

Sticky Wages, Private Consumption, and Fiscal Multipliers*

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Abstract

This paper demonstrates how adding nominal wage rigidity to a standard sticky price model can *by itself* create a mechanism by which increases in government spending cause increases in consumption. The increase in output arising from government purchases puts upward pressure on the price level. At a fixed short-run nominal wage, this bids down the real wage, which leads producers to increase labor demand. Increased labor demand allows households to both finance the tax bill associated with the government spending as well as increase their own consumption. Our approach does not rely upon existing ingredients for generating large fiscal multipliers, such as the zero lower bound, borrowing constrained households or an interaction between consumption and government purchases in the utility function.

Keywords: government spending, multipliers, sticky wages.

JEL Codes: E62, H50.

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1 Introduction

While there is a substantial body of empirical research on the size of fiscal policy multipliers, there has been relatively less theoretical work on the issue.¹ This paper presents the theoretical underpinnings of a mechanism that relies only on sticky wages and prices, which can lead to fiscal policy output multipliers that are greater than one.²

As background, we observe that two workhorse modern macroeconomic models, the neo-classical growth model and the basic New Keynesian model, as typically formulated both imply a negative consumption multiplier. Since government spending is treated as a dead-weight resource loss, the taxes required to finance the spending reduce households' after-tax wealth. The negative wealth effect leads households to reduce their consumption. A rise in government spending combined with a decline in consumption implies an output multiplier that is less than one.³

From a modeler's perspective, one way to overcome the negative wealth effect is for a government spending increase to induce a change in a relative price that encourages consumption.⁴ In our model, that price is the real wage. Specifically, our starting point is the assumption that the nominal wage is stuck at a high enough level such that equilibrium labor input is labor-demand determined.⁵ Then, an increase in the nominal price level will drive down the real wage. An increase in government spending will put the required upward pressure on the price level. The simplest explanation of the mechanism is given in Figure 1.

At a lower real wage, producers hire more workers, which increases the equilibrium labor input. If the resulting increase in labor is sufficiently large, then there will be a summed increase in producers' profits and workers' wages so that households will be able to afford greater consumption, despite the tax increase arising from government spending.⁶

¹A few of the papers estimating aggregate fiscal policy multipliers include Auerbach, and Gorodnichenko (2012), Blanchard and Perotti (2002), Hall (2009), Ramey (2011) and Ramey and Zubairy (2016).

²In this paper, we maintain a narrow focus on government spending as the source of fiscal stimulus. Also, our use of the term "fiscal policy multiplier" or multiplier will mean the effect of a one unit increase in government spending on the level of either output or private consumption.

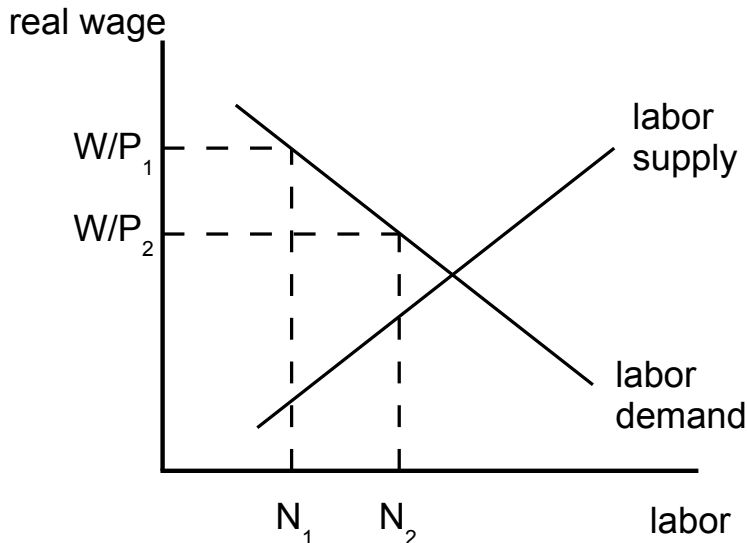
³In our literature review section, we will discuss other authors' channels that pre-date ours that can overcome the negative wealth effect. We will explain how each of these suffers from one or more deficiencies that are not present in our model.

⁴Only consumption can make the output multiplier greater than one because we work with the simple New Keynesian framework, which has neither investment nor international trade.

⁵This aspect of our framework puts it squarely in the line of "disequilibrium" macroeconomic models, such as Barro and Grossman (1971) and Mankiw and Weinzierl (2011).

⁶One might be concerned that the lower real wage would offset the higher employment to work against the increased income. This thinking is incorrect. Holding fixed the employment level, the lower real wage may lower the share of income that goes to the worker but it will increase the share of income that goes to

Figure 1: The employment effect on inflation resulting from government spending when nominal wages are rigid



Notes: W and P represent the nominal wage and the nominal price level. The economy initially has excess labor supply. An increase in government spending pushes up the price level. This decreases the real wage and increases labor input, reducing the excess supply of labor.

The often-used logic motivating government purchases during recessions is that public spending puts more income into workers' hands, allowing them to spend more. Because output is determined by the total demand for goods, rather than supply forces, the increased income will expand both equilibrium output and consumption.

In absence of other frictions, this thinking is fallacious when agents make rational consumption decisions. Since government purchases must be paid for, the unavoidable taxes (in either the present or the future) lead households to reduce consumption. Therefore, government spending will crowd-out private spending, leading to an output multiplier that is less than one. In contrast, to make the output multiplier greater than one, the economy should experience a movement in a relative price that encourages private consumption. In the simple New Keynesian model, there are two relative prices: the real wage and the real interest rate. Much existing research that generates large multipliers works through the real interest rate channel.

Under the interest rate approach, the real rate should fall in order that consumption will rise. This requires that monetary policy be passive in the sense that the government

the producer. In the New Keynesian model, workers own the firms and, thus, how the income is split is not relevant for the issue at hand.

increases the nominal interest rate in a less than one-for-one manner in response to increases in inflation. Under a passive rule, an increase in government spending drives up output which increases marginal cost. An increased marginal cost, in turn, drives up expected inflation. Under a passive monetary policy, higher inflation *reduces* the real interest rate.

While this mechanism is sufficient, it only works under a passive policy. Relying on this mechanism to explain large multipliers requires one to focus on periods when policy makers have chosen to pursue passive policies, even though these are known to have poor stabilizing properties in response to many shocks, or when the economy is stuck at the zero lower interest rate bound. In contrast, the mechanism we put forth relies on the real wage as the price channel that can generate a large multiplier. It works even when monetary policy is not passive.

Monetary policy is important in our model, because under an active policy, the real interest rate channel operates and puts downward pressure on consumption. In the theorems from our baseline models, a positive consumption multiplier will obtain when the real interest rate channel is weaker than the real wage channel.

There exists a wealth of evidence of nominal wage rigidity. Empirical microeconomic studies based on the Panel Study of Income Dynamics (PSID) or the Current Population Survey (CPS) provide strong support for nominal wage stickiness in the United States. For example, Altonji and Devereux (2000), Card and Hyslop (1997), Daly et al. (2012), Kahn (1997) and Lebow et al. (1995) examine the distribution of nominal wage changes and report a substantial spike at zero, indicating that the nominal wage stays constant over a year for many workers. The percentage of the sample with a constant wage in these studies ranges from 7% to 16%. Moreover, Gottschalk (2005) shows that the PSID data overstates the degree of nominal wage flexibility because of measurement error, and that adjusting for measurement error leads to remarkably fewer cuts in nominal wages.

Studies that use data from specific labor markets or individual firms in the United States find that the incidence of a zero nominal wage change is much more frequent than that reported in the PSID or the CPS. For instance, a telephone survey of individuals in the Washington, D.C., area in Akerlof et al. (1996) shows that 30.8% of respondents had no change in their base pay from the previous year and only 2.7% experienced wage cuts. Using data from a large financial corporation, Altonji and Devereux (2000) find that over 40% of the sample had zero nominal change; and that nominal wage cuts were received by only about 0.5% of salaried workers and 2.5% of hourly workers.

Barattieri et al. (2014) use data from the Survey of Income and Program Participation

(SIPP) and show that the nominal wage changes with an average quarterly probability ranging from 21.1% to 26.6%. Other research offers evidence of nominal sticky wages using European data. Bihan et al. (2012) obtain data from French firms and report that a wage change occurs with a quarterly frequency of around 35%. Druant et al. (2012) collect data from a firm-level survey conducted in 17 European countries. The authors find that on average, firms adjust wages every 15 months. Fehr and Goette (2005) examine Swiss data and present a spike at zero nominal wage change. The authors also show that nominal wage rigidity persist even in periods of sustained low inflation. The mean monthly frequency of nominal wage changes is reported to be 9.9% by Avouyi-Dovi et al. (2013) using French data and 12.9% by Sigurdsson and Sigurdardottir (2016) using Icelandic data.

Researchers have also conducted interview surveys to explore the underlying causes of nominal wage stickiness. Bewley (1998) shows that the major reason for employers' reluctance to cut wage is that they believe employee morale would be hurt. Blinder and Choi (1990) emphasize the perception of fair relative wages as a source of wage stickiness. Campbell and Kamlani (1997) find that downward nominal wage rigidity comes mainly from managers' concern that cutting wages would induce the most productive workers to quit.

2 The Sticky Wage Multiplier Mechanism

2.1 The Family Problem

An economy is made up of a continuum of families, each indexed by v . Time is indexed by non-negative integers. Each family engages in production, supplies and demands labor as well as consumes goods. Each family consumes all types of goods and let $C_t(v) = [\int_0^1 c_t(v)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dv]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$ denote the consumption aggregator of family v at time t , with $\varepsilon_p > 1$ denoting the elasticity of substitution between types of goods. Each family can hold bonds, $B_t(v)$, and fiat currency, $M_t(v)$, and derives utility from the real balances that its currency provides, $M_t(v)/P_t$. Money pays no interest and bonds pay a gross nominal interest rate R_t between t and $t + 1$.

The family utility function is

$$U(v) = \sum_{t=0}^{\infty} \beta^t E_0 \left[\log [C_t(v)] + \zeta \log \left[\frac{M_t(v)}{P_t} \right] \right]$$

We will sometimes use the variable r , where $r = 1 - \beta$.

In order to purchase consumption, each family supplies $N_t^s(v)$ by sending family members to enter the employ of other families. The family produces by hiring labor, $N_t^d(v)$, from other families to produce $Y_t(v) = [N_t^d(v)]^\alpha$. Let $\alpha \in (0, 1)$. The nominal wage, $\widetilde{W}_t = \xi W + (1 - \xi)P_t x$, is assumed to be a weighted average of a fixed nominal wage equal to W and a fixed real wage equal to x . That is, on the supply side, families post a nominal wage and stand willing to supply whatever labor is demanded at the fixed proportions of those two wages. Let $\xi \in [0, 1]$. Note that we have two special cases: (i) fixed nominal wages, i.e. $\xi = 1$, and (ii) fixed real wages, i.e. $\xi = 0$.

Our model of wage rigidity is deliberately a simple one. In the background, we have in mind a more general model in which employers post vacancies and meet with searching individuals and then engage in bargaining over match surpluses. As shocks hit the economy, neither member of the worker-employer matched pair may have incentive to separate so long as there is value in the existing match. A constant, i.e. rigid, wage may be a norm (as long as that wage is in the bargaining set) as an alternative to other bargaining protocols.

Nominal price setting is subject to Calvo-type rigidities: independent of past price history, a family producer cannot reset its good price with probability $\theta \in [0, 1]$. Each family sets the price of the good it sells subject to the following downward-sloping demand constraint:

$$Y_t(v) = \left[\frac{P_t(v)}{P_t} \right]^{-\varepsilon_p} Y_t^d$$

where $P_t(v)$ is the dollar price of the family v good, $P_t \equiv [\int_0^1 P_t(v)^{1-\varepsilon_p} dv]^{\frac{1}{1-\varepsilon_p}}$ is the nominal price aggregator and Y_t^d is the total demand for goods.

The family budget constraint is

$$M_t(v) + \frac{B_t(v)}{R_t} = B_{t-1}(v) + M_{t-1}(v) + \Pi_t(v) + \widetilde{W}_t N_t^s(v) - P_t C_t(v) - H_t(v)$$

where $\Pi_t(v) = P_t(v) [N_t^d(v)]^\alpha - (1 - \tau) \widetilde{W}_t N_t^d(v)$ is the profit from production, τ is a subsidy to families hiring workers and $H_t(v)$ represents the lump-sum tax collected by the government. Taxes are collected to pay for government purchases, finance the labor subsidy and potentially finance monetary policy actions.

2.2 Monetary-Fiscal Policy

The government budget constraint requires that

$$H_t + M_t^s + \frac{B_t^s}{R_t} = M_{t-1}^s + B_{t-1}^s + Z_t$$

where $Z_t = P_t G_t + \tau \widetilde{W} \bar{N}_t^s$. Let \bar{N}_t^s be average economy-wide labor supply.

Government spending evolves exogenously according to

$$G_t = \rho G_{t-1} + (1 - \rho) \bar{G} + \varepsilon_t$$

where ε_t is iid, mean zero with variance σ^2 . On occasion, we use the variable p , which is defined by $\rho = 1/(1 + p)$.

Monetary policy is conducted through an interest rate feedback rule. We assume

$$R_t = (\lambda + 1) E_t (\pi_{t+1} - 1) + \beta^{-1}$$

where $\lambda > 0$. The government stands ready to exchange money for bonds, or vice versa, at the desired interest rate R_t .

We study symmetric equilibria where each family behaves identically. Our definition of an equilibrium is standard, except that labor market clearing is replaced by a condition that profits of the families are maximized at the going market wages.

2.3 Solving the Family Problem

The family v problem is

$$\begin{aligned} \max \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \log [C_t(v)] + \zeta \log \left[\frac{M_t(v)}{p_t} \right] \right. \\ \left. + \Lambda_t(v) \left[M_{t-1}(v) + B_{t-1}(v) + \Pi_t(v) + \widetilde{W}_t N_t^s(v) - P_t C_t(v) - Z_t - M_t(v) - B_t(v)/R_t \right] \right\} \end{aligned}$$

Next, we describe the first order conditions for the family problem in a symmetric equilibrium where all families behave identically. The static consumption Euler equation is

$$\frac{1}{C_t} = \Lambda_t P_t$$

Optimal bond holdings imply

$$\Lambda_t/R_t = \beta E_t(\Lambda_{t+1})$$

The optimal choice of labor demand requires

$$mc_t(v) = \frac{\widetilde{W}_t}{\alpha P_t [N_t^d(v)]^{\alpha-1}}$$

where $mc_t(v)$ is the real marginal cost of the family firm v at time t .

When family v has the choice to reset goods prices at time t , it solves

$$\max_{P_t(v)} \sum_{i=0}^{\infty} \theta^i E_t \{ \Delta_{t,t+i}(v) [(\frac{P_t(v)}{P_{t+i}}) Y_{t+i}(v) - mc_{t+i} Y_t^d] \}$$

subject to the demand function. $\Delta_{t,t+i}(v) = \beta^i \left(\frac{C_{t+i}(v)}{C_t(v)} \right)^{-1}$ is the stochastic discount factor. Since all family firms adjusting in period t face the same problem, they will set the same optimal price, P_t^* .

The optimal pricing decision implies

$$\sum_{i=0}^{\infty} \theta^i E_t \{ \Delta_{t,t+i} [(1-\theta) \frac{P_t^*}{P_{t+i}} + \theta mc_{t+i}] \} = 0$$

Log-linearizing the above condition and $P_t \equiv [\int_0^1 P_t(v)^{1-\varepsilon_p} dv]^{\frac{1}{1-\varepsilon_p}} = [(1-\theta)P_t^{*1-\varepsilon_p} + \theta P_{t-1}^{1-\varepsilon_p}]^{\frac{1}{1-\varepsilon_p}}$ around the zero inflation steady state, we derive the New Keynesian Philips curve.

In keeping with standard practice in “cashless” New Keynesian models, we do not track the money demand Euler equation. Since money is separable in the utility function and since the government conducts monetary policy through an interest rate rule, we can pin down all variables besides the equilibrium money stock without using that equation.

Note that there is no labor supply Euler equation because, as explained above, families stand ready to supply whatever labor is desired at the going wage, \widetilde{W}_t .

Next, choose τ so that the distortion from imperfect competition is perfectly offset.

2.4 Log-Linearization

Assume $P_{-1} = W$ and $x = W/P_{-1}$. Let Equilibrium 1 denote an equilibrium of the economy where $P_t = P_{-1}$, $G_t = \bar{G}$, $C_t = \bar{C}$, and $\varepsilon_t = 0 \forall t$. Log-linearizing the inflation Euler equation

around Equilibrium 1 results in

$$\hat{\pi}_t = \kappa [(1/\alpha - 1) \hat{y}_t - \xi \hat{p}_t] + \beta E_t (\hat{\pi}_{t+1}) \quad (1)$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Solving this equation forward gives

$$\hat{\pi}_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t [(1/\alpha - 1) \hat{y}_{t+j} - \xi \hat{p}_{t+j}]$$

Apart from the \hat{p}_t term, the equation is identical to the standard New Keynesian Philips curve. In the standard New Keynesian case, either larger present or future output increases marginal cost, which leads forward-looking firms to raise their prices. The \hat{p}_{t+j} appears in our more general case because a higher price level in the future and/or the present lowers the real wage since a fraction of the wage is nominally rigid.

The log-linearized resource constraint is

$$\hat{y}_t = s \hat{c}_t + (1 - s) \hat{g}_t \quad (2)$$

After substituting the nominal rate out of the consumption Euler equation using the monetary policy rule, we have:

$$E_t (\hat{c}_{t+1}) - \hat{c}_t = \lambda E_t (\hat{\pi}_{t+1}) \quad (3)$$

The law of motion for government spending is:

$$\hat{g}_{t+1} = \rho \hat{g}_t + \varepsilon_{t+1} \quad (4)$$

We have three endogenous variables $\{\hat{c}, \hat{p}_t, \hat{y}_t\}$. We have three equations (1), (2), (3) as well as the definition of inflation, i.e., $\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$. Substituting out $\hat{\pi}_t$ and \hat{y}_t , the two dynamic equilibrium expressions are:

$$\Delta \hat{p}_t = \kappa (1/\alpha - 1) [s \hat{c}_t + (1 - s) \hat{g}_t] - \kappa \xi \hat{p}_t + \beta E_t (\Delta \hat{p}_{t+1}) \quad (5)$$

and (3).

3 Analysis of Equilibria

Next, we solve the equilibrium allocation using the method of undetermined coefficients. As a first step we provide conditions that guarantee the equilibrium is locally unique.

Theorem 1. *If the interest rate feedback parameter λ satisfies*

$$\frac{\alpha\xi}{s(1-\alpha)} < \lambda < \frac{\alpha(1-\beta+\kappa\xi)}{s\kappa(1-\alpha)} \quad (6)$$

then the equilibrium is locally unique.

Proof. In Appendix A. □

Theorem 1 provides bounds on λ required to ensure local uniqueness. The lower bound requires that λ be greater than $\alpha\xi/[s(1-\alpha)]$. In absence of sticky nominal wages, this inequality would become $\lambda > 0$. This is the celebrated “Taylor principle,” which requires that the monetary authority raise the real interest rate in response to increases in expected inflation. If the monetary authority lowers the real interest rate in response to increases in expected inflation, then self-fulfilling inflation become possible.

To understand why there is a tighter lower bound on λ in the presence of fixed nominal wages, i.e. $\xi > 0$, suppose that λ is only slightly greater than zero. Conjecture a sunspot equilibrium where agents come to believe that inflation will jump up at time zero and converge monotonically to the steady state. Because there is a slightly active rule, the increased inflation leads agents to believe that the central bank will raise the real interest rate, by a small amount, which puts downward pressure on consumption. As in the standard model, this tends works against the conjectured inflation expectations actually being realized. However, with sticky nominal wages, the inflation itself has a negative effect on marginal cost. This tends to increase output as firms sell more in the short run before prices adjust. Thus, sticky nominal wages help support sunspot fluctuations and, in turn, a more aggressive monetary policy is required to rule out these equilibria.

The upper bound on λ is required to take care of a somewhat perverse case that is present with or without fixed nominal wages. When λ is too large, there exist indeterminate equilibria where inflation exhibits one period oscillations around the steady state. This problem with overly aggressive expected inflation rules was first identified in Bernanke and Woodford (1997). For the remainder of this paper, we restrict attention to parameter configurations that satisfy (6), thus ensuring local equilibrium uniqueness.

Theorem 2. *The equilibrium inflation rate is given by $\hat{\pi}_t = \gamma \hat{g}_t$, where*

$$\gamma = \frac{\kappa(1/\alpha - 1)(1-s)(1-\rho)}{1 + \beta\rho^2 + [s\kappa(1/\alpha - 1)\lambda - (1 + \kappa\xi + \beta)]\rho} \quad (7)$$

Equilibrium consumption is given by $\hat{c}_t = \chi \hat{g}_t + \frac{\xi}{(1/\alpha - 1)s} \hat{p}_{t-1}$ where

$$\chi = \frac{(1 + \kappa\xi - \rho\beta)\gamma - \kappa(1/\alpha - 1)(1-s)}{s\kappa(1/\alpha - 1)} = \frac{(1-s)(1-\rho)/s}{1 - \rho + \frac{s\kappa(1/\alpha - 1)\lambda\rho - \kappa\xi}{1 - \beta\rho + \kappa\xi}} - \frac{1-s}{s} \quad (8)$$

Proof. In Appendix A. □

One implication of Theorem 2 is that, for any ξ , permanent shocks to government spending have no effect on either inflation or output.⁷ The intuition for these zero responses follows. Since output is unchanged, any increase in government spending happens alongside an offsetting crowd out of consumption. Suppose government spending increases and families forecast no effect on the sequence of nominal price levels following the shock. If the nominal price level path is unchanged, then the real wage will not change. Moreover, the absence of a change in inflation implies that the real interest rate remains at its initial steady state. With no change in the real interest rate, consumption going forward must be unchanged in order that the intertemporal consumption Euler equation is satisfied. This will definitely be the case if there is an immediate and permanent reduction in the level of consumption. If consumption declines by just the amount of the increase in government spending, then the total demand for goods will be unchanged. With no change of own-marginal cost, other families' prices and economy-wide demand, each family has no incentive to change its own price.

Next, the expressions for consumption and inflation in Theorem 2 simplify dramatically if government spending is iid. These become

$$\begin{aligned} \hat{\pi}_t &= \kappa(1/\alpha - 1)(1-s)\hat{g}_t \\ \hat{c}_t &= \frac{1-s}{s}\kappa\xi\hat{g}_t + \frac{\alpha\xi}{s(1-\alpha)}\hat{p}_{t-1} \end{aligned}$$

To develop an intuition for these responses, suppose that at time t , there is an unanticipated one percent increase in government spending. Then the effect on the sequence of inflation

⁷That is, if $\rho = 1$ then $\gamma = 0$ and $\hat{y}_t = 0$.

and consumption values going forward are:

$$\begin{aligned} \{\hat{\pi}_{t+j}\}_{j=0}^{\infty} &= \{\kappa(1/\alpha - 1)(1 - s), 0, 0, 0, \dots\} \\ \{\hat{c}_{t+j}\}_{j=0}^{\infty} &= \left\{ \frac{(1 - s)\kappa\xi}{s}, \frac{(1 - s)\kappa\xi}{s}, \dots \right\} \end{aligned}$$

As long as $\xi > 0$, there is an immediate and permanent increase in consumption. Since consumption growth is zero between t and $t + 1$, as well as between future periods, the real interest rate must remain at its steady state despite the spending shock as an implication of equation (3). Next, the form of the monetary policy rule implies that inflation at $t + 1$ and beyond must be at its steady-state value. The increase in consumption at $t + 1$ and beyond is due to the additional income of families from employment and profits resulting from an inflation-driven lower real wage. At time t , consumption increases in order that the families achieve intertemporal consumption smoothing.

Having explained the dynamic responses of inflation and consumption under the cases when $\rho = 0$ or $\rho = 1$, we are ready to tackle the intuition for the intermediate case of ρ between zero and one. We proceed by imagining the sequence of prices (i.e., real wage rates, real interest rates and inflation) that families expect following a government spending shock. Then, we explain how those expectations are consistent with optimization and the necessary market clearing conditions.

Suppose that, in response to the shock, families expect: (a) the real interest rate will jump up on impact and then gradually return to the steady state, (b) real income will jump up and increase as a result of higher employment (either in the form of labor income or additional profits from sales) and (c) inflation will jump up and then converge to the steady-state.

First, if families earn more period-by-period income and also see a temporarily higher real interest rate, then their optimal consumption plan will have consumption increase on impact and then converge to a new steady state from below. With higher consumption and additional government spending, families will have incentive to raise their prices, which causes inflation.

This inflation has two effects. Inflation validates families' expectations of temporarily higher real interest rates because families recognize that the central bank follows an active interest rate rule. Also, because some fraction of wages are nominally rigid, inflation also drives down the real wage path. This increases employment, which validates families' expectations of higher income.

Note that, although the government spending shock decays over time, long-run con-

sumption is permanently higher than its pre-shock steady-state value even though inflation asymptotes to its initial steady state. This permanent increase obtains because, although the inflation rate converges, the price level remains permanently above its initial value. This implies a permanently lower real wage. Note that this intuition breaks down if ξ is sufficiently close to or equals zero. We cover this case in Lemma 2 and the discussion that follows its presentation.

With the following lemma (Lemma 1), we verify a part of this intuition by proving that inflation does increase following a non-permanent government spending increase. Lemma 2 then establishes that consumption increases on impact following a non-permanent government spending shock as long as a sufficiently large fraction of wages are nominally rigid.

Lemma 1. *If $\rho < 1$, then government spending causes an increase in inflation, i.e. $\gamma > 0$.*

Proof. In Appendix A. □

Next, we provide conditions that determine the sign of the consumption response.

Lemma 2. *If*

$$\xi > \frac{r}{p\kappa} \tag{9}$$

then the consumption response on impact to a government spending shock is positive, i.e. $\chi > 0$.

Proof. In Appendix A. □

Lemma 2 provides a condition, in the case the equilibrium is locally unique, under which consumption on impact responds positively to a government spending shock. It states that the consumption response is positive if a sufficiently large fraction of wages are nominally rigid.

When there is no nominal wage rigidity, i.e. $\xi = 0$, equation (9) cannot hold and consumption does not rise in response to a government spending shock. This is consistent with the result in the the standard New Keynesian result, as described in Woodford (2011). Without (or with too little) nominal wage rigidity, there is no (or too little of a) positive income effect from increased employment. The only channel which affects consumption is the real interest rate channel. Consumption falls with the rise in the real interest rate associated with the inflation caused by government spending.

For a given positive ξ , an increase in the persistence of the government spending shock (or equivalently a decrease in p) towards 0 works against making equation (9) hold. A more

persistent increase in government spending implies a stronger contractionary real interest rate effect, because inflation is forward looking. This works against the expansionary real wage effect.

Next, an increase in r , which is approximately the time rate of discount, works against making equation (9) hold. Intuitively, the consumption boom requires permanent income to increase in response to the government spending shock. Because the expansionary effect on employment of a decrease in the real wage resulting from inflation occurs only gradually, it is crucial that families do not discount future income too quickly; otherwise, the real interest rate channel will dominate and consumption will fall on impact in response to a government spending shock.

Lemma 3. *Suppose that $\rho \in (0, 1)$ and equation (9) holds. Then, the impact response of consumption and inflation to a government spending shock is decreasing in λ , the aggressiveness of monetary policy.*

Proof. In Appendix A. □

Families have incentive to raise prices when a government spending shock increases the output that the family produces. This results in inflation. Because monetary policy is active, the central bank responds by raising the real interest rate. This puts downward pressure on consumption because of the real interest rate channel. This downward pressure is intensified when the central bank chooses a higher level of λ , implying that the consumption response on impact is reduced. A smaller increase in consumption, in turn, implies a smaller impact inflation response.

Note that even when consumption falls initially in response to a government spending increase, consumption may still eventually rise above its initial steady state. This is because the quantitative impact of inflation on the real wage builds over time following the shock.

While inflation and the real interest rate channel may drive down consumption initially, as the inflation drives up the nominal price level, this will eventually put downward pressure on real marginal cost. Eventually, the stimulative effect of the real wage channel may more than offset the real interest rate channel leading, to an increase in consumption.

Lemma 4. *Let $\rho \in (0, 1)$. Suppose $\beta > 0.5$. The impact response of inflation to a government spending shock is increasing in ρ if $\rho < 1 - \left(\frac{\kappa[s(1/\alpha-1)\lambda-\xi]}{\beta}\right)^{\frac{1}{2}}$, and is decreasing in ρ if the inequality is reversed.*

Proof. In Appendix A. □

The response of inflation on impact exhibits a hump-shaped pattern with ρ . Two forces affect current inflation: the negative wealth effect and the combined effects generated by expected inflation which includes the real interest rate channel and the real wage channel.

When ρ increases, the negative wealth effect becomes larger putting downward pressure on current inflation, while the positive combined effects generated by expected inflation become larger putting upward pressure on current inflation. So the overall response of inflation could be greater or smaller depending on the relative size of these two forces.

Initially, the combined effects increase faster, so the inflation response becomes greater. Yet, when ρ increases further, the real interest rate becomes higher, which weakens the combined effects. Moreover, the effect from the real wage channel becomes weaker because the nominal wage increases with higher inflation. Consequently, the overall response of current inflation would be smaller if ρ becomes large enough.

In addition, when λ decreases, the real interest rate channel is dampened. The peak occurs at a larger ρ . When ξ increases, the real wage channel is intensified. There is also a rightward shift of the peak. If λ approaches its lower bound (or equivalently, ξ approaches its upper bound), i.e. $\lambda = \frac{\alpha\xi}{s(1-\alpha)}$, we obtain a limiting case where the hump disappears and the size of inflation increases in ρ .

Next, we are interested in not simply the consumption multiplier on impact, but also the accumulated change in consumption for a given accumulated change in government spending.⁸ We define the cumulative discounted consumption multiplier, or simply the cumulative multiplier, as the following

$$\mu_c^{cum} = \frac{s}{1-s} \left[\frac{\sum_{t=0}^{\infty} \beta^t (d\hat{c}_t/d\hat{g}_0)}{\sum_{t=0}^{\infty} \beta^t (d\hat{g}_t/d\hat{g}_0)} \right]$$

The $s/(1-s)$ multiplicand appears because we want μ_c^{cum} to be a derivative (unit change given a unit change) as opposed to an elasticity (percentage change given a percentage change). Because \hat{g}_t is first-order autoregressive, we have $\sum_{t=0}^{\infty} \beta^t (d\hat{g}_t/d\hat{g}_0) = 1/(1-\rho\beta)$.

The final term, $\sum_{t=0}^{\infty} \beta^t (d\hat{c}_t/d\hat{g}_0)$, is the accumulated consumption response. Given the

⁸This cumulative approach for defining the multiplier is advocated, for example, by Drautzburg and Uhlig (2015) and Ramey and Zubairy (2016).

formula from Theorem 2, we have

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t (d\hat{c}_t/d\hat{g}_0) &= \sum_{t=0}^{\infty} \beta^t \left(\frac{d[\chi\hat{g}_t + \xi[s(1/\alpha - 1)]^{-1}\hat{p}_{t-1}]}{d\hat{g}_0} \right) \\
&= \frac{\chi}{1 - \rho\beta} + \frac{\alpha\xi}{s(1 - \alpha)} \sum_{t=0}^{\infty} \beta^t \left(\sum_{s=0}^{t-1} \frac{d\hat{\pi}_s}{d\hat{g}_0} \right) \\
&= \frac{\chi}{1 - \rho\beta} + \frac{\alpha\xi\gamma\beta}{s(1 - \alpha)(1 - \beta)(1 - \rho\beta)}
\end{aligned}$$

Combining terms, the cumulative multiplier is

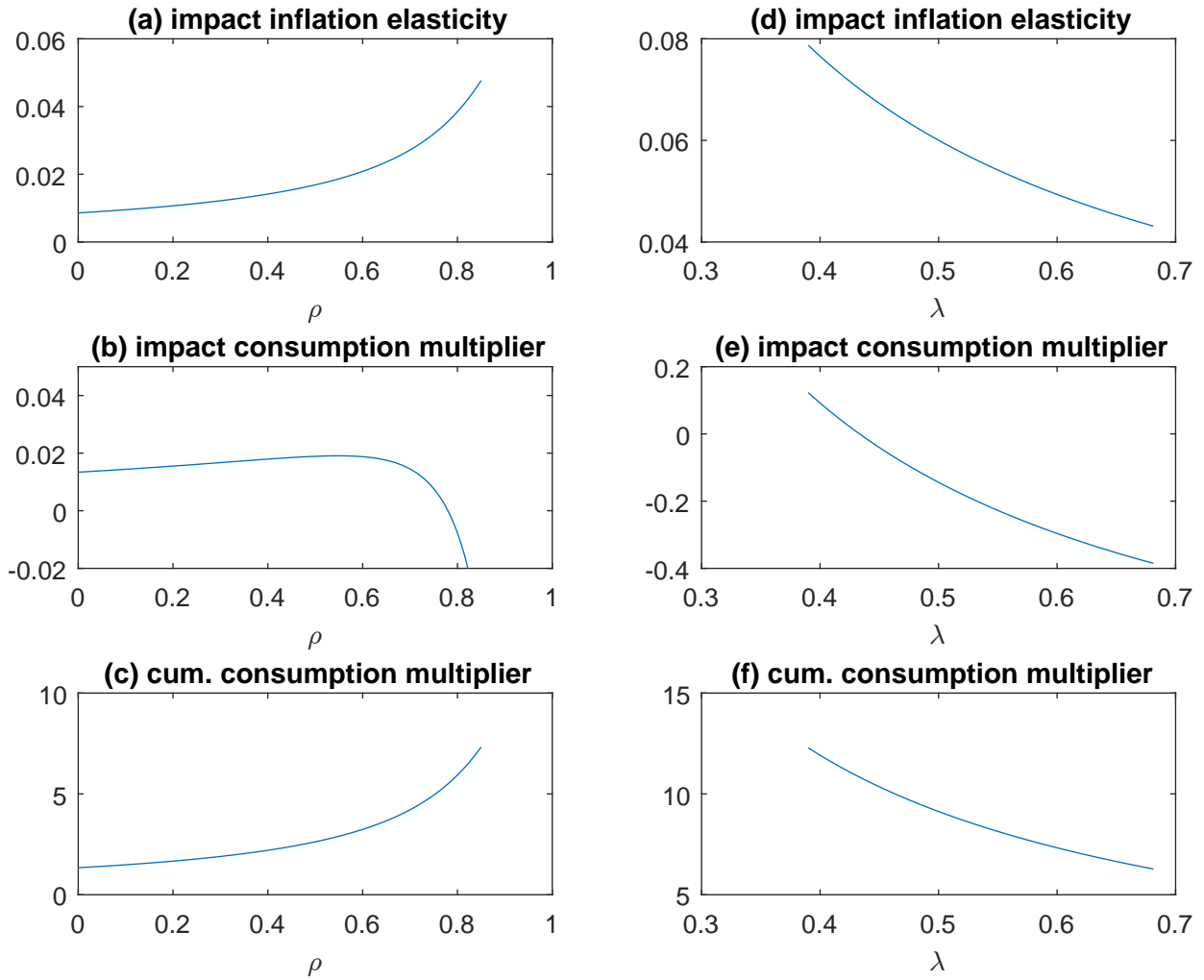
$$\mu_c^{cum} = \frac{s}{1 - s} \left[\chi + \frac{\alpha\xi\gamma\beta}{s(1 - \alpha)(1 - \beta)} \right]$$

Next, we plug specific parameters into the model and study the equilibrium response coefficients. For each parameterization reported, the equilibrium is locally unique. Most of the parameters are held fixed. The discount factor, β , is set equal to 0.99. The elasticity of output with respect to labor, α , is set to 2/3. The share of consumption in GDP, s , is assumed to be 0.8 so that the share of government spending is 0.2. The probability that a family producer cannot reset its good price, θ , is set to equal 0.75 so that the average price duration is one year. Then we have that $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} = 0.0858$. We set the weight in the nominal wage, ξ , to 0.156 so that the (quarterly) frequency of wage change, $(1 - \xi)(1 - \theta)$, is equal to 0.211 as in Barattieri et al. (2014). Figure 2 plots consumption and inflation responses for particular equilibria as we, in turn, vary ρ and λ .

Panel (a) of the figure presents the elasticity of inflation on impact to a government spending shock for alternative values of ρ . The inflation multiplier on impact is in the range of 0.009 and 0.05 and is increasing in ρ . Intuitively, a more persistent government spending shock implies that output will be above the steady state for a longer period of time, and forward-looking price setters will recognize this and initiate larger initial price increases right away.

Panel (b) plots the consumption multiplier on impact for alternative value of ρ . Over this range of ρ , the consumption multiplier is initially increasing, and then turns decreasing, as ρ becomes larger. This is consistent with Lemma 2. As government spending becomes more persistent, there is a larger inflation response. This implies a larger reduction in the real wage both today and in the future which increases the families' returns to production. Panel

Figure 2: Response of economy to a government spending shock, varying ρ and λ



(c) plots the cumulative consumption multiplier, where we discount future values of both consumption and government spending responses by β . As with the impact consumption multiplier, the cumulative consumption multiplier is increasing over this range of ρ .

The analogous information is plotted in panels (d) through (f), except we have λ varying instead of ρ . These panels demonstrate the interaction between monetary and fiscal policy in our model. As λ increases, all three responses become smaller. For panel (d), the inflation response is decreasing because the central bank is more aggressively combating inflation. As the central bank responds more aggressively, it is raising the real interest rate by a larger amount. This amplifies the contractionary intertemporal substitution effect as families reduce consumption when facing a higher real interest rate.

4 A Generalized Wage Process

With the above formulation of nominal wage rigidity, any shock that has a permanent effect on the price level will have a permanent effect on output. For example, suppose there is a transitory, positive shock to government spending. Under the conditions of Lemma 1, this drives up the price level. Although inflation might converge back to the steady state following the shock, the long-run price level would be higher. In turn, the long run real wage would be lower, which would be associated with higher long-run output.

Next, we generalize the model to undo the permanent real effects of transitory government spending shocks. Specifically, we back off the assumption of a perfectly fixed nominal wage in the first part of \widetilde{W}_t . Instead, let $\widetilde{W}_t = \xi X_t W + (1 - \xi) P_t x$, where

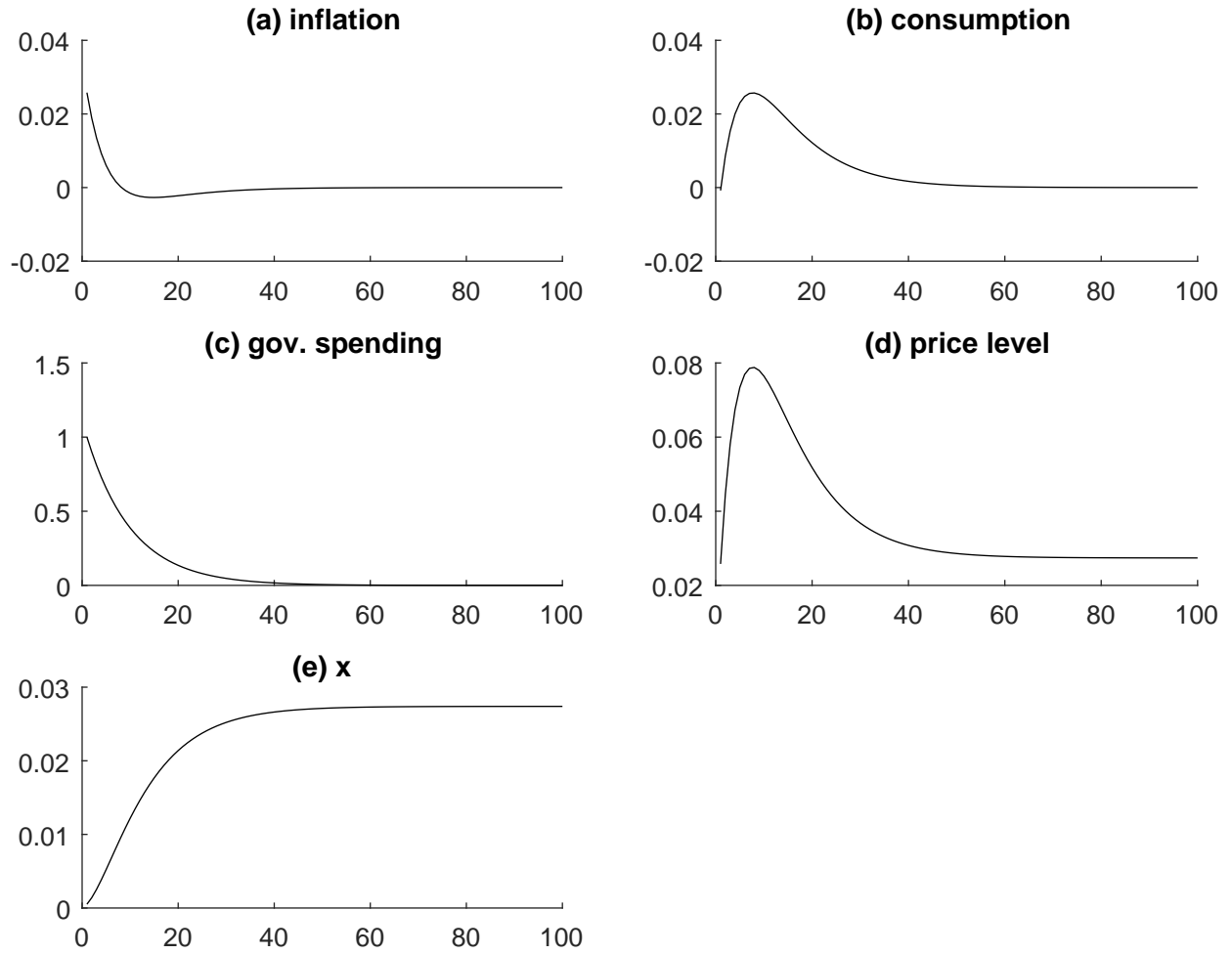
$$X_t = (P_t)^\eta (X_{t-1})^{1-\eta}$$

According to this specification, the price-wage ratio has a single steady state value and will converge to that value following a shock; however, it will experience transitory departures, changing the real marginal cost of firms.

To consider the case in which η is in the interior of the unit interval, we simulate the model. This requires us to choose specific parameter values. We set β, α, s, θ and ξ equal to their values from the previous section. We let $\eta = 0.02$, so that nominal wages adjust very slowly to price increases. Finally, let $\lambda = 0.5$ and $\rho = 0.9$. We use Dynare to compute the impulse responses of key variable to a government spending increase and plot the responses in Figure 3.

Panel (c) in Figure 3 plots the impulse response to the exogenous shock: a one percent

Figure 3: Response of economy to a government spending shock



autoregressive increase in government spending. As seen in the figure, this generates an initial increase in inflation. The increase in initial inflation drives up the price level on impact. The price level then follows a hump-shaped path.

The log real wage is then the deviation between the nominal wage, which is given in percentage deviations as x , and p . Persistent stickiness causes the nominal wage to adjust slowly while the price level rises more quickly. The net effect of these two changes is a decline in the real wage, and, therefore, marginal cost. Declining marginal cost leads firms to increase production.

Increased production puts more income into the hands of the families, which allows them to pay the tax bill associated with the increased government spending and also buy more consumption goods. In absence of the falling real wage, consumption would instead fall. This is because, *ceteris paribus*, the active monetary policy implies that increased inflation would drive up the real interest rate, leading families to reduce current consumption and increase savings.

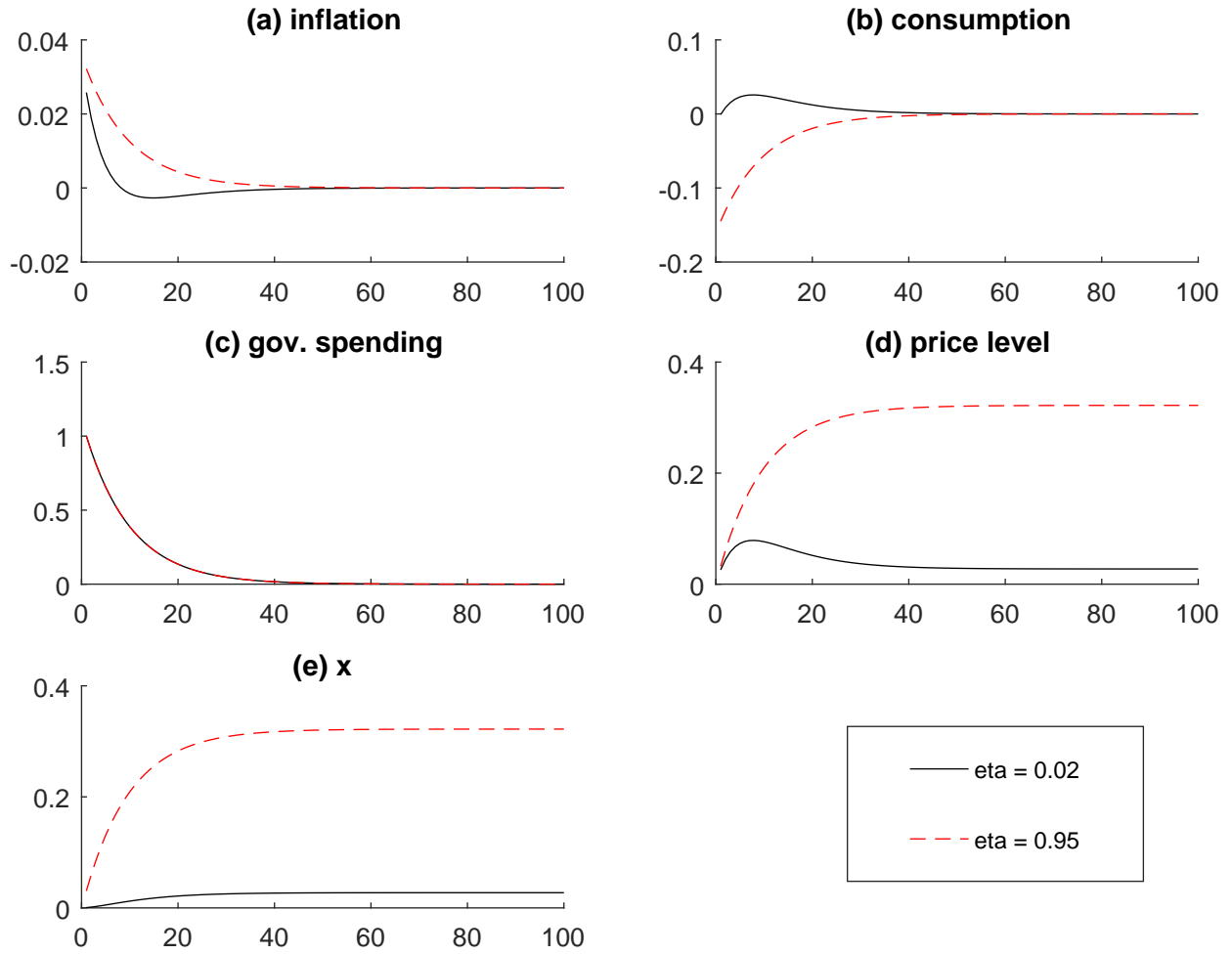
Eventually, government spending asymptotes to its steady state. This causes inflation to return to zero, which means that the price level converges to a new long run steady state. As seen in the figure, the nominal wage converges as well although this variable moves to its new steady state more slowly. Note that both x and p converge in percentage deviation terms by the same amount. This implies that the long run real wage is identical to its pre-shock level. With the real wage having converged and the real interest rate having converged (because inflation returned to zero), consumption returns to its initial steady state.

To explore the importance of the sticky wage assumption, we change the specification by increasing the speed of adjustment of the nominal wage. In the benchmark parameter, η equals 0.02, whereas, we set $\eta = 0.95$ to demonstrate the effect of faster adjustment. The other parameters are unchanged.

The solid lines in Figure 4 are the impulse responses from our benchmark case. The dashed red lines represent the fast wage adjustment case ($\eta = 0.95$). As seen in the figure, consumption falls on impact and remains below zero in the fast wage adjustment case. Inflation increases with fast wage adjustment which causes the nominal price level to increase initially. This effect tends to push down the real wage; however, because we increase η relative to the benchmark case, the nominal wage moves up towards its new steady state much more quickly. This is seen in panel (e).

This implies that the real wage increase is much more muted with fast wage adjustment. Its expansionary effect is insufficiently large to overcome the contractionary effect of the real

Figure 4: Response of economy to a government spending shock, varying the degree of wage rigidity



Notes: $\eta = 0.02$ reflects the benchmark case; $\eta = 0.95$ reflects faster nominal wage adjustment.

interest rate increase caused by inflation paired with an active interest rate rule.

Next, we consider the effect of changing the aggressiveness of monetary policy with respect to inflation, λ . For relatively small departures from the benchmark case, there is only a small effect on consumption. Therefore, we consider a very large increase and decrease from the benchmark case in order to elucidate the mechanism. The solid lines in Figure 5 reflect the benchmark case. The blue dash-dotted lines reflect the case where policy is very active, with $\lambda = 10$. In this case, the increase in inflation caused by government spending induces a very large increase in the nominal interest rate, and therefore the real interest rate. With a large interest rate increase, the standard real interest rate effect leads families to dramatically reduce consumption. The real interest rate channel initially dominates our new real wage channel. However, eventually, as the inflation rate begins to return to its steady state, the real interest rate also begins to return to its steady state.

At this point, our new proposed effect dominates as the resulting higher price level combined with a slowly adjusting nominal wage drives down the real wage. Examining panel (b), by roughly horizon twenty, consumption is above the steady state. After this, it slowly converges back to its steady state as the nominal wage “catches up” with the increase in the price level.

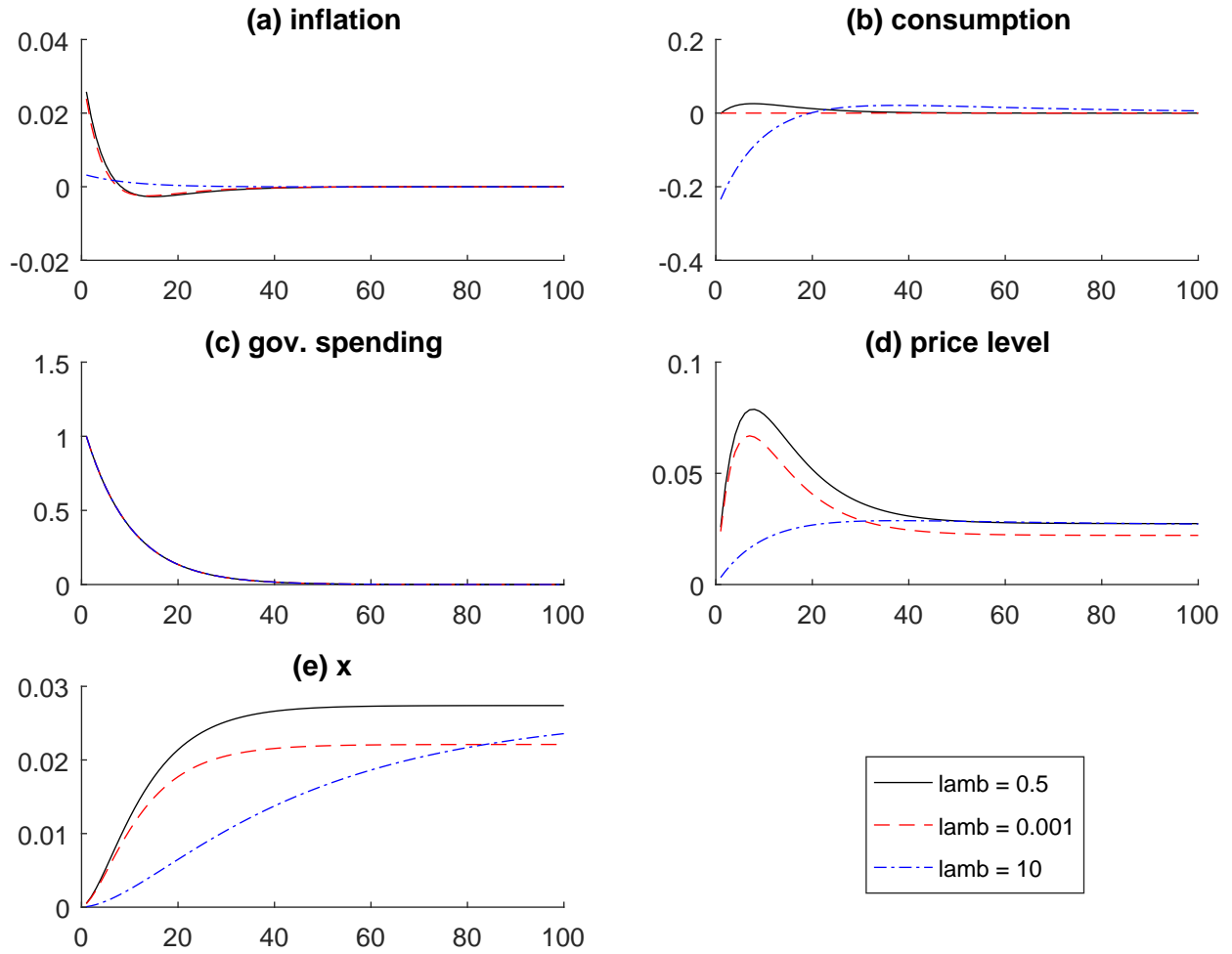
The third parameterization has $\lambda = 0.001$, which implies that interest rate policy is nearly neutral with respect to inflation. This parameterization is reflected by red dashed lines in Figure 5.

5 Discussion

5.1 Existing Research: Sticky Wages as an Element for Generating Large Multipliers

We have shown that sticky wages and prices are sufficient to generate a positive consumption multiplier and hence an output multiplier that is larger than one. Another paper that succeeds in producing a positive consumption multiplier is Rendahl (2016). The author uses an inertial labor market to achieve this goal: government spending reduces unemployment, which is persistent into the future; a brighter future increases current consumption, under the condition that the elasticity of intertemporal substitution is low. This mechanism works only when unemployment exists, meaning that the efficacy of monetary policy is constrained. In normal times (outside the liquidity trap), full employment is ensured, hence an increase in

Figure 5: Response of economy to a government spending shock, varying the responsiveness of the interest rate rule



Notes: $\lambda = 0.5$ reflects the benchmark case; $\lambda = 0.001$ reflects relatively passive policy; $\lambda = 10$ reflects relatively active policy.

government spending cannot increase employment further. Therefore, it cannot increase current consumption in normal times. Our model, instead, can generate a positive consumption multiplier without the zero lower bound.

There are other papers that are more tangentially related to our work. These papers also illustrate the transmission from higher prices to lower real wages and then to higher employment, resulting from sticky nominal wages. For example, Schmitt-Grohé and Uribe (2017) build a model in which a negative confidence shock induces a recession and a liquidity trap. Output growth gradually restores. However, because of downward nominal wage rigidity, real wages fail to fall to levels required by the recovery of full employment. Therefore, a jobless growth recovery occurs.⁹ Again, the zero lower bound on nominal interest rates, in addition to downward nominal wage rigidity, is required for the model to generate large contractions with jobless recoveries. Bordo et al. (2000) emphasize the importance of sticky wages in propagating negative monetary shocks during the great depression. The mechanism illustrated in their model is that a monetary innovation causes the price level to increase, while nominal wages respond gradually. The persistent decline in the real wage induces an increase in labor hours and output. While a positive money growth innovation under sticky wages is able to generate a large output increase, fiscal policy analysis is left untouched in this paper.

5.2 Comparing the Sticky Wage Channel to Other Approaches

Various mechanisms have been offered to explain how, within an optimizing, dynamic equilibrium model, increases in government spending might stimulate private economic activity. For various reasons, we find each of these mechanisms has one or more drawbacks.

First, as discussed in the introduction, several papers, such as Christiano (2004), Christiano et al. (2011), Eggertsson and Woodford (2006), Mertens and Ravn (2014) and Woodford (2011), have shown that under a particular class of monetary policies, the output multiplier can be greater than one in a New Keynesian model. This occurs when the central bank raises the interest rate in a less than one-for-one manner with inflation, which might arise if, for example, the policy rate were stuck at the zero lower bound. While this mechanism works, it requires monetary policy to take a particular form.

Second, a number of authors, including Bouakez and Rebei (2007), demonstrate that when private and public consumption are Edgeworth complements for households, an in-

⁹In Schmitt-Grohé and Uribe (2016), the authors investigate optimal government policies needed to eliminate the negative externality caused by downward nominal wage rigidity.

crease in government spending can increase private consumption. Intuitively, this arises because government purchases increase the marginal utility of private consumption. The government purchases therefore act as a preference shock, so that households will increase consumption even if the real interest rate and real wage were unchanged.

While it is possible to posit examples of complementarity between public and private goods, e.g. better highways may increase vacations taken and therefore vacation spending, it seems just as easy to find examples of substitutability. For example, the provision of primary and secondary education by the government is likely a substitute for private provision of those services. At a more fundamental level, by coupling a preference effect with the deadweight loss of taxation, this approach is very close to simply assuming the result.

Closely related to the private-public consumption complementarity approach to generating large multipliers is to assume that labor and private consumption are complements in the utility function. This mechanism is discussed, for example, in Hall (2009). According to this channel, the deadweight loss of government spending increases labor supply. An increase in labor supply in turn increases the marginal utility of private consumption. Households will then increase consumption, resulting in an output multiplier that is greater than one.

Whether labor and consumption are complements in production is an empirical question. It is worth noting, however, that a government's justification for public spending as stimulus to its citizenry might not be compelling. It would go something like this: "By purchasing public goods and making you poorer through taxes, you will work harder which will increase your hunger. Greater hunger will in turn boost consumption spending."

Finally, several existing explanations for a large multiplier can broadly fall under an umbrella of winner/loser mechanisms. These include, for example, Eggertsson and Krugman (2012) and Galí et al. (2007). In each case, aggregate consumption increases because the increased consumption of one group of agents outweighs the decreased consumption experienced by another group of agents. In the above examples, the winners are either borrowing constrained or consume in a rule-of-thumb manner. In each example, the losers behave as permanent income consumer.

In contrast to the above three channels for generating large fiscal multipliers, our approach does not depend upon: a particular stance of monetary policy, non-separability in preferences, or one group of agents suffering at the expense of the other group.

6 Conclusion

One hallmark of the Keynesian approach to business cycle stabilization is that wages do not clear the labor market, leading to a shortage of labor demand relative to people's willingness to work at the going wage. As discussed in the introduction, there is ample evidence in favor of a high nominal wage rigidity. Since firms care about the real, as opposed to nominal, wage when making labor decisions, the nominal price level provides an indirect channel by which the government can close the gap between labor supply and demand when wages are too high. In a model where government spending puts an upward pressure on goods prices, this spending will also drive down the real wage when nominal wages are fixed, leading to an employment boom capable of increasing consumption as well as financing the additional tax bill.

Appendix A Proofs of Theorems and Lemmas

A.1 Proof of Theorem 1

Proof. The second-order difference equation in $\hat{\pi}_t$ is given by

$$\beta E_t(\hat{\pi}_{t+2}) - [1 + \beta + \kappa\xi - s\kappa(1/\alpha - 1)\lambda] E_t(\hat{\pi}_{t+1}) + \hat{\pi}_t = \kappa(1 - s)(1/\alpha - 1)(1 - \rho)\hat{g}_t$$

The two roots of the characteristic equation are

$$e_1 = \frac{\Phi + \sqrt{\Phi^2 - 4\beta}}{2\beta}$$

and

$$e_2 = \frac{\Phi - \sqrt{\Phi^2 - 4\beta}}{2\beta}$$

where $\Phi = 1 + \beta + \kappa\xi - s\kappa(1/\alpha - 1)\lambda$. We have the following two cases to consider.

Case (i):

$$\kappa\xi - s\kappa(1/\alpha - 1)\lambda < 0 \iff \lambda > \frac{\alpha\xi}{s(1 - \alpha)}$$

If $\Phi - 2\beta > 0 \iff \lambda < \frac{\alpha(1 - \beta + \kappa\xi)}{s\kappa(1 - \alpha)}$, then the smaller root $e_2 > \frac{[1 + \beta + \kappa\xi - s\kappa(1/\alpha - 1)\lambda] - \sqrt{[1 - \beta + \kappa\xi - s\kappa(1/\alpha - 1)\lambda]^2}}{2\beta} =$

1. Hence both roots are outside the unit circle and the equilibrium is locally unique.

If $\Phi - 2\beta < 0 \iff \lambda > \frac{\alpha(1 - \beta + \kappa\xi)}{s\kappa(1 - \alpha)}$, then the larger root $-1 < e_1 < 1$ and hence the equilibrium is indeterminate.

Case (ii):

$$\kappa\xi - s\kappa(1/\alpha - 1)\lambda > 0 \iff \lambda < \frac{\alpha\xi}{s(1 - \alpha)}$$

The two roots are

$$\begin{aligned} e_1 &= 1 + \frac{\Phi - 2\beta + \sqrt{\Phi^2 - 4\beta}}{2\beta} \\ &= 1 + \frac{\Phi - 2\beta + \sqrt{(1 - \beta)^2 + 2(1 + \beta)[\kappa\xi - s\kappa(1/\alpha - 1)\lambda] + [\kappa\xi - s\kappa(1/\alpha - 1)\lambda]^2}}{2\beta} \end{aligned}$$

and

$$\begin{aligned} e_2 &= 1 + \frac{\Phi - 2\beta - \sqrt{\Phi^2 - 4\beta}}{2\beta} \\ &= 1 + \frac{\Phi - 2\beta - \sqrt{(1-\beta)^2 + 2(1+\beta)[\kappa\xi - s\kappa(1/\alpha - 1)\lambda] + [\kappa\xi - s\kappa(1/\alpha - 1)\lambda]^2}}{2\beta} \end{aligned}$$

Since

$$\begin{aligned} [1 - \beta + \kappa\xi - s\kappa(1/\alpha - 1)\lambda]^2 &= (1 - \beta)^2 + 2(1 - \beta)[\kappa\xi - s\kappa(1/\alpha - 1)\lambda] + [\kappa\xi - s\kappa(1/\alpha - 1)\lambda]^2 \\ &< (1 - \beta)^2 + 2(1 + \beta)[\kappa\xi - s\kappa(1/\alpha - 1)\lambda] + [\kappa\xi - s\kappa(1/\alpha - 1)\lambda]^2 \end{aligned}$$

we have that $e_1 > 1$ and $-1 < e_2 < 1$. Hence the equilibrium is not unique in case (ii).

Overall, λ should satisfy $\frac{\alpha\xi}{s(1-\alpha)} < \lambda < \frac{\alpha(1-\beta+\kappa\xi)}{s\kappa(1-\alpha)}$ so that the equilibrium is locally unique. \square

A.2 Proof of Theorem 2

Proof. First, substitute out \hat{y}_t from (2) into (1) and get

$$\hat{\pi}_t = \kappa\{(1/\alpha - 1)[s\hat{c}_t + (1 - s)\hat{g}_t] - \xi\hat{p}_t\} + \beta E_t \hat{\pi}_{t+1} \quad (10)$$

Update t to $t + 1$ and take expectation.

$$E_t \hat{\pi}_{t+1} = \kappa\{(1/\alpha - 1)[sE_t \hat{c}_{t+1} + (1 - s)E_t \hat{g}_{t+1}] - \xi E_t \hat{p}_{t+1}\} + \beta E_t \hat{\pi}_{t+2} \quad (11)$$

Subtract (10) from (11) and then use (3) to return a second-order difference equation in $\hat{\pi}_t$.

$$\beta E_t (\hat{\pi}_{t+2}) - [1 + \beta + \kappa\xi - s\kappa(1/\alpha - 1)\lambda] E_t (\hat{\pi}_{t+1}) + \hat{\pi}_t = \kappa(1 - s)(1/\alpha - 1)(1 - \rho) \hat{g}_t$$

Next, guess a solution that takes the form $\hat{\pi}_t = \gamma \hat{g}_t$ and plug that guess into the above equation.

$$\rho^2 \beta \gamma \hat{g}_t - \rho(1 + \beta + \kappa\xi - s\kappa(1/\alpha - 1)\lambda) \gamma \hat{g}_t + \gamma \hat{g}_t = \kappa(1 - s)(1/\alpha - 1)(1 - \rho) \hat{g}_t$$

Then, the solution for the undetermined coefficient γ is

$$\gamma = \frac{\kappa(1/\alpha - 1)(1 - s)(1 - \rho)}{1 + \beta\rho^2 + [s\kappa(1/\alpha - 1)\lambda - (1 + \kappa\xi + \beta)]\rho}$$

Using the solution for inflation as a function of government purchases, we can rewrite the inflation Euler equation as

$$(1 - \rho\beta) \gamma \hat{g}_t + \kappa\xi\gamma\hat{g}_t + \kappa\xi\hat{p}_{t-1} = s\kappa(1/\alpha - 1) \hat{c}_t + \kappa(1/\alpha - 1)(1 - s) \hat{g}_t$$

Next, we rearrange this expression to have

$$\hat{c}_t = \frac{[1 + \kappa\xi - \rho\beta] \gamma - \kappa(1/\alpha - 1)(1 - s)}{s\kappa(1/\alpha - 1)} \hat{g}_t + \frac{\alpha\xi}{(1 - \alpha)s} \hat{p}_{t-1}$$

□

A.3 Proof of Lemma 1

Proof. From equation (7) and our assumption that $\rho < 1$, $\gamma > 0$ if and only if

$$1 + \beta\rho^2 > \rho[1 + \beta + \kappa\xi - s\kappa\lambda(1/\alpha - 1)]$$

Rearranging this expression,

$$\kappa^{-1}\rho^{-1}(1 - \rho)(1 - \beta\rho) > \xi - s\lambda(1/\alpha - 1) \quad (12)$$

The term on the left-hand side of (12) is positive given our restrictions on ρ , κ and β . The term on the right-hand side is negative because of our restriction that guarantees local uniqueness. □

A.4 Proof of Lemma 2

Proof. According to equation (8), $\chi > 0$ if and only if

$$\frac{1 - \rho}{1 - \rho + \frac{s\kappa(1/\alpha - 1)\lambda\rho - \kappa\xi}{1 - \beta\rho + \kappa\xi}} > 1$$

Recall that local uniqueness requires

$$\frac{\alpha\xi}{s(1 - \alpha)} < \lambda < \frac{\alpha(1 - \beta + \kappa\xi)}{s\kappa(1 - \alpha)}$$

Given that $\lambda > \frac{\alpha\xi}{s(1-\alpha)}$, we have

$$1 - \rho + \frac{s\kappa(1/\alpha - 1)\lambda\rho - \kappa\xi}{1 - \beta\rho + \kappa\xi} > 1 - \rho + \frac{\kappa\xi\rho - \kappa\xi}{1 - \beta\rho + \kappa\xi} = \frac{(1 - \rho)(1 - \beta\rho)}{1 - \beta\rho + \kappa\xi} > 0.$$

Using $p \equiv \frac{1}{\rho} - 1$ and $r \equiv 1 - \beta$, we can rewrite (9) as $\rho < \frac{\kappa\xi}{1 - \beta + \kappa\xi}$. If $\rho < \frac{\kappa\xi}{1 - \beta + \kappa\xi}$, then

$$\lambda < \frac{\alpha(1 - \beta + \kappa\xi)}{s\kappa(1 - \alpha)} < \frac{\alpha\xi}{s(1 - \alpha)\rho}$$

Having $\lambda < \frac{\alpha\xi}{s(1 - \alpha)\rho}$, we obtain

$$s\kappa(1/\alpha - 1)\lambda\rho - \kappa\xi < 0$$

Hence

$$\frac{1 - \rho}{1 - \rho + \frac{s\kappa(1/\alpha - 1)\lambda\rho - \kappa\xi}{1 - \beta\rho + \kappa\xi}} > 1$$

□

A.5 Proof of Lemma 3

Proof. Recall that γ is

$$\gamma = \frac{\kappa(1/\alpha - 1)(1 - s)(1 - \rho)}{1 + \beta\rho^2 + [s\kappa(1/\alpha - 1)\lambda - (1 + \kappa\xi + \beta)]\rho} \quad (13)$$

Because $\rho < 1$, γ is positive. An increase in λ increases the denominator on the right-hand side of (13) and therefore decreases γ . Next, $\chi > 0$. It is given by

$$\chi = \frac{(1 + \kappa\xi - \rho\beta)\gamma - \kappa(1/\alpha - 1)(1 - s)}{s\kappa(1/\alpha - 1)}$$

Thus, a decrease in γ resulting from an increase in λ reduces χ . □

A.6 Proof of Lemma 4

Proof. Recall that γ is

$$\gamma = \frac{\kappa(1/\alpha - 1)(1 - s)(1 - \rho)}{1 + \beta\rho^2 + \rho[s\kappa(1/\alpha - 1)\lambda - (1 + \kappa\xi + \beta)]}$$

Then, we have

$$\frac{\partial \gamma}{\partial \rho} = \frac{-\kappa (1/\alpha - 1) (1 - s) [-\beta(1 - \rho)^2 - \kappa \xi + s\kappa (1/\alpha - 1) \lambda]}{\Delta^2}$$

where $\Delta = 1 + \beta\rho^2 + \rho[s\kappa(1/\alpha - 1)\lambda - (1 + \kappa\xi + \beta)]$.

The root of the equation $-\beta(1 - \rho)^2 - \kappa\xi + s\kappa(1/\alpha - 1)\lambda = 0$ is $1 - (\frac{\kappa[s(1/\alpha - 1)\lambda - \xi]}{\beta})^{\frac{1}{2}}$. Given the restriction imposed on λ that ensures a locally unique equilibrium, we can show that as long as $\beta > 0.5$, $1 - (\frac{\kappa[s(1/\alpha - 1)\lambda - \xi]}{\beta})^{\frac{1}{2}} \in (0, 1)$. Thus, if $\rho < 1 - (\frac{\kappa[s(1/\alpha - 1)\lambda - \xi]}{\beta})^{\frac{1}{2}}$, we have $\frac{\partial \gamma}{\partial \rho} > 0$. And if $\rho > 1 - (\frac{\kappa[s(1/\alpha - 1)\lambda - \xi]}{\beta})^{\frac{1}{2}}$, $\frac{\partial \gamma}{\partial \rho} < 0$. \square

Appendix B Intuition for Lemma 4

The starting point is:

$$E_t(\hat{\pi}_{t+2}) + \beta^{-1}(\kappa[s\lambda(1 - \alpha) - \xi] - 1 - \beta)E_t(\hat{\pi}_{t+1}) + \beta^{-1}\hat{\pi}_t = -\kappa(1 - s)(1 - \alpha)E_t(\Delta\hat{g}_{t+1})$$

Using lag operator notation:

$$[1 + \beta^{-1}(\kappa[s\lambda(1 - \alpha) - \xi] - 1 - \beta)L + \beta^{-1}L^2]E_t(\hat{\pi}_{t+2}) = -\kappa(1 - s)(1 - \alpha)E_t(\Delta\hat{g}_{t+1})$$

Note that the appearance of ξ only works to make the monetary policy less “active”. Other than this, it does not influence the dynamics of inflation. Therefore, define $\tilde{\lambda} = s\lambda(1 - \alpha) - \xi$.

$$[1 + \beta^{-1}(\kappa\tilde{\lambda} - 1 - \beta)L + \beta^{-1}L^2]E_t(\hat{\pi}_{t+2}) = -\kappa(1 - s)(1 - \alpha)E_t(\Delta\hat{g}_{t+1})$$

Next, factoring the lag polynomial gives us:

$$(1 - \Lambda_1 L)(1 - \Lambda_2 L)E_t(\hat{\pi}_{t+2}) = -\kappa(1 - s)(1 - \alpha)E_t(\Delta\hat{g}_{t+1})$$

It is possible to prove both roots are unstable, therefore we solve them forward.

$$(-\Lambda_1 L)(-\Lambda_1 L)^{-1}(-\Lambda_2 L)(-\Lambda_2 L)^{-1}(1 - \Lambda_1 L)(1 - \Lambda_2 L)E_t(\hat{\pi}_{t+2}) = -\kappa(1 - s)(1 - \alpha)E_t(\Delta\hat{g}_{t+1})$$

$$(\Lambda_1 L)(\Lambda_2 L)[1 - (\Lambda_1 L)^{-1}][1 - (\Lambda_2 L)^{-1}]E_t(\hat{\pi}_{t+2}) = -\kappa(1 - s)(1 - \alpha)E_t(\Delta\hat{g}_{t+1})$$

Rearranging,

$$\hat{\pi}_t = \frac{-\kappa(1-s)(1-\alpha)}{\Lambda_1\Lambda_2} [1 - (\Lambda_1 L)^{-1}] E_t \{ \Delta \hat{g}_{t+1} + (\Lambda_2)^{-1} \Delta \hat{g}_{t+2} + (\Lambda_2)^{-2} \Delta \hat{g}_{t+3} + \dots \}$$

Taking this out even further,

$$\begin{aligned} \hat{\pi}_t &= \frac{-\kappa(1-s)(1-\alpha)}{\Lambda_1\Lambda_2} E_t \{ \Delta \hat{g}_{t+1} + (\Lambda_2)^{-1} \Delta \hat{g}_{t+2} + (\Lambda_2)^{-2} \Delta \hat{g}_{t+3} + \dots \\ &\quad (\Lambda_1)^{-1} [\Delta \hat{g}_{t+2} + (\Lambda_2)^{-1} \Delta \hat{g}_{t+3} + (\Lambda_2)^{-2} \Delta \hat{g}_{t+4} + \dots] + \dots \\ &\quad (\Lambda_2)^{-1} [\Delta \hat{g}_{t+3} + (\Lambda_2)^{-1} \Delta \hat{g}_{t+4} + (\Lambda_2)^{-2} \Delta \hat{g}_{t+5} + \dots] + \dots \} \end{aligned}$$

Simplifying further

$$\begin{aligned} \hat{\pi}_t &= \frac{-\kappa(1-s)(1-\alpha)(\rho-1)}{\Lambda_1\Lambda_2} \{ 1 + \rho(\Lambda_2)^{-1} + \rho^2(\Lambda_2)^{-2} + \dots \\ &\quad (\Lambda_1)^{-1} [\rho + (\Lambda_2)^{-1}\rho^2 + (\Lambda_2)^{-2}\rho^3 + \dots] + \dots \\ &\quad (\Lambda_2)^{-1} [\rho^2 + (\Lambda_2)^{-1}\rho^3 + (\Lambda_2)^{-2}\rho^4 + \dots] + \dots \} \hat{g}_t \end{aligned}$$

Doing it again,

$$\begin{aligned} \hat{\pi}_t &= \frac{-\kappa(1-s)(1-\alpha)(\rho-1)}{\Lambda_1\Lambda_2} \{ 1 + \rho(\Lambda_2)^{-1} + \rho^2(\Lambda_2)^{-2} + \dots \\ &\quad (\Lambda_1)^{-1} [\rho + (\Lambda_2)^{-1}\rho^2 + (\Lambda_2)^{-2}\rho^3 + \dots] + \dots \\ &\quad (\Lambda_1)^{-2} [\rho^2 + (\Lambda_2)^{-1}\rho^3 + (\Lambda_2)^{-2}\rho^4 + \dots] + \dots \} \hat{g}_t \end{aligned}$$

Getting there,

$$\begin{aligned} \hat{\pi}_t &= \frac{-\kappa(1-s)(1-\alpha)(\rho-1)}{\Lambda_1\Lambda_2} \left\{ \frac{1}{1 - (\rho/\Lambda_2)} + \left(\frac{\rho}{\Lambda_1}\right) \left(\frac{1}{1 - (\rho/\Lambda_2)}\right) + \left(\frac{\rho}{\Lambda_1}\right)^2 \left(\frac{1}{1 - (\rho/\Lambda_2)}\right) + \dots \right\} \hat{g}_t \\ \hat{\pi}_t &= \frac{-\kappa(1-s)(1-\alpha)(\rho-1)}{\Lambda_1\Lambda_2} \left\{ \frac{1}{[1 - (\rho/\Lambda_2)][1 - (\rho/\Lambda_1)]} \right\} \hat{g}_t \\ \hat{\pi}_t &= -\kappa(1-s)(1-\alpha)(\rho-1) \left\{ \frac{1}{[\Lambda_1 - \rho][\Lambda_2 - \rho]} \right\} \hat{g}_t \end{aligned}$$

Appendix C Lemma 5

Lemma 5. *Let $\rho \in (0, 1)$. Let $\underline{\rho}$ denote the smaller root of the equation $\beta s \kappa (1/\alpha - 1) \lambda \rho^2 - 2\beta \kappa \xi \rho + \kappa \xi (1 + \kappa \xi + \beta) - (1 + \kappa \xi) s \kappa (1/\alpha - 1) \lambda$. If the parameter set is configured such that $\underline{\rho} > 0$ (i.e. $\lambda < \frac{(1+\kappa\xi+\beta)\alpha\xi}{(1+\kappa\xi)s(1-\alpha)}$), then the impact response of consumption to a government spending shock is increasing in ρ if $\rho < \underline{\rho}$, and it is decreasing in ρ if the inequality is reversed.*

Proof. The partial derivative of χ with respect to ρ is given by

$$\frac{\partial \chi}{\partial \rho} = \frac{1-s}{s} \left[\frac{\beta s \kappa (1/\alpha - 1) \lambda \rho^2 - 2\beta \kappa \xi \rho + \kappa \xi (1 + \kappa \xi + \beta) - (1 + \kappa \xi) s \kappa (1/\alpha - 1) \lambda}{\Omega^2 (1 - \beta \rho + \kappa \xi)^2 (1 - \rho)^2} \right]$$

where $\Omega = 1 + \frac{s \kappa (1/\alpha - 1) \lambda \rho - \kappa \xi}{(1 - \beta \rho + \kappa \xi)(1 - \rho)}$.

Let $\underline{\rho}$ and $\bar{\rho}$ denote the two roots of the equation $\beta s \kappa (1/\alpha - 1) \lambda \rho^2 - 2\beta \kappa \xi \rho + \kappa \xi (1 + \kappa \xi + \beta) - (1 + \kappa \xi) s \kappa (1/\alpha - 1) \lambda = 0$. Hence, if $\rho < \underline{\rho}$ or $\rho > \bar{\rho}$, we have $\frac{\partial \chi}{\partial \rho} > 0$. And if $\underline{\rho} < \rho < \bar{\rho}$, $\frac{\partial \chi}{\partial \rho} < 0$.

We can show that $\underline{\rho} < 1$. Moreover, $\bar{\rho} > 1$ because of our restriction that guarantees a unique equilibrium.

If $\lambda < \frac{(1+\kappa\xi+\beta)\alpha\xi}{(1+\kappa\xi)s(1-\alpha)}$, then $\underline{\rho} > 0$. Therefore, $\frac{\partial \chi}{\partial \rho} > 0$ when $\rho < \underline{\rho}$, and $\frac{\partial \chi}{\partial \rho} < 0$ when $\rho > \underline{\rho}$. \square

Establishing an intuition for Lemma 5 is challenging. If $\lambda > \frac{(1+\kappa\xi+\beta)\alpha\xi}{(1+\kappa\xi)s(1-\alpha)}$, then $\underline{\rho} < 0$. The real interest rate channel with an active monetary policy is strong enough so that the size of the response of consumption on impact decreases in $\rho \in (0, 1)$. Lemma 5 focuses on the case where the impact response of consumption exhibits a hump-shaped pattern. This pattern also results from the interactions among the negative wealth effect, the real interest rate channel and the real wage channel, as explained in Lemma 4. One thing to add is that as long as $\beta > 0.2$, the peak of the consumption response occurs at a smaller ρ than that of the inflation response does, i.e. $\underline{\rho} < 1 - \left(\frac{\kappa[s(1/\alpha-1)\lambda-\xi]}{\beta} \right)^{\frac{1}{2}}$.

Mathematically, $\frac{\partial \chi}{\partial \rho}$ can be written as

$$\frac{\partial \chi}{\partial \rho} = \frac{\alpha(1 + \kappa \xi - \beta \rho)}{\kappa s (1 - \alpha)} \frac{\partial \gamma}{\partial \rho} - \frac{\alpha \beta \gamma}{\kappa s (1 - \alpha)}$$

The first term is the influence from current inflation and it governs the effects of the real wage channel. This term varies with $\frac{\partial \gamma}{\partial \rho}$, the change in the response of contemporaneous inflation with respect to the persistency of the government spending shock. The second term is related to the size of the increase in the real interest rate because it reflects the response

of expected inflation. A higher expected inflation caused by a more persistent government spending shock would result in a larger real interest rate provided that the monetary policy is active. Consequently, a larger real interest rate prevents private consumption from increasing. Because the negative sign of the second term, the peak of the consumption response occurs earlier than that of the inflation response.

References

- Akerlof, G. A., Dickens, W. T., Perry, G. L., Gordon, R. J., and Mankiw, N. G. (1996), “The Macroeconomics of Low Inflation,” *Brookings Papers on Economic Activity*, 1996(1), 1-76.
- Altonji, J. G. and Devereux, P. J. (2000), “The Extent and Consequences of Downward Nominal Wage Rigidity,” in *Research in Labor Economics*, 383-431.
- Auerbach, A. and Gorodnichenko, Y. (2012), “Measuring the Output Responses to Fiscal Policy,” *American Economic Journal: Macroeconomics*, 4, 1-27.
- Avouyi-Dovi, S., Fougère, D., and Gautier, E. (2013), “Wage Rigidity, Collective Bargaining, and the Minimum Wage: Evidence from French Agreement Data,” *Review of Economics and Statistics*, 95(4), 1337-1351.
- Barattieri, A., Basu, S., and Gottschalk, P. (2014), “Some Evidence on the Importance of Sticky Wages,” *American Economic Journal: Macroeconomics*, 6(1), 70-101.
- Barro, R. and Grossman, H. (1971), “A General Disequilibrium Model of Income and Employment,” *American Economic Review*, 61, 82-93.
- Bewley, T. F. (1998), “Why Not Cut Pay?” *European Economic Review*, 42(3), 459-490.
- Bihan, H. L., Montornès, J., and Heckel, T. (2012), “Sticky Wages: Evidence from Quarterly Microeconomic Data,” *American Economic Journal: Macroeconomics*, 4(3), 1-32.
- Bernanke, B. and Woodford, M. (1997) “Inflation Forecasts and Monetary Policy,” *Journal of Money, Credit and Banking*, 29, 653-84.
- Blanchard, O. and Perotti, R. (2002), “An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output.” *The Quarterly Journal of Economics*, 117, 1329-1368.

- Blinder, A. S. and Choi, D. H. (1990), “A Shred of Evidence on Theories of Wage Stickiness,” *Quarterly Journal of Economics*, 105(4), 1003-1015.
- Bordo, M. D., Erceg, C. J., and Evans, C. L. (2000), “Money, Sticky Wages, and the Great Depression,” *American Economic Review*, 90(5), 1447-1463.
- Bouakez, H. and Rebei, N. (2007), “Why Does Private Consumption Rise after a Government Spending Shock?” *Canadian Journal of Economics/Revue canadienne d'économique*, 40(3), 954-979.
- Campbell, C. M. and Kamlani, K. S. (1997), “The Reasons for Wage Rigidity: Evidence from a Survey of Firms,” *The Quarterly Journal of Economics*, 112(3), 759-789.
- Card, D. and Hyslop, D. (1997), “Does Inflation Grease the Wheels of the Labor Market?” *Reducing Inflation: Motivation and Strategy*, University of Chicago Press, 1997, 71-122.
- Christiano, L. (2004), “The Zero-Bound, Zero-Inflation Targeting, and Output Collapse,” *manuscript, Northwestern University*.
- Christiano, L., Eichenbaum, M., and Rebelo, S. (2011), “When Is the Government Spending Multiplier Large?” *Journal of Political Economy*, 119(1), 78-121.
- Daly, M., Hobijn, B., Lucking, B., et al. (2012), “Why Has Wage Growth Stayed Strong?” *FRBSF Economic Letter*, 10(2).
- Drautzburg, T. and Uhlig, H. (2015), “Fiscal Stimulus and Distortionary Taxation,” *Review of Economic Dynamics*, 18, 894-920.
- Druant, M., Fabiani, S., Kezdi, G., Lamo, A., Martins, F., and Sabbatini, R. (2012), “Firms’ Price and Wage Adjustment in Europe: Survey Evidence on Nominal Stickiness,” *Labour Economics*, 19(5), 772-782.
- Eggertsson, G. B. and Krugman, P. (2012), “Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach,” *Quarterly Journal of Economics*, 127(3), 1469-1513.
- Eggertsson, G. B. and Woodford, M. (2006), “Optimal Monetary and Fiscal Policy in a Liquidity Trap,” in *NBER International Seminar on Macroeconomics 2004*, 75-144.
- Fehr, E. and Goette, L. (2005), “Robustness and Real Consequences of Nominal Wage Rigidity,” *Journal of Monetary Economics*, 52(4), 779–804.

- Galí, J., López-Salido, J. D., and Vallés, J. (2007). “Understanding the Effects of Government Spending on Consumption,” *Journal of the European Economic Association*, 5(1), 227-270.
- Gottschalk, P. (2005), “Downward Nominal-Wage Flexibility: Real or Measurement Error?” *Review of Economics and Statistics*, 87(3), 556-568.
- Hall, R. (2009), “By How Much Does GDP Rise if the Government Buys More Output?” *Brookings Papers on Economic Activity*, Fall, 232-249.
- Kahn, S. (1997), “Evidence of Nominal Wage Stickiness from Microdata,” *American Economic Review*, 87(5), 993-1008.
- Lebow, D., Stockton, D., and Wascher, W. (1995), “Inflation, Nominal Wage Rigidity, and the Efficiency of Labor Markets,” Finance and Economics Discussion Series 94-95, Washington: Board of Governors of the Federal Reserve System, Division of Monetary Affairs (October).
- Mankiw, N. G. and Weinzierl, M. (2011), “An Exploration of Optimal Stabilization Policy,” *Brookings Papers on Economic Activity*, Spring, 209-254.
- Mertens, K. and Ravn, M. (2014), “Fiscal Policy in an Expectations Driven Liquidity Trap,” *Review of Economic Studies*, 81(4), 1637-1667.
- Ramey, V. (2011), “Identifying Government Spending Shocks: It’s All in the Timing,” *Quarterly Journal of Economics*, 126(1), 51-102.
- Ramey, V. and Zubairy, S. (2016), “Government Spending Multipliers in Good Times and in Bad: Evidence from U.S. Historical Data,” *Journal of Political Economy*, forthcoming.
- Rendahl, P. (2016), “Fiscal Policy in an Unemployment Crisis,” *Review of Economic Studies*, 83(3), 1189-1224.
- Rotemberg, J. (1982), “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187-1211.
- Schmitt-Grohé, S. and Uribe, M. (2016), “Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment,” *Journal of Political Economy*, 124(5), 1466-1514.
- Schmitt-Grohé, S. and Uribe, M. (2017), “Liquidity Traps and Jobless Recoveries,” *American Economic Journal: Macroeconomics*, 9(1), 165-204.

Sigurdsson, J. and Sigurdardottir, R. (2016), "Time-Dependent or State-Dependent Wage-Setting? Evidence from Periods of Macroeconomic Instability," *Journal of Monetary Economics*, 78, 50-66.

Woodford, M. (2011), "Simple Analytics of the Government Spending Multiplier," *American Economic Journal: Macroeconomics*, 3, 1-35.