When Words Speak Louder Without Actions

Doron Levit*

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Abstract

This paper studies communication and intervention as mechanisms of corporate governance. I develop a model in which a privately informed principal can intervene in the decisions of the agent if the latter disobeys her instructions. The main result shows that intervention can prompt disobedience and harm communication. Therefore, less information can be revealed by the principal in equilibrium when the principal has the ability to intervene in the agent's decisions. In this respect, words do speak louder without actions. The model is applied to managerial leadership, corporate boards, private equity, and shareholder activism.

*The author is from the University of Pennsylvania, Wharton School, Finance Department. Email: dlevit@wharton.upenn.edu.

"Actions speak louder than words, but not nearly as often." Mark Twain

Introduction

Corporations are governed in various ways. For example, visionary managers often rely on their ability to articulate a strategy and motivate their employees to achieve their goals, while others adopt a more authoritative leadership style and frequently overrule their subordinates. Furthermore, in many corporations the board of directors not only monitors the CEO, but also advises management on strategy, crisis management, M&A, etc. The mix between board supervision and counsel varies vastly across firms. Even savvy investors such as private equity, venture capital, and activist hedge funds, who often share ideas with their portfolio companies how to add value, have different styles of governance: While some investors are quick to exercise their control rights, others tend to work more constructively with management. In all of these principal-agent situations, contracts only partially resolve the conflicts of interests, and as a result, communication (i.e., transmission of information, using words) and intervention (i.e., forcing one's will, taking actions) are the primary mechanisms of governance.

The goal of this paper is to understand the interaction between communication and intervention, and study its implications for corporate governance. There is an obvious trade off between the two mechanisms: While communication is effective only if it is persuasive, intervention is more confrontational and costly. For this reason, intervention is often used as a last resort (Simon (1947)), and the anticipation of intervention can in and of itself affect the ability to exert influence through communication. In principle, the two mechanisms can either complement or substitute one another. Which one is it? In practice, the extent to which intervention (or the threat of) can be used depends on various characteristics of the organization and its leadership. In turn, effective communication can be associated with a culture in which dialogues flourish and information flows freely. How are the two related?

To study these questions, I consider a principal-agent model with incomplete contracts and a "top-down" information structure. In the model, the optimal scale of investment (or type of project) depends on the fundamentals of the firm. The principal ("she"), who is privately informed about these fundamentals, sends the agent ("he") a message which can be interpreted as a recommendation, instructions, or a nonbinding demand. Communication is modeled as cheap talk à la Crawford and Sobel (1982). The agent has private benefits from investment in large projects (e.g., empire-building or prestige), and due to this bias, the challenge of the principal is convincing the agent to choose small projects when the fundamentals are bad.[1] In equilibrium, information is never fully revealed by the principal, who has incentives to understate the fundamentals of the firm in order to prevent over-investment. The novel feature of the model is that after communicating with the agent and observing his investment decision, the principal can intervene and adjust the size of the project. For example, the principal can monitor the agent more closely, overrule his decisions, and even replace him if needed. Since these activities require resources and attention, intervention is costly to the principal.

As one might expect, a credible threat of intervention can reinforce the agent's compliance. Intuitively, if intervention is also costly to the agent (e.g., the loss of compensation, damaged reputation, or embarrassment), then the best way to avoid the unpleasant consequences of intervention is to follow the principal's instructions. In those cases, more information can be revealed by the principal in equilibrium and the two mechanisms complement one another. Surprisingly, however, the main result of the paper demonstrates that a credible threat of intervention can in fact *weaken* the incentives of the agent to follow the principal's instructions. In those cases, intervention prompts disobedience, *less* information is revealed by the principal in equilibrium, and the two mechanisms substitute one another. More generally, communication can be more effective when the principal faces a higher cost of intervention. Moreover, the adverse effect of intervention on communication is not trivial; it can offset the value of intervention as a correction device and result with an ex-ante welfare loss for the principal. Overall, the main result of the paper shows that communication is less effective with intervention than without it, and in this respect, *words do speak louder without actions*.

How can intervention harm communication? The analysis highlights two forces that explain

[1] The same results hold if instead the agent is biased toward smaller projects (e.g., seeking the "quite life").

this result. First, the agent is exploiting the fact that intervention is costly to the principal and intentionally chooses projects that are larger than what would have been optimal even from his own perspective. By "overshooting," the agent increases the cost the principal must incur in order to bring the project to its optimal level, thereby guaranteeing that the final project would be larger than optimal. Importantly, the principal understands the incentives of the agent to overshoot, and as a result, she has even stronger incentives to pretend that the fundamentals are worse than they really are. Since the agent foresees the stronger incentives of the principal to understate the benefit from large projects, he puts even less weight on the credibility of the principal's messages. Put differently, intervention creates a *vicious cycle*: The principal's attempt to prevent the agent from preempting her expected intervention, ultimately hurts the principal by further diminishing her ability to influence the agent's decision (through communication) in the first place. Altogether, less information is revealed in equilibrium, and in this respect, intervention harms communication.

The second force is more nuanced. Notice that because of the intrinsic conflict of interests, the principal never fully communicates her private information in equilibrium. The possibility of intervention, however, allows the agent to "elicit" additional information from the principal by ignoring her messages. To understand how, note that intervention is an informed decision. The principal intervenes more forcefully when the agent's decision is detrimental, that is, when over-investment is disastrous. Only those circumstances justify incurring the cost of intervention. While the agent cannot act after this new information is revealed, he can *condition* his initial decision on the information that will be reflected by the principal's intervention.[2] In particular, if the principal does not intervene following a large investment, the agent effectively "called the principal's bluff:" he can consume his private benefits without incurring the risk of large losses due to over-investment. If instead the principal intervenes, her corrective action benefits the agent since it reverses course exactly when the consequences of over-investment are detrimental. Knowing that the principal will intervene if and only if over-investment is detrimental emboldens the agent and results with more over-investment. Indeed, by over-

[2]This reasoning is similar to the intuition behind the effect of the winner curse in common value auctions and pivotal considerations in games with strategic voting.

investing the agent forces the principal to act based on private information she was trying to conceal, which gives the agent an opportunity to condition on these informative actions. Effectively, the agent challenges the principal to *"back her words with actions,"* and this way, the agent elicits additional information. Since the agent is less receptive to the principal's messages, less information can be revealed in equilibrium through communication.

The idea that intervention can harm communication holds more generally. In particular, communication can be more effective when the principal faces a *higher cost* of intervention. Intuitively, if intervention is more costly then the principal is less likely to intervene. The first force (i.e., the reasoning behind the "vicious cycle") implies that less overshooting is therefore needed by the agent to guarantee that the final project would be larger than optimal. As a result, the principal has weaker incentives to understate the benefit from large projects, and more information can be revealed in equilibrium. According to the second force, a higher cost of intervention implies that the agent cannot rely on the principal to intervene when his actions are detrimental. At the same time, the lack of intervention following over-investment is a weaker signal that large projects are optimal. Said otherwise, the informational benefit from challenging the principal to "back her words with actions" is smaller for the agent. Since the agent is more receptive to principal's messages when the cost of intervention is larger, more information can be revealed in equilibrium .

When intervention harms communication, a non-trivial trade-off emerges: While the ability to intervene allows the principal to correct the agent's decision whenever his actions are detrimental, it also weakens her ability to influence the agent's decision in the first place. Interestingly, the second effect can be strong enough to the extent that the principal is ex-ante better off without the ability to intervene in the agent's decisions. Specifically, I derive conditions under which the ability to intervene lowers the principal's expected utility in equilibrium if and only if communication with the agent is allowed. In other words, intervention has damaging welfare implications that stem exclusively from its negative effect on the ability of the principal to send messages and communicate with the agent. The idea that communication can in and of itself reduce the value of control rights is another novel aspect of my analysis.[3]

I consider several extensions of the baseline model.[4] In particular, I explore a variant of the model in which the agent is also privately informed. Interestingly, when an informed agent disobeys the principal, the principal cannot tell for sure if it is because the agent's private information contradicts her own information or because of their conflict of interests. This force deters the principal from intervening and inevitably emboldens the agent even more. As a result, with two-sided information asymmetry, intervention is even more likely to prompt disobedience, and less information can be revealed by the principal in equilibrium. In other words, with intervention, communication is less effective when the agent is also privately informed. In addition, I consider the effect of pay for performances contracts. I show that intervention is more likely to harm communication when the agent is offered a high pay for performances. Intuitively, while a higher pay for performances mitigates the agent's bias toward over-investment (and therefore, facilitates effective communication), it has a side-effect that emboldens the agent when intervention is possible. In particular, since a higher pay to the agent comes at the expense of the principal, per unit of utility, the principal is facing a higher cost of intervention while the agent is facing a lower cost from intervention. As as result, the positive effect of a higher pay for performances on communication is weaker when the principal also has the ability to intervene. Moreover, I demonstrate that intervention can harm communication even if the principal chooses the ex-ante optimal level of pay for performances.[5]

Building on these insights, the analysis offers several novel implications.[6] For example, the model suggests that a visionary leadership and a "hands-off" managerial style are more likely to be successful in large and complex organizations, in which employees' compensation is related to firm performances. The model also predicts that corporate boards are more likely to play a significant advisory role when the CEO of the firm is powerful (e.g., holds board

[3]This result does not imply that the principal is worse off with communication. As in Crawford and Sobel (1982), in my model the sender (i.e., the principal) is ex-ante better off when more information is communicated in equilibrium. This is true whether or not the sender can intervene. See Melamud and Shibano (1991) for an alternative cheap-talk game in which the sender can be ex-ante better off in equilibrium without communication.

[4]See Section 3 for details.

[5]The analysis considers only linear contracts (i.e., equity).

[6]See Section 4 for details.

chairmanship), has less to lose from being monitored (i.e., enjoy high reputation in the labor market or a generous severance package), his pay is sensitive to performances, the number of directors is large, or when directors are diverse and busy (e.g., hold other board seats). Additionally, the model implies that private equity investors can have their voice heard more effectively when they have reputation for being non-confrontational, when their fund has a large number of portfolio companies, when they co-invest with other investors (e.g., LBOs' club deals or VCs' syndicates), or when they have better exit options from their investment (e.g., booming IPO and M&A markets). Similarly, the ease at which activist hedge funds can launch a proxy fight (e.g., due to an easier proxy access or an improved coordination among institutional investors) or the difficulty of exiting and selling their shares (e.g., due to stock illiquidity) could in fact diminish their ability to influence the policy of their target companies.

Related literature

This paper is related to the literature on incomplete contracts. Aghion and Tirole (1997) study a model with intervention and communication, and show that a commitment not to overrule the agent is beneficial because it incentivizes the agent to acquire information.[7] This hold-up problem is absent from my model. Aghion and Tirole also assume that the uninformed agent always follows the recommendations of the principal,[8] and as a result, the models offer different predictions. In particular, in Aghion and Tirole (1997) the allocation of control is irrelevant if the agent is assumed to be uninformed, while my model predicts that a commitment not to intervene can be strictly optimal in those cases. In this respect, my model shares with Crémer (1995) the idea that the principal can benefit from "letting the agent live with the consequences of his actions." However, here the mechanisms are also quite different. First, in Crémer (1995) the principal benefits from being uninformed (about the agent's ability) because

[7] Similarly, Burkart, Gromb, and Panunzi (1997) show that intervention can undermine managerial initiatives, and Adams and Ferreira (2007) show that it can disincentivize CEOs to cooperate with their board.

[8] In Aghion and Tirole (1997) there always exists a "sufficiently negative" project such that the uninformed party is always better off choosing the informed party's most favored project over the risk of choosing the "sufficiently negative" project.

it results with a *more* "aggressive" firing decision,[9] which in turn, incentivizes the agent to exert more effort. By contrast, in my model "letting the agent live with the consequences of his actions" literally means that the principal (who is always informed) commits not to intervene in the agent's decision.[10] Second, since in Crémer (1995) the principal is initially uninformed, communication plays no role in his analysis. Importantly, this strand of the literature is silent about the effect of intervention on the *quality* of communication (i.e., the flow of information), which is the main focus of my analysis. Studying the interplay between these two mechanisms not only empirically relevant, but it also highlights a novel mechanism by which the allocation of control rights affects real outcomes.

This paper is also related to the literature on delegation. Starting with Dessein (2002), a number of papers studied the trade off between delegation and communication in organizations,[11] and in particular, its applications to optimal board structure (Adams and Ferreira (2007), Chakraborty and Yilmaz (2016), Harris and Raviv (2008)) and shareholder control (Harris and Raviv (2010)). In those models, the uninformed principal delegates decision rights to the informed agent, and delegation is beneficial because it avoids the distortion of the agent's private information when he communicates with his principal. By contrast, in my model the informed principal communicates with the uninformed agent. The trade-off is between backing this communication with intervention and solely relying on communication as a governance mechanism. Moreover, while the papers above imply that the benefit for the principal from retaining decision making authority is higher when communication (by the agent) is allowed, my analysis suggests that communication by the principal can in and of itself reduce the value of control rights. For this reason, my model offers new implications for corporate governance.

Related, Matthews (1989) studies a model in which the principal has the right to veto the agent's decision following a cheap-talk communication. Importantly, in Matthews (1989) the principal's private information is about her preferences, not the (common) value of the

[9] The firing decision of the uninformed principal in Crémer (1995) is more aggressive in the sense that the agent is fired if and only if output is low, irrespective of his true ability.

[10] In Crémer (1995), the principal does not benefit from forgoing the right to fire the agent.

[11] For example, see Agastya, Bag, and Chakraborty (2014), Alonso and Matouschek (2007), Grenadier, Malenko and Malenko (2017), Mylovanov (2008), and Harris and Raviv (2005).

project/task. As such, the agent values this information only to the extent that it affects the principal's decision to exercise her veto right, and as a result, a veto threat can only improve communication.[12] Shimizu (2017) analyzes a model in which the principal can exit the relationship following a cheap-talk communication with the agent. By exiting, the principal enforces an exogenously given outcome that is detrimental to the agent. Effectively, exit is a punishment in his model, and therefore, it can only improve communication. By contrast, in my model intervention prompts disobedience and harms communication. This result hinges on two features that are missing from existing models: (i) intervention allows the principal to change the agent's decision, and (ii) intervention is based on information that the agent values.[13]

Finally, existing models in which corporate leaders have informational advantage focus on the leader's role in coordinating the various activities of the firm (e.g., Hermalin (1998); Bolton, Brunnermeier and Veldkamp (2013)).[14] My paper contributes to this literature by showing that the ease at which corporate leaders can exercise their power diminishes their ability to influence others to voluntarily follow their vision.

1 Setup

Consider a principal-agent environment in which payoffs depend on action $x \in \mathbb{R}$ and a random variable θ that has a continuous probability density function f with full support over $\left[\underline{\theta}, \overline{\theta}\right]$. I refer to x as a choice of a project or the scale of investment. The principal's payoff is given by

$$U_P(x,\theta) = U_P(\theta,\theta) - L(|x-\theta|), \tag{1}$$

[12] Notice that intervention in my model is more flexible then a right to veto. The vicious cycle that intervention creates in my model stems from this flexibility.

[13] Models in which intervention reinforces compliance or improves communication violate at least one of these assumptions (e.g., Marino, Matsusaka, and Zábojník (2010), Van den Steen (2010), and Levit (2017)).

[14] Rotemberg and Saloner (1993, 2000) also focus on the vision aspect of leadership, but without modeling a top-down communication. See Bolton, Brunnermeier, and Veldkamp (2010) for a related survey.

where $L''(\cdot) > 0$ and $L(0) = L'(0) = 0$. Notice that $U_P(x, \theta)$ obtains a maximum at $x = \theta$. Similarly, the agent's payoff is given by

$$U_A(x, \theta; b) = U_A(\theta + b, \theta; b) - T(|x - (\theta + b)|), \tag{2}$$

where $T''(\cdot) > 0$ and $T(0) = T'(0) = 0$. Notice that $U_A(x, \theta, b)$ obtains a maximum at $x = \theta + b$. Without the loss of generality I assume $b > 0$. Effectively, b captures the intrinsic conflict of interests between the principal and the agent, where a larger b implies a larger conflict. Following Grossman and Hart (1986) and Hart and Moore (1990), I assume that contracts are incomplete. In particular, the agent and the principal cannot contract over actions or the communication protocol. Alternatively, b can be interpreted as the residual conflict of interests between the principal and the agent following a contract that is agreed by both parties.[15]

The model has four stages:

1. The first stage involves communication between the principal and the agent. The principal is privately informed about θ. For simplicity, I assume that the principal perfectly observes θ while the agent is uninformed. These assumptions are relaxed in Section 3.1. Based on her private information, the principal sends the agent a message $m \in [\underline{\theta}, \overline{\theta}]$. In line with a standard cheap talk framework, the principal's information about θ is non-verifiable and the content of m does not affect the agent's or the principal's payoff directly. These assumptions capture the informal nature of communication. I denote by $\mu(m|\theta) \in [0, 1]$ the probability of sending message m conditional on θ.

2. In the second stage, the agent observes the message from the principal and chooses project x. I denote by $x(m) \in \mathbb{R}$ the decision rule of the agent conditional on message m.[16]

3. The third stage is the key departure of the model from the existing literature. The principal

[15] See Section 3.2 for the effect of pay for performances contracts on the analysis.

[16] Since the agent is indifferent with zero probability, I restrict attention to pure strategies.

observes the agent's decision and decides whether to intervene and override it. Formally, let $\Delta(x,\theta) \in \mathbb{R}$ be the principal's decision. If $\Delta = 0$ then the principal effectively did not intervene and the agent's decision is implemented. In this case, neither the principal nor the agent incurs any costs. If $\Delta \neq 0$ then the principal intervenes and the implemented project is $x - \Delta$.[17] In this case, the principal incurs a cost of $\delta C(|\Delta|)$ where $C''(\cdot) > 0$ and $C(0) = C'(0) = 0$. The agent also incurs a cost of $\tau K(|\Delta|)$ where $K''(\cdot) > 0$ and $K(0) = K'(0) = 0$. Intuitively, increasing marginal cost of intervention implies that intervention has more consequences when it is more aggressive. In this respect, Δ can be interpreted as the intensity of the principal's intervention. Notice that these functional forms assume away fixed costs from intervention, which are needed for the tractability of the model. Section 3.3 analyzes a variant of this model with a fixed intervention cost. Scalars $\delta > 0$ and $\tau > 0$ parametrize the intensity of these cost functions.[18] Crawford and Sobel (1982) is a special case of this model when intervention is prohibitively costly (i.e., $\delta = \infty$).

4. Payoffs are realized and distributed to the principal and the agent. The principal and the agent maximize their expected utilities, which are given respectively by

$$U_P(\theta, x, \Delta) = U_P(x - \Delta, \theta) - \delta C(|\Delta|) \tag{3}$$

and

$$U_A(\theta, x, \Delta; b) = U_A(x - \Delta, \theta; b) - \tau K(|\Delta|). \tag{4}$$

Solution concept

A Perfect Bayesian Equilibrium of the game consists of three parts: The principal's communication strategy $\mu^*(m|\theta)$, the agent's decision rule $x^*(m)$, and the principal's intervention policy

[17] Equivalently, the principal's intervention policy can be defined as the choice of the project that is eventually implemented, rather than the difference between the final project and the agent's decision.

[18] The analysis does not depend on the cost of intervention itself, but rather on the belief of the agent about the principal's self-perception of this cost. For example, if it is a common knowledge that the principal underestimates (overestimates) the difficulty of intervening in the agent's decision, for all purposes of the analysis, the relevant cost of intervention is lower (higher).

$\Delta^*(x,\theta)$. Specifically, the equilibrium is defined as follows: (i) For any realization $\theta \in [\underline{\theta}, \overline{\theta}]$, if m^* is in the support of $\mu(\cdot|\theta)$, then m^* maximizes the expected utility of the principal given the agent's decision rule $x^*(\cdot)$ and the intervention policy $\Delta^*(x,\theta)$; (ii) for any message m, project $x^*(m)$ maximizes the expected utility of the agent, taking into account the principal's communication strategy $\mu^*(m|\theta)$ (in order to update her prior about θ) and intervention policy $\Delta^*(x,\theta)$; (iii) for any realization $\theta \in [\underline{\theta}, \overline{\theta}]$ and $x \in \mathbb{R}$, $\Delta^*(x,\theta)$ maximizes the expected utility of the principal. Finally, all players have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium. Moreover, the agent uses Bayes' rules to update their beliefs from the principal's message about θ.

2 Analysis

This section provides the main analysis of the model. Section 2.1 studies the communication game without intervention. Section 2.2 analyzes the intervention policy of the principal. Section 2.3 analyzes the full model, where both communication and intervention are allowed. Section 2.4 studies the effect of intervention on communication and discusses the implications of the model for compliance and disobedience. Section 2.5 considers the welfare implications of intervention. All omitted proofs are given in the Appendix.

2.1 Communication without intervention

Consider the communication model when intervention is not feasible ($\delta = \infty$). Since projects are non-contractible, the agent always chooses the project that maximizes his expected utility conditional on her beliefs about θ upon message m. Communication can affect the agent's decision only by changing his beliefs. As shown by Crawford and Sobel (1982), all equilibria of this communication game are characterized by a partition of $[\underline{\theta}, \overline{\theta}]$, where the principal introduces noise into his signal by only specifying the partition element to which the realized state belongs. Specifically, let $(a_0, ..., a_N)$ denote a partition of $[\underline{\theta}, \overline{\theta}]$ with N steps where

$\underline{\theta} = a_0 < a_1 ... < a_N = \overline{\theta}$. Also, define

$$\overline{x}(\underline{a}, \overline{a}) \equiv \arg\max_x \int_{\underline{a}}^{\overline{a}} U_A(x, \theta, b)\, dF(\theta). \tag{5}$$

The following result is a variant of Theorem 1 in Crawford and Sobel.

Proposition 1 (Communication). *Consider the communication game without intervention. There exists a positive integer N^* such that for every $n \in \{1, ..., N^*\}$ there exists at least one equilibrium, where*

$$\mu^*(\overline{x}(a_{i-1}, a_i) | \theta) = 1 \text{ if } \theta \in (a_{i-1}, a_i) \tag{6}$$

$$x^*(m) = \overline{x}(a_{i-1}, a_i) \text{ if } m \in (a_{i-1}, a_i) \tag{7}$$

$$U_P(\overline{x}(a_i, a_{i+1}), a_i) - U_P(\overline{x}(a_{i-1}, a_i), a_i) = 0; \text{ for } i = 1, ..., n-1 \tag{8}$$

$$a_0 = \underline{\theta} \text{ and } a_n = \overline{\theta}. \tag{9}$$

Moreover, all other equilibria are economically equivalent to those in this class for some value of $n \in \{1, ..., N^*\}$.[19]

In equilibrium, only a finite number $n \leq N^*$ of projects are implemented with positive probability, and the states of nature for which each of these projects is best for the agent form an interval, and these intervals form a partition of $[\underline{\theta}, \overline{\theta}]$. The partition $(a_0, ..., a_N)$ is determined by (8), a well-defined second-order linear difference equation, and (9), its initial and terminal conditions. Equation (8) requires the principal to be indifferent between the associated values of x when θ falls on the boundaries between steps. Given our assumptions about U_P, this condition is necessary and sufficient for the principal's communication strategy to be a best response to $x(m)$. Similarly, (7) gives a best response of the agent to the communication strategy (6).

[19] Other equilibria are economically equivalent in the sense that the mapping between states to actions is the same. For example, instead of sending message $\overline{x}(a_{i-1}, a_i)$ if $\theta \in (a_{i-1}, a_i)$, the principal can mix and send a message that is uniformly distributed over (a_{i-1}, a_i).

2.2 Intervention

Suppose the principal observes θ and the agent's decision is x. The agent may choose $x \neq \theta$ either because he lacks sufficient information about θ or because he is biased. Either way, and regardless of the message the principal sent the agent beforehand, the principal's optimal intervention policy solves

$$\Delta^* \in \arg\max_{\Delta} \{U_P(\theta, \theta) - L(|x - \theta - \Delta|) - \delta C(|\Delta|)\}. \quad (10)$$

Note that Δ^* is a function of $x - \theta$. The next result follows.

Lemma 1. *The principal's optimal intervention policy, $\Delta^*(x - \theta)$, is given by the unique solution of*

$$L'(|x - \theta - \Delta|) = \delta C'(|\Delta|), \quad (11)$$

and it has the following properties:

(i) $\Delta^*(0) = 0$ *and if $x \neq \theta$ then $|\Delta^*(x - \theta)| < |x - \theta|$.*

(ii) $\Delta^*(\cdot)$ *is a strictly increasing and continuous function with a slope strictly smaller than one.*

(iii) If $x \neq \theta$ then $|\Delta^*(x - \theta)|$ strictly decreases in δ, $\lim_{\delta \to 0} |\Delta^*(x - \theta)| = |x - \theta|$, and $\lim_{\delta \to \infty} |\Delta^*(x - \theta)| = 0$.

According to Lemma 1 part (i), $|\Delta^*| \leq |x - \theta|$. Intuitively, by choosing $|\Delta| \leq |x - \theta|$ the principal moves the project closer to her ideal point and at the same time minimizes the cost from intervention. Part (ii) implies that $\frac{\partial |\Delta^*|}{\partial x} > 0 \Leftrightarrow x > \theta$. Intuitively, if $x > \theta$ ($x < \theta$) and x increases, then the agent's decision is moving further away from (closer to) the principal's ideal point, and as a result, the principal has stronger (fewer) incentives to incur the effort needed to mitigate the distortion in the agent's decision. In other words, the principal's intervention policy is more aggressive when the agent's decision is more detrimental. Part (iii) confirms our intuition that, everything else held equal, the principal intervenes more when it is less costly to do so.

To illustrate the effect of intervention, I will use quadratic utility and cost functions as an example throughout the analysis.[20]

Example (Quadratic functional form). *Suppose* $U_P(x,\theta) = A - (\theta - x)^2$, $U_A(x,\theta;b) = A - (x - \theta - b)^2$, *and* $C(|\Delta|) = K(|\Delta|) = \Delta^2$. *Under these assumptions,*

$$\Delta^*(x-\theta) = \frac{x-\theta}{1+\delta}. \tag{12}$$

Before proceeding with the analysis, I assume from now on that the intensity of the principal's intervention increases at a higher rate as the agent's decision is moving further away from the principal's ideal point, that is, $\frac{\partial^2 \Delta^*}{\partial^2 x} \geq 0 \Leftrightarrow x > \theta$. To guarantee this condition, which is used in the proof of Lemma 3 below, I assume

$$C'''(\cdot) \leq 0 \leq L'''(\cdot).^{21} \tag{13}$$

2.3 Communication and intervention

The principal and the agent have rational expectations about the principal's intervention policy as characterized by Lemma 1. Therefore, accounting for the optimal intervention policy, the principal's indirect utility is $V_P(x,\theta) \equiv U_P(\theta, x, \Delta^*(x-\theta))$. Since $\Delta^*(0) = 0$, the principal's indirect utility obtains its maximum at $x = \theta$.

Lemma 2. *The principal's indirect utility is a continuous single-peaked function which obtains its maximum at* $x = \theta$. *Specifically,*

$$V_P(x,\theta) = V_P(\theta,\theta) - l(|x-\theta|) \tag{14}$$

[20] Quadratic utility functions were used by Crawford and Sobel (1982) in their leading example.

[21] Assumption (13) is used for the proof (17) has a unique solution. But this assumption is not necessary. For example, the solution of (17) is unique as long as τ is sufficiently small.

where $V_P(\theta,\theta) = U_P(\theta,\theta)$,

$$l\left(|x-\theta|\right) \equiv L(|x-\theta-\Delta^*(x-\theta)|) + \delta C(|\Delta^*(x-\theta)|), \tag{15}$$

$l'\left(0\right) = 0$, *and* $l''\left(\cdot\right) > 0$.

Similarly, the agent's indirect utility is given by $U_A(\theta, x, \Delta^*(x-\theta); b)$. However, unlike the principal, the possibility of intervention changes the agent's ideal point.

Lemma 3. *The agent's indirect utility is a continuous single-peaked function which obtains its maximum at* $x = \theta + \beta$, *where*

$$\beta = \pi + \left(C'\right)^{-1}\left(\frac{L'\left(\pi\right)}{\delta}\right) \tag{16}$$

and $\pi \in (0, b]$ *is the unique solution of*

$$T'(b-\pi) = L''(\pi)\frac{\tau K'(\left(C'\right)^{-1}\left(\frac{L'(\pi)}{\delta}\right))}{\delta C''(\left(C'\right)^{-1}\left(\frac{L'(\pi)}{\delta}\right))}. \tag{17}$$

Moreover, β *strictly increases in* b.

According to Lemma 3, with intervention the agent behaves as if his bias is β. From (11) and (16), it follows that β satisfies

$$\theta + \beta - \Delta^*\left(\beta\right) = \theta + \pi. \tag{18}$$

That is, the agent's indirect utility is maximized when as a response to choosing $x = \theta + \beta$ the principal's intervention results with a project of size $\theta + \pi$. Since $\pi \in (0, b]$, the final project is smaller than the agent's original ideal point, $\theta + b$. Moreover, since $\beta > \pi$, the agent's optimal project, $\theta + \beta$, "overshoots" relative to the project that is eventually implemented by the principal. The agent's incentives to overshoot will play a key role in the analysis below.

Since there is a one-to-one mapping between β and b, the agent's indirect utility can be defined as a function of β, $V_A(x,\theta;\beta)$, where

$$V_A(x,\theta;\beta) = V_A(\theta+\beta,\theta;\beta) - t(|x-(\theta+\beta)|), \tag{19}$$

$t'(0) = 0$, and $t''(\cdot) > 0$.

Example. *Under quadratic utility and cost functions,* $\pi = b\frac{\delta^2}{\tau+\delta^2}$ *and*

$$\beta = b\frac{1+\delta}{\tau/\delta+\delta}. \tag{20}$$

Moreover, the indirect utility functions of the principal and the agent are, respectively,

$$V_P(x,\theta) = A - \frac{\delta}{1+\delta}(\theta-x)^2 \tag{21}$$

and

$$V_A(x,\theta;\beta) = A - \frac{\tau+\delta^2}{(1+\delta)^2}\left[(\theta-x+\beta)^2 + \frac{\tau}{\delta^2}\beta^2\right]. \tag{22}$$

The indirect utility functions $V_P(x,\theta)$ and $V_A(x,\theta;\beta)$ have the same generic properties as the utility functions $U_P(x,\theta)$ and $U_A(x,\theta;b)$, and therefore, the communication between the principal and the agent in equilibrium of the game with intervention features similar properties to the equilibrium of the game without intervention. The next result immediately follows from this observation.

Proposition 2. *Consider the communication game with intervention. There exists a positive integer* N^{**} *such that for every* $n \in \{1,...,N^{**}\}$ *there exists at least one equilibrium in which* $x^*(m)$ *and* $\mu^*(m|\theta)$ *are as characterized in Proposition 1, with the exception that the utility functions* $U_P(x,\theta)$ *and* $U_A(x,\theta;b)$ *are replaced everywhere by* $V_P(x,\theta)$ *and* $V_A(x,\theta;\beta)$*, respectively. Furthermore, for every* θ *and* x*, the intervention policy of the principal in equilibrium,* $\Delta^*(x-\theta)$*, is characterized by Lemma 1.*

2.4 The effect of intervention on communication

This section studies the effect of intervention on communication by comparing equilibria of a game with intervention to equilibria of a game without intervention. Although in both cases the communication between the principal and the agent features the same generic properties (e.g., the equilibrium forms a partition on $[\underline{\theta}, \overline{\theta}]$), a comparison between the two games is not immediate. Below, I show that when θ is a uniformly distributed on $[\underline{\theta}, \overline{\theta}]$, the comparison between the two games is reduced to a comparison between β and b. Note that a uniform distribution is often assumed in the cheap talk literature to gain tractability (e.g., Dessein (2002), Adams and Ferreira (2007), Chakraborty and Yilmaz (2016), and Harris and Raviv (2005, 2008, 2010)), and it is also assumed by Crawford and Sobel in their leading example.[22] I start with the following observation.

Lemma 4. *Suppose θ is uniformly distributed over $[\underline{\theta}, \overline{\theta}]$ and consider the communication game without intervention. The set of equilibria with utility functions $U_P(x,\theta)$ and $U_A(x,\theta;b)$ is identical to the set of equilibria with utility functions $V_P(x,\theta)$ and $V_A(x,\theta;b)$. Moreover, in both cases,*

$$\overline{x}\left(a_{i-1}, a_i\right) = \frac{a_{i-1}+a_i}{2} + b \quad \textit{and} \quad a_i = \frac{\overline{x}\left(a_{i-1}, a_i\right) + \overline{x}\left(a_i, a_{i+1}\right)}{2} \quad \textit{for} \quad i = 1, ..., N-1, \tag{23}$$

the elements of the partition satisfy

$$a_{i+1} - a_i = a_i - a_{i-1} - 4b, \tag{24}$$

[22] The uniform distribution assumption is mainly used in the proof of Lemma 4 to show that $\overline{x}\left(\underline{a}, \overline{a}\right) = \frac{\underline{a}+\overline{a}}{2} + b$ as long as the agent's utility is a single-peaked function with an ideal point $\theta + b$. However, it is not necessary for the main result. For example, under quadratic utility and cost functions $\overline{x}\left(\underline{a}, \overline{a}\right) = E\left[\theta | \theta \in \left(\underline{a}, \overline{a}\right)\right] + b$ even if $U_A(x,\theta;b)$ is replaced by $V_A(x,\theta;b)$. To the extent that the quality of communication decreases with the bias of the agent (e.g., under the conditions in Theorem 2 in Crawford and Sobel (1982)), Proposition 3 extends to non-uniform distributions. As another example, similar to the analysis in Section 2.4.1 below, a previous version of this paper showed that if the agent is limited to choosing between only two actions (e.g., either keep or change the status quo), then intervention can harm communication for any distribution of θ.

and the size of the partition under the most informative equilibrium is

$$N(b) = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + 2\frac{\overline{\theta} - \underline{\theta}}{b}} \right\rceil .^{23} \tag{25}$$

According to Lemma 4, the communication between the principal and the agent in equilibrium does not change if the utility functions $U_P(x, \theta)$ and $U_A(x, \theta; b)$ are replaced by $V_P(x, \theta)$ and $V_A(x, \theta; b)$, respectively. In fact, the main property that is required for this result (in addition to the uniform distribution) is that these utility functions are single-peaked with ideal points $\theta + b$ and θ, respectively. Under these assumptions, the agent's best response satisfies (23) and the solution of (8) requires (24) to hold. According to condition (24), the size of a partition element is $4b$ smaller than the size of the preceding one. Intuitively, since the agent is biased toward larger projects ($b > 0$), the challenge of the principal is convincing the agent to choose smaller projects. Indeed, the agent is worried that the principal understates the benefit from large projects. As a result, the principal has a lower credibility when she is sending a message that θ is small. The lower credibility is reflected by a larger interval, which means that less information is communicated by the principal.

An immediate corollary of Lemma 4 is that the communication stage in the equilibrium of a game with intervention when the agent's bias is b is *identical* to the communication equilibrium of a game without intervention, when the bias of the agent is β.

Corollary 1. *If θ is uniformly distributed over $[\underline{\theta}, \overline{\theta}]$ then $N^* = N(b)$ and $N^{**} = N(\beta)$.*

Since the transmitted information is more precise when the partition is finer, intervention harms (enhances) communication if $N^{**} < N^*$ ($N^{**} > N^*$).[24] Notice that the size of the largest partition that can arise in equilibrium, $N(b)$, decreases with b. Therefore, the quality of communication between the principal and the agent improves as their preferences become

[23] The notation $\lceil r \rceil$ is used to indicate the smallest integer greater than or equal to r.

[24] This definition does not take a stand on which equilibrium is selected. Crawford and Sobel (1982) proved that the equilibrium with the largest number of elements in the partition Pareto dominates any other equilibrium, and therefore, it should be selected. See also Chen, Kartik, and Sobel (2008), who provide an alternative justification for the selection of the most informative equilibrium in cheap-talk games.

more similar. Importantly, since $N^* = N(b)$ and $N^{**} = N(\beta)$, the effect of intervention on communication depends on how β compares to b. For example, note that $N(b) = 2$ if and only if $\frac{\overline{\theta}-\underline{\theta}}{12} \leq b < \frac{\overline{\theta}-\underline{\theta}}{4}$. Thus, if $\beta < \frac{\overline{\theta}-\underline{\theta}}{12}$ then $N(\beta) \geq 3$ and intervention enhances communication. However, if $\frac{\overline{\theta}-\underline{\theta}}{4} \leq \beta$ then $N(\beta) = 1$ and intervention harms communication. The next result follows.

Proposition 3. *If θ is uniformly distributed over $\left[\underline{\theta}, \overline{\theta}\right]$ then intervention harms (enhances) communication only if $\beta > b$ ($\beta < b$). Moreover, if $\beta > b$ ($\beta < b$) then there exists $\overline{\theta} - \underline{\theta} > 0$ such that intervention harms (enhances) communication.*

The observation that intervention can enhance communication is intuitive. Recall that according to Lemma 1, everything else being equal, the principal is more likely to intervene when the agent chooses a project that is further away from θ. The fear from intervention increases the incentives of the agent to choose projects that are closer to the agent's best estimate of the principal's ideal point. These incentives are particularly strong when the cost of intervention for the principal are low (small δ) and the adverse consequences for the agent are severe (large τ). If the agent is more likely to choose a project that is closer to the principal's ideal point, the principal has fewer incentives to pretend that the benefit from large projects is lower than it really is. That is, the principal does not need to manipulate her private information as much to counter the agent's bias toward larger project. Therefore, more information can be revealed in equilibrium. In this respect, intervention enhances communication.

More surprising is the observation that intervention can harm communication. Before explaining the intuition behind this result, the next lemma demonstrates that $\beta > b$ is feasible.

Lemma 5. *There are $0 < \underline{\tau} \leq \overline{\tau} < \infty$ and $0 < \underline{\delta} \leq \overline{\delta} < \infty$ such that:*

(i) If $\tau < \underline{\tau}$ or $\delta > \overline{\delta}$ then $\beta > b$. Moreover, if $\delta > \overline{\delta}$ then β is decreasing in δ and $\lim_{\delta \to \infty} \beta = b$.

(ii) If $\tau > \overline{\tau}$ or, $\tau > 0$ and $\delta < \underline{\delta}$, then $\beta < b$.

Example. *Under quadratic utility and cost functions,*

$$\beta > b \Leftrightarrow \delta > \tau. \tag{26}$$

Figure 1 plots $\beta/b = \frac{1+\delta}{\tau/\delta+\delta}$ *as a function of* δ*. For a given value of* τ*,* $\beta > b$ *whenever the corresponding curve is above the black dashed line.*

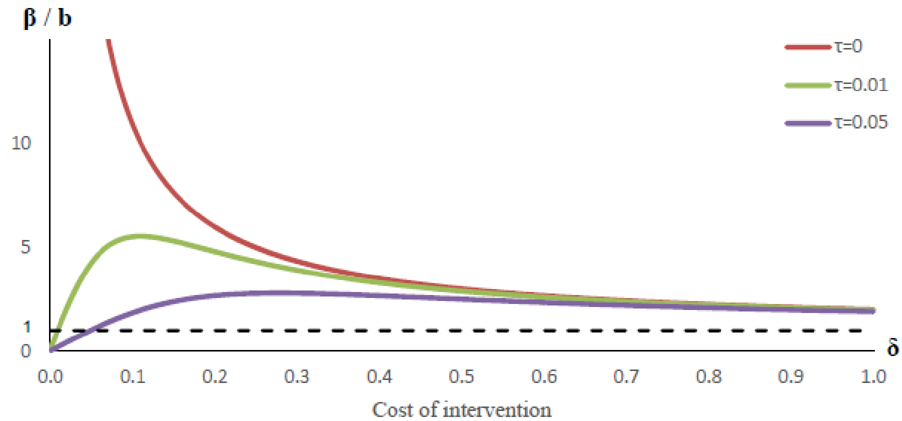

Fig. 1 - The ratio between β and b under quadratic utility and cost functions

How can intervention harm communication? There are two forces at work. First, everything else held equal, if δ is large (intervention is costly to the principal) or τ is small (intervention is not costly to the agent), the agent has incentives to choose larger projects than what he would have chosen without the possibility of intervention. Essentially, the agent is exploiting the fact that intervention is relatively more costly to the principal and intentionally "overshoots" by choosing a project that is larger than his best estimate of $\theta+b$. By doing so, the agent increases the cost that the principal has to incur in order to move the project closer to its optimal scale, θ. As a result, the project that is eventually implemented is closer to $\theta + b$. Importantly, the principal understands the incentives of the agent to overshoot. To mitigate this behavior, the principal has even stronger incentives than before to pretend that θ is smaller than it really is. However, since the agent foresees the stronger incentives of the principal to understate the benefit from large projects, he puts even less weight on the credibility of her messages. In this respect, intervention creates a vicious cycle: The principal's attempt to prevent the agent from preempting her expected intervention, ultimately hurts the principal by diminishing her ability to influence the agent's decision in the first place through communication. Altogether, less information is revealed in equilibrium, namely, intervention harms communication.

The incentives of the agent to overshoot are particularly strong when τ is small, as overshooting is less costly in those cases. Therefore, β decreases with τ. Related, the incentives of the agent to overshoot decrease with δ for large values of δ. Intuitively, if intervention is less costly to the principal, the agent can expect more intervention, and therefore, more overshooting is needed in order to guarantee that the final project is closer to $\theta + b$. Since β decreases with δ in this range, communication tends to be more effective when the principal's cost of intervention is higher. Note that if $\tau = 0$ then β decreases in δ for any value of δ. However, if $\tau > 0$ and δ is small then β actually increases with δ. Intuitively, in this range the cost of overshooting is particularly large, and since a smaller δ implies more intervention, the agent will refrain from overshooting.

The second force in play is a bit more subtle. In equilibrium, the principal does not fully reveal the value of θ at the communication stage. The agent learns that $\theta \in (a_{i-1}, a_i)$, but not the exact location of θ in this interval. Interestingly, the possibility of intervention allows the agent to "elicit" additional information from the principal. To understand how, note that intervention (and its intensity) is an informed decision. Indeed, as was argued in Lemma 1, the principal intervenes more intensively when the agent's decision is more detrimental, i.e., when $|x - \theta|$ is large. Only those circumstances justify incurring the cost of intervention. For example, recall that $\frac{a_{i-1}+a_i}{2} < \overline{x}\,(a_{i-1}, a_i) < a_i$. Therefore, the principal intervenes more when θ is closer to a_{i-1} than when it is closer to a_i. Inevitably, intervention reveals information about the location of θ in the interval (a_{i-1}, a_i), information the principal was trying to conceal when communicating with the agent. While the agent cannot act after this new information is revealed (i.e., he cannot voluntary revise his initial decision), he can *condition* his initial decision on the information that will be reflected by the principal's intervention policy. Knowing that the principal will intervene only when large projects are highly unprofitable emboldens agent, who in turn, takes more "risk" by choosing larger projects. Therefore, if τ is small, the agent has stronger incentives to “ignore” the principal’s messages, thereby relying more heavily on the information that is reflected by her intervention policy. Essentially, the agent challenges the principal to “back her words with actions”. Since the agent is less receptive to the principal’s messages, less information is revealed in equilibrium.

Remark on commitment: Throughout, it was assumed that the principal’s intervention policy is ex-post optimal. Instead, if the principal could commit to her ex-ante optimal intervention policy (i.e., $\Delta \equiv x - \theta$), the agent would have no control on the project that is eventually implemented. As long as $\tau > 0$, the agent will have strict incentives to minimize the intensity of the principal’s intervention policy by choosing a project that is as close as possible to his best estimate of θ. Effectively, the agent will behave as if he has no bias, and consequently, information will be fully revealed in equilibrium (rendering the need to intervene on the equilibrium path). Note, however, that this argument requires the agent to be believe that the principal will intervene even if it is ex-post sub-optimal. As long as the agent has some doubts, the analysis above continues to hold in the sense that a partial commitment of this sort is equivalent to assuming that the principal’s cost of intervention is lower.

2.4.1 Compliance and disobedience

This section considers a binary version of the model which will serve two purposes: (i) highlighting the agent’s informational benefit from challenging the principal to “back her words with actions;” (ii) explaining the effect of intervention on the agent’s disobedience.

Suppose $N(b) = 2$. Therefore, under the most informative equilibrium of a game without intervention there are exactly two different types of messages, and each type triggers a different project. Specifically, there is a cutoff $\theta^* \in (\underline{\theta}, \overline{\theta})$ such that the principal reveals the location of θ relative to θ^*, and if $\theta > \theta^*$ ($\theta < \theta^*$) the agent chooses project $x = x_R$ ($x = x_L$). According to Lemma 4, $x_L < \theta^* < x_R$ and $\theta^* = \frac{x_L + x_R}{2}$. Since $b > 0$, the challenge of the principal is convincing the agent to choose x_L when $\theta < \theta^*$. Since $N(b) = 2$, the agent follows these instructions in equilibrium:

$$E\left[U_A\left(x_L,\theta;b\right)|\theta<\theta^*\right]\geq E\left[U_A\left(x_R,\theta;b\right)|\theta<\theta^*\right]. \tag{27}$$

How does intervention affect the agent's incentives to choose x_L? To answer this question, consider the game with intervention, but suppose that the only feasible projects are x_L and x_R. By construction, intervention does not change the preferences of the principal; she still prefers x_L over x_R if and only if $\theta<\theta^*$. Therefore, the principal follows the same communication strategy as before. In this thought experiment, if the principal recommends on x_L (i.e., she sends a message $\theta<\theta^*$) but the agent disobeys her and chooses x_R, the principal intervenes whenever

$$-L\left(|\theta-x_L|\right)-\delta C\left(x_R-x_L\right)>-L\left(|x_R-\theta|\right). \tag{28}$$

In the Appendix I show that there exists $\theta^{**}<\theta^*$ such that (28) holds if and only if $\theta<\theta^{**}$.[25] Intuitively, the principal is willing to incur the cost of downsizing the project only if the benefit from doing so is sufficiently large. In fact, θ^{**} decreases with δ, as the willingness to intervene decreases with its cost. Suppose $\tau=0$. Expecting this intervention policy, the agent complies with the principal's instructions to choose x_L if and only if

$$E\left[U_A\left(x_L,\theta;b\right)|\theta^{**}<\theta<\theta^*\right]\geq E\left[U_A\left(x_R,\theta;b\right)|\theta^{**}<\theta<\theta^*\right]. \tag{29}$$

Intervention prompts disobedience if and only if condition (29) is violated while condition (27) holds.[26]

Proposition 4. *Suppose $N(b)=2$ and $\tau=0$. There exits $\delta_{disobedience}>0$ such that intervention prompts disobedience if and only if $\delta<\delta_{disobedience}$.*

How can intervention prompt disobedience? Recall that in equilibrium the principal reveals

[25] Implicitly, I assume that if the principal is indifferent between intervention and nonintervention, then she does not intervene.

[26] If $\tau>0$ then the agent must also account for the harm that intervention might inflict, but as long as τ is sufficiently small, intervention would still prompt disobedience.

the position of θ relative to θ^*, but she does not reveal additional information out of fear that the biased agent will choose x_R if θ is sufficiently close to θ^*. At the same time, the agent knows that by disobeying the principal he can effectively force her to reveal the location of θ relative to θ^{**}, information she was trying to conceal. Indeed, if the principal does not intervene then it must be $\theta^{**} < \theta < \theta^*$. The agent benefits from calling the principal's bluff since it allows him to "consume" his private benefits from over-investment without scarifying too much value. However, if the principal intervenes and forces the small project then it must be $\theta < \theta^{**}$. In those cases, the agent himself does not find the large project very attractive and the correction of his initial decision is in fact desirable. Either way, the agent can condition his own decision on the decision of the principal to intervene. Intervention prompts disobedience because it allows the agent to challenge the principal to "back her word with actions," which embeds an informational benefit for the agent. Notice that as δ rises, the principal is less likely to intervene (θ^{**} decreases with δ) and so the informational content of nonintervention is smaller, implying the agent is less likely to disobey. This logic explains Proposition 4.

Finally, if condition (29) is violated then the principal expects the agent to disobey her and choose the large project regardless of her instructions. Since the principal is ignored, she has weaker incentives to communicate with the agent (she is indifferent). However, if sending a message of any kind is costly, even if this cost is arbitrarily small, the principal will remain silent and the equilibrium must be uninformative. In this respect, when intervention prompts disobedience it also harms communication.

2.5 Principal's welfare

If intervention enhances communication ($\beta \leq b$) then it also increases the principal's expected utility in equilibrium. To see why, consider the following thought experiment. If the principal communicates but ex-post "surprises" the agent and does not intervene, the game is a standard communication game without intervention where the agent's bias is β instead of b. From Crawford and Sobel we know that the principal's expected utility is higher when the bias is smaller. Notice that in a game with intervention the principal also incurs the cost of intervention when she moves the project closer to θ. However, by revealed preferences, the principal must be better off with these choices. Therefore, when intervention improves communication, it necessarily increases the principal's expected utility in equilibrium.

By contrast, if intervention harms communication then a trade-off emerges: On the one hand, less information is revealed by the principal in equilibrium, which harms the principal since the agent is less likely to implement the desired project. But on the other hand, the principal has a correction device to undo the agent's bias. The next result provides sufficient conditions under which the principal is strictly better off without the option to intervene because of its adverse effect on communication. When these conditions hold, the principal's expected utility in equilibrium is higher when she is only relying on her ability to persuade the agent to follow her instructions.

Proposition 5. *Suppose θ is uniformly distributed over $\left[\underline{\theta},\overline{\theta}\right]$ and $N^* \geq 2$. Let $EV_P\left(n\right)$ and $EU_P\left(n\right)$ be the principal's expected utility in equilibrium with and without intervention, respectively, when the number of elements in the partition is n. Then, there are $0 \leq \tau_l^* < \tau_h^*$ and $0 < \delta_l^* < \delta_h^*$ such that if $\tau \in [\tau_l^*, \tau_h^*]$ and $\delta \in [\delta_l^*, \delta_h^*]$ then $N^{**} = 1$ and*

$$EU_P\left(1\right) < EV_P\left(N^{**}\right) < EU_P\left(2\right). \tag{30}$$

Under these conditions, an informative equilibrium exists if and only if the principal does not have the option to intervene, and the principal is worse off with the option to intervene if and only if an informative equilibrium is selected whenever it exists.

The starting point of Proposition 5 is the existence of an informative equilibrium in a game without intervention, which according to Lemma 4 requires $N^* \geq 2 \Leftrightarrow b < \frac{\overline{\theta}-\underline{\theta}}{4}$. This condition is necessary since otherwise intervention does not have the potential to harm communication. Proposition 5 then shows that there exists a range of τ and δ such that three additional conditions are satisfied. First, an informative equilibrium does not exist in a game with intervention, that is, $N^{**} = 1 \Leftrightarrow \frac{\overline{\theta}-\underline{\theta}}{4} \leq \beta$. Since $N^* \geq 2 > 1 = N^{**}$, intervention harms communication. Second, in the absence of communication the principal's expected utility in a game with intervention is strictly higher than in a game without intervention, that is, $EU_P(1) < EV_P(N^{**})$.[27] This condition ensures that if the principal is better off without the option to intervene, it is only because of the adverse effect of intervention on communication. Third, the principal's expected utility in a game with intervention is strictly lower than her expected utility in a game without intervention, as long as in the latter an informative equilibrium is selected, that is, $EV_P(N^{**}) < EU_P(2)$. Under these three conditions the principal is better off without the option to intervene precisely because it harms communication.[28]

Example. *Suppose θ is uniformly distributed over $\left[\underline{\theta}, \overline{\theta}\right]$. Crawford and Sobel showed that under their leading example*

$$E[(x^* - \theta)^2] = \sigma^2(n, b) + b^2, \tag{31}$$

where x^* is the action taken by the agent in equilibrium when the partition has n elements and

$$\sigma^2(n, b) = \frac{1}{12}\frac{(\overline{\theta} - \underline{\theta})^2}{n^2} + \frac{b^2(n^2 - 1)}{3}. \tag{32}$$

Therefore, under quadratic utility and cost functions

$$EU_P(n) = A - \left(\sigma^2(n, b) + b^2\right) \tag{33}$$

and

$$EV_P(n) = A - \frac{\delta}{1 + \delta}\left(\sigma^2(n, \beta) + \beta^2\right). \tag{34}$$

Figure 2 provides two examples in which $N^* = 2$ and condition (30) holds. In the right panels, $\tau = 0$. The lower right panel shows that $N^{**} = 1$ if $\delta < 1$ and $N^{**} = 2$ otherwise. The upper right panel shows that $EV_P(N^{**})$ (the red curve) is strictly greater than $EU_P(1)$ (the purple horizontal line) if and only if $\delta > 0.25$ and $EV_P(N^{**})$ is always strictly smaller than

[27] Recall that in both games there always exists an uninformative equilibrium with $N = 1$.

[28] Note that the principal can be better off without the option to intervene even in the absence of communication (i.e., $EU_P(1) > EV_P(1)$). Intuitively, intervention can be counter-productive by giving the agent strong incentives to overshoot. The principal may end up with the same implemented project, but may incur additional costs from intervention.

$EU_P(N^*)$ *(the green horizontal line). Combined, if* $\tau = 0$ *then condition (30) holds whenever* $\delta \in (0.25, 1)$. *In the left panels,* $\tau = 0.05$. *The lower left panel shows that* $N^{**} = 1$ *if and only if* $\delta \in (0.1, 0.9)$. *The upper left panel shows that* $EV_P(N^{**})$ *is always strictly greater than* $EU_P(1)$ *and* $EV_P(N^{**})$ *is smaller than* $EU_P(N^*)$ *if and only if* $\delta > 0.35$. *Combined, if* $\tau = 0.05$ *then condition (30) holds whenever* $\delta \in (0.35, 0.9)$.

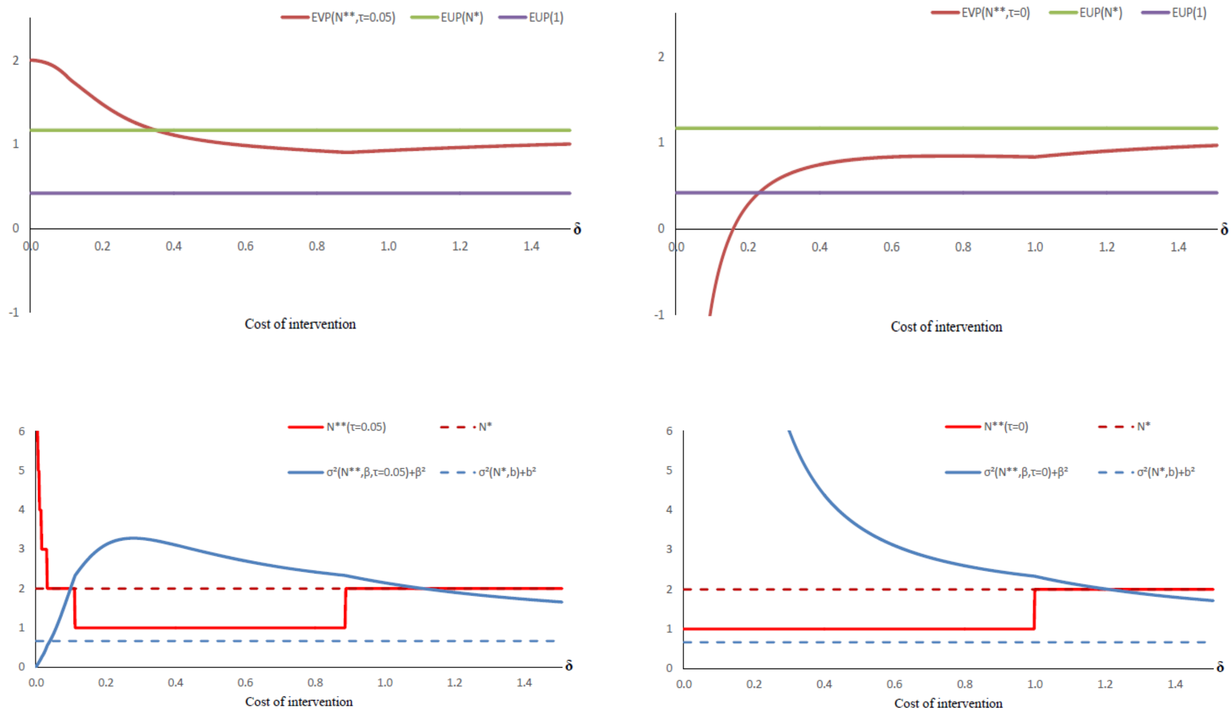

Fig. 2 Principal's expected welfare in equilibrium when $A = 2$, $\overline{\theta} - \underline{\theta} = 4$, and $b = 0.5$

Agent's welfare. *The agent expected utility in equilibrium with and without intervention when the number of elements in the partition is* n *can be similarly defined as* $EV_A(n)$ *and* $EU_A(n)$, *respectively. Under the assumptions above,*

$$EU_A(n) = A - \sigma^2(n, b) \tag{35}$$

and

$$EV_A\left(n\right) = A - \frac{\tau + \delta^2}{\left(1+\delta\right)^2}\left[\sigma^2\left(n,\beta\right) + \frac{\tau}{\delta^2}\beta^2\right]. \tag{36}$$

Figure 3 plots the agent's expected utility under the same conditions as in Figure 2. It shows that the agent can benefit from the principal's intervention, primarily because intervention is an informed decision.

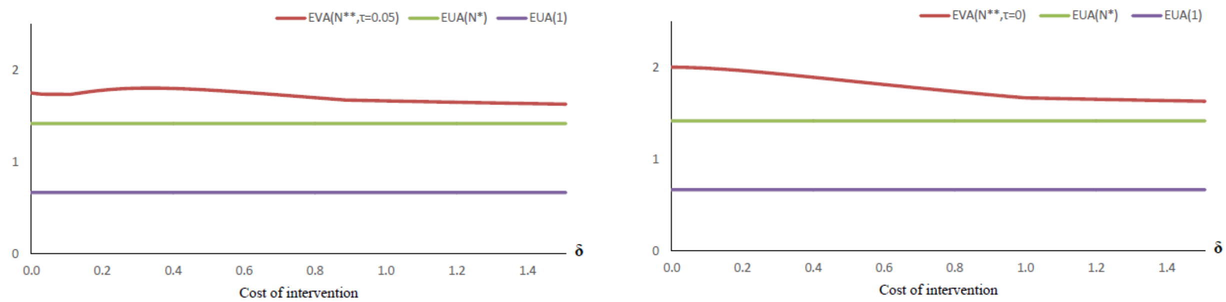

Fig. 3 Agent's expected welfare in equilibrium when $A = 2$, $\overline{\theta} - \underline{\theta} = 4$, and $b = 0.5$

3 Extensions and robustness

This section considers several extensions of the baseline model.

3.1 Informed agent

In some applications of the model, the agent may also have private information about the fundamentals θ. To consider this possibility, suppose $\theta = \theta_P + \theta_A$, where the principal is privately informed about $\theta_P \in \left[\underline{\theta}_P, \overline{\theta}_P\right]$ and the agent is privately informed about $\theta_A \in \left[\underline{\theta}_A, \overline{\theta}_A\right]$. I also assume that θ_P and θ_A are independent. Other assumptions of the model remain unchanged.

Different from the baseline model, here the agent's choice is also a function of his private information about θ_A. Without intervention, the game is identical to Harris and Raviv (2005), who showed that the set of equilibria when the agent is privately informed is equivalent to the set of equilibria when the agent is uninformed, with the exception that $x^*(\theta_A, m) = \theta_A + x^*\left(m\right)$, where $x^*\left(m\right)$ is defined in Proposition 1. In particular, the quality of communication is not affected by the agent's private information, and the largest partition that arises in equilibrium has $N\left(b, \overline{\theta}_P - \underline{\theta}_P\right)$ elements. Intuitively, the agent's optimal decision fully incorporates his private information about θ_A in a way that leaves the principal's expected utility independent of the realization of θ_A. The agent's private information, on its own, does not distort the principal's incentives to communicate her own private information.

By contrast, in a game with intervention the principal can learn about θ_A from the agent's decision. This learning channel changes the principal's intervention policy. As a result, it also changes the agent's initial decision, which in turn, affects the principal's ability to influence the agent in the first place. For simplicity, I restrict attention to quadratic utility and cost functions. In addition, I focus on linear equilibria, i.e., equilibria in which the agent's initial decision is a linear function of θ_A.[29] Under these assumptions, the following result holds.

Proposition 6. *Consider the model with quadratic utility and cost functions, and two-sided information asymmetry. The set of communication strategies that arises in a linear equilibrium of a game with intervention is identical to the set of communication strategies that arises in an equilibrium of a game without intervention (and two-sided information asymmetry) in which the agent's bias is $b\frac{1+\delta}{\delta}$. Furthermore, for every θ_P and message m, the intervention policy of the principal in equilibrium is independent of x and is given by $\Delta^* = \frac{1}{1+\delta}(\theta_P - \mathbb{E}[\theta_P|m] - \frac{1+\delta}{\delta}b)$.*

Proposition 6 shows that with intervention the agent behaves as if his bias is $b\frac{1+\delta}{\delta}$. Since $\frac{1+\delta}{\delta} > 1$, intervention always harms communication when the agent is privately informed. Note that this result holds for any $\tau > 0$. By contrast, when the agent is uninformed he behaves as if his bias is $b\frac{1+\delta}{\tau/\delta+\delta}$. Since $\frac{1+\delta}{\tau/\delta+\delta} < \frac{1+\delta}{\delta}$ for all $\tau > 0$, the agent's private information exacerbates the adverse effect that intervention has on the ability of the principal to influence the agent through communication.

To understand Proposition 6, notice that τ has no effect on the equilibrium (the informed agent behaves as if $\tau = 0$). When the agent is uninformed, larger τ weakens his incentives to choose actions that are distant from his best estimate of the principal's ideal point. However, when the agent is privately informed, choosing a distant action does not increase the intensity

[29] Notice that without intervention all equilibria are linear.

of intervention by the principal. In equilibrium, the principal learns about θ_A from the agent's decision and updates her ideal point accordingly. In particular, the principal rationally interprets distant actions as strong signals about θ_A. That is, the principal attributes the variation in observed x to unobserved variation in θ_A. For this reason, the intensity of the principal's intervention in equilibrium does not depend on x, and as such, it is beyond the agent's control. While intervention has no disciplinary effect on the agent's decision, the agent overshoots in anticipation of the principal's intervention, knowing that the principal will intervene more aggressively only if θ_P justifies doing so. For this reason, intervention harms communication.

Finally, the comparison of this analysis to the baseline model implies that, all else equal, communication is less effective when the agent is better informed. This observation also implies that the principal will intervene more intensively when the agent is better informed.

Principal's welfare with informed agent. Notice that if θ_P, the principal's private information, is uniformly distributed then the principal's expected utility under the most informative equilibrium is identical to her expected utility when θ_A is a common knowledge and $\tau = 0$. This results follows directly from Proposition 6 and from the observation that the agent's decision in equilibrium fully incorporates his private information about θ_A; the only loss of welfare stems from the noisy communication of θ_P. Therefore, under quadratic utility and cost functions, the principal's expected utility can be calculated using the expressions from the corresponding example in Section 2.5. In particular, the principal can be worse off with the ability to intervene in the agent's decision.

3.2 Pay for performances

The incompleteness of contracts plays a central role in the analysis: Actions and messages cannot be contracted. However, the value of the project (e.g., its terminal cash-flows) could in principle be contracted. For example, the principal can give the agent more skin in the game by offering him a fraction $\omega \in (0, 1)$ of $U_P(x, \theta)$ (i.e., equity contract). This section explores the effect of ω on the interaction between intervention and communication.

For this purpose, let the agent's intrinsic private benefits from investment be Bx, where $B > 0$.[30] I assume that ω does not have a direct effect on the intervention technology. Given ω, the principal's direct and indirect utility are $(1-\omega)\, U_P(x,\theta)$ and $(1-\omega)\, V_P(\theta, x; \frac{\delta}{1-\omega})$, respectively. Similarly, the agent's direct utility is $Bx + \omega U_P(x,\theta)$. Notice that $Bx - \omega L(|x-\theta|)$ is a single-peaked function that obtains its maximum at $\theta + (L')^{-1}(\frac{B}{\omega})$. Therefore, the agent's direct utility can be rewritten as $U_A(\theta + b(\omega), \theta; b(\omega)) - \omega T(|x - (\theta + b(\omega))|)$, where $b(\omega) \equiv (L')^{-1}(\frac{B}{\omega})$ and $T(\cdot)$ has the same properties as in the baseline model. Importantly, $T(\cdot)$ does not depend on ω directly. Let $\beta(\tau, \delta, b)$ be the agent's bias adjusted for intervention as defined by (16), then with pay for performances it can be written as $\beta(\omega) \equiv \beta(\frac{\tau}{\omega}, \frac{\delta}{1-\omega}, b(\omega))$. Therefore, the agent's indirect utility can be rewritten as $\omega V_A(\theta, x; \beta(\omega), \frac{\tau}{\omega})$.

Since the analysis of the baseline model is invariant to linear transformations of the indirect utility functions, the game without intervention has the same solution with the exception that b is replaced by $b(\omega)$. Similarly, the game with intervention has the same solution with the exceptions that b is replaced by $b(\omega)$, τ is replaced by $\frac{\tau}{\omega}$, and δ is replaced by $\frac{\delta}{1-\omega}$. Therefore, a change in ω has three potential effects on the analysis. First, the bias of the agent decreases with ω. Intuitively, with more skin in the game the agent internalizes more of the benefit from choosing a project of size θ, and as a result, his ideal point shifts closer to θ. Second, the agent's cost from intervention decreases in ω. Intuitively, since the actual cost from intervention is independent of ω, a larger ω implies that the agent puts more weight on maximizing $U_P(x,\theta)$. Equivalently, the cost from intervention *per unit of utility* is lower. Third, the principal's cost from intervention increases in ω. Intuitively, by giving away a larger fraction of the project to the agent, per unit of utility, the principal incurs a larger cost from intervention. Since $\frac{\delta}{1-\omega} \to \infty$ as $\omega \to 1$, the next result is a direct corollary of Lemma 5 part (i).

Proposition 7. *There is $\omega^* \in (0,1)$ such that if $\omega \in (\omega^*, 1)$ then $\beta(\omega) > b(\omega)$.*

Essentially, intervention is more likely to harm communication when the agent has a significant skin the game. While the agent's bias effectively decreases with ω, its positive effect on

[30] Similar results hold if instead these private benefits are given by $Bq(x)$ where $q' > 0$ and $q'' \leq 0$.

the principal's ability to influence the agent through communication is weaker with intervention. Indeed, a higher ω also decreases the cost from intervention for the agent and increases the cost of intervention to the principal, and as discussed in Section 2.4, intervention is more likely to harm communication when the agent's cost is small relative to the principal's cost.

Example. *Under quadratic utility and cost functions,* $b(\omega) = \frac{B}{2\omega}$*,* $\frac{\beta(\omega)}{b(\omega)} = \frac{1+\frac{\delta}{1-\omega}}{\frac{1-\omega}{\omega}\frac{\tau}{\delta}+\frac{\delta}{1-\omega}}$*, and*

$$\beta(\omega) > b(\omega) \Leftrightarrow \omega > \frac{\tau}{\tau+\delta}. \tag{37}$$

Welfare effects of pay for performances. When choosing ω the principal trades off the direct cost of giving away part of the project's value to the agent with the positive effect it may have on communication. Generally, the analysis above suggests that the optimal level of ω (from an ex-ante perspective) depends on the intervention technology (τ and δ), but it may also depend on factors which are outside of the model (e.g., agent's ability, career concerns, etc.). While the analysis of the optimal contract is beyond the scope of this paper, the example below demonstrates that intervention can harm communication even if the principal chooses ω optimally (prior to observing θ).

Example. *Figure 4 plots the principal's expected utility in equilibrium as a function of* ω*, with intervention (red curve) and without intervention (green curve), when the utility and cost functions are quadratic.*[31] *Both panels of Figure 4 shows that under the optimal* ω *the principal is better off without intervention. The optimal* ω *without intervention is* 0.48 *and it implies* $N^* = 2$*. In the left (right) panel* $\tau = 0.05$ *(*$\tau = 0$*), the optimal* ω *with intervention is* 0.38 *(*0.60*), and it implies* $N^{**} = 1$ *(*$N^{**} = 2$*). Intuitively, a larger* τ *increases the cost of giving the agent more skin in the game, as larger* ω *weakens the effect of* τ *on the incentives of the agent to comply with the principal. Notice that if* $\tau = 0.05$ *then under the optimal contract,* $N^{**} < N^*$*, that is, intervention can harm communication even if the principal chooses* ω

[31] If $U_P(x,\theta)$ is quadratic, so is the agent's utility. The principal's expected utility in equilibrium without intervention is given by $(1-\omega)EU_P(n)$ where $EU_P(n)$ is given by (33) and b is replaced by $b(\omega)$ everywhere. Similarly, the principal's expected utility in equilibrium with intervention is given by $(1-\omega)EV_P(n)$, where $EV_P(n)$ is given by (34), β is replaced by $\beta(\omega)$, and δ is replaced by $\frac{\delta}{1-\omega}$.

optimally.

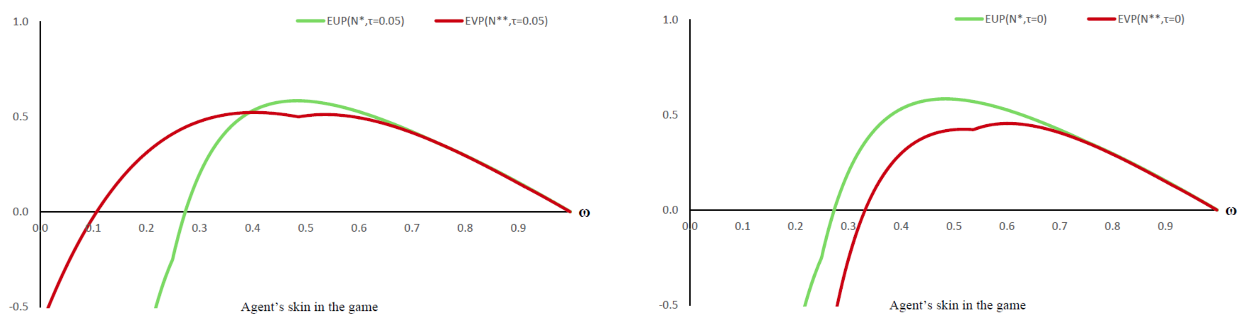

Fig. 4 Principal's expected welfare as a function of ω when $A = 2$, $\overline{\theta} - \underline{\theta} = 4$, $B = 0.5$, and $\delta = 0.4$

3.3 Fixed intervention cost

Consider an extension of the model in which the cost of intervention if fixed at $C > 0$. Namely, if the principal pays C she can freely adjust the final project at no additional cost. Since the cost of intervention is fixed, upon intervention the principal always chooses $\Delta = \theta - x$. Therefore, given x and θ, the principal intervenes if and only if

$$-L\left(|x-\theta|\right) < -L\left(0\right) - C \Leftrightarrow \theta \notin \left[x - \overline{C}, x + \overline{C}\right], \tag{38}$$

where $\overline{C} \equiv L^{-1}\left(C + L\left(0\right)\right)$.[32] I prove the following result.

Proposition 8. *Suppose $\tau = 0$ and $\overline{C} \leq b$. If θ is uniformly distributed over $\left[\underline{\theta}, \overline{\theta}\right]$ then the game with intervention does not have a partition equilibrium with more than one element. In the unique partition equilibrium the principal's messages are uninformed, the agent chooses $x^* = \max\left\{\overline{\theta}, \underline{\theta} + \overline{C}\right\}$, and the principal intervenes if and only if $\overline{C} < \overline{\theta} - \underline{\theta}$ and $\theta \in \left[\underline{\theta}, \overline{\theta} - \overline{C}\right]$.*

Intuitively, if $\overline{C} \leq b$ then the cost of intervention is relatively small and the principal intervenes whenever the project is larger than $\theta + b$. However, the principal does not intervene

[32] Implicitly, I assume that if the principal is indifferent between intervention and nonintervention, then she does not intervene.

if the project is too small, $x \in \left(\theta - \overline{C}, \theta\right)$. Since from the agent's perspective the risk of undershooting is larger than the risk of overshooting, the agent has strong incentives to choose larger projects. In response, and similar to the reasoning in baseline model, the principal has stronger incentives to understate θ, which in turn, harms her credibility. In equilibrium, no information can be revealed by the principal.

The next corollary, which follows from Proposition 8 and the observation that $N^* \geq 2$ if and only if $b \leq \frac{\bar{\theta}-\underline{\theta}}{4}$, shows that intervention can harm communication also in a setup with fixed intervention costs.

Corollary 2. *If $\overline{C} \leq b$ and $b \leq \frac{\bar{\theta}-\underline{\theta}}{4}$ then an informative equilibrium exists in a game without intervention, but it does not exist in a game with intervention.*

4 Applications

This section discusses several applications of the model.

4.1 Managerial leadership

Leadership is often defined as the ability to influence and motivate others to achieve a certain goal successfully (e.g., Hermalin (1998)). It involves articulating a strategy that is appropriate given the organization's strategic position and the environment it faces. Without the ability to persuade others to follow their vision, leaders have to choose between a compromise with an undesired outcome and exercising their authority to bring about a change. The power of leaders depends on various characteristics of the organization and its leadership. As a general message, the model suggests that the ease at which corporate leaders can exercise their power can diminish their ability to influence others to voluntarily follow their vision. In this regard, the model can be applied to study interactions between managers and their subordinates, owners of small businesses and their employees, firms and labor unions, or CEOs\headquarters and division managers.

As an example, consider the interaction between the CEO of a company (principal) and a representative division manager (agent). The firm has to decide on x, the resources to be allocated to the division (e.g., investment in physical or human capital). Investment can involve expanding to new geographical areas, introducing new products, renewing IT systems, divesting non-core assets, etc. These investments are not contractible since their attractiveness depend on a variety of macro, industry, and firm-specific factors which cannot be perfectly anticipated. The CEO has superior knowledge on the benefit from investment, θ. For example, if the proposal is to divest assets, the CEO has a better understanding of the market conditions, demand for corporate assets by investors, and the external cost of financing. If the proposal is to enter new markets, the CEO has a better knowledge of the complementarities with other products of the company, unwanted cannibalization, and alternative investment opportunities. While the CEO is interested in maximizing the value of the whole firm, the division manager is biased toward maximizing the profits of his division, and may be prone to over-invest ($b > 0$). Generally, the conflict of interests arises because the division manager is being compensated based on the profitability of his division, because of his career concerns (his skills are better reflected in the performances of his division), or due to private benefits from controlling larger assets (e.g., prestige and power).

The CEO will lay out her vision and try to persuade the division manager to follow her strategy. If she is unsuccessful, the ability of the CEO to intervene and implement the strategy in spite of the division manager's resistance (parameter δ) depends on factors such as the CEO's managerial style (e.g., hands-off approach), the CEO's characteristics (e.g., aversion to confrontation), the busyness of the CEO (e.g., the alternative cost of intervention is higher when the CEO oversees large and complex firms), and the autonomy that was granted to the division over its operations in the first place. In turn, intervention could harm the division manager's reputation, ego, or compensation ($\tau \geq 0$).

Applied to this context, the model suggests that cross sectional variation in firm and CEO characteristics that are associated with a high cost of intervention (as listed above) should be positively correlated with effective communication and visionary leadership. Effective communication can be viewed as an intangible asset or a corporate culture that creates an environment in which open dialogues can flourish. Surveys of employees' satisfaction and their view of the organization in which they are employed could be a useful source. Moreover, if communications are effective and valuable, various means of internal communications should be frequently used: in person meetings, conference calls, emails, distribution of internal memos, etc. In this regard, the model speaks to how different patterns of communication are mapped to different organizational structures and managerial styles.

4.2 Corporate boards

In a typical public corporation, the CEO runs the company on a daily basis, but the board of directors sets the strategy, approves major decisions, and has the right to replace and set the compensation of the CEO. In many cases, board members are executives in related industries, lawyers, bankers, accountants, academics, and in some cases, savvy investors such as activist hedge fund managers (Gow, Shin, and Srinivasan (2014)) or venture capitalists (even long after the IPO, see Celikyurt, Sevilir and Shivdasani (2014)). These individuals often use their business, legal, and finance expertise, to advise and direct the CEO on a variety of issues such as strategy, public relations, crisis management, and M&A. At the same time, CEOs might have a different agenda: building an empire, maintaining their reputation, or seeking the "quite life." Therefore, monitoring (intervention) and advising (communication) the CEO are among the most important duties of directors.

Intervention, however, requires coordination among directors (e.g., to avoid free-riding).[33] Therefore, the effective cost of intervention of the board as a whole is likely higher in larger boards with more diverse and busy directors. The analysis suggests that the effectiveness of the board's advisory role could be positively related to these factors. Moreover, since boards with a powerful CEO have a limited capacity of intervention, the analysis suggests that the board in those cases can in fact play a more effective advisory role. Board meeting minutes can

[33] For studies on optimal board size see Hermalin and Weisbach (1998), Harris and Raviv (2008), and Raheja (2005).

shed light on the allocation of time that directors spend during these meetings on monitoring, criticizing, and overruling the CEO, as opposed to a productive discussion which involves advice and exchanges of views. Alternatively, the effectiveness of the board monitoring can be measured indirectly by the performances of a strategic event that requires significant board input, such as an acquisition of another firm.

Moreover, the analysis suggests that the advisory role of the board is likely to be ineffective when the CEO does not suffer severe consequences form intervention (low τ), that is, if his reputation in the labor market is already established (e.g., long tenure or proximity to retirement) or if he is entitled to a generous severance package. This prediction differs from Adams and Ferreira (2007) who argue that the board's advisory role will be ineffective when the CEO dislikes board monitoring the most. Indeed, in their model there is a hold-up problem: the CEO has fewer incentives to cooperate with the board when monitoring inflicts larger costs on the CEO, and by assumption, without cooperation the board cannot advise the CEO in their model.

Finally, the analysis in Section 3.1 suggests that communication is less effective when the agent is better informed. Therefore, if the CEO has a higher expertise the model predicts that the board's advisory role is less effective (holding the board's expertise fixed). This prediction differs from Harris and Raviv (2008) and Chakraborty and Yilmaz (2016). In the former, the quality of communication from outsiders to insiders is unaffected by the expertise of insiders (given the allocation of control, which is itself endogenous), and in the latter, more information can be communicated by the board when management is also privately informed. This difference stems from the assumption in my model that the board can intervene in the CEO's decision if the CEO does not comply.[34]

[34] While the models differ in their prediction with respect to the effectiveness of communication from the board to the CEO, they all suggest that CEOs with better information should be given (weakly) more control rights.

4.3 Private equity

The implications of the model for corporate boards can also be applied to private equity. Venture capital and leveraged buyout funds typically hold board seats and other control rights in their portfolio companies, which give them the power to make strategic decisions, replace management, and even liquidate the firm (Baker and Gompers (2003), Kaplan and Strömberg (2003, 2004), Cornelli and Karakas (2015)). Importantly, these investors often provide expertise and post-investment added value to their portfolio companies. Indeed, the empirical evidence suggests that VCs provide advice and support to small entrepreneurial start-ups, help with the professionalization of the management team and the commercialization of the product, foster innovation, and improve productivity.[35] Similarly, in a typical leveraged buyout, the LBO fund appoints experts from the industry (e.g., Ex-CEOs), consultants, and its own general partners, as board members of the acquired company. Moreover, many of the large PE shops have an in-house operational research team whose purpose is to identify attractive investment opportunities, develop value creation plans for those investments, and help the fund to turnaround the operations of the target firm after the investment is made (e.g., cost-cutting, productivity improvements, repositioning, or acquisition opportunities).[36] For all of these reasons, the role of communication and intervention seems particularly important in the private equity investments.

The private equity context, however, has several unique implications that do not necessarily apply to boards of public companies. First, private equity firms tend to co-invest (i.e., club investment in leveraged buyouts and syndication in venture capital). When the deal has more than one sponsor, the investors share the cash-flows and control rights in the company, which can result in coordination problems between investors. Second, private equity firms make multiple investments. Holding the size of the fund fixed, a large number of portfolio companies increases the alternative cost of intervention in each specific portfolio company. The motives behind co-investment and diversification are likely to be related to capital constraints and risk-

[35] See Hellmann and Puri (2000, 2002); Kortum and Lerner (2000); Bottazzi, Rin, and Hellmann (2008); Chemmanur, Krishnan, and Nandy (2011); Gompers et al. (2016).

[36] See Kaplan and Strömberg (2009); Acharya et al. (2013); Gompers, Kaplan, Mukharlyamov (2016).

sharing. However, the model suggests that the cross-sectional variation in these two factors can explain the ability of private equity investors to effectively communicate, provide advice, and add value to their portfolio companies. Related, since private equity investors make multiple investments over time, they could develop reputation for either working constructively with management or being adversarial and authoritative. In the former case, intervention could be more costly since the investor's reputation is at stakes. Therefore, the model suggests that private equity investor with reputation of being "friendly" could also be more effective in advising their portfolio companies.[37] Finally, private equity are long-term investors, but they ultimately seek an exit on their investment. Exit can be a substitute for intervention if the PE investor fails to influence management. Therefore, the ease at which the PE investors can exit their investments (at better terms) has a similar effect to a higher cost of intervention. Exit from a private equity investment is easier if the IPO or the M&A market are hot. Therefore, the model predicts that the ability of private equity investors to provide advice and add value to their portfolio companies through communication is higher when these markets are booming.

4.4 Shareholder activism

In a typical campaign, the activist investor buys a sizeable stake in a public company and then engages with the management or the board of directors, expressing her dissatisfaction or view of how the company should be managed.[38] Indeed, activist hedge funds, who routinely conduct highly-detailed analysis of their investments, are likely to have information that corporate boards lack, especially if directors are lazy, busy with other activities, do not have the relevant expertise, or simply suffer from group-think.[39] Occasionally, if the company refuses to comply

[37] Baker, Gibbons, and Murphy (1999) assume that authority is non-contractible, but can be informally given through commitments enforced by reputation.

[38] There is an emerging empirical evidence that suggests that activist and institutional investors often engage in direct discussions with the management or the board of directors of their portfolio companies, mostly behind-the-scenes. For example, see Becht, Franks, Mayer, and Rossi (2009), Becht, Franks, and Grant (2017), Deloitte (2015), McCahery, Sautner, and Starks (2016).

[39] The idea that outsiders have information that insiders can learn from is central to a new literature that studies how firms use information in stock prices to make investment decisions (e.g., Bond, Edmans, and Goldstein (2012)).

with the activist's demand, the activist ends up litigating or launching a proxy fight in order to force her ideas on the company. Running a successful proxy fight, however, is costly since it requires the activist to reach out to other shareholders of the firm in order to win their vote.

Applied to this context, the analysis highlights that policies that reduce the cost of intervention for activists (e.g., the adoption of an easier proxy access) and forces that ease the coordination among shareholders (e.g., the rise of institutional/index investment or the increased influence of proxy advisory firms) can undermine the ability of activists to influence the polices of their target companies. Similarly, activists might have more influence on target companies with dispersed ownership, large market capitalization, or dual class structure. In all of these instances, intervention is likely to be more costly. Finally, and similar to the application of the model to private equity, factors that increase the ease at which activist investors can sell their shares (e.g., stock liquidity), could facilitate comminution between activist investors and their target companies.

5 Concluding remarks

Interactions between managers, directors, and investors are crucial to our understanding of how corporations are managed and governed. In many of these interactions, contracts only partially resolve the conflicts of interests, and as a result, communication and intervention become the primary mechanisms of governance. This paper sheds new light on corporate governance by analyzing a principal-agent model with incomplete contracts and a top-down information structure. Surprisingly, the main result of the paper demonstrates that a credible threat of intervention can *decrease* the incentives of the agent to follow the principal's instructions. In those cases, *intervention prompts disobedience,* communication is *less effective and less informative* with intervention than without it, and the two mechanisms substitute one another. In this respect, *words do speak louder without actions.* Building on this core insight, the analysis considers several variants of the baseline model and provides novel predictions related to managerial leadership, corporate boards, private equity, and shareholder activism.

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Appendix

Proof of Lemma 1. First, suppose $x - \theta > 0$ ($x - \theta < 0$). The principal never chooses $\Delta > x - \theta$ ($\Delta < x - \theta$), since by choosing $\Delta = x - \theta$ she not only minimizes L, but she also reduces the cost of intervention. Also, the principal never chooses $\Delta < 0$ ($\Delta > 0$), since by choosing $\Delta = 0$ not only she brings the cost of intervention to zero, but she also reduces the loss from the $L(\cdot)$ function. Therefore, $|\Delta^*| < |x - \theta|$ and $x = \theta \Rightarrow \Delta^* = 0$, as required.

Second, suppose $x - \theta \leq 0$. From the previous step it must be $x - \theta \leq \Delta \leq 0$, and therefore,

$$\Delta^* \in \arg \max_{x-\theta \leq \Delta \leq 0} \{U_P(\theta, \theta) - L(\theta - x + \Delta) - \delta C(-\Delta)\}.$$

The first order condition implies

$$-L'(\theta - x + \Delta) + \delta C'(-\Delta) = 0, \tag{39}$$

and the second order condition requires

$$-L''(\theta - x + \Delta) - \delta C''(-\Delta) < 0.$$

Since $L'' > 0$ and $C'' > 0$, the second order condition holds and Δ^* is given by the unique solution of $L'(\theta - x + \Delta) = \delta C'(-\Delta)$. Moreover, notice that if $x - \theta = 0$ then $\Delta^* = 0$, and if $x - \theta < 0$ then $x - \theta < \Delta^* < 0$. Third, suppose $x - \theta > 0$. From the initial step it must be $0 \leq \Delta \leq x - \theta$, and therefore,

$$\Delta^* \in \arg \max_{0 \leq \Delta \leq x-\theta} \{U_P(\theta, \theta) - L(x - \theta - \Delta) - \delta C(\Delta)\}.$$

The first order condition implies

$$L'(x - \theta - \Delta) - \delta C'(\Delta) = 0, \tag{40}$$

and the second order condition requires

$$-L''(x - \theta - \Delta) - \delta C''(\Delta) < 0.$$

Therefore, Δ^* is given by the unique solution of $L'(x - \theta - \Delta) = \delta C'(\Delta)$. Also, notice that $0 < \Delta^* < x - \theta$. Notice that (39) and (40) imply that Δ^* is a continuous function of $x - \theta$.

Overall, this proves (11) and completes part (i).

Consider part (ii). Define $r \equiv x - \theta$. Applying the implicit function theorem on (11),

$$\frac{\partial \Delta^*}{\partial r} = \frac{L''(|r - \Delta^*|)}{L''(|r - \Delta^*|) + \delta C''(|\Delta^*|)}. \tag{41}$$

Noting that $L'' > 0$ and $C'' > 0$ establishes $\frac{\partial \Delta^*}{\partial r} \in (0, 1)$.

Finally, consider part (iii). Recall $0 < \Delta^* \Leftrightarrow 0 < x - \theta$. Applying the implicit function theorem on (11), we have

$$\frac{\partial |\Delta^*|}{\partial \delta} = -\frac{C'(|\Delta^*|)}{L''(|x - \theta - \Delta^*|) + \delta C''(|\Delta^*|)} < 0. \tag{42}$$

Moreover, from (11) it immediately follows that if $\delta \to 0$ then $L'(|x - \theta - \Delta^*|) \to 0$. Since $L'(0) = 0$ and $L'' > 0$, if $\delta \to 0$ then $\Delta^* \to |x - \theta|$. Similarly, from (11) it immediately follows if $\delta \to \infty$ then $\frac{L'(|x-\theta-\Delta^*|)}{\delta} \to 0$. Since $C'(0) = 0$ and $C'' > 0$, if $\delta \to \infty$ then $\Delta^* \to 0$, which completes the proof. ■

Proof of Lemma 2. According to Lemma 1, Δ^* is a function of $x - \theta$ and it does not depend on θ or x in any other way. Therefore, so does $l(x - \theta) \equiv L(|x - \theta - \Delta^*(x - \theta)|) + \delta C(|\Delta^*(x - \theta)|)$. Next, I argue $l(x - \theta) = l(\theta - x)$. Indeed, suppose without the loss of generality that $x > \theta$. According to Lemma 1, $\Delta^*(x - \theta)$ is the unique solution of $L'(x - \theta - \Delta) = \delta C'(\Delta)$, and $\Delta^*(\theta - x)$ is the unique solution of $L'(x - \theta + \Delta) = \delta C'(-\Delta)$. Since both has a unique solution, it must be $\Delta^*(\theta - x) = -\Delta^*(x - \theta)$. Recall $|\Delta^*(x - \theta)| < |x - \theta|$, and therefore,

$$L(|x - \theta - \Delta^*(x - \theta)|) = \begin{cases} L(x - \theta - \Delta^*(x - \theta)) & \text{if } x > \theta \\ L(-x + \theta + \Delta^*(x - \theta)) & \text{if } x < \theta. \end{cases} \tag{43}$$

Therefore, if $x > \theta$ then

$$\begin{aligned} l(x - \theta) &= L(x - \theta - \Delta^*(x - \theta)) + \delta C(|\Delta^*(x - \theta)|) \\ &= L(x - \theta + \Delta^*(\theta - x)) + \delta C(|\Delta^*(\theta - x)|) = l(\theta - x). \end{aligned}$$

Since $l(x - \theta) = l(\theta - x)$, $l(\cdot)$ is effectively a function of $|x - \theta|$.

Finally, I prove $l'(0) = 0$ and $l'' > 0$. Without the loss of generality, suppose $x > \theta$. Then,

$$l'(x - \theta) = L'(x - \theta - \Delta^*(x - \theta))\left(1 - (\Delta^*)'(x - \theta)\right) + \delta C'(\Delta^*(x - \theta))\,(\Delta^*)'(x - \theta).$$

Since $\Delta^*(x-\theta)$ satisfies $L'(x-\theta-\Delta^*) = \delta C'(\Delta^*)$, it must be $l'(x-\theta) = L'(x-\theta-\Delta^*(x-\theta))$. Recall $\Delta^*(0) = 0$, and therefore, $l'(0) = L'(0) = 0$. To see that $l'' > 0$, notice that $l''(x-\theta) = L''(x-\theta-\Delta^*(x-\theta))\left(1-(\Delta^*)'(x-\theta)\right)$. Since $L'' > 0$, and since according to Lemma 1 $(\Delta^*)' \in (0,1)$, it follows that $l'' > 0$, as required. ■

Proof of Lemma 3. Since $\Delta^*(x-\theta)$ is a continuous function, so is $V_A(x,\theta;b)$. I prove the lemma in several steps:

1. First, I argue that if $x-\theta \leq 0$ then $\frac{\partial V_A(x,\theta;b)}{\partial x} > 0$. To see why, note that according to Lemma 1, if $x-\theta \leq 0$ then $x-\theta \leq \Delta^*(x-\theta) \leq 0$. Therefore, in this region,

$$V_A(x,\theta;b) = U_A(\theta+b,\theta) - T(\theta-x+\Delta^*(x-\theta)+b) - \tau K(-\Delta^*(x-\theta)), \tag{44}$$

and

$$\frac{\partial V_A(x,\theta;b)}{\partial x} = T'(\theta-x+\Delta^*(x-\theta)+b)(1-\frac{\partial\Delta^*}{\partial x}) + \tau K'(-\Delta^*(x-\theta))\frac{\partial\Delta^*}{\partial x}. \tag{45}$$

Recall that according to Lemma 1, $\frac{\partial\Delta^*}{\partial x} \in (0,1)$. Also note that $T'(\theta-x+\Delta^*(x-\theta)+b) > 0$ and $K'(\cdot) \geq 0$. Therefore, it must be $\frac{\partial V_A(x,\theta;b)}{\partial x} > 0$, as required.

2. Second, I argue that if $0 < x-\theta$ and $b < x-\theta-\Delta^*(x-\theta)$ then $\frac{\partial V_A(x,\theta;b)}{\partial x} < 0$. To see why, note that according to Lemma 1, $0 < x-\theta$ implies $\theta < \Delta^*(x-\theta) < x-\theta$. Since $b < x-\theta-\Delta^*(x-\theta)$, in this region,

$$V_A(x,\theta;b) = U_A(\theta+b,\theta) - T(x-\theta-b-\Delta^*(x-\theta)) - \tau K(\Delta^*(x-\theta)) \tag{46}$$

and

$$\frac{\partial V_A(x,\theta;b)}{\partial x} = -T'(x-\theta-b-\Delta^*(x-\theta))(1-\frac{\partial\Delta^*}{\partial x}) - \tau K'(\Delta^*(x-\theta))\frac{\partial\Delta^*}{\partial x}. \tag{47}$$

Since $T'(0) = 0$ and $T'' > 0$, $b < x-\theta-\Delta^*(x-\theta)$ implies $T'(b < x-\theta-\Delta^*(x-\theta)) > 0$. Therefore, in this range $\frac{\partial V_A(x,\theta;b)}{\partial x} < 0$, as required.

3. Third, suppose $0 < x-\theta$ and $x-\theta-\Delta^*(x-\theta) \leq b$. I argue that there exists $\hat{x}$ in this range such that $\frac{\partial V_A(x,\theta;b)}{\partial x}|_{x=\hat{x}} = 0$. To see why, recall $0 < x-\theta$ implies $0 < \Delta^*(x-\theta) < x-\theta$. Therefore, in this region,

$$V_A(x,\theta;b) = U_A(\theta+b,\theta) - T(\theta+b-x+\Delta^*(x-\theta)) - \tau K(\Delta^*(x-\theta)) \tag{48}$$

and

$$\frac{\partial V_A(x,\theta;b)}{\partial x} = T'(\theta+b-x+\Delta^*(x-\theta))(1-\frac{\partial\Delta^*}{\partial x}) - \tau K'(\Delta^*(x-\theta))\frac{\partial\Delta^*}{\partial x}. \tag{49}$$

Using (41) for the explicit form of $\frac{\partial \Delta^*}{\partial x}$, the equation $\frac{\partial V_A(x,\theta;b)}{\partial x} = 0$ can be rewritten as

$$T'(\theta + b - x + \Delta^*(x-\theta)) = L''\left(x - \theta - \Delta^*(x-\theta)\right) \frac{\tau K'(\Delta^*(x-\theta))}{\delta C''\left(\Delta^*(x-\theta)\right)}. \tag{50}$$

Define

$$\pi(x-\theta) \equiv x - \theta - \Delta^*(x-\theta). \tag{51}$$

Since $0 < x - \theta - \Delta^*(x-\theta) \le b$, it must be $\pi(x-\theta) \in (0,b]$. Recall that according to Lemma 1, $\Delta^*(x-\theta)$ satisfies (11). Substituting (51) into (11), and noting that in this range $0 < \Delta^*(x-\theta) < x-\theta$ gives $L'(\pi(x-\theta)) = \delta C'(x - \theta - \pi(x-\theta))$, which is equivalent to

$$x = \theta + \pi(x-\theta) + (C')^{-1}\left(\frac{L'(\pi(x-\theta))}{\delta}\right). \tag{52}$$

Using this identity, condition (50) can be rewritten as

$$T'(b - \pi(x-\theta)) = L''(\pi(x-\theta)) \frac{\tau K'((C')^{-1}(\frac{L'(\pi(x-\theta))}{\delta}))}{\delta C''((C')^{-1}(\frac{L'(\pi(x-\theta))}{\delta}))}. \tag{53}$$

Notice that the solution of (53) depends on x and θ only through $\pi(x-\theta)$. Let us rewrite (53) as (17). Recall it must be $\pi \in (0,b]$. I argue that a solution to (17) exists and it is in the interval $(0,b]$. First note that if $\pi = 0$ then the left hand side ("LHS") of (17) is $T'(b) > 0$ and the right hand side ("RHS") is $L''(0)\frac{\tau K'(0)}{\delta C''(0)} = 0$ (recall $K'(0) = 0$). Therefore, the solution must be strictly positive. Also notice that if $\pi = b$ then the LHS of (17) is $T'(0) = 0$, and the RHS is $L''(b)\frac{\tau K'((C')^{-1}(\frac{L'(b)}{\delta}))}{\delta C''((C')^{-1}(\frac{L'(b)}{\delta}))} \ge 0$. Therefore, the solution must be weakly smaller than b. Since both sides of (17) are continuous as a function of π, equation (17) has a solution in $(0,b]$. Therefore, there exists $\hat{x}$ in this range such that $\frac{\partial V_A(x,\theta;b)}{\partial x}|_{x=\hat{x}} = 0$. Notice that $\hat{x} = \theta + \pi + (C')^{-1}(\frac{L'(\pi)}{\delta})$, where π is a solution of (17). Since β is given by (16), $\hat{x} = \theta + \beta$. Notice that $\pi > 0$ implies $\beta > 0$.

4. Fourth, I argue that if (13) holds then the solution of (17) is unique and $V_A(x,\theta;b)$ obtains its maximum at $\theta + \beta$. To see why, suppose $0 < x - \theta$ and $x - \theta - \Delta^*(x-\theta) \le b$. Note that

$$\frac{\partial^2 V_A(x,\theta;b)}{\partial^2 x} = \begin{pmatrix} -T''(\theta + b - x + \Delta^*)(1 - \frac{\partial \Delta^*}{\partial x})^2 - T'(\theta + b - x + \Delta^*)\frac{\partial^2 \Delta^*}{\partial^2 x} \\ -\tau K''(\Delta^*)(\frac{\partial \Delta^*}{\partial x})^2 - \tau K'(\Delta^*)\frac{\partial^2 \Delta^*}{\partial^2 x} \end{pmatrix}. \tag{54}$$

According to (41), if $x-\theta>0$ then $\frac{\partial\Delta^*}{\partial x}=\frac{L''(x-\theta-\Delta^*)}{L''(x-\theta-\Delta^*)+\delta C''(\Delta^*)}$. Therefore,

$$\begin{aligned}\frac{\partial^2\Delta^*}{\partial^2 x} &= \frac{L'''\left(x-\theta-\Delta^*\right)(1-\frac{\partial\Delta^*}{\partial x})\delta C''\left(\Delta^*\right)-L''\left(x-\theta-\Delta^*\right)\delta C'''\left(\Delta^*\right)\frac{\partial\Delta^*}{\partial x}}{\left[L''\left(x-\theta-\Delta^*\right)+\delta C''\left(\Delta^*\right)\right]^2} \\ &= \frac{L'''\left(x-\theta-\Delta^*\right)\left(\delta C''\left(\Delta^*\right)\right)^2-\left(L''\left(x-\theta-\Delta^*\right)\right)^2\delta C'''\left(\Delta^*\right)}{\left[L''\left(x-\theta-\Delta^*\right)+\delta C''\left(\Delta^*\right)\right]^3}.\end{aligned}$$

Thus, if $C'''(\cdot)\le 0\le L'''(\cdot)$ then $\frac{\partial^2\Delta^*}{\partial^2 x}\ge 0$. Since $\frac{\partial\Delta^*}{\partial x}\in(0,1)$, it follows that $\frac{\partial^2 V_A(x,\theta;b)}{\partial^2 x}<0$. Therefore, $V_A(x,\theta;b)$ is a single-peaked function with a unique maximum. Based on the previous steps, the maximizer must be $\theta+\beta$. Moreover, it can be verified that $C'''(\cdot)\le 0\le L'''(\cdot)$ guarantees that the RHS of (17) is increasing in Δ, and therefore, the solution of (17) is unique, as required.[40]

5. Fifth, I argue that β strictly increases in b. To see why, note that from (16)

$$\frac{\partial\beta}{\partial b}=\frac{\partial\pi}{\partial b}+\frac{\frac{L''(\pi)}{\delta}\frac{\partial\pi}{\partial b}}{C''((C')^{-1}(\frac{L'(\pi)}{\delta}))}=\frac{\partial\pi}{\partial b}\left(1+\frac{\frac{L''(\pi)}{\delta}}{C''((C')^{-1}(\frac{L'(\pi)}{\delta}))}\right).$$

Since $L''>0$ and $C''>0$, the signs of $\frac{\partial\beta}{\partial b}$ and $\frac{\partial\pi}{\partial b}$ are identical. Recall that the RHS of (17) is increasing in π and the LHS is decreasing in π. Since the RHS is independent of b and the LHS increases in b, it must be that the unique solution of (17) also increases in b. That is, $\frac{\partial\pi}{\partial b}>0$, which implies $\frac{\partial\beta}{\partial b}>0$ as required. ■

Proof of Lemma 4. According to Proposition 1, the set of equilibria is determined by the functional form of (5) and the solution of (8). I will show that when θ is uniformly distributed over $\left[\underline{\theta},\overline{\theta}\right]$, these two only depend on b and $\left[\underline{\theta},\overline{\theta}\right]$, but not on the exact shape of the utility functions of the principal and the agent. If true, the set of equilibria must be identical.

To see this, suppose $U_P(x_1,\theta_0)-U_P(x_2,\theta_0)=0$ for some $\theta_0\in\left[\underline{\theta},\overline{\theta}\right]$ and $x_1<x_2$. This implies $L(|x_1-\theta_0|)=L(|x_2-\theta_0|)$. Since $L'(0)=0$ and $L''>0$, $|x-\theta|>0\Rightarrow L'(|x-\theta|)>0$. Therefore, it must be $|x_1-\theta_0|=|x_2-\theta_0|$. Consider the function $V_P(x,\theta)$. According to Lemma 2, $V_P(x,\theta)=U_P(\theta,\theta)-l(|x-\theta|)$, where $l'(0)=0$ and $l''>0$. Therefore $|x_1-\theta_0|=|x_2-\theta_0|$ implies $l(x_1,\theta_0)=l(x_2,\theta_0)$ and $V_P(x_1,\theta_0)=V_P(x_2,\theta_0)$, as required. The other direction follows similarly. Therefore, given $\overline{x}(a_{i-1},a_i)$, the solutions of (8) for U_P and for V_P are identical.

[40] Notice that assumption (13) is unnecessary. For example, if $\tau=0$ then (17) implies $\pi=b$. Moreover, the first order condition implies $T'(\theta+b-x+\Delta^*)=0$ and the second order condition, evaluated at the extreme point, becomes $-T''(\theta+b-x+\Delta^*)\left(\delta C''\left(\Delta^*\right)\right)^2$, which is always negative.

Next, consider the agent's decision. Suppose the agent learns that $\theta \in [\underline{a}, \overline{a}]$. Since $F(\theta)$ is uniform, (5) can be rewritten as

$$\overline{x}(\underline{a}, \overline{a}) \equiv \arg\min_x \frac{1}{\overline{a} - \underline{a}} \int_{\underline{a}}^{\overline{a}} T(|\theta + b - x|)\, d\theta. \tag{55}$$

Recall $T'(0) = 0$ and $T'' > 0$. I argue that as long as $T(\cdot)$ satisfies these two properties, it must be $\overline{x}(\underline{a}, \overline{a}) = \frac{\overline{a}+\underline{a}}{2} + b$. Indeed, let $z = x - b$. It can be verified that

$$\begin{aligned} \frac{\partial}{\partial z} \int_{\underline{a}}^{\overline{a}} T(|\theta - z|)\, d\theta &= \int_{\min\{z,\underline{a}\}}^{\min\{z,\overline{a}\}} T'(z-\theta)\, d\theta - \int_{\max\{z,\underline{a}\}}^{\max\{z,\overline{a}\}} T'(\theta - z)\, d\theta \\ \frac{\partial^2}{\partial^2 z} \int_{\underline{a}}^{\overline{a}} T(|\theta - z|)\, d\theta &= \int_{\min\{z,\underline{a}\}}^{\min\{z,\overline{a}\}} T''(z-\theta)\, d\theta + \int_{\max\{z,\underline{a}\}}^{\max\{z,\overline{a}\}} T''(\theta - z)\, d\theta. \end{aligned}$$

Since $T'' > 0$, the second derivative is positive. Therefore, $\frac{\partial}{\partial z} \int_{\underline{a}}^{\overline{a}} T(|\theta - z|)\, d\theta = 0$ is necessary and sufficient for the solution. The solution must satisfy

$$\int_{\min\{z,\underline{a}\}}^{\min\{z,\overline{a}\}} T'(z-\theta)\, d\theta = \int_{\max\{z,\underline{a}\}}^{\max\{z,\overline{a}\}} T'(\theta - z)\, d\theta.$$

Since $T' > 0$, the solution requires $\underline{a} < z < \overline{a}$. If $\underline{a} < z < \overline{a}$, then by integration it must be $T(z - \underline{a}) = T(\overline{a} - z)$, which implies $z = \frac{\underline{a}+\overline{a}}{2} \Leftrightarrow x = \frac{\overline{a}+\underline{a}}{2} + b$, as required. Since $t(\cdot)$ is also increasing and convex, the agent's optimal decision (given $\underline{a}$ and $\overline{a}$) when his utility is given by $U_A(x, \theta; b)$ is the same as it is when it is given by $V_A(x, \theta; b)$.

Finally, notice that $\overline{x}(a_{i-1}, a_i) = \frac{a_{i-1}+a_i}{2} + b$ and $a_i = \frac{\overline{x}(a_{i-1},a_i)+\overline{x}(a_i,a_{i+1})}{2}$ for $i = 1, ..., N-1$. Combined, the solution of (8) requires equality (24). Given the boundary conditions (9), Crawford and Sobel (1982) showed that the solution is parameterized by a_1,

$$a_i = \underline{\theta} + i \cdot (a_1 - \underline{\theta}) - 2i(i-1)\, b. \tag{56}$$

Therefore, the largest number of intervals that can be supported in equilibrium is the largest integer such that $a_1 = \underline{\theta}$ and $a_N < \overline{\theta}$, that is, $\underline{\theta} + 2N(N-1)\, b < \overline{\theta}$, which gives (25). This completes the proof. ■

Proof of Proposition 3. The first part immediately follows from the fact that $N(\cdot)$ being a decreasing function. Consider the second part. Suppose $\beta > b$, and let $N\left(b, \overline{\theta} - \underline{\theta}\right)$ be the function $N(b)$ parametrized by $\overline{\theta} - \underline{\theta}$. I prove that there exists $\overline{\theta} - \underline{\theta} > 0$ such that $N\left(\beta, \overline{\theta} - \underline{\theta}\right) <$

$N\left(b, \overline{\theta} - \underline{\theta}\right)$. Note that if $N\left(b, \overline{\theta} - \underline{\theta}\right) \geq N_0$ then $b < \frac{\overline{\theta} - \underline{\theta}}{2N_0(N_0-1)}$. Therefore, if $\beta > b$ then there exists $\overline{\theta}' - \underline{\theta}' > 0$ such that $b < \frac{\overline{\theta}' - \underline{\theta}'}{2N_0(N_0-1)} \leq \beta$, which implies $N(\beta, \overline{\theta}' - \underline{\theta}') < N_0 \leq N(b, \overline{\theta}' - \underline{\theta}')$, as required. The proof for $\beta < b$ is similar and hence omitted. ■

Proof of Lemma 5. According to Lemma 3, $\beta = \pi + (C')^{-1}\left(\frac{L'(\pi)}{\delta}\right)$ where π is the unique solution of (17). I start by showing that $\frac{\partial \beta}{\partial \tau} < 0$. Note that

$$\frac{\partial \beta}{\partial \tau} = \frac{\partial \pi}{\partial \tau} + \frac{\frac{L''(\pi)}{\delta}\frac{\partial \pi}{\partial \tau}}{C''((C')^{-1}(\frac{L'(\pi)}{\delta}))} = \frac{\partial \pi}{\partial \tau}\left(1 + \frac{\frac{L''(\pi)}{\delta}}{C''((C')^{-1}(\frac{L'(\pi)}{\delta}))}\right).$$

Since $L'' > 0$ and $C'' > 0$, the signs of $\frac{\partial \beta}{\partial \tau}$ and $\frac{\partial \Delta}{\partial \tau}$ are identical. Recall the RHS of (17) is increasing in π and the LHS is decreasing in π. Since the RHS of (17) increases in τ and the LHS is independent of τ, the unique solution of (17) is decreasing in τ. That is, $\frac{\partial \pi}{\partial \tau} < 0$, which implies $\frac{\partial \beta}{\partial \tau} < 0$ as required. Next, note that if $\tau = 0$ then $\pi = b$ and $\beta = b + (C')^{-1}\left(\frac{L'(b)}{\delta}\right) > b$. Also, from (17), if $\tau \to \infty$ then $\pi \to 0$. Since $L'(0) = C'(0) = 0$, if $\tau \to \infty$ then $\beta \to 0 < b$. Therefore, there exists $\tau^* > 0$ such that $\beta > b \Leftrightarrow \tau \leq \tau^*$. Letting $\underline{\tau} = \overline{\tau} = \tau^*$ completes the proof.

Next, note that if $\tau = 0$ then $\beta > b$ for all δ. Suppose $\tau > 0$. I argue $\lim_{\delta \to 0} \beta = 0$. Indeed, if $\delta \to 0$ then the agent can expect $\Delta^* \to x - \theta$. Since $\tau > 0$, choosing $x \neq \theta$ is inferior to $x = \theta$: in both cases project θ is implemented, but in the former case the agent incurs an additional cost of $\tau K(|x - \theta|) > 0$. Therefore, there exists $\underline{\delta} > 0$ such that if $\delta < \underline{\delta}$ then $\beta < b$.

Finally, I show that there is $\overline{\delta} > 0$ such that if $\delta > \overline{\delta}$ then $\beta > b$. Note that

$$\frac{\partial \beta}{\partial \delta} = \frac{\partial \pi}{\partial \delta} + \frac{\delta L''(\Delta)\frac{\partial \pi}{\partial \delta} - L'(\pi)}{\delta^2 C''((C')^{-1}(\frac{L'(\pi)}{\delta}))}.$$

Let $z = (C')^{-1}\left(\frac{L'(\pi)}{\delta}\right)$. Then, $\frac{\partial \beta}{\partial \delta} < 0 \Leftrightarrow \frac{\partial \pi}{\partial \delta} < \frac{L'(\pi)}{\delta^2 C''(z) + \delta L''(\pi)}$. Applying the implicit function theorem on (17),

$$\frac{\partial \pi}{\partial \delta} = -\frac{C''(z)T'(b-\pi) - \delta C'''(z)\frac{L'(\pi)}{\delta^2 C''(z)}T'(b-\pi) + L''(\pi)\tau K''(z)\frac{L'(\pi)}{\delta^2 C''(z)}}{\delta C'''(z)\frac{L''(\pi)}{\delta C''(z)}T'(b-\pi) - \delta C''(z)T''(b-\pi) - L'''(\pi)\tau K'(z) - L''(\pi)\tau K''(z)\frac{L''(\pi)}{\delta C''(z)}}$$

Using (17), $T'(b-\pi) = L''(\pi)\frac{\tau K'(z)}{\delta C''(z)}$, we get

$$\frac{\partial \pi}{\partial \delta} = \frac{C''(z)L''(\pi)\frac{\tau K'(z)}{\delta C''(z)} - C'''(z)\frac{L'(\pi)}{\delta C''(z)}L''(\pi)\frac{\tau K'(z)}{\delta C''(z)} + L''(\pi)\tau K''(z)\frac{L'(\pi)}{\delta^2 C''(z)}}{-C'''(z)\frac{L''(\pi)}{C''(z)}L''(\pi)\frac{\tau K'(z)}{\delta C''(z)} + \delta C''(z)T''(b-\pi) + L'''(\pi)\tau K'(z) + L''(\pi)\tau K''(z)\frac{L''(\pi)}{\delta C''(z)}}$$

Thus $\frac{\partial \pi}{\partial \delta} < \frac{L'(\pi)}{\delta^2 C''(z) + \delta L''(\pi)} \Leftrightarrow$

$$\frac{\frac{\tau K'(z)}{C''(z)}\left[C''(z)L''(\pi) - C'''(z)\frac{L'(\pi)}{\delta C''(z)}L''(\pi)\right]\frac{\delta C''(z)+L''(\pi)}{\delta L'(\pi)} + L''(\pi)\tau K''(z)\frac{C''(z)+\frac{L''(\pi)}{\delta}}{\delta C''(z)}}{-C'''(z)\frac{L''(\pi)}{C''(z)}L''(\pi)\frac{\tau K'(z)}{\delta^2 C''(z)} + C''(z)T''(b-\pi) + L'''(\pi)\frac{\tau}{\delta}K'(z) + L''(\pi)\tau K''(z)\frac{L''(\pi)}{\delta^2 C''(z)}} < 1$$

Note that $\lim_{\delta\to\infty}\beta = \lim_{\delta\to\infty}\pi = b$. Also, $\lim_{\delta\to\infty} z = 0$. Therefore, as $\delta \to \infty$ the LHS converges to zero. Therefore, I proved $\lim_{\delta\to\infty}\frac{\partial \beta}{\partial \delta} < 0$. Since, $\lim_{\delta\to\infty}\beta = b$, it follows that for δ sufficiently large, $\beta > b$. ■

Proof of Proposition 4. The proof has two steps. First, I prove that condition (28) holds if and only if $\theta < \theta^{**}$, where $\theta^{**} < \theta^*$ is a decreasing function of δ.[41] To see why, notice that $x_R > \theta^* > \theta$ and recall $L' \geq 0$. We can rewrite (28) as

$$\delta C(x_R - x_L) < L(x_R - \theta) - L(|x_L - \theta|). \tag{57}$$

If $x_L < \theta$ then the derivative of the RHS is $-L'(x_R-\theta) - L'(\theta - x_L) < 0$. If $x_L > \theta$ then it is $-L'(x_R-\theta) + L'(x_L-\theta)$. Since $L'' > 0$ and $x_R > x_L > \theta$, this term is also negative. Overall, the RHS is a decreasing function of θ. Therefore, there is θ^{**} such that condition (28) holds if and only if $\theta < \theta^{**}$. Since the RHS is a decreasing function of θ, θ^{**} is a decreasing function of δ. Finally, note that if $\theta = \theta^*$ then $L(x_R - \theta^*) - L(\theta^* - x_L) = 0$. Indeed, $\theta^* = \frac{x_L + x_R}{2}$, implies $x_R - \theta^* = \theta^* - x_L$. Therefore, it must be $\theta^{**} < \theta^*$, as required.

Second, note that condition (27) does not depend on δ. I prove that there exists $\delta_{disobedience} > 0$ as stated in the proposition. For this purpose, define

$$\eta(z) \equiv E\left[T(|\theta + b - x_L|) - T(|x_R - \theta - b|)\,|z < \theta < \theta^*\right]. \tag{58}$$

I prove that $\eta(z)$ is an increasing function of z. Since $\eta(z)$ is a weighted average, it is sufficient to show that $\sigma(\theta) \equiv T(|\theta + b - x_L|) - T(|x_R - \theta - b|)$ is an increasing function of θ. Recall

[41] I assume that if the principal is indifferent between intervening and not intervening, then she does not intervene. Since this indifference is a zero probability event, this assumption is immaterial.

$x_R = \frac{\theta^* + \overline{\theta}}{2} + b > \theta^* + b$. Therefore, $\theta < \theta^*$ implies $x_R - \theta - b > 0$. Then

$$\sigma'(\theta) = \begin{cases} -T'(x_L - \theta - b) + T'(x_R - \theta - b) & \text{if } \theta < x_L - b \\ T'(\theta + b - x_L) + T'(x_R - \theta - b) & \text{else.} \end{cases} \tag{59}$$

Recall $T'(0) = 0$ and $T'' > 0$. Moreover, recall $x_R > x_L$. Therefore, $\sigma'(\theta) > 0$ as required. Since $\eta(z)$ is an increasing function of z, combined with the first step, $\eta(\theta^{**})$ is a decreasing function of δ. Note that (29) holds if and only if $\eta(\theta^{**}) \leq 0$. Moreover, note that as $\delta \to 0$, we have $\theta^{**} \to \theta^*$, that is, conditional on $\theta < \theta^*$ the principal intervenes whenever the agent disobeys her. However, notice that $\theta^* = \frac{x_L + x_R}{2}$ implies $T(|\theta^* + b - x_L|) > T(|x_R - \theta^* - b|)$, which in turn implies $\eta(\theta^*) > 0$. Therefore, there exists $\delta_{disobedience} > 0$ such that $\eta(\theta^{**}) > 0$ if and only if $\delta < \delta_{disobedience}$, as required. ■

Proof of Proposition 5. According to Lemma 4 and Corollary 1, $N^* \geq 2 \Leftrightarrow b < \frac{\overline{\theta} - \underline{\theta}}{4}$ and $N^{**} = 1 \Leftrightarrow \frac{\overline{\theta} - \underline{\theta}}{4} \leq \beta$. According to Lemma 3, $\beta(\tau, \delta) = \pi(\tau, \delta) + (C')^{-1}(\frac{L'(\pi(\tau,\delta))}{\delta})$ where $\pi(\tau, \delta) \in (0, b]$ is the unique solution of (17). Note that $\overline{\theta} - E[\theta] = \frac{\overline{\theta} - \underline{\theta}}{2} > \frac{\overline{\theta} - \underline{\theta}}{4}$. I proceed in several steps.

1. First, I prove that there exist $\delta_1 > 0$ and a function $\tau_1(\cdot) > 0$ such that if $0 < \delta \leq \delta_1$ and $\tau \leq \tau_1(\delta)$ then $\beta(\tau_1(\delta), \delta) \geq \overline{\theta} - E[\theta]$ and $EV_P(N^{**}) < EU_P(1)$. Proof:

a. Since $\beta(0, \delta)$ is a decreasing function of δ where $\lim_{\delta \to 0} \beta(0, \delta) = \infty$, there exists $\delta' > 0$ such that $\beta(0, \delta) > \overline{\theta} - E[\theta] \Leftrightarrow \delta < \delta'$. Also note that $\beta(\tau, \delta)$ is a decreasing and continuous function of τ, where $\tau > 0$ implies $\lim_{\delta \to 0} \beta(\tau, \delta) = 0$. Therefore, for all $\delta < \delta_1$ there is $\tau_1(\delta) > 0$ such that if $\tau \in [0, \tau_1(\delta)]$ then $\beta(\tau, \delta) > \overline{\theta} - E[\theta]$.

b. Note that $\lim_{\delta \to 0} \tau_1(\delta) = 0$. Indeed, suppose on the contrary that $\lim_{\delta \to 0} \tau_1(\delta) = \tau_1 > 0$. In this case, $\lim_{\delta \to 0} \beta(\tau_1(\delta), \delta) = \lim_{\delta \to 0} \beta(\tau_1, \delta) = 0 < \overline{\theta} - E[\theta]$, which contradicts the definition of $\tau_1(\delta)$.

c. Before proceeding, I assume without the loss of generality that $\tau_1(\delta)$ converges to zero faster than δ. Specifically, since $\lim_{\delta \to 0} \tau_1(\delta) = 0$ and $\lim_{\delta \to 0} \beta(0, \delta) = \infty$, I can assume that $\lim_{\delta \to 0} \beta(\tau_1(\delta), \delta) = \infty$. Also, since $\tau = 0 \Rightarrow \pi = b$ for all δ, I assume without the loss of generality that $\lim_{\delta \to 0} \pi(\tau_1(\delta), \delta) = b$ and $\lim_{\delta \to 0} \frac{\partial \pi(\tau_1(\delta), \delta)}{\partial \delta} = 0$.

d. Suppose $\delta < \delta_1$ and $\tau < \tau_1(\delta)$. Since $\beta(\tau, \delta) > \overline{\theta} - E[\theta] > \frac{\overline{\theta} - \underline{\theta}}{4}$, it must be $N^{**} = 1$, and according to Lemma 4, the agent chooses $x^{**} = E[\theta] + \beta(\tau, \delta)$. Notice that $\beta(\tau, \delta) >$

$\overline{\theta} - E[\theta]$ implies $E[\theta] + \beta(\tau,\delta) - \theta > 0$ for all $\theta \leq \overline{\theta}$. Building on Lemma 2,

$$\begin{aligned} EV_P(N^{**}) &= E\left[V_P(E[\theta] + \beta(\tau,\delta), \theta)\right] \\ &= E[U_P(\theta,\theta)] - E[l(E[\theta] + \beta(\tau,\delta) - \theta) \\ &\leq E[U_P(\theta,\theta)] - l(E[E[\theta] + \beta(\tau,\delta) - \theta]) = E[U_P(\theta,\theta)] - l(\beta(\tau,\delta)), \end{aligned}$$

where the inequality follows from Jensen's inequality. The definition of $l(\cdot)$ in Lemma 2 implies

$$l(\beta) = L(|\beta - \Delta^*(\beta)|) + \delta C(|\Delta^*(\beta)|).$$

Since $\beta - \Delta^*(\beta) = \pi$,

$$l(\beta) = L(\pi) + \delta C(\beta - \pi) = L(\pi) + \delta C((C')^{-1}(\frac{L'(\pi)}{\delta})).$$

Recall $\tau \leq \tau_1(\delta)$. I argue that $\lim_{\delta\to 0} l(\beta(\tau,\delta)) = \infty$. Indeed, since $\pi \in [0,b]$ for any δ and τ, $L(\pi(\tau,\delta))$ is bounded. Also, from L'Hospital's Rule, if $\tau \leq \tau_1(\delta)$ then

$$\begin{aligned} \lim_{\delta\to 0} \frac{C((C')^{-1}(\frac{L'(\pi(\tau,\delta))}{\delta}))}{\frac{1}{\delta}} &= \lim_{\delta\to 0} \frac{C'((C')^{-1}(\frac{L'(\pi(\tau,\delta))}{\delta}))\frac{\frac{L''(\pi(\tau,\delta))\frac{\partial \pi(\tau,\delta)}{\partial\delta}\delta - L'(\pi(\tau,\delta))}{\delta^2}}{C''((C')^{-1}(\frac{L'(\pi(\tau,\delta))}{\delta}))}}{-\frac{1}{\delta^2}} \\ &= \lim_{\delta\to 0} \frac{\frac{(L'(\pi(\tau,\delta)))^2}{\delta} - L'(\pi(\tau,\delta))L''(\pi(\tau,\delta))\frac{\partial\pi(\tau,\delta)}{\partial\delta}}{C''((C')^{-1}(\frac{L'(\pi(\tau,\delta))}{\delta}))}. \end{aligned}$$

Since $\lim_{\delta\to 0}\pi(\tau,\delta) = b$ and $\lim_{\delta\to 0}\frac{\partial\pi(\tau,\delta)}{\partial\delta} = 0$, we have

$$\lim_{\delta\to 0} \delta C((C')^{-1}(\frac{L'(\pi(\tau,\delta))}{\delta})) = \lim_{\delta\to 0}\frac{1}{\delta}\frac{L'(b)^2}{C''((C')^{-1}(\frac{L'(b)}{\delta}))}.$$

Notice that C'' is a strictly positive and non-increasing function. Therefore, $\lim_{\delta\to 0}\frac{1}{\delta}\frac{L'(b)^2}{C''((C')^{-1}(\frac{L'(b)}{\delta}))} = \infty$ and $\lim_{\delta\to 0} l(\beta(\tau,\delta)) = \infty$. We conclude that if $\tau \leq \tau_1(\delta)$ then $\lim_{\delta\to 0} E\left[V_P(E[\theta] + \beta(\tau,\delta),\theta)\right] = -\infty$. Since $EU_P(1)$ is independent of τ and δ, $EV_P(N^{**}) < EU_P(1)$ as required.

2. Second, I prove that there exist $\delta_2 \in (0,\delta_1)$ and a function $\tau_2(\cdot) > 0$ such that if $0 < \delta \leq \delta_2$ and $\tau = \tau_2(\delta)$ then $\beta(\tau,\delta) = \overline{\theta} - E[\theta]$ and $EV_P(N^{**}) > EU_P(1)$. To see why, consider any $\delta_2 \in (0,\delta_1)$. Recall that $\beta(\tau,\delta)$ decreases in τ and $\lim_{\delta\to 0}\beta(\tau,\delta) = 0$ for any $\tau > 0$. Since $\delta_2 < \delta_1$ implies $\beta(0,\delta) > \overline{\theta} - E[\theta]$ for all $0 < \delta \leq \delta_2$, for every $0 < \delta \leq \delta_2$ there is a unique

$\tau_2(\delta) > 0$ such that $\beta(\tau_2(\delta), \delta) = \overline{\theta} - E[\theta]$. Therefore, $N^{**} = 1$ and

$$EV_P(N^{**}) = E[V_P(E[\theta] + \beta(\tau_2(\delta), \delta), \theta)] = E\left[V_P(\overline{\theta}, \theta)\right].$$

Notice that for any given project x, $\lim_{\delta \to 0} E[V_P(x, \theta)] = E[U_P(\theta, \theta)]$. Therefore, $\lim_{\delta \to 0} E\left[V_P(\overline{\theta}, \theta)\right] = E[U_P(\theta, \theta)]$. Since $EU_P(1)$ is independent of δ and τ, and it is strictly smaller than $E[U_P(\theta, \theta)]$, the claim is proved.

3. Third, let $\varepsilon \equiv EU_P(N^*) - EU_P(1)$. From Crawford and Sobel we know that if $N^* \geq 2$ then $\varepsilon > 0$. Notice that ε is independent of τ and δ.

4. Fourth, combined, the first and the second steps imply that for any $0 < \delta_0 \leq \delta_2$ there are $\tau', \tau'' > 0$ such that $\beta(\tau', \delta_0) > \frac{\overline{\theta} - \underline{\underline{\theta}}}{4}$, $\beta(\tau'', \delta_0) > \frac{\overline{\theta} - \underline{\underline{\theta}}}{4}$, and

$$\begin{aligned} E[V_P(E[\theta] + \beta(\tau', \delta_0), \theta)] &= EV_P(N^{**}, \tau = \tau') \\ &< EU_P(1) \\ &< EV_P(N^{**}, \tau = \tau'') = E[V_P(E[\theta] + \beta(\tau'', \delta_0), \theta)]. \end{aligned}$$

Since $E[V_P(E[\theta] + \beta(\tau, \delta), \theta)]$ is a continuous function of τ and δ, there exist $0 \leq \underline{\underline{\tau}} < \overline{\overline{\tau}}$ and $0 < \underline{\underline{\rho}} < \overline{\overline{\delta}}$ such that if $\tau \in [\underline{\underline{\tau}}, \overline{\overline{\tau}}]$ and $\delta \in [\underline{\underline{\delta}}, \overline{\overline{\delta}}]$ then $\beta(\tau, \delta) > \frac{\overline{\theta} - \underline{\underline{\theta}}}{4}$ and

$$EU_P(1) < E[V_P(E[\theta] + \beta(\tau, \delta), \theta)] < EU_P(1) + \varepsilon.$$

Noting that $EU_P(1) + \varepsilon = EU_P(N^*)$ completes the proof. ■

Proof of Proposition 6. Consider first the game without intervention. As argued by Harris and Raviv (2005), the set of equilibria in a game with two-sided information asymmetry is equivalent to the set of equilibria in a game in which only the principal has private information (as in Crawford and Sobel (1982)), with one exception. The exception is that if $x^*(m)$ is the agent's optimal choice given message m in the an equilibrium of the game with one-sided information asymmetry, then with two-sided information asymmetry the agent's optimal choice in equilibrium is $x(\theta_A, m) = \theta_A + x^*(m)$. Note that $x^*(m) = \mathbb{E}[\theta_P|m] + b$, and thus, $x(\theta_A, m) = \theta_A + \mathbb{E}[\theta_P|m] + b$ and

$$U_P(\theta_P, m) = \mathbb{E}[A - (\theta_P + \theta_A - x(\theta_A, m))^2|\theta_P, m] = A - (\theta_P - \mathbb{E}[\theta_P|m] - b)^2. \tag{60}$$

Next, consider the game with intervention. Suppose that in equilibrium the agent follows a linear strategy

$$x^*(\theta_A, m) = \alpha\theta_A + \phi(m),$$

where α is a scalar and $\phi(\cdot)$ is a real function. Conditional on θ_P, message m, and the agent's decision x, the principal solves

$$\begin{aligned}\Delta^*(x, \theta_P, m) &= \arg\max_{\Delta}\{A - \mathbb{E}[(\theta_P + H(x, \theta_A, m) - (x - \Delta))^2|\theta_P] - \delta\Delta^2\} \\ &= \frac{x - \theta_P - \mathbb{E}[H(x, \theta_A, m)]}{1 + \delta},\end{aligned}$$

where

$$H(x, \theta_A, m) = \begin{cases} \frac{x - \phi(m)}{\alpha} & \textit{if } \alpha \neq 0 \\ \theta_A & \textit{if } \alpha = 0 \end{cases}$$

is the principal's inference from the agent's decision. The agent therefore solves

$$\max_x \mathbb{E}\left[A - (\theta_P + \theta_A + b - x + \Delta^*(x, \theta_P, m))^2 - \tau\left(\Delta^*(x, \theta_P, m)\right)^2 |\theta_A, m\right] \Rightarrow \tag{61}$$

$$x^*(\theta_A, m) = \begin{cases} \frac{1}{1 + \frac{1-\alpha}{\alpha}\frac{1+\tau\frac{1-\alpha}{1+\alpha\delta}}{1+\delta}}\theta_A + \frac{\frac{1}{1+\delta}(\delta - \frac{\tau - \tau\alpha}{1+\delta\alpha})(\mathbb{E}[\theta_P|m] - \frac{\phi(m)}{\alpha}) + \frac{\phi(m)}{\alpha} + b}{1 + \frac{1-\alpha}{\alpha}\frac{1+\tau\frac{1-\alpha}{1+\alpha\delta}}{1+\delta}} & \textit{if } \alpha \neq 0 \\ \mathbb{E}[\theta_P|m] + \mathbb{E}[\theta_A] + \frac{\delta^2+\delta}{\delta^2+\tau}(\theta_A - \mathbb{E}[\theta_A] + b) & \textit{if } \alpha = 0. \end{cases} \tag{62}$$

Since $\frac{\delta^2+\delta}{\delta^2+\tau} \neq 0$, it must be $\alpha \neq 0$. Matching the coefficient of θ_A to α implies $\alpha \in \{1, \frac{\tau-\delta}{\tau+\delta^2}\}$. Notice that $\alpha = \frac{\tau-\delta}{\tau+\delta^2}$ cannot be an equilibrium. Indeed, if $\alpha = \frac{\tau-\delta}{\tau+\delta^2}$ then the second term in (62) becomes $\phi(m) + \frac{\tau-\delta}{\tau+\delta^2}b$. However, matching coefficients implies $\phi(m) + \frac{\tau-\delta}{\tau+\delta^2}b = \phi(m)$, which requires either $\frac{\tau-\delta}{\tau+\delta^2} = 0$ or $b = 0$. The former does not hold since $\alpha = \frac{\tau-\delta}{\tau+\delta^2}$ and α cannot be zero as was proven above. The latter does not hold by the assumption. Therefore, it must be $\alpha = 1$. If $\alpha = 1$ then matching the coefficient on the second term implies $\phi(m) = \mathbb{E}[\theta_P|m] + \frac{1+\delta}{\delta}b$, and hence,

$$x^*(\theta_A, m) = \theta_A + \mathbb{E}[\theta_P|m] + b\frac{1+\delta}{\delta}.$$

and

$$\Delta^*(x(\theta_A, m), \theta_P, m) = \frac{\mathbb{E}[\theta_P|m] - \theta_P + \frac{1+\delta}{\delta}b}{1+\delta}$$

Anticipating $x^*(\theta_A, m)$, the principal's expected utility conditional on θ_P and on sending message m is

$$\begin{aligned} V_P(\theta_P, m) &= \mathbb{E}\left[\begin{array}{c} A - (\theta_P + \theta_A - x(\theta_A, m) + \Delta^*(x(\theta_A, m), \theta_P, m))^2 \\ -\delta(\Delta^*(x(\theta_A, m), \theta_P, m))^2 \end{array} |m, \theta_P\right] \\ &= \mathbb{E}\left[A - \frac{\delta}{1+\delta}(\mathbb{E}[\theta_P|m] - \theta_P + \frac{1+\delta}{\delta}b)^2|m, \theta_P\right] \end{aligned}$$

Thus,

$$V_P(\theta_P, m) = A - \frac{\delta}{1+\delta}(\theta_P - \mathbb{E}[\theta_P|m] - \frac{1+\delta}{\delta}b)^2. \tag{63}$$

Recall that without intervention, the principal expected utility conditional on θ_P and on sending message m is given by (60). The only difference from (63) is that b is replaced by $\frac{1+\delta}{\delta}b$, and the entire term is scaled by $\frac{\delta}{1+\delta}$. It follows that at the communication stage, the principal behaves as if her preferences are represented by the utility function $-(\theta_P + \theta_A - x)^2$. The agent behaves as if $\delta = \infty$, $\tau = 0$, and his preferences are represented by the utility function $-(\theta_P + \theta_A + b\frac{1+\delta}{\delta} - x)^2$, which completes the proof. ■

Proof of Proposition 8. Note that $C \leq L(b) - L(0) \Leftrightarrow \overline{C} \leq b$. Suppose the agent learned that $\theta \in [\underline{a}, \overline{a}]$, where $\underline{\theta} \leq \underline{a} < \overline{a} \leq \overline{\theta}$. I start by arguing that the agent's optimal choice, denoted by $x^*(\underline{a}, \overline{a})$, is $\max\left\{\overline{a}, \underline{a} + \overline{C}\right\}$. To see why, note that the agent's expected utility conditional on x and learning $\theta \in [\underline{a}, \overline{a}]$ is

$$\begin{aligned} U_A(x, \underline{a}, \overline{a}) &= E\left[U_A(\theta + b, \theta; b) | \theta \in [\underline{a}, \overline{a}]\right] - \Pr\left[\theta \in \left[x - \overline{C}, x + \overline{C}\right] | \theta \in [\underline{a}, \overline{a}]\right] \\ &\quad \times E\left[T(|x - \theta - b|) | \theta \in \left[x - \overline{C}, x + \overline{C}\right] \cap [\underline{a}, \overline{a}]\right] - \Pr\left[\theta \notin \left[x - \overline{C}, x + \overline{C}\right] | \theta \in [\underline{a}, \overline{a}]\right] T(b) \\ &= E\left[U_A(\theta + b, \theta; b) | \theta \in [\underline{a}, \overline{a}]\right] - T(b) - \Pr\left[\theta \in \left[x - \overline{C}, x + \overline{C}\right] | \theta \in [\underline{a}, \overline{a}]\right] \\ &\quad \times E\left[T(|x - \theta - b|) - T(b) | \theta \in \left[x - \overline{C}, x + \overline{C}\right] \cap [\underline{a}, \overline{a}]\right], \\ &= E\left[U_A(\theta + b, \theta; b) | \theta \in [\underline{a}, \overline{a}]\right] - T(b) - \frac{H(x, \underline{a}, \overline{a})}{\Pr[\theta \in [\underline{a}, \overline{a}]]}. \end{aligned}$$

where

$$H(x, \underline{a}, \overline{a}) \equiv \int_{\max\{\underline{a}, x - \overline{C}\}}^{\min\{\overline{a}, x + \overline{C}\}} \left[T(|x - \theta - b|) - T(b)\right] d\theta. \tag{64}$$

Notice that $\overline{C} \leq b$ implies $\theta > x - \overline{C} \Rightarrow \theta > x - b$. Therefore,

$$H(x, \underline{a}, \overline{a}) = \int_{\max\{\underline{a}, x - \overline{C}\}}^{\min\{\overline{a}, x + \overline{C}\}} \left[T(\theta + b - x) - T(b)\right] d\theta, \tag{65}$$

which is a continuous function. Let $x^*(\underline{a},\overline{a}) \in \arg\min_x H(x,\underline{a},\overline{a})$. I prove that $x^*(\underline{a},\overline{a}) = \max\left\{\overline{a},\underline{a}+\overline{C}\right\}$. There are several steps:

1. I argue $x^*(\underline{a},\overline{a}) \in \left(\underline{a}-\overline{C},\overline{a}+\overline{C}\right)$. Indeed, if $x \leq \underline{a}-\overline{C}$ or $x \leq \overline{a}+\overline{C}$ then $H(x,\underline{a},\overline{a}) = 0$, whereas for $\varepsilon > 0$ arbitrarily small,

$$\begin{aligned} H\left(\overline{a}+\overline{C}-\varepsilon,\underline{a},\overline{a}\right) &= \int_{\overline{a}-\varepsilon}^{\overline{a}} \left[T\left(\theta-(\overline{a}-\varepsilon)+b-\overline{C}\right)-T(b)\right] d\theta \\ &< \int_{\overline{a}-\varepsilon}^{\overline{a}} \left[T\left(\overline{a}-(\overline{a}-\varepsilon)+b-\overline{C}\right)-T(b)\right] d\theta \\ &= \int_{\overline{a}-\varepsilon}^{\overline{a}} \left[T\left(\varepsilon+b-\overline{C}\right)-T(b)\right] d\theta < 0. \end{aligned}$$

2. I argue $x^*(\underline{a},\overline{a}) \leq \max\left\{\overline{a},\underline{a}+\overline{C}\right\}$. Indeed, if $\max\left\{\overline{a},\underline{a}+\overline{C}\right\} < x < \overline{a}+\overline{C}$ then

$$H(x,\underline{a},\overline{a}) = \int_{x-\overline{C}}^{\overline{a}} \left[T(\theta+b-x)-T(b)\right] d\theta$$

and

$$\begin{aligned} \frac{\partial H(x,\underline{a},\overline{a})}{\partial x} &= -\left[T\left(b-\overline{C}\right)-T(b)\right] - \int_{x-\overline{C}}^{\overline{a}} T'(\theta+b-x)\, d\theta \\ &= -T\left(b-\overline{C}\right)+T(b)-T(\overline{a}+b-x)+T\left(b-\overline{C}\right) = T(b)-T(\overline{a}+b-x). \end{aligned}$$

Since $T' > 0$, $x > \overline{a}$ implies $\frac{\partial H(x,\underline{a},\overline{a})}{\partial x} > 0$. Therefore, $\arg\min_x H(x,\underline{a},\overline{a}) \leq \max\left\{\overline{a},\underline{a}+\overline{C}\right\}$.

3. I argue $x^*(\underline{a},\overline{a}) \geq \underline{a}+\overline{C}$. Indeed, if $\underline{a}-\overline{C} < x < \underline{a}+\overline{C}$ then

$$H(x,\underline{a},\overline{a}) = \begin{cases} \int_{\underline{a}}^{\overline{a}} \left[T(\theta+b-x)-T(b)\right] d\theta & \text{if } \min\{\overline{a}-\overline{C},\underline{a}+\overline{C}\} \leq x < \underline{a}+\overline{C} \\ \int_{\underline{a}}^{x+\overline{C}} \left[T(\theta+b-x)-T(b)\right] d\theta & \text{if } \underline{a}-\overline{C} < x < \min\{\overline{a}-\overline{C},\underline{a}+\overline{C}\}. \end{cases} \quad (66)$$

Thus, if $\min\{\overline{a}-\overline{C},\underline{a}+\overline{C}\} \leq x < \underline{a}+\overline{C}$ then $\frac{\partial H(x,\underline{a},\overline{a})}{\partial x} = -\int_{\underline{a}}^{\overline{a}} T'(\theta+b-x)\, d\theta < 0$. If $\underline{a}-\overline{C} < x < \min\{\overline{a}-\overline{C},\underline{a}+\overline{C}\}$ then

$$\begin{aligned} \frac{\partial H(x,\underline{a},\overline{a})}{\partial x} &= T\left(b+\overline{C}\right)-T(b) - \int_{\underline{a}}^{x+\overline{C}} T'(\theta+b-x)\, d\theta \\ &= T\left(b+\overline{C}\right)-T(b)-T\left(x+\overline{C}+b-x\right)+T(\underline{a}+b-x) = T(\underline{a}+b-x)-T(b). \end{aligned}$$

Since $T' > 0$, in this range $\frac{\partial H(x,\underline{a},\overline{a})}{\partial x} > 0 \Leftrightarrow x < \underline{a}$. Therefore, the only candidate for the minimizer in this range is the lowest possible value of x, $\underline{a} - \overline{C}$, which yields $H(x, \underline{a}, \overline{a}) = 0$. But part #1 has already established that $\underline{a} - \overline{C} < x^*(\underline{a}, \overline{a})$. Therefore, $x^*(\underline{a}, \overline{a}) \geq \underline{a} + \overline{C}$.

4. Based on the previous two steps, $\underline{a}+\overline{C} \leq x^*(\underline{a}, \overline{a}) \leq \max\left\{\overline{a}, \underline{a} + \overline{C}\right\}$. Therefore, if $\overline{a} \leq \underline{a}+\overline{C}$ then $x^*(\underline{a}, \overline{a}) = \underline{a} + \overline{C}$. Suppose $\underline{a} + \overline{C} < \overline{a}$. I argue $x^*(\underline{a}, \overline{a}) = \overline{a}$. Indeed, if $\underline{a} + \overline{C} \leq x < \overline{a}$ then

$$H(x, \underline{a}, \overline{a}) = \begin{cases} \int_{x-\overline{C}}^{\overline{a}} [T(\theta + b - x) - T(b)]\, d\theta & \textit{if} \;\; \max\{\overline{a} - \overline{C}, \underline{a} + \overline{C}\} \leq x < \overline{a} \\ \int_{x-\overline{C}}^{x+\overline{C}} [T(\theta + b - x) - T(b)]\, d\theta & \textit{if} \;\; \underline{a} + \overline{C} \leq x < \max\{\overline{a} - \overline{C}, \underline{a} + \overline{C}\}. \end{cases} \tag{67}$$

Thus, if $\max\{\overline{a} - \overline{C}, \underline{a} + \overline{C}\} < x < \overline{a}$ then

$$\begin{aligned} \frac{\partial H(x, \underline{a}, \overline{a})}{\partial x} &= -\left[T\left(b - \overline{C}\right) - T(b)\right] - \int_{x-\overline{C}}^{\overline{a}} T'(\theta + b - x)\, d\theta \\ &= -\left[T\left(b - \overline{C}\right) - T(b)\right] - T(\overline{a} + b - x) + T\left(x - \overline{C} + b - x\right) = T(b) - T(\overline{a} + b - x). \end{aligned}$$

Since $T' > 0$, $x < \overline{a}$ implies $\frac{\partial H(x,\underline{a},\overline{a})}{\partial x} < 0$ in this range. If $\underline{a}+\overline{C} \leq x < \max\{\overline{a}-\overline{C}, \underline{a}+\overline{C}\}$ then

$$\begin{aligned} \frac{\partial H(x, \underline{a}, \overline{a})}{\partial x} &= \left[T\left(\overline{C} + b\right) - T(b)\right] - \left[T\left(b - \overline{C}\right) - T(b)\right] - \int_{x-\overline{C}}^{x+\overline{C}} T'(\theta + b - x)\, d\theta \\ &= T\left(\overline{C} + b\right) - T\left(b - \overline{C}\right) - T\left(x + \overline{C} + b - x\right) + T\left(x - \overline{C} + b - x\right) = 0. \end{aligned}$$

Since $H(x, \underline{a}, \overline{a})$ is a continuous function, it must be $x^*(\underline{a}, \overline{a}) \geq \overline{a}$, as require.

Overall, I proved $x^*(\underline{a}, \overline{a}) = \max\left\{\overline{a}, \underline{a} + \overline{C}\right\}$. Suppose on the contrary there exists a partition equilibrium with $n \geq 2$ elements. If $\theta = a_i$ then the principal must be indifferent between $x^*(a_{i-1}, a_i)$ and $x^*(a_i, a_{i+1})$,

$$\max\{-L(0) - C, -L(|x^*(a_{i-1}, a_i) - a_i|)\} = \max\{-L(0) - C, -L(|x^*(a_i, a_{i+1}) - a_i|)\}. \tag{68}$$

I prove that (68) never holds:

1. If $x^*(a_{i-1}, a_i) = a_i$ then (68) requires

$$\begin{aligned} -L(0) &= -L(|x^*(a_i, a_{i+1}) - a_i|) \Leftrightarrow L(0) = L\left(\left|\max\left\{a_{i+1}, a_i + \overline{C}\right\} - a_i\right|\right) \Leftrightarrow \\ 0 &= \left|\max\left\{a_{i+1}, a_i + \overline{C}\right\} - a_i\right|, \end{aligned}$$

which never holds given that $a_{i+1} > a_i$ and $\overline{C} > 0$.

2. If $x^*(a_{i-1}, a_i) = a_{i-1} + \overline{C}$ then

$$\begin{aligned} -L(0) - C &< -L\left(a_{i-1} + \overline{C} - a_i\right) \Leftrightarrow L^{-1}(L(0) + C) > a_{i-1} + \overline{C} - a_i \Leftrightarrow \\ \overline{C} &> a_{i-1} + \overline{C} - a_i \Leftrightarrow a_i > a_{i-1}, \end{aligned}$$

which always holds. Also note that

$$\begin{aligned} -L(0) - C &\geq -L(|x^*(a_i, a_{i+1}) - a_i|) \Leftrightarrow L(0) + C \leq L\left(\max\left\{a_{i+1} - a_i, \overline{C}\right\}\right) \Leftrightarrow \\ \overline{C} &\leq \max\left\{a_{i+1} - a_i, \overline{C}\right\} \end{aligned}$$

which always holds. Combined, (68) requires

$$\begin{aligned} -L\left(a_{i-1} + \overline{C} - a_i\right) &= -L(0) - C \Leftrightarrow L\left(a_{i-1} + \overline{C} - a_i\right) = L(0) + C \Leftrightarrow \\ a_{i-1} + \overline{C} - a_i &= \overline{C} \Leftrightarrow a_{i-1} - a_i = 0 \end{aligned}$$

which never holds.

Overall, I showed that the principal is never indifferent between $x^*(a_{i-1}, a_i)$ and $x^*(a_i, a_{i+1})$ when $\theta = a_i$, and therefore, a partition equilibrium with more than one element does not exist. Finally, note that if the equilibrium is uninformative, then the agent chooses $x^* = \max\left\{\overline{\theta}, \underline{\theta} + \overline{C}\right\}$, and the principal intervenes if and only if $\theta \notin \left[x^* - \overline{C}, x^* + \overline{C}\right]$. Since $x^* \geq \overline{\theta}$, principal intervenes if and only if $\theta < x^* - \overline{C} \Leftrightarrow \theta \in \left[\underline{\theta}, \overline{\theta} - \overline{C}\right]$. ■