Why Trade Over-the-Counter?

When Investors Want Price Discrimination*

Job Market Paper

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Abstract

Despite the availability of low-cost exchanges, over-the-counter (OTC) trading is pervasive for most assets. We explain the prevalence of OTC trading using a model of adverse selection, in which informed and uninformed investors choose to trade over-the-counter or on an exchange. OTC dealers' ability to price discriminate allows them to imperfectly cream-skim the uninformed investors from the exchange. Assets with lower adverse selection risk are predicted to have a higher fraction of trades over-the-counter, as observed in practice. The presence of an OTC market increases aggregate trade volume and reduces average bid-ask spread; nonetheless, welfare declines when adverse selection risk is low. Surprisingly, for assets that are mostly traded over-the-counter, closing the OTC market improves welfare, providing support for recent regulatory efforts to end OTC trading in those assets.

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1 Introduction

Modern exchanges are easily accessible and demand minimal transactions costs. In comparison, trading over-the-counter is costly because of the need to seek out prices and a relative lack of automation. Yet, a large portion of trades occurs over-the-counter in nearly all financial assets. Even for stocks listed on highly liquid US exchanges, 17% of trades are over-the-counter. This share is far higher in other assets, such as swaps and corporate bonds.¹

We offer an explanation for the prevalence of over-the-counter (OTC) trading in the presence of low cost exchanges. Our explanation builds on a fundamental distinction between trading on exchanges versus trading over-the-counter: on an exchange, prices are posted anonymously on a limit order book; to trade over-the-counter, an investor must request prices from a dealer, which allows the dealer to price discriminate. In equilibrium, the OTC dealer cream-skims investors who are less likely to be informed from the exchange by offering them better prices. Cream-skimming increases aggregate trade volume and decreases average bid-ask spread, yet may harm welfare. Specifically, the presence of an OTC market reduces welfare if adverse selection risk is low, and increases welfare if the risk is high. As cream-skimming concentrates the informed onto the exchange, low adverse selection risk implies a small market share for the exchange. Therefore, ending the OTC trading of an asset improves welfare if the asset is mostly traded over-the-counter.

Crucially, we can explain why OTC trading predominates in markets for standardized assets in high demand. For example, swaps are largely standardized and heavily traded on electronic platforms.² These platforms must offer the option to trade on a limit order book.³ Nevertheless, limit order books execute less than 5% of swap trades. Common justifications for OTC trading, including nonstandardization, asset complexity or regulatory barriers that

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¹ In terms of dollar value, 17% of trades in US exchange-listed equities are bilateral, and are not broker internalizations (Tuttle, 2014). Nagel (2016) shows that 95% of electronic swaps trades are over-the-counter (nonelectronic trades are entirely over-the-counter). McPartland (2016) finds that 81% of investment-grade corporate bond trades are by voice, based on survey data (and some electronic trades are over-the-counter).

² Swaps are among the most important credit derivatives with notional value of US$378 trillion (Bank for International Settlements, 2017). Worldwide, 70% of interest rate swaps and 80% of index CDSs (two most traded swaps) are electronically traded, and 95% of electronic swaps trades are over-the-counter and so are all nonelectronic trades (Nagel (2016)).

³ The Dodd-Frank Act requires electronic swaps trading platforms, called swaps execution facilities (SEFs), to offer limit order book trading. Most trades nonetheless are executed via electronic request-for-quote (eRFQs), which replicate traditional voice-based trading. Riggs, Onur, Reiffen, and Zhu (2017) gives a description of SEFs and the eRFQ protocol.
reduce demand, cannot explain why assets such as standardized swaps are predominantly traded over-the-counter.

We develop a model of venue choice in which speculators and hedgers trade an asset. Speculators are informed about the asset’s payoff and seek profit. Hedgers trade to attain idiosyncratic benefits from hedging; given a price, a hedger trades only if her hedging benefit is large enough. The investors optimally choose to trade on an exchange, to trade over-the-counter, or not to trade. On the exchange, a competitive market maker publicly posts a bid and an ask, and all buy and sell orders are executed at those prices. To trade over-the-counter, an investor incurs a cost to obtain prices from a competitive dealer. The dealer cannot observe the investor’s true type, but the dealer can price discriminate by the investor’s reputation as a hedger or a speculator. Trading on the exchange means pooling with all others who trade on the exchange, whereas trading over-the-counter implies separating from those without the same reputation.

An investor’s reputation is imperfectly informative about whether the investor is a hedger or a speculator. Any investor-specific information may affect the investor’s reputation, such as her past trading behavior, public disclosures, or the business type of the firm she represents. For example, hedge funds typically trade on proprietary information while insurance companies usually trade to hedge. These reputations are imperfect since insurance companies sometimes trade for profit, and hedge funds sometimes hedge. We refer to investors reputed to be hedgers as h-investors, and those reputed to be speculators as s-investors.

Having the OTC market raises trade volume and reduces the average bid-ask spread. Yet, welfare may decline. Cream-skimming means having the OTC market improves prices for the h-investors, while worsening the prices of the s-investors. The price improvement induces some h-investors to trade who otherwise would not, whom we call entrants. However, worse prices stops some s-investors from trading who would trade without the OTC market, called exiters. Since the exiters are offered worse prices than the entrants, the exiters’ hedging benefits must be larger: in terms of hedging benefit, the entrants make cheap substitutes for the exiters. Hence, welfare may decline even if the entrants outnumber the exiters. This conflict between

\[ \text{\footnotesize 4 In US commodity markets, Commodity Futures Trading Commission (CFTC) classifies insurance companies as hedgers. Cheng and Xiong (2014) finds that a significant proportion of orderflow by investors classified as hedgers are uncorrelated to their output fluctuations, and conclude that these investors sometimes speculate.} \]
volume and welfare shows how the goal of maximizing trade volume misaligns with efficiency.

The welfare effect of the OTC market depends on adverse selection risk, as measured by the proportion of investors who are speculators. The presence of the OTC market reduces welfare if adverse selection risk is low (such as in the swaps market), and increases welfare if adverse selection risk is high (such as in the stock market). Low adverse selection risk ensures that the h-investors receive a narrow spread with or without the OTC market. But the s-investors’ spread widens significantly in the presence of the OTC market; thus, closing the OTC market improves welfare. The opposite holds if adverse selection risk is high.

It follows that closing the OTC market increases welfare if the asset is mostly traded over-the-counter. As hedgers disproportionately trade over-the-counter, the share of trades in the OTC market is larger if a higher portion of investors are hedgers. This is the case when adverse selection risk is low, which is also when closing the OTC market improves welfare. In practice, assets less attractive to speculators are more likely to be traded over-the-counter: fixed income assets are mainly traded over-the-counter, as are swaps, bonds and repos; while equities are mostly traded on exchanges, as are stocks and equity options.\(^5\)

We examine recent policies using our modeling framework. First, the US Dodd-Frank Act’s aim of migrating swaps trades onto exchanges is consistent with welfare maximization, whereas the goal of the EU MiFID II rules to force equity trades onto exchanges is predicted to reduce welfare. Second, ending the practice of post-trade name disclosure (name give-up) in the swaps market is expected to improve welfare, even as trade volume falls and average bid-ask spread increases. Third, increased disclosures of investor-specific information, such as the proposed implementations of the blockchain and disclosure rules on investment funds, are predicted to raise bid-ask spreads on exchanges, decrease OTC spreads, and reduce welfare for the assets traditionally traded over-the-counter.

Our results are consistent with empirical evidence. Cream-skimming implies that the hedgers disproportionately trade over-the-counter, so the OTC spread is lower than the spread on the exchange. Empirically, dealers quote narrower spreads to traders who are likely to

be uninformed (Linnainmaa and Saar, 2012, Lee and Chung, 2009), trades execute at better prices over-the-counter than on exchanges (Bosetti, Gottardo, Murgia, and Pinna, 2014, Smith, Turnbull, and White, 2001), and OTC trades are less informative compared to trades on exchanges (Rose, 2014, Bessembinder and Venkataraman, 2004).

Existing models of OTC trading distinguish the OTC market from exchanges by properties other than the dealers’ ability to price discriminate. Prior literature focuses on differences in price transparency (Pagano and Roell, 1996), the sequentiality of contacting counterparties over-the-counter (Glode and Opp, 2017), or the dealers’ ability to contract with investors (Grossman, 1992) or to discriminate according to order size (Seppi, 1990, Malinova and Park, 2013). Malamud and Rostek (2017) and Babus and Parlatore (2017) analyze the welfare effects of decentralized trading, focusing on a context without adverse selection. Recent regulatory changes (such as the Dodd-Frank Act) increased the price transparency of OTC markets, and enable investors to contact multiple dealers simultaneously. Price discrimination, on the other hand, remains a robust distinction that separates the OTC markets from the exchanges.

Two papers in this literature feature endogenous choice between trading over-the-counter or on exchanges. Grossman (1992) and Seppi (1990) use price improvements in OTC markets for large block orders to rationalize OTC trading. We explain why smaller orders are traded over-the-counter despite the availability of highly liquid exchanges. For US equities, block sized orders comprise only 2.5% of OTC trades, while orders at the minimum size of 100 shares comprise 40%. Figure 1 compares order size distributions of US exchange-listed equities by protocol, with “non-ATS” indicating OTC trades.

We contribute to the literature in three ways. First, we provide an explanation for the predominance of OTC trading in standardized and heavily traded securities (such as standardized swaps). Our explanation requires neither that the OTC dealers have market power or private information, and is robust to recent regulatory efforts to increase dealer competi-

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6 A related literature analyzes the choice between a limit order book and a specialist markets (Back and Baruch, 2007, Viswanathan and Wang, 2002, Parlour and Seppi, 2003, Ready, 1999, Seppi, 1997). In these models, the specialist set one price to clear her demand, whereas orders sent to the limit order book are cleared along a supply schedule.

7 Dodd-Frank Act expanded mandatory post-trade price disclosures (via TRACE) to trades of swaps, in addition to existing disclosure requirements on corporate and municipal bond trades. Moreover, Dodd-Frank requires electronic trading of swaps, and that the OTC trading protocols on the swaps platforms allow investors to request quotes from at least three dealers simultaneously.

8 We discuss Alternative Trading Systems (ATSs) and other equities market terminology in Section 2.1.
tion and transparency. Second, we model liquidity motivated investors with heterogeneous gains from trade, and show that this heterogeneity generates rich results. For instance, due to endogenous participation by hedgers, welfare can improve as volume declines and spreads increase. Third, our results justify regulatory intervention to end OTC trading in certain assets, while showing that such intervention may be inefficient for other assets: closing the OTC market is efficient only if adverse selection risk is low. By contrast, previous papers suggest OTC trading is always inefficient (Babus and Parlatore, 2017), or that outcomes observed in practice is efficient (Glode and Opp, 2017).

Section 2 describes our model, relevant applications, and derives a unique equilibrium. Section 3 examines the welfare and market quality effects of ending OTC trading and of investor-specific information. Section 5 the robustness of our results, and Section 4 discusses policy implications.

2 A Model of Adverse Selection and Venue Choice

Three types of risk-neutral agents, investors, and competitive dealer and market maker trade an indivisible asset in an one-period game. Each investor buys or sells one unit of the asset or does not trade. After trading, the asset pays an uncertain payoff $\tilde{v}$ of 1 or $-1$ with equal probability. All distributions and the structure of the game are common knowledge.

Investors belong either to a mass $\mu$ of speculators or mass 1 of hedgers, and are indexed by
an \( i \in [0, 1 + \mu] \). The speculators receive imperfect signals about the realization \( v \) of the asset payoff \( \tilde{v} \). A speculator’s signal \( q_i \) returns the correct \( v \) with probability \( \alpha > \frac{1}{2} \), and \(-v\) with probability \( 1 - \alpha \). In contrast, the hedgers are uninformed about \( v \), and trade solely to attain hedging benefits. Each hedger is a buyer or a seller with equal probability, and she attains her hedging benefit \( b_i \) if she buys and is a buyer, or if she sells and is a seller. The hedging benefits \( \{b_i\} \) are independently and uniformly distributed over \([0, 1]\), \( b_i \overset{iid}{\sim} U[0, 1] \).

Normalizing the mass of hedgers to 1 establishes a one-to-one relationship between \( \mu \) and adverse selection risk: speculators adversely select dealers, and an investor is a speculator with probability \( \frac{\mu}{1 + \mu} \). For this reason, we write “adverse selection risk” and “\( \mu \)” interchangeably. This normalization also keeps the total attainable hedging benefit constant as \( \mu \) varies.

The dealer has informative labels about investors’ true types, and each investor knows her own label. In particular, a speculator is correctly labeled as a speculator (s-investors) with probability \( \theta \), and a hedger is labeled as a hedger (h-investors) with probability \( \gamma \). With complementary probabilities \( 1 - \theta \) and \( 1 - \gamma \), the speculator is labeled as a h-investor and the hedger as a s-investor, respectively. Therefore, labels are uninformative if \( \theta = 1 - \gamma \), are informative if \( \theta > 1 - \gamma \), and are perfectly informative if \( \theta = \gamma = 1 \). We only consider the interesting case of imperfectly informative labels, which is when \( \theta > 1 - \gamma \) and \( \theta, \gamma < 1 \). Hence, labels become more informative when \( \theta \) or \( \gamma \) increases.

An investor’s label may be interpreted as a reputation derived from public information, such as the name or business type of firm, or the dealer’s belief about the investor’s type based on relationship specific information, such as past trading history with the dealer. Examples of h-investors include insurance firms, which are known to usually trade for hedging risk, or investors whose past trades did not impose a loss on the dealer.\(^9\) Conversely, s-investors may be hedge funds, which usually trade to exploit proprietary information, or investors whose past trades imposed losses on the dealer. The labels’ informativeness \( \{\theta, \gamma\} \) indicates the accuracy of such reputation or belief. Section 5 extends the model to include any number \( N \) of labels with arbitrary levels \( \{\theta_n, \gamma_n\} \) of informativeness. Public disclosures of investor-specific information, such as inventories, investors’ strategies or past trades, is represented by an increase in \( \theta \) or \( \gamma \).

Section 2.1 gives examples of policies corresponding to changes in \( \{\theta, \gamma\} \).

\(^9\) In Section 5, we provide an example of how a dealer can separate investors into different labels based on trade history.
Two trading venues are potentially accessible: an *exchange* and an *over-the-counter* (OTC) market. The exchange and the OTC market differ by protocol and pre-trade anonymity. At the start of the game, the market maker announces bid and ask prices on the exchange. All sell orders submitted to the exchange execute at the announced bid and all buy orders execute at the ask. Prices in the OTC market are non-public. To trade over-the-counter, investors must request quotes from the dealer, who provides the requestor with a binding bid and an ask. Requesting a quote reveals the requestor’s identity, so the dealer can condition prices on the requestor’s label. We show in Section 2.2 that the dealer price discriminates against s-investors, offering a wider bid-ask spread to the s-investors than the h-investors. In our main analyses, we compare the equilibrium in which the OTC market is open alongside the exchange, versus the equilibrium where the OTC market is closed.

Investors pay an infinitesimal cost of click $\epsilon$ to request a quote. That there is a cost to trade over-the-counter means investors only request quotes if one expects OTC prices to be strictly better than prices on the exchange. The cost of click represents the physical costs involved in finding a quote, such as additional ‘clicks’ needed to request quotes, and wait times following quote requests.

An infinitesimal cost to trade over-the-counter reflects today’s OTC market structure. If the cost $\epsilon$ were sufficiently high, no investor would trade over-the-counter. As most financial assets are traded both over-the-counter and on exchanges, this cost must not be so high. Moreover, OTC trades of equities and swaps mostly occur on competitive electronic platforms, which minimize delays and search costs of OTC trading. On the over 30 electronic swaps platforms, an investor can trade on a limit order book with a single click, or trade over-the-counter with a few clicks (SIFMA, 2016). Recent regulations are forcing OTC trading onto electronic platforms: the US Dodd-Frank Act forced swaps to trade electronically; and the EU MiFID II rules will force nearly all OTC trades onto electronic platforms from 2018 (Strachan, 2014). To check robustness, we analyze noninfinitesimal $\epsilon$ in Section 5.

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10 In practice, quote requests are often one-sided (either for a buy or sell). We assume two-sided requests (called ‘request-for-market’), as one-sided quotes with a continuum of fully endogenous investors immediately reveals the true asset value $v$ to the OTC dealer. Existing models (e.g., Zhu, 2014, Kyle, 1985) allow for one-sided quotes since the total demand by hedgers in these models are exogenous and random.

11 On electronic platforms, such as those offered by TradeWeb and Bloomberg, an investor seeking to trade an asset over-the-counter simultaneously submits requests for quotes to multiple dealers of her choice, who typically respond within seconds. Hendershott and Madhavan (2015) discusses the electronic request-for-quote procedure.
Investors’ types and labels drawn
Speculators receive signals \{q_i\} about the asset payoff \(v\)

<table>
<thead>
<tr>
<th>Investors’ types and labels drawn</th>
<th>Market maker posts bid and ask on the exchange</th>
<th>Each investor may trade at a price available to her</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speculators receive signals (q_i) about the asset payoff (v)</td>
<td>Investors may request quote from dealer over-the-counter</td>
<td>Asset pays (v) per unit</td>
</tr>
</tbody>
</table>

Trading proceeds in four steps. First, the market maker posts a bid and an ask on the exchange, and investors may alternatively request quotes over-the-counter. Second, investors who requested quotes receive their OTC bids and asks. Third, each investor buys or sells at a price available to her, or does not trade. Investors act simultaneously in each of the first three steps. Fourth, the asset payoff \(v\) realizes. Figure 2 summarizes the timing of the model. In the first step, investors choose from the actions

\[
\{\text{request a quote, no quote request}\}.
\]

After the dealer responds to quote requests, investors who did not request a quote choose from

\[
\{\text{buy or sell on exchange, no trade}\}
\]

whereas investors who did request a quote choose from

\[
\{\text{buy or sell over-the-counter, buy or sell on exchange, no trade}\}.
\]

Investors prefer to trade if otherwise indifferent. Figure 3 illustrates the investors’ choice sets.

Both the market maker and the dealer are competitive, earning zero expected profit on every
trade. On the exchange, the market maker posts a bid price $\text{bid}_e$ equal to her expectation of the asset payoff $\tilde{v}$ given $\text{bid}$ and that the investor sells. For an investor who requests a quote, the dealer offers a bid $\text{bid}_o$ equal to the same expectation except additionally conditioned on the investor’s label. The ask prices $\{\text{ask}_e, \text{ask}_o\}$ are determined likewise:

\[
\begin{align*}
\text{bid}_e &= \mathbb{E}[\tilde{v} | \text{bid}_e, \text{investor sells}] \\
\text{ask}_e &= \mathbb{E}[\tilde{v} | \text{ask}_e, \text{investor sells}] \\
\text{bid}_o &= \mathbb{E}[\tilde{v} | \text{bid}_o, \text{investor sells, investor’s label}] \\
\text{ask}_o &= \mathbb{E}[\tilde{v} | \text{ask}_o, \text{investor sells, investor’s label}].
\end{align*}
\]

2.1 Empirical Applications

We now apply our model to a few real-world examples. These examples refer to different kinds of assets, characterized (in our context) by adverse selection risk $\mu$ and the information parameters $\{\theta, \gamma\}$.

Assets with high $\mu$ are those traded mainly for speculative reasons, and whose trades are informative (such as stocks, commodities, foreign exchange). We associate low $\mu$ to assets primarily traded for hedging, and whose trades are uninformative (such as swaps, corporate bonds). To be precise, $\mu$ captures an asset’s inherent attractiveness to speculators. For example, equities values are typically more volatile than the values of fixed income assets, making the former more attractive to speculators. Our model endogenizes ex-post adverse selection risk, which depends on the relative sizes of speculators and hedgers who trade.

The parameters $\{\theta, \gamma\}$ set the quality of investor-specific information available to the dealer. As $\theta$ or $\gamma$ increases, the dealer can more accurately separate the hedgers from the speculators. An increase in $\theta$ or $\gamma$ can be a consequence of investor disclosure requirements, a reduction in investor anonymity, and other changes that give more investor-specific information to dealers.

2.1.1 Regulatory trend towards eliminating OTC markets

Regulators in the US, the EU, Japan and elsewhere are encouraging centralized, on-exchange trading of equities and derivatives with the goal of eliminating OTC markets. In the US, the 2009 Dodd-Frank Act mandates electronic trading of most swaps on Swaps Execution Facilities
(SEFs), which are required to offer a limit order book trading venue (alongside OTC trading). Regulators in Japan adopted similar regulations.\textsuperscript{12} European policy-makers are going further with MiFID II rules, which seek to migrate all OTC trading of equities and derivatives onto exchanges starting 2018. MiFID II prioritizes forcing trades of equities onto exchanges by, for instance, forcing dealers to publicly post binding bid and ask for those assets (Strachan, 2014). Eliminating trading venues that allow dealers to price discriminate corresponds to closing the OTC market in terms of the model. We address the market quality and welfare implications of closing the OTC market in the following sections.

\subsection*{2.1.2 Name give up in the swaps market}

Most swaps — from over 80\% of index credit default swaps (CDSs) and 65\% of interest rate swaps (IRSs) to 45\% of single-name CDSs\textsuperscript{13} — are traded on SEFs which offer two trading methods: request-for-quote (RFQ); and all-to-all (A2A) trading. Figure 4 provides the proportions of trades on electronic platforms for certain credit derivatives.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{state_of_electronification_in_various_asset_classes.png}
\caption{State of electronification in various asset classes} \label{state_of_electronification_in_various_asset_classes}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{proportions_of_trades_on電子流電子平.png}
\caption{Proportions of trades on electronic platforms for select credit derivatives (Nagel, 2016)} \label{proportions_of_trades_on電子流電子平}
\end{figure}

The RFQ replicates traditional OTC markets, and requires an investor to (non-anonymously) request prices from dealers before she can trade. All-to-all trading occur on limit order book venues, where trading is anonymous and prices are posted publicly. Speculation is rare in

\textsuperscript{12} Duffie (2017) provides an overview of post-recession regulatory trends in financial markets. Swaps mandates in Dodd-Frank Act and corresponding regulations in Japan are discussed in Duffie (2017, Sec. 1.17).

\textsuperscript{13} Nagel (2016, p. 9) summarizes the state of electronic trading in credit derivatives markets.
the swaps market (Hilscher et al., 2015, Augustin, Subrahmanyam, Tang, and Wang, 2014), implying a low adverse selection risk $\mu$.

The swaps market practices name give-up (NGU), a trading platform rule that requires counterparties in an A2A trade to reveal their identities to each other post-trade. As the same dealers provide much of the liquidity for both OTC and A2A segments of the swaps market, NGU increases the OTC dealers’ knowledge about swaps investors’ past trades, such that banning NGU corresponds to a decrease in $\theta$ or $\gamma$.

2.1.3 Implementation of the blockchain

A blockchain is an electronic recordkeeping procedure that broadcasts every transaction across a network. Each member of a blockchain network maintains a ledger of all trades. These ledgers are periodically reconciled with one another by a public algorithm. Blockchains use transparency to generate trust in the transactions record. Even if an attempt to manipulate the record succeeds, members of the blockchain network learn that the manipulation has occurred. Thus, non-anonymity is a fundamental element of blockchains. The Depository Trust & Clearing Corporation (DTCC) is planning to transfer its Trade Information Warehouse (TIW) onto a blockchain network. The TIW is the main recordkeeping database for credit derivatives, including swaps. To protect privacy, DTCC’s plans to include only select dealer banks in TIW’s blockchain network, sharing the trade records of credit derivatives investors only to those banks. Providing investors’ trading histories to OTC dealers would imply an increase in $\theta$ or $\gamma$.

2.1.4 The equities market

Over-the-counter trading of equities occur on the ‘upstairs’ market, which executes 18% of share volume (worth $195 billion) in US exchange-listed stocks (Tuttle, 2014). Several stock exchanges maintain upstairs venues (for example, Nasdaq, LSE, TSX, Paris Bourse), where institutional investors can trade over-the-counter with dealers. Outside exchanges, investors may request quotes directly from broker-dealers. Though the upstairs market is usually described as a market for large block trades, block trades actually comprise a small proportion

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14 Malinova and Park (2016), Khapko and Zocan (2016) describes blockchains in more detail.
15 Irrera (2017) provide an overview of DTCC’s blockchain projects.
of trades upstairs. In the US, less than 2.53% of trades upstairs are for the traditional block size of 10,000 shares or greater, and the average size of trades upstairs is 368 shares against 232 shares for non-upstairs trades (Tuttle, 2013, 2014). Alternative to trading upstairs include major exchanges (limit order book venues) and Alternative Trading Systems (ATSs). The ATSs include smaller limit order book venues, brokers matching the orders of their clients, and dark pools where orders are hidden. The upstairs market corresponds to the model’s OTC market, while the equities exchanges and ATSs is represented by the model’s exchange. Equities trades tend to be speculative (Kilian and Murphy, 2014, Cheng and Xiong, 2014), so adverse selection risk $\mu$ is high.

### 2.1.5 Investment fund disclosure rules

Most financial assets are traded by investment funds, which face many disclosure rules. The investment funds include index funds that replicate equities market indices (such as, S&P500), fixed income funds that trade debt instruments (such as bonds), and hedge funds. Disclosure rules have focused on mutual funds, which must periodically disclose their portfolio composition and strategies in most jurisdictions. Recent updates to US regulations have forced the mutual funds to be more precise in their strategy disclosures (SEC, 2014). From 2013, the EU and Australia requires hedge funds to publicly disclose their leverage and trades related to risk management. Broader and more stringent disclosure requirements imply an increase in $\theta$ or $\gamma$.

### 2.2 Equilibrium

A perfect Bayesian equilibrium consists of the market maker’s and the dealer’s respective quoting strategies, investors’ trading strategies, and consistent beliefs. In equilibrium, the dealer maximizes orderflow subject to earning a zero expected profit and no investor can profitably deviate. We outline the derivation of equilibrium (Theorem 1) in this section.

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17 Investment funds refer to any firms that invest clients’ capital, then charge commission and fees on resulting returns.

18 Relevant regulations are the Alternative Investment Fund Managers Directive (AIFMD) in the EU, and Regulatory Guide 240 in Australia (both came into force in 2013). Easley, O’Hara, and Yang (2014) discusses hedge fund regulations in the US (which does not require public disclosures).
Complete proofs are presented in Appendix B.

We first derive the investors’ trading strategies when facing competitive prices. The competitive bid and ask are conditional expectations of the asset payoff $\tilde{v}$. As the asset payoff is bounded by 1 and $-1$, any ask price $ask$ is less than 1 and any bid price $bid$ is higher than $-1$. Hence, a speculator earns the profit of $\mathbb{E}[v|q_i = 1] - ask$ if she buys and $bid - \mathbb{E}[v|q_i = -1]$ if she sells. Since $q_i$ is correct with probability $\alpha$, we have

$$\mathbb{E}[v|q_i = 1] = -\mathbb{E}[v|q_i = -1] = 2\alpha - 1.$$ 

Then competitive pressure ensures $ask \leq 2\alpha - 1$ and $bid \geq -(2\alpha - 1)$.

A hedger’s expectation of $\tilde{v}$ is zero: if a hedger buys, she expects to lose $ask$, and $-bid$ if she sells. However, a hedger receives a hedging benefit $b_i$ when she trades, so the hedger buys if she is a buyer and $b_i$ is larger than her lowest ask. A hedger sells if she is a seller and $b_i$ exceeds the negative of her highest bid.

That speculators impose adverse selection leads to an intuitive outcome. Because speculators know the asset payoff $v$, the market maker and the dealer take a loss whenever they trade with a speculator. Thus, they offer better prices (higher bid and lower ask) to an investor labeled as hedgers (h-investors), who are less likely to impose such adverse selection than the investors labeled as speculators (s-investors). Consequently, h-investors wish to separate from s-investors while the s-investors prefer to pool with the h-investors. If the h-investors trade on the exchange, the s-investors can mimic them and pool. In the OTC market, the dealer sees the investor before quoting prices, making pooling infeasible. The h-investors trade over-the-counter to separate from the s-investors, while the s-investors avoid paying the cost of click by trading on the exchange. The OTC dealer thereby cream-skims the h-investors from the exchange. This cream-skimming effect causes a disproportionate share of hedgers to trade over-the-counter, clustering the speculators on the exchange.

**Proposition 1** (Cream-skimming). The h-investors only trade in the OTC market and the s-investors only trade on the exchange.

The cream-skimming result of Proposition 1 does not require the dealer to be competitive. Given prices, a monopolistic dealer expects larger profit from h-investors, whose outside option
is to trade on the exchange. Accordingly, the dealer will offer a narrower bid-ask spread to the h-investors, allowing her to cream-skim them from the exchange’s market maker. The market maker cannot compete, as reducing the exchange spread raises her expected loss against the s-investors.

Now we derive the competitive bid and ask prices consistent with investors’ strategies. Competitive prices equal conditional expectations of the asset payoff $\tilde{v}$ whose arguments differ across markets. By the Bayes’ rule, the expectation of $\tilde{v}$ is

$$E[\tilde{v}] = \Pr(v = 1|\cdot) - \Pr(v = -1|\cdot) = \frac{\Pr(\cdot|v = 1) - \Pr(\cdot|v = -1)}{\Pr(\cdot|v = 1) + \Pr(\cdot|v = -1)}.$$

The bid price $\text{bid}$ and the ask price $\text{ask}$ are

$$\text{bid} = \frac{\Pr(\cdot, \text{investor sells}|v = 1) - \Pr(\cdot, \text{investor sells}|v = -1)}{\Pr(\cdot, \text{investor sells}|v = 1) + \Pr(\cdot, \text{investor sells}|v = -1)}$$

$$\text{ask} = \frac{\Pr(\cdot, \text{investor buys}|v = 1) - \Pr(\cdot, \text{investor buys}|v = -1)}{\Pr(\cdot, \text{investor buys}|v = 1) + \Pr(\cdot, \text{investor buys}|v = -1)}.$$

Cream-skimming means only the s-investors trade on the exchange, then (·) includes that investors who trade on the exchange are s-investors and those trading over-the-counter are h-investors. If an “e” and “o” denote the exchange and the OTC market, respectively, the bid prices are

$$\text{bid}_e = \frac{\Pr(s\text{-investor sells}|v = 1) - \Pr(s\text{-investor sells}|v = -1)}{\Pr(s\text{-investor sells}|v = 1) + \Pr(s\text{-investor sells}|v = -1)}$$

$$\text{bid}_o = \frac{\Pr(h\text{-investor sells}|v = 1) - \Pr(h\text{-investor sells}|v = -1)}{\Pr(h\text{-investor sells}|v = 1) + \Pr(h\text{-investor sells}|v = -1)}.$$

As hedgers are buyers or sellers with equal probability whatever is $v$,

$$\text{bid}_e = \frac{[\Pr(s\text{-speculator sells}|v = 1) - \Pr(s\text{-speculator sells}|v = -1)] \theta \mu}{\Pr(s\text{-speculator sells}) \theta \mu + \Pr(s\text{-hedger sells})(1 - \gamma)}$$

$$\text{bid}_o = \frac{[\Pr(h\text{-speculator sells}|v = 1) - \Pr(h\text{-speculator sells}|v = -1)] (1 - \theta) \mu}{\Pr(h\text{-speculator sells}) (1 - \theta) \mu + \Pr(h\text{-hedger sells}) \gamma}.$$

Using that the speculators’ signals are independent, a mass $\alpha$ of the speculators sells and mass $1 - \alpha$ buys if $v = -1$. In addition, only the hedgers with hedging benefits larger than bid sells,
and thus:

\[ \begin{align*}
\text{bid}_e &= -\frac{(2\alpha - 1)\theta \mu}{\theta \mu + (1 - \gamma)(1 - \text{bid}_e)} \\
\text{bid}_o &= -\frac{(2\alpha - 1)(1 - \theta)\mu}{(1 - \theta)\mu + \gamma(1 - \text{bid}_o)}.
\end{align*} \]

Replacing ‘sells’ with ‘buys’ in (1) and following the same steps gives the ask prices. We soon show that the equilibrium bids and asks are unique. Then the bids and asks are symmetric around zero:

\[ \begin{align*}
\text{ask}_e &= \frac{(2\alpha - 1)\theta \mu}{\theta \mu + 1 - \text{bid}_e} = -\text{bid}_e \\
\text{ask}_o &= \frac{(2\alpha - 1)(1 - \theta)\mu}{(1 - \theta)\mu + \gamma(1 - \text{bid}_o)} = -\text{bid}_o.
\end{align*} \]

A (half) bid-ask spread \( s \) thereby characterizes the equilibrium prices through the relations

\[ \begin{align*}
\text{s}_e &= \text{ask}_e = -\text{bid}_e \\
\text{s}_o &= \text{ask}_o = -\text{bid}_o
\end{align*} \]

and (2).\(^{19}\)

The bid-ask spreads \( \{\text{s}_e, \text{s}_o\} \) are equal to the realized adverse selection risk, defined as the probability that an investor who trades is a speculator with the correct signal \( q_i = v \). Intuitively, by offering a spread \( s \) to an investor, the dealer expects the loss of \( (2\alpha - 1)(1 - s) \) if the investor is a speculator and the profit of \( s \) from a hedger if the hedger trades. The zero profit condition implies

\[
\frac{(2\alpha - 1)(1 - s) \cdot \Pr(\text{speculator} | \cdot)}{\Pr(\text{speculator} | \cdot)} = s \cdot \left[1 - \Pr(\text{speculator} | \cdot)\right].
\]

Rearranging, we get

\[
s = (2\alpha - 1) \cdot \Pr(\text{speculator} | \cdot).
\]

The spread \( \text{s}_e \) on the exchange and the OTC spread \( \text{s}_o \) each solve a fixed point problem in

\(^{19}\) Quoted and effective spreads are the same in our model.
the form of, for $\beta \in \left\{ \frac{\theta \mu}{1-\gamma}, \frac{(1-\theta)\mu}{\gamma} \right\}$,

$$s(\beta) = \frac{(2\alpha - 1)\beta}{\beta + 1 - s(\beta)}. \quad (4)$$

Equation (4) has a unique solution in the interval $[0, 2\alpha - 1]$. Choosing $\beta$ appropriately, the spreads $\{s_e, s_o\}$ are

$$s_e = \frac{1}{2} \left( 1 + \frac{\theta \mu}{1-\gamma} - \sqrt{\left(1 - \frac{\theta \mu}{1-\gamma}\right)^2 + 8(1-\alpha)\frac{\theta \mu}{1-\gamma}} \right)$$

$$s_o = \frac{1}{2} \left( 1 + \frac{(1-\theta)\mu}{\gamma} - \sqrt{\left(1 - \frac{(1-\theta)\mu}{\gamma}\right)^2 + 8(1-\alpha)\frac{(1-\theta)\mu}{\gamma}} \right). \quad (5)$$

**Theorem 1.** With the OTC market, there exists a unique equilibrium. In equilibrium, prices are characterised by spreads $\{s_e, s_o\}$ and equations (2) and (5), a speculator buys if $q_i = 1$ and sells if $q_i = -1$, and a hedger trades only if her hedging benefit is larger than her lowest spread.

The equilibrium of Theorem 1 is intuitive. The speculators trade on their private information whereas the hedgers trade only if one’s hedging benefit is larger than the trading cost. Further, the spreads are increasing in adverse selection risk, such that investors labeled as speculators pay a wider spread than those labeled as hedgers. Moreover, the h-investors receive a lower spread over-the-counter, leaving others to trade on the exchange.

Our main analyses compare the equilibrium of Theorem 1 to an equilibrium in the absence of the OTC market. Then the h-investors and the s-investors are pooled on the exchange, and there is one spread $s_e$. Steps used to derive the spreads in (5) yields that the spread $s_e$ without the OTC market is:

$$s_e = \frac{1}{2} \left( 1 + \mu - \sqrt{(1-\mu)^2 + 8(1-\alpha)\mu} \right). \quad (6)$$

**Theorem 2.** Without the OTC market, there exists a unique equilibrium. In equilibrium, prices are characterized by the spread $s_e$ in (6), a speculator buys if $v = 1$ and sells if $v = -1$, and a hedger trades only if her hedging benefit is larger than her lowest spread.
2.3 Discussion

Our form of cream-skimming does not require repeated interactions through which the dealer can discipline investors, nor that the dealer be privately informed or have the ability to contract with investors. Rather, cream-skimming arises in our setting due to self-sorting by investors: h-investors want price discrimination, leading them into the OTC market. Three existing papers provide alternative forms of cream-skimming. In Bolton, Santos, and Scheinkman (2016), dealers are privately informed about the true values of assets, and can contract with asset issuers outside exchanges, allowing the dealers to cream-skim the best assets. Desgranges and Foucault (2005) shows that a monopolistic dealer can induce investors to only submit uninformed orders by raising an investor’s spread each time the investor imposes a loss on the dealer. Easley and O’Hara (1987) features the cream-skimming of orderflows, as competing exchanges pay brokers for uninformed orders.

The modeling framework and equilibrium structure we use complement several strands of prior literature. First, microstructure models of adverse selection and venue choice either force uninformed investors to trade at particular venues (Chowdhry and Nanda, 1991, Hendershott and Mendelson, 2000), or exogenously fix their trading demands (Admati and Pfleiderer, 1988, Zhu, 2014). In our setting, the hedgers are not restricted in their venue choice or participation decisions. Second, investors in recent models with venue selection choose between an exchange with competitive prices or another venue whose price is an average of the exchange prices. All prices in our model are determined competitively. Third, we generate price discrimination as a competitive outcome of investor self-sorting. Existing models show that price discrimination may arise from monopolistic screening (Benveniste, Marcus, and Wilhelm, 1992), repeat interactions (Zhu, 2012), ordersize differences (Easley and O’Hara, 1987), or search frictions (Duffie, Garleanu, and Pedersen, 2005).

Our model relates to Glosten (1994) which predicts that, under general conditions, limit order book venues would dominate market share. We offer OTC price discrimination as an explanation for why limit order books do not dominate in practice.

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20 The alternative trading venues to an exchange in recent models include dark pools (Zhu, 2014, Brolley, 2016, Buti et al., 2017) and crossing networks (Hendershott and Mendelson, 2000, Degryse, Van Achter, and Wuyts, 2009).
2.4 Empirical Implications

We now state and evaluate the empirical implications of the equilibrium. This section focuses on the equilibrium with the OTC market as, in practice, OTC trading exists for most assets. Specifically, we answer three questions:

1. How do the spreads and trades on the exchange differ from those in the OTC market?
2. How are the exchange and the OTC market shares affected by adverse selection risk?
3. What is a test of the model?

For what follows, exchange volume is the equilibrium mass of trades executed on the exchange. The OTC market volume is analogously defined. With the OTC market, the exchange volume $V_e$ equals the total trades by the $s$-investors, whereas the OTC market volume $V_o$ is the $h$-investors’ total trades:

$$V_e = \theta \mu + (1 - \gamma)(1 - s_e)$$
$$V_o = (1 - \theta)\mu + \gamma(1 - s_o).$$

Total trade volume $V$ is the sum of $V_e$ and $V_o$. Without the OTC market, the exchange volume $\hat{V}_e$ is the total volume $\hat{V}$:

$$\hat{V}_e = \mu + 1 - \hat{s}_e = \hat{V}.\quad (8)$$

Proposition 2. Fix $\{\alpha, \theta, \gamma\}$. With the OTC market:

1. the spread $s_e$ on the exchange is strictly higher than the OTC spread $s_o$;
2. the OTC market share $V_o/V$ is strictly decreasing in adverse selection risk $\mu$; and
3. the exchange market share $V_e/V$ and the exchange spread $s_e$ are strictly increasing in $\mu$.

Our model predicts a narrower bid-ask spread in the OTC market than on the exchange, since the investors who trade over-the-counter are less likely to be speculators than those on the exchange. Empirically, trading costs are higher over-the-counter than on exchanges, and OTC trades are less informed (Westerholm, 2009, Bessembinder and Venkataraman, 2004,
Jain, Jiang, and Mcinish, 2003, Booth, Lin, Martikainen, and Tse, 2002, Smith et al., 2001, Madhavan and Cheng, 1997). For example, Rose (2014) finds higher shares of trades executed by dealers (or ‘upstairs’ trades) correspond to lower average effective spreads, while the upstairs trades are less predictive of price changes than those on the limit order book.

OTC market share is declining in adverse selection risk, as the speculators are less likely to be cream-skimmed into the OTC market. Table 1 separates asset types into ones primarily traded over-the-counter versus assets mostly traded on exchanges. Trading patterns in the table are broadly consistent with our prediction: we expect fixed income assets to attract less informed trading than equities,\(^\text{21}\) and it is the fixed income assets that are mostly OTC traded. In addition, government bond futures are primarily traded over-the-counter whereas the underlying bonds are mostly traded on exchanges. Evidence suggests speculators in government bonds mainly trade futures, not the underlying. Futures prices explain about 70\% of prices changes in 10-year maturity US treasuries (Mizrach and Neely, 2008b), Canadian government bonds (Campbell and Hendry, 2007), and the German bund (Upper and Werner, 2002).\(^\text{22}\)

Table 1 – Asset Types and Primary Trading Method in the US

<table>
<thead>
<tr>
<th>Primarily OTC Traded</th>
<th>Primarily Exchange Traded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate bonds (Biais and Green, 2007)</td>
<td>Listed equities (Tuttle, 2014)</td>
</tr>
<tr>
<td>Municipal bonds (Biais and Green, 2007)</td>
<td>Equity options</td>
</tr>
<tr>
<td>Government bonds</td>
<td>(Nybo, Sears, and Wade, 2014)</td>
</tr>
<tr>
<td>(Mizrach and Neely, 2008a)</td>
<td>Government bond futures</td>
</tr>
<tr>
<td>Credit default swaps (Riggs et al., 2017)</td>
<td>(Mizrach and Neely, 2008a)</td>
</tr>
<tr>
<td>Interest rate swaps (Nagel, 2016)</td>
<td>Exchange traded funds (Stafford, 2016)</td>
</tr>
<tr>
<td>Repos (Han and Nikolaou, 2016)</td>
<td></td>
</tr>
<tr>
<td>Foreign exchange</td>
<td></td>
</tr>
<tr>
<td>(Rime and Schrimpf, 2013)</td>
<td></td>
</tr>
</tbody>
</table>

Proposition 2 can explain why a smaller proportion of US exchange-listed equities are traded over-the-counter (17\% by dollar volume Tuttle, 2014) than in the corresponding options (42\% Nybo et al., 2014). Evidence suggests adverse selection risk \(\mu\) is lower in options: trade volumes is larger for option than equities, yet options prices follow changes in equity prices (Muravyev,

\(^{21}\) By design, fixed income assets have less volatile prices than equities. Since speculators profit on the difference between current and future prices, we expect more informed trading in equities. For instance, stock prices are informative about prices of credit default swaps prices but not vice versa (Hilscher et al., 2015).

Pearson, and Paul Broussard, 2013, Chakravarty, Gulen, and Mayhew, 2004), consistent with less informed trading in options. If \( \mu \) is lower in options, Proposition 2 predicts higher OTC market share in options, as seen empirically.

That the OTC market share declines in adverse selection risk \( \mu \) implies the exchange market share \( V_e/V \) increases in the exchange spread \( s_e \), for \( V_e/V \) and \( s_e \) are both increasing in \( \mu \). A simple test follows. Our model predicts a positive correlation between the proportion of trades on exchange and the exchanges’ spreads. Example 1 illustrates this correlation.

**Example 1.** Suppose \( \alpha = 0.8 \) and \( \gamma = \theta = 0.6 \). We independently draw 100 values of \( \mu \) from the distribution \( \mathbb{U}[0, 1] \). On each draw, \( s_e \) and \( V_e/V \) are computed, which are averaged across the 100 draws. We repeat this process 100 times, then plot the results in Figure 5a. Figure 5b plots the exchange market share \( V_e/V \) and its spread \( s_e \) as \( \mu \) varies.

![Figure 5 – Example 1](image)

On right (Figure 5a): we set \( \gamma = \theta = 0.6 \) and independently draw 100 values of \( \mu \) from the distribution \( \mathbb{U}[0, 1] \). For each draw, we compute \( s_e \) and \( V_e/V \) then average them across the 100 draws. This process is repeated 100 times and plotted.

Equivalent plot to Figure 5a is easily produced using transactions data that contains quantity, prices and an indicator of whether a trade was executed over-the-counter or on an exchange. In the US, Financial Industry Regulatory Agency (FINRA) collects such data for exchange-listed equities, although it is not publicly accessible.

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23 Muravyev et al. (2013) finds that 10% of price changes in US stocks and options first occur in stock prices, and (Chakravarty et al., 2004) reports 10 to 17% for the same figure (which vary across stocks).

3 Welfare Analysis

Now we analyze how the OTC market and investor anonymity affect welfare and market quality. We measure welfare by the investors’ total payoffs, and market quality by total trade volume and average bid-ask spread.

This section states three main results. First, having the OTC market may reduce welfare yet raise volume and narrow the average spread. Second, the OTC market harms welfare when its market share is high. Third, reducing anonymity in the presence of the OTC market has the same qualitative effects as having the OTC market. The third result has implications for current policy debates, which we discuss in Section 4.

3.1 Welfare and the average bid-ask spread

Welfare is the expected sum of all agents’ payoffs. Competitive prices imply zero profit for the dealer and the market maker, and that the speculators′ profit equals the hedgers′ trading losses. Then welfare equals the total hedging benefit attained through trade. With the OTC market, an investor who trades pays either the OTC spread $s_o$ if she is a h-investor or the exchange spread $s_e$ if she is a s-investor. Hence, welfare $W$ is given by

$$ W = (1 - \gamma) \int_{s_o}^{\infty} s \, ds + \gamma \int_{s_{o}}^{\infty} s \, ds, $$

which simplifies to

$$ W = \frac{1}{2} \left[ 1 - \gamma s_{o}^{2} - (1 - \gamma) s_{e}^{2} \right]. \quad (9) $$

Without the OTC market, every investor is offered the same spread $s_{e}$, and welfare $\hat{W}$ equals

$$ \hat{W} = \frac{1}{2} \left[ 1 - \hat{s}_{e}^{2} \right]. \quad (10) $$

Average spread is the trade volume-weighted mean of spreads. The average spread $\hat{s}$ without the OTC market is trivially $\hat{s}_{e}$, while the average spread $\bar{s}$ with the OTC market is the volume-
weighted mean of \( s_o \) and \( s_e \):

\[ \bar{s} = \frac{V_o}{V} s_o + \frac{V_o}{V} s_o. \]  

(11)

An increase in total volume and a decrease in the average spread are equivalent. Due to competitive prices, the the dealer’s and the market maker’s total revenue, the average spread times the total volume, must equal their total loss, the amount of speculators’ trades times the expected loss on each trade against a speculator:

\[ V \cdot \bar{s} = \mu \cdot (2\alpha - 1), \]  

(12)

implying that the average spread is decreasing in total volume. In subsequent sections, we write ‘total volume increases’ and ‘the average spread decreases’ interchangeably.

### 3.2 Effects of having the OTC market

The following proposition compares the equilibria with the OTC market (Theorem 1) to the one without (Theorem 2). We say ‘having the OTC market increases welfare’ if welfare \( W \) with the OTC market is larger than \( \hat{W} \) without. Analogously, having the OTC market decreases average spread (total volume) if \( \bar{s} (V) \) is below \( \hat{\bar{s}} (\hat{V}) \).

**Proposition 3** (Cheap substitution). Fix \( \{\alpha, \theta, \gamma\} \). There exists \( \mu^* > 0 \) such that having the OTC market:

1. decreases welfare if \( \mu \leq \mu^* \) and increases welfare if \( \mu > \mu^* \); and
2. increases total volume and decreases the average spread.

Figure 6 illustrates Proposition 3 numerically, when \( \alpha = 0.9 \) and \( \theta = \gamma = 0.6 \). The figure plots the change in welfare, total volume and the average spread from introducing the OTC market for varying levels of adverse selection risk \( \mu \). Red dotted lines represent zero change: above a red line indicates a higher value with the OTC market than without. We observe that having the OTC market increases total volume and reduces the average spread, while it increases welfare only at low \( \mu \).
Proposition 3 states that the OTC market can harm welfare yet increase total volume. This is because volume and average spreads weights trades equally, but welfare depends on the hedging benefits that each trade attains. Having the OTC market narrows the spread for the h-investors which induces hedgers, called *entrants*, who otherwise would *not* trade to do so. Meanwhile, the s-investors are offered a wider spread such that some hedgers, called *exiters*, do not trade though they *would* without the OTC market. The total volume increases whenever the entrants outnumber the exiters. However, the exiters have individually larger hedging benefits than the entrants: in terms of the hedging benefit, the entrants make *cheap substitutes* for the exiters, meaning welfare can decline even if there are more entrants. Figure 7 illustrates why the entrants are cheap substitutes.

All investors face the spread $\hat{s}_e$ without the OTC market, so the marginal hedger who trades has a hedging benefit of $e$. Having the OTC market means the h-investors receive a lower spread $s_h < \hat{s}_e$ and the s-investors get a higher spread $s_e > \hat{s}_e$; therefore, the marginal hedger has a hedging benefit of $d$ if she is a h-investor, or $f$ if she is a s-investor. Because $d$ is lower than $f$, the entrants trade only at spreads below $\hat{s}_e$ while the exiters would trade even at $\hat{s}_e$. Thus, each entrant’s hedging benefit must be smaller than the exiters’ benefits, such that the entrants are cheap substitutes for the exiters.

Having the OTC market harms welfare when adverse selection risk $\mu$ is low and improves welfare when $\mu$ is high. We use Figure 8 to aid exposition.

When adverse selection risk is low, the spread $\hat{s}_e$ is low without the OTC market, which leaves little scope for the h-investors’ spread to fall but large scope for the s-investors’ spread to widen. The left diagram in Figure 8 illustrates this intuition. Conversely, $\hat{s}_e$ is high if $\mu$ is
Figure 7 – Cheap Substitution

On the vertical axis is the hedging benefits and on the horizontal axis is the spread. The diagonal line marks the hedging benefit of the marginal hedger who trades given that spread. For example, at the spread \( \hat{s}_e \), the marginal hedger has a hedging benefit of \( e \).

Figure 8 – Welfare Effects of Having the OTC market

In each triangle, on the vertical axis is the hedging benefits and on the horizontal axis is the spread. The diagonal line marks the hedging benefit of the marginal hedger who trades given that spread. For example, at the spread \( \hat{s}_e \), the marginal hedger has a hedging benefit of \( e \).

high, so the potential price improvement for the h-investors is large while the s-investors’ spread cannot be much wider. This intuition is shown on the right diagram, in which a red vertical line marks the largest possible competitive spread.\(^{25}\) Moreover, having the OTC market increases total volume, since proportionally more hedgers receive a reduced spread than the speculators.

\(^{25}\) The largest possible competitive spread is \( [2\alpha - 1] \). As the dealer expects a loss of \( [2\alpha - 1] \) on each trade with a speculator, the limit of competitive spreads when \( \mu \) becomes unboundly large is \( [2\alpha - 1] \).
Determining the level of adverse selection risk is difficult in practice. By contrast, calculating the OTC market share $V_o/V$ using regulatory data is easy, as trade reports must state whether the trade was executed over-the-counter.\textsuperscript{26} The next proposition gives a condition on $V_o/V$ for the OTC market to improve welfare.

**Proposition 4.** Fix $\{\alpha, \theta, \gamma\}$ and suppose the OTC market exists. There exists $v_o^* > 0$ such that having the OTC market decreases welfare if OTC market share $V_o/V > v_o^*$, and increases welfare if $V_o/V < v_o^*$.

The OTC market improves welfare only if the OTC market share $V_o/V$ is low. That the speculators are less likely to be cream-skimmed into the OTC market implies $V_o/V$ is high when adverse selection risk $\mu$ is low, which is when having the OTC market harms welfare. An implication is that closing the OTC market for assets mostly traded over-the-counter, and keeping it for assets mostly traded on exchanges, would improve welfare. Referring to Table 1, closing the OTC market is closed for swaps is consistent with increasing welfare, whereas the OTC market should be kept for listed equities.

Equilibrium outcome that the speculators always trade does not drive our results. Appendix A shows our results still hold when the speculators’ demand is arbitrarily elastic with respect to their spreads. Our results hold as they only require that the equilibrium spreads increase whenever the ex ante share of investors who are speculators increases. Even with elastic speculator demand, a higher ex ante share of speculators raises adverse selection risk, and therefore the equilibrium spreads.

### 3.3 Effects of reducing investor anonymity

Investor anonymity impedes dealers’ ability to accurately discern speculators from hedgers. The model captures this aspect of anonymity by imperfect informativeness of investors’ labels. Changes that increase public disclosures (e.g., mutual fund disclosure rules) or sharing of private information (e.g., blockchain adoption) reduces investor anonymity. Less investor anonymity is represented here as an increase in the informativeness of investors’ labels or, equivalently, an increase in $\{\theta, \gamma\}$. We focus, without loss of generality, on $\theta$ below.

\textsuperscript{26} In the US, FINRA maintains the OTC Reporting Facility (ORF) that records off-exchange trades of exchange-listed equities. European Market Infrastructure Regulation (EMIR) requires trade reports to state if the trade was executed over-the-counter.
Proposition 5. Fix \( \{\alpha, \gamma\} \). With the OTC market, there exists \( \mu^*(\theta) > 0 \) such that, when \( \theta \) increases:

1. the OTC spread decreases and the exchange spread increases;
2. welfare decreases if \( \mu < \mu^*(\theta) \) and increases if \( \mu > \mu^*(\theta) \); and
3. total volume increases and the average spread decreases.

Reducing anonymity has analogous effects on welfare and market quality as introducing the OTC market. Less anonymity can reduce welfare while increasing total volume; specifically, it harms welfare when adverse selection risk \( \mu \) is low, and always increases total volume. As anonymity declines, the h-investors’ spreads improve and the s-investors’ spreads worsen. These are also the effects from having the OTC market, and the subsequent intuitions are the same. In the following section, we examine current policy debates using Proposition 5.

4 Implications for Recent Policies

We revisit the policies described in Section 2.1 using light of the finds in preceding sections.

4.1 Aims of Dodd-Frank Act and MiFID II

Both Dodd-Frank and MiFID II aim to migrate trading away from OTC markets. However, Dodd-Frank covers the swaps market while MiFID II targets nearly all financial securities, with the equities market as the priority for ending OTC trading. Swaps markets are predominated by investors with hedging motives (e.g., insurance companies), so \( \mu \) is low. Then we predict that closing the OTC market for swaps is likely to benefit welfare (Proposition 3). Conversely, 83% dollar value of equities trades are nonOTC, which suggests \( \mu \) is high and thus eliminating the OTC market in equities is likely to harm welfare (Proposition 4). In sum, the aims of Dodd-Frank and MiFID II to migrates trades in swaps and other assets that face low adverse selection (e.g., corporate bonds) are predicted to improve welfare, but the priority of MiFID II to eliminate OTC trading in equities is likely to harm welfare.
4.2 Abolishing name give up in swaps market

Dealers and swaps trading platforms insist on maintaining name give up (NGU) whereas buy-side firms strongly oppose the practice. Dealers claim NGU reduces their risk as it provides information about the dealers’ on-exchange trades. In addition, dealers claim eliminating NGU would discourage dealers from liquidity provision due to the rise in risk. Buy-side firms claim NGU leaks their private information to dealers, as the firms’ on-exchange trades are disclosed to their counterparties (who are usually dealers). The firms in particular suggest that NGU undermines swaps exchanges, where only 5% of swaps trades occur (McPartland, 2014).

Abolishing name give up reduces dealers’ information about investors past trades, corresponding to a decrease in $\theta$ or $\gamma$. Given low adverse selection $\mu$ in the swaps market, we predict that eliminating NGU will benefit welfare even as trade volume decreases and the average spread widens. Specifically on swaps exchanges, both trade volume and average spread will improve. Our results reconcile dealers’ claim that abolishing NGU will harm liquidity with buy-side claims that NGU harms swaps exchanges and market participants at large. The results also explain the swaps platforms’ insistence on keeping NGU. Name give up raises total trade volume and the platforms charge a fee per trade.

4.3 Implementation of the blockchain

Depository Trust & Clearing Corporation (DTCC), authority for reporting in trades of credit derivatives, is implementing an update to the repository for credit default swaps (CDSs) and repurchase agreements (repos) based on the blockchain technology. This new repository will employ large broker-dealer banks and other financial institutions as members of a blockchain network, and only these members will partake in the collection and reconciliation of transactions in CDSs and repos.

The broker-dealer banks are the primary dealers in CDS and repo markets, which corresponds to an increase in $\theta$ and $\gamma$. Our model would then predict that the OTC spreads will decline, and the spreads on exchanges will increase, with the new repository. For welfare, as

27 Buy-side firms requested the US Commodity Futures Trading Commission (CFTC) to ban NGU in 2014. CFTC declined to take action in 2015, then a group of buy-side firms sued leading swaps platforms, citing NGU as one of grievances. This lawsuit was settled out of court in 2016. Managed Funds Association (2015) summarizes positions for and against NGU.
CDSs and repos are mostly traded over-the-counter, the new repository is predicted to reduce welfare, though increase total volume and reduce the average spread (Proposition 5).

Other plans to apply blockchains typically focus on mainly OTC traded assets, markets in which potential cost savings from automation are larger.\textsuperscript{28} We provide a information-based reason to believe adopting the blockchain to such assets may reduce welfare.

4.4 Investment fund disclosure rules

Investment disclosure rules increase investor-specific information available to dealers, represented by an increase in $\theta$ and $\gamma$. For example, forcing mutual funds to release more precise summaries of their strategies (e.g., US SEC’s 2014 disclosure guidance) reduces noise in the public perception of the funds’ true strategies. Similarly, rules requiring hedge funds to publicly disclose leverage and risk management practices (e.g., EU AIFMD, Australia’s Regulatory Guide 240), helps to separate hedge funds that rarely hedge apart from those that often do. Empirical evidence suggests such disclosures by investment funds are informative.\textsuperscript{29} Then we expect fund disclosure rules to reduce the spreads over-the-counter and raise spreads on exchanges for assets frequently traded by investment funds. Further, as trades by investment funds comprise a larger proportion of trades for an asset, the increase in $\theta$ and $\gamma$ from disclosure rules would be higher. Thus, we predict that the effect of fund disclosure rules on spreads will be larger for assets in which a higher share of trades are by the affected investment funds.

The welfare consequences of fund disclosure rules would also vary by asset. If a disclosure rule targets funds that tend to trade equities (e.g., index mutual funds), we expect welfare to improve (Proposition 5). But we expect welfare to decrease if the rule targets funds that trade mostly OTC assets, such as swaps.

5 Robustness

In this section, we emphasize the generality of cream-skimming. To do so, we generalize the model of Section 2 (the main model) to an arbitrary partition of investors, noninfinitesimal

\textsuperscript{28} Edelen (2016), Johnson (2016) describe ongoing blockchain projects in the financial industry. \textsuperscript{29} Agarwal, Mullally, Tang, and Yang (2015) shows more frequent mutual fund disclosures reduced the returns of more informed funds. Brown, Goetzmann, Liang, and Schwarz (2008) finds that a temporary hedge fund disclosure rule reduced information asymmetry between small and large investors.
cost $\epsilon > 0$ to trade over-the-counter, and hedging benefits drawn from any distribution that meets mild conditions.

5.1 Cream-skimming with arbitrary partitions of investors

We now define the $N$-Label Model. The model generalizes the distribution of hedging benefits to any distribution $F$ that satisfies some regularity conditions that the cdf and pdf of $F$ be differentiable, and that the pdf be not too high.

**Definition 1.** The $N$-Label Model is the model of Section 2 altered in three ways:

1. hedging benefits $\{b_i\} \overset{iid}{\sim} F$;
2. investors are not labeled $\{s-, h-\}$ and, instead, each investor has one of $N \geq 2$ labels; and
3. cost of click $\epsilon$ can be any positive number.

The cdf $F(\cdot)$ of $F$ satisfies $F(0) < 1$.

We index the $N$ labels by $n \in \{1, \ldots, N\}$. Label $n$ consists of mass $\theta_n$ of speculators and $\gamma_n$ of hedgers. The labels are ascendingly ordered in the proportion $\theta_n/\gamma_n$ of speculators with each label, such that $n' > n$ implies $\theta_{n'}/\gamma_{n'} > \theta_n/\gamma_n$.

**Proposition 6.** We let $N^*(\epsilon) < N$ be a positive integer-valued function as defined in Appendix C.4. There exists a unique equilibrium of the $N$-Label Model in which every investor with label $n \leq N^*(\epsilon)$ only trades over-the-counter, and all other investors only trade on the exchange.

Proposition 6 states that, in the unique equilibrium of the $N$-Label Model, the OTC dealer cream-skims investors in the groups with the lowest adverse selection risk from the exchange. The equilibrium of Proposition 6 is constructed iteratively. Start in the case where every investor trades on the exchange. Clearly, investors with label 1 (label 1 investors) have most to gain from deviating to the OTC market, as label 1 investors have the smallest adverse selection risk (and so are subsidizing investors of all other labels). If the cost $\epsilon$ to access the OTC market is not too high, label 1 investors trade in the OTC market instead. Among the investors who remain on the exchange, label 2 investors have most to gain from deviating and,
if \( \epsilon \) is not too high, they too rather trade in the OTC market. This process stops once even investors with the most to gain finds \( \epsilon \) too high for a profitable deviation. Thus, investors of sufficiently good labels are cream-skimmed to the OTC market, while the rest stay on the exchange.

The \( N \)-Label Model subsumes numerous extensions of action sets, information structure and dynamics in the baseline model. For example, an extension in which investors choose from \( X \geq 1 \) ordersizes is equivalent to the \( N \)-Label Model with \( N = 2X \). If, in addition, investors can make one of \( Y \) types of public disclosures, we get \( N = 2 \cdot X \cdot Y \). Similarly, the dealer may partition investors by their trading histories, where investors who were more likely to impose losses on the dealer in the past are more likely to impose adverse selection than other investors.

5.2 Learning investors’ trading motives over time

A competitive dealer and an investor trades an asset on \( T \) occasions, indexed by \( t \in \{1, \ldots, T\} \). The asset pays a cashflow \( \tilde{v}_t \) of 1 or \(-1\) at the end of each period. The investor is a speculator with probability \( \frac{\mu}{\mu + \mu} \) or a hedger with complementary probability \( \frac{1}{\mu + \mu} \). Each \( t \), a speculator buys if the realized cashflow \( v_t \) is 1 and sells if \( v_t = -1 \). A hedger buys or sells with equal probability. Suppose the investor earned a profit in \( \tau \) of \( T \) occasions. As a speculator always makes a profit, \( \tau < T \) implies the investor is a hedger. Otherwise, the investor is a speculator with probability \( \frac{\mu}{\mu + 2 - \tau} \), i.e., denoting by \( \beta_{\tau,T} \) the probability that the investor is a speculator,

\[
\beta_{\tau,T} = \begin{cases} 
0 & \text{if } \tau < T \\
\frac{\mu}{\mu + \frac{1}{2\tau}} & \text{otherwise}.
\end{cases}
\]

If this investor is the one with whom the dealer has traded most often, the dealer partitions investors into \( T + 1 \) groups: investors who made a loss in a trade; and investors who traded with the dealer in \( \tau \in \{1, \ldots, T\} \) occasions and made a profit on each occasion. Mapping this partition into the \( N \)-Label Model, \( N = T + 1 \) and

\[
\left\{ \frac{\theta_1}{\gamma_1}, \frac{\theta_2}{\gamma_2}, \ldots, \frac{\theta_{T+1}}{\gamma_{T+1}} \right\} = \{0, \beta_{1,1}, \ldots, \beta_{T,T}\}.
\]
Appendix

A Speculators with Elastic Aggregate Demand

In this appendix, we extend the model of Section 2 to make the speculators’ aggregate demand elastic with respect to spreads (as is the hedgers’ demand). Our analyses show that a variant of Proposition 3 holds with this extension.

The model is altered in two ways. First, speculators are assigned types \( \{\alpha_i\} \), where \( \alpha_i \sim \mathcal{U}[0, 1/\eta], \eta \geq 1 \). A speculator trades only if her type \( \alpha_i \) is larger than her smallest half spread. Thus, the speculators’ equilibrium trading strategies are now similar to those of the hedgers. The types \( \{\alpha_i\} \) represent differences in, for example, information acquisition costs or risk aversion among the speculators. Second, the speculators’ signal precision \( \alpha \) is set to one. This change simplifies the model without loss of generality.

Parameter \( \eta \) determines the elasticity of the speculators’ demand. Given a spread, the proportion of speculators who trade is decreasing in \( \eta \). As \( \eta \) approaches infinity, this proportion becomes arbitrarily close to zero for any positive spread.

Theorem A.0.1. With the OTC market, there exists a unique equilibrium. In equilibrium:

1. the s-investors only trade on the exchange and the h-investors only trade in the OTC market;
2. a speculator buys (sells) if (i) her signal equals \( 1 \) (\(-1\)), and (ii) her entry cost is less than \( 1 - s_i \), where \( s_i \) is the speculator’s smallest half spread;
3. a hedger buys (sells) if she is a buyer (seller) and her lowest spread is less than her hedging benefit;
4. the dealer assigns the probability \( \theta \mu / (1 - \gamma) \) that a s-investor who requests a quote is a speculator; and
5. prices are characterized by the half spread \( s_o \) in the OTC market and \( s_e \) on the exchange, where
   \[ s_o = \frac{(1-\theta)\mu}{1 + (1-\theta)\mu \gamma \eta} \quad \text{and} \quad s_e = \frac{\theta \mu}{1 + \frac{\theta \mu}{1 - \gamma} \eta}. \]

Theorem A.0.2. Without the OTC market, there exists a unique equilibrium. In equilibrium:

1. a speculator buys (sells) if (i) her signal equals \( 1 \) (\(-1\)), and (ii) her entry cost is less than \( 1 - s_i \), where \( s_i \) is the speculator’s smallest half spread;
2. a hedger buys (sells) if she is a buyer (seller) and her lowest spread is less than her hedging benefit;
3. Prices are characterized by the half spread $\hat{s}_e$ on the exchange, where

$$
\hat{s}_e = \frac{\mu}{1 + \mu \eta}.
$$

**Proposition A.0.1.** Fix $\{\eta, \theta, \gamma\}$. There exists $\mu^{**}$ such that having the OTC market:

1. decreases welfare if $\mu \leq \mu^{**}$ and increases welfare if $\mu > \mu^{**}$; and
2. does not change the total trade volume or the average bid-ask spread.

Theorems A.0.1 and A.0.2 are proved in the same way as Theorems 1 and 2 (in Appendices B.1 and B.2), and Appendix C.5 gives the proof of Proposition A.0.1.

From Proposition A.0.1, having the OTC market (i) can harm welfare while total volume and average spread are unchanged, and (ii) harms welfare when most investors are hedgers. These results are analogous to those of Proposition 3 and so are the associated intuitions. The welfare results in Proposition 3 hold as they only require that the equilibrium spreads increase when the ex ante share of investors who are speculators increases. Whether the speculators’ demands are elastic or not does not affect this relationship between equilibrium spreads are ex ante share of speculators.

**B Proofs of Equilibrium**

**B.1 Proof of Proposition 1**

We proceed as follows. Lemma B.1.1 puts a bounds on prices, and Corollary B.1.1 provides investors’ trading strategies. Lemma B.1.3 establishes a rule for determining equilibrium spreads. With these results, Lemmas B.1.2 and B.1.4 show that no equilibrium violates Proposition 1. To complete the proof, Lemma B.1.5 shows there exists an equilibrium that satisfies Proposition 1.

**Lemma B.1.1.** Denote hedger $i$’s lowest ask price by $ask_i$ and her highest bid price by $bid_i$. Then, $(2\alpha - 1) \leq ask_i \leq 0$ and $-(2\alpha - 1) \leq bid_i \leq 0$.

**Proof.** A speculator who receives the signal $q_i = 1$ expects the payoffs

$$
\begin{cases}
2\alpha - 1 - ask_i, & \text{if } i \text{ buys} \\
bid_i - (2\alpha - 1), & \text{if } i \text{ sells}
\end{cases}
$$

and, if $q_i = -1$, the payoffs are

$$
\begin{cases}
-(2\alpha - 1) - ask_i, & \text{if } i \text{ buys} \\
bid_i + 2\alpha - 1, & \text{if } i \text{ sells}
\end{cases}
$$

A hedger who is a buyer expects the payoff $b_i - ask_i$ if she buys and $bid_i$ if she sells, while a seller expects the payoff $-ask_i$ if she buys and $b_i + bid_i$ if she sells.
If \( \text{bid}_i > 0 \), an hedger sells with probability \( \frac{1 + \text{ask}_i + \text{bid}_i}{2} \) (all sellers and buyers with \( \text{bid}_i - \text{ask}_i < \text{bid}_i \)) and a speculator sells if her signal \( q_i = -1 \) (if \( \text{bid}_i > 2(2\alpha - 1) - \text{ask}_i > 0 \), the speculator always sells). Then the expected payoff from selling to an investor is at most \(-\text{bid}_i < 0\), so \( \text{bid}_i \leq 0 \). An analogous argument gives \( \text{ask}_i \geq 0 \).

When \( \text{bid}_i < -(2\alpha - 1) \), a speculator never sells then competitive prices imply \( \text{bid}_i = 0 \), a contradiction. A symmetric argument yields \( \text{ask}_i \leq 2\alpha - 1 \).

**Corollary B.1.1.** A hedger buys if \( \text{bid}_i \leq \text{ask}_i \) and she is a buyer, and sells if \( \text{bid}_i \geq \text{bid}_i \) and she is a seller. A speculator buys if \( q_i = 1 \) and sells if \( q_i = -1 \).

**Proof.** Lemma B.1.1 and inspection of investors’ payoffs in the proof of Lemma B.1.1 gives the desired result.

**Lemma B.1.2.** If an \( \ell \)-investor \( \ell \in \{s, h\} \) trades in a market, \( \ell \)-investors only trade in that market.

**Proof.** Given \( \ell \), there are four cases to rule out as supportable in equilibrium.

1. \( \text{bid}_e = \text{bid}_o \) and some \( \ell \)-investors sell in each market;
2. \( \text{ask}_e = \text{ask}_o \) and some \( \ell \)-investors buy in each market;
3. \( \text{bid}_e > \text{bid}_o, \text{ask}_e > \text{ask}_o \), a positive mass of \( \ell \)-investors sell on the exchange and buy OTC, and no other \( \ell \)-investors trade; and
4. \( \text{bid}_e < \text{bid}_o, \text{ask}_e < \text{ask}_o \), a positive mass of \( \ell \)-investors buy on the exchange and sell OTC, and no other \( \ell \)-investors trade.

In case (i), the dealer can post the bid \( \text{bid}_e < \text{bid} < \text{bid}_e + \epsilon \), attract all \( \ell \)-investors who wants to sell, and earn a strictly positive profit. Case (ii) is ruled out analogously. In case (iii), every \( \ell \)-investor who wants to buy does so over-the-counter. But then, the dealer can post the ask \( \text{ask}_o < \text{ask} < \text{ask}_o + \epsilon \) on the exchange, attract all \( \ell \)-investors who wants to buy OTC, and earn a strictly positive profit. Cases (iv) is ruled out analogously.

**Lemma B.1.3.** For investors who consist of a mass \( \alpha_s \mu \) of speculators and a mass \( \alpha_h \) of hedgers, competitive prices are uniquely characterized by:

\[
s \left( \frac{\alpha_s \mu}{\alpha_h} \right) := \text{ask} = -\text{bid} = \frac{1}{2} \left( 1 + \frac{\alpha_s \mu}{\alpha_h} - \sqrt{\left( 1 - \frac{\alpha_s \mu}{\alpha_h} \right)^2 + 8(1 - \alpha) \frac{\alpha_s \mu}{\alpha_h}} \right). \tag{B.1.1}
\]

**Proof.** Competitive price setting implies,

\[
\text{ask} = \mathbb{E}[\hat{\rho} | \text{buys}].
\]
From Corollary B.1.1, a speculator buys only if \( q_i = 1 \) and a hedger buys with probability \( \frac{1}{2}(1 - \text{ask}) \). Then, applying the Bayes’ rule,

\[
\text{ask} = \frac{(2\alpha - 1) \frac{\alpha_s \mu}{\alpha_h} + \frac{\alpha_s \mu}{\alpha_h}}{\frac{\alpha_s \mu}{\alpha_h} + 1 - \text{ask}}
\]

whose solutions are

\[
\left\{ \frac{1}{2} \left( 1 + \frac{\alpha_s \mu}{\alpha_h} \pm \sqrt{\left( 1 - \frac{\alpha_s \mu}{\alpha_h} \right)^2 + 8(1 - \alpha) \frac{\alpha_s \mu}{\alpha_h}} \right) \right\}
\]

If sign of the third term is a “+”, \( \text{ask} \geq 1 \) which contradicts Lemma B.1.1. We check that if the sign is a “−”, \( 0 < \text{ask} \leq (2\alpha - 1) \). That \( \text{ask} > 0 \) is trivial, and \( \text{ask} \leq (2\alpha - 1) \) if and only if

\[
1 + \frac{\alpha_s \mu}{\alpha_h} - \sqrt{\left( 1 - \frac{\alpha_s \mu}{\alpha_h} \right)^2 + 8(1 - \alpha) \frac{\alpha_s \mu}{\alpha_h}} \leq 2(2\alpha - 1).
\]

Rearranging, squaring both sides and cancelling terms, we get:

\[
4(2\alpha - 1)^2 \leq 4(2\alpha - 1)
\]

which is true.

By symmetry, \( \text{ask} = -\text{bid} \).

\[\square\]

**Lemma B.1.4.** There is no equilibrium in which \( s \)-investors trade over-the-counter or \( h \)-investors trade on the exchange.

The proof of Lemma B.1.4 uses that an equilibrium strategy profile and beliefs (together an “assessment”) are consistent. An assessment \( A \) is consistent if there exists a sequence of assessments \( \{A^n\} \) that converge to \( A \), with the property that, for each \( A^n \): (i) the strategy profile is completely mixed; and (ii) beliefs are derived from the Bayes’ rule (Osborne and Rubinstein, 1994, Definition 224.2).

Now we show that an assessment, in which all investors only trade on the exchange, is not consistent. Perturb investors’ strategies by a small probability \( \varepsilon \) to make them completely mixed. Given this perturbation, a speculator buys with probability \( [1 - \varepsilon] \) if \( v = 1 \) and with probability \( \varepsilon \) if \( v = -1 \), while a hedger buys with probability \( [1 - \varepsilon] \) if she is a buyer and with probability \( \varepsilon \) if a seller. The Bayes’ rule implies

\[
se = \frac{(1 - 2\varepsilon)(2\alpha - 1)\mu}{\mu + 1 - se}.
\]

As \( \varepsilon \) approaches zero, \( se \) approaches \( s(\mu) \) (defined in (B.1.1)).

The \( h \)-investors’ OTC spread \( s_h^o \) with perturbed strategies solves

\[
s_h^o = \frac{(1 - 2\varepsilon)(2\alpha - 1)(1 - \theta)\mu}{(1 - \theta)\mu + (1 - s^o_h + 2\varepsilon^2 s^o_h)\gamma}
\]

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whose limit on the right-hand-side as \( \varepsilon \) approaches zero is 

\[
\frac{(1-\theta)\mu}{(1-\theta)\mu + (1-s_0^s)\gamma}.
\]

Then \( s_0^h = s\left(\frac{(1-\theta)\mu}{\gamma}\right) \), such that \( s_0^h < s_e \) and the h-investors wish to deviate. For brevity, we do not repeat analogous steps in the following proofs.

**Proof.** Given Lemma B.1.2, we only need to rule out three cases:

(i) investors only trade on the exchange;

(ii) investors only trade OTC; and

(iii) s-investors only trade OTC, and h-investors only trade on the exchange.

We already showed case (i) is not consistent. Analogous steps show that case (ii) is not consistent. For case (iii), \( s_0^s > s_e \), so the s-investors wish to deviate.

**Lemma B.1.5.** There exists an equilibrium in which: s-investors do not request quotes and only trade on the exchange; and h-investors only trade over-the-counter.

**Proof.** Consistent beliefs imply \( s_0^s < s_e \), so the h-investors cannot profitably deviate. If s-investors who trade over-the-counter pays the \( s_0^s = s_e \) but needed to pay \( \varepsilon \) to request a quote, so s-investors cannot profitably deviate.

### B.2 Proofs of Theorems 1 and 2

Two lemmas in addition to Appendix B.1 prove Theorem 1. Lemma B.2.1 determines equilibrium bid-ask spreads \( s_e \) and \( s_o \) using Proposition 1 and Lemma B.1.3. Lemma B.2.2 provides the unique, consistent off-equilibrium beliefs. This equilibrium is unique as Corollary B.1.1 and Proposition 1 uniquely characterize investors’ equilibrium strategy profile, while Lemmas B.2.1 and B.2.2 give the unique equilibrium prices and beliefs.

**Lemma B.2.1.** In equilibrium, the OTC spread \( s_o \) and the exchange spread \( s_e \) are characterized by equation (5).

Lemma B.2.1 follows immediately from Lemma B.1.3 and Proposition 1.

**Lemma B.2.2.** In the off-equilibrium event that a s-investor requests a quote, the dealer believes this investor is randomly drawn from among all s-investors. The dealer quotes the spread \( s_0^s = s_e \) to s-investors.

**Proof.** Perturbing the strategy profile by probability \( \varepsilon \), the likelihood of a s-investor being a speculator is

\[
\frac{\varepsilon(2\alpha - 1)\theta \mu}{\varepsilon \theta \mu + \varepsilon(1 - \gamma)} = \frac{(2\alpha - 1)\theta \mu}{\theta \mu + 1 - \gamma}.
\]

Thus, the OTC dealer solves

\[
s_0^s = \frac{\theta \mu}{(2\alpha - 1)\theta \mu + (1 - \gamma)(1 - s_0^s)}
\]

which is the same problem solved by the market maker on the exchange, so that \( s_0^s = s_e \).
B.3 Proof of Proposition 2

Proof of Item 1, Proposition 2. Choosing $\beta \in \{\theta \mu / (1 - \gamma), (1 - \theta) \mu / \gamma\}$ appropriately, the derivative of $s \in \{s_e, s_o\}$ with respect to $\mu$ is

$$s' (\beta) = 1 + \frac{1 - \beta - 4(1 - \alpha)}{\sqrt{(1 - \beta)^2 + 8(1 - \alpha) \beta}}.$$

This derivative is strictly positive if and only if

$$1 > 2(1 - \alpha)$$

which is true whenever $\alpha < 1.$

Proof of Items 2 and 3, Proposition 2. It is sufficient to show that $V_e / V$ is strictly increasing in $\mu$. The proportion $V_e / V$ equals

$$\frac{V_e}{V} = \frac{(1 - \gamma)(1 - s_e) + \theta \mu}{1 - \gamma s_o - (1 - \gamma) s_e + \mu}.$$  

Rearranging, we get

$$\frac{V_e}{V} = \frac{1}{1 + \gamma(1 - s_o) + (1 - \theta) \mu (1 - \gamma)(1 - s_e) + \theta \mu}.$$

Define

$$G := \gamma (1 - s_o) + (1 - \theta) \mu (1 - \gamma)(1 - s_e) + \theta \mu.$$

We only need to show that $G$ is decreasing in $\mu$. The derivative of $G$ derivative is negative if and only if

$$-\frac{\partial s_o}{\partial \mu} + \frac{(1 - \theta) \mu}{\gamma} < 0.$$

As $(1 - \theta) \mu / \gamma < \theta \mu / (1 - \gamma)$ and $1 - s_o > 1 - s_e$, the proof is complete if $\partial s_o / \partial \mu > \partial s_e / \partial \mu$. This is true since $s'(\beta)$ is strictly concave, $s''(\beta) < 0$, which implies $s'((1 - \theta) \mu / \gamma) > s'(\theta \mu / (1 - \gamma)).$ 

C Proofs of Welfare Analyses

C.1 Preliminaries

We provide some general results that are used in the subsequent proofs. In this appendix, we assume the hedgers’ hedging benefits $\{b_i\}$ are drawn from a distribution $F$, with cdf $F(\cdot)$ and pdf $f(\cdot)$, and we consider the following fixed point problem: for $0 < \eta < 1$,

$$s = \frac{\eta \mu}{\mu + 1 - F(s)}.$$  

(C.1.1)
The solution $s(\mu)$ of (C.1.1) and $F$ satisfy Assumption C.1.1.

**Assumption C.1.1.** For all $s \in [0, 1]$, $F(s)$ and $f(s)$ are continuously differentiable, and the solution $s(\mu)$ of (C.1.1) is continuously differentiable and strictly increasing.

Assumption C.1.1 implies two properties of $s(\mu)$:

1. $s(\mu)$ is a bijection; and
2. $s'(\mu) = \frac{1}{\mu'(s)}$.

The next lemma provides a sufficient condition for the welfare result of Proposition 5.

**Lemma C.1.1.** We set $\theta' > \theta$, and define the functions

\[
S(\mu) := s(\mu) s'(\mu) f(s(\mu)),
\]

\[
\Delta W := \gamma \int_{s(\frac{1-\theta\mu}{\gamma})}^{s(\frac{1-\theta\mu'}{\gamma})} x f(x) \, dx - (1 - \gamma) \int_{s(\frac{\theta\mu}{1-\gamma})}^{s(\frac{\theta\mu'}{1-\gamma})} x f(x) \, dx
\]

Assume $S(\mu)$ is continuously differentiable and strictly ‘n’-shaped. The function $S(\mu)$ is strictly ‘n’-shaped if there exists $\hat{\mu} > 0$ such that $S(\mu)$ is strictly increasing for all $\mu < \hat{\mu}$ and strictly decreasing for all $\mu > \hat{\mu}$. Then, there exists $\hat{\mu}$ such that $\Delta W(\mu) < 0$ if $\mu < \hat{\mu}$ and $\Delta W(\mu) > 0$ if $\mu > \hat{\mu}$.

**Proof.** $\Delta W$ expands to

\[
\gamma \int_{s(\frac{1-\theta\mu}{\gamma})}^{s(\frac{1-\theta\mu'}{\gamma})} S(x) \, dx - (1 - \gamma) \int_{s(\frac{\theta\mu}{1-\gamma})}^{s(\frac{\theta\mu'}{1-\gamma})} S(x) \, dx \quad (C.1.2)
\]

First, we show that there exists $\mu_0 > 0$ such that $\Delta W < 0$ if $\mu < \mu_0$. As $S$ is continuous and ‘n’-shaped, there is an unique $\mu_0 > 0$ that sets $S\left(\frac{\theta\mu_0}{1-\gamma}\right) = S\left(\frac{(1-\theta)\mu_0}{1-\gamma}\right)$. Figure C.1 illustrates the choice of $\mu_0$.

![Figure C.1 – Choosing $\mu_0$](image-url)
This choice of $\mu_0$ implies $S(x)$ is strictly decreasing for $x \leq \frac{(1-\theta)\mu_0}{\gamma}$. Recall $\theta > 1 - \gamma$, then $\frac{\theta'\mu_0}{1-\gamma} > \frac{(1-\theta)\mu_0}{\gamma}$. Hence, $S\left(\frac{\theta'\mu_0}{1-\gamma}\right) > S\left(\frac{(1-\theta)\mu_0}{\gamma}\right) > S\left(\frac{(1-\theta')\mu_0}{\gamma}\right)$. Given $x = \mu_0$, an upper bound on the first term of (C.1.2) is

$$S\left(\frac{(1-\theta)\mu_0}{\gamma}\right)(\theta' - \theta)\mu_0.$$ 

A lower bound on the second term is

$$S\left(\frac{\theta'\mu_0}{1-\gamma}\right)(\theta' - \theta)\mu_0,$$

which is equal to the first term’s upper bound. The ‘n’-shape assumption on $S$ implies $S\left(\frac{\theta'\mu_0}{1-\gamma}\right) > S\left(\frac{(1-\theta)\mu_0}{\gamma}\right)$ for all $\mu < \mu_0$, so $\Delta W < 0$ if $\mu < \mu_0$.

Second, we show there is a $\mu_1 > 0$ such that $\Delta W > 0$ if $\mu > \mu_1$. By an argument symmetric to the one above, there exists $\mu_1 > 0$ that solves $S\left(\frac{(1-\theta')\mu_1}{\gamma}\right) = S\left(\frac{\theta\mu_1}{1-\gamma}\right)$. Then we construct bounds on the terms of (C.1.2) as above to show that $\Delta W > 0$ if $\mu > \mu_0$. Figure C.2 illustrates the choice of $\mu_1$.

![Figure C.2 – Choosing $\mu_1$](image)

Third, we show that $\Delta W$ is strictly increasing in $\mu \in (\mu_0, \mu_1)$. If $\mu$ is in the interval $(\mu_0, \mu_1)$ and increases by a small increment, the change in $\Delta W$ is proportional to

$$(1-\theta)S\left(\frac{(1-\theta)\mu}{\gamma}\right) - \theta' S\left(\frac{\theta'\mu}{1-\gamma}\right) + \theta S\left(\frac{\theta\mu}{1-\gamma}\right) - (1-\theta')S\left(\frac{(1-\theta')\mu}{\gamma}\right) \quad (C.1.3)$$

We know

$$S\left(\frac{\theta'\mu}{1-\gamma}\right) < S\left(\frac{\theta\mu}{1-\gamma}\right)$$

$$S\left(\frac{(1-\theta')\mu}{\gamma}\right) < S\left(\frac{(1-\theta)\mu}{\gamma}\right)$$

$$S\left(\frac{\theta'\mu}{1-\gamma}\right) < S\left(\frac{(1-\theta)\mu}{\gamma}\right),$$
so a lower bound for the sum of the first and the last terms in (C.1.3) is:

$$(\theta' - \theta)S \left( \frac{(1 - \theta)\mu}{\gamma} \right).$$

A lower bound for the sum of the second and the third terms is:

$$-(\theta' - \theta)S \left( \frac{\theta'\mu}{1 - \gamma} \right)$$

and $S \left( \frac{(1 - \theta)\mu}{\gamma} \right) > S \left( \frac{\theta'\mu}{1 - \gamma} \right)$ such that $\Delta W$ must be strictly increasing in $\mu \in (\mu_0, \mu_1)$. Then $\hat{\mu}$ is unique.

The following lemma establishes a sufficient condition for the total volume and the average spread results of Proposition 5.

**Lemma C.1.2.** We set $\theta' > \theta$, and define the functions

$$T(\mu) := s'(\mu) f(s(\mu))$$

$$\Delta V := \gamma \int_s^{s \left( \frac{(1 - \theta)\mu}{\gamma} \right)} f(x) \, dx - (1 - \gamma) \int_s^{s \left( \frac{\theta'\mu}{1 - \gamma} \right)} f(x) \, dx$$

Assume $T(\mu)$ is continuously differentiable and strictly decreasing in $\mu$. Then $\Delta V < 0$.

*Proof.* $\Delta V$ expands to

$$\gamma \int_s^{s \left( \frac{(1 - \theta)\mu}{\gamma} \right)} T(x) \, dx - (1 - \gamma) \int_s^{s \left( \frac{\theta'\mu}{1 - \gamma} \right)} T(x) \, dx. \quad (C.1.4)$$

As $T(x)$ is strictly decreasing, an upper bound of the first term in (C.1.4) is:

$$T \left( \frac{(1 - \theta)\mu}{\gamma} \right) (\theta' - \theta) \mu$$

An upper bound for the second term is:

$$-T \left( \frac{\theta\mu}{1 - \gamma} \right) (\theta' - \theta) \mu.$$

By assumption, $T \left( \frac{\theta\mu}{1 - \gamma} \right) > T \left( \frac{(1 - \theta)\mu}{\gamma} \right)$ such that $\Delta V < 0$. \qed

### C.2 Proofs of Propositions 3 to 5

The Lemmas C.1.1 and C.1.2 of the preceding appendix proves Proposition 5, which implies Proposition 3.

We first show that the distribution $\mathbb{U}[0, 1]$ and the equilibrium spreads $\{s_e, s_o\}$ satisfy Assumption C.1.1. Setting $F = \mathbb{U}[0, 1]$, the cdf and the pdf of $F$ are continuously differentiable in the support $[0, 1]$. From equation (5), the spreads $\{s_e, s_o\}$ are continuously differentiable.
Proof of Proposition 5. Proposition 2 implies \( s(\hat{\beta}) \) is strictly increasing. Then, \( s_o = s(\theta \mu/(1 - \gamma)) \) must be strictly increasing in \( \theta \), and \( s_o = s((1 - \theta)\mu/\gamma) \) is strictly decreasing in \( \theta \).

By Lemma C.1.1, showing that \( s(\hat{\beta})s'(s(\hat{\beta})) \) is ‘n’-shaped proves the welfare result. Note \( f(\cdot) = 1 \) and, by rearranging equation (4), we find the inverse function \( \hat{\beta}(s) \) of \( s(\beta) \) as

\[
\hat{\beta}(s) = \frac{(1 - s)s}{2\alpha - 1 - s}
\]

whose derivative is

\[
\hat{\beta}'(s) = \frac{1 - 2s}{2\alpha - 1 - s} + \frac{(1 - s)s}{(2\alpha - 1 - s)^2}.
\]

Using the property \( s'(\hat{\beta}) = \frac{1}{\hat{\beta}'(s)} \):

\[
s(\beta)s'(\beta) = \frac{s(\beta)}{\hat{\beta}'(s)} = \left( \frac{1 - 2s}{2\alpha - 1 - s} + \frac{1 - s}{(2\alpha - 1 - s)^2} \right)^{-1}. \tag{C.2.1}
\]

The term inside the bracket can be rewritten as

\[
\frac{1 - s}{(2\alpha - 1 - s)} - \frac{1}{2\alpha - 1 - s} + \frac{1 - s}{(2\alpha - 1 - s)^2}
\]

Dividing the first term by \( \frac{2\alpha - 1}{2\alpha - 1} \), then adding and subtracting \( s \) from the numerator, we get:

\[
\frac{1}{(2\alpha - 1)s} + \frac{2(1 - \alpha)}{(2\alpha - 1)s(2\alpha - 1)} - \frac{1}{2\alpha - 1 - s} + \frac{1 - s}{(2\alpha - 1 - s)^2}.
\]

Combining the last two terms:

\[
\frac{1}{(2\alpha - 1)s} + \frac{2(1 - \alpha)}{(2\alpha - 1)s(2\alpha - 1)} + \frac{2(1 - \alpha)}{(2\alpha - 1 - s)^2}. \tag{C.2.2}
\]

Equation (C.2.2) is a sum of strictly convex functions, implying that it is strictly convex. Moreover, (C.2.2) approaches infinity as \( s \downarrow 0 \) or \( s \uparrow 2\alpha - 1 \). Thus, as \( s(\beta) \) is strictly increasing, there exists \( \hat{\beta} \) such that (C.2.2) is strictly decreasing for all \( \beta < \hat{\beta} \) and strictly increasing for all \( \beta > \hat{\beta} \). Then substituting (C.2.2) into (C.2.1) gives that \( s(\beta)s'(\beta)f(s(\hat{\beta})) \) is ‘n’-shaped.

By Lemma C.1.2, showing that \( s'(\beta)f(s(\beta)) \) is strictly decreasing proves the total volume result. As \( f(\cdot) = 1 \), \( s'(\beta)f(s(\beta)) = \frac{1}{\beta'(s)} \). Because \( s'(\beta) > 0 \), we only need to show that \( s(\beta) \) is strictly concave, which is true if \( \beta(s) \) is strictly convex. Since \( s(\beta) \) is a bijection and \( s'(\beta) > 0 \), \( \beta'(s) > 0 \) so that we only need to show \( \beta''(s) \) is strictly positive, where

\[
\beta''(s) = \frac{2(2\alpha - 1)}{(2\alpha - 1 - s)^2} \left( \frac{1 - s}{2\alpha - 1 - s} - 1 \right).
\]

Then \( \beta''(s) \) is strictly positive if and only if \( \alpha < 1 \), which is true.
The average spread $\bar{s}$ result is implied by the zero profit condition

$$V \cdot \bar{s} = (2\alpha - 1)\mu$$

and that closing the OTC market strictly reduces the total trade volume $V$. □

To prove Proposition 3, we establish the equivalence between the equilibrium (i) with the OTC market and $\theta = 1 - \gamma$, and (ii) without the OTC market. Under (i), all investors receive the spread $\hat{s}_e$, and no investor requests a quote in the OTC market to avoid the cost of click $\epsilon$. Under (ii), again the investors are offered the spread $\hat{s}_e$ and no investor trades over-the-counter.

**Proof of Proposition 3.** Choose $\gamma_0 \in [0, 1)$ and $\theta_0 \in (1 - \gamma_0, 1]$. The equilibrium without the OTC market is equivalent to the one with the OTC market and $\theta = 1 - \gamma$. Setting $\gamma = \gamma_0$, $\theta = 1 - \gamma_0$ and $\theta' = \theta_0$, and applying Proposition 5 proves Proposition 3. □

**Proof of Proposition 4.** Propositions 2 and 3 immediately imply Proposition 4. □

### C.3 Useful Results

This section provides results that are useful for the proof of Proposition 6 (in Appendix C.4).

We start by providing a condition on the distribution $F$ of hedging benefits $\{b_i\}$ that ensures competitive bid-ask spread $s$ is continuous and single valued.

**Assumption C.3.1.** Distribution $F$ has continuously differentiable cdf $F(\cdot)$ and pdf $f(\cdot)$. For all $s < 1$, $F(s) < 1$, and

$$s \frac{1 - F(s)}{2\alpha - 1 - s}$$

is increasing in $s \in [0, 1]$.

All results in this and subsequent sections use Assumption C.3.1.

**Lemma C.3.1.** Consider the fixed point problem

$$s = \frac{(2\alpha - 1)\beta}{\beta + 1 - F(s)}$$

where $\beta > 0$ and $1 > \alpha > 1/2$. There is a unique solution to (C.3.2) in $[0, 1]$.

**Proof.** Rewrite equation (C.3.2) to:

$$\beta = \frac{s}{2\alpha - 1 - s} [1 - F(s)]$$

Given $s' \in [0, 1)$, $s = s'$ solves (C.3.2) if and only if $s = s'$ solves (C.3.3). By Assumption C.3.1, $\beta(s)$ is continuously differentiable and strictly increasing in $s$. Suppose $s', s'' \in [0, 1)$, $s = s'$ is a solution to (C.3.3), and there exists $s'' \neq s'$ such that $s = s''$ is a solution to (C.3.3).
Then, $\beta(s') = \beta(s'')$, which contradicts strict monotonicity of $\beta(s)$. Lastly, a solution to (C.3.3) strictly in the interval $[0, 1]$ exists since the right term in (C.3.3) goes to infinity as $s \uparrow 2\alpha - 1 < 1$, and to zero as $s \downarrow 0$. \hfill \square

For the following results, define $s(\beta)$ as the smallest solution to (C.3.2), and $\beta^{-1}(\beta(s))$ as the inverse function of $\beta(s)$.

**Lemma C.3.2.** The inverse function $\beta^{-1}(\beta)$ is differentiable, strictly increasing, and $\beta^{-1}(0) = 0$.

**Proof.** From the proof of Lemma C.3.1, $\beta(s)$ is continuously differentiable and strictly increasing; in particular, $\beta(s)$ is a bijection. Thus, $\beta^{-1}(\beta)$ exists, and is differentiable and strictly increasing. As $\beta(s)$ is a bijection and $\beta(s) = 0$ iff $s = 0$, $\beta^{-1}(0) = 0$. \hfill \square

**Lemma C.3.3.** The solution $s(\beta)$ of equation (C.3.2) is differentiable and strictly increasing.

**Proof.** By Lemma C.3.1, $s(\beta) = \beta^{-1}(\beta)$, where $\beta^{-1}(\beta)$ is differentiable and strictly increasing for all $s \in (0, 2\alpha - 1)$. Since $s(\beta)$ must be in the interval $(0, 2\alpha - 1)$ for all $\beta > 0$, $s(\beta)$ is differentiable and strictly increasing. \hfill \square

**Lemma C.3.4.** Define $s_\delta(\beta)$ as the solution to the fixed point problem

$$s = \frac{\beta}{\beta + 1 - F(s + \delta)}$$

for some $\delta > 0$. Then, for all $\delta > 0$,

$$s_\delta(\beta) > s(\beta).$$

and

$$\lim_{\delta \to 0} s_\delta(\beta) = s(\beta).$$

**Proof.** Define $\beta_\delta(s)$ as the inverse function of $s_\delta(\beta)$. Then, for $s \in [0, 1)$, (recall $s(\beta)$ in equation (C.3.3))

$$\beta_\delta(s) = \frac{s}{2\alpha - 1 - s} [1 - F(s + \delta)] > \beta(s)$$

which has all the same properties as $\beta(s)$ if

$$\frac{1}{s(2\alpha - 1 - s)} \leq \frac{f(s + \delta)}{1 - F(s + \delta)}.$$ (C.3.4)

Since $\frac{1}{s(1-s)} > \frac{f(s)}{1-F(s)}$ and $\{f(\cdot), F(\cdot)\}$ are continuously differentiable, $\lim_{\delta \to 0} s_\delta(\beta) = s(\beta)$ and (C.3.4) holds whenever $\delta$ is small enough. As $\beta_\delta(s) > \beta(s)$, $s_\delta(\beta) > s(\beta)$. \hfill \square

**Corollary C.3.1.** Distribution $F = U[0, 1]$ satisfies Assumption C.3.1.
Proof. Corresponding cdf and pdf are

\[ F(s) = s \quad \text{and} \quad f(s) = 1 \]

which are continuously differentiable. Substituting the cdf and the pdf into (C.3.1), Assumption C.3.1 is satisfied.

C.4 Proof of Proposition 6

We first define key functions which we later show to be equilibrium objects. Lemma C.4.1 provides a monotonicity result on equilibrium spreads, and Lemma C.4.2 derives a useful ordering of labels. Lemmas C.4.3 and C.4.4 shows that the equilibrium of Proposition 6 exists; then Lemmas C.4.5 and C.4.6 establish the uniqueness of this equilibrium.

The following proofs rely on the results of Appendix C.3. In particular, we define \( s_e(n) \) as the smallest solution to the fixed point problem (C.3.2). The solution \( s_e(n) \), by Lemma C.3.1, is the unique competitive spread given the ex ante ratio of speculators to hedgers.

Using this definition of \( s_e(n) \), we define \( s_o(n) \) as the spread on the exchange given that all investors with labels \( \{n, \ldots, N\} \) trade on the exchange and no other investors do. We define \( s_o(n) \) as the OTC spread for investors with the label \( n \), if \( n \)-investors only trade over-the-counter:

\[
\begin{align*}
  s_e(n) &:= s \left( \frac{\sum_{i=n}^{N} \theta_n}{\sum_{i=n}^{N} \gamma_n} \right) \\
  s_o(n) &:= s \left( \frac{\theta_n}{\gamma_n} \right)
\end{align*}
\]  

(C.4.1)

Function \( N^\ast(\epsilon) \) denotes the label with the highest proportion of speculators for which \( s_e(n) \) is larger than \( s_o(n) \) plus the cost of click \( \epsilon \). As we soon show, investors with labels up to (and including) \( N^\ast(\epsilon) \) trade over-the-counter.

\[
N^\ast(\epsilon) := \max \{ n | s_e(n') - s_o(n') \geq \epsilon, \forall n' \leq n \}
\]  

(C.4.2)

Lemma C.4.1. \( s_e(n) \) and \( s_o(n) \) are decreasing in \( n \).

Proof. By Lemma C.3.3 and that, for all \( n' > n \), \( \theta_{n'}/\gamma_{n'} > \theta_n/\gamma_n \), \( s_o(n) \) is decreasing in \( n \). For \( s_e(n) \), we need to show, for all \( n' > n \),

\[
\frac{\sum_{i=n}^{N} \theta_n}{\sum_{i=n}^{N} \gamma_n} \leq \frac{\sum_{i=n'}^{N} \theta_{n'}}{\sum_{i=n'}^{N} \gamma_{n'}}
\]

which expands to

\[
\frac{\theta_n + \ldots + \theta_{n'} + \ldots + \theta_N}{\gamma_n + \ldots + \gamma_{n'} + \ldots + \gamma_N} \leq \frac{\theta_{n'} + \ldots + \theta_N}{\gamma_{n'} + \ldots + \gamma_N}.
\]
Rearranging, we get
\[ \frac{\theta_n + \cdots + \theta_{n'-1}}{\gamma_{n'} + \cdots + \gamma_N} \leq \frac{\gamma_n + \cdots + \gamma_{n'-1}}{\gamma_{n'} + \cdots + \gamma_N}. \]

Expanding this inequality,
\[ (\theta_n + \cdots + \theta_{n'-1})(\gamma_{n'} + \cdots + \gamma_N) \leq (\theta_{n'} + \cdots + \theta_N)(\gamma_n + \cdots + \gamma_{n'-1}) \quad (C.4.3) \]

By definition, for any \{m, m', m''\} such that \( m < m' \leq m'' \leq N \), \( \frac{\theta_m}{\gamma_m} \leq \frac{\theta_{m''}}{\gamma_{m''}} \) which implies \( \theta_m \gamma_{m''} \leq \theta_{m''} \gamma_m \). Setting \( m'' \) as each of elements in \{m', \ldots, N\}, we find
\[ \theta_m(\gamma_{m'} + \cdots + \gamma_N) \leq (\theta_{m'} + \cdots + \gamma_N) \gamma_m. \quad (C.4.4) \]

Applying \( (C.4.4) \) iteratively to \{n, n+1, \ldots, n' - 1\} on each side of \( (C.4.3) \) shows that the inequality in \( (C.4.3) \) holds.

\[ \square \]

**Lemma C.4.2.** If \( s_\varepsilon(n) - s_o(n) \geq \varepsilon \),

1. for all \( n' \geq n \), \( s_\varepsilon(n) - s_o(n') \leq \varepsilon \); and
2. for all \( n'' \leq n \), \( s_\varepsilon(n) - s_o(n'') \geq \varepsilon \).

**Proof.** Lemma C.4.1 implies \( s_o(n') \leq s_o(n) \leq s_o(n'') \), which gives the desired result. \[ \square \]

**Lemma C.4.3.** If \( N^*(\varepsilon) \) exists, investors with labels \{1, \ldots, N^*(\varepsilon)\} only trade over-the-counter, and other investors only trade on the exchange.

**Proof.** Note that consistent beliefs imply investors with label \( n \) receive the spread \( s_o(n) \) over-the-counter. We then only need to show:

(i) for any \( n'' < N^*(\varepsilon) \), \( s_o(N^*(\varepsilon) + 1) - s_o(n'') \geq \varepsilon \);

(ii) for any \( n' > N^*(\varepsilon) \), \( s_e(N^*(\varepsilon) + 1) - s_o(n') \leq \varepsilon \); and

(iii) \( s_e(N^*(\varepsilon) + 1) - s_o(N^*(\varepsilon)) \geq \varepsilon \).

Point (i) is true by the definition of \( N^*(\varepsilon) \) in equation \( (C.4.2) \). For (ii), Lemma C.4.2 implies \( s_o(n') \leq s_o(N^*(\varepsilon)) \) such that \( s_e(N^*(\varepsilon)) - s_o(N^*(\varepsilon)) \geq \varepsilon \rightleftharpoons s_e(N^*(\varepsilon) + 1) - s_o(n') > \varepsilon \). Similarly, for (iii), we get \( s_e(N^*(\varepsilon)) - s_o(N^*(\varepsilon)) \geq \varepsilon \rightleftharpoons s_e(N^*(\varepsilon) + 1) - s_o(N^*(\varepsilon)) > \varepsilon \). Now we show \( N^*(\varepsilon) < N \): if \( N^*(\varepsilon) = N \) and investors with label \( N \) only trade on the exchange, \( s_e(n) = s_o(n) \) and label \( N \) investors do not wish to deviate; but, if \( N^*(\varepsilon) = N \) and label \( N \) investors only trade in the OTC, consistency implies \( s_e(n) = s(\mu) \leq s(\frac{\theta_{\varepsilon}}{\gamma_N} \mu) = s_o(n) \), and so label \( N \) investors strictly prefer to deviate. \[ \square \]

**Lemma C.4.4.** The cutoff \( N^*(\varepsilon) \) exists, and \( N^*(\varepsilon) < N \).

**Proof.** Consider the following procedure:
1. Suppose investors with labels in \( \{ n, ..., N \} \) only trade on the exchange, and all other investors only trade in the OTC.

2. If \( s_e(n) - s_o(n) \geq \epsilon \), set \( n = n + 1 \) and return to step 1; or

3. if \( s_e(n) - s_o(n) < \epsilon \), set \( N(\epsilon) = n - 1 \) and stop.

Starting this procedure with initial value \( n = 1 \) always yields an output \( N(\epsilon) < N \) that satisfies the definition of \( N^*(\epsilon) \).

Following lemmas establish uniqueness.

**Lemma C.4.5.** If some \( n \) investors trade in a market, every \( n \) investor trade in that market or do not trade.

**Proof.** Define the set \( \{ n_1, ..., n_r, ... \} \) of labels for which there exists investors who trade in both markets. Moreover, we let \( \{ \theta_{n_r}, \gamma_{n_r} \} \) be the mass of speculators and hedgers labeled \( n_r \) who requests a quote in the OTC and \( \{ \theta_{n_r}^o := 1 - \theta_{n_r}, \gamma_{n_r}^o := 1 - \gamma_{n_r} \} \). Clearly, \( \epsilon > 0 \) means

\[
s_o(n_r) < s_e \left( \frac{\theta_{n_r} \mu}{\gamma_{n_r}} \right) < s_e
\]

which (by Lemma C.4.1) implies

\[
\frac{\theta_{n_r}^o}{\gamma_{n_r}^o} < \frac{\theta_{n_r}}{\gamma_{n_r}} < \frac{1 - \theta_{n_r}^o}{1 - \gamma_{n_r}^o}.
\]

Then, the dealer can slightly decrease \( s_e \), attract every investor who trades in the OTC, and earn a strictly positive profit.

**Lemma C.4.6.** If \( n < n' < N \) and investors labeled \( n \) trade on the exchange, investors labeled \( n' \) trade on the exchange.

**Proof.** Since \( n < n' \), by Lemmas C.4.1 and C.4.5,

\[
s_e - s_o(n) \leq \epsilon
\]

and, by definition,

\[
s_o(n) \leq s_o(n').
\]

Together, these inequalities imply

\[
s_e - s_o(n') \leq \epsilon
\]

such that investors labeled \( n' \) prefers to trade on the exchange.
C.5 Proof of Proposition A.0.1

We first prove the volume result which, by (12), implies the average spread result. Total trade volume without the OTC market equals

\[(1 - \eta \hat{s}_e)\mu + 1 - \hat{s}_e = 1 + \mu - (1 + \eta \mu)\hat{s}_e.\]

With the OTC market, the total volume is

\[1 + \mu - \eta \mu [(1 - \theta)s_o + \theta s_e] - [\gamma s_o + (1 - \gamma)s_e].\]

Subtracting the volume without the OTC market from the one with, then substituting spreads given in Theorems A.0.1 and A.0.2 yields zero.

We use Lemma C.1.1 to prove the welfare result. Equilibrium spreads solve the fixed point problems in the form

\[s = \frac{\beta}{\beta + 1 - \frac{F(s)}{G(s)}}\]

for \(\beta \in \{\mu, \frac{\theta \mu}{1 - \gamma}, \frac{(1 - \theta)\mu}{\gamma}\}\). Setting \(1 - L(s) = [1 - F(s)]/[1 - G(s)]\), the fixed point problems become

\[s = \frac{\beta}{\beta + 1 - L(s)}\]  \hspace{1cm} (C.5.1)

where \(L(s) = [F(s) - G(s)]/[1 - G(s)]\). Here, \(F(s) = s\) and \(G(s) = \eta s\) such that

\[L(s) = -\frac{(\eta - 1)s}{1 - \eta s}.\]

Then \(L\) and its derivative are continuously differentiable, which satisfies Assumption C.1.1.

Now we need only that \(S(\beta)\) is ‘n’-shaped as defined in Lemma C.1.1. In this case, \(S(\beta) = s(\beta)s'(\beta)\). Rearranging (C.5.1), we get

\[\beta(s) = \frac{s}{1 - \eta s}\]

which is strictly monotone. Thus, \(s(\beta)\) is a bijection and \(s'(\beta) = 1/\beta'(s) = (1 - \eta s)^2\). This means

\[S = s(1 - \eta s)^2\]

whose derivative equals \(2\eta s(1 - \eta s)\), which is strictly positive if and only if \(s < 1/(2\eta)\). As \(s(\beta)\) is strictly increasing, \(S(\beta)\) is then ‘n’-shaped and we have our desired result.
References


Hendershott, T., D. Li, D. Livdan, and N. Schurhoff. 2015. Relationship Trading in OTC Markets.


Rime, D., and A. Schrimpf. 2013. The anatomy of the global FX market through the lens of the 2013 Triennial Survey. BIS Quarterly Review.


