Collusion through Coordination on Announcements*

Joseph E. Harrington, Jr.† and Lixin Ye‡

26 September 2017

Abstract

Motivated by some recent collusive practices that do not seem to constrain the prices that sellers offer, a theory of collusion is developed based on interpreting firms’ actions as announcements about cost. By coordinating their announcements, firms are able to produce supracompetitive prices by influencing buyers’ conduct rather than constraining sellers’ prices.

*The comments of Matt Backus, Martin Peitz, Patrick Rey, and participants at the 2016 Hal White Antitrust Conference (Washington, D.C.), 2016 UBC Industrial Organization Conference (Kelowna, British Columbia), 2017 MaCCI Summer Institute in Competition Policy (Romrod, Germany), and a seminar at the Norwegian School of Economics are gratefully acknowledged, as is the extremely able research assistance of Ben Rosa and Xingtan (Ken) Zhang. The first author recognizes the financial support of the National Science Foundation (SES-1148129).

†Patrick T. Harker Professor, Department of Business Economics & Public Policy, The Wharton School, University of Pennsylvania, Philadelphia, PA 19104, harrij@wharton.upenn.edu

‡Department of Economics, Ohio State University, Columbus, OH 43210, ye.45@osu.edu
1 Introduction

Collusion involves firms coordinating their conduct so that, as long as all firms comply with how they agreed to behave, supracompetitive prices and profits will result. The challenge resides in ensuring that all firms comply. To achieve that end, cartels monitor for compliance and, when there is evidence of non-compliance, impose a punishment in order to provide incentives to comply.

In posted price markets (such as most retail markets), coordinated conduct typically takes the form of agreeing to charge prices above competitive levels and then monitoring prices for compliance. Examples include collusion among retail gasoline stations (Clark and Houde, 2011), retail pharmacies (Chilet, 2016), and fine arts auction houses (Mason, 2004). For many cartels in intermediate goods markets, coordination is again on price but compliance is more problematic because, given prices can be privately negotiated, monitoring of prices is difficult. For this reason, cartels also commonly coordinate on a market allocation scheme, and then monitor compliance with respect to that scheme. For example, cartels in citric acid, lysine, and vitamins agreed to sales quotas, and monitoring involved comparing actual sales with agreed-upon sales.¹

While coordination on prices is most common, firms can instead coordinate on an allocation of customers to cartel members, with the understanding that a cartel member does not supply customers that it has not been assigned. An allocation could take the form of exclusive territories whereby only a single cartel member is allowed to sell to customers in a particular region, in which case it will act like a monopolist (subject to not setting the price so high that another cartel member wants to undercut it). One implementation of an exclusive territories approach is the home-market principle, whereby each cartel member is allocated its home market.² In the context of auctions, it could mean allocating an item or contract to a particular member of the bidding ring, as with bidding rings in auctions for construction contracts (Kawai and Nakabayashi, 2015) and stamps (Asker, 2010). Alternatively, coordination could assign existing customers to firms. Recently, a number of high-tech companies were prosecuted for coordinating on a “no-poaching” agreement in which each agreed not to try to hire other companies' employees.³ As long as all firms complied


with the no-poaching agreement, each firm would pay wages below competitive levels.

These are just some of the ways in which firms can suppress competition by coordinating their conduct. The feature that we want to emphasize is that success occurs as long as all firms comply with the agreed-upon conduct because coordination directly constrains competition, whether it means the price that a firm charges or the customers that a firm supplies. The challenge is whether firms will act as agreed. As a result, the theory of collusion has focused on the characterization of effective monitoring and severe punishments.

In contrast to those canonical cases of collusion, there are some collusive practices for which coordinated conduct does not directly constrain competition, in which case it is not apparent that compliance is sufficient to produce supracompetitive outcomes. First, some cartels coordinate on list prices but not on discounts, which means firms do not coordinate on transaction prices. While it is easy to monitor and ensure that all firms set the agreed-upon list price, collusion could prove ineffective due to firms competing in discounts off of list prices. In fact, discounts were common in some of the cases involving coordination on list prices. That coordination on list prices presents a puzzle is evident from this observation by a member of the thread cartel which took the more common path of coordinating on transaction prices:4

[A cartel member] explained that list prices have more of a political importance than a competitive one. Only very small clients pay the prices contained in the lists. As the official price lists issued by each competitor are based on large profit margins, customers regularly negotiate rebates, but no clear or fixed amount of rebates is granted. ... Therefore, the list prices are essentially “fictitious” prices ... while [rebates] were discussed and agreed during the meetings.

A second set of collusive practices has firms coordinate on a surcharge for an input, such as fuel in markets for transportation services. Cartel members were essentially agreeing on how they wrote up the invoice - there would be a line assigning a part of the transaction price to this surcharge - and not coordinating on the transaction price itself. Collusion could prove ineffective due to firms competing in the non-surchARGE component of the transaction price, while complying by charging the agreed-upon surcharge. In Section 2, some of the cases involving coordination on list prices and surcharges are reviewed.

---

The contribution of this paper is providing an explanation for how these collusive practices could be effective. Contrary to the usual perspective of collusion - which focuses on how a collusive practice impacts sellers’ conduct - our approach takes account of how buyers’ conduct is impacted. The theory developed here is that these collusive practices work, not so much because they influence what prices sellers propose to buyers, but rather because they influence what prices buyers propose to sellers. As reviewed in Section 2, all of these cases have occurred in intermediate goods markets for which buyer-seller negotiation is the norm. Our argument is that coordination on list prices and surcharges is effective because it influences buyers’ beliefs in the negotiation process, and it is the manipulation of those beliefs that results in supracompetitive prices. In fact, our theory will have sellers offering the same prices as under competition, in which case the impact of collusion is entirely on the prices that buyers offer and are willing to accept.5

The theory focuses on the information about a seller’s cost that is conveyed by its list price or surcharge. While recognizing that list prices and surcharges can be more than information, the model parsimoniously isolates attention on the informational component by assuming that firms make cheap talk announcements about their costs. More specifically, the model assumes that there are two sellers, each of which receives some information about its cost which takes the form of a distribution on cost. Sellers then make announcements - such as in the form of list prices - about whether it is a low-cost or a high-cost type. Buyers decide which seller with which to negotiate based on the announcements. When a buyer shows up at a seller to negotiate, a seller learns its cost at that moment which is a draw from its distribution. Buyers are heterogeneous in their values and in how many sellers they approach to negotiate. As a tractable representation of buyer-seller negotiations, a buyer is modelled as conducting a second-price auction with a reserve price in which case the sellers that are invited to a buyer’s auction represent the sellers with which a buyer negotiates.

When sellers are competing, sufficient conditions are provided for a separating equilibrium to exist whereby a seller’s announcement reveals its cost type to buyers. Collusion has sellers using a pooling strategy in which they coordinate on announcements that signal they are high-cost types. These coordinated announcements induce buyers to set a high reserve price (or, in other words, negotiate less aggressively). Buyers recognize the possibility that sellers may be colluding and thus

5That sellers’ prices are exactly the same under competition is likely due to the particular modelling of the negotiation process. With other models of negotiation, sellers’ prices could also be influenced, but that does not affect the main takeaway of the paper which is that buyers’ conduct is impacted by sellers’ collusive practices.
that a high-cost announcement may not signal that a seller is a high-cost type. A novel feature of the model is encompassing the descriptively realistic assumption that buyers are uncertain about whether sellers are competing or colluding.

In viewing list prices and surcharges as just cheap talk messages, the model is stylized but has the benefit of generality in that it encompasses any variable that can convey cost information. Though the theory does not address why firms would choose list prices or surcharges as the vehicle to manipulate buyers’ beliefs about cost, they are natural candidates because they are a feature of the competitive process and are most likely perceived by buyers to be influenced by cost (indeed, surcharges are expressed to be associated with some input).\textsuperscript{6} Furthermore, for the markets we have in mind, treating list prices and surcharges as cheap talk is probably a reasonable approximation. If buyers can always anticipate discounts off of list prices then list prices as an upper bound on a seller’s negotiated price is not a binding constraint.\textsuperscript{7} The argument for surcharges being cheap talk is perhaps even more compelling. At most, it provides a lower bound on the total price (equal to the surcharge) but that is surely a non-binding constraint. In any case, our analysis shows that the information in list prices and surcharges is sufficient for coordination on them to produce supracompetitive prices.

This paper offers the first theory of collusion based on influencing buyers’ conduct, and it offers an explanation for why some recent collusive practices are effective even if they do not constrain the prices that sellers offer. Section 2 reviews some legal cases in which firms coordinated their list prices or surcharges. Section 3 describes the stage game model and relates it to past work, and Section 4 presents the candidate strategy profile. There are two steps to developing the theoretical argument. The first step is establishing an endogenous connection between announcements and final transaction prices; that is performed in Section 5. The second step is showing that firms can jointly raise profits by coordinating their announcements; that is done in Sections 6-7. Section 8 illustrates how this theoretical insight can provide some guidance in antitrust cases, while Section 9 concludes with some research directions. Unless otherwise noted, proofs are in the appendix.

\textsuperscript{6}Note that it is illegal for firms to explicitly coordinate their conduct in any manner that raises transaction prices. Hence, sellers are no less open to prosecution by coordinating on literal announcements about cost than they are by coordinating on list prices or surcharges. Thus, concerns about prosecution will not determine the vehicle used to influence buyers’ beliefs.

\textsuperscript{7}The previous quotation from the thread cartel highlights the "fictitious" nature of list prices.
2 Cases

*Reserve Supply v. Owens-Corning Fiberglas* (1992) is a private litigation case involving collusion in the market for fiberglass insulation. Two of the top three suppliers were accused of coordinating their list prices over 1979-83. The plaintiffs and defendants disagreed whether the alleged coordination could have resulted in supracompetitive transaction prices:

Reserve points to Owens-Corning and CertainTeed’s practices of maintaining price lists for products and ... asserts that these lists have no independent value because no buyer in the industry pays list price for insulation. Instead, it claims that the price lists are an easy means for producers to communicate and monitor the price activity of rivals by providing a common starting point for the application of percentage discounts. ... Owens-Corning and CertainTeed counter by arguing that the use of list prices to monitor pricing would not be possible because the widespread use of discounts in the industry ensures that list prices do not reflect the actual price that a purchaser pays.\(^8\)

The Seventh Circuit Court expressed skepticism with regards to the plaintiffs’ argument:

We agree that the industry practice of maintaining price lists and announcing price increases in advance does not necessarily lead to an inference of price fixing. ... [T]his pricing system would be, to put it mildly, an awkward facilitator of price collusion because the industry practice of providing discounts to individual customers ensured that list price did not reflect the actual transaction price.\(^9\)

In a case involving the market for urethane, plaintiffs claimed:

[T]hroughout the alleged conspiracy period, the alleged conspirators announced identical price increases simultaneously or within a very short time period. ... [P]urchasers could negotiate down from the increased price. But the increase formed the baseline for negotiations. ... [T]he announced increases caused prices to rise or prevented prices from falling as fast as they otherwise would have.\(^10\)

---

\(^8\) Reserve Supply v. Owens-Corning Fiberglas 971 F. 2d 37 (7th Cir. 1992), para 61.

\(^9\) Ibid, para. 62.

Supporting the alleged effect of list prices on transaction prices were internal memos from defendant Dow Chemical, such as:

In March 2002, Dow touted “Recent Successes,” emphasizing a class-wide price increase: “We announced 10 cts on Polyols March 1. We announced 15 cts on TDI March 1, 2002. It’s Working!!!!!!!”\(^{11}\)

The Tenth Circuit Court quoted the District Court in supporting the plaintiffs:

The court reasoned that the industry’s standardized pricing structure - reflected in product price lists and parallel price-increase announcements - “presumably established an artificially inflated baseline” for negotiations. Consequently, any impact resulting from a price-fixing conspiracy would have permeated all polyurethane transactions, causing market-wide impact despite individualized negotiations.\(^{12}\)

Turning to surcharges, over 40 air cargo companies participated in an agreement to coordinate fuel surcharges from late 1999 to early 2006. Inadvertently laying the groundwork for collusion, the International Air Transport Transportation (IATA) proposed a rule in 1997 for linking a fuel surcharge to a fuel price index.

When IATA discontinued publishing the fuel index, the draft resolution served as a model for airlines to set up their own FSC [fuel surcharge] mechanisms. As a consequence there was little difference between the various FSC mechanisms set up by the airlines and the application of them led to most airlines having index systems providing for similar levels of FSC with little or no difference as to the timing of the trigger in practice. These FSC index arrangements were generally published on the internet. There are a number of airlines who did not set up their own FSC mechanism but simply relied upon the mechanism published by another airline.\(^{13}\)

The surcharge was initially as low as four cents per kilogram and ultimately reached 72 cents per kilogram (LeClair, 2012). Guilty pleas led to fines of around $3 billion and customer damages.

\(^{11}\)Ibid, p. 15.

\(^{12}\)In Re: Urethane Antitrust Litigation, No. 13-3215 (10th Cir. Sep. 29, 2014); p. 7.

\(^{13}\)European Commission Decision, 11.09.2010, Case COMP/39258 - Airfreight, ¶103.
exceeding $1.2 billion.\textsuperscript{14} The collection of damages means there was an estimated overcharge and, therefore, coordination on fuel surcharges affected transaction prices.

In on-going private litigation, four class I railroads have been accused of coordinating their fuel surcharges starting in 2003.

The barrier to this plan [to coordinate fuel surcharges], according to plaintiffs, was that the great majority of rail freight transportation contracts already included rate escalation provisions that weighted a variety of cost factors, including fuel, based on an index called the All Inclusive Index (the “AII”). The railroad trade organization known as the Association of American Railroads (“AAR”), which is dominated by the four defendants, publishes this index. ... Plaintiffs allege that the defendants conspired to remove fuel from the AII so that they could apply a separate “fuel surcharge” as a percentage of the total cost of freight transportation.\textsuperscript{15}

The plaintiffs alleged that railroads’ conduct became coordinated after the AAR moved to this All Inclusive Index Less Fuel (AIILF):

\[\text{Although the railroads’ surcharges had varied in the past, from July 2003 onward the western railroads imposed identical surcharges. And from March 2004, three months after the December announcement of the AIILF, the eastern railroads imposed identical fuel surcharges. Plaintiffs further assert that it is unlikely that the eastern and western defendants would independently impose identical fuel surcharges, because fuel cost as a percentage of operating cost and fuel efficiency differed widely among the defendant railroads.}\]

The fuel surcharge was 0.4 percent of the base rate for each dollar that the price of oil on the West Texas Intermediate index exceeded $23 per barrel.\textsuperscript{17} The Surface Transportation Board ruled that

\textsuperscript{15}In re Rail Freight Surcharge Antitrust Litig., 587 F.Supp.2d 27, 30 (2008), United States District Court, District of Columbia. November 7, 2008
\textsuperscript{17}In re Rail Freight Surcharge Antitrust Litig., U.S. District Court for the District of Columbia, Opinion, June 21, 2012, p. 11.
Because railroads rely on differential pricing, under which rates are dependent on factors other than costs, a surcharge that is tied to the level of the base rate ... stands virtually no prospect of reflecting the actual increase in fuel costs.\textsuperscript{18}

Over 2001-07, fuel surcharges exceeded the rise in fuel costs by 55 percent.\textsuperscript{19}

Fuel is not only the only input for which there has been illegal coordination on surcharges. Six manufacturers of motive power batteries in Belgium were found guilty of coordinating on a common surcharge for lead.\textsuperscript{20} The cartel lasted from 2004 to 2011, and ended with an application for leniency.

A final example of coordinated announcements is a cement cartel in the United Kingdom.\textsuperscript{21} Annually, cement suppliers sent letters to their customers announcing price increases. However, prices were then individually negotiated with customers and the full price increase was rarely implemented. The Competition and Markets Authority concluded that firms coordinated their price announcement letters but noted

that firms generally fail to achieve the prices set out in the price letters, in part because of the rebates offered to large customers. This failure to achieve “list” prices suggests that prices are not simply fixed through this mechanism [that is, price announcement letters].\textsuperscript{22}

In commenting on the UK cement case, the head of Compass Lexecon’s London office posed the question: “How do price announcements help firms coordinate on prices if prices are ultimately individually negotiated?”\textsuperscript{23} It is to that question that we now turn.

\section*{3 Model}

Consider a market with two sellers offering identical products. A seller may be one of two types, $L$ or $H$, and type $L$ occurs with probability $q$. Sellers’ types are independent. A type $t$ seller’s

\textsuperscript{18}Surface Transportation Board Decision, STB Ex Parte No. 661 Rail Fuel Surcharges, Decided: January 25, 2007, p. 6.
\textsuperscript{19}USDA: Study of Rural Transportation Issues, June 03, 2010
\textsuperscript{20}Belgian Competition Authority, Press Release, N° 4/2016, 23 February 2016
\textsuperscript{21}“Aggregates: Report on the market study and proposed decision to make a market investigation reference,” Office of Fair Trading, OFT1358, August 2011.
\textsuperscript{22}Ibid, p. 53.
\textsuperscript{23}“Exchange of Information: Current Issues,” 30 April 2014, Allen & Overy, Brussels.
unit cost is assumed to be a random draw from the cdf $F_t : [\underline{c}_t, \bar{c}_t] \to [0, 1], t \in \{L, H\}$. $F_t$ is continuously differentiable with positive density everywhere on $(\underline{c}_t, \bar{c}_t)$. The inverse hazard rate function, $h_t(c) \equiv F_t(c)/F'_t(c)$, is assumed to be non-decreasing, $h'_t(c) \geq 0$, which holds for most of the common distributions such as uniform, normal, exponential, logistic, chi-squared, and Laplace. The two cost distributions are ranked in terms of their inverse hazard rates: $h_L(c) > h_H(c)$ for all $c \in (\underline{c}_t, \bar{c}_t)$. Note that the latter condition implies $F_H$ first-order stochastically dominates $F_L$ and, consequently, we will refer to a type $L$ seller as a low-cost type and a type $H$ seller as a high-cost type.

There is a continuum of buyers. Each buyer is endowed with a per unit valuation $v \in [\underline{v}, \bar{v}]$ and volume $z \in [\underline{z}, \bar{z}]$ (that is, the number of units demanded). Buyers also differ according to whether they solicit offers from either 1 or 2 sellers. What exactly it means to “solicit” an offer is described below. A fraction $\gamma \in [0, 1]$ of sellers solicit an offer from a single seller and a fraction $1 - \gamma$ from two sellers. A buyer’s per unit valuation is assumed to be independent of its volume and how many offers are solicited. Valuations are distributed according to the cdf $G : [\underline{v}, \bar{v}] \to [0, 1]$, where $G$ is continuously differentiable with positive density everywhere on $(\underline{v}, \bar{v})$. A buyer’s volume is allowed to be correlated with how many offers are solicited, and let $\mu^w$ be the expected volume of a buyer who solicits $w$ offers. Normalizing total market volume to one, define

$$b \equiv \frac{\gamma \mu^1}{\gamma \mu^1 + (1 - \gamma) \mu^2}$$

as the fraction of market volume that is from buyers who solicit an offer from one seller, and $1 - b$ as the fraction of market volume that is from buyers who solicit an offer from two sellers. The ensuing analysis depends on $\gamma$, $\mu^1$, and $\mu^2$ only through $b$.

The modelling of the interaction between buyers and sellers is intended to capture many intermediate goods markets for which buyers are industrial customers. Sellers first make some announcement informative of their costs which could be a list price, surcharge, or some other variable. After observing those announcements, each buyer approaches either 1 or 2 sellers to negotiate. A buyer who approaches two sellers is presumed to engage in an iterative bargaining process whereby she uses an offer from one seller to obtain a better offer from the other seller. Rather than explic-

---

24 Though it would be preferable to endogenize the number of sellers that are solicited by a buyer, it is assumed to be exogenous for reasons of tractability. This specification could be trivially rationalized by assuming buyers incur a cost to negotiating with each seller, they vary in this cost, and the cost is independent of a buyer’s valuation. Some buyers have very low cost and thus negotiate with both sellers, while other buyers have a high enough cost that it is optimal to only negotiate with one seller.
itly model that process, we will use the second-price auction with a reserve price as a metaphor for it. More specifically, a buyer “invites” \( w \) sellers to the auction, where \( w \in \{1, 2\} \). The buyer sets a reserve price and the \( w \) sellers submit bids which, in equilibrium, will equal their cost. We have buyers choose a publicly observed reserve price so they are not passive, which better mimics negotiation. A transaction occurs if the lowest bid is below the buyer’s reserve price. In the case of having chosen just one seller, the mechanism is equivalent to the buyer making a take it or leave it offer. Announcements, such as list prices, are presumed to be chosen less frequently than negotiated prices and this has the implication that a seller knows its cost type when it makes its announcement but does not know its actual cost until the time of negotiation. In practice, this uncertainty about future cost may be due to volatility in input prices or not knowing the opportunity cost of supply because future inventories or capacity constraints are uncertain.

The extensive form is as follows:

- **Stage 1:** Sellers draw types from \( \{L, H\} \) (which is private information to each seller) and choose announcements from \( \{l, h\} \).

- **Stage 2:** Buyers learn their valuations and volumes and observe sellers’ announcements. If a buyer is specified as approaching only one seller then it chooses a seller.\(^{25}\)

- **Stage 3:** Each seller realizes its cost. If a seller is type \( t \) then its cost is a draw from \( [c_t, \overline{c}_t] \) according to \( F_t \).

- **Stage 4:** For each buyer, the \( w \in \{1, 2\} \) selected sellers participate in a second-price auction with a reserve price. Each seller submits a bid. Transactions and transaction prices are determined as follows:

  - If there are two sellers in the auction and: i) both bids are below the reserve price then the buyer buys from the seller with the lowest bid at a price equal to the second lowest bid; ii) one bid is below the reserve price and the other bid is above the reserve price then the buyer buys from the seller with the lowest bid at a price equal to the reserve price; iii) both bids are above the reserve price then there is no transaction.

\(^{25}\)While a buyer’s valuation is private information, results are robust to assuming that a buyer’s volume is private or public information. If volume distinguishes small and large buyers then assuming it is observed by sellers is more natural.
– If there is one seller in the auction and: i) the bid is below the reserve price then the buyer buys from the seller at the reserve price; ii) the bid is above the reserve price then there is no transaction.

A strategy for a seller is a pair of functions: an announcement function and a bid function. The announcement function maps from \( \{L, H\} \) to \( \{l, h\} \) and thus has a seller select an announcement based on its cost type. In the event a seller is matched with a buyer, a bid function assigns a bid depending on the seller’s cost type, the seller’s cost, the other seller’s announcement, and whether the buyer matches with one or two sellers. The weakly dominant bidding strategy for a seller is to bid its cost. From hereon, we will think of a strategy for a seller as an announcement function and a bid function that has its bid equal to its cost. For a buyer who matched with one seller, a strategy selects a seller and a reserve price conditional on the announcements and the buyer’s valuation and volume (though the latter variable will not matter). If the buyer is matched with two sellers then a strategy selects a reserve price conditional on the announcements and the buyer’s valuation and volume. The solution concept is perfect Bayes-Nash equilibrium.

**Related Literature**  Our model is closely related to models of directed search in a market setting. Directed search is present here in that announcements (e.g. list prices) may induce buyers to negotiate with certain sellers. The paper closest to ours is Menzio (2007), who considers cheap talk in a search model of a competitive labor market. Employers have private information about the quality of their vacancies and can costlessly communicate with unemployed workers before they engage in an alternating offer bargaining game to determine the wage. Under certain conditions, there exists an equilibrium in which cheap-talk messages about compensation is correlated with actual wages and, therefore, serve to direct the search of workers. Our theory encompasses similar forces to those present in Menzio (2007) though in the context of an imperfectly competitive product market setting.

Our paper is also related to indicative bidding, which serves as the basis for shortlisting bidders in a two-stage auction procedure. Ye (2007) shows there does not exist a symmetric separating equilibrium bid function in indicative bidding; hence, the most “qualified” bidders may not be selected for the final stage. By restricting indicative bids to a finite domain, Quint and Hendricks (2015) explicitly models indicative bidding as cheap talk with commitment, and show that a symmetric equilibrium exists in weakly-monotone strategies. But again, the highest-value bidders are not always selected, as bidder types pool over a finite number of bids. Announcements in our
setting are like indicative bids in those settings. However, unlike in their analysis, in our setting the trading mechanism depends on the announcement in that the announcement affects a buyer’s reserve price as well as the seller that the buyer selects. As a result, a separating equilibrium in the cheap-talk stage becomes possible.

Our model is also related to work on bargaining and cheap talk. Farrell and Gibbons (1989) consider a buyer and a seller with private valuations who choose cheap talk messages, after which each agent proposes a price. If the buyer’s price exceeds the seller’s price then a transaction takes place at a price equal to the average of the two prices, otherwise there is no transaction. In contrast, our model has multiple sellers, only sellers choose cheap talk messages, and cheap talk messages affect not only the price proposed by a buyer but also the seller with which a buyer negotiates. Seller competition using cheap talk messages is absent from Farrell and Gibbons (1989).

Finally, our paper differs from all the papers mentioned above in that our model will be the stage game in an infinitely repeated setting.\footnote{Given that one announcement vehicle is list prices, let us mention that there is a small body of work that encompasses list prices and discounts off of list price. Chen and Rosenthal (1996), Raskovich (2007), García Díaz, Hernán González, and Kujal (2009), and Lester, Visschers, and Woldt (2015) have sellers post a list price which is subsequently followed by either discounts or negotiation. Those papers do not consider collusion and the driving forces to their analyses are distinct from that which is operative in our model. Gill and Thanassoulis (2016) do consider collusion but assumes firms coordinate on both list and discounted prices.}

4 Strategies Under Competition and Collusion

Suppose firms are competing which means that their strategies form a perfect Bayes-Nash equilibrium for the one-shot game. As this is a cheap talk game, there are always pooling equilibria which, in our setting, means uninformative announcements about cost.\footnote{With those equilibria, a seller’s strategy is to choose $h$ with some probability $s \in [0,1]$ if type $L$ or $H$, and then bid its realized cost if selected by a buyer. A buyer’s beliefs on a seller’s type are the prior beliefs: If $l$ or $h$ is observed then a seller is type $L$ with probability $q$. A buyer chooses an optimal reserve price based on those prior beliefs.} We will focus on equilibria in which a seller’s announcement is informative of its cost type as that will prove to be a necessary condition for collusion to be effective. Hence, consider sellers using the separating strategy that has a low-cost (high-cost) type choose a low-cost (high-cost) announcement:

$$\phi(t) = \begin{cases} l & \text{if } t = L \\ h & \text{if } t = H \end{cases}$$  \hspace{1cm} (1)
Alternatively, firms are colluding and, in that case, a seller make a high-cost announcement regardless of its type:

\[ \psi(t) = \begin{cases} h & \text{if } t = L \\ h & \text{if } t = H \end{cases} \]  

(2)

The value of coordinating their announcements in this manner is explained below.

A critical element to the ensuing theory is the descriptively realistic assumption that buyers are uncertain whether sellers are competing or colluding.\(^{28}\) Buyers assign probability \(\kappa\) (for the German “kartell”) that firms are colluding and using (2), and probability \(1 - \kappa\) that firms are competing and using (1). Buyers are not naive in that they recognize that collusion is possible and how collusion operates. However, as colluding sellers hide their illegal activities, buyers are left uncertain regarding the existence of a cartel. Consistent with the low level of documented cartels, it is presumed that \(\kappa > 0\) but small.\(^{29}\) It will become clear where we use the presumption that \(\kappa\) is small. Buyers are assumed to live for only one period and do not observe the history.\(^{30}\)

Given these beliefs on collusion, a buyer’s beliefs as to sellers’ types given their announcements can be derived. When buyers observe either or both sellers choosing a low-cost announcement, they infer that firms are competing. Letting \(m_i\) denote the message and \(t_i\) denote the type of firm \(i\), respectively, posterior beliefs (conditional on announcements) are:

\(^{28}\)That other agents - whether buyers, the competition authority, or potential entrants - are uncertain about whether market outcomes are the product of competition or collusion is assumed, for example, in Harrington (1984), Besanko, and Spulber (1989, 1990), LaCasse (1995), Schinkel and Tuinstra (2006), and Souam (2001).

\(^{29}\)Cartel duration data is consistent with buyers assigning a low probability to collusion. If buyers strongly suspected collusion, it would be in their best interests to report those suspicions to the competition authority, which would imply cartel duration is short. To the contrary, cartels typically operate for many years before they are discovered and are most often not reported by buyers. Average duration for discovered cartels is around six years (Harrington and Wei, 2017), with some cartels operating for decades before being discovered (Levenstein and Suslow, 2006). One of the few data sets reporting how a cartel was discovered found that only eight out of 47 cartels convicted by the Antitrust Division of the U.S. Department of Justice were reported by a customer or learned about through private litigation (Hay and Kelley, 1974). For the many years that a cartel operated prior to being prosecuted, it must then be the case that buyers did not believe there was a cartel.

\(^{30}\)Though this assumption is inconsistent with them being industrial buyers, it allows us to avoid a difficult dynamic problem. If buyers were long-lived or observed the history then they would update their beliefs over time regarding the hypothesis that there is collusion. While characterizing buyers’ beliefs over time is not a problem in and of itself, colluding sellers would take into account how their current actions (both with regards to announcements and bids) impact buyers’ beliefs and the future value of collusion. Thus, it now becomes a dynamic game between buyers and sellers. That is clearly a setting worth examining but is one we leave to future research.
• If \((m_1, m_2) = (l, l)\) then firms are competing and \(\Pr(t_i = L \mid (m_1, m_2) = (l, l)) = 1, i = 1, 2\).

• If \((m_i, m_j) = (l, h)\) then firms are competing and \(\Pr(t_i = L \mid (m_i, m_j) = (l, h)) = 1, \Pr(t_j = L \mid (m_i, m_j) = (l, h)) = 0, i \neq j, i, j = 1, 2\).

However, when buyers observe both sellers choosing a high-cost announcement, they do not know whether sellers are competing (and are high-cost types) or are colluding. Bayesian updating implies:

\[
\Pr(t_i = L \mid (m_1, m_2) = (h, h)) = \frac{\kappa q}{\kappa q + (1 - q)}, \quad i = 1, 2. \tag{3}
\]

With these beliefs on sellers’ types, the next step is to derive a buyer’s reserve price. Let \(R_{m_1, m_2}^w(v)\) denote the optimal reserve price when a buyer’s valuation is \(v\), announcements are \((m_1, m_2)\), and the buyer approaches \(w\) sellers. (As a buyer’s payoff is linear in its volume \(z\), the optimal reserve price does not depend on \(z\), and so that term is suppressed.) If \((m_1, m_2) \in \{(l, l), (l, h), (h, l)\}\) then sellers are inferred to be competing in which case a seller’s announcement fully reveals its type. When a buyer approaches only one seller, she will randomly choose a seller when \((m_1, m_2) = (l, l)\) and choose the seller with the low-cost announcement when \((m_1, m_2) \in \{(l, h), (h, l)\}\). Hence, in all cases, a buyer’s beliefs on the seller’s cost (and bid) is \(F_L\). It follows that the optimal reserve price is:

\[
R_{m_1, m_2}^1(v) \equiv \arg \max z(v - R)F_L(R), \quad \forall (m_1, m_2) \in \{(l, l), (l, h), (h, l)\}. \tag{4}
\]

If a buyer instead solicits bids from two sellers, she infers the sellers’ types are \((\phi^{-1}(m_1), \phi^{-1}(m_2))\) where recall \(\phi\) is a seller’s strategy under competition (see (1)). It follows that

\[
R_{m_1, m_2}^2(v) \equiv \arg \max_{R} z \int_{R_{\phi^{-1}(m_1)}}^{R} \int_{c_1}^{R} (v - c_2) \, dF_{\phi^{-1}(m_2)}(c_2) \, dF_{\phi^{-1}(m_1)}(c_1) \\
+ z \int_{R_{\phi^{-1}(m_1)}}^{R} \int_{c_2}^{R} (v - c_1) \, dF_{\phi^{-1}(m_2)}(c_1) \, dF_{\phi^{-1}(m_2)}(c_2) \\
+ z(v - R) \left[ \left(1 - F_{\phi^{-1}(m_2)}(R)\right)F_{\phi^{-1}(m_1)}(R) + \left(1 - F_{\phi^{-1}(m_1)}(R)\right)F_{\phi^{-1}(m_2)}(R) \right]. \tag{5}
\]

Now suppose \((m_1, m_2) = (h, h)\) so buyers remain uncertain regarding whether firms are competing or colluding. Given posterior beliefs (3) as to a seller’s type, a buyer believes a seller chooses its cost according to the mixture cdf \(F_\kappa\):

\[
F_\kappa \equiv \left(\frac{\kappa q}{\kappa q + (1 - q)}\right) \circ F_L + \left(\frac{1 - q}{\kappa q + (1 - q)}\right) \circ F_H.
\]
It follows that:
\[
R^{1}_{hh}(v) \equiv \arg \max_{R} z (v - R) F_{\kappa}(R),
\]
(6)
and
\[
R^{2}_{hh}(v) \equiv \arg \max_{R} \int_{c_{L}}^{R} \int_{c_{1}}^{R} (v - c_{2}) dF_{\kappa}(c_{2}) dF_{\kappa}(c_{1})
+z \int_{c_{L}}^{R} \int_{c_{2}}^{R} (v - c_{1}) dF_{\kappa}(c_{1}) dF_{\kappa}(c_{2})
+z (v - R) 2 (1 - F_{\kappa}(R)) F_{\kappa}(R).
\]
(7)
where this expression uses the assumption \(c_{L} \leq c_{H}\).

When a buyer approaches one seller, Lemma 1 shows that the optimal reserve price is higher when both sellers post high-cost announcements (and thus may be colluding) than when one or both sellers posts a low-cost announcement (in which case sellers are competing).

**Lemma 1** \(R^{1}_{hh}(v) > R^{1}_{ll}(v) (= R^{1}_{lh}(v)), \forall v.\)

For when a buyer approaches both sellers, Lemma 2 shows that the optimal reserve price is increasing in how many sellers posted high-cost announcements. This result does require that the probability of colluding \(\kappa\) is not too high. Otherwise, depending on the prior beliefs on sellers’ cost types (i.e., the value of \(q\)), it is possible that \(R^{2}_{hh}(v) < R^{2}_{lh}(v).\)\(^{31}\) However, \(R^{2}_{hh}(v), R^{2}_{lh}(v) > R^{2}_{ll}(v)\) regardless of \(\kappa.\)

**Lemma 2** If \(\kappa\) is sufficiently small then \(R^{2}_{hh}(v) > R^{2}_{lh}(v) > R^{2}_{ll}(v), \forall v.\)

## 5 Competition

The objective of this section is to show that announcements can be informative under competition. Coordinating on announcements cannot be profitable unless announcements are impactful with regards to transaction prices, which requires that announcements are perceived by buyers as containing information when firms compete. In determining when a separating equilibrium (under

---

\(^{31}\)For example, when \(\kappa = 1, R^{2}_{hh}(v)\) is based on each seller having a low-cost distribution with probability \(q.\) In comparison, \(R^{2}_{lh}(v)\) is based on one seller having a low-cost distribution for sure and the other seller having a high-cost distribution for sure. The relationship between those reserve prices is ambiguous. It is possible that our qualitative conclusions hold even when \(R^{2}_{lh}(v) > R^{2}_{hh}(v)\) but it is difficult to speculate as the characterization of equilibrium is different.
competition) exists, the analysis will examine when $b = 1$ (so the entire market volume is from buyers who negotiate with one seller), $b = 0$ (all buyers negotiate with both sellers), and finally the general case of $b \in [0, 1]$.

### 5.1 All Buyers Negotiate with One Seller

Suppose $b = 1$ so that all buyers approach only one seller. Let us derive the conditions for sellers’ competitive strategy (1) to be part of a perfect Bayes-Nash equilibrium. We have already dealt with a buyer’s beliefs and strategy and just need to derive conditions for a seller’s strategy to be optimal.

A low-cost type seller prefers to choose message $l$ (as prescribed by the competitive strategy) and signal it is a low-cost type if and only if

$$\left( \frac{q}{2} + 1 - q \right) \int_{\mathbb{L}} \int_{\mathbb{L}} \left( R_{lh}^l(v) - c \right) dF_L(c) dG(v)$$

On the LHS of the inequality is the payoff from choosing $l$ (which uses the property, $R_{lh}^l(v) = R_{lh}^l(v)$). A seller posting $l$ is chosen for sure by the buyer when the other seller posted $h$, which occurs when the other seller is type $H$ (and that occurs with probability $1 - q$); and is chosen with probability $1/2$ when the other seller posted $l$, which occurs when the other seller is type $L$ (and that occurs with probability $q$). Thus, a seller who chooses a low-cost announcement is approached by a buyer with probability $q/2 + 1 - q$. In that case, the buyer offers a price of $R_{lh}^l(v)$ and the seller accepts the offer if its realized cost is less than $R_{lh}^l(v)$. If the seller selects a high-cost announcement then it is approached by the buyer with probability $1/2$ in the event that the other seller also posted a high-cost announcement, and is not approached when the other seller posted a low-cost announcement. Hence, a seller with announcement $h$ assigns probability $(1 - q)/2$ to being approached by a buyer and, in that situation, is offered $R_{hh}^l(v)$.

If instead a seller is a high-cost type then it prefers to choose $h$ if and only if

$$\left( \frac{1 - q}{2} \right) \int_{\mathbb{L}} \int_{\mathbb{L}} \left( R_{hh}^l(v) - c \right) dF_H(c) dG(v)$$

The expressions are the same as in (8) except that the inequality is reversed and the cost distribution is $F_H$ instead of $F_L$. 

17
When a buyer selects one seller with which to negotiate, a seller’s announcement plays two roles. First, it affects the likelihood that a seller is selected by a buyer. By conveying it is low cost with announcement $l$, a seller is selected with probability $1 - (q/2)$, while the probability is only $(1 - q)/2$ if it conveys it is high cost with announcement $h$. This effect is referred to as the inclusion effect in that a low-cost announcement makes it more likely a buyer includes a seller in the negotiation process. A low-cost announcement signals a seller has a low-cost distribution in which case it is more likely to accept the buyer’s offer. The inclusion effect makes a low-cost announcement attractive because it induces more buyers to approach a seller and thereby results in more sales. However, there is a countervailing effect from a seller posting conveying that message, which is that a buyer negotiates more aggressively knowing it is more likely the seller’s cost is low given it conveyed it is a low-cost type. Referred to as the bargaining effect, it manifests itself by a buyer making a lower offer (in the form of a lower reserve price) in response to a low-cost announcement.\footnote{Though not labelling them as such, the inclusion and bargaining effects are present in Menzio (2007) in the context of a labor market with search.}

In sum, a low-cost announcement makes it more likely that a buyer negotiates with a seller but then the buyer will demand a lower price in those negotiations. Announcements can be informative because only a low-cost seller is willing to accept more aggressive buyers in exchange for attracting more buyers. Theorem 3 offers conditions such that an equilibrium with informative announcements exists.\footnote{For reasons of economizing on the analysis, the proofs of Theorems 3 and 4 are combined.}

**Theorem 3** If $b = 1$ then there exists $q$ and $\bar{q}$ such that a separating equilibrium exists if and only if $q \in \left[\frac{1}{2}, \bar{q}\right]$.

The probability that the other seller is a low-cost type cannot be too low ($q > \tilde{q}$), so that a low-cost seller prefers a low-cost announcement in order to compete with a possible low-cost rival, nor too high ($q < \tilde{q}$), so that a high-cost seller does not prefer a low-cost announcement in order to compete with a possible low-cost rival. In Section 7, we offer a parametric model for which $0 < \frac{1}{2} < \tilde{q} < 1$ and, therefore, a separating equilibrium exists.

### 5.2 All Buyers Negotiate with Both Sellers

When all buyers approach both sellers ($b = 0$), separating equilibria do not exist. The expected profit per unit to a seller of type $t_1$ whose announcement is $m_1$ (and thus inferred to be $\phi^{-1}(m_1)$)
when the other seller’s type and announcement are \(t_2\) and \(m_2\), respectively, is

\[
B(m_1, t_1; m_2, t_2) \equiv \int_{\underline{c}_2}^{\bar{c}_2} \int_{\underline{c}_1}^{\bar{c}_1} \left( \min \left\{ R^2_{m_1 m_2} (v), c_2 \right\} - c_1 \right) \times \frac{dF_{t_1} (c_1)}{dF_{t_2} (c_2)} dG (v),
\]

and the function is referred to as \(B\) because a buyer approaches both sellers. Recall that a buyer’s optimal reserve price is \(R^2_{m_1 m_2} (v)\) given announcements \(m_1\) and \(m_2\). If seller 1’s bid (= cost) is less than \(\min \left\{ R^2_{m_1 m_2} (v), c_2 \right\} - c_1\), then a buyer with valuation \(v\) buys from seller 1 and pays a price equal to \(\min \left\{ R^2_{m_1 m_2} (v), c_2 \right\}\). Hence, the probability that seller 1 makes a sale is weakly increasing in the reserve price \(R^2_{m_1 m_2} (v)\), as is the profit conditional on making a sale which equals \(\min \left\{ R^2_{m_1 m_2} (v), c_2 \right\} - c_1\). For realizations of \(c_2\) and \(v\) such that \(R^2_{m_1 m_2} (v) < c_2\), both are strictly increasing in the reserve price. \(B(m_1, t_1; m_2, t_2)\) is then increasing in the reserve price.

If seller 2 uses (1) then seller 1’s expected payoff from announcement \(m_1\) is

\[
q B(m_1, t_1; l, L) + (1 - q) B(m_1, t_1; h, H).
\]

Given \(B(m_1, t_1; m_2, t_2)\) is increasing in the reserve price, Lemma 2 implies

\[
qB(h, t_1; l, L) + (1 - q) B(h, t_1; h, H) > qB(l, t_1; l, L) + (1 - q) B(l, t_1; h, H), \quad t_1 \in \{L, H\}.
\]

A seller then prefers to convey it is high cost regardless of its type. Hence, a separating equilibrium does not exist.

With buyers approaching both sellers, a seller’s announcement does not affect the probability of being selected – so there is no inclusion effect – but it does affect how aggressively a buyer negotiates. A seller will always want to signal it is more likely to have a high-cost distribution because it induces a buyer to set a higher reserve price. When all buyers negotiate with both sellers, announcements are then uninformative.\(^{34}\)

### 5.3 General Case

Thus far, it has been shown that a separating equilibrium may exist when \(b = 1\), and only pooling equilibria exist when \(b = 0\). The next result considers when buyers are heterogeneous regarding

\(^{34}\)By a similar argument, one can show that semi-pooling equilibria do not exist.
how many sellers are approached.\textsuperscript{35}

**Theorem 4** If $\kappa$ is sufficiently small and a separating equilibrium exists for $b = 1$ then there exists $b^* \in (0, 1)$ such that a separating equilibrium exists if and only if $b \in [b^*, 1]$.

Announcements about cost can be informative when they influence a buyer’s decision as to which seller to approach to negotiate a deal, which we have referred to as the inclusion effect. A low-cost seller can find it worthwhile to make a low-cost announcement because the resulting increase in the number of buyers it attracts offsets the enhanced aggressiveness of those buyers. For equilibrium announcements to be informative, there must then be enough volume from one-seller buyers ($b$ is sufficiently high) so that the inclusion effect is sufficiently strong.

In concluding this section, let us present a seller’s expected profit under competition prior to learning its type:

$$
E [\pi^\text{comp}] \equiv b \left[ q^2 (1/2) A(l, L; l) + q(1-q) A(l, L; h) + (1-q)^2 (1/2) A(h, H; h) + (1-b)[q^2 B(l, L; l, L) + q(1-q) B(l, L; h, H) + q(1-q) B(h, H; l, L) + (1-q)^2 B(h, H; h, H)] \right]
$$

where $A(m_1, t_1; m_2)$ is the expected profit per unit to a seller when a buyer approaches only that seller.\textsuperscript{36} The first bracketed expression pertains to the fraction $b$ of market volume from buyers who negotiate with only one seller. With probability $q$, the seller is low cost and chooses announcement $l$ which signals to buyers it has a low-cost distribution. From these buyers, it will attract half of them when the other seller also chooses a low-cost announcement (which occurs with probability $q$) and all of them when the other seller chooses a high-cost announcement (which occurs with probability $1-q$). In that case, the expected profit earned on each unit is $A(l, L; l) (= A(l, L; h))$. Now suppose this seller is a high-cost type, which occurs with probability $1-q$, and thereby chooses announcement $h$. For the buyers who approach only one seller, the seller will not attract any of them when the other seller chose a low-cost announcement, and will get half of them when the other seller chooses a high-cost announcement. A high-cost announcement then attracts, in expectation,

\textsuperscript{35}$\kappa$ is required to be sufficiently small in Theorem 4 in order for the relationship between reserve prices in Lemma 2 to hold.

\textsuperscript{36}The expression for $A(m_1, t_1; m_2)$ is provided in the proof of Theorem 4. As expected profit does not depend on the other seller’s type, $t_2$ is absent from $A(m_1, t_1; m_2)$. 

20
(1 − q)/2 of those buyers, and the seller earns expected profit of $A(h, H; h)$ per unit. The second bracketed expression in (11) is the expected profit coming from the fraction $1 − b$ of market volume from buyers who negotiate with both sellers, where $B(m_1, t_1; m_2, t_2)$ is weighted by the probability that sellers are types $(t_1, t_2)$.

6 Collusion

Having established that announcements can impact transaction prices when firms compete, we now turn to allowing firms to collude. After evaluating when coordination on list prices is profitable in Section 6.1, we characterize conditions under which collusion is supported by strategies in an infinitely repeated game in Section 6.2.

6.1 Profitability of Coordination on Announcements

The expected profit of a seller from using the collusive strategy (2), and coordinating on high-cost announcements, is

$$E\left[\pi^{\text{coll}}\right] \equiv b[q(1/2)A(h, L; h) + (1 − q)(1/2)A(h, L; h)]$$

$$+ (1 − b)[q^2B(h, L; h, L) + q(1 − q)B(h, L; h, H)]$$

$$+ q(1 − q)B(h, H; h, L) + (1 − q)^2B(h, H; h, H)].$$

For the fraction $b$ of market volume from buyers who approach one seller, each seller will end up negotiating with half of those buyers and earn expected profit per unit of $A(h, t; h)$ when its type is $t$. For the fraction $1 − b$ of market volume from buyers who bargain with both sellers, a seller earns $B(h, t_1; h, t_2)$ per unit when its type is $t_1$ and the other seller’s type is $t_2$.

Subtracting (11) from (12) and re-arranging, the incremental profit from collusion is:

$$E\left[\pi^{\text{coll}}\right] − E\left[\pi^{\text{comp}}\right]$$

$$= b\left\{\left(\frac{q}{2}\right)A(h, L; h) + \left(\frac{1−q}{2}\right)A(h, H; h) \right.$$

$$− \left(\frac{q^2}{2}\right)A(l, L; l) − q(1 − q)A(l, L; h) − \left(\frac{(1−q)^2}{2}\right)(1/2)A(h, H; h)\right\}$$

$$+ (1 − b)[q^2[B(h, L; h, L) − B(l, L; l, L)] + q(1 − q)[B(h, L; h, H) − B(l, L; h, H)]$$

$$+ q(1 − q)[B(h, H; h, L) − B(h, H; l, L)] + (1 − q)^2[B(h, H; h, H) − B(h, H; h, H)]\right\}.$$
that term yields
\[
\left( \frac{q^2}{2} \right) [A(h, L; h) - A(l, L; l)] + \left( \frac{q(1-q)}{2} \right) [A(h, L; h) - A(l, L; h)]
\]
\[
+ \left( \frac{q(1-q)}{2} \right) [A(h, H; h) - A(l, L; h)]
\]
(14)

When both sellers are high-cost types then, whether colluding or not, they make high-cost announcements. Given expected profit is the same under collusion and competition, there is no term in (14) corresponding to the event when both are high-cost types. The first term in (14) pertains to when both sellers are low-cost types which occurs with probability \( q \). In that case, a seller attracts half of the buyers under both collusion and competition, and makes additional expected profit per buyer under collusion equal to
\[
A(h, L; h) - A(l, L; h)
\]
(15)

The first term in (15) is when the seller’s cost is less than \( R_{hh}^1(v) \). As collusion has both sellers choosing a high-cost announcement (rather than a low-cost announcement when competing), a seller ends up selling at \( R_{hh}^1(v) \) instead of \( R_{lh}^1(v) \). Because buyers set a higher reserve price compared to when firms do not coordinate their announcements, the seller earns higher profit of \( R_{hh}^1(v) - R_{lh}^1(v) \) conditional on selling, which we refer to as the price-enhancing effect. The second term in (15) is when the seller’s cost lies in \( [R_{lh}^1(v), R_{hh}^1(v)] \). Choosing a low-cost announcement under competition would result in not making a sale because the seller’s bid (which equals its cost) would exceed the buyer’s reserve price of \( R_{hh}^1(v) \). In contrast, under collusion, sellers choose high-cost announcements which induces a buyer to set the higher reserve price of \( R_{hh}^1(v) \) and, given it exceeds the seller’s cost, results in a transaction at a price of \( R_{hh}^1(v) \). Thus, collusion produces profit of \( R_{hh}^1(v) - c \), while competition would have yielded zero profit. Interestingly, collusion allows a Pareto-improving transaction to take place that would not have occurred under competition because collusion causes buyers to bargain less aggressively. This effect we refer to as the transaction-enhancing effect.

Next consider when the seller is a low-cost type and the other seller is a high-cost type. Under competition, the seller attracts all buyers and earns \( A(l, L; h) \) per unit, while under collusion it earns a higher profit per unit of \( A(h, L; h) \) but only attracts half of the buyers. The second term in (14) captures the half of the market that the seller attracts under both collusion and competition.
On those buyers, the profit per unit is higher by \( A(h, L; h) - A(l, L; h) \), and the associated profit gain is \( b(1/2) [A(h, L; h) - A(l, L; h)] \). However, this gain is offset by an expected loss of \( b(1/2)A(l, L; h) \) corresponding to the half of buyers who no longer solicit a bid from the seller under collusion. That profit loss appears in the third term in (14). But the seller gets those lost buyers back when the tables are turned and it is now a high-cost type and the other seller is a low-cost type. In that event, it would not have attracted any buyers under competition but gets half of the buyers under collusion and earns expected profit of \( b(1/2)A(h, H; h) \). That profit gain is also in the third term in (14). Hence, the net profit impact is \( b(1/2) [A(h, H; h) - A(l, L; h)] \), which gives us the third term in (14). Referred to as the business-shifting effect, it is the change in profit associated with half of the buyers no longer soliciting a bid from a firm when it is a low-cost type (under competition) and now soliciting a bid when it is a high-cost type (under collusion). This profit change could be positive or negative. While, ceteris paribus, it is better for a seller to attract a buyer when it is a low-cost type, the buyer’s reserve price is lower. If the third term is non-negative then (14) is positive which means collusion increases expected profit earned on buyers who solicit one offer. If the third term is negative then the sign of (14) is ambiguous.

Returning to the incremental profit from collusion in (13), the second bracketed expression pertains to the fraction \( 1 - b \) of market volume from buyers who solicit bids from both sellers. \( B(h, t_1; h, t_2) - B(\phi(t_1), t_1; \phi(t_2), t_2) \) is the difference in expected profit per unit for a type \( t_1 \) seller under collusion and under competition. It can be shown that

\[
B(h, t_1; h, t_2) - B(\phi(t_1), t_1; \phi(t_2), t_2) = \int_\nu \int_{\nu_1} R_{hh}^2(v) dF_{t_1} \left( c_2 - R_{\phi(t_1)\phi(t_2)}^2(v) \right) dF_{t_2} (c_2) dG(v)
\]

When \((t_1, t_2) = (H, H)\), all four terms are zero because, whether colluding or competing, they announce they are high-cost types so the outcome is the same. For any other type pairs, each of these four terms is positive as long as \( \kappa \) is sufficiently small so that \( R_{hh}^2(v) > R_{lh}^2(v) \). The first and third terms are driven by the price-enhancing effect: Collusion raises the buyer’s reserve price...
which increases the price seller 1 receives from $R_{\phi(t_1)\phi(t_2)}^2(v)$ to $c_2$ (in the first term) and to $R_{hh}^2(v)$ (in the third term). The second and fourth terms capture the transaction-enhancing effect: By inducing the buyer to have a higher reserve price of $R_{hh}^2(v)$, seller 1 sells for a price of $c_2$ (in the second term) and $R_{hh}^2(v)$ (in the fourth term). There is no business-shifting effects given that these buyers solicit bids from both sellers. Coordination on list prices then always increases profits earned from buyers who solicit bids from both sellers.

$E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}]$ is a weighted average of (16) with weight $1 - b$, which was just shown to be positive, and (14) with weight $b$, for which the sign is ambiguous. It then follows that if $E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}] > 0$ for $b = 1$ then $E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}] > 0$ for all values of $b$. $E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}] > 0$ for $b = 1$ if and only if (14) is positive, which is true if $q$ is sufficiently close to 1. In sum, sufficient conditions for collusion to be more profitable than competition (for all values of $b$) are $\kappa$ sufficiently small and $q$ sufficiently close to 1.

While the focus has been on evaluating the gain in profit from collusion, it is worth noting that the preceding analysis raises the possibility that collusion could also raise expected total surplus. The transaction-enhancing effect is welfare-improving as it expands the set of Pareto-improving transactions and thereby increases both a seller’s profit and a buyer’s net surplus. The welfare effect of the business-shifting effect is ambiguous. That the cost of the seller is higher on average under collusion lowers expected surplus - both by making a transaction less likely and reducing surplus in the event of a transaction - but, holding cost constant, transactions are more likely because the buyer’s reserve price is higher. Finally, the price-enhancing effect is welfare-neutral as it is a transfer from buyers to sellers. In sum, collusion reduces the likelihood that it is the lower cost supplier that makes a sale, which reduces expected surplus, but facilitates trade by making buyers less aggressive, which raises expected surplus. The net effect on welfare depends on which effect dominates. We will return to this issue in Section 7.

6.2 Coordination on Announcements as an Equilibrium

Though announcements do not constrain transaction prices, coordination on high-cost announcements influences transaction prices because it induces buyers to negotiate less aggressively as they believe sellers have high cost (in expectation). The impact on buyers’ bargaining behavior is manifested with a higher reserve price which benefits a seller in two ways. First, for those transactions that would have occurred whether firms colluded or competed, a seller receives a higher price because a buyer’s reserve price is higher. Second, the higher reserve price means that a buyer is less
likely to cause bargaining to break down which implies a seller earns positive profit under collusion (and the buyer earns positive surplus) because a transaction is consummated that would not have taken place under competition.

While we have shown that coordinating on high-cost announcements can be attractive to sellers, it has not yet been established that it is an equilibrium for sellers to collude. A variant of the usual Folk Theorem arguments suffices to establish the stability of collusion. Suppose the situation between buyers and sellers repeats itself infinitely often and $\delta \in (0,1)$ is the common discount factor of sellers. Each period, a seller acquires some partial private information on its cost for the upcoming period. This information acquisition is represented by a seller learning its type. With that knowledge, it then chooses its announcement. Each period it receives new information about its cost which is represented by independently drawing a new type.\footnote{If a period is, say, a quarter then a firm knows its cost distribution for the next three months and, based on those beliefs, chooses an announcement. Over the ensuing three months, a seller gets a cost draw whenever a buyer arrives at the seller and it is that cost that is relevant when bargaining with the buyer.}

In order to close the model, one additional layer will be added in order to endogenize the probability that buyers attach to firms colluding, $\kappa$. Suppose there is an exogenous Markov process by which a cartel is born (so firms adopt the collusive strategy) and dies (so firms revert to using the competitive strategy). Let $f$ (for “form”) denote the probability that a cartel forms out of a competitive industry, and $d$ (for “die”) denote the probability that a cartel dies and transforms into a competitive industry.\footnote{While it would be appealing to endogenize cartel birth and death, such a task is beyond the scope of this project. There is very little theoretical research that endogenizes cartel formation and collapse within an infinitely repeated game. With a Bertrand price game, stochastic demand can cause cartel collapse when it results in the lack of existence of collusive equilibria; see Rotemberg and Saloner (1986). That research does not model cartel formation. Harrington and Chang (2009, 2015) assume exogenous cartel birth, as done here, and endogenize cartel death with stochastic demand in the context of the Prisoners’ Dilemma.} Assume time is $-\infty, \ldots, 0, \ldots, +\infty$ and we are at time $t = 0$. As buyers live for only one period and do not observe the history, the probability they assign to firms colluding is the steady-state probability that there is a cartel, which is defined by

$$\kappa = \kappa (1 - d) + (1 - \kappa) f \iff \kappa = \frac{f}{f + d}.$$ 

The strategy profile for sellers is as follows. If sellers are in the competitive state then each chooses an announcement according to the separating (stage game) strategy (1). If sellers are in the cartel state and: i) they have always chosen high-cost announcement $h$ while in the cartel state then, as described in (2), they choose announcement $h$ regardless of type; and ii) for any other
history, they revert to the competitive state and choose an announcement according to (1). Once in
the competitive state - whether due to exogenous collapse or a deviation (which will not occur in equilibrium) - firms have a probability \( f \) in each period of transiting to the cartel state.39

Let \( V^{\text{coll}} \) denote the value (i.e., expected present value of profits) to a seller when in the cartel state, and \( V^{\text{comp}} \) denote the value in the competitive state. They are recursively defined by:

\[
V^{\text{coll}} = E[\pi^{\text{coll}}] + (1 - d)\delta V^{\text{coll}} + d\delta V^{\text{comp}} \quad (17)
\]

\[
V^{\text{comp}} = E[\pi^{\text{comp}}] + (1 - f)\delta V^{\text{comp}} + f\delta V^{\text{coll}}. \quad (18)
\]

Solving (17)-(18) yields

\[
V^{\text{coll}} = \frac{(1 - (1 - f)\delta) E[\pi^{\text{coll}}] + d\delta E[\pi^{\text{comp}}]}{(1 - \delta)(1 - \delta(1 - d - f))}. \quad (19)
\]

\[
V^{\text{comp}} = \frac{(1 - \delta(1 - d)) E[\pi^{\text{comp}}] + f\delta E[\pi^{\text{coll}}]}{(1 - \delta)(1 - \delta(1 - d - f))} \quad (20)
\]

Using (19)-(20) and simplifying, the incremental value to being in the cartel state is:

\[
V^{\text{coll}} - V^{\text{comp}} = \frac{E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}]}{1 - \delta(1 - d - f)}. \quad (21)
\]

Given the strategy for the infinitely repeated game, the equilibrium conditions for firms to collude are:

\[
b \left( \frac{1}{2} \right) A(h, t; h) + (1 - b) \left[ q B(h, t; h, L) + (1 - q) B(h, t; h, H) \right] \\
+ \delta \left( (1 - d)V^{\text{coll}} + dV^{\text{comp}} \right) \geq b A(l, t; h) + (1 - b) \left[ q B(l, t; h, L) + (1 - q) B(l, t; h, H) \right] + \delta V^{\text{comp}}, \quad t \in \{ L, H \}. \quad (22)
\]

The expression on the LHS of the inequality is the payoff to choosing a high-cost announcement (as prescribed by the collusive strategy), and on the RHS is the payoff from instead choosing a low-cost announcement. Note that when a seller deviates by choosing a low-cost announcement, it is ensured of attracting all buyers because the other seller is anticipated to post a high-cost announcement. Hence, we have \( b A(l, t; h) \) on the RHS and \( b \left( \frac{1}{2} \right) A(h, t; h) \) on the LHS.

39 Alternatively, we could assume that reaching the competitive state because of a deviation results in a per period probability \( g \) of returning to the cartel state and allow \( g \) to differ from \( f \). For example, \( g = 0 \) captures infinite reversion to a stage game Nash equilibrium. As ensuing results are robust to \( g \in [0, f] \), it is assumed \( g = f \) in order to reduce notation and make for simpler expressions.
Rearranging (22) and substituting using (21), (22) becomes:

$$
\begin{align*}
    \left( \frac{\delta(1-d)}{1-\delta(1-d-f)} \right) & \left( E[\pi^{\text{coll}}] - E[\pi^{\text{comp}}] \right) \\
    \geq & \; b \left[ A(l, t; h) - \left( \frac{1}{2} \right) A(h, t; h) \right] \\
    & + (1 - b) \left[ q (B(l, t; h, L) - B(h, t; h, L)) + (1 - q) (B(l, t; h, H) - B(h, t; h, H)) \right].
\end{align*}
$$

(23)

If collusion is more profitable than competition (so that the LHS is positive) then this equilibrium condition always holds for a high-cost type.\(^{40}\) Under competition, a high-cost type’s current expected profit is lower when it chooses a low-cost announcement, and that remains the case when firms coordinate their announcements. Hence, short-run profit (as well as the continuation payoff) is \textit{lower} to a high-cost type if it were to deviate to a low-cost announcement. In contrast, it is possible for a low-cost type to earn higher current expected profit by deviating to a low-cost announcement and attracting all buyers. For (23) to be assured of holding for a low-cost type, the LHS must then be sufficiently great.

The LHS of (23) is the difference in the future value between setting the collusive announcement \(h\) and deviating with announcement \(l\). If we let the probability of cartel birth and death become very small and firms to become very patient then

$$
\lim_{d, f \to 0, \delta \to 1} \frac{\delta(1-d)}{1-\delta(1-d-f)} = +\infty.
$$

Thus, as long as collusion is profitable, \(E[\pi^{\text{coll}}] > E[\pi^{\text{comp}}]\), it is an equilibrium for firms to coordinate on high-cost announcements when cartel birth and death are sufficiently rare and firms are sufficiently patient.\(^{41}\)

Summarizing the analysis of Section 6, coordination on announcements requires that collusion is \textit{feasible} (i.e., announcements are informative under competition, which requires \(q \in [\underline{q}, \overline{q}]\), see Theorem 3), collusion is \textit{profitable} (i.e., \(E[\pi^{\text{coll}}] > E[\pi^{\text{comp}}]\), which requires (13) to be positive), and collusion is \textit{stable} (i.e., it is an equilibrium outcome in an infinitely repeated game, which requires (23) to hold). In the next section, we show that these conditions are satisfied for a particular class of distributions on costs and values.

\(^{40}\)It can be shown that the second bracketed term on the RHS is negative for both cost types, and the first term is negative for a high-cost type. As then the RHS is negative for a high-cost type, (23) holds because the LHS is positive.

\(^{41}\)While admittedly speculative, the evidence suggests that \(d\) and \(f\) are low. A very small fraction of markets have documented cartels, which is consistent with a low value for \(f\). At the same time, the estimated annual probability of death is around 0.17 (Harrington and Wei, 2015). If a period is a quarter then this translates into \(d = 0.046\), which is reasonably low in absolute terms but probably high relative to the probability of cartel formation.
7 Collusion for a Class of Parametric Distributions

Assume $b = 1$ (so all buyers negotiate with one seller) and $\kappa = 0$ (so the prior probability of collusion is zero). By the analysis in Section 6, the ensuing results will approximate the case when $b$ is close to one and $\kappa$ is close to zero.\(^{42}\) Suppose valuations and costs have support $[0, 1]$. Valuations are uniformly distributed: $G(v) = v$. The cdf for a low-cost type is $F_L(c) = e^\alpha$ and for a high-cost type is $F_H(c) = e^\beta$, where $0 < \alpha < \beta$ so the inverse hazard rate ranking is satisfied: $h_L(c) = c/\alpha > c/\beta = h_H(c)$. Recall that a seller is a low-cost type with probability $q$.\(^{43}\)

**Theorem 5** Under the assumptions of Section 7, collusion is feasible if and only if

\[
q(\alpha, \beta) \equiv \frac{\beta^{\beta+1}}{(\beta+1)^{\beta+1}} - 2\frac{\alpha^{\alpha+1}}{(\alpha+1)^{\alpha+1}} \leq q \leq \frac{\beta^{\beta+1}}{(\beta+1)^{\beta+1}} - 2\frac{\alpha^{\alpha+1}}{(\alpha+1)^{\alpha+1}} \equiv \bar{q}(\alpha, \beta) \tag{24}
\]

and is profitable if $\alpha < 1$.

Given these distributions, the necessary and sufficient conditions for a separating equilibrium to exist that are provided in Theorem 3 take the form in (24). It can be shown that $\alpha < \beta$ implies the RHS of (24) exceeds the LHS. For example, $[q(0.5, 2), \bar{q}(0.5, 2)] = [0.453, 0.857]$. A sufficient condition for collusion to be profitable is that the low-cost distribution is concave, $\alpha < 1$. Collusion is stable as long as $\delta$ is close to one.

Figure 1: Range of Values for $q$ for which Collusion is Feasible and Profitable

\(^{42}\)If $\kappa > 0$ or $b < 1$ then there is no longer closed-form solutions for optimal reserve prices and, therefore, no closed-form solutions for $q$ and $\bar{q}$.

\(^{43}\)The proofs of the all results in this section are provided in an Online Appendix.
For when \((\alpha, \beta) \in [0, 1] \times [0, 2]\), Figure 1 reports the range of values for \(q, \widetilde{q}(\alpha, \beta) - q(\alpha, \beta)\), such that collusion is feasible and profitable (where the latter holds because \(\alpha < 1\)).\(^{44}\) Depending on the values for \((\alpha, \beta)\), there can be a wide range of values for \(q\) such that firms can effectively and profitably coordinate their announcements.

Let us now examine whether collusion can be welfare-improving. Let \(\Delta(q)\) denote the difference between expected total surplus under collusion and under competition, where its dependence on \(q\) is made explicit.\(^{45}\) For \((\alpha, \beta) \in [0, 1] \times [0, 2]\), Figure 2 reports the maximum welfare difference,

\[
\overline{\Delta}(\alpha, \beta) \equiv \max \{ \Delta(q) : q \in [q(\alpha, \beta), \bar{q}(\alpha, \beta)] \},
\]

and the minimum welfare difference,

\[
\underline{\Delta}(\alpha, \beta) \equiv \min \{ \Delta(q) : q \in [q(\alpha, \beta), \bar{q}(\alpha, \beta)] \}.
\]

Figure 2 shows that \(\overline{\Delta}(\alpha, \beta) > 0\) for most values of \((\alpha, \beta)\) so collusion improves welfare for some values of \(q\). In addition, for some values of \((\alpha, \beta)\), \(\overline{\Delta}(\alpha, \beta) > 0\) so welfare is higher under collusion for all values of \(q\) (for which collusion is feasible and profitable). By reducing the aggressiveness of buyers, collusion enhances the total surplus in the market by resulting in more Pareto-improving transactions and that can more than compensate for the higher cost under collusion.

Figure 2: Welfare Difference Between Collusion and Competition

\(^{44}\) \(q(\alpha, \beta)\) and \(\bar{q}(\alpha, \beta)\) are constrained to lie in \([0, 1]\). Hence, more exactly, Figure 1 reports \(\max \{ \min \{\bar{q}(\alpha, \beta), 1\} - 0 \} - \min \{ \max \{q(\alpha, \beta), 0\} - 1 \} \).

\(^{45}\) In the Online Appendix, the expression for \(\Delta(q)\) is provided.
8 Tying Theory to Cases: Antitrust Guidance

The feasibility of collusion relies on the existence of a separating equilibrium under competition so that announcements are informative and, therefore, coordination on high-cost announcements can affect buyers’ beliefs and thereby lead them to negotiate less aggressively. Recall that critical to the existence of a separating equilibrium is the inclusion effect, for it leads a low-cost seller (but not a high-cost seller) to choose a low-cost announcement in order to attract more buyers, even though buyers will bargain more aggressively. In our simple set-up, the inclusion effect is stronger when more buyers negotiate with only one of the two sellers.

For the purpose of drawing some insight into the market conditions conducive to coordination on announcements, let us extrapolate our finding by conjecturing that the inclusion effect is generally stronger when buyers negotiate with a smaller fraction of sellers. As each negotiation takes time and effort, a buyer is likely to negotiate with a smaller fraction of sellers when a market has more sellers. One also expects buyers to negotiate with fewer sellers when the magnitude of the expenditure over which negotiation is taking place is smaller, as then the extent of possible savings from negotiating with multiple sellers is less. Thus, there should be a strong inclusion effect in markets with many sellers, small buyers, and the input is a minor raw material in a buyer’s production process. Assuming the logic of our result extends, coordination on announcements would be an effective form of collusion in those markets. In contrast, the inclusion effect is likely to be weak in a market with few sellers and large buyers for a high-expenditure item (such as some complex piece of equipment). In those markets, coordination on announcements is unlikely to be effective.

To illustrate how this intuition can be applied to assess the credibility of a claim that coordination on list prices is an effective collusive strategy, let us consider two cases: large turbine generators around 1960 and urethane around 2000. A turbine generator is a device that converts mechanical energy into electrical energy, and the particular market under consideration is for large turbine generators purchased by electric utilities.46 It is a substantial custom-made piece of equipment which could cost in excess of $10 million in the early 1960s (which is around $80 million in 2017 dollars). At the time, General Electric and Westinghouse were the only producers of turbine generators. In light of the item’s high expense to a buyer and the presence of only two sellers, it is quite likely that a buyer would negotiate with both sellers. In such a market, our theory suggests

---

46 For details of the turbine generators market and legal case, see Sultan (1974) and Harrington (2011).
that list prices (being the form of announcements) would be uninformative since most buyers would negotiate with both sellers. Therefore, coordinating on list prices would be an ineffective method of collusion because a seller’s list price would have little effect on a buyer’s beliefs about a seller’s cost and thus have minimal impact on bargaining and final transaction prices. While GE and Westinghouse did collude in this market, it is notable that they did so by first coordinating on a policy of not offering discounts in which case list prices became actual transaction prices. GE then acted as a price leader on list prices. Absent a move to a “no negotiation” policy, the theory of this paper suggests that coordination on list prices would have been ineffective.

Returning to a case discussed in Section 2, the urethane market would appear to have features more conducive to coordination on list prices. Polyurethanes are used in various consumer and industrial products including mattress foams, insulation, sealants, and footwear. BASF, Bayer, Dow Chemical, Huntsman, and Lyondell either pled guilty or were convicted of coordinating their list prices over 2000-03. Their market shares in the sub-markets for polyether polyols, toluene diisocyanate (TDI), and methylene diphenyl diisocyanate (MDI) are shown in Figure 3. In contrast to turbine generators with two sellers, buyers of various categories of polyurethane had four or five suppliers from which to choose. While we do not have any data on the level of expenditure for a buyer, it is probably not a big ticket item like a large turbine generator. It would then seem unlikely that a buyer would negotiate with all or almost all sellers. If that is so then the inclusion effect might be significant enough to support informative list prices which would be the basis for firms effectively colluding by coordinating their list prices.

Figure 3: Market Shares in Urethane


47 The ensuing facts are from In re: Urethane Antitrust Litigation, 768. F.3d 1245 (10th Cir. 2014) and Class Plaintiffs’ Response Brief, In re: Urethane Antitrust Litigation, (10th Cir.), February 14, 2014.

48 Cartel members controlled the entire market for MDI and TDI and 79% of the market for polyether polyols.
9 Concluding Remarks

This paper offers the first theory of collusion based on influencing buyers’ conduct. It was shown that coordination on announcements - such as through list prices and surcharges - can be an effective form of collusion even though sellers are left unconstrained in the final prices they offer buyers. By coordinating on announcements that convey they have high cost, sellers can induce buyers to bargain less aggressively and that will deliver supracompetitive prices. Notably, sellers continue to bargain in a competitive manner. We also offer some initial insight for what types of markets are suitable for this type of collusive practice.

In concluding, we offer two directions for future research. The model assumed that buyers live for only one period and lack information prior to their arrival in the market. This assumption was made for purposes of tractability in order to focus on the phenomenon of coordination on announcements. Of course, industrial buyers live for many periods which then means there is a game between buyers and sellers. Buyers could use past data to assess whether firms are colluding, and sellers (if they are colluding) could adjust their behavior in order to balance higher profits earned from collusion against buyers becoming more confident that there is collusion. The latter is detrimental to a cartel because it means buyers will negotiate more aggressively if they think high-cost announcements are less likely to be driven by high cost, and there is the possibility that buyers may pursue private litigation to claim damages. There is no research that models equilibrium behavior in a dynamic game in which colluding sellers try to avoid detection and buyers try to detect collusion. While technically challenging, it is a research direction that is likely to shed new light on cartel dynamics.49

A natural question to pose is why, in practice, sellers choose to coordinate on list prices or surcharges rather than go that additional step and coordinate on final prices, especially given that explicit coordination of any form that impacts final prices is per se illegal. The answer may lie with a cartel’s concern about detection. For two reasons, detection by customers or the competition authority may be less likely when sellers only coordinate on list prices, surcharges, or some other form of announcements. First, coordinating on final prices along with a market allocation (and the monitoring of sales) requires more extensive and frequent communication among cartel members

49Besanko and Spulber (1990) consider a static game of incomplete information between a (possible) cartel and customers who are trying to determine whether there is a cartel. Harrington (2004, 2005) and Harrington and Chen (2006) examine the impact of detection for the price path in a dynamic setting but customers are represented by a detection technology and thus are not strategic.
which enhances the chances of the cartel’s discovery. Second, buyers might be less inclined to think that sellers are colluding when they offer different final prices, even though their list prices or surcharges are similar. A topic for future research is to understand when firms prefer to coordinate on final prices and when they instead prefer to coordinate on list prices, surcharges, or some other form of announcements that impact final prices.

10 Appendix

Proof of Lemmas 1 and 2: First, it can be verified that given \( h_L(c) > h_H(c) \), we have \( h_L(c) > h_\kappa(c) > h_H(c) \) if \( \kappa \in (0, 1) \).

To show Lemma 1, the first-order conditions of (4) and (6) are given by

\[
v - R_{m_1m_2}^1(v) = h_L(R_{m_1m_2}^1(v)), \quad \forall (m_1, m_2) \in \{(l, l), (l, h), (h, l)\}
\]

\[
v - R_{hh}^1(v) = h_\kappa(R_{hh}^1(v))
\]

It is easily verified that

\[
R_{m_1m_2}^{1\prime}(v) = \frac{1}{1 + h_L'(R_{m_1m_2}^1(v))} > 0, \quad \forall (m_1, m_2) \in \{(l, l), (l, h), (h, l)\}
\]

So \( R_{m_1m_2}^1(v) \) is increasing in \( v \), \( \forall (m_1, m_2) \in \{(l, l), (l, h), (h, l)\} \).

To show that \( R_{hh}^1(v) > R_{ll}^1(v)(= R_{lh}^1(v) = R_{hl}^1(v)) \) \( \forall v \), suppose the negation so that \( R_{hh}^1(v) \leq R_{ll}^1(v) \) for some \( v \). It follows that

\[
0 \leq -(R_{hh}^1(v) - R_{ll}^1(v)) = h_\kappa(R_{hh}^1(v)) - h_L(R_{ll}^1(v)) \leq h_\kappa(R_{hh}^1(v)) - h_L(R_{hh}^1(v)) < 0
\]

which is a contradiction.

Next to show Lemma 2, when \( (m_1, m_2) \in \{(l, l), (h, h)\} \), the first-order condition from (5) and (7) are given by

\[
v - R_{ll}^2(v) = h_L(R_{ll}^2(v)) \quad (25)
\]

\[
v - R_{hh}^2(v) = h_\kappa(R_{hh}^2(v)) \quad (26)
\]

So we have \( R_{ll}^2(v) = R_{ll}^1(v) \) and \( R_{hh}^2(v) = R_{hh}^1(v) \). When \( (m_1, m_2) \in \{(l, h), (h, l)\} \), say, when \( (m_1, m_2) = (l, h) \), the first-order condition from (5) becomes
In that case, a type announcement) and a fraction buyers who approach only one seller (and that those buyers will choose the seller with the low-cost seller’s announcement as buyers believe …rms are competing:

When the seller chooses a low-cost announcement, its expected payoﬀ is independent of the other seller’s announcement, for if it is a low-cost message then buyers believe sellers are competing and when the seller’s announcement conveys it is high cost then the payoﬀ does depend on the other.

Given the assumption that \( h_L (R^2_{th}) > h_H (R^2_{th}) \), we have

\[
(v - R^2_{th}) - h_L (R^2_{th}) < 0 < (v - R^2_{th}) - h_H (R^2_{th}),
\]

As \( (v - R^2_{th}) - h_L (R^2_{th}) = 0 \) (from (25)) then (27) implies \( (v - R^2_{th}) - h_L (R^2_{th}) < (v - R^2_{th}) - h_L (R^2_{th}) \).

As \( (v - R^2_{hh}) - h_\kappa (R^2_{hh}) = 0 \) (from (26)) then (27) implies \( (v - R^2_{hh}) - h_\kappa (R^2_{hh}) < (v - R^2_{hh}) - h_H (R^2_{hh}) \). Those two conditions imply \( R^2_{th} + h_L (R^2_{th}) > R^2_{th} + h_L (R^2_{th}) \) and \( R^2_{hh} + h_\kappa (R^2_{hh}) > R^2_{hh} + h_H (R^2_{hh}) \). When \( \kappa \) is sufﬁciently small, we have \( R^2_{hh} + h_\kappa (R^2_{hh}) > R^2_{hh} + h_H (R^2_{hh}) > R^2_{hh} + h_H (R^2_{hh}) \) by continuity of \( h_\kappa (R^2_{hh}) \) in \( \kappa \).

Given that \( h^l_1(z) \geq 0 \), we have the strict monotonicity of \( z + h_1(z), t \in \{L, H\} \). Thus \( \exists \pi > 0 \) such that if \( \kappa \in [0, \pi] \) then \( R^2_{hh} (v) > R^2_{th} (v) > R^2_{hh} (v), \forall v \).

**Proofs of Theorem 3 and 4:** Let us ﬁrst prove Theorem 4. Define \( A(m_1, t_1; m_2) \) to be the expected proﬁt per unit to a seller of type \( t_1 \) whose announcement is \( m_1 \) when the other seller’s announcement is \( m_2 \) and a buyer approaches only that seller:

\[
A(m_1, t_1; m_2) \equiv \int_{m_1}^{\pi} \int_{L}^{R^2_{m_1m_2}(v)} (R^1_{m_1m_2} (v) - c) dF_{t_1} (c) dG (v).
\]  

When the seller chooses a low-cost announcement, its expected payoff is independent of the other seller’s announcement as buyers believe ﬁrms are competing: \( A(l, t_1; l) = A(l, t_1; h) \). However, when the seller’s announcement conveys it is high cost then the payoff does depend on the other seller’s announcement, for if it is a low-cost message then buyers believe sellers are competing and when it is a high-costs message then buyers are uncertain about whether they face competition or collusion: \( A(h, t_1; l) \neq A(h, t_1; h) \).

When it chooses its announcement, a seller knows that a fraction \( b \) of market volume is from buyers who approach only one seller (and that those buyers will choose the seller with the low-cost announcement) and a fraction \( 1 - b \) of market volume is from buyers who approach both sellers. In that case, a type \( L \) seller optimally chooses announcement \( l \) if and only if

\[
W(l, L, b) \equiv b \left[ \left( \frac{q}{2} \right) A(l, L; l) + (1 - q) A(l, L; h) \right] \\
+(1 - b) \left[ qB(l, L; l, L) + (1 - q) B(l, L; h, H) \right] \\
\geq b \left( \frac{1 - q}{2} \right) A(h, L; h) + (1 - b) \left[ qB(h, L; l, L) + (1 - q) B(h, L; h, H) \right] \equiv W(h, L, b).
\]
A type $H$ seller optimally chooses announcement $h$ if and only if

$$W(h, H, b) \equiv b \left( \frac{1-q}{2} \right) A(h, H; h) + (1 - b) [qB(h, H; l, L) + (1 - q)B(h, H; H)]$$  \hspace{1cm} (30)$$

$$\geq b \left[ \left( \frac{q}{2} \right) A(l, H; l) + (1 - q)A(l, H; h) \right] + (1 - b) [qB(l, H; l, L) + (1 - q)B(l, H; h, H)]$$

$$\equiv W(l, H, b)$$

From Section 4.2, if $b = 0$ then (29) does not hold (as a type $L$ seller prefers to choose announcement $h$) though (30) does hold. Suppose that (29)-(30) are satisfied when $b = 1$. Combining these conditions for $b = 0$ and $b = 1$ delivers:

$$W(l, L, 1) - W(h, L, 1) > 0 > W(l, L, 0) - W(h, L, 0)$$  \hspace{1cm} (31)$$

$$W(h, H, 1) - W(l, H, 1) > 0 > W(h, H, 0) - W(h, H, 0)$$

By the linearity of the conditions in (31) with respect to $b$, it follows that there exists $b^* \in (0, 1)$ such that (29)-(30) hold if and only if $b \in [b^*, 1]$.

Turning to the proof of Theorem 3, set $b = 1$. Using (28) in (29)-(30), those conditions can be re-arranged to conclude that a separating equilibrium exists if and only if $q \in [\underline{q}, \bar{q}]$ where

$$\frac{\int_{V}^{\pi} \sum_{v_r} R_{hh} (v) (R_{hh} (v) - c) dF_L (c) dG (v) - 2 \int_{V}^{\pi} \sum_{v_r} R_{hl} (v) (R_{hl} (v) - c) dF_L (c) dG (v)}{\int_{V}^{\pi} \sum_{v_r} R_{hh} (v) (R_{hh} (v) - c) dF_L (c) dG (v) - \int_{V}^{\pi} \sum_{v_r} R_{hl} (v) (R_{hl} (v) - c) dF_L (c) dG (v)} \equiv \frac{q \leq q \leq \bar{q}}{\underline{q} \leq q \leq \bar{q}}$$  \hspace{1cm} (32)$$

References


[7] Chilet, Jorge Alé, "Gradually Rebuilding a Relationship: Collusion in Retail Pharmacies in Chile," The Hebrew University, April 2016.


