When to Introduce Electronic Trading Platforms in Over-the-Counter Markets?

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Abstract

I study a hybrid over-the-counter (OTC) market structure in which traders have the choice of obtaining an asset either in a bilateral market or on an electronic trading platform. In a hybrid market (HM), turnover is higher and expected prices are lower than in a pure bilateral market (PBM). I present sufficient conditions under which dealer profits are higher in the HM than in the PBM and vice versa. Dealers can increase their profits in the HM by colluding to keep their activity on the platform at a certain level. The model also delivers several other empirical implications regarding prices, trading volume and the traders' choices of trading venue under the two different market structures.

1 Introduction

Electronic trading platforms are currently transforming the way investors trade in over-the-counter (OTC) markets. They were also the cause of an EU investigation[1] and a class action in the U.S. resulting in a \$1.87bn settlement in 2015.[2] In this class action, a number of buy-side investors accused 13 dealer banks, along with Markit and the ISDA, of colluding to keep competition from electronic platform providers out of the market. Besides a large fee, the resulting settlement included promises by the defendants to promote

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[1] For details regarding the investigation that started in 2011, see White and Bodoni (2015) and White (2016).

[2] See Drucker and Voris (2015) and McLannahan and Rennison (2015) for details.

electronic trading in the future. The settlement was welcomed by investors and regulators as a significant step toward market efficiency. The rather reluctant introduction of electronic trading in the CDS market stands in sharp contrast to trading practices in the bond market. The first electronic trading platform for bonds has been established in 1988[3] and new bond trading platforms easily gain traction.[4]

What do these disputed trading platforms do? In general, electronic trading platforms are meant to facilitate trading in OTC markets by increasing competition among dealers and leading to higher price transparency. A trader who wants to buy a corporate bond, most of which are not traded on exchanges, usually has the choice between (i) calling a single dealer to ask for a price at which this dealer is willing to trade a certain quantity of the bond and (ii) submitting a so-called request-for-quote (RFQ) via an electronic trading platform to multiple dealers at once. A dealer who has received an RFQ may or may not reply by quoting a price at which the dealer is willing to trade a certain quantity of the bond. If some of the contacted dealers reply to an RFQ, the trader can pick the most attractive quote and trade the bond at this price. Thus, submitting an RFQ on an electronic trading platform means performing a first-price auction among the contacted dealers.

A number of questions arise from the above-mentioned anecdotes on the CDS and bond markets. Why have electronic trading platforms been used in the bond market for such a long time, while dealers were very reluctant to introduce them in the CDS market? How do electronic trading platforms affect the ability of the market to match buyers and sellers of financial assets? How do dealers' quoting strategies change when there is a platform introduced in an OTC market?

To address these questions, I set up a model of a hybrid market (HM) in which traders have the choice between trading an asset in a bilateral market or on an electronic trading platform. In the bilateral market, each dealer has to be contacted separately, whereas a trading platform allows to contact all dealers at once. The assumptions on dealers and traders are as in Duffie et al. (2016), who model a pure bilateral market (PBM). There is a continuum of traders who want to buy one unit of an asset. A fraction of these traders find search costless and the other traders have positive search costs. Traders with search costs are called slow and traders without search costs are called fast. A finite number of dealers can provide the asset at some cost.

One can assess the effects of introducing an electronic trading platform in an OTC market by comparing the equilibrium in an HM (modeled in this paper) with the equilibrium in a PBM (modeled in Duffie et al. (2016)). It turns out that introducing an electronic trading platform has two main effects on the payoffs of

[3]See MTS's timeline: http://www.mtsmarkets.com/about-us/company-timeline.

[4]See Marsh and Detrixhe (2015).

dealers and traders. First, a platform lowers average prices charged to both kinds of traders. Second, the market participation of slow traders may increase in the HM compared to the PBM. Thus, traders always prefer the HM to the PBM in this model, since they potentially get more of the asset at lower prices. This implication is consistent with the above-mentioned anecdote about the CDS market in which traders wanted to have an HM.

On the other hand, the effect of trading platforms on dealer profits is not clear. In the HM, dealers may sell more units of the asset due to higher market participation of the slow traders, but they also receive a lower average markup for each traded asset. This paper gives sufficient conditions under which the effect of an increased volume on dealer profits dominates the effect of lower markups and vice versa. Intuitively, the volume effect dominates when market participation of slow traders is low and the profit from selling to fast traders is low as well. This happens if the number of dealers is large. The price effect dominates if slow traders already participate fully in the market. This happens if search costs become small or if the fraction of fast traders becomes large.

If one views institutional investors as fast traders and retail investors as slow traders, then these model implications are also consistent with the above-mentioned anecdotes about the CDS and bond markets. There seem to be more dealers trading in bond markets than in CDS markets[5] and the fraction of retail investors in bond markets is plausibly higher than the fraction of retail investors in CDS markets.[6] The model in this paper suggests that it may be profitable to introduce a platform in bond markets while it may not be profitable to do so in CDS markets. This may explain why electronic trading platforms have been used in bond markets for many years, while they have been introduced in CDS markets only recently and under pressure from regulators.

Economically, an electronic trading platform is a commitment device. Dealers may want to introduce an electronic trading platform because increased competition enables them to credibly commit to lower markups and thereby attract more slow traders. Even if dealers are willing to introduce a platform, they might still want to coordinate and keep their response rates on the platform rather low in order to maximize their profits.

In the given model, an HM always seems to be preferred to a PBM from a social perspective, since potentially more beneficial trades are realized in the HM while the total surplus per trade is assumed to be

[5]For instance, there are more liquidity providers on the Tradeweb platform for most fixed-income securities than on the respective platform for CDSs:
http://www.tradeweb.com/Institutional/Markets/.

[6]While not offering CDS platform access to retail investors, Tradeweb offers access to fixed-income platforms to retail investors. See http://www.tradeweb.com/Retail/.

constant. The only reason why some assets are traded in a PBM then seems to be that dealers earn higher profits under this particular market structure. Before jumping to that conclusion, however, one has to note that there are some limitations of the model.

The most important limitations of the model are as follows. There is no uncertainty about the traders' value of the asset and the dealers' costs of providing the asset. Furthermore, information about the asset is symmetric. As Hendershott and Madhavan (2015) argue, information leakage may incentivize investors to trade bilaterally as opposed to trading on an electronic trading platform. Information leakage is not modeled in this paper. Arguably, information asymmetries may vary among different asset classes. U.S. Treasury bonds are perhaps an asset class with relatively small information asymmetries, while information asymmetries are stronger for junk bonds. The model presented in this paper can be thought of as referring to markets like the market for U.S. Treasuries.

This paper proceeds as follows. After a review of the related literature in Section 2, the model of the pure bilateral market of Duffie et al. (2016) will be partially reviewed in Section 3. Section 4 extends the model from Section 3 by introducing an electronic trading platform as a second trading venue. Section 5 presents sufficient conditions under which dealers prefer the HM to the PBM and vice versa. Several other empirical predictions will also be found in Section 5. Section 6 presents some numerical results and investigates the dealers' incentives to coordinate and choose collectively a profit-maximizing response rate on the trading platform. Most proofs are found in Appendix 7. The Internet Appendix contains omitted proofs as well as details on numerical solutions. Internet Appendix A looks at an alternative model setup where quoting on the platform and in the bilateral market is mutually exclusive.

2 Related Literature

This paper heavily draws on results and techniques of Duffie et al. (2016). The latter develop a model of bilateral dealer markets in which traders have to contact dealers sequentially to obtain prices for an asset. They combine random-pricing strategies from consumer search models[7] with insights on optimal search by Weitzman (1979) and show that a benchmark can improve both dealer profits and welfare. With a benchmark, dealers commit to lower markups and traders with search costs may increase their market participation. Zhu (2012) models a bilateral OTC market under the assumption that a dealer will renegotiate an offer if a trader comes back to him after having visited other dealers. These models of bilat-

[7]See for example Varian (1980), Burdett and Judd (1983), Stahl (1989) and Janssen et al. (2005, 2011).

eral OTC markets have to be distinguished from intensity-driven seach models as for instance Duffie et al. (2005), Weill (2007), Lagos and Rocheteau (2009), Gârleanu (2009), Lagos et al. (2011), Feldhütter (2005), Pagnotta and Philippon (2011) or Lester et al. (2015). Glebkin (2016) considers an intensity-driven search model with an RFQ trading protocol.

Random pricing plays also a role in limit order book markets. While Glosten (2015), Biais et al. (2000, 2013) or Back and Baruch (2013) derive well-behaved supply schedules, Dennert (2015) presents a model in which market makers set prices randomly and a pure-strategy equilibrium does not exist. Baruch and Glosten (2015) use mixed strategies to explain flickering quotes. Jovanovic and Menkveld (2015) and Yueshen (2015) consider the case in which market makers' participation is uncertain. An important insight from these papers is that uncertain presence of other market makers leads to positive expected profits. I will show an analogous result to be true in the case of electronic trading platforms in OTC markets: Uncertain presence of dealers on an electronic trading platform prevents Bertrand competition and leads to positive expected profits for dealers on the platform.

The interaction between different trading venues has been studied in various papers. Parlour and Seppi (2003) study the competition between two different types of exchanges. Vayanos and Weill (2008) show that search frictions in both spot and repo market can lead to the on-the-run phenomenon. Praz (2015) models the prices of an asset that is traded on both a liquid (exchange) market and an illiquid (OTC) market.

A key theme in this paper, namely that market structures develop endogenously depending on characteristics of market participants, is already found in Biais and Green (2007). The latter suggest that the migration of bond trading from exchanges to the OTC market was related to the growing importance of institutional investors in bond markets. King et al. (2012) provide a historical overview of the foreign exchange market and describe how this market has become more transparent in recent years. Various empirical studies have examined the choices between different kinds of trading venues. Bessembinder and Venkataraman (2004) examine equities and the choice between trading in an upstairs search market and trading on an electronic stock exchange. Barclay et al. (2006) look at inter-dealer brokerage for U.S. Treasuries and the choice between trading via electronic limit order books and trading via voice-based systems. Hendershott and Madhavan (2015) look at the choice between trading a bond in a bilateral OTC market and going to an OTC electronic trading trading platform, i.e. the choices a trader faces in an HM structure considered in this paper. Trading costs differ across various trading venues. Bessembinder and Kaufman (1997) compare trading costs across different exchanges. Barclay et al. (2003) and Conrad et al. (2003) compare trading costs on traditional exchanges to trading costs in different electronic markets. Edwards et al. (2007) and Goldstein et al. (2007)

present evidence regarding trading costs of corporate bonds in OTC markets. Harris and Piwowar (2006) compare costs of trading municipal bonds to costs of trading equities on exchanges. Green et al. (2007) estimate dealers' bargaining power via a structural model. They suggest that dealers exercise substantial market power.

3 Recap: The Pure Bilateral Market

I will consider a PBM as modeled in Duffie et al. (2016) with no uncertainty about the dealers' cost. For expositional purposes, I will briefly review the main results regarding the equilibrium in the PBM below. Afterwards, I will present a model of the HM in which there is an electronic trading platform as an additional trading venue.

There are $\mathbb{N} \ni N > 1$ risk-neutral dealers and a continuum of risk-neutral traders with measure 1. Traders want to buy an asset to which they attribute value $v \in \mathbb{R}$. Dealers can provide the asset at cost $c \in \mathbb{R}$. A fraction $\mu \in (0, 1)$ of traders is fast and does not find search costly. The other traders are slow and have to pay cost $s > 0$ when contacting a dealer. Traders are called "fast" or "slow", because the cost associated with search can be interpreted as the cost of waiting. It is assumed that where v, c, μ and s are common knowledge. In the following it will also be assumed that $v > c + s$, which means that market entry by slow traders is efficient.

In a PBM, traders have to contact each of the N dealers separately to obtain a quote. A dealer does not observe whether the trader who contacts him is fast or slow. After each contact with a dealer, a trader has the option to buy the asset at one of the quotes obtained up to that point, leave the market without buying the asset or continue to search.

Duffie et al. (2016) derive the following equilibrium. Fast traders will always contact all dealers in the PBM. Slow traders will enter the market with a probability $\gamma^{PBM} \in (0, 1]$ and follow a reservation price strategy with reservation price r^{PBM}. Each dealer independently draws a price p from a continuous distribution H^{PBM} without atoms or gaps and finite support $[\underline{p}^{PBM}, r^{PBM}]$. On its support, this distribution is given by

$$H^{PBM}(p) = 1 - \left(\frac{\gamma^{PBM}(1 - \mu)(r^{PBM} - p)}{N\mu(p - c)} \right)^{1/(N-1)}.$$

The distribution H^{PBM} is determined such that a dealer is indifferent between quoting any price in the support of H^{PBM}. The slow traders' reservation price satisfies

$$r^{PBM} = \int_{\underline{p}^{PBM}}^{r^{PBM}} p \, \mathrm{d}H^{PBM}(p) + s.$$

The reservation price in the PBM satisfies an indifference condition. When a slow trader is offered the reservation price, he is indifferent between accepting the offer and continuing to search in the PBM. The reservation price r^{PBM} is a strictly monotone increasing function in γ^{PBM} with $\lim_{\gamma^{PBM} \to 0} r^{PBM}(\gamma^{PBM}) = c + s$.

Finally, the slow traders' probability of market entry is given by

$$\gamma^{PBM} = \begin{cases} 1 & \text{if } r^{PBM}(1) \leq v \\ x & \text{else,} \end{cases}$$

where x uniquely solves $r^{PBM}(x) = v$ and satisfies $x \in (0, 1)$. Thus, when a slow trader expects a non-negative profit of entering the PBM even if all other slow traders enter with probability one, he enters with probability 1 as well. Otherwise, there is a unique equilibrium value $\gamma^{PBM} \in (0, 1)$ such that expected payoffs of entering the PBM are zero.

4 A Model of the Hybrid Market

In this section I present the setup for the model of the hybrid market structure. The setup builds on the model of Duffie et al. (2016) that was partially reviewed in Section 3. In the model of the HM, I will make the same assumptions on traders and dealers as those laid out in Section 3. In the HM, however, traders have the choice between using two different trading venues:

- **Bilateral market:** As it was the case in section 3, a trader may contact each of the N dealers individually and ask for a quote. After each contact with a dealer, a trader may buy the asset at one of the prices obtained so far, leave the market or continue searching in the bilateral market or on the electronic trading platform (see next bullet point). Notice that continuing to search in the bilateral market is only feasible if there are dealers who have not yet been contacted individually by the trader. Similarly, continuing to search on the platform is only feasible if the trader has not used the platform before. It is assumed that each dealer can be contacted at most once by every trader. Slow traders have to pay cost s each time they contact a dealer.
- **Electronic trading platform:** Traders can also submit a so-called "request-for-quote" (RFQ) via

an electronic trading platform to all N dealers at once. Every time a trader submits an RFQ on the electronic trading platform, each dealer can decide to respond to this RFQ and provide a quote at which he is willing to sell the asset. Thus, the trader gets a number $n \in \mathbb{N}$ of quotes, with $0 \leq n \leq N$. The case that $n = 0$ is quite possible as Hendershott and Madhavan (2015) show empirically. Each dealer independently responds to an RFQ with probability $\eta \in (0, 1)$. After having received the n quotes, a trader can either decide to buy the asset at the most attractive quote he has received so far, continue searching in the bilateral market or go out of the market without buying the asset. Continuing to search in the bilateral market is only feasible there are dealers who have not been contacted individually by the trader yet. Slow traders have to pay cost s when submitting an RFQ. Fast traders find it costless to use the platform and submit an RFQ. Each trader is allowed to submit at most one RFQ via the electronic trading platform.

Each dealer has a separate trading desk for the bilateral market and for the platform. The objective of each trading desk is to maximize its own profit.

Traders can visit the N dealers in the bilateral market and use the platform in any order. Potentially, they can contact all available dealers and also go once to the platform to request quotes. Thus, there are potentially $N+1$ periods in which traders can search for quotes. After that they can either choose the lowest offered quote and buy the asset wherever they found this offer or they can choose not to buy the asset if the offered quotes were too expensive. In the event that a trader has received several lowest quotes and he decides to buy the asset, he is equally likely to choose any one of those lowest quotes. Traders can terminate their search early or stay out of the market completely. Terminating the search early or staying out of the market may be optimal for slow traders who find search costly. The dealers' RFQ response rate η is exogenously given and can represent the cost of paying attention as in Yueshen (2015). Risk management considerations might also prevent dealers from responding to every RFQ. Even though η is treated as exogenous most of the time here, some results in this paper suggest that dealers have incentives to endogenously keep η low in order to increase their profits. Since dealers are identical, I will assume that the dealers' quoting strategies are symmetric on both the platform and in the bilateral market. As in the PBM described in Section 3, a dealer does not observe the trader's type and does not know how many dealers the trader has contacted before. Therefore, dealers do not observe any information on which they can condition their quotes. The prices each dealer quotes in the bilateral market can therefore be assumed to be independent and identically distributed. Similarly, the prices each dealer quotes on the electronic trading platform can be assumed to be independent and identically distributed. Notice, however, that dealers' quoting strategies in the bilateral market may be different from those they use on the electronic trading platform.

A dealer would never want to quote a price below his cost c, since doing so may result in losses when a trader buys the asset from that dealer. Quoting below c is clearly never optimal. Without loss of generality it can therefore be assumed that a dealer quotes according to a probability distributions with support in $[c, \infty)$. This means the following:

1. In the bilateral market, dealers quote prices drawn independently from a distribution function $H : \mathbb{R} \to [0, 1]$ with support $\operatorname{supp} H \subset [c, \infty)$.

2. On the electronic trading platform, dealers quote independently from other dealers according to a distribution function $G : \mathbb{R} \to [0, 1]$ with support $\operatorname{supp} G \subset [c, \infty)$.

Since the dealers' strategies are symmetric, H and G are the same for all dealers. Traders are assumed to make their decision about market entry and exit independently. Pairwise independence of these decisions and the dealers' quotes on the different trading venues makes the exact law of large numbers by Sun (2006) applicable when referring to prices and quantities of traders in the market.

This section proceeds as follows. First, I will derive the traders' optimal search strategies conditional on the dealers' quoting strategies. A search strategy specifies where to start the search, when to accept an offer, when to leave the market as well as when and where to continue to search. Second, I will derive the dealers' optimal quoting strategies given that the traders use specific search strategies. Third, I will derive conditions, under which the dealers' quoting strategies and the traders' search strategies are simultaneously optimal. This means that given the traders' search strategies, the dealers' quoting strategies are optimal and vice versa. In other words, I determine conditions under which an equilibrium exists.

4.1 Traders

The equilibrium search strategies are different for slow and fast traders. Since fast traders can costlessly canvass the entire market, a fast trader will always take advantage of this ability. Obtaining another quote can potentially result in a lower price a trader has to pay for the asset, while the other offers received before remain valid.[8] The exact order in which a fast trader contacts dealers or goes to the platform does not matter, since the received offers are independent from each other. A fast trader will choose the lowest offer received

[8] This particular assumption is a simplification of the real trading mechanism. An offer by a contacted dealer will in general be valid only for a very short moment. If a trader goes back to a previously contacted dealer, this dealer might very well charge a higher price. Zhu (2012) models search in a bilateral dealer market if dealers might change their quote when a trader returns after having contacted other dealers.

from the N dealers in the bilateral market and from the n dealers who responded to the request-for-quote on the platform.

Slow traders on the other hand have to carefully consider whether entering the market or continuing the search is worth the cost s. Slow traders also have to determine the exact order in which they go to the platform or contact dealers in the bilateral market.

In order to determine the slow traders' optimal search strategy, one needs to take a closer look at the dealers' quoting strategies. I will denote the random variable that has the distribution function H and represents the price a dealer quotes in the bilateral market by p_b. The lowest quote a trader gets on the platform conditional on the event that at least one dealer responds to the RFQ is denoted by q. Since each dealer independently responds to an RFQ with probability η and quotes according to the probability distribution G, the random variable q is distributed according to the distribution function F defined by

$$F(x) := \frac{1-(1-\eta G(x))^N}{1-(1-\eta)^N}.$$

Since $\operatorname{supp} G \subset [c,\infty)$, one obtains $\operatorname{supp} F \subset [c,\infty)$. One can now define the following *reservation prices* r_b and r_p as the solutions to the following equations:

$$r_b := \mathbb{E}(\min(p_b, r_b)) + s, \tag{1}$$

$$r_p := (1-(1-\eta)^N) \cdot \mathbb{E}(\min(q, r_p)) + (1-\eta)^N r_p + s. \tag{2}$$

Since both H and F have a support that is bounded from below, solutions to (1) and (2) always exist. The interpretation of r_b is as follows. Assume a slow trader has the opportunity to buy the asset at the price r_b. Then he would be just indifferent between buying the asset at that price and contacting a dealer in the bilateral market to search for a better price. An analogous interpretation holds for r_p. When defining r_p one only has to take account of the possibility that a trader may not get a response at all when submitting an RFQ on the electronic trading platform. The latter event happens with probability $(1-\eta)^N$. The first term in (2) denotes the expected price improvement if the trader searches on the platorm and at least one dealer responds to an RFQ. The second term in (2) says that there will be no price improvement if no dealer responds to the RFQ. Together with the search cost s, these terms have to add up to r_p, if the trader is indifferent between buying the asset at r_b or searching on the platform. A slow trader is indifferent between buying the asset at the price r_b and contacting a dealer in the bilateral market. One can now solve the slow traders' search problem as in Weitzman (1979). The solution is summarized in the following

Lemma 1. *Given the above assumptions on the prices p_b in the bilateral market and the lowest quote q on the platform, the reservation prices r_b and r_p as defined in equations (1) and (2) are well-defined for $s > 0$ and it is optimal for slow traders to start their search on the platform if*

$$v \geq r_b > r_p. \tag{3}$$

Furthermore, if condition (3) holds there is the following optimal continuation rule: If a trader received no quote on the platform or a lowest quote greater than r_b, the trader will continue to search in the bilateral market until he finds a quote less than or equal to r_b.

If $r_b = v$ and a slow trader did not receive a quote less than r_b on the platform, then the slow trader is indifferent between continuing to search in the bilateral market and terminating the search. In this case, continuing to search with any probability $\gamma \in [0, 1]$ is optimal for slow traders.

Lemma 1 states that as long as (3) holds, slow traders start their search on the platform and then follow a reservation price strategy with constant reservation price no matter how G and H look like in particular. Notice that the slow traders' threshold below which they accept an offer is the same on the platform and in the bilateral market. If slow traders are indifferent between continuing the search in the bilateral market and terminating the search, any probability $\gamma \in [0, 1]$ of continuing the search is optimal. It will turn out later, however, that this probability affects the dealers' optimal quoting strategies. When determining an equilibrium, γ will be uniquely dertermined such that the slow traders' search strategy is consistent with the dealers' quoting strategies.

The proof of Lemma 1 only requires that dealers quote independently on both trading venues and all dealers use the same distributions G and H when making offers.

The following statement gives a lower bound for r_b.

Lemma 2. *The above stated assumptions on H imply $r_b \geq c + s$, where r_b is defined as in (1). The strict inequality $r_b > c + s$ holds if the probability of the event $\{p_b > c\}$ is positive.*

Lemma 2 states that r_b is always strictly greater than c. This fact will be used in the next section when determining the dealers' optimal quoting strategies. The inequality $r > c$ will imply that dealers make a positive profit on both the platform and in the bilateral market.

4.2 Dealers

This section deals with the optimal strategies for dealers given that traders behave as discussed in Section 4.1. The fraction μ of fast traders plays two important roles. First, a positive fraction of fast traders eliminates the possibility of a situation in which all dealers in the bilateral market charge the monopoly price r. Such a situation is also known as the Diamond (1971) paradox. Second, fast traders connect the two trading venues, since they search for the best price in the overall market. Thus, a dealer making a decision on either trading venue has to consider what is happening on the other trading venue. The first result in this section gives a first characterisation of the dealers' pricing strategies if slow traders use a reservation price strategy with reservation price r as described in Lemma 1. It follows from standard search-theoretic arguments as the ones given in Varian (1980) which have also recently been applied to pure bilateral dealer markets by Duffie et al. (2016).

Lemma 3. *Let the slow traders start their search on the platform and let them use a reservation price strategy with reservation price r. Let the slow traders' probability γ of continuing the search in the bilateral market after not having received a satisfactory offer on the platform be positive.*

Then, neither G nor H can have any atoms. Neither in the bilateral market nor on the platform, a dealer ever quotes a price greater than r or less than or equal to c. A dealer on the platform faces a mass k_p of slow traders given by

$$k_p := 1 - \mu.$$

A dealer in the bilateral market faces a mass k_b of slow traders given by

$$k_b := (1 - \eta)^N \gamma(1 - \mu)/N.$$

One consequence of the fact that dealers never quote a price above r is that slow traders will buy the asset at the first price they receive. A slow trader therefore contacts at most one dealer in the bilateral market. Lemma 3 states that the two distribution functions G and H are continuous given that slow traders use the stated reservation price strategy. Dealers are only willing to randomize prices if the expected payoffs from quoting these prices are the same. Otherwise, it would be optimal not to quote prices with lower payoffs. These indifference conditions will determine the distributions H and G. So far we know from Lemma 3 that the suprema of the supports of G and H have to be less than or equal to r. The indifference conditions in the following result also imply that the suprema of the supports are equal to r.

Lemma 4. *Let slow traders start their search on the platform and use the reservation price r. For all prices p in the support of H it has to hold that*

$$(p-c)\left[k_b + \mu(1-H(p))^{N-1}\left(1-\eta G(p)\right)^N\right] = (r-c)k_b. \tag{4}$$

For all prices p in the support of G it has to hold that

$$(p-c)\left[k_p\left(1-\eta G(p)\right)^{N-1} + \mu(1-H(p))^N\left(1-\eta G(p)\right)^{N-1}\right] = (1-\eta)^{N-1}(r-c)k_p. \tag{5}$$

The suprema of the supports of both G and H are equal to the slow traders' reservation price r.

The intuition behind the condition in equation (4) is similar to the intuition in the case of a pure bilateral market as in Duffie et al. (2016). One option for the dealer is to quote the slow traders' reservation price r and get only slow traders as customers, since the fast traders almost surely get a better offer elsewhere. The expected profit in this case is expressed by the right-hand side of equation (4). Another option is to lower the price and also have a chance of selling to fast traders. The expected profit in this case is expressed by the left-hand side of equation (4). There is one difference between (4) and an analogous condition for a pure bilateral market (without coexisting platform). In a hybrid market, a dealer also has to take the probability $(1-\eta G(p))^N$ of not being undercut by dealers on the platform into account.

Equation (5) expresses a similar trade-off for the dealers on the platform. A dealer on the platform can only sell, if he is not undercut by other dealers on the platform. This event happens with probability $(1-\eta G(p))^{N-1}$. The dealer then has to be indifferent between selling at price r only to slow traders and selling at a lower price also with positive probability to fast traders. Note that it has to be that case that $0 < \eta < 1$. If $\eta = 1$ and the dealers' presence on the platform is certain, the dealers will engage in Bertrand competition and drive the price on the platform to c. This insight is analogous to an insight found for example in Yueshen (2015) that uncertain presence of market makers in a limit order book market leads to positive expected profits for the market makers.

The result that the support of the price distribution in the bilateral market has upper bound r is also similar to previous results in the literature, as those of Duffie et al. (2016) or Varian (1980). However, it is far from trivial that these results also hold in the setup considered here. While other studies as the above-mentioned only consider a single type of market, considering two types of markets leads to some complications. In a single type of market, having a supremum of the support of the price distribution lower than r cannot be optimal, since a deviating dealer could increase the price without decreasing turnover. Showing that the upper bound of the support of the price distribution in a joint market as the HM also has to be r is more complex. For example, the trade-off faced by a dealer on the platform in a hybrid market is increasing the price for slow traders versus loosing potential customers to the bilateral market. It turns out, however, that the price effect dominates and that there is only an equilibrium if the upper bounds of the supports of the price distributions are equal to r.

So far we only know the upper bounds of the supports of H and G. Making a claim about the lower bounds is a bit trickier. It will turn out that the equations determining the lower bounds depend on which of the two lower bounds is larger. So far, one can only say that the two lower bounds are never equal.

Lemma 5. *Given the assumptions in Lemma 4, the lower bounds of the supports of G and H cannot be equal.*

Once one has a particular order of the lower bound $\underline{p}^b$ of the support of H and the lower bound $\underline{p}^p$ of the support of G, one can derive conditions that describe the bounds' locations. I first consider the case in which $\underline{p}^b < \underline{p}^p$.

Lemma 6. *Let the assumptions in Lemma 4 hold and let there be two functions H, G that solve equations (4) and (5) such that $\underline{p}^b < \underline{p}^p$.*

Then the restriction of H on the interval $[\underline{p}^b, \underline{p}^p]$ can be expressed in closed form by

$$H_{[\underline{p}^b,\underline{p}^p]}(p) = 1 - \left(\frac{(r-p)k_b}{(p-c)\mu}\right)^{1/(N-1)}. \tag{6}$$

The lower bound of the support of H is also given explicitly by

$$\underline{p}^b = c + \frac{(r-c)k_b}{k_b+\mu}. \tag{7}$$

The lower bound of the support of G is characterized by the fixed-point equation

$$\underline{p}^p = c + \frac{(1-\eta)^{N-1}(r-c)k_p}{k_p + \mu(1 - H_{[\underline{p}^b,\underline{p}^p]}(\underline{p}^p))^N}. \tag{8}$$

Given that

$$\frac{\gamma(1-\eta) - \gamma(1-\eta)^N}{N - \gamma(1-\eta)^N} < \mu \tag{9}$$

holds, a unique solution to (8) exists that is indeed greater than $\underline{p}^b$. If on the other hand

$$\mu < \frac{1-\eta-(1-\eta)^N}{\frac{N}{\gamma}+(1-\eta)-(1-\eta)^N} \tag{10}$$

holds, a solution to (8) with $\underline{p}^p > \underline{p}^b$ cannot exist.

Lemma 7 does not state when solutions to 4 and 5 with $\underline{p}^b < \underline{p}^p$ exist. However, if solutions exists, then the lower bounds of the supports of H and G are under technical conditions uniquely determined. Lemma 7 gives a sufficient and a necessary condition for solutions to 4 and 5 with $\underline{p}^b < \underline{p}^p$ to exist. To make further statements about the existence of such solutions, it is necessary to examine H and G on their whole support. If one assumes $\underline{p}^b > \underline{p}^p$, analogous statements are true.

Lemma 7. *Let the assumptions in Lemma 4 hold and let there be two functions H, G that solve equations (4) and (5) such that $\underline{p}^p < \underline{p}^b$. Then the restriction of G on the interval $[\underline{p}^p, \underline{p}^b]$ can be expressed in closed form by*

$$G_{[\underline{p}^p,\underline{p}^b]}(p) = \frac{1}{\eta} - \frac{1}{\eta}\left(\frac{(1-\eta)^{N-1}(r-c)k_p}{(p-c)}\right)^{1/(N-1)}. \tag{11}$$

The lower bound of the support of G is also given explicitly by

$$\underline{p}^p = c + (1-\eta)^{N-1}(r-c)k_p. \tag{12}$$

The lower bound of the support of H is the unique solution to the fixed-point equation

$$\underline{p}^b = c + \frac{(r-c)k_b}{k_b + \mu(1-\eta G_{[\underline{p}^p,\underline{p}^b]}(\underline{p}^b))^N}. \tag{13}$$

Given that

$$\frac{\gamma(1-\eta)-\gamma(1-\eta)^N}{N-\gamma(1-\eta)^N} > \mu \tag{14}$$

holds, a solution to (13) exists that is indeed greater than $\underline{p}^p$. The technical condition

$$\mu < \frac{(N-1)\gamma}{N^2}(1-\eta) \tag{15}$$

ensures that this solution is unique. If on the other hand

$$\mu > \frac{1-(1-\eta)^{N-1}}{\frac{N}{\gamma}(1-\eta)^{N-1}-(1-\eta)^{N-1}} \tag{16}$$

holds, a solution to (13) with $\underline{p}^b > \underline{p}^p$ cannot exist.

Similar comments as those made with respect to Lemma 6 apply also to Lemma 7. So far we know when the distributions G, H are well-defined on an interval of prices for which one distribution function is always zero. The next result describes when the distribution functions are well-defined for all other prices.

Lemma 8. *If*

$$\frac{\gamma}{N}(1-\eta)^{-N/(N-1)} < 1, \tag{17}$$

then for each price $p \in [\max(\underline{p}^p, \underline{p}^b), r]$ there exist $H(p), G(p)$, such that equations (4) and (5) hold. If

$$\mu < \frac{(N-1)^2}{(2N-1)(1-\eta)^{-\frac{N^2}{N-1}}+(N-1)^2} \tag{18}$$

(in the case that $\underline{p}^p < \underline{p}^b$) or

$$\frac{\gamma(1-\eta)-\gamma(1-\eta)^N}{N(1-\eta)^N-\gamma(1-\eta)^N} < \mu < \frac{(N(N-1)^2+\gamma(2N-1))(1-\eta)^N-(2N-1)(1-\eta)\gamma}{(N(N-1)^2+\gamma(2N-1))(1-\eta)^N} \tag{19}$$

(in the case that $\underline{p}^b < \underline{p}^p$), then these $H(p)$ and $G(p)$ are unique and continuous functions of p. If

$$N > (1-\eta)^{1-N}, \tag{20}$$

and

$$(1-\mu)N > 1, \tag{21}$$

then these functions are strictly monotone increasing in p.

4.3 Equilibrium

In the last two subsections the optimal strategies for dealers and traders have been determined. It has also been established that the dealers' strategies are well-defined under some fairly general conditions. In equilibrium these strategies have to be consistent with each other. In particular, traders have to choose their reservation price r and their probability of continuing their search γ consistent with the distributions G and H as stated in Lemma 1. Since traders would never buy the asset at a price greater than v, it must hold in equilibrium that $r \leq v$. In equilibrium, it must also hold that $\gamma = 1$, if slow traders make a positive expected profit in the bilateral market. If the expected profit in the bilateral market is zero, i.e. $r = v$, slow traders are indifferent between any $\gamma \in (0, 1]$. The dealers then have to choose the distributions H and G based on the traders choices of r and γ. Finally, the existence of an equilibrium requires that (3) holds, i.e. given the dealers choices of G and H it must indeed be optimal for traders to start their search on the platform and they must get a non-negative expected payoff from searching in the bilateral market. The following definition summarizes this reasoning.

Definition 1. *A Perfect Bayesian Nash Equilibrium with symmetric strategies satisfies the following:*

- *Fast traders visit every dealer in the bilateral market and the platform.*
- *Slow traders follow a reservation price strategy. The reservation price r solves (1).*
- *Slow traders start their search on the platform and continue their search in the bilateral market with probability γ if they do not find a satisfactory quote on the platform. This probability satisfies*

$$\gamma = \begin{cases} 1 & \textit{if } r < v \textit{ or} \\ x \in (0, 1] & \textit{such that } r = v. \end{cases}$$

- *The dealers quote according to continuous gapless distributions that satisfy (4) and (5). The suprema of the supports of these distributions are equal to r.*
- *The reservation prices for the bilateral market and platform satisfy (3).*

I first show that it is indeed possible for traders to choose a reservation price r such that this reservation price equals r_b as defined in (1). Then I show that this reservation price is strictly monotone increasing in γ. Monotonicity of r in γ will imply the uniqueness of the slow traders equilibrium choice of γ. One has to keep in mind that there are two regimes of dealers' strategies. In regime 1, considered in Lemma 6, it holds that $\underline{p^b} < \underline{p^p}$ and in regime 2, considered in Lemma 7, it holds that $\underline{p^p} < \underline{p^b}$. Note that if $r < v$ than traders earn a positive expected profit from entering the bilateral market. In this case $\gamma = 1$ is the only optimal probability with which to continue the search. If $r = v$, then the traders are indifferent between any $\gamma \in [0, 1]$. However, it will turn out that given the other parameters there is only one unique $\gamma \in (0, 1]$ such that $r = v$ indeed holds.

Lemma 9. *Let the sufficient conditions in Lemma 8 and Lemma 6 (for regime 1) or Lemma 7 (for regime 2), respectively, for some $\gamma \in (0, 1]$ hold and let*

$$\mu < \frac{(N-1)\left(\eta + N(1-\eta)^N - 1\right)}{N\left(-\eta + N(1-\eta)^N - (1-\eta)^N + 1\right)}. \tag{22}$$

Then, holding γ fixed, there are for each regime unique functions H and G and reservation price r, such that

$$r = s + \int_{\underline{p^b}}^{r} p \, \mathrm{d}H(p) \tag{23}$$

holds and the functions G and H satisfy equations (4) and (5) on their support.

Moreover, the traders' reservation price r is continuous and strictly monotone increasing in γ. In regime 1, one has $\lim_{\gamma \to 0} r = c + s$.

Lemma 9 states that the slow traders' reservation price for a fixed γ is well-defined. This result is different from the result in Lemma 1, that the solution to equation (1) well-defined. The distribution H is fixed in (1), while H depends on r in equation (23). The fact that the reservation price that solves (23) is strictly monotone increasing in γ implies that any equilibrium value of γ is unique. If $\gamma = 1$ and $r \leq v$, the equilibrium value of γ is unique by the construction of the equilibrium. If $r = v$ for a $\gamma < 1$, one would have $r \neq v$ for any other $\gamma < 1$ and one would also have $r > v$ if $\gamma = 1$. Thus, any other choice of γ is not admissible in an equilibrium.

Lemma 9 also suggests how an equilibrium might be constructed. If slow traders enter with $\gamma = 1$ and find out that $r > v$, they might gradually reduce their γ until $r = v$ holds. This solution is guaranteed to exist in regime 1, since $\lim_{\gamma \to 0} r = c + s$. In regime 2, the distributions H and G might not be defined anymore if γ is too low, because condition (14) will be violated.

The economic intuition for the effect of γ on the reservation price is rather simple. By reducing the market entry in the bilateral market, slow traders increase the proportions of fast traders in the market.

This increases competition among dealers and lowers prices.

One can now finally show when two different types of equilibria exist. For regime 1, the following holds.

Proposition 1. *Let (9), (17), (19), (20), (21) and (22) hold for the slow traders choice of* γ*. Then dealers can choose* $\underline{p^b} < \underline{p^p}$*, where* $\underline{p^b}$ *is determined by (7) and* $\underline{p^p}$ *is determined by (8). On the interval* $[\underline{p^b}, \underline{p^p}]$ *the distribution function* H *is determined by (6). On the interval* $[\underline{p^p}, r]$ *there are unique monotone increasing functions* G*,* H *that satisfy (4) and (5), where* r *is the unique reservation price for the bilateral market that solves (23). Let the slow traders' choice of* γ *be such that either* $\gamma = 1$ *and* $r < v$ *or* $\gamma \in (0, 1]$ *and* $r = v$ *and let slow traders follow a reservation price strategy with reservation price* r*, starting with their search on the platform. Fast traders visit every trader in the bilateral market and also visit the platform.*

If $r_p < r_b$*, where* r_p *and* r_b *are defined as in (1) and (2), then the above described strategies of dealers and traders constitute a unique Perfect Bayesian Nash Equilibrium with* $\underline{p^b} < \underline{p^p}$ *in which dealers follow symmetric strategies and slow traders start their search on the platform.*

For regime 2 similar statements are true.

Proposition 2. *Let (14),(15), (17), (18), (20), (21) and (22) hold for the slow traders choice of* γ*. Then dealers can choose* $\underline{p^p} < \underline{p^b}$*, where* $\underline{p^p}$ *is determined by (12) and* $\underline{p^b}$ *is determined by (13). On the interval* $[\underline{p^p}, \underline{p^b}]$ *the distribution function* G *is determined by (11). On the interval* $[\underline{p^b}, r]$ *there are unique monotone increasing functions* G*,* H *that satisfy (4) and (5), where* r *is the unique reservation price for the bilateral market that solves (23). Let the slow traders' choice of* γ *be such that either* $\gamma = 1$ *and* $r < v$ *or* $\gamma \in (0, 1]$ *and* $r = v$ *and let slow traders follow a reservation price strategy with reservation price* r*, starting with their search on the platform. Fast traders visit every trader in the bilateral market and also visit the platform.*

If $r_p < r_b$*, where* r_p *and* r_b *are defined as in (1) and (2), then the above described strategies of dealers and traders constitute a unique Perfect Bayesian Nash Equilibrium with* $\underline{p^p} < \underline{p^b}$ *in which dealers follow symmetric strategies and slow traders start their search on the platform.*

The condition $r_p < r_b$ in Propositions 1 and 2 has to be verified numerically in most cases. This is not a big problem, since I showed that the functions based on which r_p and r_b are calculated are well-defined. In some cases, it is possible to give explicit sufficient conditions under which $r_p < r_b$ holds. In some cases one might know r_b, for instance if parameters are such that $\gamma = 1$ is not possible in equilibrium and therefore $r_b = v$. In these cases, the following result will be useful.

Lemma 10. *Let the slow traders' reservation price be* $r = r_b$*. Let* γ*,* c*,* v*,* η*,* s*,* μ *and* N *be such that dealers can quote prices according to strictly monotone increasing distributions described in Lemma 4. Then there is* *an upper bound for the expected best quote on the platform conditional on at least one response to the RFQ. This upper bound is given by*

$$\mathbb{E}(q) := \int_{\underline{p}^p}^{r} p \,\mathrm{d}F(p) \leq c + (r-c)(1-\eta)^{N-1} \frac{N\eta}{1-(1-\eta)^N}. \tag{24}$$

There is a corresponding lower bound given by

$$\mathbb{E}(q) \geq c + (r-c)(1-\mu)(1-\eta)^{N-1} \frac{N\eta}{1-(1-\eta)^N} \tag{25}$$

The inequality $r_p < r_b$ holds if

$$s < (1-(1-\eta)^N)(r - \mathbb{E}(q)). \tag{26}$$

Whenever I mention "equilibrium in the HM" in the rest of the paper, I will mean one of the types of equilibria described in Propositions 1 and 2.

5 Endogenous Market Design

Now that the equilibrium in the hybrid market is characterized, I want to deal with the question when such a hybrid market arises if dealers can choose the market structure under which the asset is traded. I will let the dealers choose between two options: operate a pure bilateral market (PBM) or operate a hybrid market described in section 4.

I will now consider how the introduction of a trading platform changes the behavior of market participants. The following result states how the slow traders' market participation is affected by the platform. Using Lemma 9, I will let $r(\gamma)$ be the unique reservation price of slow traders that solves equation (23) given that slow traders choose the probability of entry in to the HM equal to γ.

Lemma 11. *If equilibria in the HM and PBM exist, then the equilibrium values γ and γ^{PBM} satisfy the following.*

- *If $\gamma^{PBM} = 1$, then $\gamma = 1$.*
- *If $\gamma^{PBM} < 1$, then*

$$\gamma = \begin{cases} 1 & \textit{if } r(1) \leq v \\ x & \textit{else,} \end{cases}$$

where x solves $r(x) = v$ and satisfies $x > \gamma^{PBM}$.

Lemma 11 states that the introduction of a platform always increases the probability of market entry into the bilateral market of slow traders if $\gamma^{PBM} < 1$. It will turn out that due to this result, dealers want to introduce a platform to increase their turnover in some cases. However, a platform also affects prices at which they may sell the asset to both fast and slow traders.

Lemma 12. *If equilibria in the HM and PBM exist, then the expected price charged to a slow trader conditional on market entry is always lower in the HM than in the PBM. Also the expected price charged to a fast trader is lower in the HM than in the PBM.*

From a dealer's perspective, there are two competing effects that determine the profitability of introducing a platform: a volume effect and a price effect.

Volume effect: Let

$$\lambda_{slow} := (1-\mu)\Big((1-(1-\eta)^N) + (1-\eta)^N\gamma\Big)$$

denote the total mass of slow traders buying an asset in the HM. A slow trader finds a quote on the platform with probability $(1-(1-\eta)^N)$ and he contacts a dealer in the bilateral market with probability $(1-\eta)^N\gamma$. By the exact law of large numbers, the expected masses are equal to the actual masses of slow trades in the two trading venues. The total mass of slow traders in the PBM is given by $(1-\mu)\gamma^{PBM}$.

Lemma 11 states that the market participation of slow traders in the bilateral market goes up if a platform is introduced and $\gamma^{PBM} < 1$. Thus, the total amount of slow traders the dealers are contacted by increases, since $\gamma > \gamma^{PBM}$ implies $\lambda_{slow} > (1-\mu)\gamma^{PBM}$. Fast traders always buy the asset. Dealers' total turnover is therefore higher in the HM than in the PBM, if $\gamma^{PBM} < 1$.

Price effect: Lemma 12 states that the prices charged in expectation to both types of traders are lower in the HM than in the PBM. Therefore, it is not clear yet, whether a platform may increase or decrease dealer profits.

Both of these model implications on the prices and on the trading volume should be testable.

Testable Implication 1. *Define the total volume as the sum of transactions in the bilateral market and on the platform. If a platform is introduced into a bilateral market, total volume goes up.*

If one interprets fast and slow traders as institutional investors and retail investors, one obtains the following testable implication.

Testable Implication 2. *If a platform is introduced in a bilateral market, average markups for retail investors decrease and average markups for institutional investors decrease.*

It may seem intuitive that markups decrease and traded volume increases after an additional trading venue has been introduced, since an increased number of quotes in the market seem to be equivalent to more competition. However, in Internet Appendix A it is shown that this is not the case. In the Internet Appendix, I consider a slightly modified setup in which the total number of active trading desks stays the same in the PBM and HM. In this setup, the total number of quotes may be lower in the HM, while markups for slow traders nevertheless decrease compared to the PBM. The way dealers compete on the platform plays a significant role in increasing the slow traders' market participation. While there may be many quotes in a bilateral market, those quotes are usually given to fast traders. Slow traders are thus left in a poor bargaining position. A platform is an opportunity for slow traders to obtain multiple quotes and to increase their bargaining power.

In general, the model has to be solved numerically to determine when the volume effect dominates the price effect and vice versa. However, sometimes it is possible to assess when which effect dominates without using numerical methods. The following result follows directly from Lemma 12.

Proposition 3. *If equilibria in the HM and PBM exist such that $\gamma^{PBM} = 1$, dealer profits are lower in the HM than in the PBM.*

Corollary 1. *If $s \to 0$ or $\mu \to 1$ while holding other parameters constant, dealer profits are higher in the PBM than in the HM, given that an HM equilibrium exists.*

Corollary 1 directly yields another testable implication.

Testable Implication 3. *Suppose dealers have sufficient influence on the market structure under which a specific asset is traded. Then an HM should become less likely if many of the traders interested in that asset are institutional investors.*

According to Proposition 3, it must be the case that $\gamma^{PBM} < 1$, if dealers's profits are higher in an HM equilibrium than in the corresponding PBM equilibrium. The conditions stated in Corollary 1 ensures that $\gamma^{PBM} = 1$ and dealer profits are lower in the HM than in the PBM. Quite intuitively, one gets full market participation by slow traders, if search costs go to zero or competition introduced by fast traders makes dealers quote very attractive prices.

In the following, I will look for conditions under which equilibria in the HM and PBM exist such that the dealer profits are higher in the HM. Since we know that $\gamma^{PBM} < 1$ in these cases, it seems natural to look for equilibria in which $\gamma \to 0$ and $\gamma^{PBM} \to 0$. If furthermore $\mu \to 0$, then dealer profits in the PBM also go to zero. Now, let furthermore $1 > (1-\eta)^N > 0$ hold. In the HM, the mass of slow traders in the market is at least equal to $1-(1-\eta)^N$. This number is equal to the probability that at least one dealer quotes a price on the platform, in which case a slow trader would buy the asset. Equations (12) and (8) imply that $\underline{p}^p \geq c + (1-\eta)^{N-1}(r-c)(1-\mu) > c$, since $r = v$ if $\gamma < 1$. It follows that dealers make a positive profit in the HM while profits go to zero in the PBM.

It remains to show that such an equilibrium indeed exists.

Proposition 4. *Let $c, v \in \mathbb{R}$ and $s > 0$, with $v > c + s$. Let $K_1, K_2 \in (0, \infty)$ be two constants such that*

$$s < c(1-e^{-K_2}) + e^{-K_2}(v-c)\frac{K_2}{1-e^{-K_2}}. \tag{27}$$

Set $\mu = \frac{K_1}{N}$ and set $\eta = \frac{K_2}{N}$. As $N \to \infty$, an equilibrium in the HM exists such that $\underline{p}^b < \underline{p}^p$ (regime 1) and dealers make a higher profit in the HM than in the PBM.

Note that $K_2 = \eta N$ in Proposition 4 denotes the expected number of responses to an RFQ on the platform. The restriction on η in Proposition 4 has to be made for technical reasons in order to keep the expected number of dealers who respond to an RFQ positive and finite. Analogously, the restriction on s is made for technical reasons to ensure the existence of an equilibrium. In essence, Proposition 4 says that if N becomes large and μ becomes small, an HM is more profitable for dealers than a PBM. In fact, it is already sufficient to let $N \to \infty$ while keeping μ constant to make dealers prefer the HM to the PBM.

Proposition 5. *Let $c, v \in \mathbb{R}$, $\mu \in (0,1)$ and $s > 0$, with $v > c + s$. Let $K \in (0, \infty)$ be a constants such that*

$$s < c(1-e^{-K}) + e^{-K}(v-c)\frac{K}{1-e^{-K}}. \tag{28}$$

Set $\eta = \frac{K}{N}$. As $N \to \infty$, an equilibrium in the HM exists such that $\underline{p}^b < \underline{p}^p$ (regime 1) and dealers make a higher profit in the HM than in the PBM.

Based on Proposition 5, one can formulate the next empirical prediction.

Testable Implication 4. *If dealers have sufficient influence on the market structure, an HM should become more likely if there are many dealers.*

Proposition 5 shows that it is not necessary to let $\mu \to 0$ in order to make dealers prefer an HM to a PBM. However, the equilibrium constructed in Proposition 4 is nevertheless interesting, since it allows to determine several statistics in closed-form.

Corollary 2. *Consider the equilibrium characterized in Proposition 4. The expected price a slow trader gets in any bilateral market (PBM or bilateral part of the HM) is equal to $v - s$. The expected price a trader gets on the platform is given by*

$$\mathbb{E}(q) = c + (v - c)e^{-K_2}\frac{K_2}{1 - e^{-K_2}}.$$

Corollary 2 and L'Hospital's rule give $\mathbb{E}(q) \to v$ as $K_2 \to 0$ and $\mathbb{E}(q) \to c$ as $K_2 \to \infty$. This is quite intuitive, since as $K_2 \to 0$, a dealer is most likely alone on the platform if he responds to an RFQ. Then it is most profitable to give a quote equal to v. As $K_2 \to \infty$, competition among dealers drives the best quote down to the dealers' cost c. Suppose now that dealers might collude on the expected number of responses a trader gets for an RFQ. Then dealers might choose the profit-maximizing η that is given in closed form for the type of equilibrium characterized in Proposition 4.

Corollary 3. *Consider the equilibrium characterized in Proposition 4. Then the dealers' collective profit $\Pi^d(K_2)$ in the HM for a given choice of K_2 is given by*

$$\Pi^d(K_2) = K_2(v - c)e^{-K_2}.$$

This profit is maximized if the expected number of responses to an RFQ is given by

$$K_2^* = 1.$$

The result in Corollary 3 states that the profit-maximizing response rate does not depend on any parameters other than the number of dealers N. Intuitively, the expected profit-maximizing number of dealers on the platform is equal to 1, since each dealer would like to be alone on the platform in order to charge the monopoly price r.

Finally, one can observe that a platform may lead to a separation of fast and slow traders.

Corollary 4. *In the equilibrium characterized in Proposition 4, fast traders buy the asset in the bilateral market with a probability that goes to 1 as $N \to \infty$.*

The last result is of course very specific to the equilibrium described in Proposition 4. In general, a slightly weaker claim is true.

Proposition 6. *Consider any equilibrium in the HM. Fast traders are more likely to trade in the bilateral market than slow traders are. Slow traders are more likely to trade on the platform than fast traders are.*

Proposition 6 says that an HM partially separates fast and slow traders. In the special case discussed in Corollary 4, this separation becomes perfect. Proposition 6 also yields another testable prediction.

Testable Implication 5. *In any hybrid market, the ratio of trades executed on a platform to trades executed in the bilateral market is higher for retail investors than for institutional investors.*

Propositions 3, 4 and 5 show that dealers can be better in the PBM, but sometimes they might also be better off in the HM. A simple consequence of Lemma 11 and Lemma 12 is that both kinds of traders prefer the HM to the PBM.

Proposition 7. *If equilibria in the HM and PBM exist, both fast and slow traders are better off in the HM than in the PBM.*

The introduction of a platform not only affects market participation and expected values of the quotes, but also other characteristics of prices in the bilateral market. Especially if market participation of slow traders is still low in the HM, prices in the bilateral market are more dispersed in the HM than in the PBM. In order to see this it is useful to take a look at the lower bounds of the price distributions in the two different bilateral markets.

Proposition 8. *Suppose equilibria in the HM and PBM exist. Then* $\underline{p}^b < \underline{p}^{PBM}$.

It now follows that the range of possible prices in the bilateral market is greater in the HM than in the PBM if $\gamma < 1$.

Corollary 5. *Suppose equilibria in the HM and PBM exist such that* $\gamma < 1$. *Then* $r - \underline{p}^b > r^{PBM} - \underline{p}^{PBM}$.

The size of the support of the price distributions can be viewed as a kind of price dispersion. Then Corollary 5 says that there is in some sense more price dispersion in the bilateral market in the HM than in the PBM if $\gamma < 1$. This claim does not necessarily hold anymore if $\gamma = 1$, since the upper bound of the support of H might be lower than the upper bound of the support of H^{PBM} in this case.

Assuming, one knows that market participation by retail investors is limited, one could empirically test the statement in Corollary 5.

Testable Implication 6. *If market participation by retail investors is limited, the introduction of an electronic trading platform increases price dispersion in the bilateral market.*

From Lemma 7 in Section 4.2, one obtains the following Corrollary.

Corollary 6. *There is a threshold value $\overline{\mu}$ that only depends on η and N, such that a regime-2 equilibrium cannot exist if $\mu > \overline{\mu}$. Any HM equilibrium then must satisfy $\underline{p}^b < \underline{p}^b$.*

In any regime-1 equilibrium, the range of possible prices is greater in the bilateral market than on the platform. One can therefore use Corollary 6 to make the following empirical prediction.

Testable Implication 7. *Consider a hybrid market and hold the number of dealers and the RFQ response rate fixed. Then price dispersion is greater in the bilateral market than on the platform, if the fraction of institutional investors is sufficiently high.*

A consequence of Lemma 11 is that turnover in the whole market cannot be lower in the HM than in the PBM. However, turnover can be lower in some part of the HM than in the PBM. In fact, the following result holds.

Proposition 9. *If equilibria in the HM and PBM exist, then both fast and slow traders are less likely to trade in the bilateral market of the HM than they are to trade in the PBM. Therefore, turnover is lower in the bilateral market of the HM than in the PBM.*

Proposition 9 is quite intuitive, since one expects that trading migrates from the bilateral market to the platform in the HM. However, it is again not a trivial result, since it was argued above that overall market participation is always strictly higher in the HM than in the PBM if $\gamma^{PBM} < 1$. Thus, additional trading on the platform overcompensates the loss in turnover in the bilateral market in this case.

Proposition 9 gives the following testable implication.

Testable Implication 8. *If a platform is introduced in a pure bilateral market, turnover in the bilateral market decreases.*

To conclude this section, all empirical predictions are summarized. These predictions can be categorized as either high-level predictions or micro-level predictions. The high-level predictions comprise statements on when which market structure should arise and statements on price and volume levels in different trading venues. The micro-level predictions include statements on how the price distributions look like in the different trading venues or where different investors most likely trade. In the following overview, numbers in parentheses refer to the testable implications in the above text.

1. **High-level predictions:** The model implies that overall volume increases if a platform is introduced in a PBM (1), while prices become more attractive for both types of traders (2). One further obtains

conditions, under which a platform should not be profitable for dealers. In particular, a platform should not be profitable if the fraction of institutional investors becomes very large (3). On the other hand, a platform should become profitable for dealers, if the number of dealers becomes very large (4).

2. **Micro-level predictions:** Institutional investors execute more of their trades in the bilateral market than retail investors do (5). Under the natural assumption that the market participation of retail investors is limited, a platform increases price dispersion in the bilateral market (6). If the fraction of institutional investors is not too small, price dispersion on the platform is lower than that in a coexisting bilateral market (7). Even though total turnover increases in the HM compared to the PBM, turnover in the bilateral market decreases, if a platform is introduced in a PBM (8).

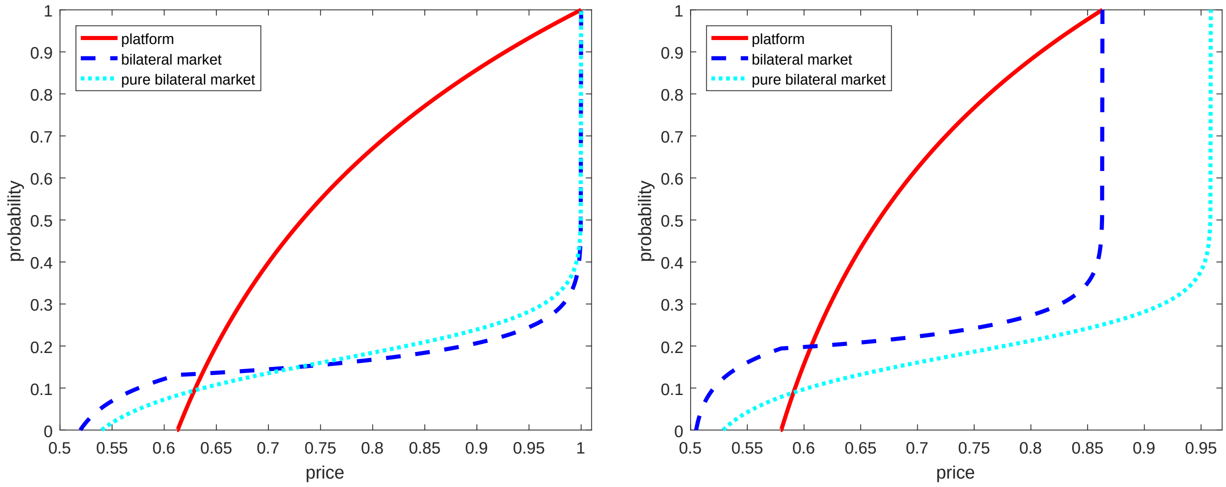
6 Numerical Results

In this section I will give examples for regime-1 and regime-2 equilibria described in Propositions 1 and 2, respectively. For each regime, there are equilibria in which dealers prefer the HM and equilibria in which dealers prefer the PBM. Finally, I will show that even if dealers introduce a platform, there is still potential for collusion: If dealers collectively hold back their response rate on the platform, they may increase their profits. It is explained in Internet Appendix 7 how these numerical results were derived.

6.1 Regime-1 Equilibria

I will first consider equilibria of the type described in Proposition 1. Figure 1 shows the probability distribution for prices in both HM and PBM such that $\underline{p}^b < \underline{p}^p$. Each panel considers a different set of parameters. The only difference, however, between the exogenous parameters in Panel A and Panel B of Figure 1 is that $\mu = 0.1$ in Panel A and $\mu = 0.5$ in Panel B. Due to this difference, dealer profits are higher in the HM than in the PBM in Panel A, whereas dealer profits are lower in the HM than in the PBM in Panel B.

Intuitively, a higher fraction of fast traders leads to more competition among the dealers. If there are more fast traders, dealers find it more profitable to lower prices in order to attract fast traders. Therefore, slow traders will enter the bilateral market with a higher probability if μ is larger. This effect is the same in the HM as in the PBM. However, if the slow traders' market participation in the PBM is already fairly high, a platform might introduce too much competition among the dealers and lower prices too much. The slow traders' market participation rate cannot exceed 1, but prices will always become lower if a platform is introduced.



Panel A.   Panel B.

Figure 1: **Regime-1 equilibria.** The distributions in Panel A correspond to HM and PBM equilibria for the following exogenous parameters: $N = 15$, $v = 1$, $c = 0.5$, $s = 0.08$, $\mu = 0.1$ and $\eta = 0.1$. Endogenous parameters are $\gamma \approx 0.329$ and $\gamma^{PBM} \approx 0.145$. Dealers' joint profits are 0.185 in the HM equilibrium and 0.065 in the PBM equilibrium. Therefore, dealers prefer the HM. The distributions in Panel B correspond to HM and PBM equilibria for the following exogenous parameters: $N = 15$, $v = 1$, $c = 0.5$, $s = 0.08$, $\mu = 0.5$ and $\eta = 0.1$. Endogenous parameters are $\gamma = 1$ and $\gamma^{PBM} = 1$. Dealers' joint profits are 0.010 in the HM equilibrium and 0.229 in the PBM equilibrium. Therefore, dealers prefer the PBM.

In Panel A of Figure 1, the fraction of fast traders is sufficiently low for slow traders not to enter the bilateral market fully in neither the PBM nor the HM equilibrium. In both the HM and the PBM, slow traders' entry in the bilateral market is chosen such that $r = v$ holds. As Lemma 11 states, that means $\gamma > \gamma^{PBM}$. The range of prices in the bilateral market is greater in the HM than in the PBM. This is consistent with Corollary 5.

In Panel B of Figure 1, the fraction of fast traders is sufficiently high for slow traders to fully enter the bilateral market in both the HM and the PBM equilibrium. The reservation price of slow traders is even lower in the HM than in the PBM, because dealers in the bilateral market of the HM have incentives to quote lower prices in order to compete with the platform. Due to full market entry by slow traders, Corollary 5 is not applicable. One can see that in fact the range of possible prices in the PBM is greater than that of the bilateral market in the HM.

A look at Table 1 further clarifies why a platform increases dealer profits in Panel A of Figure 1, but not in Panel B. In Panel A, market participation of slow traders was only around 14.5% in the PBM, whereas around 79.4% + 6.8% = 86.2% of slow traders buy an asset in the HM. In this case, it pays off to lower expected prices in order to get a considerable increase in turnover.

Panel B of Table 1 shows the statistics corresponding to the equilibrium in Panel B of Figure 1. In Panel A,

Table 1: Prices and market participation for regime 1

Panel a: Dealers prefer HM

	slow traders		fast traders		
	E(price)	P(trade)	E(price)	P(trade)	RP
Platform	0.714	0.794	0.682	0.072	0.799
Bilateral mkt	0.920	0.068	0.552	0.928	1.000
PBM	0.920	0.145	0.604	1.000	1.000

Panel b: Dealers prefer PBM

	slow traders		fast traders		
	E(price)	P(trade)	E(price)	P(trade)	RP
Platform	0.653	0.794	0.630	0.023	0.748
Bilateral mkt	0.783	0.206	0.517	0.977	0.863
PBM	0.879	1.000	0.580	1.000	0.959

Note: The information in this table corresponds to the equilibria described in Figure 1. Panel (a) of this Table refers to panel (a) of Figure 1. Panel (b) of this table refers to panel (b) of Figure 1. For each type of trader, the first column denotes the expected price this type of trader gets in the different trading venues, conditional on trading there. The second column for each type of trader shows the probability that this type of trader will trade in a certain venue. The last column shows the reservation price for each trading venue. This reservation price solves (1) if a bilateral market is considered and (2) if the platform is considered.

market participation of slow traders was only around 10% in the PBM, whereas around 67.2%+4.4% = 71.6% of slow traders buy an asset in the HM. In this case, it pays off to lower expected prices in order to get a considerable increase in turnover. Slow traders fully participate in the PBM. By introducing a platform, the dealers cannot increase their market participation as much as they did in the previous case from Panel A. Instead, dealers only reduce prices. Additionally, having a higher fraction of fast traders μ makes the impact of slow traders on dealer profits less important. Overall, a platform has a negative effect on dealer profits in this case.

In both equilibria considered here, fast traders mostly trade in the bilateral market and slow traders mostly trade on the platform. Thus, one can verify that the claim in Proposition 6 holds in this specific example. As stated in Proposition 9, both types of traders reduce their activity in the bilateral market if a platform is introduced. The fact that the reservation price for the platform is lower than the reservation price for the bilateral market in both equilibria means that it is indeed optimal for slow traders to start their search on the platform.

6.2 Regime-2 Equilibria

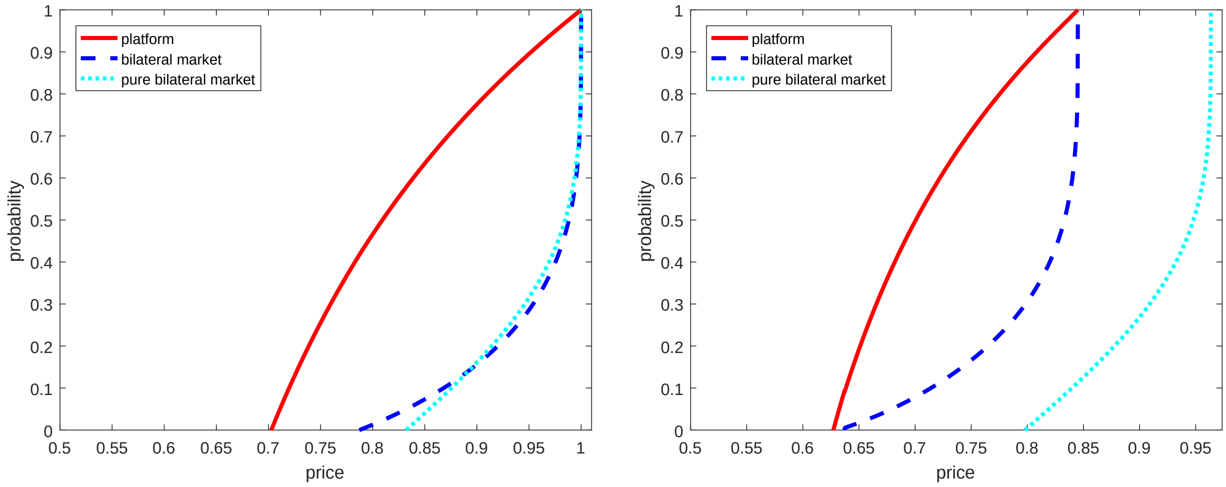
I will now consider equilibria of the type described in Proposition 2. Figure 2 shows again two examples for equilibria in the HM and the PBM. Here it holds that $\underline{p}^p < \underline{p}^b$. The difference between the set of exogenous parameters in Panel A and Panel B of Figure 2 is that μ is higher in Panel B. As a result, the slow traders' market participation in the PBM equilibrium of Panel A is rather low, whereas slow traders fully enter the PBM in Panel B.

One can see in Figure 2 that the shape of the price distributions in the bilateral markets is different from that of the distributions in Figure 1. An intuitive explanation for that result goes as follows. In the equilibria considered in Figure 1, a dealer in a bilateral market had both an incentive to quote low prices to attract fast traders and to quote high prices to obtain a high profit margin when selling to slow traders. However, as stated in Corollary 6, a regime-2 equilibria only exist if the fraction of fast traders μ is sufficiently low. In this case the incentive to quote low prices is much smaller and dealers mostly quote high prices in the bilateral market. Otherwise, anlalogous comments as those made regarding Figure 1 also apply to Figure 2. In Panel A of Figure 2, the presence of a platform increases the range of prices quoted in the bilateral market. Considering Panel B, note that an increase in μ brings to lower bounds of the supports of the price distributions of the bilateral market and the platform closer together. If μ would increase significantly more, a regime-2 equilibrium would no longer exist.

Table 2 shows that only 10% of slow traders enter the PBM in Panel A of Figure 2, but all slow traders enter the PBM in Panel B. By introducing a platform, the dealers cannot increase their market participation as much as they did in the previous case from Panel A. Instead, dealers only reduce prices.The overall effect on dealer profits of introducing the platform is negative in Panel B of Figure 2.

An important difference between the regime 1 and regime 2 equilibria discussed in this section is that even fast traders execute most of their trades on the platform in the equilibria described in Figure 2 and Table 2. The result stated in Proposition 6 remains true, however. Fast traders are more likely than slow traders to trade in the bilateral market, whereas slow traders are more likely than fast traders to trade on the platform.

One can verify that the reservation price for the platform is lower than the reservation price for the bilateral market in the HM in Table 2. This shows that it is indeed optimal for slow traders to start their search on the platform.



Panel A. Panel B.

Figure 2: Regime-2 equilibria. The distributions in Panel A correspond to HM and PBM equilibria for the following exogenous parameters: $N = 5$, $v = 1$, $c = 0.5$, $s = 0.04$, $\mu = 0.01$ and $\eta = 0.2$. Endogenous parameters are $\gamma \approx 0.135$ and $\gamma^{PBM} \approx 0.100$. Dealers' joint profits are 0.225 in the HM equilibrium and 0.050 in the PBM equilibrium. Therefore, dealers prefer the HM. The distributions in Panel B correspond to HM and PBM equilibria for the following exogenous parameters: $N = 5$, $v = 1$, $c = 0.5$, $s = 0.04$, $\mu = 0.1$ and $\eta = 0.2$. Endogenous parameters are $\gamma = 1$ and $\gamma^{PBM} = 1$. Dealers' joint profits are 0.230 in the HM equilibrium and 0.417 in the PBM equilibrium. Therefore, dealers prefer the PBM.

6.3 Collusion in the HM

In the above analysis, the dealers' response rate η was held fixed. Given this fixed response rate, it has been argued that dealers may chose either the HM or the PBM as a the overall market structure, depending on where dealer profits are higher.

This section argues that once an HM has been established, there is still some potential for dealers to use their influence on the market structure toward their benefit. It has already been established by Proposition 4 and Corollary 3 that in some special cases, there is a profit-maximizing response rate to an RFQ on the platform. In the following, I want to illustrate that dealers can increase their profits by limiting their activity on the platform even when the number of dealers N is small. Moreover, the optimal response rates in the following examples seem close to the theoretically optimal response rate in the equilibrium described in Proposition 4.

I will briefly revisit the two examples from Section 6.1 and Section 6.2 in which dealers could increase their profits by introducing a platform. Panel A of Figure 3 corresponds to the equilibrium described in Panel A of Figure 1. In Figure 3 however, dealers choose different levels of η. One can check that the conditions mentioned in Proposition 1 hold for the stated parameters and the shown values of η and γ. From

Table 2: Prices and market participation for regime 2

Panel a: Dealers prefer HM

	slow traders		fast traders		
	E(price)	P(trade)	E(price)	P(trade)	RP
Platform	0.802	0.672	0.774	0.537	0.842
Bilateral mkt	0.960	0.044	0.876	0.463	1.000
PBM	0.960	0.100	0.895	1.000	1.000

Panel b: Dealers prefer PBM

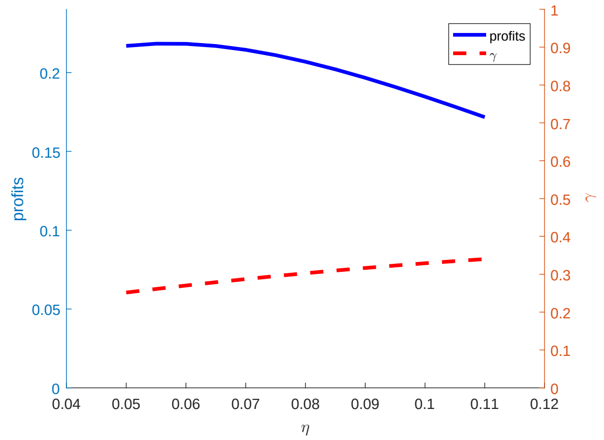
	slow traders		fast traders		
	E(price)	P(trade)	E(price)	P(trade)	RP
Platform	0.697	0.672	0.672	0.456	0.748
Bilateral mkt	0.805	0.328	0.716	0.544	0.845
PBM	0.924	1.000	0.859	1.000	0.964

Note: The information in this table corresponds to the equilibria described in Figure 2. Panel (a) of this Table refers to panel (a) of Figure 2. Panel (b) of this table refers to panel (b) of Figure 2. For each type of trader, the first column denotes the expected price this type of trader gets in the different trading venues, conditional on trading there. The second column for each type of trader shows the probability that this type of trader will trade in a certain venue. The last column shows the reservation price for each trading venue. This reservation price solves (1) if a bilateral market is considered and (2) if the platform is considered.

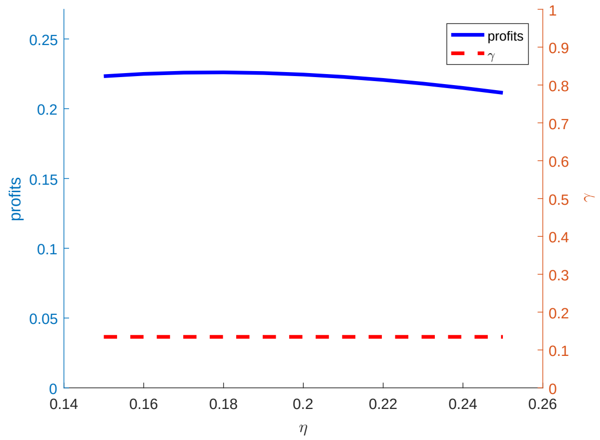
the dealers' point of view, there seems to be an optimal η approximately equal to $0.067 \approx 1/N$. Thus, the optimal response rate derived for the equilibrium of the type described in Proposition 4 is also optimal for the equilibrium considered here. Dealer profits decrease if dealers choose a response rate sufficiently higher than 0.067 even though slow traders' equilibrium choice of γ increases. From the dealers' perspective, it is profitable to reduce turnover in order to charge higher prices. From the social perspective, this is inefficient behavior, since in the given setup all trades are beneficial: Traders value the asset at $v > c + s$.

Panel B of Figure 3 shows a similar picture. Here I consider the equilibrium from Panel A in Figure 2 and let the dealers' choice of η vary. The conditions mentioned in Proposition 2 hold for the stated parameters and the shown values of η and γ. The dealers' optimal response rate in this equilibrium seems to be approximately equal to 0.18, which is slightly different from $0.2 = 1/N$, but in the same order of magnitude. The slow traders' equilibrium choice of γ increases in η as in Panel A. However, this increase is not very strong. Limiting the RFQ response rate is again inefficient, since total turnover decreases.

In order for dealers to achieve these optimal response rates, it is necessary to coordinate. From an individual dealer's point of view, it is always better to respond to an RFQ than not to respond, since a dealer can expect a positive payoff from responding. In practice, one might see RFQ response rates slightly



Panel A.



Panel B.

Figure 3: Dealer profits and the choice of η. In Panel A, the solid blue line shows dealer profits for various choices of η, where the other parameters are as in Panel A of Figure 1. One can see that the profits are highest if $\eta \approx 0.067 \approx 1/N$. The slow traders' equilibrium choice of γ (dashed red line) increases with the dealers' choice of η. In Panel B, the solid blue line shows dealer profits for various choices of η, where the other parameters are as in Panel A of Figure 2. One can see that the profits are highest if $\eta \approx 0.18$. The increase in the slow traders' equilibrium choice of γ (dashed red line) is barely noticeable.

higher than $1/N$ just because dealers cannot collude perfectly.[9]

7 Conclusions

This paper presents a model that describes the behavior of dealers and traders in a hybrid market structure consisting of a bilateral market and an electronic trading platform. Building on results of Duffie et al. (2016) who model a pure bilateral market, this paper establishes conditions under which dealers earn higher profits with one particular market structure than with the other. If dealers have a substantial influence on trading mechanisms in OTC markets, this model might explain why some assets are traded in an HM and why some assets are only traded in a PBM: Dealers prefer it that way. In fact, the theoretical predictions of the model seem to be consistent with anecdotes regarding the CDS and bond markets mentioned in the introduction. Additionally, the model presented here implies that dealers have an incentive to collude and reduce their response rate to RFQs in order to increase their profits. One should note that this model neglects potentially important information leakage on electronic trading platforms as discussed in Hendershott and Madhavan (2015). In the model presented in this paper, only the information regarding the traders' types is asymmetric.

[9] Here, I do not consider how dealers might collude. However, Schneider et al. (2016) state in footnote 43 that dealers do not always seem to intend to be competitive on the platform. It would not be hard to construct a repeated game in which traders want to buy an asset in every period and other dealers could punish a deviating dealer such that collusion is sustained in equilibrium.

Other forms of information asymmetry may have significant effects on equilibrium trading behavior that are not modeled in this paper.

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Appendix A

This appendix contains all proofs.

Proof of Lemma 1. One can rewrite equations (1) and (2) the following way:

$$s = \int_c^{r_b} (r_b - x)\,\mathrm{d}H(x) =: \varphi_b(r_b),$$

$$s = (1-(1-\eta)^N)\int_c^{r_p} (r_p - x)\,\mathrm{d}F(x) =: \varphi_p(r_p).$$

The integrals in the definitions of φ_b and φ_p indeed exist, since

$$\int_c^{r_b} (r_b - x)\,\mathrm{d}H(x) \le (r_b - c)$$

and

$$(1-(1-\eta)^N)\int_c^{r_p} (r_p - x)\,\mathrm{d}F(x) \le (1-(1-\eta)^N)(r_b - c).$$

The functions $\varphi_p, \varphi_b : \mathbb{R} \to \mathbb{R}$ are continuous and strictly monotone increasing. Furthermore, $\lim_{x\to\infty} \varphi_i(x) = \infty$ and $\varphi_i(c) = 0$ for $i \in \{p, b\}$. Thus, there exist unique $r_b, r_p \in \mathbb{R}$ such that $\varphi_b(r_b) = s$ and $\varphi_p(r_p) = s$ for $s > 0$. This means that the solutions to equations (1) and (2) exist and are unique.

Next, one has to show that given that the platform has been contacted, the stated continuation rule is optimal. Suppose the trader received no quote on the platform at all. Then it is clearly optimal to continue to search in the bilateral market, since $r_b \leq v$ and the definition of r_b implies

$$\mathbb{P}(p_b \leq v)\mathbb{E}(v - p_b | p_b \leq v) - s = v - \mathbb{E}(\min(p_b, v)) - s \geq 0.$$

The last equation means that the trader would get a non-negative payoff from searching in the bilateral market, since the the expected payoff from being able to buy the asset for a price less than v outweighs the search cost s. Thus, it is always at least as good to continue to search as to terminate the search.

Analogously, suppose the trader received a quote $p \geq r_b$ as a response to an RFQ on the platform. Then it is also optimal to continue to search in the bilateral market, since $r_b \leq v$ and the definition of r_b implies

$$\mathbb{P}(p_b \leq v)\mathbb{E}(v - p_b | p_b \leq v) - s = v - \mathbb{E}(\min(p_b, v)) - s \geq 0,$$

if $p > v$. This means that the expected payoff from being able to buy the asset for a price less than v outweighs the search cost s. If $p \leq v$, one gets

$$\mathbb{P}(p_b \leq p)\mathbb{E}(v - p_b | p_b \leq p) - s = p - \mathbb{E}(\min(p_b, p)) - s \geq 0,$$

Thus, the potential price improvement outweighs the search cost s.

On the other hand, if the trader has received a quote $p < r_b \leq v$ as a response to an RFQ on the platform, the definition of r_b implies $p - \mathbb{E}(\min(p_b, p)) - s < 0$, i.e. search costs are larger than the potential benefits of obtaining a better price.

It remains to show that prioritizing the platform is indeed the optimal strategy. This will be shown by induction over the number m of dealers left to contact in the bilateral market. If $m = 0$ (for instance, because all dealers have been contacted), one can verify analogously to the arguments above that it is optimal to go to the platform if no quote less than r_p has been received until that point, since $r_p \leq v$. Suppose that $m \geq 0$ and let it be optimal to go to the platform if no price offer less than r_p has been received. It has to be shown that it is optimal to go to the platform if $m + 1$ dealers remain uncontacted in the bilateral market and no quote less than r_p has been received so far. This implies that the trader should start his search on the platform, when N dealers are uncontacted in the bilateral market.

Let y be the best current quote at which the trader can buy the asset. If $r_b > r_p > y$, search costs will be greater than the expected price improvement in both the bilateral market and on the platform. The trader should stop searching and buy the asset for the price y and not prioritize the bilateral market over the platform. If $r_b > y \geq r_p$, search costs will be greater than the expected price improvement in the bilateral market. On the other hand, the price improvement on the platform will be greater than the search costs and it is therefore optimal to go to the platform.

Now, let $y \geq r_b > r_p$ or assume that no quote has been received so far. Both going to the bilateral market and going to the platform is at least as good as not to search. In this case, it is not trivial that prioritizing the platform is optimal.

Let B denote the expected payoff a trader gets if he goes to the bilateral market first and receives the quote p_b. If $p_b < r_p$, it is clearly optimal to buy the asset at the price p_b. If $p_b \geq r_p$, the inductive hypothesis states that it is optimal to continue to search on the platform and receive the quote q with probability $\omega := 1 - (1-\eta)^N$. If $q \leq p_b$ and $q < r_b$, the trader buys the asset at price q. Let $q > p_b$ or assume no quote has been received on the platform. Then the trader will buy the asset at price p_b, if $p_b < r_b$ or otherwise continue to search in the bilateral market.

Let A be the expected payoff the trader gets if he starts to search on the platform and receives the quote q with probability ω. If $q < r_b$, it is clearly optimal to buy the asset at the price q. If $q \geq r_b$ is optimal to continue to search in the bilateral market and receive the quote p_b. If $q \leq p_b$ and $q < r_b$, the trader buys the asset at price p_b. Let $q > p_b$ or assume no quote has been received on the platform. Then the trader will buy the asset at price p_b, if $p_b < r_b$ or otherwise continue to search in the bilateral market.

If both $p_b > r_b$ and no quote $q \leq r_b$ has been received, the trader will keep searching after having visited both the platform and one dealer in the bilateral market. The probability of this outcome and the value of future search opportunities are independent of the order in which one visited the trading venues. The continuation value in this event multiplied by the probability of occurrence of the event is equal to some number X (the exact value is not important since it will cancel out later).

One can now calculate the payoffs according to the above reasoning

$$
\begin{aligned}
A &= -s+\omega\mathbb{P}(q\le r_p)\cdot\mathbb{E}(v-q|q\le r_p)+\omega\mathbb{P}(r_p<q\le r_b)\cdot\mathbb{E}(v-q|r_p<q\le r_b)\\
&\quad +(1-\omega\mathbb{P}(q\le r_b))\cdot(-s+\mathbb{P}(p_b<r_b)\cdot\mathbb{E}(v-p_b|p_b<r_b))+X.
\end{aligned}
$$

Rewriting equations (1) and (2) and using the expressions $s=\omega\mathbb{P}(q\le r_p)\cdot\mathbb{E}(r_p-q|q\le r_p)$ and $s=\mathbb{P}(p_b\le r_p)\cdot\mathbb{E}(r_p-p_b|p_b\le r_p)$, one gets

$$
\begin{aligned}
A &= -s+\underbrace{\omega\cdot\mathbb{P}(q\le r_p)\cdot\mathbb{E}(r_p-q|q\le r_p)}_{=s}+\omega\cdot\mathbb{P}(q\le r_p)\cdot(v-r_p)\\
&\quad +\omega\cdot\mathbb{P}(r_p<q\le r_b)\cdot\mathbb{E}(v-q|r_p<q\le r_b)\\
&\quad +(1-\omega\cdot\mathbb{P}(q\le r_b))\cdot(-s+\underbrace{\mathbb{P}(p_b<r_b)\cdot\mathbb{E}(r_b-p_b|p_b<r_b)}_{=s}+\mathbb{P}(p_b<r_b)\cdot(v-r_b)+X\\
&= \omega\cdot\mathbb{P}(q\le r_p)\cdot(v-r_p)+\omega\cdot\mathbb{P}(r_p<q\le r_b)\cdot\mathbb{E}(v-q|r_p<q\le r_b)\\
&\quad +(1-\omega\cdot\mathbb{P}(q\le r_b))\cdot\mathbb{P}(p_b<r_b)\cdot(v-r_b)+X.
\end{aligned}
$$

Analogously, one gets

$$
\begin{aligned}
B &= -s+\mathbb{P}(p_b\le r_p)\cdot\mathbb{E}(v-p_b|p_b\le r_p)\\
&\quad +\mathbb{P}(r_p<p_b\le r_b)\cdot\Big\{-s+\omega\cdot\mathbb{P}(q\le r_p)\cdot\mathbb{E}(v-q|q\le r_p)\\
&\quad +\omega\cdot\mathbb{P}(r_p\le q,p_b<r_b)\cdot\mathbb{E}(v-\min(q,p_b)|r_p\le q,p_b<r_b)\\
&\quad +(1-\omega\cdot\mathbb{P}(q\le r_b))\cdot\mathbb{E}(v-p_b)|r_p\le p_b<r_b)\Big\}\\
&\quad +\mathbb{P}(r_b<p_b)\cdot\Big\{-s+\omega\cdot\mathbb{P}(q<r_p)\cdot\mathbb{E}(v-q|q<r_b)\\
&\quad +\omega\cdot\mathbb{P}(r_p\le q<r_b)\cdot\mathbb{E}(v-q|r_p\le q<r_b)\Big\}+X\\
&= \mathbb{P}(p_b\le r_b)\cdot(v-r_b)+\mathbb{P}(r_p<p_b)\cdot\omega\cdot\mathbb{P}(q\le r_p)\cdot(v-r_p)\\
&\quad +\mathbb{P}(r_p\le p_b<r_b)\cdot\omega\cdot\mathbb{P}(r_p\le q<r_b)\cdot\mathbb{E}(v-\min(q,p_b)|r_p\le q,p_b<r_b)\\
&\quad -\mathbb{P}(r_p\le p_b<r_b)\cdot\omega\cdot\mathbb{P}(q\le r_b)\cdot\mathbb{E}(v-p_b|r_p<p_b\le r_b)\\
&\quad +\mathbb{P}(r_b<p_b)\cdot\omega\cdot\mathbb{P}(r_p<q\le r_b)\cdot\mathbb{E}(v-q|r_p<q\le r_b)+X.
\end{aligned}
$$

After some further manipulations one gets

$$
\begin{aligned}
A - B &= (r_b - r_p) \cdot \mathbb{P}(p_b \leq r_p) \cdot \omega \cdot \mathbb{P}(q \leq r_p) \\
&\quad + (r_b - \mathbb{E}(p_b | r_p \leq p_b < r_b)) \cdot \mathbb{P}(r_p < p_b \leq r_b) \cdot \omega \cdot \mathbb{P}(q \leq r_p) \\
&\quad + (r_b - \mathbb{E}(q | r_p \leq q < r_b)) \cdot \omega \cdot \mathbb{P}(r_p < q \leq r_b) \cdot \mathbb{P}(p_b \leq r_p) \\
&\quad + \{ \ \mathbb{E}(\min(q, p_b) | r_p \leq q, p_b < r_b) - \mathbb{E}(q | r_p \leq q < r_b) - \mathbb{E}(p_b | r_p \leq p_b < r_b) \\
&\quad + r_b \ \} \cdot \mathbb{P}(r_p < p_b \leq r_b) \cdot \omega \cdot \mathbb{P}(r_p < q \leq r_b) \\
&> 0,
\end{aligned}
$$

since

$$
\begin{aligned}
\mathbb{E}(\min(q, p_b) | r_p \leq q, p_b < r_b) &= r_b + \mathbb{E}(\min(q - r_b, p_b - r_b) | r_p \leq q, p_b < r_b) \\
&\geq r_b + \mathbb{E}(\min(q - r_b + p_b - r_b) | r_p \leq q, p_b < r_b) \\
&= \mathbb{E}(q | r_p \leq q < r_b) + \mathbb{E}(p_b | r_p \leq p_b < r_b) - r_b.
\end{aligned}
$$

Thus, contacting the platform first is optimal.

If $r_b = v$ and a slow trader did not receive a quote less than r_b, it follows from (1) that he neither gains nor looses anything by contacting a dealer in the bilateral market. Continuing with any probability is optimal. □

Proof of Lemma 2. Rewriting (1) gives

$$
s = \mathbb{E}(\max(r_b - p_b, 0)).
$$

The right-hand side of the last equation is continuous and monotone increasing in r_b. One has $\mathbb{E}(\max(c - p_b, 0)) = 0$, since no dealer quotes below his cost. Thus, $r_b > c$ for $s > 0$. Equation (1) now gives

$$
r_b := \mathbb{E}(\min(p_b, r_b)) + s \geq c + s.
$$

the last inequality is strict, if prices are greater than c with positive probability. □

Proof of Lemma 3. First, there is no demand from traders, if a dealer quotes a price higher than r. On the other hand, the dealers' profit is zero or negative if he quotes a price at or below his cost c. Now, let the dealer quote a price p with $c < p \leq r$. Lemma 2 ensures the existence of such a p. Now the dealer gets a positive profit on both the platform and on the bilateral market. This can be seen as follows. With probability $(1-\eta)^{N-1}$ the dealer is alone on the platform. If a slow trader requested the quote, the dealer is able to sell the asset at price r due to the slow traders' reservation price strategy. Thus, the dealers' profits are at least $(1-\eta)^{N-1}(1-\mu)(p-c) > 0$ on the platform in the event that the dealer charges p. The same logic can be applied to the bilateral market in which case the dealer would get at least $(1-\eta)^{N-1}\gamma(1-\mu)(p-c)/N > 0$, since slow traders will not get a satisfactory offer on the platform with a probability greater than or equal to $(1-\eta)^{N-1}$. Then they continue to search with probability $\gamma > 0$ in the bilateral market where they choose a particular dealer with probability $1/N$. Thus, the optimally quoted prices are greater than c and less than or equal to r.

$k_p = 1 - \mu$ is clear from the slow traders' strategy of starting the search on the platform. Since dealers never quote a price greater than r, the slow traders always buy the asset when they receive a quote on the platform. The only case in which they continue their search in the bilateral market is when they do not get a quote on the platform. Then they continue to search with probability $\gamma > 0$ and buy the asset from the first dealer they contact in the bilateral market. The exact law of large numbers implies $k_b = (1-\eta)^N\gamma(1-\mu)/N$, since there are (uncountably) infinitely many slow traders.

It can furthermore be shown that there cannot be atoms in the distribution G of prices quoted on the platform, no matter how the distribution H of prices in the bilateral market looks like. Suppose there is a price p with $r \geq p > c$ that is quoted on the platform with probability $\rho > 0$. A single dealer can now profitably deviate from this strategy as follows. Since the number of prices charged with positive probability must be countable, one can find for each $\delta > 0$ an ε_δ, such that $\delta \geq \varepsilon_\delta > 0$ and the price $p - \varepsilon_\delta$ is charged with probability zero. A dealer can now charge price $p - \varepsilon_\delta$ with probability ρ and charge price p with probability zero. Using the fact that $\lim_{\delta \to 0} G(p - \varepsilon_\delta) = G(p) - \rho$, one can express the difference Δ_p in profits between quoting $p - \varepsilon_\delta$ and quoting p for small δ as follows. The considered dealer only makes a positive profit if no other dealer on the platform quotes a lower price. If no other dealer quotes a lower price, there might be $j = 0, 1, ..., N-1$ dealers who quote price p. The calculation below considers the cases in which j dealers quote price p on the platform separately.

I will allow H to have atoms. To simplify the algebra, I will introduce the function $D : \mathbb{Z} \times \mathbb{R} \to \mathbb{R}$. I define $D(j, p)$ as the expected quantity of fast traders the considered dealer on the platform faces when he quotes p, j other dealers on the platform also quote p and no other dealer on the platform quotes a lower price. $D(j, p)$ cannot be increasing in p. As the $j + 1$ dealers raise their quote, no dealer in the bilateral market becomes more likely to quote a higher price than the dealers on the platform do. D is decreasing in j, since sharing of the total demand reduces the demand for each individual dealer, if j increases.

If H has no atom at price p then one has

$$D(j,p) := \frac{\mu}{j+1}(1-H(p))^N.$$

The last expression says that the $j+1$ dealers who quote p on the platform share equally the demand by fast traders, in case all dealers on in the bilateral market quote a higher price.

If H has an atom at price p, then price p is quoted with some probability $\rho > 0$ in the bilateral market and D is given by

$$D(j,p) := \sum_{k=0}^{N} \binom{N}{k} (1-H(p))^{N-k} \rho^k \frac{\mu}{j+1+k}.$$

If k dealers in the bilateral market quote p and all other dealers in the bilateral market quote a higher price, the dealers quoting p will share the demand equally.

Now I will show that there is a profitable deviation if the equilibrium choice of G has atoms. The first two terms in the first equation below look at the event in which the considered dealer quotes the lowest price on the platform and no other dealer quotes p, which happens with probabilities $(1-\eta G(p-\varepsilon_\delta)-\eta\rho)^{N-1}$ and $(1-\eta G(p))^{N-1}$, respectively. The last two terms look at the events in which $j > 0$ other dealers quote p. If the considered dealer quotes p as well, he will only get a fraction $1/(j+1)$ of the trader's demand in expectation. If the considered dealer quotes $p-\varepsilon_\delta$, he will still get the full demand.

$$
\begin{aligned}
\Delta_p &= (1-\eta G(p-\varepsilon_\delta)-\eta\rho)^{N-1}(p-\varepsilon_\delta-c)(k_p+D(0,p-\varepsilon_\delta)) \\
&\quad -(1-\eta G(p))^{N-1}(p-c)(k_p+D(0,p)) \\
&\quad +\sum_{j=1}^{N-1}\binom{N-1}{j}(1-\eta G(p-\varepsilon_\delta)-\eta\rho)^{N-1-j}(\eta\rho)^j(p-\varepsilon_\delta-c)(k_p+D(0,p-\varepsilon_\delta)) \\
&\quad -\sum_{j=1}^{N-1}\binom{N-1}{j}(1-\eta G(p))^{N-1-j}(\eta\rho)^j(p-c)(\frac{k_p}{j+1}+D(j,p)) \\
&\geq (1-\eta G(p-\varepsilon_\delta)-\eta\rho)^{N-1}(p-\varepsilon_\delta-c)(k_p+D(0,p)) \\
&\quad -(1-\eta G(p))^{N-1}(p-c)(k_p+D(0,p)) \\
&\quad +\sum_{j=1}^{N-1}\binom{N-1}{j}(1-\eta G(p-\varepsilon_\delta)-\eta\rho)^{N-1-j}(\eta\rho)^j(p-\varepsilon_\delta-c)(k_p+D(j,p)) \\
&\quad -\sum_{j=1}^{N-1}\binom{N-1}{j}(1-\eta G(p))^{N-1-j}(\eta\rho)^j(p-c)(\frac{k_p}{j+1}+D(j,p)) \\
&\to \sum_{j=1}^{N-1}\binom{N-1}{j}(p-c)\frac{jk_p}{j+1}(1-\eta G(p))^{N-1-j}(\eta\rho)^j>0 \quad \text{as } \delta\to 0.
\end{aligned}
$$

Thus, the proposed deviation is profitable. In equilibrium, G cannot have any atoms. The calculation shows that the increase in profits is possible because ties can be avoided and the dealer gets the full demand of the trader, when he would have had to split the demand in expectations with other dealers.

Analogously, one can show that there cannot be any atoms in the distribution of prices in the bilateral market. The difference Δ_b in profits between quoting $p-\varepsilon_\delta$ and quoting p in this case is

$$
\begin{aligned}
\Delta_b &= (1-H(p-\varepsilon_\delta)-\rho)^{N-1}(p-\varepsilon_\delta-c)(k_b+(1-\eta G(p-\varepsilon_\delta))^N\mu) \\
&\quad -(1-H(p))^{N-1}(p-c)(k_b+(1-\eta G(p)^N)\mu) \\
&\quad +(1-(1-H(p-\varepsilon_\delta))^{N-1})(p-\varepsilon_\delta-c)k_b \\
&\quad -(1-(1-H(p)+\rho)^{N-1})(p-c)k_b \\
&\quad +\sum_{j=1}^{N-1}\binom{N-1}{j}(1-H(p-\varepsilon_\delta)-\rho)^{N-1-j}\rho^j(p-\varepsilon_\delta-c)(k_p+(1-\eta G(p-\varepsilon_\delta))^N\mu) \\
&\quad -\sum_{j=1}^{N-1}\binom{N-1}{j}(1-H(p))^{N-1-j}\rho^j(p-c)(k_p+(1-\eta G(p))^N\frac{\mu}{j+1})
\end{aligned}
$$

The first two terms look at the event in which the considered dealer has the lowest quote in the bilateral market and no other dealer quotes p. This event happens with probability $(1 - H(p - \varepsilon_\delta) - \rho)^{N-1}$ and $(1 - H(p))^{N-1}$, respectively. The next two terms look at the event in which the considered dealer does not have the lowest quote, i.e. at least one other dealer has a strictly lower quote. In this case, the dealer can only sell to slow traders. The last two terms look at the event in which the considered dealer has the lowest quote and $j > 0$ other dealers quote p. This means that there are ties if the dealer quotes p as well and the demand by fast traders will be split in expectation equally among those $j+1$ dealers. Since G is continuous, the event that the lowest quote on the platform is equal to p has probability zero. Thus, demand does not have to be split with dealers on the platform. A dealer always gets the full demand of a slow trader in the bilateral market. One has $\lim_{\delta \to 0} H(p - \varepsilon_\delta) = H(p) - \rho$. Thus, as $\delta \to 0$, the first four terms in the previous equation vanish and the difference between the last two terms is positive, since $N \geq 2$:

$$\lim_{\delta \to 0} \Delta_b = \sum_{j=1}^{N-1} \binom{N-1}{j} (1 - H(p))^{N-1-j} \rho^j (p - c)(1 - \eta G(p))^N \frac{j\mu}{j+1} > 0.$$

Thus, the proposed deviation is profitable. Again, the profitable deviation exists because ties with other dealers can be avoided. In equilibrium, H cannot contain any atoms. □

Proof of Lemma 4. If a dealer is in the bilateral market, then for every p in the support of H the expected profit

$$\Pi_b(p) = (p - c)\left[k_b + \mu(1 - H(p))^{N-1} \sum_{j=0}^{N-1} \binom{N-1}{j} (1-\eta)^{N-1-j} \eta^j (1 - G(p))^j\right]$$

must be constant. Using the binomial formula $(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$, the previous equation simplifies to

$$\Pi_b(p) = (p - c)\left[k_b + \mu(1 - H(p))^{N-1} \left(1 - \eta G(p)\right)^N\right].$$

Suppose there is a $\overline{p} < r$ such that $H(\overline{p}) = 1$. Then a dealer could profitably deviate from this strategy by quoting for instance r with positive probability. The difference in expected profits would be

$$\Pi_b(r) - \Pi_b(\overline{p}) = (r - \overline{p})k_b > 0.$$

Thus, $H(\overline{p}) < 1$ for $\overline{p} < r$. By Lemma 3, no dealer quotes a price above r. Therefore, r must be the supremum of the support of H. Lemma 3 also states that H cannot have any atoms. Dealers are only willing to quote according to a continuous distribution that has a support with supremum r if profits for prices in the support of H are equal to $\Pi_b(r)$. When quoting a price equal to r, a dealer only sells to slow traders, since other dealers almost surely quote a lower price if H is a continuous distribution. The indifference condition determining H is therefore

$$(p-c)\left[k_b + \mu(1-H(p))^{N-1}\left(1-\eta G(p)\right)^N\right] = (r-c)k_b.$$

This is equation (4). If a dealer is on the platform, then for every p in the support of H the expected profit

$$\Pi_p(p) = (p-c)\left[k_p \sum_{j=0}^{N-1} \binom{N-1}{j} (1-\eta)^{N-1-j}\eta^j(1-G(p))^j + \mu(1-H(p))^N \sum_{j=0}^{N-1} \binom{N-1}{j} (1-\eta)^{N-1-j}\eta^j(1-G(p))^j\right]$$

must be constant. Using the binomial formula, this equation can again be simplified to

$$\Pi_p(p) = (p-c)\left[k_p\left(1-\eta G(p)\right)^{N-1} + \mu(1-H(p))^N\left(1-\eta G(p)\right)^{N-1}\right].$$

Suppose there is a $\overline{p} < r$ such that $G(\overline{p}) = 1$. Then a dealer could profitably deviate from this strategy by quoting for instance r with positive probability. This can be seen as follows. From (4) and $G(p) = 1$ for $p \geq \overline{p}$ it follows that

$$(r-c)k_b = (r-c)k_p\frac{\gamma}{N}(1-\eta)^N \geq (\overline{p}-c)(k_b + (1-\eta)^N\mu(1-H(\overline{p}))^{N-1}),$$

where the last inequality is an equality whenever $\overline{p}$ is in the support of H. Further algebraic manipulation gives

$$
\begin{aligned}
\Pi_p(r) &= (r-c)k_p(1-\eta)^{N-1} \\
&\geq \frac{N}{\gamma(1-\eta)}(\overline{p}-c)(k_b+(1-\eta)^N\mu(1-H(\overline{p}))^{N-1}) \\
&= \frac{N}{\gamma(1-\eta)}(\overline{p}-c)(k_p\frac{\gamma}{N}(1-\eta)^N+(1-\eta)^N\mu(1-H(\overline{p}))^{N-1}) \\
&= (\overline{p}-c)(k_p(1-\eta)^{N-1}+(1-\eta)^{N-1}\frac{N}{\gamma(1-H(\overline{p}))}\mu(1-H(\overline{p}))^N) \\
&> (\overline{p}-c)(1-\eta)^{N-1}(k_p+\mu(1-H(\overline{p}))^N) \\
&= \Pi_p(\overline{p}).
\end{aligned}
$$

The strict inequality in the above calculation holds, since $H(\overline{p})<1$ for $\overline{p}<r$, $0<\gamma\leq 1$ and $N>1$. It is therefore more profitable to quote r than to quote $\overline{p}$. Thus, it must be the case that $G(\overline{p})<1$ for $\overline{p}<r$. The supremum of the support of G is therefore given by r, since no dealer quotes above r. The equilibrium profit that results from quoting prices in the support of G on the platform is determined by the profit that results from quoting r:

$$
(p-c)\left[k_p\left(1-\eta G(p)\right)^{N-1}+\mu(1-H(p))^N\left(1-\eta G(p)\right)^{N-1}\right]=(1-\eta)^{N-1}(r-c)k_p.
$$

This is equation (5), which has to hold for prices in the support of G.

□

Proof of Lemma 5. Proof by contradiction. Let $\underline{p^p}$ be the lower bound of the support of G and let $p=\underline{p^b}$ be the lower bound of the support of H.

Suppose $p=\underline{p^b}=\underline{p^p}$, i.e. $G(p)=H(p)=0$. Then equation (4) gives

$$
\mu=\frac{r-p}{p-c}k_b=\frac{r-p}{p-c}\gamma\frac{1}{N}(1-\eta)^Nk_p.
$$

However, Equation (5) gives

$$
\mu=(1-\eta)^{N-1}\frac{r-p}{p-c}k_p.
$$

Clearly,

$$
\frac{r-p}{p-c}\gamma\frac{1}{N}(1-\eta)^Nk_p<(1-\eta)^{N-1}\frac{r-p}{p-c}k_p.
$$

Thus, $p = \underline{p}^b = \underline{p}^p$ must be wrong, i.e. it must be the case that $\underline{p}^p \neq \underline{p}^b$. $\square$

Proof of Lemma 6. If $\underline{p}^b < \underline{p}^p$, then $G(p) = 0$ for all $p < \underline{p}^p$. This simplifies equation (4) which has to be satisfied for all prices $p \in [\underline{p}^b, \underline{p}^p]$ that are in the support of H:

$$(p-c)\left[k_b + \mu(1-H(p))^{N-1}\right] = (r-c)k_b.$$

Solving for H gives

$$H_{[\underline{p}^b,\underline{p}^p]}(p) = 1 - \left(\frac{(r-p)k_b}{(p-c)\mu}\right)^{1/(N-1)}.$$

The lower bound of the support of H is determined by $H_{[\underline{p}^b,\underline{p}^p]}(p) = 0$. This means

$$\underline{p}^b = c + \frac{(r-c)k_b}{k_b+\mu}.$$

The lower bound of the support of G is the price $\underline{p}^p$ for which $G(\underline{p}^p) = 0$ and equation (5) still hold. Since $\underline{p}^b < \underline{p}^p$, it holds that $H(\underline{p}^p) = H_{[\underline{p}^b,\underline{p}^p]}(\underline{p}^p) > 0$. Setting $G = 0$ in equation (5), plugging in $H_{[\underline{p}^b,\underline{p}^p]}$ for H and solving for p gives

$$\underline{p}^p = c + \frac{(1-\eta)^{N-1}(r-c)k_p}{k_p + \mu(1-H_{[\underline{p}^b,\underline{p}^p]}(\underline{p}^p))^N}.$$

Suppose now that condition (9) holds. To show that equation (8) indeed has a solution, I define the continuous function

$$\varphi^G(p) := (p-c)(k_p + \mu(1-H_{[\underline{p}^b,\underline{p}^p]}(p))^N) - (1-\eta)^{N-1}(r-c)k_p.$$

Then $\underline{p}^p$ solves (8) if and only if $\varphi^G(\underline{p}^p) = 0$. In order to continue with the proof of the existence of a unique solution it helps to perform a change of variables. I first define the function $X_G : [\underline{p}^b, r] \to [0,1]$ by $X_G(p) = \frac{r-p}{p-c}\frac{k_b}{\mu}$. Note that $X_G(\underline{p}^b) = 1$ and $X_G(r) = 0$. Since the function X_G is monotone decreasing on its domain, it is invertible. Let $P_G = X_G^{-1}$. Then $P_G(x) = \frac{rk_b + cx\mu}{\mu x + k_b}$.

The equation $\varphi^G(p) = 0$ has a unique solution $\underline{p}^p \in [\underline{p}^b, r]$ if and only if

$$\varphi^G \circ P_G(x) = \frac{(r-c)k_b}{\mu x + k_b}\left(k_p + \mu x^{N/(N-1)}\right) - (1-\eta)^{N-1}(r-c)k_p = 0$$

has a unique solution $x^* \in [0,1]$. Using the fact that $k_b = (1-\eta)^N \frac{\gamma}{N}$, I rewrite $\varphi^G \circ P_G(x) = 0$ into a fixed-point equation:

$$x = \frac{\gamma(1-\eta)}{N\mu}\left(k_p + \mu x^{N/(N-1)}\right) - \frac{k_p\gamma(1-\eta)^N}{\mu N} =: \kappa_G(x).$$

This equation has a unique solution if the therein defined function κ_G is a contraction. Let $x \in [0,1]$. Then, $\kappa_G(x) > 0$. Furthermore,

$$\kappa_G(x) < \frac{\gamma(1-\eta)}{N\mu}\left(k_p + \mu\right) - \frac{k_p\gamma(1-\eta)^N}{\mu N} < 1,$$

since $(1-\eta)^{N-1}k_p + \mu\frac{N}{\gamma(1-\eta)} < 1$ is equivalent to (9). This can be checked by using $k_p = 1-\mu$ and simplifying terms. Thus, the inclusion $\kappa_G([0,1]) \subseteq [0,1]$ holds. Calculating the derivative of κ_G gives

$$\kappa_G'(x) = \frac{(1-\eta)\gamma}{(N-1)}x^{1/(N-1)} \leq \frac{(1-\eta)\gamma}{(N-1)}x^{1/(N-1)} < 1.$$

Also, the derivative is clearly non-negative. Thus, κ_G is a contraction and by the Banach fixed-point theorem, $\kappa_G(x) = x$ has a unique solution in $[0,1]$. By the above arguments, this proves that a solution $\underline{p}^p \in [\underline{p}^b, r]$ to (8) exists and that it is unique. Because of Lemma 3, it cannot be the case that $\underline{p}^p = \underline{p}^b$. Therefore, $\underline{p}^p > \underline{p}^b$.

Lastly, it remains to show that there is no solution $\underline{p}^p$ to (8) with $\underline{p}^p > \underline{p}^b$ if (10) holds. Suppose now that (10) holds and $\underline{p}^p > \underline{p}^b$. The following inequality is equivalent to (10), as can be checked by using $k_p = 1-\mu$ and simplifying:

$$k_p(1-\eta)^{1-N} > k_p + \frac{N}{\gamma}(1-\eta)^{-N}\mu.$$

Therefore,

$$\underline{p}^b = c + \frac{(r-c)k_b}{k_b+\mu} = c + \frac{(r-c)k_p}{k_p + \frac{N}{\gamma}(1-\eta)^{-N}\mu} > c + \frac{(r-c)k_p}{k_p(1-\eta)^{1-N}} = c + \frac{(r-c)(1-\eta)^{N-1}k_p}{k_p} > \underline{p}^p,$$

where the last inequality follows from (8) and the fact that $\underline{p}^b < \underline{p}^p$ implies $(1 - H_{[\underline{p}^b,\underline{p}^p]}(\underline{p}^p)) > 0$. However, the last result gives the contradiction $\underline{p}^b > \underline{p}^p$. Therefore, there cannot be a solution to (8) with $\underline{p}^b < \underline{p}^p$ if (10) holds.

$\square$

Proof of Lemma 7. If $\underline{p}^p < \underline{p}^b$, then $H(p) = 0$ for all $p < \underline{p}^b$. This simplifies equation (5) which has to be satisfied for all prices $p \in [\underline{p}^p, \underline{p}^b]$ that are in the support of G:

$$(p-c)\left[k_p\left(1-\eta G(p)\right)^{N-1} + \mu\left(1-\eta G(p)\right)^{N-1}\right] = (1-\eta)^{N-1}(r-c)k_p.$$

Solving for G gives

$$G_{[\underline{p}^p,\underline{p}^b]}(p) = \frac{1}{\eta} - \frac{1}{\eta}\left(\frac{(1-\eta)^{N-1}(r-c)k_p}{(p-c)}\right)^{1/(N-1)}.$$

The lower bound of the support of G is determined by $G_{[\underline{p}^p,\underline{p}^b]}(p) = 0$. This means

$$\underline{p}^p = c + (1-\eta)^{N-1}(r-c)k_p.$$

The lower bound of the support of H is the price p^b for which $H(\underline{p}^b) = 0$ and equation (4) still hold. Since $\underline{p}^p < \underline{p}^b$ it holds that $G(\underline{p}^b) = G_{[\underline{p}^p,\underline{p}^b]}(\underline{p}^b) > 0$. Setting $H = 0$ in equation (5), plugging in $G_{[\underline{p}^b,\underline{p}^p]}$ for G and solving for p gives

$$\underline{p}^b = c + \frac{(r-c)k_b}{k_b + \mu(1-\eta G_{[\underline{p}^p,\underline{p}^b]}(\underline{p}^b))^N}.$$

In order to see the different roles of conditions (14) and (15), I prove the existence and uniqueness of the solution to (13) separately. Suppose now that condition (14) holds. To show that equation (13) indeed has a solution I define the continuous function

$$\varphi^H(p) := (p-c)(k_b + \mu(1-\eta G_{[\underline{p}^p,\underline{p}^b]}(p))^N) - (r-c)k_b.$$

Then

$$\begin{aligned}
\varphi^H(\underline{p}^p) &= (\underline{p}^p - c)(k_b + \mu(1-\eta G_{[\underline{p}^p,\underline{p}^b]}(\underline{p}^p))^N) - (r-c)k_b \\
&= (1-\eta)^{N-1}(r-c)k_p(k_b+\mu) - (r-c)k_b \\
&< 0.
\end{aligned}$$

Using $k_p = 1-\mu$ and $k_b = k_p\frac{\gamma}{N}(1-\eta)^N$, one can check by simplifying some terms that

$$(1-\eta)^{N-1}(r-c)k_p(k_b+\mu) < (r-c)k_b \Leftrightarrow \frac{\gamma(1-\eta)-\gamma(1-\eta)^N}{N-\gamma(1-\eta)^N} > \mu.$$

On the other hand one gets

$$\begin{aligned}
\varphi^H(r) &= (r-c)(k_b + \mu(1-\eta G_{[\underline{p^p},\underline{p^b}]}(r))^N) - (r-c)k_b \\
&= (r-c)(k_b + \mu(1-\eta)^N) - (r-c)k_b \\
&> 0.
\end{aligned}$$

Since φ^H is continuous, there is at least one $\underline{p^b} \in (\underline{p^p}, r)$ such that $\varphi^H(\underline{p^b}) = 0$. Since $\underline{p^b}$ solves (8) if and only if $\varphi^H(\underline{p^b}) = 0$, the existence of a solution to (13) greater than $\underline{p^p}$ is shown.

I now show that the solution to (13) is unique given that (15) holds in addition to (14). I first define the function $X_H : [\underline{p^p}, r] \to [0,1]$ by $X_H(p) = \frac{(1-\eta)^{N-1}(r-c)k_p}{p-c}$. Note that $X_H(\underline{p^p}) = 1$ and $X_H(r) = (1-\eta)^{N-1}k_p$. Since the function X_H is monotone decreasing on its domain, it is invertible. Let $P_H = X_H^{-1}$. Then $P_H(x) = \frac{(1-\eta)^{N-1}(r-c)k_p}{x} + c$.

The equation $\varphi^H(p) = 0$ has a unique solution $\underline{p^b} \in (\underline{p^p}, r)$ if and only if

$$\varphi^H \circ P_H(x) = \frac{(1-\eta)^{N-1}(r-c)k_p}{x}\left(k_b + \mu x^{N/(N-1)}\right) - (r-c)k_b = 0$$

has a unique solution $x^* \in ((1-\eta)^{N-1}k_p, 1)$. I will show that there is a unique solution in $x^* \in [0,1]$ if (15) holds, i.e. x^* is the only solution in an even larger interval. Using the fact that $k_b = (1-\eta)^N \frac{\gamma}{N}$, I rewrite $\varphi^H \circ P_H(x) = 0$ into a fixed-point equation:

$$x = (1-\eta)^{N-1}k_p + \mu\frac{N}{\gamma(1-\eta)}x^{N/(N-1)} =: \kappa_H(x).$$

This equation has a unique solution if the therein defined function κ_H is a contraction. Let $x \in [0,1]$. Then, clearly $\kappa_H(x) > 0$. Furthermore,

$$\kappa_H(x) < (1-\eta)^{N-1}k_p + \mu\frac{N}{\gamma(1-\eta)} < 1,$$

since $(1-\eta)^{N-1}k_p + \mu\frac{N}{\gamma(1-\eta)} < 1$ is equivalent to (14). This can be checked by using $k_p = 1-\mu$ and simplifying terms. Thus, the range of κ_H is in its domain. Calculating the derivative gives

$$\kappa_H'(x) = \mu\frac{N^2}{(N-1)\gamma(1-\eta)}x^{1/(N-1)} \le \mu\frac{N^2}{(N-1)\gamma(1-\eta)} < 1,$$

where the last inequality follows from (15). Also, the derivative is clearly non-negative. Thus, κ_H is a contraction and by the Banach fixed-point theorem $\kappa_H(x) = x$ has a unique solution in $[0, 1]$. By the above arguments, this proves uniqueness of the solution to (13).

Lastly, it remains to show that there is no solution $\underline{p^b}$ to (13) with $\underline{p^b} > \underline{p^p}$ if (16) holds. Suppose now that (16) holds and $\underline{p^b} > \underline{p^p}$. The following inequality is equivalent to (16) as can be checked by using $k_p = 1 - \mu$ and simplifying:

$$k_p(1-\eta)^{N-1} + \frac{N}{\gamma}(1-\eta)^{N-1}\mu > 1.$$

Therefore,

$$\underline{p^p} = c + \frac{(1-\eta)^{N-1}(r-c)k_p}{1} > c + \frac{(1-\eta)^{N-1}(r-c)k_p}{k_p(1-\eta)^{N-1} + \frac{N}{\gamma}(1-\eta)^{N-1}\mu} = c + \frac{(r-c)k_b}{k_b + \mu(1-\eta)^N} > \underline{p^b},$$

where the last inequality follows from (13) and the fact that $\underline{p^b} < \underline{p^p}$ implies $(1 - \eta G_{[\underline{p^p},\underline{p^b}]}(\underline{p^b})) > (1-\eta)$. However, the last result gives the contradiction $\underline{p^p} > \underline{p^b}$. Therefore, a solution to (13) with $\underline{p^p} < \underline{p^b}$ is not possible if (16) is true. $\square$

Proof of Lemma 8. First I introduce the new variables $X = 1 - H(p)$ and $Y = 1 - \eta G(p)$. The equations (4) and (5) can be rewritten as

$$(p-c)\left[k_b + \mu X^{N-1} Y^N\right] = (r-c)k_b \tag{29}$$

and

$$(p-c)\left[k_p Y^{N-1} + \mu X^N Y^{N-1}\right] = (1-\eta)^{N-1}(r-c)k_p. \tag{30}$$

Equation (29) can be used to express X in terms of Y:

$$X = \left(\frac{(r-p)k_b}{(p-c)\mu Y^N}\right)^{1/(N-1)}. \tag{31}$$

Plugging (31) into (30) gives

$$(p-c)\left[k_p Y^{N-1} + \mu\left(\frac{(r-p)k_b}{(p-c)\mu Y^N}\right)^{N/(N-1)} Y^{N-1}\right] = (1-\eta)^{N-1}(r-c)k_p. \tag{32}$$

From $p \geq \max(\underline{p}^p, \underline{p}^b)$ and $\underline{p}^b \geq c + \frac{(r-c)k_b}{k_b+\mu}$ it follows that $\frac{(r-p)k_b}{(p-c)\mu} \leq 1$. Using this result and plugging $Y = 1-\eta$ into (32) gives for $p < r$ that

$$
\begin{aligned}
& (p-c)\left[k_p Y^{N-1} + \mu \left(\tfrac{(r-p)k_b}{(p-c)\mu Y^N}\right)^{N/(N-1)} Y^{N-1}\right] \\
= \quad & (p-c)\left[k_p(1-\eta)^{N-1} + \mu \left(\tfrac{(r-p)k_b}{(p-c)\mu}\right)^{N/(N-1)} (1-\eta)^{(-2N+1)/(N-1)}\right] \\
\leq \quad & (p-c)k_p(1-\eta)^{N-1} + (r-p)k_b(1-\eta)^{(-2N+1)/(N-1)} \\
= \quad & (p-c)k_p(1-\eta)^{N-1} + (r-p)k_p(1-\eta)^{N-1}\tfrac{\gamma}{N}(1-\eta)^{-N/(N-1)} \\
< \quad & (1-\eta)^{N-1}(r-c)k_p,
\end{aligned}
$$

where the last inequality follows from (17).

Furthermore, the left-hand side of equation (32) goes to infinity as $Y \to \infty$. Since the left-hand side of equation (32) is continuous in Y, there is for every $p \in [\max(\underline{p}^p, \underline{p}^b), r)$ a $Y^* \in (1-\eta, \infty)$, such that (32) holds. Equation (31) gives then the appropriate $X^* \in (0, (1-\eta)^{-N/(N-1)}$, such that X^* and Y^* solve (29) and (30) for the particular p. Thus, existence of solutions to (29) and (30) is shown.

To show that these solutions are unique, I use the implicit function theorem. Under conditions, the implicit function theorem states the following. For each considered p^* and X^* and Y^* that solve (29) and (30) there is an open interval containing p^*, such that there are unique functions X and Y. Furthermore, $(p, X(p), Y(p))$ solve 29) and (30) for all p in that open interval.

It has already been shown that there exist appropriate X^* and Y^* for each $p^* \in [\max(\underline{p}^p, \underline{p}^b), r)$ such that (29) and (30) hold. The implicit function theorem now ensures that unique functions X and Y exist in an open interval around each of those p^*. These open intervals must overlap partly, since there is such an open interval for each $p \in [\max(\underline{p}^p, \underline{p}^b), r)$. Furthermore, the union of those intervals must contain $[\max(\underline{p}^p, \underline{p}^b), r)$. This implies that there exist unique functions X and Y such that $(p, X(p), Y(p))$ for $p \in [\max(\underline{p}^p, \underline{p}^b), r)$ solve (29) and (30).

Differentiating the left-hand sides of equations (29) and (30) with respect to X and Y gives the following Jacobian:

$$
J := (p-c)\begin{pmatrix} \mu(N-1)X^{N-2}Y^N & \mu N X^{N-1}Y^{N-1} \\ \mu N X^{N-1}Y^{N-1} & (N-1)Y^{N-2}(k_p + \mu X^N) \end{pmatrix}.
$$

Using $k_p = 1 - \mu$, its determinant is given by

$$\det J = (p-c)^2(\mu(N-1)^2 X^{N-2} Y^{2N-2}(k_p + \mu X^N) - \mu^2 N^2 X^{2N-2} Y^{2N-2}).$$

The condition that has to hold to apply the implicit function theorem is that this determinant must not be zero for $p \in [\max(\underline{p}^p, \underline{p}^b), r)$ and the respective X and Y that solve (29) and (30). It holds that $\det J > 0$ exactly if

$$(N-1)^2(1 - \mu + \mu X^N) - \mu N^2 X^N > 0. \tag{33}$$

Since $X < (1-\eta)^{-N/(N-1)}$ the last inequality holds if

$$\mu < \frac{(N-1)^2}{(2N-1)(1-\eta)^{-\frac{N^2}{N-1}} + (N-1)^2}.$$

Thus, the unique functions X and Y exist if (18) holds.

One can determine a different bound on X by using $p > \underline{p}^p \geq c + (1-\eta)^{N-1}(r-c)k_p$ and $Y > 1 - \eta$. Then equation (31) gives

$$X \leq \left(\frac{\gamma - \gamma(1-\mu)(1-\eta)^{N-1}}{\mu N (1-\eta)^{N-1}} \right)^{1/(N-1)} < 1,$$

if $\mu > \frac{\gamma(1-\eta) - \gamma(1-\eta)^N}{N(1-\eta)^N - \gamma(1-\eta)^N}$. The last condition is stated on the left-hand side in (19). Since $0 < X < 1$, it holds that $X^N < X^{N-1}$ and therefore (since $N > 1$):

$$(N-1)^2(1-\mu+\mu X^N) - \mu N^2 X^N > (N-1)^2 \left(1 - \mu + \frac{\gamma - \gamma(1-\mu)(1-\eta)^{N-1}}{N(1-\eta)^{N-1}} \right) - N \frac{\gamma - \gamma(1-\mu)(1-\eta)^{N-1}}{(1-\eta)^{N-1}}.$$

The expression on the right hand side in the last inequality is greater than zero exactly when

$$\mu < \frac{(N(N-1)^2 + \gamma(2N-1))(1-\eta)^N - (2N-1)(1-\eta)\gamma}{(N(N-1)^2 + \gamma(2N-1))(1-\eta)^N},$$

as stated on the right-hand side in (19). Thus, condition (33) holds and unique functions X and Y also exist if (19) holds.

So far it has been shown that X and Y are unique and continuous (since differentiable) on the half-open interval $[\max(\underline{p}^p, \underline{p}^b), r)$. In fact, they are also continuous on the closed interval $[\max(\underline{p}^p, \underline{p}^b), r]$ as can be seen as follows. For $p = r$, equations (29) and (30) are uniquely solved by choosing $X = 0$ and $Y = 1 - \eta$. Now let $p < r$ and $p \to r$. Then it follows from (29) that $X^{N-1}Y^N \to 0$, i.e. either $Y \to 0$ or $Y \to 0$ or both. If $Y \to 0$, then (30) would not hold, therefore $X \to 0$. Now it follows from (30) that $Y \to 1 - \eta$, i.e. X and Y are continuous on $[\max(\underline{p}^p, \underline{p}^b), r]$.

I will now show that X and Y are monotone decreasing if condition (20) holds. If X and Y are strictly decreasing, then H and G are strictly increasing. Monotonicity of X and Y is again shown by using the implicit function theorem. Since the unique existence of X and Y is under the given conditions shown, the derivative of X and Y is given by

$$\begin{pmatrix} X'(p) \\ Y'(p) \end{pmatrix} = -J^{-1}(X, Y, p) f_p(X, Y, p),$$

where J is the above defined Jacobian matrix and f_p is the derivative of the left-hand sides of equations (29) and (30) with respect to p. This means

$$f_p(X, Y, p) = \begin{pmatrix} Y^N \mu X^{N-1} + \frac{\gamma(1-\eta)^N(1-\mu)}{N} \\ (1-\mu)Y^{N-1} + X^N \mu Y^{N-1} \end{pmatrix},$$

where I used again that $k_p = 1 - \mu$ and $k_b = \gamma(1-\eta)^N(1-\mu)/N$. This gives

$$\begin{pmatrix} X'(p) \\ Y'(p) \end{pmatrix} = -(p-c)^{-2} \begin{pmatrix} \frac{X^{1-N}Y^{-N}\left(\left(X^N-1\right)\mu+1\right)\left(NX^N\mu Y^N+(N-1)X\gamma(1-\eta)^N(\mu-1)\right)}{N\mu\left(-\mu X^N+N^2(\mu-1)+\mu+2N\left(\left(X^N-1\right)\mu+1\right)-1\right)} \\ \frac{Y^{1-N}\left(Y^N\left(N(\mu-1)+\left(X^N-1\right)\mu+1\right)-X\gamma(1-\eta)^N(\mu-1)\right)}{-\mu X^N+N^2(\mu-1)+\mu+2N\left(\left(X^N-1\right)\mu+1\right)-1} \end{pmatrix}.$$

After some algebraic manipulation one gets, since $p > c$, that

$$X'(p) < 0 \Leftrightarrow \eta < 1 - \left(\frac{\mu N X^{N-1} Y^N}{\gamma(1-\mu)(N-1)}\right)^{1/N} \tag{34}$$

and

$$Y'(p) < 0 \Leftrightarrow \eta > 1 - \left(\frac{Y^N\left((1-\mu)(N-1) - \mu X^N\right)}{\gamma(1-\mu)X}\right)^{1/N}. \tag{35}$$

It has already been stated that $X(r) = 0$ and $Y(r) = 1 - \eta$. If X and Y are in fact strictly monotone decreasing then by definition of $\underline{p}^p$ and $\underline{p}^b$ it must be the case that $X(p) \in (0, 1]$ and $Y(p) \in (1 - \eta, 1]$ for every $p \in [\max(\underline{p}^p, \underline{p}^b), r)$.

Using these bounds on X and Y and condition (21) which ensures that the term in the parentheses is positive, the restriction on η in (35) holds if

$$\eta > 1 - \left(\frac{(1-\eta)^N((1-\mu)(N-1)-\mu)}{\gamma(1-\mu)}\right)^{1/N} \Leftrightarrow (1-\eta) < (1-\eta)\left(\frac{(1-\mu)(N-1)-\mu}{\gamma(1-\mu)}\right)^{1/N},$$

which always holds if (21) holds. Thus, Y is strictly monotone decreasing given that (21) holds.

To examine when the condition stated in (34) holds, one first notes that by (31) it is the case that $X^{N-1}Y^N = \frac{(r-p)k_b}{(p-c)\mu}$. Using the fact that $p \geq \underline{p}^p \geq c + (1-\eta)^{N-1}(r-c)k_p$ one gets

$$X^{N-1}Y^N \leq \frac{\gamma(1-\mu) - (1-\eta)^N\gamma(1-\mu)}{N\mu}.$$

Thus, the condition stated in (34) holds if

$$\eta < 1 - \left(\frac{(1-\eta)-(1-\eta)^N}{N-1}\right)^{1/N} \Leftrightarrow N > (1-\eta)^{1-N}.$$

Therefore, (20) ensures that also X is monotone decreasing.

All properties of X and Y proved here directly translate into the equivalent properties of H and G, with the only difference that the latter are by construction of X and Y monotone increasing.

□

Proof of Lemma 9. Under the stated conditions, a fixed γ and a fixed r there are by Lemma 8, Lemma 6 and Lemma 7 two unique functions G and H for each regime that satisfy equations (4) and (5) on their support.

Now, I show that at least one solution r to equation (23) exists for a fixed γ.

Define

$$z := \left(\frac{(r-p)\gamma(1-\eta)^N(1-\mu)}{N\mu(p-c)(1-\eta G(p))^N}\right)^{1/(N-1)}.$$

Then

$$p = \frac{r\gamma(1-\mu)(1-\eta)^N + cz^{N-1}N\mu(1-\eta G(p))^N}{N\mu(1-\eta G(p))^N z^{N-1} + \gamma(1-\mu)(1-\eta)^N}.$$

Substituting into (23) gives

$$r = s + \int_0^1 \frac{r\gamma(1-\mu)(1-\eta)^N + cz^{N-1}N\mu(1-\eta G(p(z)))^N}{N\mu(1-\eta G(p(z)))^N z^{N-1} + \gamma(1-\mu)(1-\eta)^N}\,\mathrm{d}z,$$

where I used an expression for H which comes from equation (31). Defining

$$\alpha(\gamma, r) := \int_0^1 \left(\frac{N\mu(1-\eta G(p(z)))^N z^{N-1}}{\gamma(1-\mu)(1-\eta)^N} + 1\right)^{-1} \mathrm{d}z$$

(G still depends on r), one can rewrite the last expression as

$$r = c + \frac{s}{1-\alpha(\gamma, r)}. \tag{36}$$

Therefore, solution to (23) exists if there is a solution r to

$$(1-\alpha(\gamma, r))(r-c) = s$$

for some fixed γ. The left-hand side of the previous equation is continuous and goes to zero as $r \to c$, since $\alpha(\gamma, r) \in (0,1)$ for any $r > c$ and $\gamma \in (0,1]$. As $r \to \infty$, the left-hand side of the previous equation goes to infinity, since

$$\alpha(\gamma, r) \leq \overline{\alpha}(\gamma) := \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma(1-\mu)} + 1\right)^{-1} \mathrm{d}z < 1$$

for any $\gamma \in (0,1]$. Thus, since $s > 0$, there is at least one solution r to (23) for a fixed γ.

In order to show that the solution r to equation (23) is unique, I show that $\frac{\partial}{\partial r}\int_{\underline{p^b}}^{r} p\,\mathrm{d}H(p) < 1$. Then $\varphi_r(r,\gamma) := \int_{\underline{p^b}}^{r} p\,\mathrm{d}H(p) - r$ defines a decreasing function that is monotone in r and any solution to $\varphi_r(r,\gamma) = 0$ is unique. I rewrite the integral in (23) using the change of variables $X = 1 - H(p)$, i.e $\int_{\underline{p^b}}^{r} p\,\mathrm{d}H(p) = \int_0^1 p(X)\,\mathrm{d}X$. In the last equation, $p(X)$ is the price that corresponds to a specific value of X. This price is uniquely defined, since X (as a function of p) is invertible if H is monotone increasing.

If $\frac{\partial p(X)}{\partial r} < 1$ for all $X \in (0,1)$, then $\frac{\partial}{\partial r}\int_0^1 p(X)\,\mathrm{d}X < 0$ by the Leibnitz integral rule. In the following, I use $Y := 1 - G$ and $Y(X) := 1 - G(p(X))$. I consider two cases. First, let $\underline{p^b} < \underline{p^p}$. Setting $Y = 1$ in equation (31) gives $p(X) = \frac{rk_b + c\mu X^{N-1}}{\mu X^{N-1} + k_b}$ for $p \in [\underline{p^b}, \underline{p^p}]$. Taking the derivative gives $\frac{\partial}{\partial r}p(X) < 1$. Second, to complete the proof, I consider the case in which either $\underline{p^p} < \underline{p^b}$ or $p \in [\underline{p^p}, r]$. Just rearranging equation (31) gives $p(X) = \frac{rk_b + c\mu X^{N-1}Y(X)^N}{\mu X^{N-1}Y(X)^N + k_b}$. Since Y is a function of X and is also affected by r, calculating the derivative of p with respect to r is trickier in this case. A way to take care of the joint dynamics of p, r and Y is to apply the implicit function theorem once more. I view p and Y as implicit functions of X and r and then calculate the derivative of p with respect to r for a given value of X, exactly as I did before in the case in which $Y = 1$.

I first calculate the derivatives of the left-hand sides of equations (29) and (30) with respect to p and Y as shown in the Jacobian M.

$$M := \begin{pmatrix} Y^N \mu X^{N-1} + \frac{\gamma(1-\eta)^N(1-\mu)}{N} & N(p-c)X^{N-1}Y^{N-1}\mu \\ (1-\mu)Y^{N-1} + X^N \mu Y^{N-1} & (N-1)(p-c)Y^{N-2}\left(\mu X^N - \mu + 1\right) \end{pmatrix}.$$

calculating the determinant

$$\det M = \frac{(c-p)Y^{N-2}\left(\mu\left(X^N - 1\right) + 1\right)\left(\mu N X^N Y^N + \gamma(\mu-1)(N-1)X(1-\eta)^N\right)}{NX}$$

and rearranging terms, one gets that $\det M > 0 \Leftrightarrow \mu < \frac{\gamma(N-1)X(1-\eta)^N}{NX^NY^N + \gamma NX(1-\eta)^N - \gamma X(1-\eta)^N}$. Since one knows from (29) that $X^{N-1}Y^N = \frac{(r-p)k_b}{(p-c)\mu}$ one can substitute this into the previous inequality, solve for μ and get $\det M > 0 \Leftrightarrow \mu < 1$. Thus, the determinant of M is always nonzero, since $\mu < 1$. The implicit function therefore exists.

The derivative of the left-hand sides minus the right-hand sides of equations (29) and (30) with respect to r are give by f_r defined as

$$f_r := \begin{pmatrix} -\frac{\gamma(1-\eta)^N(1-\mu)}{N} \\ -(1-\eta)^{N-1}(1-\mu) \end{pmatrix}.$$

Then, the derivatives of p and Y with respect to r are given by $-M^{-1}f_r$. The first row of the latter vector expresses the derivative of p as an implicit function of X with respect to r.

$$\frac{\partial p(X)}{\partial r} = -\frac{(\mu-1)(1-\eta)^{N-1}\left(\mu N^2 Y X^N + \gamma(\eta-1)\mu(N-1)X^{N+1} - \gamma(\eta-1)(\mu-1)(N-1)X\right)}{\left(\mu\left(X^N-1\right)+1\right)\left(\mu N X^N Y^N + \gamma(\mu-1)(N-1)X(1-\eta)^N\right)}. \quad (37)$$

One can verify that this derivative is defined whenever $\det M \neq 0$. Furthermore, one can verify that $\frac{\partial p(X)}{\partial r} \to 1$ as $X \to 0$. In order to show that $\frac{\partial p(X)}{\partial r} < 1$ for all $X \in (0, 1]$ I show that the numerator in (37) increases faster than the denominator. The denominator is always negative as can be seen by comparing it to the determinant of M, which is positive. A faster increasing numerator and a negative denominator implies a decreasing overall fraction. Since numerator and denominator are equal at $X = 0$, one has $\frac{\partial p(0)}{\partial r} = 1$.

Thus, the claim $\frac{\partial p(X)}{\partial r} < 1$ for all $X \in (0,1]$ follows.

The difference Δ between the numerator and denominator can be expressed as

$$\Delta = \mu N X^{N-1}\left((1-\mu)NY(1-\eta)^{N-1} - Y^N\left(1-\mu\left(1-X^N\right)\right)\right).$$

Differentiating with respect to X gives

$$\begin{aligned}\frac{\partial \Delta}{\partial X} &= \mu(N-1)NX^{N-2}\left((1-\mu)NY(1-\eta)^{N-1} - Y^N\left(1-\mu\left(1-X^N\right)\right)\right) - \mu^2 N^2 X^{2N-2} Y^N \\ &\geq \mu(N-1)NYX^{N-2}\left((1-\mu)N(1-\eta)^{N-1} - \frac{\mu N}{N-1} - 1\right),\end{aligned}$$

where the inequality holds since $X, Y \leq 1$. The last expression is greater than zero if (22) holds.

It is thus shown that the derivative of the expected price in the bilateral market with respect to r is less than one. By the above arguments, any solution to (23) must be unique.

I will now prove that the solution r to (23) is strictly monotone increasing in γ. The first step in the following argument is to show that the expected price in the bilateral market is increasing in γ. I consider again two cases. First, let $\underline{p}^b < \underline{p}^p$. Setting $Y = 1$ in equation (31) gives $X(p) = \left(\frac{(r-p)k_b}{(p-c)\mu}\right)^{1/(N-1)}$ for $p \in [\underline{p}^b, \underline{p}^p]$. Taking the derivative gives $\frac{\partial}{\partial \gamma}X(p) > 0 \Leftrightarrow \frac{\partial}{\partial \gamma}H(p) < 0$. Second, to complete the proof, I consider the case in which either $\underline{p}^p < \underline{p}^b$ or $p \in [\underline{p}^b < r]$. Define

$$f_g := \begin{pmatrix} \frac{(p-r)(1-\eta)^N(1-\mu)}{N} \\ 0 \end{pmatrix}$$

as the derivatives of the left-hand sides minus the right-hand sides of equations (29) and (30) with respect to γ. Viewing X and Y (as defined above) as implicit functions of γ I can calculate their respective derivatives with respect to γ by $-J^{-1} \cdot f_g$, where J is defined as in the proof of lemma 8. This gives the following derivative of X with respect to γ:

$$\frac{\partial X(p)}{\partial \gamma} = \frac{1}{\det J}\frac{(1-\mu)(N-1)(p-c)(1-\eta)^N Y^{N-2}(r-p)\left(\mu\left(X^N-1\right)+1\right)}{N},$$

with $X, Y \in [0,1]$. Under the conditions stated in Lemma 8, one has $\det J > 0$ (as shown in the proof of Lemma 8). Thus, $\frac{\partial X}{\partial \gamma} > 0$, since the second fraction in the above expression is positive. This implies $\frac{\partial H(p)}{\partial \gamma} < 0$. Thus, by first order stochastic dominance it must hold for any two γ_1 and γ_2 with $0 < \gamma_1 < \gamma_2 \leq 1$

and a fixed r_1 that

$$\int p \, \mathrm{d}H_{\gamma_1,r_1}(p) < \int p \, \mathrm{d}H_{\gamma_2,r_1}(p),$$

where the subscripts of H indicate the γ and r based on which the dealers formed their price distributions. It has thus been shown that the expected price is decreasing in γ. Suppose now that r_1 solves $r_1 = \int p \, \mathrm{d}H_{\gamma_1,r_1}(p)$. Then $\frac{\partial}{\partial r} \int p \, \mathrm{d}H(p) < 1$ implies that the r_2 that solves $r_2 = \int p \, \mathrm{d}H_{\gamma_2,r_2}(p)$ must satisfy $r_2 > r_1$. Thus, it is shown that the reservation price that solves (31) is strictly monotone increasing in γ.

The continuity of the solution r to (23) in γ follows from equation (36), since α is a continuous function because it is an integral over a finite interval with a continuous and bounded integrand.

In regime 1, it is possible for slow traders to let $\gamma \to 0$ without violating condition (9), given that (9) holds for some $\gamma > 0$. I will now show that $r \to c + s$ as $\gamma \to 0$.

Since $G(p) \in [1-\eta, 1]$ one has $\alpha(\gamma) \geq \underline{\alpha}(\gamma) := \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma(1-\mu)(1-\eta)^N} + 1 \right)^{-1} \mathrm{d}z$. This gives

$$\lim_{\gamma \to 0} r \geq c + \lim_{\gamma \to 0} \frac{s}{1 - \underline{\alpha}(\gamma)} = s + c, \tag{38}$$

since $\lim_{\gamma \to 0} \underline{\alpha}(\gamma) = 0$. On the other hand, one has $\alpha(\gamma) \leq \overline{\alpha}(\gamma) := \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma(1-\mu)} + 1 \right)^{-1} \mathrm{d}z$. This gives

$$\lim_{\gamma \to 0} r \leq c + \lim_{\gamma \to 0} \frac{s}{1 - \overline{\alpha}(\gamma)} = s + c, \tag{39}$$

since $\lim_{\gamma \to 0} \overline{\alpha}(\gamma) = 0$. Taking (39) and (39) together, one gets $\lim_{\gamma \to 0} r = c + s$. This completes the proof of the lemma.

□

Proof of Proposition 1. The fast traders' strategy is clearly optimal, since fast traders do not pay any costs when searching, while always enjoying the benefit of potentially finding a better price. Lemma 1 states that it is optimal for slow investors to start searching on the platform. If $r = v$, then slow traders are indifferent between continuing to search in the bilateral market or terminating the search if they didn't find a satisfactory offer on the platform. However, Lemma 9 implies that there is a unique γ such that $r = v$. If $r < v$, slow traders have a strict preference to continue to search in the bilateral market and $\gamma = 1$ is the optimal choice. Thus, γ is uniquely determined. According to Lemma 6 and Lemma 8 unique and monotone increasing functions H and G exist on the specified intervals. It remains to show that dealers cannot improve their payoff by quoting a price outside the support of the respective distribution or by changing the probabilities with which they quote a specific price. First, just changing the probabilities with which a price in the support is quoted has no effect on the dealers' profits, since by construction of H and G in Lemma 3, the profit is the same for each price on the respective support. Quoting above r gives no profit, while for other prices positive profit is possible. Thus, if one can show that the profit of quoting a price $p < \underline{p}^p$ on the platform gives a profit less than quoting a price $p \in [\underline{p}^p, r]$ and quoting a price $p < \underline{p}^b$ in the bilateral market gives a profit less than quoting a price $p \in [\underline{p}^p, r]$, optimality of the dealers' strategies is shown.

First, it is obvious that the expected profit from quoting $p < \underline{p}^b$ in the bilateral market must be lower than the profit from quoting $p = \underline{p}^b$, since expected turnover stays the same while the price decreases. It follows that quoting $p < \underline{p}^p$ in the bilateral market gives an expected profit less than any profit resulting from quoting a price in the support of H.

Second, I take κ_G from the proof of Lemma 6. It has been shown that there is a unique x^* such that $x^* = \kappa_G(x^*)$. Now $\kappa'_G < 1$ implies

$$\frac{(r-c)k_b}{\mu x + k_b}\left(k_p + \mu x^{N/(N-1)}\right) - (1-\eta)^{N-1}(r-c)k_p < 0$$

for $x > x^*$. Plugging back in the appropriate definition of p gives

$$(p-c)\left[k_p + \mu(1-H(p))^N\right] < (1-\eta)^{N-1}(r-c)k_p$$

with $p \in [\underline{p}^b, \underline{p}^p)$. Clearly, quoting a price below $\underline{p}^b$ yields even lower profits, since turnover cannot increase any further. Thus, optimality of the dealers' strategies is shown. Both dealers and traders' strategies are uniquely determined if slow traders start the search on the platform and dealers' follows symmetric strategies with $\underline{p}^b < \underline{p}^p$. Therefore, there cannot be another equilibrium with these properties. This concludes the proof.

□

Proof of Proposition 2. Analogously to Proposition 1, this proposition follows from Lemma 1, Lemma 7, Lemma 8 and Lemma 9. It only remains to show that dealers cannot improve their payoff by quoting a price outside the support of the respective distribution or by changing the probabilities with which they quote a specific price. First, just changing the probabilities with which a price in the support is quoted has no effect on the dealer profits, since by construction of H and G in Lemma 3, the profit is the same for each price on the respective support. Quoting above r gives no profit, while for other prices positive profit is possible. Thus, if one can show that the profit of quoting a price $p < \underline{p}^b$ in the bilateral market gives a profit less than quoting a price $p \in [\underline{p}^b, r]$ and quoting a price $p < \underline{p}^p$ on the platform gives a profit less than quoting a price $p \in [\underline{p}^p, r]$, optimality of the dealers' strategies is shown.

First, it is obvious that the expected profit from quoting $p < \underline{p}^p$ on the platform must be less than the profit from quoting $p = \underline{p}^p$, since expected turnover stays the same while the price decreases. It follows that quoting $p < \underline{p}^p$ on the platform gives an expected profit less than any profit resulting from quoting a price in the support of G.

Second, I take κ_H from the proof of Lemma 7. It has been shown that there is a unique x^* such that $x^* = \kappa_H(x^*)$. Now $\kappa_H' < 1$ implies

$$\frac{(1-\eta)^{N-1}(r-c)k_p}{x}\left(k_b + \mu x^{N/(N-1)}\right) - (r-c)k_b < 0$$

for $x > x^*$. Plugging back in the appropriate definition of p gives

$$(p-c)\left[k_b + \mu\left(1-\eta G(p)\right)^N\right] < (r-c)k_b$$

with $p \in [\underline{p}^p, \underline{p}^b)$. Clearly, quoting a price below $\underline{p}^p$ yields even lower profits, since turnover cannot increase any further. Thus, optimality of the dealers' strategies is shown. Both dealers and traders' strategies are uniquely determined if slow traders start the search on the platform and dealers' follows symmetric strategies with $\underline{p}^p < \underline{p}^b$. Therefore, there cannot be another equilibrium with these properties. This concludes the proof.

□

Proof of Lemma 10. I first derive the upper bound expressed in (24). Conditional on responding, each dealer on the platform quotes according to the distribution G characterized in Lemma 4. Since each dealer only responds with probability η, the lowest quote conditional on at least one response is distributed according to the distribution F defined by

$$F(p) := \frac{1-(1-\eta G(p))^N}{1-(1-\eta)^N}.$$

Rewriting equation (5), one gets for $p \in [\underline{p}^p, r]$ that

$$\begin{aligned} G(p) &= \frac{1}{\eta} - \frac{1}{\eta}\left(\frac{(1-\eta)^{N-1} \cdot (r-c)}{(p-c)} - \frac{\mu}{k_p}(1-H(p))^N(1-\eta G(p))^{N-1}\right)^{1/(N-1)} \\ &\geq \max\left[0, \frac{1}{\eta} - \frac{1}{\eta}\left(\frac{(1-\eta)^{N-1} \cdot (r-c)}{(p-c)}\right)^{1/(N-1)}\right] \\ &=: \underline{G}(p) \end{aligned}$$

and therefore

$$\begin{aligned} F(p) &\geq \underline{F}(p) \\ &:= \frac{1-(1-\eta\underline{G}(p))^N}{1-(1-\eta)^N} \\ &= \max\left[0, \frac{1-\left(\frac{(1-\eta)^{N-1}\cdot(r-c)}{(p-c)}\right)^{N/(N-1)}}{1-(1-\eta)^N}\right]. \end{aligned}$$

It follows that $\int_{\underline{p}^p}^r p\,\mathrm{d}F(p) \leq \int_{\underline{p}^p}^r p\,\mathrm{d}\underline{F}(p)$ by first-order stochastic dominance. Performing the change of variables $p = c + \frac{(r-c)(1-\eta)^{N-1}}{(1-(1-(1-\eta)^N)z)^{(N-1)/N}}$, one gets

$$\begin{aligned} \int_{\underline{p}^p}^r p\,\mathrm{d}\underline{F}(p) &= \int_0^1 \left(c + \frac{(r-c)(1-\eta)^{N-1}}{(1-(1-(1-\eta)^N)z)^{(N-1)/N}}\right)\mathrm{d}z \\ &= c + (r-c)(1-\eta)^{N-1}\frac{N\eta}{1-(1-\eta)^N}. \end{aligned}$$

It follows that $\int_{\underline{p}^p}^r p\,\mathrm{d}F(p) \leq c + (r-c)(1-\eta)^{N-1}\frac{N\eta}{1-(1-\eta)^N}$.

The corresponding lower bound for prices on the platform is derived analogously. Since $H \geq 0$, one gets by rewriting equation (5) for $p \in [\underline{p}^p, r]$ that

$$
\begin{aligned}
G(p) &= \frac{1}{\eta} - \frac{1}{\eta}\left(\frac{(1-\eta)^{N-1}\cdot(r-c)}{(p-c)} - \frac{\mu}{k_p}(1-H(p))^N(1-\eta G(p))^{N-1}\right)^{1/(N-1)} \\
&\leq \min\left\{\max\left[0, \frac{1}{\eta} - \frac{1}{\eta}\left(\frac{(1-\eta)^{N-1}\cdot(r-c)\cdot k_p}{(p-c)}\right)^{1/(N-1)}\right], 1\right\} \\
&=: \overline{G}(p)
\end{aligned}
$$

and therefore

$$
\begin{aligned}
F(p) &\geq \overline{F}(p) \\
&:= \frac{1-(1-\eta\overline{G}(p))^N}{1-(1-\eta)^N}.
\end{aligned}
$$

Performing a change of variables, one gets

$$
\int_{-\infty}^{r} p \,\mathrm{d}F(p) \geq \int_{-\infty}^{r} p \,\mathrm{d}\overline{F}(p) = c + k_p(r-c)(1-\eta)^{N-1}\frac{N\eta}{1-(1-\eta)^N},
$$

from which (25) follows by substituting $k_p = 1-\mu$.

Next, I show that if (26) holds, then $r_p < r_b$ holds as well. Let φ_p be the function defined in the proof of Lemma 1. Then $r_b > r_p \Leftrightarrow \varphi_p(r) > s$, since $r_b = r$, $\varphi_p(r_p) = s$ and φ_p is strictly monotone increasing. Thus, $r_b > r_p$ is equivalent to

$$
s < \varphi_p(r) = (1-(1-\eta)^N)\int_{-\infty}^{r}(r-x)\,\mathrm{d}F(x) = (1-(1-\eta)^N)(r-\mathbb{E}(q)),
$$

since the dealers do not give quotes above r on the platform.

$\square$

Proof of Lemma 11. Define $\alpha(\gamma, r)$ and $\overline{\alpha}(\gamma)$ as in the proof of Lemma 9. It is shown in Duffie et al. (2016) that the reservation price in the PBM satisfies $r^{PBM}(\gamma^{PBM}) = c + \frac{s}{1-\overline{\alpha}(\gamma^{PBM})}$. Using equation (36) in the proof of Lemma 9 one gets

$$
r(\gamma) = c + \frac{s}{1-\alpha(\gamma, r)} < c + \frac{s}{1-\overline{\alpha}(\gamma)} = r^{PBM}(\gamma), \tag{40}
$$

since $\alpha(\gamma, r) < \overline{\alpha}(\gamma)$ for any $r > c + s$. The latter holds, since by strict monotonicity $G(p) < 1$ for at least some subset of $[\underline{p}^b, r]$ with positive measure. Thus, holding the probability of entry of slow traders fixed, the reservation price of slow traders is lower in the HM.

Let $\gamma^{PBM} = 1$. Then

$$v \geq r^{PBM}(1) > r(1)$$

and by the characterization of the equilibria in the HM from Propostions 1 and 2 it follows that $\gamma = 1$ is the unique equilibrium of entry of slow traders in the HM.

Let $\gamma^{PBM} < 1$. Then

$$v = r^{PBM}(\gamma^{PBM}) > r(\gamma^{PBM}).$$

By Propostions 1 and 2 it follows that $\gamma = \gamma^{PBM}$ is not a possible equilibrium probability of entry for slow traders in the HM. Propositions 1 and 2 state that the unique equilibrium of entry γ satisfies

$$\gamma = \begin{cases} 1 & \text{if } r(1) \leq v - s \\ x & \text{else} \end{cases},$$

where x solves $r(x) = v$. It remains to show that $x > \gamma^{PBM}$. The latter can be seen as follows. Lemma 9 states that the reservation price r is strictly monotone increasing in γ. Thus, if $v = r(x) > r(\gamma^{PBM})$, it must be the case that $x > \gamma^{PBM}$. □

Proof of Lemma 12. First, I consider slow traders. I will show that, conditional on having at least one quote on the platform, the best quotes on the platform are on average lower than quotes obtained from a dealer in the HM bilateral market. If prices in the HM bilateral market are on average not higher than in the PBM, it follows that slow traders can always expect a lower price in the HM than in the PBM, since they will buy the asset with positive probability on the platform.

By Propostions 1 and 2, condition (3) holds in the HM equilibrium and therefore $r_p < r_b$. By Lemma 4 and Lemma 1, there are no quotes above r_b on neither trading venue in the HM. Let p_b denote the random price in the HM bilateral market and let q denote the best quote on the platform conditional on at least one response to an RFQ. Equation (23) states $r_b = \mathbb{E}(p_b) + s$. Rewriting equation (2) gives

$$
\begin{aligned}
s &= (1-(1-\eta)^N)\int_{\underline{p}^p}^{r_p}(r_p-p)\,\mathrm{d}F(p) \\
&< (1-(1-\eta)^N)\int_{\underline{p}^p}^{r_b}(r_b-p)\,\mathrm{d}F(p) \\
&= (1-(1-\eta)^N)(\mathbb{E}(p_b)+s-\mathbb{E}(q)).
\end{aligned}
$$

This implies

$$(1-\eta)^n s < (1-(1-\eta)^N)(\mathbb{E}(p_b)-\mathbb{E}(q)).$$

Therefore, it holds that $\mathbb{E}(p_b) > \mathbb{E}(q)$. It remains to show that prices in the bilateral market are on average not higher in the HM than in the PBM.

If $\gamma^{PBM} = 1$ it follows from Lemma 11 that $\gamma = 1$. In the proof of Lemma 11 it was shown that $r(\gamma) < r^{PBM}(\gamma)$ for all $\gamma \in (0,1]$. Therefore, $r^{PBM} > r$. This is equivalent to the claim that $\int p\,\mathrm{d}H^{PBM}(p) > \int p\,\mathrm{d}H(p)$.

If on the other hand $\gamma^{PBM} < 1$ and $\gamma = 1$, it follows from monotonicity of r in γ (stated in Lemma 9) that $r^{PBM} > r$. Thus, the claim holds also in this case.

If $\gamma^{PBM} < 1$ and $\gamma < 1$ then it must be the case that $v = r^{PBM} = r$. Thus, the expected price in the bilateral maket is the same in HM and PBM. The claim that the price in the HM is not higher holds also in this case. By the above argument, it follows that slow traders can always expect a better price in the whole HM than in the PBM.

Now I consider fast traders. In the HM, fast traders choose the lowest price among all quotes on the platform and in the bilateral market. In the PBM, fast traders choose the lowest price among the quotes from all dealers. I will show that the lowest quote in the bilateral market of the HM is on average lower than the lowest quote in the PBM. Then it follows that the lowest quote in the whole HM is also lower than the lowest quote in the PBM.

I will use the above-proved fact that the expected price quoted by one dealer in the bilateral market in the HM is not higher than the expected price a dealer quotes in the PBM. This means $r - c < r^{PBM} - c$. Using the expressions for r and r^{PBM} from equation (40) in the proof of Lemma 11 now gives

$$\frac{s}{1-\alpha(\gamma,r)} \leq \frac{s}{1-\overline{\alpha}(\gamma^{PBM})} \Leftrightarrow \alpha(\gamma,r) \leq \overline{\alpha}(\gamma^{PBM}).$$

Due to different choices entry probabilities in th HM and the PBM, the last inequality is not trivial. Using the respective definitions of $\alpha(\gamma, r)$ and $\overline{\alpha}(\gamma^{PBM})$ from the proof of Lemma 9 gives

$$\int_0^1 \left(\frac{N\mu(1-\eta G(p(z)))^N z^{N-1}}{\gamma(1-\mu)(1-\eta)} + 1 \right)^{-1} dz \leq \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma^{PBM}(1-\mu)(1-\eta)} + 1 \right)^{-1} \mathrm{d}z. \tag{41}$$

Denote $\varphi_{HM} : \mathbb{R} \to [0,1]$, defined by

$$\varphi_{HM}(p) := 1 - (1 - H(p))^N$$

and $\varphi_{PBM} : \mathbb{R} \to [0,1]$, defined by

$$\varphi_{PBM}(p) := 1 - (1 - H^{PBM}(p))^N.$$

Performing the substitution $z = \left(\frac{(r-p)\gamma(1-\eta)^N(1-\mu)}{(r-c)N\mu(1-\eta G(p(z)))^N} \right)^{1/(N-1)}$, one can write the expected price a fast trader gets in the bilateral market in the HM as $\int p \, \mathrm{d}\varphi_{HM}(p) = r\beta + c(1-\beta)$, where

$$\beta := \int_0^1 \left(\frac{N\mu(1-\eta G(p(z)))^N z^{N-1}}{\gamma(1-\mu)(1-\eta)^N} + 1 \right)^{-1} N z^{N-1} \, \mathrm{d}z.$$

Performing analogously the substitution $z = \left(\frac{(r-p)\gamma^{PBM}(1-\mu)}{(r-c)N\mu} \right)^{1/(N-1)}$, one can write the expected price a fast trader gets in the PBM as $\int p \, \mathrm{d}\varphi_{PBM}(p) = r^{PBM}\overline{\beta} + c(1-\overline{\beta})$, where

$$\overline{\beta} := \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma^{PBM}(1-\mu)} + 1 \right)^{-1} N z^{N-1} \, \mathrm{d}z.$$

It has already been shown that $r^{PBM} \geq r$. From the above discussion it now follows that fast traders get a better price in the HM than in the PBM if $\beta < \overline{\beta}$. The latter is indeed the case as can be seen as follows. Let Φ be a function defined by

$$\Phi(z) := \left(\frac{N\mu(1-\eta G(p(z)))^N z^{N-1}}{\gamma(1-\mu)(1-\eta)^N} + 1 \right)^{-1} - \left(\frac{N\mu z^{N-1}}{\gamma^{PBM}(1-\mu)} + 1 \right)^{-1}.$$

Then equation (41) implies $\int_0^1 \Phi(z) \, \mathrm{d}z \leq 0$. The case $\Phi(x) = 0$ for all $x \in (0,1)$ is not possible since G is monotone increasing. Thus, $\Phi(x) < 0$ for some $x \in (0,1)$. If $\Phi(x) \leq 0$ for all $x \in (0,1)$ then clearly $\int_0^1 \Phi(z) N z^{N-1} \, \mathrm{d}z < 0$ and therefore $\beta < \overline{\beta}$ and the claim follows. Let $\Phi(x) > 0$ for some $x \in (0,1)$. Since $G \circ p$ is strictly monotone decreasing in z, there is a unique $x^* \in (0,1)$ such that $\Phi(x) < 0$ for $0 \leq x < x^*$

and $\Phi(x) < 0$ for $x^* < x \leq 1$. Then

$$\begin{aligned} \beta - \overline{\beta} &= \int_0^1 \Phi(z) N z^{N-1} \, \mathrm{d}z \\ &< \int_0^1 \Phi(z) N (x^*)^{N-1} \, \mathrm{d}z \\ &\leq 0 \end{aligned}$$

Thus, in any case it holds that $\beta < \overline{\beta}$. As discussed above it follows that fast traders can always expect a better price in the HM than in the PBM. □

Proof of Proposition 3. The dealers' collective expected profit is by definition the product of turnover times the difference of the expected price and the per asset. By the exact law of large numbers, the expected profit is equal to the actual profit. If $\gamma^{PBM} = 1$, there is full market participation by both kinds of traders. Turnover therefore cannot be higher in the HM than in the PBM. Since, according to Lemma 12, prices are lower for both fast and slow traders in the HM, collective profits must be lower in the HM. □

Proof of Corollary 1. As stated in the proof of Lemma 11, one has by definition that

$$r^{PBM}(\gamma^{PBM}) = c + \frac{s}{1 - \overline{\alpha}(\gamma^{PBM})} < c + \frac{s}{1 - \overline{\alpha}(1)} = c + \frac{s}{1 - \int_0^1 \left(\frac{N \mu z^{N-1}}{(1-\mu)} + 1 \right)^{-1} \mathrm{d}z}. \tag{42}$$

If $s \to 0$, the last expression in (42) converges to c. If $\mu \to 1$, the last expression in (42) converges to $c + s < v$. In both cases, the slow traders' equilibrium entry decision is $\gamma^{PBM} = 1$, since $r^{PBM}(1) < v$. The claim follows now from Proposition 3.

□

Proof of Proposition 4. There are three things to show. First, it has to be shown that $\gamma, \gamma^{PBM} \to 0$ as $N \to \infty$. Second, all conditions mentioned in Proposition 1 have to be satisfied as $N \to \infty$. Third, dealer profits have to be higher in the HM than in the PBM as $N \to \infty$.

I start by showing that under the given parameters with $N \to \infty$, it must be the case that both γ and γ^{PBM} go to zero as $N \to \infty$ if equilibria in HM and PBM exist. This result is independent of how the functions G looks like in particular. As shown in the proof of Lemma 9 and stated in the proof of Lemma 11, one has $r^{PBM} = c + \frac{s}{1-\overline{\alpha}(\gamma^{PBM})}$ and $r > c + \frac{s}{1-\underline{\alpha}(\gamma)}$. By the Lebesgue Dominated Convergence Theorem one gets for $\mu = \frac{K_1}{N}$, $\eta = \frac{K_2}{N}$ and fixed γ, γ^{PBM} that

$$\begin{aligned}
\lim_{N\to\infty} \underline{\alpha}(\gamma) &= \lim_{N\to\infty} \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma(1-\mu)(1-\eta)^N} + 1 \right)^{-1} dz \\
&= \int_0^1 \left(\frac{K_1 \lim_{N\to\infty} z^{N-1}}{\gamma e^{-K_2}} + 1 \right)^{-1} \mathrm{d}z = 1
\end{aligned}$$

and

$$\begin{aligned}
\lim_{N\to\infty} \overline{\alpha}(\gamma^{PBM}) &= \lim_{N\to\infty} \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma^{PBM}(1-\mu)} + 1 \right)^{-1} dz \\
&= \int_0^1 \left(\frac{K_1 \lim_{N\to\infty} z^{N-1}}{\gamma^{PBM}} + 1 \right)^{-1} \mathrm{d}z = 1.
\end{aligned}$$

It follows that $r, r^{PBM} \to \infty$ for $N \to \infty$ if γ, γ^{PBM} were held fixed. Clearly, $\gamma = 1$ or $\gamma^{PBM} = 1$ is not possible in equilibrium. As stated in Lemma 9 and Section 3, r and r^{PBM} are monotone increasing in γ and γ^{PBM}, respectively. In order to ensure $r = v$ and $r^{PBM} = v$, it has to be the case that $\gamma, \gamma^{PBM} \to 0$ as $N \to \infty$.

Now, I will check the conditions stated in Proposition 1 for $N \to \infty$. Condition (9) holds for, since

$$\frac{\gamma(1-\eta) - \gamma(1-\eta)^N}{N - \gamma(1-\eta)^N} \frac{1}{\mu} = \frac{\gamma(1-K_2/N) - \gamma(1-K_2/N)^N}{K_1 - \gamma(1-K_2/N)^N K_1/N} \to 0 \quad \text{as } N \to \infty.$$

Condition (17) holds in the limit, since $\frac{\gamma}{N}(1-K_2/N)^N \to 0$ as $N \to \infty$.

It can be similarly verified that (19) holds, since the left-hand side of the inequality divided by μ goes to zero and the right-hand side of the inequality divided by μ goes to infinity as $N \to \infty$.

Condition (20) holds, since $(1-\eta)^{N-1} \to e^{-K_2}$ and condition (21) holds if $N \to \infty$, since $(1-\eta)^{1-N} \to e^{K_2}$.

Also condition (22) holds, as it can be verified that $\frac{(N-1)\left(\eta+N(1-\eta)^N-1\right)}{N\left(-\eta+N(1-\eta)^N-(1-\eta)^N+1\right)} \to 1$ as $N \to \infty$.

In order to show that $r_p < r_b$ as $N \to \infty$, I will use the fact that $\gamma \to 0$ implies by the construction of the equilibrium in Proposition 1 that $r = r_b \to v$. Then Lemma 10 gives a sufficient condition on s that will ensure $r_p < r_b$. Setting $r = v$, obtaining an upper bound for the expected best quote on the platform from (24) and substituting into (26) gives

$$s < (1-(1-\eta)^N)\left(c + (v-c)(1-\eta)^{N-1}\frac{N\eta}{1-(1-\eta)^N}\right) \to c(1-e^{-K_2}) + e^{-K_2}(v-c)\frac{K_2}{1-e^{-K_2}}.$$

Therefore, $r_p < r_b$ as $N \to \infty$ if (27) holds.

Lastly, it has to be shown that dealer profits are larger in the HM than in the PBM. This can be readily seen, since the expected price a dealer quotes on a platform is greater than $\underline{p}^p$. By equations (12) and (8) one has $\underline{p}^p \geq c + (1-\eta)^{N-1}(r-c)(1-\mu) \to c + (v-c)e^{-K_2} > c$ as $N \to \infty$. The mass of slow traders on the platform goes to $1 - e^{-K_2}$ as $N \to \infty$ while both the mass of slow traders and the mass of fast traders goes to zero in the PBM as $N \to \infty$. Since the price a dealer quotes in the PBM is bounded from above by v, dealers' collective profits go to zero in the PBM. In the HM however, dealers' collective profits are greater than or equal to $e^{-K_2}(1-e^{-K_2})(v-c) > 0$ as $N \to \infty$. □

Proof of Proposition 5. The proof is analogous to that of Proposition 4. However, some arguments have to be slightly modified. First, it has to be shown that $\gamma, \gamma^{PBM} \to 0$ as $N \to \infty$. Second, all conditions mentioned in Proposition 1 have to be satisfied as $N \to \infty$. Third, dealer profits have to be higher in the HM than in the PBM as $N \to \infty$. The last step involves showing that profit from selling to fast traders goes to zero in the PBM.

I start by showing that under the given parameters with $N \to \infty$, it must be the case that both γ and γ^{PBM} go to zero as $N \to \infty$ if equilibria in HM and PBM exist. As shown in the proof of Lemma 9 and stated in the proof of Lemma 11, one has $r^{PBM} = c + \frac{s}{1-\overline{\alpha}(\gamma^{PBM})}$ and $r > c + \frac{s}{1-\underline{\alpha}(\gamma)}$. One has $Nz^{N-1} \to 0$ for all $z \in (0,1)$ and $N \to \infty$. By the Lebesgue Dominated Convergence Theorem one therefore gets for $\eta = \frac{K_2}{N}$ and fixed $\mu, \gamma, \gamma^{PBM}$ that

$$\begin{aligned}
\lim_{N\to\infty} \underline{\alpha}(\gamma) &= \lim_{N\to\infty} \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma(1-\mu)(1-\eta)^N} + 1\right)^{-1} \mathrm{d}z \\
&= \int_0^1 \left(\frac{\lim_{N\to\infty} N\mu z^{N-1}}{\gamma(1-\mu)e^{-K}} + 1\right)^{-1} \mathrm{d}z = 1
\end{aligned}$$

and

$$\begin{aligned}\lim_{N\to\infty}\overline{\alpha}(\gamma^{PBM}) &= \lim_{N\to\infty}\int_0^1\left(\frac{N\mu z^{N-1}}{\gamma^{PBM}(1-\mu)}+1\right)^{-1}\mathrm{d}z \\ &= \int_0^1\left(\frac{\lim_{N\to\infty}N\mu z^{N-1}}{\gamma^{PBM}(1-\mu)}+1\right)^{-1}\mathrm{d}z=1.\end{aligned}$$

It follows that $r, r^{PBM}\to\infty$ for $N\to\infty$ if γ,γ^{PBM} were held fixed. Clearly, $\gamma=1$ or $\gamma^{PBM}=1$ is not possible in equilibrium. As stated in Lemma 9 and Section 3, r and r^{PBM} are monotone increasing in γ and γ^{PBM}, respectively. In order to ensure $r=v$ and $r^{PBM}=v$, it has to be the case that $\gamma,\gamma^{PBM}\to 0$ as $N\to\infty$.

Now, I will check the conditions stated in Proposition 1 for $N\to\infty$. Condition (9) holds for, since

$$\frac{\gamma(1-\eta)-\gamma(1-\eta)^N}{N-\gamma(1-\eta)^N}\to 0 \quad \text{as } N\to\infty.$$

Condition (17) holds in the limit, since $\frac{\gamma}{N}(1-K/N)^N\to 0$ as $N\to\infty$.

It can be similarly verified that (19) holds for large N, since the left-hand side of the inequality goes to zero and the right-hand side of the inequality goes to one as $N\to\infty$.

Condition (20) holds, since $(1-\eta)^{N-1}\to e^{-K}$ and condition (21) holds if $N\to\infty$, since $(1-\eta)^{1-N}\to e^{K}$. Also condition (22) holds, as it can be verified that $\frac{(N-1)\left(\eta+N(1-\eta)^N-1\right)}{N\left(-\eta+N(1-\eta)^N-(1-\eta)^N+1\right)}\to 1$ as $N\to\infty$.

In order to show that $r_p<r_b$ as $N\to\infty$, I will use the fact that $\gamma\to 0$ implies by the construction of the equilibrium in Proposition 1 that $r=r_b\to v$. Then Lemma 10 gives a sufficient condition on s that will ensure $r_p<r_b$. Setting $r=v$, obtaining an upper bound for the expected best quote on the platform from (24) and substituting into (26) gives

$$s<(1-(1-\eta)^N)\left(c+(v-c)(1-\eta)^{N-1}\frac{N\eta}{1-(1-\eta)^N}\right)\to c(1-e^{-K})+e^{-K}(v-c)\frac{K}{1-e^{-K}}.$$

Therefore, $r_p<r_b$ as $N\to\infty$ if (28) holds.

Lastly, it has to be shown that dealer profits are larger in the HM than in the PBM. To this end, I first show that the expected price a fast trader is charged in the PBM goes to c as $N\to\infty$. Since slow traders' market participation goes to zero, it then follows that dealers do not make any profit in the PBM. Showing that dealers' profit is positive in the HM gives the result.

As stated in Section 3, the dealers in the PBM quote according to the distribution function H^{PBM}, with $\operatorname{supp} H^{PBM} = [\underline{p}^{PBM}, r^{PBM}]$. The infimum of the support $\underline{p}^{PBM}$ is determined by

$$1 - \left(\frac{\gamma^{PBM}(1-\mu)(r^{PBM} - \underline{p}^{PBM})}{N\mu(\underline{p}^{PBM} - c)} \right)^{1/(N-1)} = 0 \Leftrightarrow \underline{p}^{PBM} = c + \frac{(r^{PBM} - c)\gamma^{PBM}(1-\mu)}{N\mu + \gamma^{PBM}(1-\mu)}.$$

One therefore obtains $\underline{p}^{PBM} \to c$ as $N \to \infty$. Since H^{PBM} is strictly monotone increasing, the best offer a fast dealer gets converges to c in probability as $N \to \infty$. Since the support of H^{PBM} is bounded, the expected price, a fast trader has to pay also converges to c as $N \to \infty$.

It is now established that dealers' combined profits go to zero as $N \to \infty$. Analogously to the proof of Proposition 4, it remains to show that the combined profits are strictly positive in the HM. The expected price a dealer quotes on a platform is greater than $\underline{p}^p$. By equations (12) and (8) one has $\underline{p}^p \geq c + (1-\eta)^{N-1}(r-c)(1-\mu) \to c + (v-c)e^{-K}(1-\mu) > c$ as $N \to \infty$. The mass of slow traders on the platform goes to $1 - e^{-K}$ as $N \to \infty$. In the HM, dealers' collective profits are therefore greater than or equal to $e^{-K}(1-e^{-K})(v-c)(1-\mu) > 0$ as $N \to \infty$. □

Proof of Corollary 2. Since in equilibrium $\gamma, \gamma^{PBM} < 1$, the expected prices in the two types of bilateral markets follow from $r = v = r^{PBM}$. Since $\mu, \eta \to 0$, $N\eta = K_2$ and $(1-\eta)^N \to e^{-K_2}$ it follows from (24) and (24) that

$$\mathbb{E}(q) = c + (v-c)e^{-K_2}\frac{K_2}{1-e^{-K_2}}$$

as $N \to \infty$. □

Proof of Corollary 3. The dealers' collective profit is equal to the profit from slow traders on the platform as $\mu, \gamma \to 0$. Regarding the mass of slow traders on the platform, it holds that $1-(1-\eta)^N \to 1 - e^{-K_2}$. Multiplying this mass with the difference of the expected price from Corollary 2 and the dealers' cost c gives the expression for $\Pi(K_2)$.

Maximizing this expression with respect to K_2 gives the resulting K_2^*. □

Proof of Corollary 4. Since $\underline{p}^b < \underline{p}^p$, there is a positive probability $\varepsilon > 0$ that a dealer in the bilateral market gives a better quote than any dealer on the platform. The probability that none of the N dealers gives a quote better than any dealer on the platform is then given by $(1-\varepsilon)^N$. As $N \to \infty$, this probability goes to zero. This means that fast traders will get a better quote in the bilateral market than on the platform with probability one as $N \to \infty$.

Since $\gamma \to 0$ as $N \to \infty$, slow traders enter the bilateral market with probability zero and thus do not trade there. $\square$

Proof of Proposition 6. In the HM, slow traders will always buy the asset on the platform if they receive a price. Therefore, slow traders buy the asset on the platform with probability $1-(1-\eta)^N$ and buy the asset in the bilateral market with probability $\gamma(1-\eta)^N$. Fast traders, however, will buy the asset wherever they find the best quote in the whole market. Thus, even if there is a price on the platform, there is a positive probability, that they find a better price in the bilateral market, since the supports of G and H overlap. Fast traders will therefore buy the asset with probability $\Psi < 1-(1-\eta)^N$ on the platform and buy the asset with probability $1-\Psi > \gamma(1-\eta)^N$ in the bilateral market, since they will for sure buy the asset somewhere in the HM. $\square$

Proof of Proposition 7. Lemma 12 states that both fast and slow traders have to pay a lower expected price for the asset. It is clear that fast traders are better off in the HM, since a fast trader always buys an asset. A slow trader is not less likely to buy an asset in the HM, since $\gamma \geq \gamma^{PBM}$ implies $1-(1-\eta)^N + \gamma(1-\eta)^N \geq \gamma^{PBM}$. Thus, slow traders receive at least the sane quantity of the asset for a lower price per asset. $\square$

Proof of Proposition 8. I first show that

$$\gamma < \gamma^{PBM} \frac{(1-\eta G(\underline{p}^b))^N}{(1-\eta)^N}. \tag{43}$$

Suppose that the latter inequality does not hold. Then calculating the reservation prices in the HM and PBM as in (40) in the proof of Lemma 11 gives

$$r(\gamma) > c + \frac{s}{1-\int_0^1 \left(\frac{N\mu(1-\eta G(\underline{p}^b))^N z^{N-1}}{\gamma(1-\mu)(1-\eta)^N}+1\right)^{-1} \mathrm{d}z} \geq c + \frac{s}{1-\int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma^{PBM}(1-\mu)}+1\right)^{-1} \mathrm{d}z} = v.$$

This is a contradiction, because Lemma 12 and the definitions of the reservation prices as expected price minus search cost imply $r(\gamma) \leq r^{PBM}(\gamma^{PBM})$.

The expression for H^{PBM} in Section 3 gives

$$\underline{p}^{PBM} = \frac{N\mu c + \gamma^{PBM}(1-\mu)v}{N\mu + \gamma^{PBM}(1-\mu)}. \tag{44}$$

Suppose $\underline{p}^b < \underline{p}^b$. Then $G(\underline{p}^b) = 0$. Now (7), (43) and (44) imply that $\underline{p}^b < \underline{p}^{PBM}$

Suppose $\underline{p}^p < \underline{p}^b$. Now (13), (43) and (44) imply that $\underline{p}^b < \underline{p}^{PBM}$

□

Proof of Corollary 5. If $\gamma < 1$, Lemma 11 implies $\gamma^{PBM} < 1$. This in turn implies $r = v = r^{PBM}$. The claim now follows directly from Proposition 8. □

Proof of Corollary 6. Set $\overline{\mu} := \frac{1-(1-\eta)^{N-1}}{N(1-\eta)^{N-1}-(1-\eta)^{N-1}}$. Then it follows from condition (10) that if $\mu > \overline{\mu}$, a regime-2 equilibrium cannot exist. □

Proof of Proposition 9. In the PBM, fast traders buy the asset with probability 1. In the HM, however, there is a positive chance that the lowest quote on the platform is lower than any price a dealer in the bilateral market quotes. This can be seen, since the support if G and H always overlap.

Suppose that slow traders would not trade less in the the bilateral market of the HM than in the PBM. This would mean $\gamma(1-\eta)^N \geq \gamma^{PBM}$. It follows that

$$\begin{aligned}
\int_0^1 \left(\frac{N\mu(1-\eta G(p(z)))^N z^{N-1}}{\gamma(1-\mu)(1-\eta)^N} + 1 \right)^{-1} \mathrm{d}z \quad &> \quad \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma(1-\mu)(1-\eta)^N} + 1 \right)^{-1} \mathrm{d}z \\
&\geq \quad \int_0^1 \left(\frac{N\mu z^{N-1}}{\gamma^{PBM}(1-\mu)} + 1 \right)^{-1} \mathrm{d}z
\end{aligned}$$

Defining r and r^{PBM} as in the proof of Lemma 11, one now gets $r^{PBM} < r$. If $\gamma^{PBM} = 1$, one clearly has a contradiction, since γ cannot be greater than one. If $\gamma^{PBM} < 1$, it must be the case that $r^{PBM} = v$. But then the above results imply $r > v$. This cannot hold in equilibrium since slow traders would make a negative expected profit in the bilateral market. This contradiction proves that slow traders trade less in the bilateral market of the HM than in the PBM. □

Appendix B

Here I describe how I find numerical solutions to equations (4) and (5) and how to calculate some statistics shown in section 6.

Case 1: $\underline{p}^b < \underline{p}^p$

The endogenous parameters $\underline{p}^p$ and $\underline{p}^p$ and the distribution of prices in the bilateral market for $p \in [\underline{p}^b, \underline{p}^p]$ are given by Lemma 6.

Rearranging (4) and (5), one gets the following fixed-point equations that can be used to determine $H(p)$ and $G(p)$ for each $p \in [\underline{p}^p, r]$:

$$H(p) = 1 - \left(\frac{(r-p)k_b}{(p-c)\mu(1-\eta G(p))^N} \right)^{1/(N-1)}. \tag{45}$$

$$G(p) = \frac{1}{\eta} - \frac{1}{\eta} \left[\frac{(1-\eta)^{N-1} \cdot (r-c)}{(p-c)} - \frac{\mu}{k_p}(1-H(p))^N (1-\eta G(p))^{N-1} \right]^{1/(N-1)}. \tag{46}$$

An indifference condition between continuing to search in the bilateral market and taking an offer on the platform determines r as in (1):

$$r = s + \int_{\underline{p}^b}^{r} p \, \mathrm{d}H(p). \tag{47}$$

The parameter γ is chosen such that the slow traders do not make losses when going into the bilateral market. If expected profits in the bilateral market are positive, then $\gamma = 1$, i.e. slow traders surely continue their search when not having found a satisfactory offer on the platform as stated in Lemma 1.

Equations (7),(8),(6),(45),(46) and (47) constitute a system of fixed-point equations that can be solved numerically to obtain the distributions of the prices in the two trading venues.

The assumed reservation price strategy in which slow traders go first to the platform is only appropriate if the expected profit from first going to the platform is greater than the expected profit from first going to the bilateral market:

$$(1-(1-\eta)^N) \int_{\underline{p}^p}^{r} (v-p) \, \mathrm{d}F(p) - s + \gamma(1-\eta)^N \int_{\underline{p}^b}^{r} (v-p-s) \, \mathrm{d}H(p) \geq \int_{\underline{p}^b}^{r} (v-p-s) \, \mathrm{d}H(p), \tag{48}$$

where

$$F(p) := \frac{1-(1-\eta G(p))^N}{1-(1-\eta)^N}$$

is the conditional price distribution on the platform if there is at least one dealer submitting a quote.

Once one has obtained the equilibrium price distributions on the trading venues, one can calculate various statistics. The expected payoff for slow traders was already used in equation (48). The expected payoff for fast traders can be calculated as follows. Since fast traders choose the lowest price in the entire market, they draw prices from the following distribution M, where

$$M(p) = 1 - (1 - \eta G(p))^N (1 - H(x))^N.$$

Fast traders therefore get

$$\int_{\underline{p}^b}^{r} (v - p) \, \mathrm{d}M(p).$$

Slow traders will always get a satisfactory offer on the platform if there is at least one dealer submitting a quote. Therefore, by the exact law of large numbers, the quantity of slow traders trading on the platform is

$$\lambda_p^{slow} := (1 - \mu)(1 - (1 - \eta)^N).$$

If slow traders do not find a satisfactory offer on the platform they will surely find a satisfactory offer in the bilateral market if they happen to choose to continue their search. The quantity of slow traders in the bilateral market is therefore

$$\lambda_b^{slow} := \gamma(1 - \mu)(1 - \eta)^N.$$

Fast traders always trade. Their choice of trading venue depends on where they find the lowest quote. Let p_p be the lowest price on the platform. Then the probability that the lowest quote is on the platform is

$$\begin{aligned} P(\text{trade on platform}) &= \int_{\underline{p}^p}^{r} P(\text{trade on platform} | p_p = x) \, \mathrm{d}P(p_p \leq x) \\ &= \int_{\underline{p}^p}^{r} (1 - H(x))^N \, \mathrm{d} \left(1 - (1 - \eta G(x))^N \right). \end{aligned}$$

Analogously, one can calculate the price that fast traders on average have to pay if they trade on the platform as

$$\mathbb{E}(p_p|\text{trade on platform}) = \int_{\underline{p}^p}^{r} x(1-H(x))^N \,\mathrm{d}\left(1-(1-\eta G(x))^N\right).$$

The price that fast traders on average have to pay if they trade in the bilateral market is given by

$$\mathbb{E}(p_b|\text{trade in bil. mkt.}) = \int_{\underline{p}^b}^{r} x(1-\eta G(x))^N \,\mathrm{d}\left(1-(1-H(x))^N\right).$$

The quantity of fast traders trading on the platform is now

$$\lambda_p^{fast} := \mu P(\text{trade on platform}).$$

And the quantity of fast traders trading in the bilateral market is

$$\lambda_b^{fast} := \mu(1-P(\text{trade on platform})).$$

Having the quantities of slow traders in the two trading venues, one can calculate the dealers' collective payoff as

$$\lambda_p^{slow}\left(\int_{\underline{p}^p}^{r} p \,\mathrm{d}F(p) - c\right) + \lambda_b^{slow}\left(\int_{\underline{p}^b}^{r} p \,\mathrm{d}H(p) - c\right) + \mu\left(\int_{\underline{p}^b}^{r} p \,\mathrm{d}M(p) - c\right)$$

Case 2: $\underline{p}^b > \underline{p}^p$

In this case, the model is solved analogously. The endogenous parameters $\underline{p}^p$ and $\underline{p}^p$ and the distribution of prices on the platform for $p \in [\underline{p}^p, \underline{p}^b]$ are given by Lemma 7.

The parameter γ is again chosen in accordance with Lemma 1.

Equations (12),(13),(11),(45),(46) and (47) constitute a system of fixed-point equations that can be solved numerically to obtain the distributions of the prices in the two trading venues.

The descriptive statistics are computed analogously as well.

A A Setup in which Quoting on the Platform and in the Bilateral Market is Mutually Exclusive

Some of the key results of this paper are driven by the fact that the introduction of a trading platform increases competition among the dealers. For example, the claim in Lemma 12 and Lemma 11 that expected markups decline after the introduction of a trading platform are analogous the fact that prices in an oligopolistic market decline if the number of sellers increases. Some readers may have the concern that it is in some sense trivial that the introduction of a platform leads to a decline in markups and an increase in volume, since the number of active trading desks doubles after a platform has been introduced (each dealer has an active trading desk both in the bilateral market and on the platform). There are two points to be made to address this concern.

First, in standard oligopolistic markets, sellers are usually not trapped in a bad equilibrium in which it would be optimal to decrease prices to increase traded volume. Intead, oligopolistic sellers usually could increase profits by colluding to charge higher prices. However, in this paper it is argued that dealers in an OTC market may charge suboptimally high markups and the competition from introducing a platform may increase the dealers' profits (see Proposition 4 and Proposition 5).

Second, the key results in this paper are not driven by the fact that there is more quoting activity in the HM compared to the PBM. To make this point clear, I am considering a modified setup of the model of the main text in this section. In the following, I am assuming that a dealer cannot simultaneously quote on the platform and in the bilateral market after a platform has been introduced. Thus, the number of active trading desks will not increase after a platform has been introduced. I will provide a numerical example in which it is nevertheless the case that the dealers' profits will be higher in the HM than in the PBM. This example shows that it is not the number of active trading desks that drives the key results in this paper. It will even turn out that the total number of quotes that are provided in the market is lower in the HM than in the PBM. It thus seems that the way dealers compete on the platform is the driving force that leads to a an increase in traded volume. In the PBM a lot of quoting activity is concentrated towards the fast traders, whereas slow traders trade with the first dealer they meet (if they trade at all). However, once a platform is introduced, even slow traders have the chance of obtaining multiple quotes.

Suppose that there are $2N$ dealers in a PBM (with $\mathbb{N} \ni N \geq 2$ as before). If a platform is introduced in this market a dealer can no longer operate both on the platform and the bilateral markets. Instead each dealer is exogenously assigned to one of the two trading venues (e.g. by a regulator). Specifically, I will consider the case in which N dealers quote on the platform and N dealers quote in the bilateral market. The other assumptions from the main text of the paper still hold. This means the fraction of fast traders is given by μ and each dealer in on the platform quotes independently with probability η. Equations (4) and (5) still characterize the quoting strategies of the dealers on the platform and in the bilateral market, since exactly N dealers will operate on each venue. (In the main text it was assumed that dealers operate separate trading desks. Conceptually, it does not make a difference whether one talks about different trading desks or different dealers, since both are assumed to maximize their own profit.) Similarly, the traders' search strategies are identical to the ones derived in the main text of the paper. Table 3 shows two equilibria for the HM as described in Proposition 1. It has been solved numerically as described in Internet Appendix 7. The equilibrium in the PBM is determined as described in Section 3, with the only difference that $2N$ instead of N traders are active in the PBM.

Note that by Lemma 4 and equations (4) and (5) a dealer's profit on the platform is equal to $(r-c)k_b$ and a dealer's profit in the bilateral market in the HM conditional on quoting (which happens with probability η) is given by $(r-c)k_p(1-\eta)^{N-1}$. A similar argument from Duffie et al. (2016) implies that a dealer's profit in the PBM is given by $(r^{PBM}-c)\gamma^{pbm}(1-\mu)/N$. Thus, the dealers' joint payoffs are given by

$$\text{Joint dealer payoff} = \begin{cases} N\eta(r-c)k_p(1-\eta)^{N-1} & \text{(HM platform)} \\ N(r-c)k_b & \text{(HM platform)} \\ 2(r^{PBM}-c)\gamma^{PBM}(1-\mu) & \text{(PBM)} \end{cases} \cdot$$

Adopting the notation from Internet Appendix 7 and defining

$$M^{PBM}(p) := 1-(1-H^{PBM}(p))^{2N},$$

one can express the payoffs for the traders π_t as follows:

$$\pi_t = \begin{cases} (1-(1-\eta)^N)\left(v-\int p\,\mathrm{d}F(p)\right)-s+\gamma(1-\eta)^N\left(v-\int p\,\mathrm{d}H(p)-s\right) & \text{Slow (HM)} \\ v-\int p\,\mathrm{d}M(p)-c & \text{Fast (HM)} \\ \gamma^{PBM}\left(\int p\,\mathrm{d}H^{PBM}(p)-c\right) & \text{Slow (PBM)} \\ v-\int p\,\mathrm{d}M^{PBM}(p)-s & \text{Fast (PBM)} \end{cases}.$$

In the HM, there are ηN quotes on the platform in expectation conditional on at least one response to an RFQ. This event happens with probability $1-(1-\eta)^N$. Additionally, there are $N\mu$ quotes to fast traders in the bilateral market and $(1-\eta)^N\gamma(1-\mu)$ quotes to slow traders in the bilateral market.

In the PBM one has $2N\mu$ quotes to fast traders and $\gamma^{PBM}(1-\mu)$ quotes to slow traders. To sum up, one has

$$\text{Quotes} = \begin{cases} N(\eta(1-(1-\eta)^N) + \mu + k_b) & \text{in HM} \\ 2N\mu + \gamma^{PBM}(1-\mu) & \text{in PBM} \end{cases}.$$

Table 3: Two equilibria

	Equilibrium 1	Equilibrium 2
Exogenous Parameter:		
N	70	70
μ	0.8	0.8
η	0.02	0.01
s	0.03	0.03
c	0.5	0.5
v	1	1
Endogenous Parameter:		
γ	1	1
r	0.84	0.85
γ^{PBM}	0.11	0.11
r^{PBM}	1	1
Joint dealer payoffs:		
HM platform	0.023	0.024
HM bilateral market	0.016	0.035
PBM	0.011	0.011
Trader payoffs:		
Slow (HM)	0.272	0.174
Fast (HM)	0.498	0.497
Slow (PBM)	0	0
Fast (PBM)	0.499	0.499
Quotes:		
HM	57.11	56.45
PBM	112.02	112.02

The only difference between the two equilibria in Table 3 lies in the value of η. In equilibrium 1, η is higher, making the platform more attractive than the bilateral market for dealers. In equilibrium 2, dealers make more profits in the bilateral market. In both equilibria, all dealers' profits are higher compared to the

PBM.

An interesting feature of the setup considered here is that fast traders may actually prefer a PBM, since their payoff is higher in the PBM than in the HM in Table 3. Slow traders prefer the HM as in the main text. Overall, the HM leads to a higher overall welfare, since both dealers and slow traders are significantly better off in the HM, while the loss of fast traders is very small.

Note that overall quoting activity is higher in the PBM, since fast traders visit all dealers. In the HM, fast traders visit fewer dealers, but slow traders obtain more quotes, which strengthens their bargaining position. Consequently, markups for slow traders are lower in the HM. Due to lower markups, the slow traders' market participation is higher in the HM. This higher market participation is also the reason why dealers make higher profits in the HM.