture, he explored the socially optimal level of law enforcement expenditures. Recent evidence suggests that individual or private expenditures against crime (or *vigilance*) are sizeable and even greater than public expenditures; see, e.g., Anderson (2012).<sup>1,2</sup> However, understanding the determinants of crime and vigilance, and how they affect one another, is a problem that has not received much attention in the literature. This is surprising, especially from a policy analysis viewpoint, as in general, any policy directed at alleviating crime, such as investing in policing or raising penalties, not only impacts criminal behavior, but also vigilant behavior, which in turn determines the expected gains from crime.

This paper develops a new strategic model of crime, centered on the competition between optimizing criminals and potential victims. Our model gives a new vehicle for thinking about crime and its determinants, showing precisely how the different forces interact. In our model, criminals choose whether to attempt a crime, whereas their innocent rivals decide how vigilant to be. While formulated as a game, we show how this framework generates a supply *and* demand for crime in an implicit market that essentially realizes Becker's partial equilibrium vision for crime. Our theory predicts a wide range of equilibrium variables: the crime rate, the attempted crime rate, the deterrence rate, and the potential victim's expenses. We also create a graphical analysis for understanding not only comparative statics but also welfare gains and losses, and explore the levels of crime and vigilance that respectively minimize potential victims' losses and social costs.

Chance is a crucial aspect of crime; few of us are victims in any month. According to the Bureau of Justice Statistics, in 2013, only 9% of 11.5 million households experienced one or more property victimizations. Our model captures the stochastic nature of crime firstly by assuming random encounters of criminals and victims. Randomness also plays a key role since not all attempted crimes succeed. One often fails to break into a house or steal a car. The failure chance, or the *deterrence rate*, is endogenous and it rises in vigilance expenses at decreasing rates. We believe this notion of deterrence is new in the economics of crime literature,<sup>3</sup> wherein deterrence is the act of preventing an attempted crime from happening;

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<sup>1</sup>Over and above property losses, Anderson (2012) estimates \$480 billion spent on private vigilance efforts, in addition to public expenditures. He estimates the cost of crime to be around \$1 trillion in the US.

<sup>2</sup>This has become even more transparent since the domestic security changes post 9-11.

<sup>3</sup>A recent test verified that TSA security deters only 5% of criminal attempts; see [www.vox.com/2016/5/17/11687014/tsa-against-airport-security](http://www.vox.com/2016/5/17/11687014/tsa-against-airport-security).

here we focus on the act of preventing the success of a criminal attempt.

As in Becker, we assume that potential criminals have an extensive margin — whether to engage in crime. Doing so risks one’s outside option, say one’s career. A greater punishment makes the life of crime less appealing, and so formally raises the outside option. We assume that criminals vary by their criminal costs. As a result, the supply of criminals is increasing in how lucrative it is, and decreasing in the value of noncriminal life endeavors. Since we wish to isolate the missing and important strategic contest between criminals and their target victims, we do not model the law enforcement optimization and embed any source of legal costs into the criminal cost.

All told, we explore the equilibrium of a model in which a continuum of potential criminals each chooses whether to engage in crime, while a continuum of potential victims each elects how vigilant to be. The number of criminals and per-criminal offenses then fix the attempted crime rate, while vigilance expenses determine the deterrence rate. Potential victims are hurt by a greater attempted crime rate, while greater vigilance frustrates criminals. In our *crime equilibrium*, everyone optimizes taking other actions as given. The crime rate is then a derived quantity, reflecting the undeterred attempted crimes.

We re-interpret our crime equilibrium as a competitive equilibrium, partially depicted in a metaphorical supply and demand framework. Because of the natural asymmetry of the market, deterrence plays the role of a price for potential criminals, while the attempted crime rate plays the same for potential victims. Optimal criminal behavior induces a *downward-sloping supply of attempted crime*, as fewer crimes are committed with greater deterrence. Similarly, optimal vigilant behavior induces an *upward-sloping demand for deterrence*, since a greater victimization chance elicits a more vigilant response. The supply-demand crossing yields a unique *market-clearing deterrence and attempted crime rate*, rendering unambiguous all comparative statics analyses we perform (Theorem 1).

We focus on property crime. This excludes victimless crimes, like prostitution or drug dealing, but may subsume violent crime, if one imagines that victims lose much more than perpetrators gain from violence. For such situations, our equilibrium offers predictions for changes in all equilibrium variables as the payoff stakes change for victims or criminals, as legal penalties change, and new or cheaper vigilance technologies are introduced. As with standard partial equilibrium analysis, each change affects supply or demand or both.

OVERVIEW OF THE RESULTS. To see how we diverge from Becker, consider his most basic claim that aggregate and individual criminal activity should fall in expected legal penalties. By contrast, since we assume heterogeneous criminals, a higher expected legal penalty deters entry. The supply curve shifts down, but demand is unaffected. So greater

punishment discourages private vigilance, raising the marginal profitability of every offense. On balance, there are fewer criminals, but each is more poorly deterred to make up for the greater punishment. Moreover, Proposition 1 finds that the crime rate is *hump-shaped* as a function of expected penalties, resembling what Ayres (2016) calls a *Laffer curve for crime*. For some intuition, consider two extreme cases. If legal penalties are very high, the attempted crime rate is very low, and decreases in legal penalties lead to proportionately large increases in attempted crimes, raising the crime rate. At the opposite extreme, if penalties are very low, deterrence rate is very high, and increases in legal penalties lead to proportionately large decreases in deterrence, lowering the crime rate. These results also hold for changes in the criminal material gain, or in the apprehension chance.<sup>4</sup> In Proposition 2, we provide a test to determine whether the Laffer curve for crime slopes downward.

Since our centerpiece is the endogenous vigilance by potential victims, we explore how vigilance technology innovations, or changes in property losses, impact the crime equilibrium. Unlike legal penalties, cheaper vigilance or higher losses raise demand, but leave supply unaffected. Proposition 3 uncovers a *vigilance magnification effect* that reflects the competitive nature of crime: When the marginal costs of vigilance decrease, potential victims grow more vigilant. But since criminals respond to variations in deterrence, vigilance increases more and the attempted crime rate decreases less than what a decision theory model crime would predict. Altogether, we find fewer criminals, each with greater chance of failing, so that the crime rate drops.

In §7, we provide an equilibrium analysis for the Lojack vigilance technology improvement for cars, explored in Ayres and Levitt (1998). We assume that potential victims can purchase Lojack at a fixed price, securing high deterrence. Since potential victims vary by their vigilance costs, some opt for Lojack, and others do not. As a result, the aggregate demand curve is higher, and the new equilibrium level of crime is lower. We find Lojack users confer a positive spillover on those who do not, by lowering their attempted crime rate, and provide a conceptual measure of the external benefit in our supply and demand framework.

A major source of exogenous variation across crimes is the value of property to victims or criminals. At one extreme is money, worth the same to victims and criminals, and at the opposite are very personal goods, or violent crime. But arguably most property crimes entail a positive but partial *criminal markdown* — since owners invariably prefer their property more than others do, and because laundering spoils of crime is by no means

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<sup>4</sup>Becker (1968) argues (on p. 188) that improved law enforcement has an ambiguous effect on total offenses. Becker also presumes that this would be partially offset by a drop in private expenditures; see also Philipson and Posner (1996); Levitt (1997); McCrary (2002); Di Tella and Schargrodsky (2004); Nagin (2013). In fact, by Proposition 1, while better policing does displace private vigilance, it can still reduce the crime rate.

easy. The markdown on automobiles might be around 80%, for instance. Proposition 4 fixes the criminal markdown, and explores a simultaneous increase in the stakes of the game for everyone: property losses and criminal gains. This naturally applies to a fixed class of goods, as the value of the good rises for owner and criminal alike. This change impacts both sides of the market: Facing a greater loss, potential victims grow more vigilant, while criminals respond to the increased stakes with more offenses. The demand and supply curves both increase, and thus the deterrence rate rises. While vigilance expenses and deterrence always rise, *the attempted crime rate and crime rate are both hump-shaped*. For instance, if we think of cars, the theory suggests that most attempted thefts and actual thefts occur for cars in the mid-range price segment, which seems to be in line with causal empiricism.

The supply and demand graphical framework is prized in economics not only for its easy comparative statics analysis, but also for its transparent depiction of welfare gains and losses. In our space of deterrence and attempted crime rates, the mass of successful crimes is an area. Since criminals and victims value stolen goods differently, we must scale supply and demand curves separately. When we do so in §5, we find that the area under the falling supply curve and above the equilibrium deterrence represents the producer surplus of crime — namely, the criminal profits of the inframarginal criminals. More subtly, the consumer surplus of potential victims is not the area over the rising demand curve below the market deterrence, but instead the area under this curve. This twist arises because “trade” here is not win-win. Rather, criminal offenses constitute an economic good for criminals, but a bad for potential victims.

In the same vein, we offer a contribution to rent-seeking (Tullock, 1967), since criminals expend resources to procure victims’ property, and victims invest resources to forestall this. Crime can thus be viewed as a set of rent-seeking contests. We have discovered that this offers a clear example of the famous Tullock Paradox — namely, that social costs may be less than the magnitude of potential transfers (the stolen property). This owes to the random nature of crime. We give a precise measure of the Tullock gap in terms of primitives. Only in the case of no criminal markdown, no diminishing returns to vigilance, and no criminal and victim heterogeneity does the paradox disappear.

We then turn to finding the optimal levels of crime and vigilance in two benchmarks. If the objective is to minimize potential victims’ expected losses, as in a *neighborhood watch*, then it is optimal to raise vigilance expenditures from their equilibrium level in order to have more deterrence and less attempted crimes. The reason is that in a neighborhood watch, the social planner coordinates the behavior of potential victims and thereby is able to alter the attempted crime rate. Like in a monopsony, the optimal deterrence is given by the intersection of the demand curve and marginal supply curve, while the supply curve

determines the attempted crime rate. Because potential victims do not internalize how their joint behavior affect crime (they are both anonymous and negligible), this outcome is not an equilibrium. Thus, *to restore efficiency, policies must be targeted to the demand side*. For instance, a Pigouvian subsidy aimed to reduce vigilance expenses can restore efficiency. Now, if the objective is to minimize social costs, then the analysis is more subtle and the answer depends critically on the convexity of the criminal outside option distribution.

LITERATURE REVIEW. Aside from first treating criminals as maximizing agents, Becker (1968) introduced an aggregate supply of crime with intuitive derivatives in fundamentals. Ehrlich (1981) later posited an intuitive demand for crime, where the net payoff per offense acts as the price.<sup>5</sup> So his supply curve slopes up, and demand slopes down. He might be the first to think of the number of crimes as an equilibrium object. By contrast, we formulate the criminal game, specifying all payoff functions. Thus, our supply explicitly arises from the criminal maximization, and our demand is derived from the victim's optimization problem. Moreover, Ehrlich's verbal story suggests that victims' vigilance costs directly reduce criminal material payoffs.<sup>6,7</sup> By contrast, we assume that the main effect of vigilance, like burglar systems, is to deter crimes (maybe with a delay, as with Lojack). In so doing, we take seriously the randomness of crime by modeling both probabilistic encounters and stochastic deterrence.

Knowles et al. (2001) may be the first to study a population game of crime. Like Becker, they focus on the interaction between police and potential criminals, and seek to identify whether police stops exhibit racial or statistical discrimination. Their potential criminals are heterogeneous and decide whether to carry contraband. The probability of a police search of their vehicle incentivizes their decision to carry drugs. The models differ in most respects but share the feature that the criminal extensive margin and police search decision (like our vigilance) are mutual equilibrium best replies.<sup>8</sup>

There are small literatures that focus on how the observability of vigilance might divert crime to less protected targets (Shavell, 1991; Koo and Png, 1994; Hotte and Van Ypersele, 2008), on the levels of vigilance when the property value is unknown to the criminals (Baumann and Friehe, 2013), and on the interplay between optimal law enforcement and rational victim behavior (Ben-Shahar and Harel, 1995; Hylton, 1996; Garoupa, 2003).

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<sup>5</sup>He offers a verbal reduced-form logic: "the demand schedule for offenses represents the average potential payoff per offense at alternative frequencies of offenses". Ehrlich (2010) is a recent survey. Hotte et al. (2003) develop a model of criminals and victims. While their supply follows from criminals' entry, their demand reflects the criminals' optimization. So their market equilibrium takes vigilance as given.

<sup>6</sup>He writes: "Burglar alarm systems, guards, . . . all serve the similar purpose of decreasing the gross loot per offense, or increasing the cost and effort to the offender of committing the offense."

<sup>7</sup>In this case, the interaction becomes sequential and does not admit a supply and demand approach.

<sup>8</sup>In the same spirit, Persico (2002) performs a theoretical analysis of the effects of police fairness on crime.

The empirical literature on crime has raced far ahead of the theory, and we cannot do justice to it. There are many great surveys and analyses, like Levitt (2004).<sup>9</sup> But our equilibrium model should allow us to speak to many of them, and we cite them in the text, as our theorems permit. Our framework allows us to explore the effect of more police (Levitt, 1997; McCrary, 2002) or greater legal penalties (Levitt, 2004), the spillover effects of vigilance (Ayres and Levitt, 1998), the investments in private security (Clotfelter, 1978), and a neighborhood watch (Cook and MacDonald, 2011).

In §2, we set up the model; in §3, we construct the graphical apparatus, and characterize the unique equilibrium. In §4, we explore the comparative statics of the crime equilibrium. In §5, we study the social costs of crime, and in §6 we derive the optimal levels of crime and vigilance. In §7, we analyse the positive spillovers of Lojack, and in §8 we discuss some extensions. Section §9 concludes. All omitted proofs are in the Appendix.

## 2 The Model

PLAYERS, MATCHING, AND DETERRENCE. Consider an economy in which the ownership of a single good is dispersed among a large population of risk-neutral agents. We will call those endowed with the good *potential victims*, and those not so endowed *potential criminals*. We explore a novel competition for ownership of the goods, as some potential criminals may desire to steal the good. Thus, our focus is on property crime, such as theft, burglary, etc., and it excludes victimless crimes such as speeding, pollution, and tax evasion, and so on. We explore a static game, with the standard understanding that it can also be viewed as the steady-state of a dynamic game.

Potential criminals are heterogeneous, and shall be labeled by their *cost*  $c \geq 0$ . This subsumes the expected foregone income from the legal sector, which varies across potential criminals, and legal punishment. For instance, in Becker (1968), the latter is the probability of capture times the monetary equivalent of the punishment. In general, potential criminals treat  $c$  as the fixed cost of their illicit activities. For simplicity, the *cost mass distribution*  $F$  has a density  $f(c) \equiv F'(c) > 0$  on  $[0, \infty)$ . Since we assume entry of new criminals,  $F$  need not be a probability distribution with unit mass. We say that criminals have *higher costs* with  $F_H$  than  $F_L$  if  $F_H(c) < F_L(c)$  all  $c > 0$ .<sup>10</sup>

We follow Becker in assuming that potential criminals simply have an extensive margin — they choose whether to enter and attempt a single crime. If so, they are also a *criminal*. For now, we shall think of the length of time of our equivalent steady state model as short enough

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<sup>9</sup>The normative approach to crime and public enforcement is surveyed in Polinsky and Shavell (2000).

<sup>10</sup>This extends first order stochastic dominance, applying it to distributions with different masses.

that committing at most one crime is realistic. In section §8, we discuss what happens if the number of crimes is also a decision margin.

Stealing a good entails a deadweight loss, since owners invariably value their property more than others do. If criminals must further launder their spoils, and so such losses are amplified, since laundering is by no means easy. We ignore complementarities among crimes, and assume criminals focus on the theft of a specific good. So motivated, we describe goods by pairs  $(m, M)$ , where  $M > 0$  is the potential victim's *property loss*,<sup>11</sup> and  $m > 0$  the *criminal's gain*. For instance, auto theft has a large *markdown*  $1 - m/M \gg 0$ , jewel theft a smaller markdown, and money has no markdown ( $m = M$ ). Despite our property crime focus, one could think that a violent crime has a high markdown  $1 - m/M \approx 1$ . While it is invariably the case for almost all studied good that  $m \leq M$ , we will not impose this, to as not to complicate the statement of some results.

Criminals randomly meet potential victims as part of a decentralized random matching model. All potential victims are criminal targets with the same endogenous *attempted crime rate*  $\alpha \geq 0$ , namely, the expected number of attempted crimes per capita. Chance plays two roles in our model. Whether one is a victim of an attempted crime and whether an attempted crime succeeds are both random events. Potential victims can probabilistically deter a crime by placing bars on windows, installing home alarms, or by mental alertness.<sup>12</sup> We call all such actions *vigilance*, subsumed by a scalar  $v \geq 0$ . Our model is stationary, and thus vigilance is a flow cost, or the annuity value of initial one-shot costs.<sup>13</sup> Vigilance is a direct subtraction from utility.

Any attempted crime against a potential victim fails with *individual deterrence rate*  $\delta \in [0, 1]$ . Potential victims are indexed by privately-observed heterogeneous vigilance cost<sup>14</sup> indices  $\xi \in [0, 1]$ , so that  $v = V(\delta|\xi)$  is the cost of producing individual deterrence rate  $\delta \geq 0$ . The types  $\xi$  have a probability distribution  $G$  and density  $g(\xi) \equiv G'(\xi) > 0$  on  $[0, 1]$ . We assume a monotone and strictly convex cost  $V_\delta(\delta|\xi), V_{\delta\delta}(\delta|\xi) > 0$ , with  $V(0|\xi) = V_\delta(0|\xi) = 0$  and  $V_\delta(1|\xi) < \infty$ . Also, potential victims with higher cost types  $\xi$  face greater costs and marginal costs, namely,  $V_\xi(\delta|\xi) > 0$  and  $V_{\delta\xi}(\delta|\xi) > 0$ . We further assume that  $V_\delta(\delta|\xi)$  is supermodular in  $(\delta, \xi)$ .

While we have assumed that marginal vigilance costs  $V_\delta$  increase, we now ensure that they are log-concave, and therefore never jump up. More precisely,  $[\log(V_\delta)]_{\delta\delta} \leq 0$ . This also forces a well-defined limit  $\lim_{\delta \downarrow 0} V_{\delta\delta}(\delta|\xi)/V_\delta(\delta|\xi) > 1$ . Any convex geometric function

<sup>11</sup>If property is insured, we think of  $M$  as the *net loss after any insurance payments*.

<sup>12</sup>Each adult spends one minute and 50 seconds locking and unlocking doors each day (Anderson, 2012).

<sup>13</sup>As usual for partial equilibrium analysis, our static analysis intuitively applies to the stationary problem.

<sup>14</sup>We subsume all heterogeneity into the vigilance cost function for simplicity.

$V(\delta|\xi) = (1 + \xi)\delta^\gamma$ ,  $\gamma \geq 1$ , meets all these properties.

Let  $\delta(\xi)$  denotes the individual deterrence rate of potential victim  $\xi$  conjectured by potential criminals. The criminal cannot observe  $\xi$ , and so believes that his attempt will fail with probability  $\Delta \equiv \int_0^1 \delta(\xi)dG(\xi) \in [0, 1]$ . This is the (average) *deterrence rate*. The *crime rate* is  $\kappa \equiv \int_0^1 \alpha(1 - \delta(\xi))dG(\xi) \equiv \alpha(1 - \Delta) \in [0, 1]$ .

OPTIMIZATIONS AND EQUILIBRIUM. Since crimes are undeterred attempted crimes, a potential victim with individual deterrence  $\delta$  has expected property losses  $\alpha(1 - \delta)M$ . Potential victims seek to minimize their expected total property losses of crime plus vigilance  $v$ . But since  $v = V(\delta|\xi)$  is increasing for all  $\xi$ , we can simply think of the victim as choosing the deterrence rate  $\delta \in [0, 1]$  and express *losses from crime* for a type  $\xi$  potential victim as:<sup>15</sup>

$$\mathcal{L}(\delta, \alpha, \xi) \equiv \alpha(1 - \delta)M + V(\delta|\xi) \tag{1}$$

Reflecting an underlying competitive assumption that no potential victim impacts the attempted crime rate  $\alpha$ , potential victims choose  $\delta$  to minimize (1), taking  $\alpha$  as given. In this respect, vigilance (or private defense) is different from law enforcement (or public defense), which surely has aggregate effects (Polinsky and Shavell, 2000). To understand the impact of vigilance on crime, we consider exogenous policing.<sup>16</sup>

As in Becker (1968), potential criminals face a binary decision: attempt a crime or not. A potential criminal with opportunity cost  $c$  cannot observe the deterrence rates produced by potential victims, and so he takes the average deterrence rate  $\Delta$  as given, and attempts a crime with gain  $m$  if and only if his expected *net criminal profits*  $\Pi(\Delta|c) \geq 0$ , where:<sup>17</sup>

$$\Pi(\Delta|c) \equiv (1 - \Delta)m - c. \tag{2}$$

Naturally, the *marginal potential criminal*  $\bar{c}$  earns no net profits from crime.

Notice the embedded causation. The potential victims' payoffs reflect the criminal behavior, via the attempted crime rate, and the criminal payoffs are only impacted by the

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<sup>15</sup>With risk averse potential victims, and actuarially fair insurance prices, the problem is slightly more complex — for a victim  $\xi$  chooses  $\delta \in [0, 1]$  to *maximize*  $u(w - \alpha(1 - \delta)M) - V(\delta, \xi)$ , for some increasing and strictly concave  $u(\cdot)$ , and large enough income  $w > 0$ . This is similar to Ehrlich and Becker (1972) who study the relation between (competitive) market insurance and self protection.

<sup>16</sup>A model with both endogenous public and private responses to crime would shed light on the classic debate of public versus private law enforcement (Becker and Stigler, 1974; Landes and Posner, 1975), and is an important next step. But modeling these two interacting forces simultaneously clouds the effects of vigilance on crime, which is our novel focus.

<sup>17</sup>In §??, we allow criminals to choose the expected quantity of offenses  $\lambda \geq 0$  over their lifetime. In Smith and Vásquez (2015) we explore a richer, yet more complicated problem, in which criminals choose also the *caliber* of their offenses, affecting the deterrence rate of an attempted crime.

potential victims deterrence. The only spillovers of potential victims onto each other will be indirectly channeled by the impact on criminal behavior, and conversely, criminals only affect other criminals indirectly via the potential victims vigilance.

A *crime equilibrium* is a pair  $(\delta^*(\cdot), \bar{c}^*)$  such that (i) each potential victim  $\xi \in [0, 1]$  expends vigilance  $v^*(\xi) = V(\delta^*(\xi)|\xi)$ , where the individual deterrence  $\delta^*(\xi)$  minimizes expected losses  $\mathcal{L}(\delta, \alpha^*, \xi)$  in (1), given the attempted crime rate  $\alpha^* = F(\bar{c}^*)$ , and (ii) a potential criminal attempts a crime if and only if his cost is  $c \leq \bar{c}^*$ , where profits  $\Pi(\Delta^*|\bar{c}^*) = 0$  in (2), for  $\Delta^* = \int_0^1 \delta^*(\xi) dG(\xi)$ .

In particular, a potential criminal chooses whether to attempt a crime expecting an equilibrium deterrence rate, and a potential victim selects his vigilance anticipating the attempted crime rate.<sup>18</sup> The simultaneity of Nash equilibrium reflects the unobserved nature of vigilance, and we revisit the issue of partially observed vigilance in §8.

### 3 A Supply and Demand Equilibrium Analysis

Consider the potential victims' optimization problem (1). Given an attempted crime rate  $\alpha \geq 0$ , the *optimal individual deterrence function*  $\bar{\delta}(\alpha|\xi) \in [0, 1]$  minimizes  $\mathcal{L}(\delta, \alpha|\xi)$ . Clearly, if there is no attempted crime, then individuals elect no vigilance, or  $\bar{\delta}(0|\xi) = 0$ . If there is an interior solution  $\bar{\delta}(\alpha|\xi) \in (0, 1)$ , then it obeys the first-order condition for (1):

$$\mathcal{L}_\delta(\bar{\delta}, \alpha|\xi) = -\alpha M + V_\delta(\bar{\delta}|\xi) = 0 \tag{3}$$

The second order condition holds by strict cost convexity of  $V$ . Finally, observe that the marginal losses from crime are negative when  $\alpha > V_\delta(1|\xi)/M$ , and so the optimal deterrence is interior when  $0 < \alpha < V_\delta(1|\xi)/M$ , and obeys (3). The optimal individual deterrence function  $\bar{\delta}(\alpha|\xi)$  optimally rises from zero as  $\alpha$  rises. Moreover, potential victims with higher indices  $\xi$  have higher marginal vigilance costs  $V_\delta(\cdot|\xi)$  and so choose lower deterrence, i.e.,  $\bar{\delta}_\xi(\alpha|\xi) < 0$ .<sup>19</sup>

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<sup>18</sup>Even if victims believed that the attempted crime rate is random with mean  $\alpha$ , since expected losses (1) are linear in the attempted crime rate, the optimal choice of deterrence would be unaffected. The same holds for criminals if they believed that the deterrence rate is stochastic with mean  $\Delta$ .

<sup>19</sup>Our model with cost heterogeneity can subsume property value heterogeneity when  $V(\delta|\xi)$  is multiplicative, such as  $(1 + \xi)V(\delta)$ . For if we imagine a model in which potential victims face the same vigilance cost function but differ on their property values so that a type  $\xi$  value his property by  $M/(1 + \xi)$ , then the optimal behavior of potential victims is the same as one where they only differ by their cost function  $(1 + \xi)V(\delta)$ .

The market *deterrence demand*, given the attempted crime rate  $\alpha$ , obeys:

$$\mathcal{D}(\alpha) \equiv \int_0^1 \bar{\delta}(\alpha|\xi) dG(\xi) \quad (4)$$

This deterrence demand vanishes with no attempted crime,  $\mathcal{D}(0) = 0$ , and is continuous and upward sloping (see Figure 1), because crime is an economic bad. It hits perfect deterrence  $\mathcal{D}(\alpha) = 1$  once every single potential victim chooses perfect deterrence, namely, when  $\alpha \geq V_\delta(1|1)/M$ . We therefore call this the *attempted crime ceiling*. As potential victims take more precautions when they risk losing a more valuable good, the demand  $\mathcal{D}$  increases in the loss  $M$ , rising towards perfect deterrence as  $M \uparrow \infty$ .

So inspired, we can compute the elasticity of the deterrence demand function  $\mathcal{D}(\cdot)$  in (4) as a weighted average of the elasticities of individual deterrence demand functions  $\bar{\delta}(\cdot|\xi)$ :<sup>20</sup>

$$\mathcal{E}_\alpha(\mathcal{D}) = \int_0^1 \frac{\bar{\delta}(\alpha|\xi)}{\mathcal{D}(\alpha)} \cdot \frac{\alpha \bar{\delta}_\alpha(\alpha|\xi)}{\bar{\delta}(\alpha|\xi)} dG(\xi).$$

Now, differentiating  $\bar{\delta}(\cdot|\xi)$  yields:

$$\alpha \bar{\delta}_\alpha(\alpha|\xi) = \alpha M / V_{\delta\delta}(\bar{\delta}|\xi) = V_\delta(\bar{\delta}|\xi) / V_{\delta\delta}(\bar{\delta}|\xi), \quad (5)$$

where we used the FOC (3) and its implicit differentiation. So demand  $\mathcal{D}$  is more elastic — or steeper when plotted in  $(\alpha, \Delta)$ -space — when marginal vigilance cost  $V_\delta$  is (uniformly) less elastic across all types  $\xi$ .

We now derive the supply function of crime given a deterrence  $\Delta$ . Since the marginal potential criminal is  $\bar{c} \equiv (1 - \Delta)m$  by (2), the *attempted crime supply* is:

$$\mathcal{A}(\Delta) \equiv F((1 - \Delta)m). \quad (6)$$

The supply curve simply reflects the differentiable and increasing map from the marginal criminal  $\bar{c}$  to the attempted crime rate  $\alpha = F(\bar{c})$ . As noted after (3), if the deterrence  $\Delta$  is less than one, the attempted crime rate is at most  $V_\delta(1|1)/M$ , and so is boundedly finite (as in Figure 1).

The supply of attempted crime is therefore downward sloping, falling in the deterrence, vanishing as  $\Delta \uparrow 1$  (see Figure 1). The supply increases in the criminal gain  $m$ , vanishing when  $m = 0$ , and also increasing in the opportunity costs  $F$ . The supply  $\mathcal{A}$  inherits properties of the opportunity cost distribution  $F$  — for instance, it is more elastic with a greater hazard

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<sup>20</sup>We define the *elasticity of a function*  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  with respect to  $x \in \mathbb{R}$  by  $f \mapsto \mathcal{E}_x(f) \equiv xf_x/f$ .

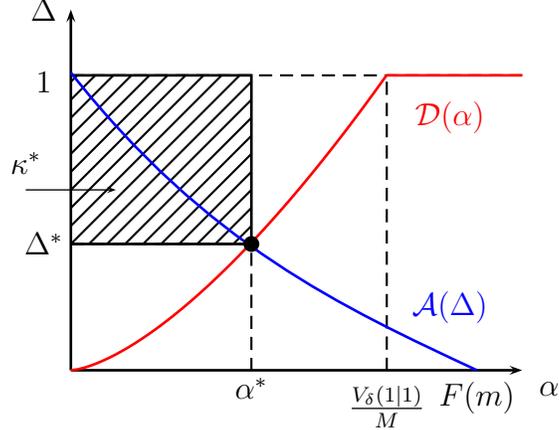


Figure 1: **Existence and Uniqueness of a Crime Equilibrium.** Deterrence demand slopes upward, rising from  $(0, 0)$  to  $(V_\delta(1|1)/M, 1)$ , and attempted crime supply slopes down from  $(0, 1)$  to  $(F(m), 0)$ . The equilibrium crime rate  $\kappa^*$  is the dashed area  $(1 - \Delta^*)\alpha^*$  of undeterred attempted crimes. We observe that  $\alpha^*$  is strictly below the attempted crime ceiling  $V_\delta(1|1)/M$ .

rate  $f/F$ , since  $\mathcal{E}_\Delta(\mathcal{A}) = -\mathcal{E}_c(F)\Delta/(1 - \Delta)$ .

In our model, deterrence plays the role of a demand quantity, and the attempted crime rate  $\alpha$  as a price. But conversely, deterrence acts as supply price, and the attempted crime rate  $\alpha$  as a supply quantity. Thus while a more elastic supply curve in Figure 1 is more horizontal per usual, a more elastic demand curve in Figure 1 is more vertical!

The falling supply and rising derived demand for attempted crime intersect at a unique deterrence rate  $\Delta^* \in (0, 1)$ , and this equilibrates the crime market. Figure 1 depicts this equilibrium, in which the crime rate  $\kappa^*$  is the shaded area  $(1 - \Delta^*)\alpha^*$ .

**Theorem 1.** *There exists a unique crime equilibrium  $(\delta^*(\cdot), \bar{c}^*)$ .*

## 4 Comparative Statics

We now use the supply and demand framework to perform comparative statics.

### 4.1 Changing Supply, and the Criminal Laffer Curve

We first explore changes that affect *just* the criminal supply side of the market. For instance, the criminal gain  $m$  may rise, if the demand for stolen good increases. Or the cost distribution  $F$  may fall: For instance, the expected legal penalties of crime may fall due to more policing or harsher punishment. Or, perhaps a new after-school basketball program raises the value of the outside options.

To state our theorem precisely, we continuously index the cost distribution  $F(c|\varphi)$  so that costs are higher with *greater* values of  $\varphi \in [0, 1]$ . Suppose that  $F(m|1) = 0$  so that attempting a crime is never profitable when  $\varphi = 1$ , even if deterrence is zero. On the other end, assume that  $F(0|0)$  is high enough, and specifically,  $F(0|0) \geq V_\delta(1|1)/M$  so that in equilibrium crimes are attempted even if deterrence is perfect.<sup>21</sup> This captures a world, say, where many criminal outside options are negative, due to terrible outside options for the criminal.

**Proposition 1 (Supply Shifts).** *If potential criminals' costs fall ( $\varphi$  falls), or the criminal gain  $m$  increases, then the vigilance  $v^*$ , deterrence rate  $\Delta^*$ , and attempted crime rate  $\alpha^*$  all increase. The crime rate is a hump-shaped function of  $m$  and  $\varphi$ , vanishing when  $m = 0$  or  $\varphi = 1$  and also as  $m \uparrow \infty$  or if  $\varphi = 0$ . If marginal vigilance costs  $V_\delta$  are more elastic, then the deterrence  $\Delta^*$  increases less, and  $\alpha^*$  increases more.*

The last claim reflects how demand is less elastic with elastic marginal vigilance costs  $V_\delta$ , and so the demand quantity increases less whenever supply increases, and demand price increases more.

Due to equilibrium feedback effects, some new predictions emerge. For instance, Becker (1968) found that the level of crime falls in the anticipated expected punishment and foregone legal income. He argued that: “a rise in the income available in legal activities would reduce the incentives to enter illegal activities and thus would reduce the level of offenses” (p. 177). But in our equilibrium setting, that is only true for the attempted crime rate. Meanwhile, potential victims respond to variations in  $\alpha$ , and raising deterrence as  $\alpha$  rises. For some intuition, let us consider two extreme cases. In the Police State, massive legal penalties hold the attempted crime rate  $\alpha^*$  very low, and so decreases in legal penalties lead to proportionately large increases in  $\alpha^*$ , and thus the crime  $\kappa^* = (1 - \Delta^*)\alpha^*$  rises (moving from left to right in panel (b)). At the opposite extreme, in the Wild West, policing is very slight, and crime is held at bay by deterrence — the quick draw. In this case, the nondeterrence rate  $1 - \Delta^*$  is extremely low, and so increases in legal penalties lead to proportionately large increases  $1 - \Delta^*$ , and thus the crime rate  $\kappa^* = (1 - \Delta^*)\alpha^*$  rises (moving from right to left in panel (b)). In general, with log-concave marginal vigilance costs, the equilibrium crime rate is a hump-shaped function of the expected punishment, resembling what Ayres (2016) calls a *Laffer curve for crime*.<sup>22</sup>

*Proof of Proposition 1:* Assume that the criminal cost index  $\varphi$  falls. In Figure 2, as  $\varphi$

<sup>21</sup>We are essentially allowing  $F(\cdot|\varphi)$  to have a mass point at  $c = 0$  for indices  $\varphi$  close to zero.

<sup>22</sup>Building on our equilibrium logic, he provides more discussion about this Laffer curve for crime and a connection to privacy precaution in internet markets.

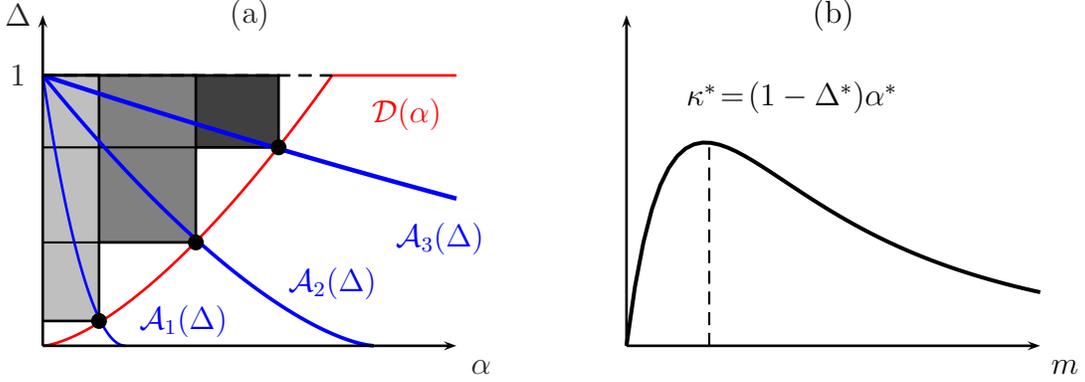


Figure 2: **Proposition 1: Deterrence and Attempted Crime Rates.** When criminal gains  $m$  rise, supply increases, and the deterrence rate  $\Delta^*$  and attempted crime rate  $\alpha^*$  rise (panel (a)). But the crime rate  $\kappa^*$  is non-monotone (panel (b)): For log-concave marginal vigilance costs  $V'$ , it is hump-shaped and so highest for intermediate costs or gains. The crime rate  $\kappa^*$  is also hump-shaped as a function of criminal costs  $\varphi$ . This plot is similar, but with  $1 - \varphi$  on the horizontal axes, and so the crime rate vanishes at  $\varphi = 1$ .

risers through  $\varphi_1 < \varphi_2 < \varphi_3$ , the supply of attempted crime locus (6) shifts right from  $\mathcal{A}_1$  to  $\mathcal{A}_2$  to  $\mathcal{A}_3$ , while the demand curve  $\mathcal{D}$  holds constant. Thus, the attempted crime rate  $\alpha^*$  and the deterrence rate  $\Delta^*$  unambiguously rise: More crimes are attempted, but each succeeds less often, as potential victims grow more vigilant.

The effect on the crime rate  $\kappa^* = (1 - \Delta^*)\alpha^*$  is unclear, a priori. Let us consider a specific potential victim  $\xi$  with individual demand  $\bar{\delta}(\alpha|\xi)$ . Recall from consumer theory that along a demand curve, total revenue increases in quantity (falls in price) in price iff the demand is elastic. In our environment, think of  $\bar{\kappa}m = (1 - \bar{\delta}(\alpha|\xi))\alpha m$  as the total criminal revenue, and  $(1 - \bar{\delta}(\alpha|\xi))m$  as the price. Then we conclude that the individual crime rate  $\bar{\kappa}$  falls in  $\alpha$  along the demand curve if  $(1 - \bar{\delta}(\alpha|\xi))\alpha m$  rises in  $(1 - \bar{\delta}(\alpha|\xi))m$ . This reduces to<sup>23</sup>

$$\mathcal{E}_{(1-\bar{\delta}(\alpha|\xi))m}((1 - \bar{\delta}(\alpha|\xi))\alpha m) = 1 + \mathcal{E}_{(1-\bar{\delta}(\alpha|\xi))m}(\alpha) = 1 - (1 - \bar{\delta}(\alpha|\xi))/(\alpha\bar{\delta}_\alpha(\alpha|\xi)) > 0. \quad (7)$$

We argue that the left side of (7) is monotone increasing in  $\alpha$ , and respectively negative and positive for low and high values of  $\alpha$ . For the monotonicity,  $(1 - \bar{\delta}(\alpha|\xi))$  is decreasing in  $\alpha$ , while we claim that  $\alpha\bar{\delta}_\alpha(\alpha|\xi)$  is increasing in  $\alpha$ . Indeed, recall that  $\alpha\bar{\delta}_\alpha(\alpha|\xi) = V_\delta(\bar{\delta}|\xi)/V_{\delta\delta}(\bar{\delta}|\xi)$ , by (5). Also, since for each  $\xi$  marginal costs  $V_\delta$  are log-concave,  $V_\delta(\bar{\delta}|\xi)/V_{\delta\delta}(\bar{\delta}|\xi)$  is monotone in  $\bar{\delta}$ , and so in  $\alpha$ .

Altogether, the left side of (7) is monotone in  $\alpha$ , positive at  $\alpha = V_\delta(1|\xi)/M$  (since  $\bar{\delta}(\alpha|\xi) = 1$ ), and negative as  $\alpha \downarrow 0$ , since  $\bar{\delta}(0|\xi) = 0$  and  $\lim_{\delta \downarrow 0} V_\delta/V_{\delta\delta} < 1$  by assumption,

<sup>23</sup>Recall that the elasticity of a product is the sum of the elasticities.

so that  $\lim_{\alpha \downarrow 0} \alpha \bar{\delta}_\alpha(\alpha|\xi) < 1$  by (5). In other words, the left side of (7) is positive for  $\alpha > \hat{\alpha}$  and negative for  $\alpha < \hat{\alpha}$ , for some  $\hat{\alpha} \in (0, V_\delta(1|\xi)/M)$ . The individual crime rate is thus a hump-shaped function of the attempted crime rate  $\alpha$ , peaking at  $\hat{\alpha}$ .

We have shown that each individual crime rate  $\alpha(1 - \bar{\delta}(\alpha|\xi))$  is hump-shaped. In the appendix, we show that the aggregate crime rate  $\kappa = \int_0^1 \alpha(1 - \bar{\delta}(\alpha|\xi))dG(\xi)$  is hump-shaped as well. This is subtle because, e.g., the sum of two hump-shaped functions can have two humps.

To finalize the proof, consider the extremes of  $\varphi$ . We have assumed that when  $\varphi = 1$ , all criminals have costs  $c > m$ . Thus, there are no attempted crimes, and so  $\alpha^* = 0$ , and thus  $\kappa^* = 0$ . Moreover,  $F(0|0) \geq V_\delta(1|1)/M$  and thus the mass of attempted crimes is at least the attempted crime ceiling, whereupon everyone chooses perfect deterrence, and so  $\Delta^* = 1$ . All told, the crime rate  $\kappa^*$  vanishes at  $\varphi = 0, 1$ , and since  $\alpha^*$  increases in  $\varphi$ , the crime rate  $\kappa^*$  is hump-shaped in the cost index  $\varphi$ .

Similar logic established a hump-shape as the criminal gain  $m$  rises, but the crime rate  $\kappa^*$  only tends to zero as  $m \uparrow \infty$ .  $\square$

That the crime rate  $\kappa$  eventually falls in criminal costs is intuitive and consistent with some findings in the empirical literature that study the relation between unemployment and property crime (Raphael and Winter-Ebmer, 2001; Levitt, 2004; Chalfin and McCrary, 2017). For higher unemployment is tantamount to lower costs of crime.<sup>24</sup> Yet our theory suggests that when unemployment is high enough, the endogenous response of potential victims could lead to lower crime rates. We next pursue a test for whether this laffer curve is downward sloping.

Recalling (7), the crime rate falls along the demand curve when demand is elastic, if we think of  $(1 - \mathcal{D}(\alpha))m$  as the price. Equivalently,  $(1 - \mathcal{D}(\alpha))/(\alpha \mathcal{D}'(\alpha)) < 1$ . Since  $\Delta = \mathcal{D}(\alpha)$ , we can rewrite this condition as  $\kappa M / (\alpha^2 M \mathcal{D}'(\alpha)) < 1$ . Call  $\bar{v}$  the *total vigilance expenses* and differentiate  $\mathcal{D}(\alpha)$  in (4) to get:

$$\begin{aligned} \alpha^2 M \mathcal{D}'(\alpha) &= \alpha M \int_0^1 \frac{V_\delta(\bar{\delta}|\xi)}{V_{\delta\delta}(\bar{\delta}|\xi)} dG(\xi) \geq \int_0^1 V_\delta(\bar{\delta}|\xi) \frac{V_\delta(\bar{\delta}|\xi)}{V_{\delta\delta}(\bar{\delta}|\xi)} dG(\xi) \\ &= \int_0^1 V(\bar{\delta}|\xi) \frac{V_\delta(\bar{\delta}|\xi)/V(\bar{\delta}|\xi)}{V_{\delta\delta}(\bar{\delta}|\xi)/V_\delta(\bar{\delta}|\xi)} dG(\xi) \geq \int_0^1 V(\bar{\delta}|\xi) dG(\xi) = \bar{v}, \end{aligned}$$

where we log-differentiated the FOC (3) to get the first equality; used that optimal vigilance demands  $\alpha M \geq V_\delta(\bar{\delta}(\alpha|\xi)|\xi)$  for all  $\xi$  to get the first inequality; and finally used that  $(V_\delta/V)/(V_{\delta\delta}/V_\delta) \geq 1$  as is well-known that if  $\log V_\delta$  is concave in  $\delta$ , so is  $\log V$ .<sup>25</sup>

Using this into the above inelasticity inequality, we see that:

<sup>24</sup>For as Aristotle wrote, “Poverty is the parent of crime”.

<sup>25</sup>This is a consequence of Prékopa’s Theorem (Prékopa, 1973).

**Proposition 2.** *The crime rate decreases in the criminal gains  $m$ , and increases in the criminal costs  $\varphi$ , if average property losses  $\kappa M$  are less than vigilance expenses  $\bar{v}$ .*

Proposition 2 provides a useful test based on observables. Then, *if expected property losses are less than vigilance costs, then the crime rate falls along the demand curve.* For a concrete application, consider Lojack.<sup>26</sup> According to Ayres and Levitt (1998), the property loss of vehicles with Lojack was roughly \$1000. They estimate a theft rate in Lojack cities of 0.025, and a \$97 annuity equivalent of the \$600 Lojack installation fee. This exceeds the expected property loss of \$25, and so  $\kappa M < V$ .

In general, the falling portion of our Laffer curve is contrary to the message of Becker (1968) that crime globally falls in punishment. This difference reflects the importance of our modeling market response.

## 4.2 Changing Demand

We now study exogenous changes that affect directly the demand side or the incentives of potential victims. For example, a greater property loss  $M$  increases the individual deterrence demand  $\bar{\delta}$ , but now via a greater marginal efficacy of vigilance (as  $\mathcal{L}_{\Delta M} < 0$ ). As seen in left panel of Figure 3, the deterrence demand shifts up, while the supply curve is unaffected. The crime rate thus unambiguously falls because fewer potential criminals enter, and each attempt is deterred with a higher chance.

Another reason why demand might shift is a vigilance technology change. New deterrence technologies affect only the demand side of the market. For instance, van Ours and Vollaard (2015) find a large reduction of car theft in the Netherlands after the EU mandated that all new cars have electronic engine immobilizers.<sup>27,28</sup> In the same spirit, these authors also found in 2011 that burglary greatly fell in the Netherlands after a mandate for burglary-proof windows and doors in all home constructions. We capture such regulations by a “price floor”, since they mandate a higher deterrence level than people optimally desire. This naturally reduces crime rates, as seen in the right panel of Figure 3.

**Proposition 3.** *If deterrence demand increases — for instance, if property losses  $M$  rise, or marginal vigilance costs  $V_\delta$  fall — then the deterrence rate rises, and the attempted and*

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<sup>26</sup>The LoJack Stolen Vehicle Recovery System is an aftermarket vehicle tracking system hidden inside the vehicle, that allows vehicles to be tracked by police, with the aim of recovering them in case of theft.

<sup>27</sup>Farrell et al. (2011) find that the dramatic drop in auto theft over the last 20 years in the US, Britain and Australia is negatively correlated with an increase in anti-theft devices for cars.

<sup>28</sup>Using the FBI Uniform Crime Report Data 2012, the auto theft rate has crashed 47% since 2003 in the U.S., since newer cars have theft-immobilizer devices or part markings. By contrast, larceny and burglary are respectively down 10% and 19% in this same time span.

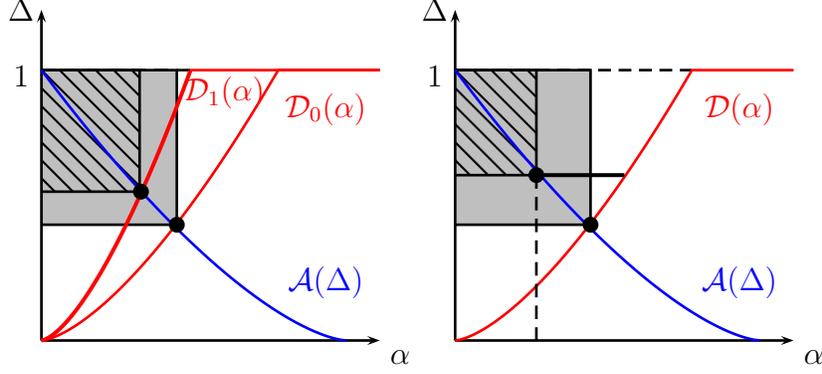


Figure 3: **Property Losses, and Mandated Engine Immobilizers.** LEFT: When property losses rise from  $M_0$  to  $M_1$ , the deterrence demand shifts left from  $\mathcal{D}_0$  to  $\mathcal{D}_1$ , while attempted crime supply  $\mathcal{A}$  is unaffected; thus, the attempted crime rate falls and deterrence rises. RIGHT: More mandated vigilance is similar to the analysis of a price floor, and reduces attempted and actual crimes.

*actual crime rates fall.*

### 4.3 Changing Supply and Demand

Now we explore what happens when the stakes of the game rise. That is, what happens when the property value rise for potential victims and also for criminals? For example, more luxury cars are more valuable for the owner and the auto thief. Likewise, in more expensive neighborhoods there are more valuable objects to steal and lose. In general, moving from low to high stakes affects both the demand and supply curves.

Parametrize property values  $(m, M) = ((1 - \mu)t, t)$  for  $t > 0$ . We say that the stakes of the game increase if  $t > 0$  rises.

**Proposition 4.** *The attempted crime rate and crime rate are both hump-shaped as the stakes of the game increase. The deterrence rate and vigilance expenses increase in  $t$ .*

Our theory provides an explanation for why luxury vehicles are typically the least likely to be stolen (reference).<sup>29</sup> Or why rich neighborhoods usually exhibit low crime rates (reference). Furthermore, the theory suggests that the levels of crime and attempted crimes are the highest in neither too rich nor poor neighborhoods, and that auto thieves are after cars in the mid-range, which seems to be the case (reference).

*Proof of Proposition 4:* By Theorems 1–3, when  $(m, M)$  rise along a constant markdown, supply  $\mathcal{A}$  shifts up and right, while demand  $\mathcal{D}$  shifts up and left. As seen in the left and

<sup>29</sup>While cars vary by their ease of theft, it is reasonable to presume that auto makers install the anti-theft devices that the consumer would have desired.

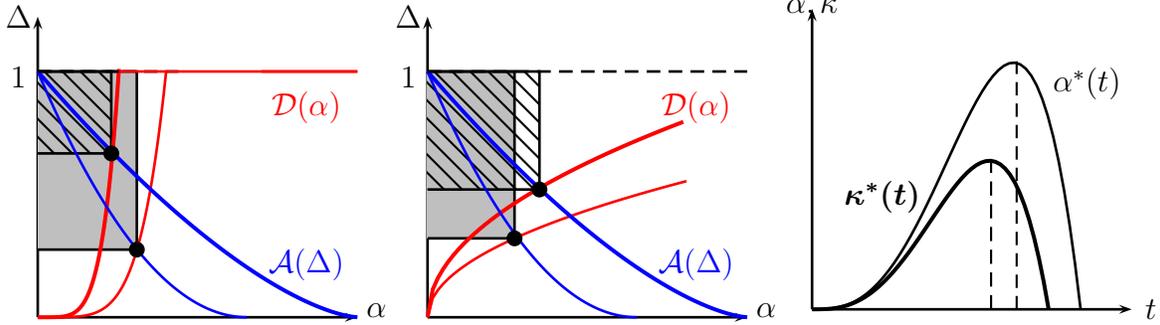


Figure 4: **Theorem 4: Greater Criminal Gain and Property Losses Raise Deterrence.** All panels assume iso-elastic and convex vigilance costs  $V(\Delta) = \Delta^\gamma, \gamma > 1$ . Fix the markdown  $\mu \in (0, 1)$  and assume that  $(M, m) = (t, (1 - \mu)t)$  rise in  $t$ . **LEFT:** When vigilance costs are not too convex (i.e.,  $\gamma \in (1, 2)$ ), demand shifts up more than supply, and the attempted crime rate falls. **MIDDLE:** When vigilance cost are sufficiently convex (i.e.,  $\gamma > 2$ ), supply shifts up more than demand does, resulting in more attempted crimes. **RIGHT:** This panel fixes the convexity of  $V$  and shows that in general the attempted crime rate and crime rate vanish for low and high stakes  $t$ , suggesting that the highest levels of attempted and actual crimes occur for intermediate stake levels.

middle panels of Figure 4, the deterrence rate unambiguously rises. However, *the attempted crime rate rises when supply  $\mathcal{A}$  shifts up more than demand  $\mathcal{D}$* , which is the case iff (7) holds, by Claim A.4. As seen in the middle panel of Figure 4, when vigilance costs are strictly convex, inequality (7) holds (fails) for low (high) deterrence, and so for low (high) enough stakes, since the equilibrium deterrence rises with greater stakes. Finally, the crime rate. Obviously, when (7) fails, the crime rate falls, since there fewer attempts (Claim A.4) and each is deterred with greater chance (see Figure 4, left panel); however, as Claim A.5 shows, the crime rate rises when the stakes are low enough.  $\square$

## 5 The Social Costs of Crime

The thrust of Becker was a treatise on the social costs of crime, focusing on the optimal public expenditures against crime. But private vigilance expenses is also an important source of social costs that have received much less attention among economists.

We now explore the welfare loss of crime, owing to criminals opportunity costs, vigilance costs, and the transfer loss of theft, namely,  $\kappa\mu M$ , where  $\mu = 1 - m/M$  is the criminal markdown. We ignore the unmodeled policing expenditures. Since the respective marginal criminal and attempted crime rate obey  $\bar{c} = (1 - \Delta)m$  and  $\alpha = F(\bar{c})$ , total criminals costs

are the sum of their opportunity costs  $\int_0^{\bar{c}} cf(c)dc$ , or:

$$\int_0^{\bar{c}} cf(c)dc = (1 - \Delta)mF(\bar{c}) - \int_0^{\bar{c}} F(c)dc = \alpha(1 - \Delta)m - \int_0^{\bar{c}} F(c)dc.$$

Next, the total vigilance costs of crime are  $\int_0^1 V(\bar{\delta}(\alpha|\xi)|\xi)dG(\xi)$ . Thus, given the crime rate  $\kappa = (1 - \Delta)\alpha$ , the *social costs of crime* are:

$$\kappa\mu M + \kappa(1 - \mu)M - \int_0^{\bar{c}} F(c)dc + \int_0^1 V(\bar{\delta}(\alpha|\xi)|\xi)dG(\xi). \quad (8)$$

As seen in Figure 5, the first term is the transfer loss of theft. The next two terms are gross criminal costs. For if all criminals shared a common opportunity costs, then the third term vanishes, while the first term is obviously gross criminal gains, and therefore balances criminal losses, by free entry. Criminal heterogeneity is accounted for by the third term. For a criminal  $c \in [0, \bar{c})$  makes profits  $\bar{c} - c$ , and thus total profits across all criminals amount to  $\bar{c}F(\bar{c}) - \int_0^{\bar{c}} cf(c)dc = \int_0^{\bar{c}} F(c)dc$ . So the third term in (8) subtracts aggregate criminal profits owing to heterogeneity. So the social costs of crime (8) can be reinterpreted as potential victims' expected losses (summing the first two and last terms) less criminal profits. If potential criminals are homogeneous, the social cost of crime corresponds to potential victims' losses.

What happens to the social costs when legal penalties increase? Because of the criminal Laffer curve, the answer is not obvious, for greater penalties reduce vigilance expenses but may raise expected property losses  $\kappa M$ , by Theorem 1. However, with homogeneous criminals, if penalties are high enough, then social costs unambiguously fall with legal penalties. But, with heterogeneous criminals, it is unclear how social costs vary in legal penalties — for given the competitive nature of crime, policies that help potential victims tend to hurt criminals, and so the sign of the net effect is ambiguous.

Now let us suggestively depict the social costs in our supply and demand framework. In any crime equilibrium  $(\Delta^*, \bar{c}^*, \alpha^*, \kappa^*)$ , we argue that *criminal profits*  $\int_0^{\bar{c}^*} F(c)dc$  are captured by the area above equilibrium deterrence  $\Delta^*$  and below the value-scaled supply curve  $\mathcal{A}^S(\Delta)m$  in Figure 5. Indeed, consider the marginal criminal  $\bar{c}(\Delta) \equiv (1 - \Delta)m$ . Therefore,

$$\int_0^{\bar{c}^*} F(c)dc = \int_1^{\Delta^*} F(\bar{c}(\Delta))\bar{c}_\Delta(\Delta)d\Delta = \int_{\Delta^*}^1 \mathcal{A}(\Delta)m d\Delta,$$

for  $\mathcal{A}(\Delta) \equiv F(\bar{c}(\Delta))$  by (6), and  $\bar{c}_\Delta = -m$ .

Next, we depict vigilance expenses. For simplicity, let us focus on a crime equilibrium in

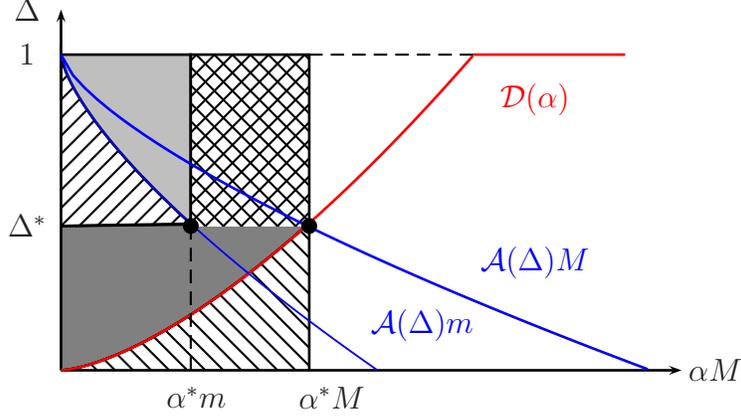


Figure 5: **Per Capita Social Costs of Crime.** The areas are measured in dollars after we scale the units on the horizontal axis by  $M$ . The crosshatched region represents the transfer loss  $\kappa\mu M$ . The NW-dashed region is criminal profits, the light-shaded region is criminal costs, and the dark-shaded region vigilance costs  $V(\Delta^*)$ . The dashed area *below demand*  $\mathcal{D}(\cdot)$  is the potential victims' surplus, i.e., gains  $\alpha^*\Delta^*M$  less total vigilance costs.

which all types  $\xi$  optimally choose imperfect deterrence, i.e.,  $\bar{\delta}(\alpha^*|\xi) < 1$ . Using (3):

$$\begin{aligned} v^* &= \int_0^1 V(\bar{\delta}(\alpha^*|\xi)|\xi)dG(\xi) = \int_0^1 \int_0^{\alpha^*} V_\delta(\bar{\delta}(\alpha|\xi)|\xi)\bar{\delta}_\alpha(\alpha|\xi)d\alpha dG(\xi) \\ &= \int_0^1 \int_0^{\alpha^*} \alpha M \bar{\delta}_\alpha(\alpha|\xi)d\alpha dG(\xi) \end{aligned}$$

where we used that  $\bar{\delta}(0|\xi) = 0 = V_\delta(0|\xi)$  in the second equality, and the FOC (3) in the third one. Now integrate by parts to get:  $\int_0^{\alpha^*} \alpha \bar{\delta}_\alpha(\alpha|\xi)d\alpha = \alpha^* \bar{\delta}(\alpha^*|\xi) - \int_0^{\alpha^*} \bar{\delta}(\alpha|\xi)d\alpha$ . Therefore,

$$\begin{aligned} v^*/M &= \int_0^1 \int_0^{\alpha^*} \alpha \bar{\delta}_\alpha(\alpha|\xi)d\alpha dG(\xi) = \int_0^1 \alpha^* \bar{\delta}(\alpha^*|\xi)dG(\xi) - \int_0^1 \int_0^{\alpha^*} \bar{\delta}(\alpha|\xi)d\alpha dG(\xi) \\ &= \int_0^1 \alpha^* \bar{\delta}(\alpha^*|\xi)dG(\xi) - \int_0^{\alpha^*} \int_0^1 \bar{\delta}(\alpha|\xi)dG(\xi)d\alpha = \alpha^* \Delta^* - \int_0^{\alpha^*} \mathcal{D}(\alpha)d\alpha \\ &= \int_0^{\alpha^*} [\Delta^* - \mathcal{D}(\alpha)]d\alpha = \int_0^{\alpha^* M} [\Delta^* - \mathcal{D}(\tilde{\alpha}/M)]d\tilde{\alpha}/M \end{aligned}$$

where we have changed variables so that  $\tilde{\alpha} = \alpha M$ . Altogether,  $v^* = \int_0^{\alpha^* M} [\Delta^* - \mathcal{D}(\alpha)]d(\alpha M)$ , namely, *vigilance expenditures  $v^*$  equals the area below equilibrium deterrence  $\Delta^*$  and above the demand  $\mathcal{D}(\alpha)$ .*<sup>30</sup>

Since the demand curve  $\mathcal{D}$  depicts potential victims' reservation price at any quantity, the area over it and below  $\Delta = \Delta^*$  captures a cost, and their surplus is thus the triangular

<sup>30</sup>If in equilibrium some types  $\xi$  choose perfect deterrence, then this area would be an upper bound for the actual vigilance costs.

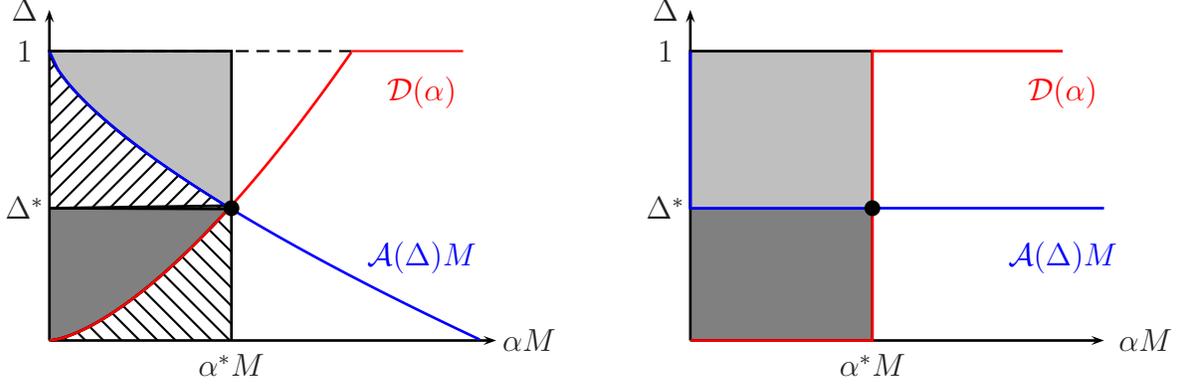


Figure 6: **Gains Dissipate with Linear Vigilance and Homogeneous Criminals and Victims.** Both panels assume no markdown  $\mu = 0$  (i.e.,  $m = M$ ). At left, we depict the Tullock paradox. The light shaded region is criminal costs and the dark-shaded area is the vigilance costs, as in Figure 5. This is less than the left rectangle, namely, the value  $\alpha^*M$  of potential gains of attempted crime. The difference is the NW-dashed area — profits owing to criminal heterogeneity — and the SE-dashed area below the rising demand curve. The right panel considers the extreme case with homogeneous criminals with opportunity cost  $\bar{c}^*$  — and so a supply  $\mathcal{A}(\Delta) = \infty$  for  $\Delta \leq \Delta^*$ , and 0 otherwise — and linear vigilance  $V(\Delta) = \Delta V(1)$  — and so a vertical demand. Here, social costs equal potential gains.

area *below* his demand. On the other hand, since crime is an economic bad for victims but an economic good for criminals, criminals’ profits are the area below supply and above  $\Delta = \Delta^*$ .

For another lesson, recall that Becker (1968) argues that in a competitive theft market, the total criminal costs should approximate the market value of the property loss (Becker, 1968, page 171, footnote 3). While he ignores vigilance costs, Tullock (1967) had already crystalized a more general insight that rent-seeking behavior by all agents involved might well dissipate the gains — at least absent a transfer loss of theft ( $m = M$ ). On the other hand, his “Tullock paradox” (Tullock, 1980) later observed that rent-seeking expenditures are often swamped by the potential gains. This paradox occurs here, and we see in Figure 6 that it owes precisely to the strict convexity of the vigilance costs, and to criminal and potential victim heterogeneity. The presence of police may be another reason for the Tullock Paradox. But since we are not modeling police behavior, it is not clear how to think of this case.

## 6 Optimal Levels of Crime and Vigilance

In this section, we explore the optimal levels of crime and vigilance. We consider two criteria: (i) Potential victim’s expected losses minimization, and (ii) social costs minimization.

## 6.1 Potential Victim Losses Minimization: Neighborhood Watch

If individuals start a Watch program, then this is tantamount to a coordination of vigilance efforts to minimize average total losses from crime. In this case, we can think of a social planner (e.g., the president of the neighborhood association) who is in charge of minimizing potential victims' expected losses. On one hand, the planner would like to minimize the deterrence rate in order to minimize vigilance costs. On the other hand, he would like to raise the deterrence rate to discourage criminal activity. In general, the planner anticipates how changes in deterrence affect the attempted crime rate, contrary to potential victims who take the attempted crime rate as given. We assume that the planner chooses  $\Delta^e \in [0, 1]$ , the socially optimal or *efficient deterrence rate*.

Before characterizing  $\Delta^e$ , let us define the *inverse deterrence demand function*  $\mathcal{D}^{-1} \equiv \mathcal{A}^D: [0, 1] \rightarrow [0, \bar{\alpha}]$  where  $\mathcal{D}(\mathcal{A}^D(\Delta)) = \Delta$ , and  $\bar{\alpha} = V_\delta(1|1)/M$  is the attempted crime rate ceiling (defined in §3).<sup>31</sup> So  $\mathcal{A}^D(\Delta)$  is the attempted crime rate that induces a deterrence demand  $\Delta$ , and individual demands  $\delta^D(\Delta|\xi)$  solving  $\mathcal{A}^D(\Delta)M = V_\delta(\delta^D(\Delta|\xi)|\xi)$  for each  $\xi \in [0, 1]$ .<sup>32</sup> It is easy to check that  $\delta^D(\cdot|\xi)$  is monotone and  $\delta^D(0|\xi) = 0$  for all  $\xi$ .

The planner's problem is to find  $\Delta^e$  that solves:

$$\min_{\Delta \in [0,1]} \mathcal{A}(\Delta)(1 - \Delta)M + \int_0^1 V(\delta^D(\Delta|\xi)|\xi)dG(\xi).$$

Notice that the efficient deterrence  $\Delta^e$  must be positive, since raising deterrence from zero is costless at the margin (since  $V_\delta(0|\xi) = 0$  for all  $\xi$ ). Also,  $\Delta^e < 1$ , since a marginal drop of deterrence from  $\Delta = 1$  has no marginal impact on the crime rate  $\mathcal{A}(\Delta)(1 - \Delta)$ . Thus, the efficient deterrence is positive and imperfect, i.e.,  $\Delta^e \in (0, 1)$ , and thus it must solve the following first-order condition:

$$\mathcal{A}(\Delta^e)M - \mathcal{A}'(\Delta^e)(1 - \Delta^e)M = \int_0^1 V_\delta(\delta^D(\Delta^e|\xi)|\xi)\delta_\Delta^D(\Delta^e|\xi)dG(\xi) = \mathcal{A}^D(\Delta^e)M,$$

where we used the definition of  $\delta^D$  and that  $\int_0^1 \delta_\Delta^D(\Delta|\xi)dG(\xi) = 1$ , as  $\int_0^1 \delta^D(\Delta|\xi)dG(\xi) = \Delta$ . Altogether, the above first-order condition turns to:

$$\mathcal{A}(\Delta^e) (1 + \mathcal{E}_{1-\Delta}(\mathcal{A})) = \mathcal{A}^D(\Delta^e). \tag{9}$$

In other words, the efficient deterrence is given by the intersection between the deterrence demand curve and the marginal crime rate  $[\mathcal{A}(\Delta)(1 - \Delta)]_\Delta$ . As seen in Figure 7, the resulting

<sup>31</sup>This inverse function is well-defined, since  $\mathcal{D}(\cdot)$  is strictly increasing for all attempted crime rates  $\alpha \leq \bar{\alpha}$ .

<sup>32</sup>For simplicity, we assume that the FOC (3) holds for all potential victim types  $\xi$ .

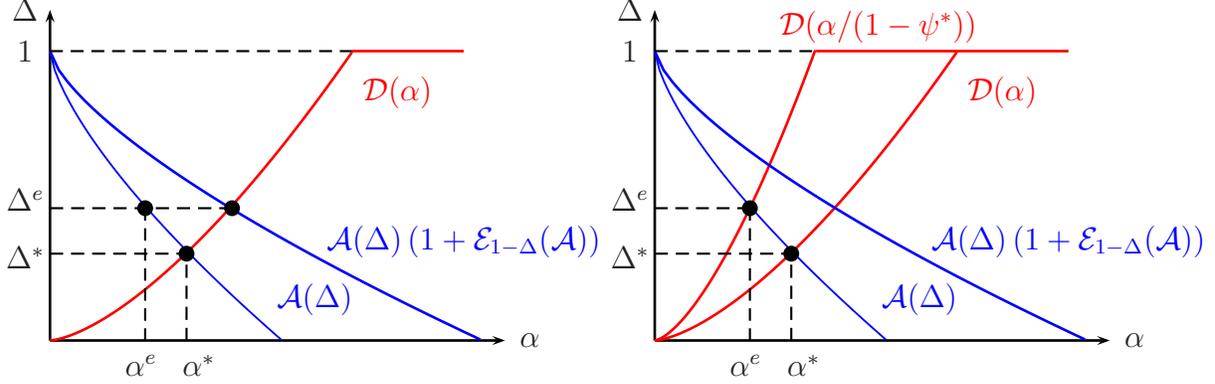


Figure 7: **The Efficient Deterrence and Attempted Crime Rates.** LEFT: The efficient deterrence level is given by the intersection of the inverse demand  $\mathcal{A}^D$  and marginal crime rate  $\mathcal{A}(1 + \mathcal{E}_{1-\Delta}(\mathcal{A}))$ . The marginal crime rate lies above the attempted crime supply  $\mathcal{A}$ , and so the respective efficient deterrence and attempted crime rate are greater and lower than the decentralized ones. RIGHT: A Pigovian subsidy  $\psi^*$  on vigilance costs will implement the efficient level of deterrence and crime.

attempted crime rate  $\alpha^e$  is given by the supply curve  $\alpha^e \equiv \mathcal{A}(\Delta^e)$ . Notice the analogy with monopoly theory where a monopolist chooses how much to produce by equalizing marginal costs (supply) and marginal revenues, whereas the final price is fixed by the demand curve.

The divergence between the efficient attempted crime rate and the equilibrium one depends critically on the elasticity of the supply curve. Indeed, rearranging (9) we find the percentage deviation of supply and demand obeys:

$$\frac{\mathcal{A}^D(\Delta^e) - \mathcal{A}(\Delta^e)}{\mathcal{A}(\Delta^e)} = \mathcal{E}_{1-\Delta}(\mathcal{A}). \quad (10)$$

Since  $\mathcal{E}_{1-\Delta}(\mathcal{A}) > 0$ , the efficient deterrence will exceed the decentralized one. This difference is greater the more elastic the attempted supply is, or equivalently, the more heterogeneous potential criminals are. Indeed, when criminals are homogeneous, the supply of crime is perfectly elastic in deterrence, and so  $\Delta^e = \Delta^*$  and  $\alpha^e = \alpha^*$ .

**Proposition 5.** *Consider a decentralized crime equilibrium  $(\Delta^*, \alpha^*, \kappa^*)$ . The solution to the planner's problem  $(\Delta^e, \alpha^e, \kappa^e)$  entails more deterrence  $\Delta^e > \Delta^*$ , and fewer attempted crimes  $\alpha^e < \alpha^*$  and successful crimes  $\kappa^e < \kappa^*$ .*

Figure 7 illustrates the proof of Theorem 5. First, since the elasticity  $\mathcal{E}_{1-\Delta}(\mathcal{A}^S) > 0$ , the marginal supply curve is above the supply curve for all deterrence  $\Delta < 1$ . Thus, the efficient deterrence rate  $\Delta^e$  must be greater than the equilibrium one  $\Delta^*$ . Second, the marginal criminal  $\bar{c}^e = (1 - \Delta^e)m < \bar{c}^* = (1 - \Delta^*)m$ . Next, since the supply  $\mathcal{A}^S(\Delta)$  falls in deterrence, we have  $\alpha^e = \mathcal{A}^S(\Delta^e) < \mathcal{A}^S(\Delta^*) = \alpha^*$ . Finally, since the efficient solution

dictates a lower attempted crime rates and higher deterrence rate, the number of successful attempted crimes  $\kappa^e = \alpha^e(1 - \Delta^e)$  is less than  $\kappa^* = \alpha^*(1 - \Delta^*)$ .

The identified market failure is due to *lack of coordination* of potential victims. Since each of them is atomless and behaves independently of others, each believes that their vigilance behavior will have no impact on the rate at which they encounter criminals. However, as is well-known, the planner's solution is not an equilibrium, for if the attempted crime rate is  $\alpha^e$ , potential victims would demand deterrence  $\mathcal{D}(\alpha^e) < \Delta^e$ . So how can the planner implement the efficient solution? One way is via a Pigouvian subsidy.<sup>33</sup>

Suppose that the government can subsidy vigilance expenses by paying a fraction  $\psi \in [0, 1]$  of vigilance expenses so that vigilance costs fall from  $V(\delta|\xi)$  to  $(1 - \psi)V(\delta|\xi)$ . Potential victims expected losses then turn to  $\alpha(1 - \delta)M + (1 - \psi)V(\delta|\xi)$ , and the optimal deterrence  $\bar{\delta}(\alpha/(1 - \psi)|\xi)$  obeys the first-order condition:  $(1 - \psi)V_\delta(\bar{\delta}|\xi) = \alpha M$ . The deterrence demand turns to  $\mathcal{D}(\alpha/(1 - \psi)) = \int_0^1 \bar{\delta}(\alpha/(1 - \psi)|\xi)dG(\xi)$ , while the attempted supply of crime is unchanged. As seen in the right panel of Figure 7, the planner can implement the efficient solution with a *Pigovian subsidy*  $\psi^*$  obeying<sup>34</sup>

$$\mathcal{D}(\alpha^e/(1 - \psi^*)) = \Delta^e.$$

Altogether, we see that in order to implement the efficient levels of crime and deterrence, the government need not to rely on greater legal punishments, but rather it should design policies that directly help potential victims, and indirectly hurt criminals.

## 6.2 Social Costs Minimization

Now suppose that the social planner chooses deterrence  $\Delta^s \in [0, 1]$  to minimize social costs, namely, total potential victim losses minus criminal profits. The attempted crime rate is then given by  $\alpha^s = \mathcal{A}(\Delta^s)$ . As in the previous section, we work with the inverse demand function  $\mathcal{A}^D(\Delta)$  and the induced individual demand curves  $\delta^D(\Delta|\xi)$ , obeying  $\int_0^1 \delta^D(\Delta|\xi)dG(\xi) = \Delta$ . From equation (8) in §5, the social costs of crime as a function of deterrence  $\Delta$  are:

$$\mathcal{A}(\Delta)(1 - \Delta)M + \int_0^1 V(\delta^D(\Delta|\xi)|\xi)dG(\xi) - \int_0^{(1-\Delta)m} F(c)dk.$$

<sup>33</sup>Another way could be by investing in more policing; however, this seems less obvious as policing also affects the incentives of potential criminals, and thereby the marginal crime rate curve.

<sup>34</sup>This subsidy  $\psi^*$  exists, since  $\mathcal{D}(\alpha^e/(1 - \psi)) < \Delta^e$  for  $\psi = 0$ , and  $\mathcal{D}(\alpha^e/(1 - \psi)) \rightarrow 1 > \Delta^e$  as  $\psi \rightarrow 1$ .

By analogous reasons, the optimal deterrence must be interior  $\Delta^s \in (0, 1)$ , and so it must solve the first-order condition:

$$\begin{aligned} \mathcal{A}(\Delta^s)M - \mathcal{A}'(\Delta^s)(1 - \Delta^s)M &= \int_0^1 V_\delta(\delta^D(\Delta^s|\xi)|\xi)\delta_\Delta^D(\Delta^s|\xi)dG(\xi) + F[(1 - \Delta)m]m \\ &= \mathcal{A}^D(\Delta^s)M + \mathcal{A}(\Delta^s)m. \end{aligned}$$

But since  $m = (1 - \mu)M$ , where  $\mu$  is the criminal markdown, the FOC can be rewritten as:

$$\mu\mathcal{A}(\Delta^s) - \mathcal{A}'(\Delta^s)(1 - \Delta^s) = \mathcal{A}^D(\Delta^s). \quad (11)$$

Comparing the FOCs (9) and (11), we see that the left side of (11) is less than  $\mathcal{A}(\Delta^s) - \mathcal{A}'(\Delta^s)(1 - \Delta^s)$  (since  $\mu < 1$ ), and thus *the planner mandates less deterrence and thereby more attempted crimes compared to a neighborhood watch*, namely,  $\Delta^s < \Delta^e$  and  $\alpha^s > \alpha^e$ . The reason is that now the marginal cost of deterrence accounts not only for vigilance expenses but also criminal profits. Hence, raising deterrence is marginally more costly than in a neighborhood watch. But could  $\Delta^s$  be lower than its decentralized counterpart  $\Delta^*$ ?

By standard supply and demand logic, the excess of supply  $\mathcal{A}(\Delta) - \mathcal{A}^D(\Delta)$  vanishes when  $\Delta = \Delta^*$ , and so the social cost minimizer deterrence  $\Delta^s > \Delta^*$  iff  $\Delta^s$  induces excess of supply, i.e.,  $\mathcal{A}(\Delta^s) > \mathcal{A}^D(\Delta^s)$ . Looking at the left side of (11), we see that

$$\mu\mathcal{A}(\Delta^s) - \mathcal{A}'(\Delta^s)(1 - \Delta^s) > \mathcal{A}(\Delta^s) \iff \frac{-\mathcal{A}'(\Delta^s)(1 - \Delta^s)}{\mathcal{A}(\Delta^s)} = \frac{\bar{c}^s f(\bar{c}^s)}{F(\bar{c}^s)} > 1 - \mu,$$

where  $\bar{c}^s \equiv (1 - \Delta^s)m$ . In other words, the optimal deterrence  $\Delta^s > \Delta^*$  when the supply elasticity  $\mathcal{E}_{1-\Delta}(\mathcal{A})$  is greater than  $1 - \mu$ . For example, this the case when the cost mass distribution  $F$  is convex, for then  $cf(c)/F(c) \geq 1 > 1 - \mu$  for all  $c > 0$ . However, when the distribution  $F$  is “too concave,” then the planner would dictate less deterrence and more attempted crimes than their decentralized counterparts. In fact, if  $F(c) = c^\varrho$  with  $\varrho \leq 1 - \mu$ , then  $\Delta^s < \Delta^*$  and  $\alpha^s > \alpha^*$ . As seen in Figure 8, criminal profits become relatively less important to social planner when the distribution  $F$  is convex.

## 7 Vigilance Spillover Effects: Lojack

Ayres and Levitt (1998) find that cars equipped with Lojack enjoy a recovery rate of around 90%. We can interpret this in our model as a deterrence rate of 90%. But more subtly, they found that even though no more than 2% of cars had Lojack, the overall auto theft rate per capita (in Lojack cities) fell 17% after four years of its introduction. Identifying these

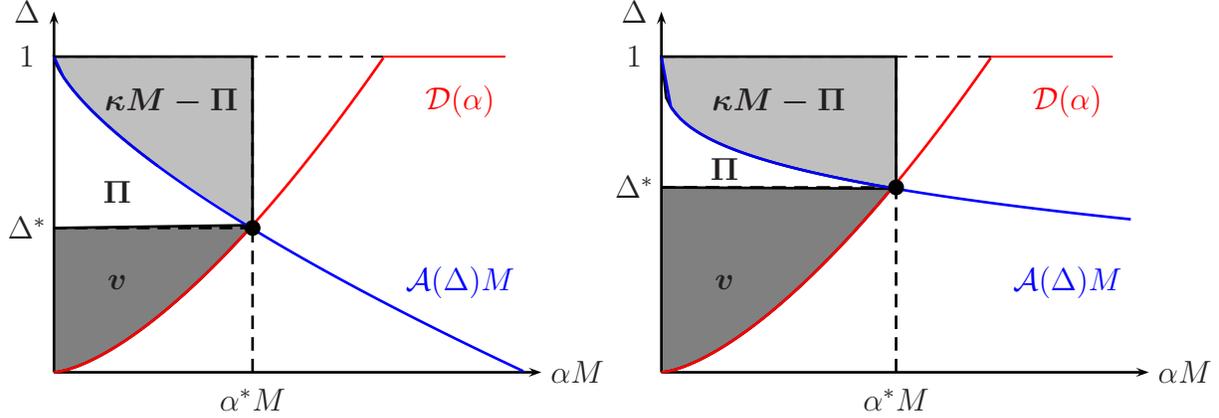


Figure 8: **Social Cost Minimization.** Both panels assume no markdown  $m = M$ . When the mass distribution is more convex, the supply curve is more elastic, reducing the criminal profits. Thus, expected losses minus criminal profits  $\kappa M - \Pi$  become more relevant in the optimization, inducing the social planner to choose a level of deterrence higher than  $\Delta^*$ .

positive spillovers was the clever finding.<sup>35</sup> Our framework is flexible and can be readily adapted to this case.

Suppose potential victims can buy Lojack at a fixed price  $p_L \in (V(1|0), V(1|1))$ . The bounds on  $p_L$  ensures that respective low cost and high cost potential victims would not and would find profitable to adopt Lojack to secure perfect deterrence.

Since Lojack is unobservable to criminals, it ideally fits our theory. For all potential victims face the same attempted crime rate  $\alpha$ , independent of their adoption choices. for simplicity, let us assume that Lojack secures perfect deterrence. This simplifies the logic, for now the only losses with Lojack are the costs  $p_L$ . So given  $\alpha$ , a potential victim  $\xi$  buys Lojack iff their minimized expected losses (1) (without Lojack) obey  $\bar{\mathcal{L}}(\alpha|\xi) \equiv \min_{\Delta} \mathcal{L}(\Delta, \alpha|\xi) \geq p_L$ . The higher cost potential victims  $\xi$  enjoy the highest minimized expected losses  $\bar{\mathcal{L}}$ , and so have the highest marginal gains from buying Lojack. Specifically, for an intermediate Lojack price  $p_L$ , some but not all purchase Lojack, and all potential victims  $\xi \geq \underline{\xi}(\alpha, p_L)$  do so, where  $\underline{\xi}(\alpha, p_L)$  is the indifferent type.<sup>36</sup> Clearly, when the attempted crime rate  $\alpha$  vanishes,  $\bar{\mathcal{L}}(\alpha|\xi) = 0$ , no one buys Lojack. By continuity, no one does for small enough  $\alpha < \underline{\alpha}(p_L)$ , where  $\underline{\xi}(\underline{\alpha}(p_L), p_L) = 1$ , and  $\underline{\xi} = \underline{\xi}(\alpha, p_L) \in (0, 1)$  for all  $\alpha > \underline{\alpha}(p_L)$ .<sup>37</sup>

<sup>35</sup>Ayres and Levitt (1998) estimate an external benefit of Lojack users exceeding \$1300 annually. By the same logic, people with home security systems confer a positive externality on those with just the stickers for them. Without stating so, Ayres and Levitt (1998) clearly had in mind a strategic model of crime. They assumed that criminals cared about average deterrence and accordingly adjusted their attempted crime rate. They argue: “An individual car owner’s decision to install Lojack only trivially affects the likelihood of his or her own vehicle being stolen since thieves base their decisions on mean Lojack installation rates” (p. 45).

<sup>36</sup>Specifically, if  $\bar{\mathcal{L}}(\alpha|0) \leq p_L \leq \bar{\mathcal{L}}(\alpha|1)$ , then  $\bar{\mathcal{L}}(\alpha|\underline{\xi}(\alpha, p_L)) \equiv p_L$  for some threshold index  $\underline{\xi}(\alpha, p_L) \in [0, 1]$ .

<sup>37</sup>Notice that even if the attempted crime rate is greater than its ceiling, only some potential victims get

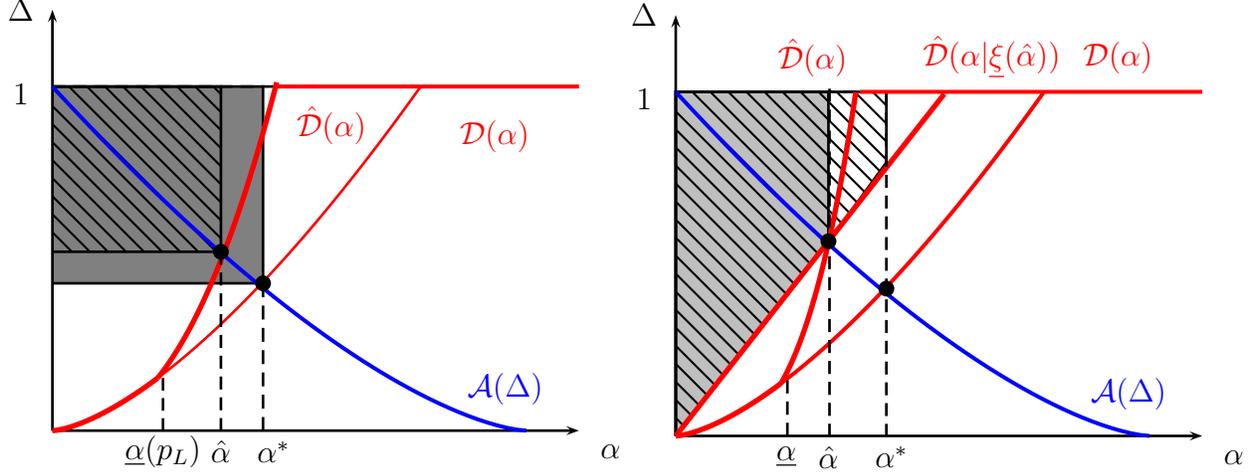


Figure 9: **Lojack.** When Lojack is available, high cost potential victims  $\xi$  start buying at the attempted crime rate  $\underline{\alpha}$ , and increasingly so as  $\alpha$  rises; therefore, the new deterrence demand  $\hat{\mathcal{D}}$  increasingly diverges from  $\mathcal{D}$  for  $\alpha \geq \underline{\alpha}$  at left. This reduces the crime rate as well as the attempted crime rate (from dark shaded to slashed dark shaded), by Proposition 3.

The right panel focuses on the nonadopters of Lojack (i.e., types  $\xi \leq \underline{\xi}$ ), for whom the attempted crime rate falls  $\hat{\alpha}$  to  $\alpha^*$ , and total losses fall from the shaded to the slashed shaded region. The shaded uncrossed difference of these areas is the external benefit of Lojack.

So for any attempted crime rate  $\alpha$ , since potential victims  $\xi \geq \underline{\xi}(\alpha, p_L) \in (0, 1)$  adopt Lojack, we can write the *deterrence demand when Lojack is available*  $\hat{\mathcal{D}}(\alpha)$  as:

$$\hat{\mathcal{D}}(\alpha) = G(\underline{\xi})\mathcal{D}_{NL}(\alpha|\underline{\xi}) + [1 - G(\underline{\xi})] \quad (12)$$

where  $\mathcal{D}_{NL}(\alpha|\underline{\xi}) = \int_0^{\underline{\xi}} \bar{\delta}(\alpha|\xi) dG(\xi)/G(\underline{\xi})$ , and  $\underline{\xi} = \underline{\xi}(\alpha, p_L)$ . Clearly, the deterrence demand when Lojack is available is at least the deterrence demand (4), that is,  $\hat{\mathcal{D}}(\alpha) \geq \mathcal{D}(\alpha)$  with strict inequality when the attempted crime rate  $\alpha > \underline{\alpha}$ . As seen in the left panel of Figure 9, *the new equilibrium entails more deterrence and fewer attempted and actual crimes.*

Now we measure the external benefit of Lojack. To this end, fix its price  $p_L$ . Let  $\alpha^*$  and  $\hat{\alpha}$  be the attempted crime rates respectively before and after Lojack was available (so  $\alpha^*, \hat{\alpha} > \underline{\alpha}$ ). Next, for a fixed type  $\xi$ , the *conditional deterrence demand* is  $\hat{\mathcal{D}}(\alpha|\xi) \equiv G(\xi)\mathcal{D}_{NL}(\alpha|\xi) + [1 - G(\xi)]$ , where  $G(\xi)$  is the mass of non-adopters, whom we assume choose imperfect deterrence.<sup>38,39</sup> As seen in the right panel of Figure 9, the deterrence demand  $\hat{\mathcal{D}}(\alpha) \geq$

Lojack (i.e.,  $\underline{\xi}(\alpha, p_L) > 0$ ), since  $\bar{\mathcal{L}}(\alpha|0) = V(1|0) < p_L < \bar{\mathcal{L}}(\alpha|1) = V(1|1)$ .

<sup>38</sup>In other words,  $\bar{\delta}(\alpha^*|\xi) < 1$ . This case arises when  $\bar{\delta}(\hat{\alpha}|\xi) \leq \bar{\delta}(\hat{\alpha}|0) < 1$  for all  $\xi \leq \underline{\xi}(\hat{\alpha})$ , where we suppress the dependence of  $\underline{\xi}$  in  $p_L$  to reduce notation.

<sup>39</sup>In general, there may be some non adopters that choose perfect deterrence when Lojack is available, i.e., when the attempted crime rate  $\alpha = \hat{\alpha}$ . In this case, there is no external benefit for them, as Lojack does not trigger a change in their behavior. This case emerges when  $\bar{\delta}(\hat{\alpha}|0) = 1$ , for then there exists a type

$\hat{\mathcal{D}}(\alpha|\underline{\xi}(\hat{\alpha}))$  iff  $\alpha \geq \hat{\alpha}$ , since more potential victims buy Lojack when  $\alpha \geq \hat{\alpha}$ .

The *external benefit* created by Lojack adopters  $\xi > \underline{\xi}(\hat{\alpha})$  is the total reduction in minimized losses of nonadopters  $\xi \leq \underline{\xi}(\hat{\alpha})$ . This external benefit is positive since  $\alpha^* > \hat{\alpha}$ , and equals

$$\int_0^{\underline{\xi}(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^*} \bar{\mathcal{L}}_{\alpha}(\alpha|\xi) d\alpha dG(\xi) = \int_0^{\underline{\xi}(\hat{\alpha})} \int_{\hat{\alpha}}^{\alpha^*} [1 - \bar{\delta}(\alpha|\xi)] M d\alpha dG(\xi) = \int_{\hat{\alpha}}^{\alpha^*} [1 - \hat{\mathcal{D}}(\alpha|\underline{\xi}(\hat{\alpha}))] M d\alpha,$$

by applying the Envelope Theorem to (1), and substituting (12). This area can be depicted in our supply and demand framework; as seen in the right panel of Figure 9, the *external benefit corresponds to the area of the non deterrent rate*  $[1 - \hat{\mathcal{D}}(\alpha|\underline{\xi}(\hat{\alpha}))]$  *between*  $\hat{\alpha}$  *and*  $\alpha^*$ .

For an intuition of why this area captures the external benefit, recall from §5 that vigilance expenses coincide with the area above the demand curve and below the equilibrium deterrence. One can see that, conditional on  $\xi \leq \underline{\xi}$ , average vigilance expenses obey

$$\int_0^{\hat{\alpha}} \left[ \hat{\mathcal{D}}(\hat{\alpha}|\underline{\xi}(\hat{\alpha})) - \hat{\mathcal{D}}(\alpha|\underline{\xi}(\hat{\alpha})) \right] d\alpha.$$

Thus, the average losses of those individuals is the sum of this term and  $[1 - \mathcal{D}_{NL}(\hat{\alpha}|\underline{\xi})]\hat{\alpha}$ , which is the crime rate faced by those individuals. This corresponds to the grey area in Figure 9. By the same logic, the average losses of non-adopters when the attempted crime rate is  $\alpha^*$  is given by the dashed area. The external benefit is the subtraction of these areas.

To finalize this section, we ask what if the cost of Lojack  $p_L$  falls? The answer is simple, since for any attempted crime rate, more potential victims would adopt Lojack — i.e., the marginal type  $\underline{\xi}$  falls. Thus, the deterrence demand  $\hat{\mathcal{D}}(\alpha|\underline{\xi})$  in (12) would increase, and the attempted crime rate and crime rate would decrease.

## 8 Extensions (in progress)

## 9 Concluding Remarks

Becker's insight was the maximizing criminal. Ehrlich (1981) added that potential victims likewise respond to incentives. So inspired, we have devised a simple model in which: (1) pairwise random matching of criminals and potential victims produces attempted crimes; (2) not all crimes succeed; and (3) deterrence is probabilistic, rising in vigilance. We have

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$\xi_0 \in (0, \underline{\xi}(\hat{\alpha}))$  such that all types  $\xi \leq \xi_0$  choose  $\bar{\delta}(\hat{\alpha}|\xi) = 1$ , enjoying no external benefit. Thus, the critical set of non adopters that benefits from adopters is  $[\xi_0, \underline{\xi}(\hat{\alpha})]$ . Our analysis is valid then by integrating on this domain.

precisely formulated the payoff functions of criminals and victims, and how their actions impact one another. Our framework proved to be tractable and rich, amenable to perform comparative statics and explore diverse extensions. By allowing potential victims to optimally respond, we account for the parry and thrust of crime, for victims and criminals both respond when change befalls either party, or when law enforcement adjusts. Understanding these forces help sharpen policy intervention by focusing on crowding out effects. One approach to crime prevention is increased legal penalties, but one could also help individuals to deter crime. Our analysis highlights differences between these approaches and shows the importance of keeping more careful track of other data variables such as the attempted crime rate. Given the scarcity of micro data, understanding the problem conceptually becomes even more important and informative to policy makers. We hope our research inspires new ideas and clarifies situations and concepts tantamount to crime.

## A Omitted Proofs

### A.1 Existence and Uniqueness: Proof of Theorem 1

First, since the demand  $\mathcal{D}(\alpha)$  is strictly increasing in  $\alpha$  for  $\alpha \leq V_\delta(1, 1)/M$ , we can define the inverse demand map  $\mathcal{A}^D(\Delta) \in [0, V_\delta(1, 1)/M]$ , obeying  $\mathcal{D}(\mathcal{A}^D(\Delta)) \equiv \Delta$ . Next, define the *excess of supply*  $\mathcal{ES}(\Delta) \equiv \mathcal{A}(\Delta) - \mathcal{A}^D(\Delta)$ . Clearly, when deterrence is perfect,  $\mathcal{A}(1) = 0 < \mathcal{A}^D(1) = V_\delta(1, 1)/M$ . Also, since the demand  $\mathcal{D}(0) = 0$ , the inverse obeys  $\mathcal{A}^D(0) = 0 < \mathcal{A}(0) = F(m)$ . Altogether, the excess of supply obeys:  $\mathcal{ES}(1) < 0 < \mathcal{ES}(0)$ . Thus, by the Intermediate Value Theorem (IVT), there exists  $\Delta^* \in (0, 1)$  such that the excess of supply vanishes  $\mathcal{ES}(\Delta^*) = 0$ , or  $\mathcal{A}(\Delta^*) = \mathcal{A}^D(\Delta^*)$ . Finally, since supply falls and demand rises in deterrence,  $\Delta^*$  is unique because  $\mathcal{ES}(\Delta)$  strictly falls in  $\Delta$ .  $\square$

### A.2 Greater Punishment: Proof of Proposition 1 Finished

Define the *individual crime rate function*  $\bar{\kappa}(\alpha, \xi) \equiv \alpha(1 - \bar{\delta}(\alpha, \xi))$  for victim  $\xi$ , given  $\alpha$ . Notice that  $\bar{\kappa}(0, \xi) = 0$ , and also  $\bar{\kappa}(\alpha, \xi) = 0$  for all  $\alpha \geq \bar{\alpha}_\xi$ , where  $\bar{\alpha}_\xi \equiv V_\delta(1, \xi)/M$  is the *attempted crime rate ceiling for victim*  $\xi$ . Also, the *critical attempted crime rate*  $\hat{\alpha}_\xi \in (0, \bar{\alpha}_\xi)$  is the one that maximizes the crime rate  $\bar{\kappa}(\alpha, \xi)$ , for all  $\xi$ .

**Claim A.1.** *For all  $\xi$ , the individual crime rate function  $\bar{\kappa}(\alpha, \xi)$  is quasi-concave in  $\alpha$ .*

*Proof:* Twice differentiate the FOC (3) in  $\alpha$  to get  $V_{\delta\delta}\bar{\delta}_\alpha = M$  and then  $V_{\delta\delta\delta}(\bar{\delta}_\alpha)^2 + V_{\delta\delta}\bar{\delta}_{\alpha\alpha} = 0$ .

$$V_{\delta\delta}\bar{\delta}_\alpha = V_\delta/\alpha \quad \text{and} \quad \bar{\delta}_{\alpha\alpha} = -V_{\delta\delta\delta}\bar{\delta}_\alpha^2/V_{\delta\delta} \quad (13)$$

substituting from the FOC (3). Now, when  $\alpha < \bar{\alpha}_\xi$ ,

$$\bar{\kappa}_{\alpha\alpha} = -2\bar{\delta}_\alpha - \alpha\bar{\delta}_{\alpha\alpha} \leq 0 \iff \frac{V_{\delta\delta\delta}\alpha\bar{\delta}_\alpha}{V_{\delta\delta}} \leq 2 \iff \frac{V_{\delta\delta\delta}}{V_{\delta\delta}} \frac{V_\delta}{V_{\delta\delta}} \leq 2.$$

By log-concavity of  $V_\delta$ , the last inequality holds even with 1 on the right side. Thus,  $\bar{\kappa}$  is quasi-concave in  $\alpha$ , because it is concave in  $\alpha < \bar{\alpha}_\xi$  and then zero for  $\alpha \geq \bar{\alpha}_\xi$ .  $\square$

**Claim A.2.** *Then the critical attempted crime rate  $\hat{\alpha}_\xi$  increases in the index  $\xi$ .*

*Proof:* By definition  $\hat{\alpha}_\xi = \arg \max_\alpha \bar{\kappa}(\alpha, \xi)$ . By Topkis' Theorem,<sup>40</sup>  $\hat{\alpha}_\xi$  increases in  $\xi$  if  $\bar{\kappa}_{\alpha\xi}(\alpha, \xi) > 0$ . We argue that this is positive, namely, that  $\bar{\kappa}(\alpha, \xi) \equiv \alpha(1 - \bar{\delta}(\alpha, \xi))$  has a positive cross partial  $\bar{\kappa}_{\alpha\xi} = -\bar{\delta}_\xi - \alpha\bar{\delta}_{\alpha\xi}$ .

Now log-differentiate the  $\alpha$  derivative  $M = V_{\delta\delta}\bar{\delta}_\alpha$  of the FOC (3) in  $\xi$  to get

$$0 = \frac{V_{\delta\delta\delta}\bar{\delta}_\xi}{V_{\delta\delta}} + \frac{V_{\delta\delta\xi}}{V_{\delta\delta}} + \frac{\bar{\delta}_{\alpha\xi}}{\bar{\delta}_\alpha} \implies \frac{-\bar{\delta}_{\alpha\xi}}{\bar{\delta}_\alpha} = -\frac{V_{\delta\delta\delta}}{V_{\delta\delta}} \cdot \frac{V_{\delta\xi}}{V_{\delta\delta}} + \frac{V_{\delta\delta\xi}}{V_{\delta\delta}} = \frac{V_{\delta\xi}}{V_{\delta\delta}} \left( \frac{V_{\delta\delta\xi}}{V_{\delta\xi}} - \frac{V_{\delta\delta\delta}}{V_{\delta\delta}} \right),$$

where we used  $\bar{\delta}_\xi = -V_{\delta\xi}/V_{\delta\delta}$ , found by differentiating (3). Since  $\alpha\bar{\delta}_\alpha = V_\delta/V_{\delta\delta} > 0$  by (13), we conclude that  $\bar{\kappa}_{\alpha\xi} > 0$  iff

$$\frac{-\bar{\delta}_{\alpha\xi}}{\bar{\delta}_\alpha} > \frac{\bar{\delta}_\xi}{\alpha\bar{\delta}_\alpha} \iff \frac{V_{\delta\xi}}{V_{\delta\delta}} \left( \frac{V_{\delta\delta\xi}}{V_{\delta\xi}} - \frac{V_{\delta\delta\delta}}{V_{\delta\delta}} \right) > \frac{-V_{\delta\xi}}{V_{\delta\delta}} \frac{V_{\delta\delta}}{V_\delta} \iff \frac{V_{\delta\delta\xi}}{V_{\delta\xi}} > \frac{V_{\delta\delta\delta}}{V_{\delta\delta}} - \frac{V_{\delta\delta}}{V_\delta}$$

where last right side is nonpositive (as  $V_\delta$  is log-concave in  $\delta$ ) and  $V_{\delta\delta\xi}, V_{\delta\xi} > 0$ .  $\square$

We need to show that the crime rate function  $\tilde{\kappa}(\alpha) \equiv \int_0^1 \bar{\kappa}(\alpha, \xi) dG(\xi)$  is quasi-concave in  $\alpha$ . Choi and Smith (2017) [CS17] provide a condition that ensures that the weighted sum of quasi-concave functions is quasi-concave.

To this end, decompose the individual crime rate function  $\bar{\kappa} = \bar{\kappa}^I + \bar{\kappa}^D$  into its increasing and decreasing portions,  $\bar{\kappa}^I$  and  $\bar{\kappa}^D$ , respectively, where  $\bar{\kappa}^I$  and  $-\bar{\kappa}^D$  are monotone, and  $\bar{\kappa}^I$  (resp.  $-\bar{\kappa}^D$ ) is constant right (resp. left) of the peak (argmax) of  $\bar{\kappa}$ . When these functions are differentiable, for any  $\xi, \xi'$ , we say that  $-\bar{\kappa}^D(\cdot, \xi)$  grows proportionally faster than  $\bar{\kappa}^I(\cdot, \xi')$ , if  $\bar{\kappa}^I(\cdot, \xi')$  is more risk averse than  $-\bar{\kappa}^D(\cdot, \xi)$ , namely:

$$-\bar{\kappa}_{\alpha\alpha}^I(\alpha, \xi')/\bar{\kappa}_\alpha^I(\alpha, \xi') \geq -\bar{\kappa}_{\alpha\alpha}^D(\alpha, \xi)/\bar{\kappa}_\alpha^D(\alpha, \xi). \quad (14)$$

More generally, for any  $\alpha_3 \geq \alpha_2 \geq \alpha_1$ :

$$[\bar{\kappa}^I(\alpha_2, \xi') - \bar{\kappa}^I(\alpha_1, \xi')][\bar{\kappa}^D(\alpha_2, \xi) - \bar{\kappa}^D(\alpha_3, \xi)] \geq [\bar{\kappa}^I(\alpha_3, \xi') - \bar{\kappa}^I(\alpha_2, \xi')][\bar{\kappa}^D(\alpha_1, \xi) - \bar{\kappa}^D(\alpha_2, \xi)] \quad (15)$$

<sup>40</sup>See Topkis (1998).

By Proposition 1 in CS17, the crime rate  $\tilde{\kappa}$  is quasi-concave in  $\alpha$  iff  $-\bar{\kappa}^D(\cdot, \xi)$  grows proportionally faster than  $\bar{\kappa}^I(\cdot, \xi')$ , for all  $\xi', \xi$ .

**Claim A.3.**  $-\bar{\kappa}^D(\cdot, \xi)$  grows proportionally faster than  $\bar{\kappa}^I(\cdot, \xi')$ , for all  $\xi, \xi'$ .

*Proof:* Whenever  $\bar{\kappa}^D(\cdot, \xi)$  or  $\bar{\kappa}^I(\cdot, \xi')$  is constant, (15) holds, since both sides vanish.

Assume first  $\xi' \leq \xi$ . Then the claim holds — for if  $\alpha < \hat{\alpha}_\xi$ , then  $-\bar{\kappa}^D(\alpha, \xi)$  is constant, and if  $\alpha \geq \hat{\alpha}_\xi$ , then  $\bar{\kappa}^I(\alpha, \xi')$  is constant (recalling that  $\alpha_\xi \geq \hat{\alpha}_{\xi'}$ , by Claim A.2).

The case  $\xi' > \xi$  is trickier, as both  $\bar{\kappa}^I(\alpha, \xi')$  and  $-\bar{\kappa}^D(\alpha, \xi')$  are increasing on an interval, namely, for  $\alpha \in [\hat{\alpha}_\xi, \min\{\bar{\alpha}_\xi, \hat{\alpha}_{\xi'}\}]$ . Here,  $-\bar{\kappa}^D(\cdot, \xi)$  and  $\bar{\kappa}^I(\cdot, \xi')$  are differentiable, and so we can use criterion (14). But on this interval,  $\bar{\kappa}^I(\alpha, \xi) = \bar{\kappa}(\alpha, \xi)$  is increasing and concave, whereas  $\bar{\kappa}^D(\alpha, \xi') = \bar{\kappa}(\alpha, \xi')$  is increasing and concave, since  $\bar{\kappa}(\cdot, \xi)$  and  $\bar{\kappa}(\cdot, \xi')$  are concave, by Claim A.1. As a result, so  $\bar{\kappa}^I(\cdot, \xi')$  is more risk averse than  $-\bar{\kappa}^D(\cdot, \xi)$ :

$$-\bar{\kappa}_{\alpha\alpha}^I(\alpha, \xi')/\bar{\kappa}_\alpha^I(\alpha, \xi') > 0 > -\bar{\kappa}_{\alpha\alpha}^D(\alpha, \xi)/\bar{\kappa}_\alpha^D(\alpha, \xi). \quad \square$$

### A.3 Greater Gains and Losses: Proof of Proposition 4 Finished

Fix the markdown  $\mu = 1 - m/M \in (0, 1)$ , and parameterize  $M = t$  and  $m = (1 - \mu)t, t > 0$ .

**Claim A.4.** *When the stakes  $t$  rise, the attempted crime rate rises when supply  $\mathcal{A}^S$  shifts up more than demand  $\mathcal{A}^D$ , which is the case if and only if (7) holds.*

*Proof:* First, by Theorems 1–3, when  $t$  rises, supply  $\mathcal{A}^S$  shifts up and right, while demand  $\mathcal{A}^D$  shifts up and left. Next, fix  $\alpha > 0$  and log-differentiate supply  $\mathcal{A}^S(\Delta^S(t)|t) \equiv F[(1 - \Delta^S(t))(1 - \mu)t] \equiv \alpha$  and demand  $\mathcal{A}^D(\Delta^D(t)|t) \equiv V'(\Delta^D(t))/t \equiv \alpha$  in  $t > 0$  to get:

$$\frac{d\Delta^S}{dt} = \frac{1 - \Delta}{t} \quad \text{and} \quad \frac{d\Delta^D}{dt} = \frac{V' 1}{V'' t}.$$

Comparing the above expressions yields  $d\Delta^S/dt > d\Delta^D/dt$  iff  $1 - \Delta > V'/V''$ , or (7).  $\square$

**Claim A.5.** *If the stakes  $t$  rise, the crime rate  $\kappa$  rises when stakes  $t$  are low enough.*

*Proof:* Assume that the stakes  $t$  are low enough so that (7) holds. By Claim A.4, the attempted crime rate  $\alpha$  rise, i.e.,  $\alpha_t > 0$ . Next, log-differentiate  $\mathcal{A}^S(\Delta^S(t)|t) \equiv \alpha$  and  $\mathcal{A}^D(\Delta^D(t)|t) \equiv \alpha$  in  $t > 0$  and arrange terms to get:

$$\begin{bmatrix} (1 - \Delta)\frac{V''}{V'} & -1 \\ -\frac{f}{F}(1 - \Delta)(1 - \mu)t & -1 \end{bmatrix} \begin{bmatrix} \frac{\Delta_t}{1 - \Delta} \\ \frac{\alpha_t}{\alpha} \end{bmatrix} = \begin{bmatrix} \frac{1}{t} \\ -\frac{f}{F}(1 - \Delta)(1 - \mu) \end{bmatrix}.$$

Solve for  $\Delta_t/(1 - \Delta)$  and  $\alpha_t/\alpha$  using Cramer's Rule:

$$\frac{\Delta_t}{1 - \Delta} = \frac{\frac{1}{t} + \frac{f}{F}(1 - \Delta)(1 - \mu)}{(1 - \Delta) \left( \frac{V''}{V'} + \frac{f}{F}(1 - \mu)t \right)} \quad \text{and} \quad \frac{\alpha_t}{\alpha} = \frac{\frac{f}{F}(1 - \Delta)(1 - \mu) \left( (1 - \Delta) \frac{V''}{V'} - 1 \right)}{(1 - \Delta) \left( \frac{V''}{V'} + \frac{f}{F}(1 - \mu)t \right)}.$$

Now, log-differentiate the crime rate  $\kappa(t) = (1 - \Delta(t))\alpha(t)$  in  $t$  to see that  $\kappa(t)$  rises iff  $\Delta_t/(1 - \Delta) < \alpha_t/\alpha$ . Using the above expressions and doing some algebra, the crime rate rises iff

$$\frac{f}{F}(1 - \Delta)(1 - \mu)t \left( (1 - \Delta) \frac{V''}{V'} - 2 \right) > 1. \quad (16)$$

We will show that the above inequality (16) holds for small enough  $t$ . Suppose that  $t \rightarrow 0$ . First, the equilibrium deterrence vanish  $\Delta(t) \rightarrow 0$  as  $t \rightarrow 0$ , for the demand and supply loci tend to the origin. Second, since the distribution  $F$  is strictly increasing and satisfies  $F(0) = 0$ , we have that the elasticity  $kf(c)/F(c) \rightarrow 1$  as  $c \rightarrow 0$ , and thus:

$$\frac{f[(1 - \Delta(t))(1 - \mu)t]}{F[(1 - \Delta(t))(1 - \mu)t]}(1 - \Delta(t))(1 - \mu)t \rightarrow 1 \quad \text{as} \quad t \rightarrow 0.$$

Third, since  $V'' > 0 = V'(0)$  for  $\Delta > 0$ , we have  $\Delta V''(\Delta)/V'(\Delta) \rightarrow 0$  as  $\Delta \rightarrow 0$ , and so the marginal cost elasticity  $\Delta(t)V''(\Delta(t))/V'(\Delta(t)) \rightarrow 1$  as  $t \rightarrow 0$ . This implies that  $(1 - \Delta(t))V''(\Delta(t))/V'(\Delta(t)) \uparrow \infty$  as  $t \rightarrow 0$ . Altogether, the left side of inequality (16) explodes as  $t$  vanish, and so the crime rate rises as  $t \rightarrow 0$ .  $\square$

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