Trading Complex Risks

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Abstract

Complex risks differ from simple risks in that agents facing them only possess imperfect information about the underlying objective probabilities. This paper studies how complex risks are priced by and shared among heterogeneous investors in a Walrasian market. I apply decision theory under ambiguity to derive robust predictions regarding the trading of complex risks in the absence of aggregate uncertainty. I test these predictions in the laboratory. The experimental data provides strong evidence for theory’s predicted reduction in subjects’ price sensitivity under complex risks. While complexity induces more noise in individual trading decisions, market outcomes remain theory-consistent. This striking feature can be reconciled with a random choice model, where the bounds on rationality are reinforced by complexity. When moving from simple to complex risks, equilibrium prices become more whereas risk allocations become less sensitive to noise introduced by imperfectly rational subjects. Markets’ effectiveness in aggregating beliefs about complex risks is determined by the trade-off between reduced price sensitivity and reinforced bounded rationality. Moreover, my results imply that complexity has similar but more pronounced effects on market outcomes than ambiguity induced by conventional Ellsberg urns.

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1. Introduction

Financial markets have incurred a dramatic increase in complexity over the past decades. Successive market integration and ongoing financial innovation both have expanded and complicated the universe of tradable risks. The soaring levels of securitized contingent claims, a prominent example of the latter, are generally believed to have catalyzed what eventually turned into the Great Recession.\(^1\) Meanwhile, the implications of this rising complexity in traded assets’ inherited risk structure are still poorly understood.

Alongside information aggregation, financial markets’ essential raison d’\^etre is their efficiency in allocating tradable risks to those with the highest risk bearing capacities. Efficient risk sharing is, however, not prevailing unconditionally in such markets. Greenwald and Stiglitz (1986) show that when either information is imperfect or markets are incomplete, competitive market allocations are generally not constrained Pareto efficient.

In this paper I study how ‘complex risks’ are priced by and shared among heterogeneous agents. In contrast to simple risks, I regard a given asset’s payoff distribution as complex, if agents only possess imperfect information about the underlying objective probabilities. Starting from a complete market, I focus on the role of imperfect information on the pricing and sharing of tradable risks when aggregate endowments are constant. I thus deliberately abstract from financial innovation’s potential market completion effects in order to highlight complexity’s informational role on market efficiency.\(^2\)

My analysis rests on two integral parts: First, within a simple economy without aggregate uncertainty, I analyze competitive trading of both simple and complex risks via a complete Walrasian market. In order to account for complexity effects on agents’ trading behavior, I apply decision theory under ambiguity, equivalently often referred to as Knightian uncertainty (Knight, 1921). Second, by conducting a laboratory experiment, I test the derived clear-cut predictions empirically.

Given market completeness, the absence of aggregate risk implies the existence of individual trading strategies that leave agents perfectly hedged against future consumption risk. Relying on ambiguity theory, I show that, for complexity-averse agents, competitive supply and demand become less price sensitive under complex risks, as agents become more reluctant to deviate from

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\(^1\) See, e.g., Ghent et al. (2014) for a complexity-controlled performance analysis of mortgage-backed securities.

\(^2\) In reality, most markets, including those for financial assets, can hardly be characterized as being complete in a static sense, i.e., in the absence of retrading opportunities. Hence, the financial innovation industry’s touted services towards market completion have to be evaluated against dynamic completeness as developed in Kreps (1982) and Duffie and Huang (1985). Assuming dynamic completeness, the existence of a Radner equilibrium (Radner, 1972) crucially depends on agents’ ability of perfect foresight, i.e., to perfectly forecast today all future prices depending on information revealed tomorrow. Asparouhove et al. (2016) experimentally show how the inability of perfect foresight can cause considerable deviations from equilibrium prices. Thus, one reasonable concern implied by the increasing complexity of traded risks is that agents lacking the required resources to fully understand their complicated nature may fail to correctly forecast future price movements.
their perfect hedging strategy. This is intuitive: If risks are complex, the agents not only demand compensation for the risk they are bearing from not playing their perfect hedging strategy, but also for the uncertainty about the actual riskiness induced by such an action.

Experimental asset market equilibria corroborate complexity aversion-implied trading behavior, i.e., I find complexity to (locally) reduce supply and demand price elasticity. At the market level, complex risks are generally mispriced, but equally well shared relative to simple risks. At the individual level, complexity causes more mistakes in subjects’ trading decisions, where mistakes are defined as adopting strictly dominated strategies as implied by theory.

A comparison of both frequencies and distributions of dominated actions under simple and complex risks confirms that subjects’ trading strategies become increasingly noisy under the latter. Strikingly, as the number of subjects becomes larger, this noise cancels out in equilibrium and theory-consistent risk allocations prevail. This can be explained by a random choice model, where the relative likelihood of a given action is increasing in its ambiguity theory-based utility.

Overall, markets’ effectiveness in aggregating individual beliefs about complex risks is determined by the trade-off between reduced price sensitivity and increased severity of bounded rationality (more dominated actions). Accounting for subjective beliefs, I find that, despite reinforced bounds to rational behavior, markets’ prove remarkably effective in pricing complex risks. Beyond binding limits to rationality, their information aggregation is impaired, while optimal risk sharing still prevails. Finally, my results indicate that complex risks have similar but more pronounced implications on market outcomes than ambiguity induced by conventional Ellsberg urns.

In contrast to a situation with known payoff distributions, complexity is introduced by providing subjects instead with the formal definition of the underlying process in addition to a dynamic visualization of its past trajectory. Thus, the presence of complex risks requires subjects to deductively determine traded assets’ payoff distributions by processing complex information. The advantage of this implementation is the simple structure of the complicated but yet well-defined problem at heart. In fact, it requires solving a stochastic differential equation, which, although technically doable by hand, turns out to be infeasible for the vast majority of subjects. However, the problem’s simple formulation together with the visualized information of one random realization allows one to appraise—with more or less certainty—the apparently objective underlying risk.

In summary, the term complexity will henceforth refer to a complete Walrasian market for a risky asset whose true payoff distribution is not known with certainty, thereby imposing complex risks on utility maximizing traders. Although this notion of complexity is arguably specific, it

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3 This is not surprising given the means at hand and the limited time available during the experiment. Presenting subjects with an obviously solvable but complicated problem represents the design’s integral treatment.
naturally extends to real world financial markets’ inherent purpose of multidimensional risk sharing.\(^4\)

In the absence of perfect information, subjects possess a more or less precise estimate of the relevant payoff distribution, i.e., are faced with a smaller or wider set of possible priors. Considering the imperfection of the available information, “it would be irrational for an individual who has poor information about her environment to ignore this fact and behave as though she were much better informed” (Epstein and Schneider, 2010, p. 5). Trading decisions under complex risks are henceforth analyzed by applying two seminal ambiguity models in financial economics: a generalization of the multiple-priors model by Gilboa and Schmeidler (1989), and the smooth ambiguity model by Klibanoff et al. (2005). While the former implies kinked ambiguity preferences, the latter allows for smooth ambiguity effects.

The intuition behind both models is simple. If agents are averse to perceived ambiguity, they, ceteris paribus, prefer to avoid being exposed to imperfectly understood risks. When starting from a zero ambiguity exposure, this leads to a no-trade interval.\(^5\) For nonzero initial endowments in the risky asset, as pointed out by Dow and da Costa Werlang (1992), engaging in trade is generally still optimal. In my model economy, incentives to trade stem from nontradable but hedgeable consumption risk. In short, under both models, agents’ price sensitivity of their perfect hedging strategy decreases in the presence of complex risks. Intuitively, being completely hedged insures agents not only against risk but also against potential complexity-induced ambiguity. The main difference between the two models lies in their implied conditions for mispricing. Within the smooth ambiguity model, incorrect beliefs immediately impact equilibrium prices, whereas this does not unconditionally hold for the multiple-priors model. Overall, my empirical evidence speaks in favor of kinked preferences as embedded in the latter.

The merits of taking the study of how individual decision making aggregates to market outcomes to the lab are manifold. First, by design, the lab easily allows for the construction of a complete market. Moreover, the experimenter can exercise full control over each market participants’ information set and how their individual decisions interact towards equilibrium. Second, the laboratory environment offers the unique virtue of measuring subjects’ beliefs, in particular their expectations, which most often constitutes an impossibility when confronted with real world data. Third, treatment effects under investigation can be analyzed in isolation,

\(^4\) There is a vast scientific literature on various notions of (financial) complexity. In computer science and machine learning one distinguishes, e.g., between computational complexity (required resources), sample complexity (minimum number of draws), and Kolmogorov complexity (minimum descriptive length) of problem solving. The herein considered form of complexity is somewhat different in that it directly relates to the analysis of pricing and risk sharing in a financial market. Interestingly enough, recent contributions in decision science provide evidence for commonalities between the human brain and computer algorithms solving and reacting to problems with varying levels of complexity (see, e.g., Bossaerts and Murawski (2016)).

\(^5\) For example, this phenomenon serves Dimmock et al. (2016) in explaining known household portfolio puzzles, e.g., the equity home bias.
while controlling for any kind of endogeneity concerns. Thus, the lab enables a direct comparison between simple versus complex risks, while comparing the latter to the ‘pure ambiguity’ case usually associated with Ellsberg (1961)’s urn experiment. Once the unobservability of expectations and the inability to monitor strategic uncertainty underlying field data are taken into serious account, the advantages of full laboratory control become evident.

My design directly builds on the experimental setup proposed by Biais et al. (2017). Relying on a two-state world with two nonredundant assets (a risk-free bond and a risky stock), it offers the simplest possible setting to test the seminal general equilibrium theory by Debreu (1959) and Arrow (1964). Controlling for subjects competitive behavior, Biais et al. (2017) find market outcomes to be consistent with the theory of complete and perfect markets: On average, (simple) risk is perfectly shared and only aggregate risk is priced. Therefore, Biais et al. (2017)’s parameter-free test of the most fundamental asset pricing theory constitutes the ideal benchmark upon which the trading of simple and complex risks can be compared. Moreover, its simple market-clearing pricing scheme based on individual supply and demand functions can be controlled for any kind of strategic uncertainty. This constitutes an impracticality in the context of the continuous double auction that is normally used in experimental asset market studies.

This paper relates to three distinctive strands of the literature. First, a growing literature investigates the drivers and implications of financial complexity both from a theoretical as well as an empirical perspective. Ellison (2005) and Gabaix and Laibson (2006) demonstrate theoretically that inefficient levels of financial complexity can prevail in a competitive equilibrium. Carlin (2009) finds that financial complexity is an increasing function in the degree of competition among financial institutions. Carlin and Manso (2011) show how educational initiatives aiming to foster financial literacy may eventually cause welfare diminishing obfuscation, i.e., the strategic acceleration of complexity by financial service providers in order to preserve industry rents (see Ellsion and Ellison (2009)). From an investor’s view, Brunnermeier and Oehmke (2009) discuss three different ways to deal with complexity: (i) applying separation results, (ii) relying on models, or (iii) via standardization. Arora et al. (2011) illustrate how the usage of computationally complex derivatives may worsen asymmetric information costs. Célier and Vallée (2017) empirically test the implications of the Carlin (2009) model and indeed find complexity to be increasing in issuer competition. Furthermore, several studies analyze the steadily growing market for complex securities, in particular their pricing, historical performance, as well as the characteristics of the involved issuers and investors (Henderson and

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6 For their most general predictions, Biais et al. (2017) only rely on first order stochastic dominance. When allowing for deviations from their symmetric payoff distribution, my analysis assumes expected utility maximization instead.
Pearson (2011), Ghent et al. (2014), Griffin et al. (2013), Sato (2014), and Amromin et al. (2011)). Relying on expected utility theory, Hens and Rieger (2008) moreover reject the often claimed market completing effect of structured products. In summary, there exists both theoretical and accumulating empirical evidence that financial institutions rely on a continuing increase in complexity to shield industry rents from competitors and learning by investors rather than to create higher quality products. My paper complements this literature by investigating rising complexity’s implications on agents’ trading behavior.

Second, the herein presented analysis naturally relates to experimental studies on trading ambiguous or complex assets. Implementing a continuous double auction of state-contingent claims based on an Ellsberg urn, Bossaerts et al. (2010) analyze how subjects’ ambiguity aversion affects asset prices and final portfolio holdings. Similar to the no-trade result, they find that, for certain subsets of prices, ambiguity-averse agents prefer to hold nonambiguous portfolios. Furthermore, Bossaerts et al. (2010) show how, in the presence of aggregate risk, sufficiently ambiguity-averse investors indirectly impact asset prices by altering the per capita risk to be shared among marginal investors.

Carlin et al. (2013) study how computational complexity alters bidding behavior in a deterministic environment. They find higher complexity to increase volatility, lower liquidity, and decrease trade efficiency, i.e., to reduce gains from trade. Moreover, Carlin et al. (2013) provide evidence that, additionally to any noise arising from estimation errors, traders’ bidding strategies are influenced by a complexity-induced adverse selection problem. Intuitively, given traders’ private values of the tradable asset are affiliated, the fear of winner’s curse, i.e., to systematically lose by trading against a better informed counterparty, leads traders to submit more conservative ask and bid quotes.

Asparouhove et al. (2015) show how ambiguity preferences can explain asset prices under asymmetric reasoning. They consider a continuous double auction of arrow securities, where, midway through the auction, agents are confronted with an involved updating problem regarding the relative likelihood of the underlying states. In line with Fox and Tversky (1995)’s comparative ignorance proposition, Asparouhove et al. (2015) argue that agents perceive irreconcilable post-updating market prices as ambiguous. Hence, if ambiguity-averse, agents who apply incorrect reasoning become price-insensitive. Consistent with ambiguity aversion, the more price-sensitive agents there exist, the less severe is the experimentally documented mispricing.

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7 In the experimental design by Carlin et al. (2013) participants trade different assets whose values have to be determined deductively by solving systems of linear equations, where the authors differentiate between simple and complex computational problems.

8 The updating task in Asparouhove et al. (2015)’s experimental design is an adaptation of the famous ‘Monty Hall problem’.
Third, this paper also relates to an emerging literature comparing individuals’ preferences towards pure Ellsberg-like ambiguity and complex risk(s), where, as in this paper, the latter is uniquely defined by an objective probabilistic structure. The findings in Halevy (2007) give support to a close relation between individuals’ ability to correctly reduce compound lotteries and their attitudes to pure ambiguity. The vast majority of subjects (95%) who failed to disentangle compound objective lotteries, displayed nonambiguity-neutral behavior.

In their recent paper, Armantier and Treich (2016) provide strong empirical evidence for “a tight link between attitudes toward ambiguity and attitudes toward complex risk” (Armantier and Treich, 2016, p. 5). In their ambiguity treatment, subjects are confronted with lotteries whose outcomes depend on draws from an opaque Ellsberg urn, while complex risks are represented by lotteries that get settled by simultaneous draws from multiple transparent urns. Based on estimated certainty equivalents for both lottery types, Armantier and Treich (2016) elicit ambiguity as well as complex risk premiums. They find a strong positive correlation between the two premiums across subjects.

The remainder of the paper is organized as follows. Section 2 introduces the model economy and develops the necessary theory for generating predictions about trading both simple and complex risks. Section 3 describes the experimental design and confronts the theoretical predictions with the data. Section 4 concludes.

2. Theory

This section introduces the model economy for which I thereafter study individual trading behavior conditional on agents’ information quality. If risks are simple, implications of varying risk-preferences are analyzed within the classical framework of expected utility. In contrast, if risks are complex, individual preferences are adjusted to account for their imperfect information. The first case provides a clear-cut benchmark to which complexity-induced implications can be compared to.

2.1. Model

I start from the simple economy of Biais et al. (2017). In the two-period interpretation of this trading economy, $t \in \{1, 2\}$, uncertainty gets resolved in the second period, where there are two possible states of the world, $\Omega = \{u, d\}$. The probability of reaching state $u$ is denoted by $\pi$, i.e. $P(\omega=u) = \pi$ and $P(\omega=d) = 1 - \pi$, respectively. Contrary to Biais et al. (2017), I allow for
any nontrivial binary payoff distribution $\pi \in (0, 1)$.$^9$ This generalization is crucial, given agents’ subjective probability estimates in the context of complex risks.

The economy offers access to a complete asset market, where shares of a risky asset (stock) can be traded in exchange for units in the risk-free asset (numéraire). The stock pays a state-dependent dividend $X$ per share in $t = 2$, but nothing beforehand. The dividend fully transfers the stock’s final value to shareholders, i.e., after payments have been made, all shares expire worthless (no continuation value). Without loss of generality, it is assumed that $X(u) > X(d)$. The time difference between $t = 1$ and $t = 2$ is considered to be very short, allowing to abstract from any time discounting. Therefore, in-between periods, the risk-free asset simply serves as pure storage device (cash) that does not pay any interests.

There is an infinite number of agents populating the economy. I denote the unbounded set of agents by $I$. Agent $i \in I$ is endowed with nonnegative holdings in the risk-free asset $B_i$, $S_i$ shares of the risky stock, and some state-contingent non-tradable income $I_i(\omega)$ paid out in $t = 2$ only. Moreover, every agent belongs to one of two types, i.e., either she is allowed to buy shares (potential buyer) or to sell them (potential seller). There exist as many buyers as sellers and their respective endowments are identical within each type. Every agent only cares about her utility of consumption $C_i(\omega)$ in $t = 2$, where consumption is given by the sum of the final holdings in the risk-free asset, dividend payments, and nontradable income. In the first period, potential buyers and sellers are able to trade shares via a call-market in order to maximize their increasing utility from consumption $U_i(C_i)$ in the second period.

Finally, agents’ state-contingent income is set to exactly offset the aggregate consumption risk caused by the stock’s dividend payments. If aggregate endowments are constant, I show that for risk-averse agents, the rational expectation equilibrium is independent from potentially heterogeneous attitudes towards simple consumption risks. In particular, under simple risks, the stock market-clearing price and quantity remain unaffected by the shape of agents’ utility functions $(U_i)_{i \in I}$. If, despite the income $I_i(\omega)$, aggregate risk prevailed, market equilibrium would necessarily reflect agents’ (average) risk preferences.$^{10}$

2.2. Trading Simple Risks: Expected Utility

In the presence of simple risks, since the value of $\pi$ is common knowledge, agents possess perfect information regarding the stock’s payoff distribution. According to classical consumption-based

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$^9$ Biais et al. (2017) only consider the symmetrical case, i.e., $\pi = 1/2$. Imposing symmetry has the advantage of delivering robust predictions even under the inapplicability of expected utility theory, i.e., by only assuming the absence of first order stochastically dominated actions.

$^{10}$ Constantinides (1982) shows that if agents with different risk attitudes all maximize expected utility subject to a common prior, equilibrium prices can always be rationalized in a representative agent framework. Hence, in the absence of complex risks, market equilibrium can be explained by the risk preferences of this representative agent.
asset pricing theory, the stochastic discount factor then corresponds to the representative agent’s marginal rate of intertemporal substitution. In Biais et al. (2017)’s simple economy, with consumption restricted to $t = 2$, expected utility theory implies an equilibrium stock price $P$ equal to the stock’s normalized expected payoff weighted by her marginal utilities across states.\textsuperscript{11} By choosing $(I_i(\omega))_{i \in \mathcal{I}}$ such that aggregate consumption risk vanishes, the interconnectedness between $P$ and marginal utilities disappears, allowing for predictions robust to any parameterization of any set of increasing utility functions $(U_i)_{i \in \mathcal{I}}$. The results in this subsection correspond to generalizations of Biais et al. (2017) to values of $\pi \neq 1/2$.

Recalling the two-state nature of the economy in $t = 2$, one can write agent $i$’s expected utility from consumption as

$$E[U_i(C_i(\omega))] = \pi U_i(C_i(u)) + (1 - \pi) U_i(C_i(d))$$

$$= \pi U_i \left( \mu_i + \sqrt{\frac{1 - \pi}{\pi}} \sigma_i \right) + (1 - \pi) U_i \left( \mu_i - \sqrt{\frac{1 - \pi}{\pi}} \sigma_i \right),$$

where $\mu_i \equiv \pi C_i(u) + (1 - \pi) C_i(d)$ and $\sigma_i^2 \equiv \pi(1 - \pi)(C_i(u) - C_i(d))^2$. Thus, any agent’s expected utility can be rewritten as a function of her expected consumption, the standard deviation of consumption across states, and the probability $\pi$.

In the absence of aggregate risk, i.e., if aggregate endowment across $S_i X(\omega)$ and $I_i(\omega)$ is constant, there must exist a tradable quantity $\hat{Q}$ at which every seller and buyer is perfectly hedged against any consumption risk in $t = 2$. If agents are risk-averse, i.e., whenever $U_i$ is strictly concave for every agent $i$, there exists a unique equilibrium for the call-market in $t = 1$.

**Proposition 1.** If $U_i$ is differentiable and strictly concave $\forall i \in \mathcal{I}$, and there exists a tradable quantity $\hat{Q}$ such that every seller and buyer is perfectly hedged, i.e., $\sigma_i = 0, \forall i \in \mathcal{I}$, then seller $i$’s supply and buyer $j$’s demand curve for the risky asset have the unique intersection point $(E[X], \hat{Q}), \forall \{i, j\} \subset \mathcal{I}$.

**Proof.** For proof see Appendix A.

The driving force behind Proposition 1 is the strict concavity of the utility functions, i.e., agents aversion to consumption risk. To see this, it is helpful to separately consider the shape of both

\textsuperscript{11} When deciding on her optimal trading strategy $Q$ in $t = 1$, the representative agent solves the following problem (where $Q > 0$ implies buying)

$$\max_Q E[U_i(C_i(\omega))] \quad \text{s.t.} \quad C_i(\omega) = (S_i + Q)X(\omega) + (B_i - QP) + I_i(\omega),$$

maximizing her expected utility from consumption in $t = 2$ subject to her budget constraint (neglecting any borrowing constraints). Hence, the first order condition yields

$$P = E \left[ \frac{U_i'(C_i(\omega))}{E[U_i'(C_i(\omega))]} X(\omega) \right].$$
seller i’s supply and buyer j’s demand curve for the risky stock.

First, note that for a price equal to one share’s expected dividend, seller i’s expected consumption in Eq. (1) is independent of the number of shares sold. Since seller i is risk-averse, for \( P = E[X] \) she will therefore always decide to sell exactly \( \hat{Q} \) shares and thereby be perfectly hedged against future fluctuations in consumption. However, for \( P < E[X] \) (\( P > E[X] \)) her expected consumption only increases, if she sells less (more) than \( \hat{Q} \) shares. Because she is only willing to incur risk, i.e., deviate from selling \( \hat{Q} \) shares, if appropriately compensated in return, her supply curve must lie somewhere in the lower left and upper right quadrant of the price-quantity space shown in Subfigure (a) of Figure 1.

Second, note that for \( P = E[X] \), similarly buyer j’s expected consumption in Eq. (1) is independent of the number of shares bought. Given her risk-aversion, she chooses to buy exactly \( \hat{Q} \) shares for \( P = E[X] \), and more (less) than \( \hat{Q} \) shares if \( P < E[X] \) (\( P > E[X] \)), as illustrated in Subfigure (b) of Figure 1. Thus, when there is no aggregate risk, seller i’s supply and buyer j’s demand curve exhibit the unique intersection point \( (E[X], \hat{Q}) \) as depicted in Subfigure (c).

Interestingly enough, depending on the shape of \( U_i \), a large opposite income effect can dominate the corresponding substitution effect of a given price change. Hence, seller i’s supply or buyer j’s demand curve can effectively be nonmonotonic within the respective dominating quadrants of the \( PQ \)-plane. The following remark provides an example of a nonmonotonic supply curve.

**Remark 1.** Suppose, seller i’s utility function is defined piecewise as follows

\[
U_i(C) = \begin{cases} 
    c_1 \frac{C^{1-\epsilon}}{1-\epsilon}, & \text{for } 0 \leq C \leq \bar{C}, \\
    c_2 - e^{-\alpha C}, & \text{for } \bar{C} \leq C,
\end{cases}
\]

where \( \alpha > \epsilon > 0 \) and \( \epsilon \) small, \( \alpha > 0 \), and \( c_1 \) and \( c_2 \) are positive constants such that \( U_i \) is differentiable \( \forall C \geq 0 \). For certain parameter pairs \((\alpha, \pi)\), seller i’s supply curve can be nonmonotonic over a nonempty subset of \( P \).

**Proof.** For proof see Appendix A.

Figure D.1 in the Appendix D shows an example of a nonmonotonic supply curve for similar parameter values as in the actual experiment. The intuition for this exemplary nonmonotonicity effect is simple. For every seller and any given \( \hat{Q} \), both \( C(d) \) and \( C(u) \) are strictly increasing in \( P > 0 \). If prices are high enough, seller i’s higher CARA coefficient \( \alpha \), relevant for \( C(\omega) > \bar{C} \), can dominate her lower CRRA coefficient \( \epsilon \). Thus, for even higher prices, she is willing to bear less and less risk, causing her supply curve to decrease until it eventually reaches \( \hat{Q} \), and thereby completely eliminating her consumption risk.
Absence of Risk Aversion

In case agents are not averse to consumption risk, for all \( P \neq E[X] \), an even stricter separation between dominating and dominated strategies than shown in Figure 1 applies. From the proof of Proposition 1 it directly follows that whenever \( U_i \) is either linear or convex, seller \( i \) always strictly prefers to sell zero shares for \( P < E[X] \). In contrast, for \( P > E[X] \), her expected utility is maximized if and only if she sells her full initial endowment in shares. The symmetric behavior applies to risk-neutral and risk-loving buyers, respectively.

For \( P = E[X] \), risk-neutral agents are indifferent between trading \( \hat{Q} \) shares or any other quantity, whereas risk-loving agents are indifferent between trading zero shares or the maximum number possible. In summary, as long as they do not consistently choose among their set of indifferent strategies in an asymmetric manner, the equilibrium in Figure 1 remains unaffected by a nonzero mass of nonrisk-averse agents.
2.3. Trading Complex Risks: Heterogeneous Complexity Preferences

When agents’ information regarding the distribution of $X(\omega)$ is imperfect, I consider the associated consumption risk to be (more) complex. In the presence of such complex risks, rationality in decision making requires some form of acknowledgment regarding the information’s inherent degree of (im)precision. The literature provides a vast number of models intended to account for individuals’ degree of confidence in their relative likelihood estimates. In the following, I analyze individual trading of complex risks within two classes of seminal ambiguity models: multiple-priors utility and the ‘smooth ambiguity’ model proposed by Klibanoff et al. (2005). In the former, agents’ information quality has a ‘first order’ effect on their trading decision (change in mean), whereas for the latter, lower information precision increases the total amount of perceived ‘risk’ (see Epstein and Schneider (2010)). For multiple-priors utility, there exists a direct mapping to rank-dependent expected utility, which I briefly discuss.

Multiple-priors Utility

Agents facing complex risks are unable to determine $\pi$ with certainty, but rather consider several payoff distributions possible. Hence, intuitively, when making their trading decisions, they are guided by a set of potential probability laws. I denote agent $i$’s subjective set of possible priors on the state space $\Omega$ by $C_i$.

Based on this idea of multiple priors, Gilboa and Schmeidler (1989) axiomatize a multiple-priors maxmin decision rule that assumes infinite ambiguity-aversion. In order to allow for a full spectrum of ambiguity preferences, I employ the generalization proposed by Ghirardato et al. (2004), the so-called $\alpha$-maxmin model, instead. Assuming the set $C_i$ of subjective priors to be convex, agent $i$’s utility from consumption in $t = 2$ is then given by

$$U_i(C_i(\omega)) = \alpha_i \min_{\pi \in C_i} \left( E[U_i(\pi)] \right) + (1 - \alpha_i) \max_{\pi \in C_i} \left( E[U_i(\pi)] \right),$$

(2)

where, as before, $E[U_i(\pi)] = \pi U_i(C_i(u)) + (1 - \pi) U_i(C_i(d))$, $U_i$ is a differentiable and strictly concave utility function, and $\alpha_i \in [0, 1]$. First, note the straightforward interpretation of Eq. (2). On the one hand, the cardinality or wideness of $C_i$ measures agent $i$’s ambiguity perception: The bigger her set of subjective priors, the more ambiguity she perceives. On the other hand, her preferences towards ambiguity are expressed by $\alpha_i$: If $\alpha_i > 1/2$, she puts more weight on the minimal expected utility, implying ambiguity-aversion. In contrast, if $\alpha_i < 1/2$ ($\alpha_i = 1/2$), then she is ambiguity-loving (ambiguity-neutral). For their axiomatization, Gilboa and Schmeidler (1989) assume maximal ambiguity-aversion, i.e., $\alpha_i = 1$. Second, whenever $C_i$ is a singleton, Eq. (2) reduces to Eq. (1) with subjective probability $\pi_i$, i.e., Eq. (2) converges to subjective...
Table I. Agent types with multiple-priors utility

<table>
<thead>
<tr>
<th>ambiguity-averse</th>
<th>correct ($\pi \in B_i$)</th>
<th>incorrect ($\pi \notin B_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes ($\alpha_i &gt; 1/2$)</td>
<td>Type AC</td>
<td>Type AI</td>
</tr>
<tr>
<td>no ($\alpha_i \leq 1/2$)</td>
<td>Type NC</td>
<td>Type NI</td>
</tr>
</tbody>
</table>

Notes: In the presence of complexity-induced ambiguity, I distinguish between four different types of agents with multiple-priors utility. Agent $i$ can either be ambiguity-averse or does not dislike ambiguity. Additionally, she can either apply correct or incorrect reasoning when processing her imperfect information about $\pi$. Expected utility as $C_i \equiv \pi_i$. For ease of notation, I furthermore rely on the following definition:

$$E_i[X] := \alpha_i E_i[X] + (1 - \alpha_i) E_i[X],$$

where $E_i[X] \equiv E\pi_i[X]$ with $\pi_i := \arg \min_{\pi \in C_i} \mu_i(\pi)$, and $E_i[X] \equiv E\pi_i[X]$ with $\pi_i := \arg \max_{\pi \in C_i} \mu_i(\pi)$.

When risks are complex, agents perceive ambiguity regarding the probability $\pi$. In order to analyze individual trading behavior within the $\alpha$-maxmin model, a case-by-case analysis is required, whereby agent $i$ can behave differently from agent $j$ in two dimensions: First, agent $i$ is either averse to ($\alpha_i > 1/2$) or not disliking ($\alpha_i \leq 1/2$) perceived ambiguity. Second, she can either have correct or incorrect beliefs about the true payoff probability $\pi$. More precisely, I classify agent $i$ as having incorrect beliefs, if $\pi$ is not sufficiently close to the midpoint of her set of priors, i.e., if $\pi \notin B_i \subset C_i$, where $B_i$ itself depends on her ambiguity-aversion:

$$B_i = \begin{cases} [\pi_M - \Delta(2\alpha - 1), \pi_M + \Delta(2\alpha - 1)], & \text{for } \alpha > \frac{1}{2}, \\ \pi_M & \text{for } \alpha \leq \frac{1}{2}, \end{cases}$$

where $\pi_M$ denotes the midpoint of $C_i$ with length (or maximum difference) $2\Delta$. We note that $B_i \rightarrow C_i$ as $\alpha_i \rightarrow 1$ and $B_i \rightarrow \pi_M$ as $\alpha_i \rightarrow 1/2$. Table I summarizes the four possible combinations of types.

Price Sensitivity

In order to deduce the effect(s) of complexity-driven ambiguity on agents’ trading behavior, the different types presented in Table I have to be considered separately. I start with the first row of Table I. If aggregate endowments are constant, any risk-averse agent, as shown above, prefers to trade exactly $\hat{Q}$ shares for $P = E[X]$. Now, given their distaste for the perceived ambiguity regarding $\pi$, agents of type AC and AI eventually both prefer to trade $\hat{Q}$ for prices significantly different from $E[X]$. More precisely, for any given degree of risk-aversion, the subset of prices for which they wish to be perfectly hedged against consumption risk is increasing in both their
ambiguity aversion and ambiguity perception.

**Proposition 2.** In the presence of perceived ambiguity and if there exists a tradable quantity \( \hat{Q} \) such that \( \sigma_i = 0 \; \forall i \in I \), then agents of types AC and AI exhibit constant supply or demand curves over closed subsets of \( P \). Their absolute price elasticity is a decreasing function in both \( \alpha_i \) and the cardinality/length of \( C_i \).

**Proof.** For proof see Appendix A.

In case of no aggregate risk, it holds for any seller \( i \) that \( \pi_i < \pi_i \) for \( Q < \hat{Q} \) and \( \pi_i > \pi_i \) for \( Q > \hat{Q} \), respectively. Intuitively, if seller \( i \) is hedged against varying consumption by selling exactly \( \hat{Q} \) shares, her expected consumption \( \mu_i \) decreases in \( 1 - \pi_i (\pi_i) \) whenever she sells less (more) than \( \hat{Q} \) shares. Analogously, for any buyer \( j \) it holds that \( \pi_j > \pi_j \) for \( Q < \hat{Q} \) and \( \pi_j < \pi_j \) for \( Q > \hat{Q} \), respectively. These shifts in relative size of \( \pi_i \) and \( \pi_i \) around \( \hat{Q} \) in combination with ambiguity-aversion are the driving force behind Proposition 2.

To foster the reader’s intuition, the result in Proposition 2 is illustrated in Figure 2 from the perspective of an ambiguity-averse seller—the analogous reasoning also applies to any ambiguity-averse buyer. First, due to seller \( i \)’s risk-aversion, it can be shown (see proof of Proposition 2) that for \( P = E_i[X] \), selling exactly \( \hat{Q} \) shares strictly dominates trading any other quantity of the risky asset. Moreover, given Eq. (2), she is only willing to sell less than \( \hat{Q} \) shares for prices strictly below \( E_i[X] \) (see proof of Proposition 2). This is illustrated in Subfigure (a) of Figure 2. Analogously, seller \( i \) only agrees to sell more than \( \hat{Q} \) shares in return for \( P > E_i[X] \) (see Subfigure (b)). Second, due to the above discussed order effect of \( \pi_i \) and \( \pi_i \), it follows that the lower price bound \( L \) in Subfigure (a) and the upper price bound \( U \) in Subfigure (b) do not coincide. Therefore, putting everything together, the piecewise constant supply curve depicted in Subfigure (c) prevails, where seller \( i \)’s supply of the risky asset is constant over the closed subset \([L, U]\).

In comparison to the analysis under simple risks in Section 2.2, a nice and intuitive interpretation of Proposition Eq. (2) emerges. Since agents of types AC and AI are averse to ambiguity, selling or buying \( \hat{Q} \) shares becomes even more attractive compared to situations with objective payoff distributions. By trading exactly \( \hat{Q} \) units of the risky asset, agents are not only able to avoid risk, but additionally to dispose any exposure to perceived ambiguity. Trading \( \hat{Q} \) shares hence simultaneously corresponds to the perfect hedging strategy against both risk and ambiguity. In return for this dual insurance, agents are willing to forego potential gains from trade.

I now turn to the second row in Table I. For nonambiguity-averse agents, there are two cases to be distinguished among. First, if \( \alpha_i \) equals \( 1/2 \), agent \( i \) is ambiguity-neutral. For a
seller with $\alpha_i = 1/2$, $L$ and $U$ in Figure 2 coincide, i.e., under complex risks, she behaves as a subjective expected utility-maximizer. The analogous argument applies for an ambiguity-neutral buyer. Second, if $\alpha_i < 1/2$, agent $i$ is ambiguity-loving. The same reasoning as in the proof of Proposition 2 implies that for an ambiguity-loving seller, it holds that $L > U$. Hence, when risks are complex, there exists a certain price between $U$ and $L$ for which she is indifferent between gaining exposure to ambiguity from selling less or more than $\tilde{Q}$ shares. At or precisely beyond this threshold, her supply curve therefore exhibits a discontinuity, i.e., jumping from strictly below to strictly above $\tilde{Q}$. For prices below and above the threshold, her supply curve’s price elasticity increases in comparison to simple risks. Again, the analogous argument can be made for an ambiguity-loving buyer.

12 This can be interpreted as the natural counterpart of ambiguity-averse sellers’ piecewise flat supply curves.
Mispricing and Suboptimal Risk Sharing

How complex risks are priced and shared in equilibrium, crucially depends on agents’ beliefs regarding $\pi$. If aggregate endowments are constant, the risky asset is mispriced whenever the market-clearing price deviates from its expected dividend. Furthermore, the absence of aggregate risk in combination with a complete market allows for perfect risk sharing. Hence, whenever the market-clearing quantity (per capita) is different from $\hat{Q}$, consumption risk is only suboptimally shared among risk-averse agents. I therefore subsequently refer to the market-clearing price and quantity for simple risks, i.e., $(E[X], \hat{Q})$, as benchmark equilibrium.

Nonambiguity-loving agents ($\alpha_i \geq 1/2$) with correct beliefs ($\pi \in B_i$) never cause any mispricing or incomplete risk sharing, simply because their supply or demand curves always go through (see above) the benchmark equilibrium. Due to the jump of their supply (demand) curve between $U$ and $L$, an ambiguity-loving seller (buyer) almost surely never chooses to sell (buy) $\hat{Q}$ shares at $P = E[X]$, independently of her beliefs regarding $\pi$. While it is clear why ambiguity-neutral agents with incorrect beliefs provoke mispricing and suboptimal risk sharing, due to their piecewise constant supply and demand curves, this is, however, less clear for ambiguity-averse agents whose $B_i$ does not contain $\pi$.

**Proposition 3.** In the presence of perceived ambiguity and if there exists a tradable quantity $\hat{Q}$ such that $\sigma_i = 0 \forall i \in I$, then any nonzero mass of type AI sellers (buyers) moves aggregate supply (demand) away from the market equilibrium under simple risks.

**Proof.** For proof see Appendix A.

Figure 3 illustrates the mechanics behind Proposition 3 for the simplified case of only three sellers and buyers, respectively. Subfigure (a) depicts the exemplary supply curves (for a given discrete price grid) from three different sellers. Assuming type NC to be ambiguity-neutral, she chooses—in line with her correct beliefs—to sell $\hat{Q}$ shares for $P = E[X]$. Due to type AC’s pronounced ambiguity-aversion, her supply curve is constant over a considerable subset of prices (delimited by circles). Importantly, since $\pi \in B_{AC}$, its constant part still contains the benchmark equilibrium. In contrast, the constant piece of type AI’s supply curve (delimited by squares) does not include the point $(E[X], \hat{Q})$. Hence, neither the length of $C_{AI}$ nor the degree of her ambiguity-aversion $\alpha_{AI} > 1/2$ are sufficient to prevent that $\pi \not\in B_{AI}$. Therefore, based on incorrect beliefs, her supply draws the average supply curve (solid line) away from the benchmark equilibrium.

For simplicity, all three buyers in Subfigure (b) are assumed to hold correct beliefs such that their demand curves all contain the benchmark equilibrium. This ensures that any mispricing and incomplete risk sharing in equilibrium is solely driven by the AI-type seller’s supply curve in
Notes: For the $\alpha$-maxmin model (Eq. (2)), this figure illustrates how ambiguity-averse agents with incorrect beliefs can cause mispricing and suboptimal risk sharing of complex risks in equilibrium (Proposition 3). Subfigure (a) shows three exemplary supply curves of one ambiguity-neutral (type NC) and two ambiguity-averse (type AC and AI) sellers. All exemplary buyers in Subfigure (b) are assumed to be nonambiguity-loving and to have correct beliefs. Subfigure (c) finally shows, how the incorrect beliefs of seller AI cause mispricing and incomplete risk sharing of complex risks in equilibrium. Due to the absence of aggregate consumption risk, both distortions are unambiguously defined and measurable.

Subfigure (a). The solid line constitutes the resulting average demand curve. Finally, Subfigure (c) depicts the market-clearing price $P^*$ and quantity $Q^*$ (per capita) that corresponds to the intersection of the average supply and demand curves. Due to seller AI’s underestimation of $\pi$, the market-clearing price is smaller than the stock’s expected dividend, implying mispricing equal to $|P^* - E[X]|$. Furthermore, the average market-clearing quantity of shares is greater than $\hat{Q}$, i.e., in equilibrium, agents do not share complex risks perfectly.

Intuitively, Proposition 3 establishes a condition under which ambiguity-induced price insensitivity is sufficiently large in order to offset any equilibrium effects of incorrect beliefs about complex risks. Given the midpoint of agent $i$’s set of priors $C_i$, the more ambiguity-averse she is,
i.e., the larger her $\alpha_i$, the wider becomes the subset of payoff distributions $B_i$ for which incorrect beliefs do not cause any deviations from the benchmark equilibrium. Note that for any $\alpha_i < 1$, the subset $B_i$ in Eq. (4) is strictly smaller than $C_i$. Thus, as long as agent $i$ is not maximally ambiguity-averse, requiring the true payoff distribution $\pi$ to be contained in $C_i$ is not sufficient for precluding differences between simple and complex equilibria.

From Multiple-priors to Rank-dependent Expected Utility

Since the seminal work by Tversky and Kahneman (1992), cumulative prospect theory has become the most prominent alternative to expected utility for modeling decision making under uncertainty. Therefore, a reasonable question likely asked by the reader is the following: How do trading decisions under complex risks from agents with rank-dependent utility differ from the herein presented analysis? For binary acts, e.g., trading the above risky asset, Chateauneuf et al. (2007) show that their proposed ‘neo-additive’ decision weights allow for a one-to-one correspondence from $\alpha_i$ and $C_i$ in Eq. (2) to (i) a likelihood sensitivity index and (ii) a pessimism (optimism) index as generally used in rank-dependent expected utility models.\(^\text{13}\)

Smooth Ambiguity Preferences

Proposition 2’s somehow extreme result of (local) perfect price inelasticity is arguably linked to the kinked preferences induced by the maxmin property of Eq. (2). In order to support the generalizability of the result’s qualitative finding, I henceforth analyze individual trading behavior under the ‘smooth ambiguity’ model by Klibanoff et al. (2005). Adopting the above notation, agent $i$’s utility from consumption in $t = 2$ can then be written as

$$U_i(C_i(\omega)) = \int_{\Delta(\Omega)} \phi_i(E[U_i(\tilde{\pi})]) \, d\mu_i(\tilde{\pi}),$$

where $\Delta(\Omega)$ is the simplex of all possible payoff distributions on $\Omega$, $\mu_i$ is agent $i$’s subjective probability measure on $\Delta(\Omega)$, and $\phi_i$ is a continuous, strictly increasing, real-valued function.

Eq. (5) has an intuitive interpretation: On the one hand, the more payoff distributions exhibit a nonzero probability mass under $\mu_i$, the bigger agent $i$’s set of possible priors. On the other hand, the curvature of $\phi_i(\cdot)$ expresses her ambiguity preferences: As for utility functions in the presence of risk, concavity of $\phi_i(\cdot)$ implies ambiguity-averse, linearity ambiguity-neutral, and convexity ambiguity-loving preferences. Hence, similar to the $\alpha$-maxmin model in Eq. (2), the smooth ambiguity model allows for a separation between the level of ambiguity perceived

\(^{13}\) In rank-dependent expected utility models, the likelihood sensitivity index measures the steepness of the probability weighting function and the optimism (pessimism) index its intersection point with the 45-degree line.
Figure 4. Supply curve of ambiguity-averse seller with smooth preferences

Notes: This figure shows the decreased price elasticity of the supply curve for complex risks implied by the smooth ambiguity model (Eq. (5)) for a risk-averse and ambiguity-disliking seller $i$.

by agent $i$ as well as her general preferences towards ambiguity per se. For ease of notation and analog to Eq. (3), I rely on the following definition:

$$E_i[X] := \int_{\Delta(\Omega)} E_{\hat{\pi}}[X] d\mu_i(\hat{\pi}),$$

where $E_{\hat{\pi}}[X]$ denotes the expected payoff of the risky asset based on $P(\omega = u) = \hat{\pi}$ and $P(\omega = d) = 1 - \hat{\pi}$, respectively.

Proposition 4. Let $\mu_i(\hat{\pi})$ be the normalized Lebesgue measure on agent $i$’s set of possible priors $[\underline{\pi}, \overline{\pi}] \subset [0, 1]$, i.e., $\mu_i(\hat{\pi}) := \frac{1}{\overline{\pi} - \underline{\pi}} d\hat{\pi} \forall \hat{\pi} \in [\underline{\pi}, \overline{\pi}]$. In the presence of perceived ambiguity and if there exists a tradable quantity $\hat{Q}$ such that $\sigma_i = 0 \forall i \in \mathcal{I}$, then

(i) agent $i$’s price elasticity is an increasing function in the second order derivative of $\phi_i(\cdot)$.

(ii) any nonzero mass of sellers (buyers) for whom $\frac{\overline{\pi} + \underline{\pi}}{2} \neq \pi$ moves aggregate supply (demand) away from the benchmark equilibrium under simple risks.

Proof. For proof see Appendix A. $\square$

As implied by the proof of Proposition 4, with utility as in Eq. (5), any agent’s supply (demand) curve goes through $(\hat{Q}, E_i[X])$. Thus, independently of her ambiguity preferences, she always finds it optimal to sell (buy) $\hat{Q}$ shares for a price $P$ equal to her subjective expected payoff per share given her subset of priors.

For prices below and above $E_i[X]$, Figure 4 exemplary illustrates how imperfect information about $\pi$ affects an ambiguity-averse seller’s supply curve. The demand curve for any ambiguity-averse buyer behaves analogously. If, under complex risks, seller $i$ dislikes any perceived ambiguity regarding $\pi$, selling $\hat{Q}$ shares generally becomes more attractive than under
simple risks. Due to her smooth distaste for ambiguity, i.e., the concavity of \( \phi_i(\cdot) \), she smoothly decreases her supply’s price elasticity for prices different from \( E_i[X] \), as displayed in Figure 4. However, in contrast to Figure 2, her supply curve never becomes perfectly inelastic for any interior nonempty subset of prices.

In case seller \( i \) is ambiguity-loving, i.e., \( \phi_i(\cdot) \) is convex, the slope of her supply curve amplifies when moving form simple to complex risks. Comparing Figure 4 to Subfigure (a) in Figure 1 moreover shows how increasing complexity under Eq. (5) manifests itself similarly as a shift in sellers’ risk aversion under Eq. (1): If seller \( i \) is ambiguity-averse, she is always willing to accept a lower \( \mu_i \) in return for a gradual reduction in \( \sigma_i \).

For equally probable priors, the second part of Proposition 4 states that whenever there is a critical mass of agents for whom \( \pi \) is different from their respective midpoint of priors, they shift aggregate supply (demand) away from the benchmark equilibrium. Under the smooth ambiguity model, the pricing and allocation of complex risks is therefore more sensitive to agents’ ex-ante beliefs than under kinked ambiguity-preferences. For smooth preferences, ambiguity-induced price insensitivity can never offset a critical mass’ distorting equilibrium effects of incorrect beliefs, no matter how small the respective deviations relative to \( \pi \) are.

**Summary**

In contrast to the smooth ambiguity model, the pricing of complex risks by ambiguity-averse agents with multiple-priors utility is less sensitive to incorrect beliefs. Within the multiple-priors model, the necessary mispricing condition requires the exclusion from a set, i.e., \( \pi \notin \mathcal{B}_i \), instead of a pointwise deviation from \( \pi \). Another implication of its piecewise constant supply (demand) curve is the arising possibility of multiple equilibria. In an economy with three-dimensional heterogeneous agents, i.e., with respect to their beliefs as well as their preferences towards risk and ambiguity, multiple equilibria are nevertheless unlikely to prevail. For instance, if the supply curve of a given mass of sellers equals \( \hat{Q} \) for a nonsingleton subset of prices, a nonzero mass of sellers whose supply is not constant over the same subset is sufficient for the average supply curve to be nonconstant.

In general, within both models, complex risks have similar implications for individual trading behavior and aggregate market outcomes:

1. If averse to complexity-induced ambiguity, the price sensitivity of agents with nonsingleton priors decreases under complex risks.

2. Agents with nonsingleton priors can cause mispricing and trade towards suboptimal allocations of complex risks.
These two implications are not independent. A decrease in price sensitivity around the perfect hedging quantity $\tilde{Q}$ reduces the potential for imperfectly shared idiosyncratic risks. This is intuitive, because, under complex risks, ambiguity-averse agents always prefer to trade (close to) $\tilde{Q}$ shares for a wider range of prices. However, for smooth ambiguity preferences, a reduction in price sensitivity does not attenuate the degree of mispricing induced by imperfect information (see above).

2.4. Price-taking Behavior, Asymmetric Information, and Strategic Uncertainty

Before turning to the experimental test of the above theory, three potentially interfering effects need to be addressed more carefully. First, my model economy assumes infinitely many agents. When implementing it in the laboratory, complying with this particular assumption constitutes an apparent impossibility. I meet this practical constraint by running all sessions with a relatively high number of at least 16 subjects.\footnote{This minimum number is in line with the average number of 17.6 subjects per session in Biais et al. (2017).} Moreover, I alternate between two different pricing schemes: market-clearing—as persistently assumed above—and random price draws (see below). Comparing subjects’ supply and demand functions between these two pricing schemes allows me to control for their price-taking behavior.

Second, and more importantly, depending on how agents self-assess their information processing capabilities relative to others, they might perceive considerable information asymmetries in the presence of complex risks. In a Grossman and Stiglitz (1980) rational-expectation equilibrium, market-clearing prices imperfectly reflect informed traders’ costly information about the risky stock’s expected payoff. Applied to my setting, there exists a dominant strategy for (completely) uninformed agents whose implications are in line with the ambiguity preference-based theory above: Agents who perceive themselves as uninformed (i.e., face too high information processing costs) and simultaneously believe markets to generate, at least partially, information efficient prices should always submit perfectly inelastic supply (demand) functions, i.e., $Q_i(P) = \tilde{Q}$ $\forall P$.

Note, however, that in contrast to Grossman and Stiglitz (1980), I require some unobservable heterogeneity in agents’ information processing abilities (costs) in order to prevent market-clearing prices to be fully informative.\footnote{Whenever agent $i$ believes that there is a nonzero mass of agents submitting supply (demand) functions based on relatively less informative beliefs, she finds herself better off trading according to her more informative beliefs.} Otherwise, given the conditionality of agents’ supply (demand) functions on market-clearing prices, no one has an incentive to process the complex information in the first place. Thus, Grossman and Stiglitz (1980)’s informational efficiency paradox would prevail.
Third, any further potential implications caused by strategic uncertainty must be accounted for. In a trading game such as the one considered herein, agent i generally faces strategic uncertainty about the behavior of the remaining —i traders. Whenever agent i forms subjective beliefs about her opponents’ actions, these beliefs—whether rationalizable or not—may affect her trading decisions ex-ante.

Alternating between market-clearing and random price draws not only allows for testing the price-taking hypothesis, but additionally enables me to control for any potential effects from either perceived asymmetric information or strategic uncertainty.

3. Experiment

In this section I first present the parameterization of the model economy, motivate and illustrate the chosen lab implementation of complex risks, and provide a detailed overview of the conducted sessions. The collected data is then analyzed on both the aggregate as well as on the individual level. For the latter, I construct two different measures of price sensitivity. The variant discrepancy between individual behavior and aggregate outcomes under simple versus complex risks can be reconciled with varying bounds on quasi-rational choice. Finally, markets’ general effectiveness in aggregating traders’ imperfect information about complex risks is evaluated.

3.1. Design and Sessions Overview

The selection process of the model parameters is twofold. On the one hand, the distribution of the stock’s binary dividend needs to be fixed. In order to control for a natural focal point effect, I alternate between two values of \( \pi \), i.e., \( \pi \in \{1/3, 1/2\} \). Furthermore, to simplify calculations of expected payoffs, I set the stock’s dividend \( X(\omega) \) equal to ECU 150 (experimental currency units) in state \( u \) and ECU 0 in state \( d \), respectively.

On the other hand, agents’ endowments need to be as such that aggregate consumption is constant across states. Table II presents the endowments for both sellers and buyers that independently apply at the beginning of every trading round. Note, in the presence of equally many sellers as buyers, consumption risk is zero on the aggregate level. In particular, if any seller \( i \) and any buyer \( j \) agree to trade \( \hat{Q} = 2 \) shares at a price per share of \( P \), both are perfectly hedged with constant consumption equal to ECU 300 + 2\( P \) and ECU 600 − 2\( P \), respectively. The symmetry between sellers’ and buyers’ potential overall consumption is intentional. When comparing local sensitivities between their supply and demand, symmetric function arguments
Table II. Endowments for sellers and buyers

<table>
<thead>
<tr>
<th></th>
<th>Seller</th>
<th>Buyer</th>
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<tbody>
<tr>
<td>Stock</td>
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<td>0</td>
</tr>
<tr>
<td>Bond</td>
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<td>300</td>
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<tr>
<td>Cont. income $I(\omega)$</td>
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<td></td>
</tr>
<tr>
<td>State $u$: $I(u)$</td>
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<td>0</td>
</tr>
<tr>
<td>State $d$: $I(d)$</td>
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<td>300</td>
</tr>
<tr>
<td>Agg. endowment</td>
<td>constant</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table shows the endowments for sellers and buyers, respectively, that apply at the beginning of every independent trading round. All figures except the number of shares are in experimental currency units (ECU).

allow me to isolate and solely analyze preference driven differences.\(^{16}\)

Complex versus Simple Risks in the Laboratory

When implemented in the laboratory, complex risks need to satisfy two necessary conditions in order to comply with the above definition:

(i) they have to follow an *objective* underlying probability distribution, and

(ii) subjects have to be aware of the problem’s well-defined nature and the existence of its *unique* solution.

Moreover, when aiming for informative empirical data, the (imperfect) information about complex risks should

(iii) not be too complex, i.e., imposing at least some nontrivial restrictions on subjects’ sets of priors, but

(iv) still be complex enough such that subjective priors neither are singletons.

I argue that the following implementation satisfies (i)–(iv). Consider the geometric Brownian motion shown in Subfigure (a) of Figure 5. In the ‘complexity treatment’, subjects were provided with both the *dynamic* visualization of a *reference* path between $t = 0$ and $t = 1$, as well as the *formal* specification of the stochastic differential equation governing its evolution. In order to map a continuous process $S_t$ into the required binary payoff distribution,\(^{17}\) a simple threshold approach was applied. More specifically, whenever the reference path in $t = 2$ was greater or equal than a predefined threshold $L$, i.e., if $S_2 \geq L$, the risky stock paid a dividend $X(u)$ equal to

\(^{16}\)Despite the symmetry in total consumption, endowment effects and reference-dependent preferences (see, e.g., Kahneman et al. (1991)) could of course still be at play here. However, I find no such evidence in my experimental data.

\(^{17}\)Not to be confused with seller $i$’s share endowment $S_i$ in Section 2.
Notes: This figure shows the information about complex risks subjects were provided with during the experiment. For the first stage, Subfigure (a) presents an example of the information displayed on subjects’ screens when asked to enter their supply (demand) schedules. Whenever the blue reference path ends up in the green region, the stock pays a dividend per share equal to ECU 150 (experimental currency units) and zero otherwise. Given the here considered parameterization, Appendix B shows that the former probability equals $1/2$. For the second stage, Subfigure (b) presents a possible realization of the process and the stock’s corresponding payoff per share.

150 and zero otherwise. As demonstrated in Appendix B, the problem of determining $P(S_2 \geq L)$ as in Figure 5 can be solved with a back-of-the-envelope calculation applying Itô calculus.

When submitting their respective supply (demand) functions during the first stage of trading rounds with complex risks, subjects were presented the type of information as exemplary displayed in Subfigure (a) of Figure 5. While doing so, they were given the possibility to repeatedly observe the reference path’s dynamic evolution between $t = 0$ and $t = 1$. Across complex trading rounds, two different parameterizations of $S_t$ were used—one for $\pi = 1/3$ and one for $\pi = 1/2$, respectively—whereas the realized path was unique to every round. At the second stage, subjects were informed about their number of shares sold (bought) and were presented with the realization of $S_2$ as, e.g., shown in Subfigure (b).

For submitting their supply (demand) schedules, subjects faced—similar as in Biais et al. (2017)—a predefined price vector. The increment of the uniformly spaced price vector was set to five ECU, i.e., for every $P \in \{0, 5, 10, ..., 145, 150\}$, subjects were asked to choose the preferred number of shares to be sold (bought). These quantities were entered with a precision of two decimal places.

\footnote{In order to minimize the number of necessary keyboard entries, the decision process was divided into two substages (see the experimental instructions in Appendix E for details).}
1 Ball is randomly drawn.

$$\begin{align*}
\text{Dividend} &= 150 \text{ if } G \\
\text{Dividend} &= 0 \quad \text{if } R
\end{align*}$$

1 Ball is randomly drawn.

$$\begin{align*}
\text{Dividend} &= 150 \text{ if } G \\
\text{Dividend} &= 0 \quad \text{if } R
\end{align*}$$

(a) Pre-trading screenshot for simple risks  
(b) Pre-trading screenshot for ambiguous risks

**Figure 6. Simple and ambiguous risks in the laboratory**

*Notes:* This figure shows the information about simple and ambiguous risks subjects were provided with at the first stage during the respective trading rounds of the experiment. Whenever the randomly drawn ball is green, the stock pays a dividend per share equal to ECU 150 (experimental currency units) and zero otherwise. In contrast to simple risks in Subfigure (a), the distribution of green and red balls in Subfigure (b) is arbitrary.

In order to more precisely test the above theoretical predictions, it is helpful to control for subjects’ beliefs about complex risks. This is achieved in the following way. During the first stage of complex trading rounds, subjects were additionally asked to provide their point estimate regarding the stock’s expected payoff per share. Independently of subjective preferences, the thereby elicited point estimates allow to anchor subjects’ individual sets of priors.

In contrast, during the first stage of trading rounds with *simple* risks, subjects *knew* the exact probability of the stock paying a dividend equal to 150. For the case where $$\pi = \frac{1}{2}$$, subjects were confronted with an urn containing 15 green and 15 red balls, as depicted in Subfigure (a) of Figure 6. At the second stage, the color of one randomly drawn ball was revealed. Whenever this ball happened to be green, the stock paid a dividend per share equal to ECU 150 and zero otherwise. Finally, as a control treatment, the tradable risks of the last trading round were purely ambiguous. Instead of a ‘transparent urn’, subjects were confronted with the Ellsberg (1961)-like urn shown in Subfigure (b), whose composition of green and red balls was unknown.

**Sessions Structure, Incentivization, and Subject Summary Statistics**

Table III provides an overview of the six sessions conducted in the ‘Laboratory for Experimental and Behavioral Economics’ at the University of Zurich during fall 2016. The number in parenthesis indicates the number of subjects in a given session. Each session consisted of ten independent trading rounds. All subjects only participated in one session. For every single trading round, Table III lists the actual payoff distribution, the nature of the underlying consumption

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19 Depending on subjects’ respective preferences, the risky asset’s expected payoff under complex risks is either defined by the mean of Eq. (3) for trading more or less than $Q$ shares, or by Eq. (6), respectively.
## Table III. Sessions overview

<table>
<thead>
<tr>
<th>Round</th>
<th>Session 1 (#16)</th>
<th>Session 2 (#18)</th>
<th>Session 3 (#16)</th>
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<tbody>
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<td>π Type Pricing</td>
<td>π Type Pricing</td>
<td>π Type Pricing</td>
</tr>
<tr>
<td>2</td>
<td>high C (P) MC</td>
<td>high C (P) random</td>
<td>high C (P) random</td>
</tr>
<tr>
<td>3</td>
<td>low C (P) MC</td>
<td>low C (P) random</td>
<td>low C (P) random</td>
</tr>
<tr>
<td>4</td>
<td>1/2 C MC</td>
<td>1/4 C random</td>
<td>1/3 C MC</td>
</tr>
<tr>
<td>5</td>
<td>1/3 C random</td>
<td>1/2 C MC</td>
<td>1/2 C random</td>
</tr>
<tr>
<td>6</td>
<td>1/2 S MC</td>
<td>1/2 S random</td>
<td>1/3 S MC</td>
</tr>
<tr>
<td>7</td>
<td>1/3 S random</td>
<td>1/3 S random</td>
<td>1/3 S random</td>
</tr>
<tr>
<td>8</td>
<td>ambig A MC</td>
<td>ambig A random</td>
<td>ambig A MC</td>
</tr>
</tbody>
</table>

Notes: This table provides an overview of the six conducted sessions. Each session consisted of ten independent trading rounds. The number in parenthesis indicates the number of subjects in a given session. For every session, the first column lists the actual payoff distribution, the second column the nature of the underlying consumption risk (simple (S) versus complex (C)), and the third column the applied pricing scheme (market clearing (MC) versus random price draw (random)). The ‘high’ (‘low’) π refers to an integer parameterization of the stochastic reference path $S_t$ that results in a probability $\mathbb{P}(S_2 > L)$ of 84.21% (15.89%), and ‘P’ denotes a practice round. To control for potential ‘comparative ignorance effects’ (see Fox and Tversky (1995)), the sequential ordering of simple and complex risks is reversed between the first three and the last three sessions.

In each session, after the ten trading rounds shown in Table III, subjects were additionally presented with two lotteries, each based on one of the two urns in Figure 6. For both lotteries, their certainty equivalents were elicited via Abdellaoui et al. (2011)’s computerized iterative choice list method. Importantly, the lotteries’ payoffs were chosen such that they matched the range of possible consumption levels in each of the previous trading rounds (see Figure D.2 in...
Appendix D). Overall, one session lasted approximately 90 minutes.

At the end of every session, one out of the seven nonpractice trading rounds or one of the two lottery outcomes was randomly chosen, each with equal probability. Subjects then were paid either their final wealth of the selected trading round or the outcome of the selected lottery, both divided by twelve. Additionally, if their point estimate regarding $\pi$ was correct (within ±3%), they earned an extra three Swiss francs, whenever the corresponding trading round was selected for payment. On average, participants received 38.40 Swiss francs, with a maximum of CHF 50 and a minimum of CHF 25.

Recruited subjects were students from either the University of Zurich or ETH Zurich, majoring in economics, business, mathematics, physics, engineering, or computer science, respectively. Their respective role of either a buyer or a seller was randomly assigned at the beginning of the experiment and thereafter retained throughout all trading rounds. The instructions provided to subjects acting as sellers are provided in Appendix E.20

Table IV presents the average values (proportions) of certain socioeconomic variables collected via a short questionnaire following the main experiment. Risk aversion is measured as the normalized difference between the simple lottery’s expected payoff and subjects’ respective certainty equivalents. A value of one (minus one) denotes maximum (minimum) risk aversion,21 a value of zero implies risk-neutrality. The total sample’s average risk aversion of 0.060 corresponds to a constant relative risk aversion (CRRA) coefficient of 0.684.22 Ambiguity aversion is defined as the individual differences in certainty equivalents for the simple and ambiguous lottery. Hence, a positive value indicates ambiguity aversion. A standard randomization check reveals no significant indications of an unbalanced sample.23

3.2. Aggregate Market Outcomes

Figure 7 shows average supply and demand curves across sessions with identical chronology of simple versus complex risks. For trading rounds with complex risks, the vertical solid (dotted) line indicates sellers’ (buyers’) average point estimate of the risky asset’s expected payoff. In general, subjects overestimate the latter,24 where in three of four cases, sellers’ average estimate exceeds the one of buyers.25 Focusing on average supply and demand curves around (average)

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20 Analogous instructions were provided to subjects acting as buyers and are available upon request.

21 According to Abdellaoui et al. (2011)’s iterative choice list method.

22 My estimate of average relative risk aversion is in line with the experimental literature: see Holt and Laury (2002) for binary lotteries, Goeree et al. (2002) for private value auctions, Goeree et al. (2003) for generalized matching pennies games, and Goeree and Holt (2004) for one-shot matrix games. Similarly, Biais et al. (2017) find the representative investor’s CRRA coefficient to approximately equal 0.5.

23 Throughout the entire paper, I report two-sided $p$-values.

24 One possible explanation is that subjects fail to account for the second order effect due to the nonzero quadratic variation of the Brownian motion $W_t$.

25 This could be due to an ‘indirect’ endowment effect.
### Table IV. Summary statistics and randomization check

<table>
<thead>
<tr>
<th>Variable</th>
<th>Total sample ($N = 98$)</th>
<th>Sellers ($N = 49$)</th>
<th>Buyers ($N = 49$)</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>23.674 (3.008)</td>
<td>23.837 (3.287)</td>
<td>23.510 (2.724)</td>
<td>0.689</td>
</tr>
<tr>
<td>Gender</td>
<td>0.337 (0.475)</td>
<td>0.286 (0.456)</td>
<td>0.388 (0.492)</td>
<td>0.393</td>
</tr>
<tr>
<td>UZH (ETH)</td>
<td>0.582 (0.496)</td>
<td>0.653 (0.481)</td>
<td>0.510 (0.505)</td>
<td>0.219</td>
</tr>
<tr>
<td># semesters</td>
<td>3.806 (2.827)</td>
<td>3.633 (2.928)</td>
<td>3.980 (2.742)</td>
<td>0.365</td>
</tr>
<tr>
<td>Knowledge BM</td>
<td>0.459 (0.501)</td>
<td>0.367 (0.487)</td>
<td>0.551 (0.503)</td>
<td>0.105</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>0.060 (0.265)</td>
<td>0.087 (0.294)</td>
<td>0.035 (0.232)</td>
<td>0.328</td>
</tr>
<tr>
<td>CRRA-γ</td>
<td>0.684 (3.358)</td>
<td>1.045 (4.415)</td>
<td>0.323 (1.740)</td>
<td>0.335</td>
</tr>
<tr>
<td>Ambiguity aversion</td>
<td>0.101 (0.245)</td>
<td>0.067 (0.225)</td>
<td>0.133 (0.262)</td>
<td>0.133</td>
</tr>
</tbody>
</table>

**Notes:** This table reports means and standard deviations (in parenthesis) in the total sample and across sellers and buyers, respectively. $p$-values for the null hypothesis of perfect randomization are listed in the last column (Wilcoxon signed rank tests for interval variables and Yates (1934)’ corrected $\chi^2$ tests for binary variables). ‘Age’ is reported in years. ‘Gender’ and ‘UZH’ are dummy variables indicating female subjects and students from the University of Zurich (versus ETH), ‘# semesters’ denotes the number of completed semesters. ‘Knowledge BM’ is a dummy variable equal to one for subjects who have heard about the mathematical object ‘Brownian motion’ before. Risk aversion is measured as the normalized difference ($\in [-1,1]$) between the simple lottery’s expected payoff and subjects’ respective certainty equivalents. CRRA-γ denotes the corresponding constant relative risk aversion coefficient. Ambiguity aversion is measured as the individual differences in certainty equivalents between the simple and ambiguous lottery.

---

expected payoffs, price sensitivities locally decrease for all four cases in Figure 7 when moving from simple to complex risks.

Table V reports average market clearing prices and quantities in concordance with Figure 7, i.e., across sessions with the same sequential order of simple and complex trading rounds. Moreover, column three and four of Table V list the degree of mispricing and suboptimal risk sharing according to the respective definitions in Section 2. As anticipated, mispricing is clearly less pronounced under simple than under complex risks. Notably, however, even under complex risks, the price of the risky stock seems relatively reasonable (average deviation from expected payoffs of approximately 14%). Both simple and complex risks are well shared. Strikingly, in three out of four cases, the degree of risk sharing is higher or equal for complex relative to simple trading rounds.

In order to better visualize local differences in price sensitivity, I adjust the average supply and demand curves under complex risks in Figure 7 to control for subjective beliefs. Essentially,
Figure 7. Average supply and demand

Notes: This figure shows the average supply and demand curves across subjects and trading rounds. In the top (bottom) row, averages are computed across sessions where complex (simple) trading rounds are followed by simple (complex) trading rounds. In the left (right) column, averages are computed across trading rounds where $\pi$ is equal to $1/2$ ($1/3$). The black horizontal line (first from the left) corresponds to the risky asset’s expected payoff. The solid horizontal line indicates sellers’ average point estimate of the risky asset’s expected payoff under complex risks, whereas the dotted horizontal line corresponds to the average of buyers’ respective point estimates. In the lower left plot, the two exactly coincide.

The price grid, against which each individual curve gets plotted, is adapted such that average payoff estimates coincide with risky assets’ true expected payoffs (see Appendix C for details). Figure 8 presents the adjusted supply and demand curves under complex risks. In contrast to Figure 7, Figure 8 allows for a direct comparison of price sensitivities. For prices close to but below $E[X]$, all four supply curves for simple risks lie below the respective supply curves for complex risks, only to cross the latter for prices close to but (generally) higher than $E[X]$ (vertical lines in Figure 8). The opposite holds true for the two demand curves where $\pi$ equals 1/2 (left column of Figure 8). For $\pi$ equal to 1/3, demand curves coincide for very low prices, but are higher in the case of complex risks for prices around and above $E[X]$.

For a more systematic investigation of the empirical supply and demand functions depicted in Figure 8, I plot the respective averages across all sessions (to further reduce noise) together with their respective error bounds, indicating standard errors of the mean. The resulting supply and demand curves are shown in Figure 9. The above described pattern now manifests itself.
Table V. Average market clearing prices and quantities

<table>
<thead>
<tr>
<th></th>
<th>Market clearing</th>
<th>Mispricing</th>
<th>Suboptimal risk sharing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P^*$</td>
<td>$Q^*$</td>
<td>$</td>
</tr>
<tr>
<td>Simple risks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi = \frac{1}{2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sessions 1-3</td>
<td>76.87</td>
<td>2.10</td>
<td>1.87</td>
</tr>
<tr>
<td>Sessions 4-6</td>
<td>79.47</td>
<td>2.33</td>
<td>4.47</td>
</tr>
<tr>
<td>$\pi = \frac{1}{3}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sessions 1-3</td>
<td>52.82</td>
<td>2.01</td>
<td>2.82</td>
</tr>
<tr>
<td>Sessions 4-6</td>
<td>46.89</td>
<td>2.00</td>
<td>3.11</td>
</tr>
<tr>
<td>Complex risks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi = \frac{1}{2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sessions 1-3</td>
<td>80.30</td>
<td>1.94</td>
<td>5.30</td>
</tr>
<tr>
<td>Sessions 4-6</td>
<td>80.98</td>
<td>2.10</td>
<td>5.98</td>
</tr>
<tr>
<td>$\pi = \frac{1}{3}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sessions 1-3</td>
<td>63.47</td>
<td>1.99</td>
<td>13.47</td>
</tr>
<tr>
<td>Sessions 4-6</td>
<td>56.98</td>
<td>2.15</td>
<td>6.98</td>
</tr>
</tbody>
</table>

Notes: This table reports average market clearing prices and quantities across sessions with the same sequential order of trading rounds involving simple and complex risks, respectively. Moreover, the measures of mispricing and suboptimal risk sharing as defined in Section 2 are listed in column three and four.

more clearly. For $\pi$ equal to $\frac{1}{2}$ (left column of Figure 9), the average supply (demand) for simple risks crosses the respective supply (demand) for complex risks from below (above). For $\pi$ equal to $\frac{1}{3}$, the same is true for sellers, whereas for buyers, average demands converge at a price close to the risky stock’s expected payoff. Furthermore, in all four cases, there is a clear difference in price sensitivity around prices equal to expected values.

The statistical significance of the differences in Figure 9 is tested by conducting a Wilcoxon signed-rank test, where, in the case of complex risks, interpolated quantities are used. The results thereof are plotted in Figure D.3 in Appendix D. As conjectured, the average supply curves are statistically different for prices below and above expected payoffs. In case of $\pi$ equal to $\frac{1}{2}$, the same statistically significant hump-shaped pattern around $E[X]$ is observed for average demand curves. For $\pi$ equal to $\frac{1}{3}$, their $p$-values are close to 0.1 below $E[X]$, temporarily increase around $E[X]$, and decrease again sharply to values close to zero thereafter.

Naturally, an analogous analysis lends itself to contrast subjects’ behavior between the two applied pricing mechanisms: market clearing and random price draws. Figure 10 presents the respective supply and demand curves averaged across complex trading rounds with equal pricing schemes. Overall, average supply and demand for complex risks look very similar between the two pricing mechanisms. The $p$-values of the corresponding Wilcoxon signed-rank test are
Figure 8. Average Supply and Demand Adjusted for Subjective Beliefs

Notes: This figure shows the average adjusted supply and demand curves across subjects and trading rounds. Average curves for complex risks are adjusted as described in Appendix C in order to account for deviations of average beliefs from the true underlying payoff distribution. In the top (bottom) row, averages are computed across sessions where complex (simple) trading rounds are followed by simple (complex) trading rounds. In the left (right) column, averages are computed across trading rounds where $\pi$ is equal to $1/2$ ($1/3$). The black horizontal line corresponds to the risky asset’s expected payoff.

3.3. Individual Behavior

Aggregate market outcomes appear to corroborate the predictions from theory: Equilibrium quantities are less price-sensitive under complex than simple risks, thereby mitigating the suboptimality in the former’s allocation. I subsequently turn to the analysis of individual trading behavior. Therefore, I propose two measures of local price sensitivity at the subject level.

First, from a quantity perspective, I count for each subject $i$ the number of consecutive prices plotted in Figure D.4 in Appendix D. The patterns in Figure D.4 indicate that there exists no statistical evidence against the hypothesis of a globally (across pricing schemes) adopted price-taking behavior. Hence, neither the limited number of subjects, nor asymmetric information, nor strategic uncertainty has an effect on local price sensitivity under complex risks.
Figure 9. Differences in average supply and demand for simple and complex risks

Notes: This figure shows the average adjusted supply and demand curves across subjects and trading rounds. Average curves for complex risks are adjusted as described in Appendix C in order to account for deviations of average beliefs from the true underlying payoff distribution. In the top (bottom) row, average supply (demand) curves are computed across all sessions. In the left (right) column, averages are computed across trading rounds where \( \pi \) is equal to \( \frac{1}{2} \) (\( \frac{1}{3} \)).

for which her submitted supply (demand) schedule equals \( \hat{Q} \) shares, i.e.,

\[
\mathcal{M}_i^1 := |\{Q = \hat{Q}\}_i|,
\]

where the bars denote the cardinality of the considered set. When determining \( \mathcal{M}_i^1 \), I focus on subjects who adopt the perfect hedging strategy at least once. Note that this is a direct implication of risk- and ambiguity-aversion. Additionally, subjects who adopt the perfect hedging strategy more than once but for nonconsecutive prices, i.e., whose supply (demand) functions must be nonmonotonic, are excluded.\(^{26}\)

Second, from a price sensitivity perspective, I compute the slope of each subject \( i \)'s supply

\(^{26}\) As demonstrated in Figure D.1 in Appendix D, there exist strictly concave (piece-wise defined) utility functions that imply nonmonotonic supply (demand) curves. Therefore, I also compute \( \mathcal{M}_i^1 \) across all subjects, even allowing for zero values. None of the herein presented qualitative results are affected.
Figure 10. Average supply and demand across pricing schemes

Notes: This figure shows the average adjusted supply and demand curves for complex risks across subjects and the two different pricing schemes: market clearing (MC) and random price draws (random). Average curves are adjusted as described in Appendix C in order to account for deviations of average beliefs from the true underlying payoff distribution. In the top (bottom) row, average supply (demand) curves are computed across all sessions. In the left (right) column, averages are computed across complex trading rounds where $\pi$ is equal to $1/2$ ($1/3$).

(demand) function at her individual point estimate $E_i[X]$, i.e.,

$$\mathcal{M}_i^2 := \Delta Q_i(E_i[X]),$$  \hspace{1cm} (8)

where for sellers

$$\Delta Q_i(E_i[X]) \equiv Q_i(P_{i+2}) - Q_i(P_i),$$

and for buyers

$$\Delta Q_i(E_i[X]) \equiv Q_i(P_{f-2}) - Q_i(P_f),$$

respectively, with $P_i$ ($P_f$) denoting the last (first) price strictly below (above) seller (buyer) $i$’s point estimate $E_i[X]$. The ‘±2’ in the index ensures that $P_i < E_i[X] < P_{i+2}$ for sellers and $P_{f-2} < E_i[X] < P_f$ for buyers, respectively. By construction, $\mathcal{M}_i^2$ can only be computed if $P_{i+2}$ ($P_f$) is smaller or equal to $\max(P) = 150$. Under simple risks it holds of course that $E_i[X] = E[X] \forall i \in I$. Moreover, $\mathcal{M}_i^2$ can be interpreted as a less extreme measure of price
Figure 11. Individual measures of price sensitivity

Notes: This figure shows average individual trading behavior under simple and complex risks across all subjects. Subfigure (a) plots the average consecutive price range for which subjects adopt the perfect hedging strategy, i.e., aiming to trade $Q$ shares (see Eq. (7)). Subfigure (b) plots the average slope of subjects’ supply and demand curves at their individual point estimates of the risky asset’s expected payoff (see Eq. (8)). Error bars indicate standard errors of the mean.

sensitivity than $M^1$, where the latter only accounts for perfect price inelasticity.

Figure 11 displays the between-treatment average values of $M^1$ and $M^2$ across all subjects. Subfigure (a) plots the average frequency with which the perfect hedging strategy is adopted. The average price range for which subjects choose to trade $\hat{Q}$ shares increases by 0.225 under complex relative to simple risks ($p$-value = 0.726, $t$-test). Average slopes of pooled supply and demand curves as defined in Eq. (8) are plotted in Subfigure (b). Price sensitivity locally decreases by 0.232 when moving from complex to simple risks ($p$-value = 0.003, $t$-test).

The results presented in Figure 11 are somewhat inconclusive. While, from a price sensitivity perspective, the empirical evidence conforms well to the theoretical predictions, from a pure quantity perspective, no significant increase in the average frequency of the perfect hedging strategy is observed. One can think of two possible reasons: (i) subjects exhibit smooth ambiguity preferences instead of multiple-priors utility, or (ii) subjects fail to trade in their best interest when risks are too complex.

The second argument requires some more elaboration. There exists no theory-consistent reason, why agents failing to trade $\hat{Q}$ shares at a price equal to $E_i[X]$ should adopt the per-
fect hedging strategy more frequently under complex relative to simple risks. Put differently, increasing complexity of traded risks may tighten the bounds on traders’ rationality as, thereby (partially) detracting the explanatory power of the proposed preference-based theory.

**Complexity Bounds on Rationality**

In order to control for varying bounds on rationality, I follow Biais et al. (2017) by contrasting individual trading data to a setting of bounded rationality in the spirit of McKelvey and Palfrey (1995, 1998)’s quantal response models.27 As proposed by Luce (1959), I hereafter assume that agent i’s trading decisions follow a random choice model. Specifically, for a given price \( P \), her probability density of trading \( Q_j \) shares under simple risks is given by

\[
f_i(Q_j|P) = \frac{\bar{\psi}_i(E[U_i(Q_j|P)])}{\int \bar{\psi}_i(E[U_i(Q|P)])dQ},
\]

(9)

where \( \bar{\psi}_i(\cdot) \) denotes an increasing differentiable function and \( Q \) runs from zero to the maximum number of tradable shares.

Since \( \bar{\psi}_i(\cdot) \) is increasing in \( E[U_i(Q_j|P)] \), Eq. (9) implies that the likelihood with which agent \( i \) decides to sell (buy) \( Q_j \) shares is also increasing in \( E[U_i(Q_j|P)] \). In other words, the higher the expected utility from trading \( Q_j \) shares for a price \( P \), the greater the probability that agent \( i \) actually ends up doing so. Hence, the lower the slope of \( \bar{\psi}_i(\cdot) \), the more frequently she deviates from her optimal strategy, i.e., the more severe are the bounds on her rationality.

As in Biais et al. (2017), applying bounded rational behavior as formalized in Eq. (9) to the above theory of trading simple risks imposes the following three implications:28

S1 For \( P = E[X] \), the distribution of supplied and demanded shares has a unique mode at \( \hat{Q} \).

S2 For \( P < E[X] \), the distribution of supplied and demanded shares is asymmetric around \( \hat{Q} \) and decreasing above (below) \( \hat{Q} \) for sellers (buyers).

S3 For \( P > E[X] \), the distribution of supplied and demanded shares asymmetric around \( \hat{Q} \) and decreasing below (above) \( \hat{Q} \) for sellers (buyers).

According to Proposition 1, every risk-averse agent whose rationality is bounded as in Eq. (9) most likely aims to trade \( \hat{Q} \) shares for \( P = E[X] \). Similarly, such risk-averse sellers and buyers more often adopt dominating instead of dominated strategies. Moreover, for a given price, the

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27 A somewhat stricter caveat as in Biais et al. (2017) applies: In my competitive setting with sufficiently imperfect price informativeness (see above), agents solely trade according to their own beliefs. By differentiating between market-clearing and random pricing, I control for any deviations from such individual behavior.

28 In contrast to Biais et al. (2017), risk-aversion, i.e., the accordance of agents’ expected utilities with second order stochastic dominance, is a necessary condition for the decreasing quantity distributions for prices different than \( E_i[X] \). If \( \pi = 1/2 \), which always holds in Biais et al. (2017), first order stochastic dominance is sufficient.
Figure 12. Supply distribution for prices equal to expected payoffs

Notes: This figure shows the number of shares supplied by sellers for prices equal to (estimated) expected payoffs. The empirical distributions are computed across subjects and sessions. The left (right) plot contrasts average distributions between simple and complex trading rounds with $\pi$ equal to $\frac{1}{2}$ ($\frac{1}{3}$). If, under complex risks, sellers’ point estimate $E_i[X]$ lies between two elements of the predefined price vector, linearly interpolated quantities are reported.

larger the distance between the chosen quantity and the corresponding set of dominating trades, the less frequently should they opt for the former. Because of the randomness implied by Eq. (9), all three implications are convergence results. Hence, if subjects’ behavior is indeed governed by Eq. (9), whether the limited number of subjects in my sample suffices to yield according results remains an empirical question.

Similarly, under complex risks, agent $i$’s decides to trade $Q_j$ shares for a price $P$ with probability

$$f_i(Q_j|P) = \frac{\psi_i(E_i[U_i(Q_j|P)])}{\int \psi_i(E_i[U_i(Q|P)]) dQ}, \quad (10)$$

where, again, $\psi_i(\cdot)$ denotes an increasing differentiable function. There are two differences between Eq. (9) and Eq. (10): On the one hand, given the complexity of traded risks under Eq. (10), agents rely on their individual expectation operator $E_i[\cdot]$. On the other hand, due to the different informational precision levels at hand, $\psi_i(\cdot)$ and $\psi_i(\cdot)$ most likely compose different functions. In particular, postulating more severe (global) bounds on agent $i$’s rationality in the
presence of complex risks is equivalent to

\[ \overline{\psi_i}(x) > \underline{\psi_i}(x) \quad \text{and} \quad \overline{\psi_i'}(x) > \underline{\psi_i'}(x) \quad \forall x \in E_i[U_i(\cdot)], \]

(11)

which implies more frequent deviations from her optimal trading strategy. Eq. (11) translates to the following three implications regarding individual trading behavior under complex risks:

C1 For \( P = E_i[X] \), the distribution of supplied and demanded shares still exhibits a unique mode at \( \hat{Q} \), but is more dispersed than under simple risks.

C2 For \( P < E_i[X] \), the distribution of supplied and demanded shares is less asymmetric around \( \hat{Q} \) and decreases less above (below) \( \hat{Q} \) for sellers (buyers) than under simple risks.

C3 For \( P > E_i[X] \), the distribution of supplied and demanded shares is less asymmetric around \( \hat{Q} \) and decreases less below (above) \( \hat{Q} \) for sellers (buyers) than under simple risks.

Analogously to S1–S3, the three distribution results C1–C3 hold if the number of agents behaving according to Eq. (10) goes to infinity.

Figure 12 presents the supply distribution for \( P = E_i[X] \). While integer numbers are more frequently supplied than fractions of shares, both distributions are roughly symmetric around \( \hat{Q} = 2 \) shares, constituting the clear mode under simple risks and complex risks with \( \pi = 1/2 \) (left plot in Figure 12). When moving from simple to complex risks, the frequency of the perfect hedging strategy decreases sharply, i.e., from 0.694 to 0.235 for \( \pi = 1/2 \) and from 0.469 to 0.163 for \( \pi = 1/3 \) (\( p \)-values for differences = 0.000, \( t \)-test). In the case of \( \pi = 1/3 \), the frequencies of the most extreme deviations from \( \hat{Q} \) increase considerably under complex risks. These results are in line with both implications S1 and C1.

Figure 13 depicts the distribution of shares supplied for \( P < E_i[X] \) and \( P > E_i[X] \), respectively. Under both simple and complex risks, supplies of less (more) than \( \hat{Q} \) shares clearly occur most often for low (high) prices. Additionally, except for complex risks with \( \pi = 1/3 \) (upper right plot in Figure 13), the frequency of supplying more (less) than \( \hat{Q} \) shares is decreasing (increasing) in \( Q_i \) for \( P < E_i[X] \) (\( P > E_i[X] \)), with generally lower frequency levels under simple risks. The supply distributions presented in Figure 13 reconcile well with the above proposed implications. First, subjects more often choose dominating instead of dominated actions, where the occurrence of the latter is decreasing in their inferiority (see implications C2–C3). Second, under complex risks, subjects deviate more frequently from utility-maximizing actions than under simple risks (S2–S3). The analogous analysis of the corresponding demand distributions (see Figure D.5 and Figure D.6 in Appendix D) reveals similar evidence in support of S1–S3 and C1–C3 for buyers.

In summary, my empirical findings reconcile well with the random choice models postulated
Notes: This figure shows the number of shares supplied by sellers for prices different from expected payoffs. The empirical distributions between simple and complex risks are computed across subjects and sessions. In the top (bottom) row, total supplies for prices below (above) \( E[X] \) are reported. The left (right) column shows average supply distributions across trading rounds with \( \pi \) equal to \( \frac{1}{2} \) (\( \frac{1}{3} \)).

Figure 13. Supply distribution for prices different from expected payoffs

In Eq. (9) and Eq. (10): Complexity tightens the bounds on risk-averse agents’ rational behavior, where, rationality under complex risks is defined in line with decision theory under ambiguity. A simple counting exercise further underpins this hypothesis. Figure 14 shows the distributions of dominated action frequencies between risk types. As expected, subjects more frequently fall for dominated trading strategies if risks are complex. Although, as can be deduced from Figure D.7 in Appendix D, some limited learning takes place while trading complex risks.

Once varying levels of rationality are controlled for, the inconclusiveness regarding the above two price sensitivity measures disappears. Figure 15 shows the between-treatment average values of \( M^1 \), where two different rationality conditions are applied. In Subfigure (a), averages are only computed for subjects who prefer to be perfectly hedged for \( P = E[X] \) under complex risks. For these subjects, the price range for which they supply (demand) \( \tilde{Q} \) shares increases by 5.172 under complex relative to simple risks (\( p \)-value = 0.000, \( t \)-test).

In contrast, Subfigure (b) plots averages computed across potentially different subjects: Under simple risks, only nondominated supply and demand curves (as presented in Figure 14)
**Notes:** This figure shows the distributions of dominated action frequencies across all subjects. Under simple risks, dominated actions correspond to offered (demanded) quantities above (below) $\hat{Q}$ shares for $P < E[X]$ and vice versa for $P > E[X]$. Under complex risks, dominated actions correspond to offered (demanded) quantities above (below) $\hat{Q}$ shares for $P < E_{i}[X]$ and vice versa for $P > E_{i}[X]$. Note that in the presence of complex risks, the price thresholds depend on subjects’ individual point estimates.

are considered. Accordingly, the corresponding average for complex risks is solely based on nondominated supply and demand curves equal to $\hat{Q}$ at $P = E_{i}[X]$. Relying on these conditions, the average cardinality of continuous prices for which the perfect hedging strategy is adopted increases by 3.853 under complex risks ($p$-value = 0.012, $t$-test). Hence, for both cases in Figure 15, the conditional $\mathcal{M}^{1}$ measure relates strikingly well with the theory’s predictions, particularly so with those implied by kinked ambiguity preferences.

**Regression Analysis and Comparison to Ambiguity**

For a full regression analysis, I additionally include the remaining data from each session’s last trading round (see Table III), where tradable risks are based on the draw from the nontransparent Ellsberg (1961)-like urn depicted in Figure 5. Since subjects’ beliefs in these rounds are unknown, classifying individual trades involving ambiguous risks into dominated and nondominated actions is no longer possible.
Figure 15. Conditional frequency of perfect hedging strategy

Notes: This figure shows the average frequency of the perfect hedging strategy under simple and complex risks, conditional on rational trading behavior. Both subfigures plot conditional average cardinalities of the consecutive price ranges for which subjects supply (demand) $Q$ shares (see Eq. (7)). In Subfigure (a), averages are only based on subjects who, under complex risks, supply (demand) $Q$ shares at $P = E_i[X]$. In Subfigure (b), the average value for simple risks is computed across all nondominated (see Figure 14) supply (demand) curves. The corresponding average for complex risks is determined across all nondominated supply (demand) curves that additionally go through $Q$ at $P = E_i[X]$. Error bars indicate standard errors of the mean.

Table VI reports OLS coefficient estimates of the following pooled regression model:

$$M_{1r}^{1.2} = \beta_0 + \beta_1 Complexity_r + \beta_2 Ambiguity_r + \beta_3 RA_i + \beta_4 (AA_i \times Complexity_r) + \beta_5 (AA_i \times Ambiguity_r) + bX_{ir} + \epsilon_{ir}, \quad (12)$$

where the dependent variable is either one of the above introduced sensitivity measures for subject $i$ in trading round $r$. $Complexity_r$ and $Ambiguity_r$ are dummy variables indicating trading rounds with complex and ambiguous risks, respectively. $RA_i$ and $AA_i$ measure subject $i$’s risk and ambiguity aversion (see Table III). Finally, $X_{ir}$ contains socio-economic and trading round specific control variables, and $\epsilon_{ir}$ denotes the idiosyncratic error term. Corrected standard errors clustered at the subject level are reported in parenthesis.

The reported coefficients in Table VI affirm the findings from Figure 11. The second and fourth columns show that complex risks significantly decrease the slope of local supply (demand)
<table>
<thead>
<tr>
<th></th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_1$</th>
<th>$M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>3.974$^a$</td>
<td>0.661$^a$</td>
<td>3.413</td>
<td>0.685$^b$</td>
</tr>
<tr>
<td></td>
<td>(0.511)</td>
<td>(0.096)</td>
<td>(2.373)</td>
<td>(0.335)</td>
</tr>
<tr>
<td>Complexity (dummy)</td>
<td>0.408</td>
<td>-0.240$^a$</td>
<td>-0.645</td>
<td>-0.249$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.646)</td>
<td>(0.065)</td>
<td>(0.709)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>Ambiguity (dummy)</td>
<td>1.617$^c$</td>
<td>0.017</td>
<td>1.727$^c$</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.959)</td>
<td>(0.123)</td>
<td>(0.955)</td>
<td>(0.122)</td>
</tr>
<tr>
<td>RA (risk aversion)</td>
<td>5.389$^b$</td>
<td>-0.216</td>
<td>5.302$^b$</td>
<td>-0.263</td>
</tr>
<tr>
<td></td>
<td>(2.111)</td>
<td>(0.252)</td>
<td>(2.137)</td>
<td>(0.229)</td>
</tr>
<tr>
<td>AA (ambig. aversion)× Complexity</td>
<td>1.935</td>
<td>-0.142</td>
<td>0.305</td>
<td>-0.061</td>
</tr>
<tr>
<td></td>
<td>(1.978)</td>
<td>(0.156)</td>
<td>(1.817)</td>
<td>(0.150)</td>
</tr>
<tr>
<td>AA × Ambiguity</td>
<td>5.360</td>
<td>-0.299</td>
<td>4.573</td>
<td>-0.216</td>
</tr>
<tr>
<td></td>
<td>(3.940)</td>
<td>(0.334)</td>
<td>(3.847)</td>
<td>(0.294)</td>
</tr>
<tr>
<td>Order × Complexity</td>
<td>-</td>
<td>-</td>
<td>2.269$^b$</td>
<td>0.005</td>
</tr>
<tr>
<td>Gender</td>
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<td>-</td>
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<td>(0.087)</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>(0.830)</td>
<td>(0.098)</td>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>465</td>
<td>653</td>
<td>465</td>
<td>653</td>
</tr>
</tbody>
</table>

Notes: This table reports OLS coefficient estimates. The dependent variables are unconditional measures of local price sensitivity. $M_1$ denotes the cardinality of consecutive prices for which subjects adopt the perfect hedging strategy, i.e., aiming to trade $Q$ shares. $M_2$ measures the average slope of subjects’ supply and demand curves at their individual point estimates of the risky asset’s expected payoff. ‘Complexity’ and ‘Ambiguity’ are dummy variables indicating trading rounds with complex and ambiguous risks, respectively. ‘Risk aversion’ measures the normalized difference between the simple lottery’s expected payoff and subjects’ respective certainty equivalents. The first two interaction terms control for different effects of ‘Ambiguity aversion’ across trading rounds with simple and complex risks, where ambiguity aversion is measured as the difference between subjects’ certainty equivalents for the simple and the ambiguous lottery. The term ‘Order × Complexity’ interacts the dummy variable ‘Order’, indicating sessions where complex risks were proceeded by simple risks, with complex trading rounds. ‘Gender’ is a dummy variable indicating female subjects. ‘Controls’ comprise subjects’ age, their attended university, and number of completed semesters. Furthermore, ‘Controls’ contain subjects’ self-evaluated understanding and difficulty level of the assigned task (measured by integers from one to five) and two additional dummy variables controlling for their familiarity and knowledge regarding the Brownian motion. Numbers in parenthesis denote robust standard errors clustered at the subject level. Superscripts $^a$, $^b$, and $^c$ indicate statistical significance at the 1%, 5%, and 10%-level, respectively.

by approximately 0.25 on average. As revealed by columns one and three, no such unconditional effect is observed on subjects’ adopted frequency of the perfect hedging strategy. Nevertheless, as expected, the latter is higher in rounds with ambiguous risks and increases with subjects’ risk aversion. When regressing local slopes on the ambiguity dummy and risk aversion, both coefficients exhibit the anticipated sign but lack statistical significance.

Controlling for a potential order effect, I find evidence for Fox and Tversky (1995)’s comparative ignorance effect: If complex risks are proceeded by simple risks, the adopted perfect
hedging frequency increases significantly. Furthermore, female subjects more often follow the perfect hedging strategy and submit significantly less (locally) sensitive supply (demand) functions. This contrasts the findings in Borghans et al. (2009) that men require higher compensation for the introduction of ambiguity than do women.

Despite having the expected sign in all eight cases, none of the estimates of $\beta_4$ and $\beta_5$ as defined in Eq. (12) are statistically significant. Under kinked ambiguity preferences, an affection for ambiguous risks potentially causes supply (demand) discontinuities. Although technically in line with the above predictions, such discontinuities likely augment the inherent noise level of empirically observed supply (demand) functions. Therefore, I reestimate Eq. (12) for only nonambiguity-averse subjects. The corresponding coefficient estimates are reported in Table D.1 in Appendix D.

The results with respect to the complexity dummy as well as subjects’ risk aversion remain qualitatively the same. Similar holds true for the above described order and gender effects. However, four of the eight coefficients interacted with ambiguity aversion become statistically significant (at least at the 5%-level). Strikingly, both magnitudes and statistical significance are clearly higher for the trading round with ambiguous risks relative to those with complex risks (despite the latter’s fourfold larger sample size).

In summary, these findings point towards three important implications. First, ambiguity preferences appear to possess the highest explanatory power in the presence of purely ambiguous rather than complex risks. This comes as little surprise, but reassures the design’s effectiveness in translating ambiguity preferences into measurable model-based trading predictions. Second, and more importantly, complex risks reduce local supply (demand) slopes more substantially than does pure ambiguity. In particular, this can be seen from comparing the coefficient estimates for the complexity and ambiguity dummies and recalling the small values of the applied ambiguity aversion measure. Third, in synthesis, while ambiguity preference-based theories explain individual behavior under complex risks reasonably well, the aversion to pure ambiguity underestimates the latter’s impact on market outcomes.

Given the fundamental difference regarding the existence of a uniquely defined risk structure, it is not surprising that the magnitudes of subjects’ reactions to complex and ambiguous risks are different. Even though their beliefs under pure ambiguity are unknown, a similar analysis as presented in Figure 12 lends itself as a simplified comparison of relative bounded rationality. For both simple and ambiguous risks, Figure D.8 in Appendix D presents the joint distributions of supplied and demanded shares at a price of ECU 75. Assuming subjects adopt the natural reference point of a fifty-fifty chance under pure ambiguity, there is no evidence that pure ambiguity has any hampering effect on subjects’ rationality.
3.4. Market’s Effectiveness in Aggregating Complex Information

In light of the attained insights regarding individual trading behavior, I finally move back to an equilibrium perspective by returning to this paper’s underlying elementary question: How well are financial markets suited to cope with complexity? In particular, are they capable of efficiently allocating complex risks at informative prices? I investigate this question by dissecting both the equilibration processes and their respective outcomes of the above asset markets.

Figure 16 displays bootstrapped distributions of aggregate market outcomes. All densities are based on ten thousand resamples of 49 individual supply and demand functions. For any given resample, average supply and demand are crossed and linearly interpolated market-clearing prices \( P^* \) and quantities \( Q^* \) deduced.

Comparing estimated densities between simple and complex risks unveils three striking characteristics of market equilibrium. First, and not surprisingly, both distributions of \( P^* \) under simple risks are closer to and more centered around \( E[X] \) than those under complex risks. Second, and contrary to market-clearing prices, the centers of both \( Q^* \) distributions under complex risks are remarkably close to \( \hat{Q} = 2 \) shares, i.e., the perfect hedging strategy. In case of \( \pi = 1/2 \) (lower left plot in Figure 16), complex risks are even more efficiently shared than simple risks. Both observations are in line with actual market outcomes reported in Table V.

Third but foremost, the relative variation between simple and complex risks is much larger for market-clearing prices than for market-clearing quantities. Given their predicted decrease in supply and demand sensitivity, this observation aligns well with the above ambiguity preference-based theories. Figure D.9 in Appendix D furthermore illustrates how these relative variations in \( P^* \) and \( Q^* \) depend on the underlying resampling size. All variability ratios are considerably stable in the number of traders. At the maximum resampling size, both standard deviations of \( P^* \) under complex risks are still more than twice as high as under simple risks. In contrast, standard deviations of market clearing quantities are consistently much closer for simple and complex risks. In the limit, the variation in \( Q^* \) under complex risks only exceeds the one under simple risks by approximately 30% for \( \pi = 1/2 \) and less than 10% for \( \pi = 1/3 \). Hence, throughout its equilibration path, the variation in markets’ risk sharing ability are remarkably similar for simple and complex risks.

In order to conclusively evaluate markets’ ability to (partially) aggregate agents’ subjective information about complex risks, simply referring to the results shown in Figure 16 would not be fair. The above distributions of \( P^* \) under complex risks do not yet take into account the dispersion of subjects’ beliefs regarding the risky asset’s true expected payoff. Therefore, I
Figure 16. Bootstrapped equilibrium distributions

Notes: This figure shows bootstrapped densities of market-clearing prices and quantities for simple and complex risks. Every average supply and demand curve is based on resampling 49 individual supply and demand schedules (same resampling size under simple and complex risks). For each pair of averaged supply and demand, the linearly interpolated market-clearing price and quantity are computed. Repeating this procedure ten thousand times yields the depicted estimated densities of equilibrium prices (top row) and quantities (bottom row). The left (right) column shows bootstrapped densities for trading rounds with $\pi$ equal to $\frac{1}{2}$ ($\frac{1}{3}$).

propose to consider the following ratio of standard deviations instead:

$$\text{Std}(P^*)-\text{Ratio} = \sqrt{\frac{\text{Var}(P^*_c)}{\text{Var}(P^*_s + E^*_c[X])}},$$  \hspace{1cm} (13)

where $P^*_s$ ($P^*_c$) denotes the market-clearing price for simple (complex) risks, and $E^*_c[X]$ indicates subjects’ average estimate of $E[X]$ under complex risks. When comparing variations in $P^*_c$ to those in $P^*_s$, Eq. (13) actually controls for the fluctuations of subjects’ point estimates by adding $E^*_c[X]$ to the variance in its denominator.

Whether the ratio in Eq. (13) is eventually greater or smaller than unity, i.e., whether markets efficiently aggregate complex risks or not, is again an empirical question. From a theoretical perspective, however, the answer is: it depends. The decisive factor is whichever of the following trade-off effects dominates: increased severity of bounded rationality versus reduced price sensitivity. In the absence of both effects, the ratio in Eq. (13) should equal one. Whenever
risk-averse agents’ behave fully rationally, $P_s^*$ coincides with $E[X]$ and is thus deterministic. Moreover, if agents are neutral to complexity-induced ambiguity, $Var(P_c^*)$ exactly corresponds to $Var(E_c^*[X])$, since the market-clearing price always equals the average of agents’ expected asset payoff.

In the presence of ambiguity aversion and a thereby implied decrease in local price sensitivity under complex risks, $Var(P_c^*)$ may fall below $Var(E_c^*[X])$, thereby pushing Eq. (13) downwards. To see this, consider two different agents: an ambiguity-neutral seller and an infinitely ambiguity-averse buyer. Facing complex risks, the seller shall belief that $E[X] \in [a, b]$ with uniform probability, while the buyer believes that $E[X]$ is uniformly distributed over $[\frac{a+b}{2}, b]$, where $a < b$. Hence, the seller’s supply curve goes through $(\frac{a+b}{2}, \hat{Q})$, whereas the buyer’s demand curve is completely flat at $\hat{Q}$ over the nonempty subset of prices $[\frac{a+b}{2}, b]$. Therefore, the unique trading equilibrium equals $(\frac{a+b}{2}, \hat{Q})$ with $P_c^* = \frac{a+b}{2}$ and $E_c^* = \frac{3a+5b}{8}$. However, if instead the buyer believes that $E[X] \in [a, \frac{a+b}{2}]$, $P_c^*$ would still equal $\frac{a+b}{2}$, but $E_c^*$ would jump to $\frac{5a+3b}{8}$. In contrary, given more severe bounds on rationality under complex risks, i.e., in case Eq. (11) applies, the noise in $P_c^*$ relative to $P_s^*$ increases, which ultimately pushes Eq. (13) upwards.

Both cases are present in the experimental data. Figure 17 shows the respective values of Eq. (13), conditional on subjects’ maximum number of dominated actions (see Figure 14). Unconditionally, the standard deviation ratio for $\pi = \frac{1}{2}$ lies below one (0.813), whereas for $\pi = \frac{1}{3}$ it exceeds one (1.170). This is in line with the observations from Figure 12 and Figure 13: Relative to $\pi$ equal to one half, the number of strongly dominated actions is substantially higher for $\pi$ equal to one third, implying more severe bounds on subjects’ rationality in the latter case. As shown in the right plot of Figure 17, the ratio for $\pi = \frac{1}{3}$ is decreasing in the strictness of the applied rationality constraint. Focusing on subjects who completely abstain from any dominated actions, it eventually also falls below unity.

To sum up, markets’ prove to be notably efficient in pricing and sharing complex risks, despite increased noise levels in individual trading behavior. Nevertheless, beyond binding limits to bounded rationality, their information aggregation is severely impaired, while their risk sharing ability yet prevails.

4. Concluding Remarks

In this paper, I study how complex but purely objective risks are traded in a competitive asset market. Relying on decision theory under ambiguity, the paper provides a novel perspective on agents’ trading behavior in the presence of imperfectly understood uncertainty. In his seminal work, Ellsberg (1961) himself characterizes ambiguity as “a quality depending on the amount,
Figure 17. Relative variability of market-clearing prices

Notes: Conditioning on subjects’ maximum number of dominated actions (see Figure 14), this figure shows the ratio

\[
\text{Std}(P^*)\text{-Ratio} = \sqrt{\frac{\text{Var}(P^*_c)}{\text{Var}(P^*_s + E^*_c[X])}},
\]

where \(P^*_s\) (\(P^*_c\)) denotes the market-clearing price for simple (complex) risks, and \(E^*_c[X]\) indicates subjects’ average estimate of \(E[X]\) under complex risks. Both estimates \(P^*_s\) and \(P^*_c\) are bootstrapped based on resampling and averaging individual supply and demand schedules. For each pair of averaged supply and demand, the linearly interpolated market-clearing price is computed. The respective resample size is set to the minimum number of sellers or buyers who satisfy the given rationality condition (maximum allowed number of dominated actions). This procedure is repeated ten thousand times. The left (right) plot shows standard deviation ratios for trading rounds with \(\pi\) equal to \(\frac{1}{2}\) (\(\frac{1}{3}\)).

Type, and ‘unanimity’ of information, and giving rise to one’s degree of ‘confidence’ in an estimate of relative likelihoods” (Ellsberg, 1961, p. 657)—an interpretation that advocates a bridging of ambiguity models and financial markets for increasingly complex assets.

In the absence of aggregate risk, the controlled setting of Biais et al. (2017) offers an ideal experimental framework to distinctively test for complexity’s impact on individual trading decisions and aggregate market outcomes. Starting from Debreu (1959) and Arrow (1964)’s seminal benchmark for simple risks, ambiguity preference-based predictions possess significant explanatory power regarding both adopted trading strategies and equilibrium allocations under complex risks. In general, I find asset markets to prove remarkably effective in pricing complex risks and
even more robust in sharing them optimally across risk-averse investors. However, the former quality crucially depends on the severity by which complexity curtails agents’ rationality under the perceived ambiguity of complex risks.
References


Appendix A: Proofs

Proof of Proposition 1. Here I prove the case if agent $i$ is a seller. In the case of a buyer, the analogous reasoning applies. Relying on the identities in Eq. (1), any agent $i$’s expected utility from consumption can be rewritten as (neglecting the subscript $i$)

$$
E[U(C(\omega))] = \pi U \left( \mu + \sqrt{\frac{1 - \pi}{\pi}} \sigma \right) + (1 - \pi) U \left( \mu - \sqrt{\frac{\pi}{1 - \pi}} \right)
\triangleq f(\mu, \sigma, \pi),
$$

and since $U$ is increasing it follows that

$$
\frac{\partial f}{\partial \mu} = \pi U' \left( \mu + \sqrt{\frac{1 - \pi}{\pi}} \sigma \right) + (1 - \pi) U' \left( \mu - \sqrt{\frac{\pi}{1 - \pi}} \right)
> 0,
$$

(A.1)

and from decreasing marginal utility from consumption that

$$
\frac{\partial f}{\partial \sigma} = \pi U' \left( \mu + \sqrt{\frac{1 - \pi}{\pi}} \sigma \right) \sqrt{\frac{1 - \pi}{\pi}}
+ (1 - \pi) U' \left( \mu - \sqrt{\frac{\pi}{1 - \pi}} \sigma \right) \left( -\sqrt{\frac{\pi}{1 - \pi}} \right)
= \sqrt{\pi(1 - \pi)} U' \left( \mu + \sqrt{\frac{1 - \pi}{\pi}} \sigma \right) - \sqrt{\pi(1 - \pi)} U' \left( \mu - \sqrt{\frac{\pi}{1 - \pi}} \sigma \right)
< 0.
$$

(A.2)

When selling $Q$ shares for a price equal to $P$, the seller’s consumption in $t = 2$ equals

$$
C(u) = (S - Q)X(u) + (B + QP) + I(u)
$$

(A.3)

in state $u$, and

$$
C(d) = (S - Q)X(d) + (B + QP) + I(d)
$$

(A.4)

in state $d$.

Let us now denote the expected asset payoff $E[X]$ by $P^*$, i.e.,

$$
P^* := \pi X(u) + (1 - \pi)X(d).
$$

Furthermore, we define $\hat{Q}$ as the quantity for which $\sigma^2 = 0$, i.e.,

$$
\sigma^2 = 0 \iff C(u) = C(d).
$$
\[(S - Q)X(u) + I(u) = (S - Q)X(d) + I(d)\]
\[\Leftrightarrow \hat{Q} = S + \frac{I(u) - I(d)}{X(u) - X(d)},\]  
(A.5)

where we assume that \(\hat{Q} > 0\). From the definition of \(\mu\) together with (Eq. (A.3)) and (Eq. (A.4)) we get
\[\frac{\partial \mu}{\partial Q} = \pi(P - X(u)) + (1 - \pi)(P - X(d)),\]
and thus
\[
\begin{align*}
\frac{\partial \mu}{\partial Q} &= \begin{cases} 
< 0 & \text{if } P < P^*, \\
= 0 & \text{if } P = P^*, \\
> 0 & \text{if } P > P^*.
\end{cases}
\end{align*}
\]
(A.6)

First, strict concavity now implies
\[
E[U(C(\omega))] < U\left(\pi \mu + \sqrt{\pi(1 - \pi)\sigma} + (1 - \pi)\mu - \sqrt{(1 - \pi)\pi \sigma}\right) = U(\mu),
\]
hence, from (Eq. (A.5)) and (Eq. (A.6)) it follows that, \(\forall \pi \in (0, 1), (P^*, \hat{Q})\) strictly dominates all other points on the line \((P^*, Q)\).

Second, (Eq. (A.1)) & (Eq. (A.6)) together with (Eq. (A.2)) & (Eq. (A.5)) imply that

(i) for any given price \(P < P^*\), any point in the upper left quadrant of Subfigure (a) of Figure 1 is strictly dominated by \((P, \hat{Q})\);

(ii) for any given price \(P > P^*\), any point in the lower right quadrant of Subfigure (a) of Figure 1 is strictly dominated by \((P, \hat{Q})\).

Hence, \(\forall \pi \in (0, 1)\), the seller’s supply curve has to lie somewhere in the lower left and upper right quadrant and has to go through the point \((P^*, \hat{Q})\). This completes the proof.

Proof of Remark 1. Since \(\epsilon\) can be arbitrarily small, I directly consider the limit \(\epsilon \to 0\), i.e.,
\[
\lim_{\epsilon \to 0} U_i(C) = c_1 C, \text{ for } 0 \leq C \leq \overline{C}.
\]
The corresponding first and second derivatives of \(U_i(C)\) are
\[
\lim_{\epsilon \to 0} U'_i(C) = c_1, \text{ and } \lim_{\epsilon \to 0} U''_i(C) = 0.
\]
For \(C \geq \overline{C}\), the respective derivatives are
\[
U'_i(C) = \alpha e^{-\alpha C}, \text{ and } U''_i(C) = -\alpha^2 e^{-\alpha C}.
\]
The following conditions ensure the differentiability of $U(C)$ at $C$:

$$c_1C = c_2 - e^{-\alpha C} \iff c_2 = c_1C + e^{-\alpha C},$$  \hspace{1cm} (A.7)

$$c_1 = \alpha e^{-\alpha C}.$$  \hspace{1cm} (A.8)

Given Eq. (A.3) and Eq. (A.4), the FOC for $E[U(C(\omega))]$ with respect to $Q$ implies

$$\pi U'(C(u))(P - X(u)) + (1 - \pi)U'(C(d))(P - X(d)) = 0.$$  \hspace{1cm} (A.9)

Taking the first derivative of the LHS of Eq. (A.9) with respect to $P$ yields

$$\pi U'(C(u)) + (1 - \pi)U'(C(d)) + \pi U''(C(u))Q(P - X(u)) + (1 - \pi)U''(C(d))Q(P - X(d)).$$

Since $\frac{\delta}{\delta Q}$(LHS of Eq. (A.9)) < 0 $\forall (P, Q) \in \mathbb{R}^2_0$, agent $i$’s supply curve is decreasing in $P$ if $\frac{\delta}{\delta P}$(LHS of Eq. (A.9)) < 0. For the here considered utility function, this is the case whenever

$$c_1 < \frac{1 - \pi}{\pi} \alpha e^{\alpha C(d)} (\alpha Q(P - X(d)) - 1).$$

Together with Eq. (A.8), this implies that for high enough prices, i.e., if

$$P > X(d) + \frac{1 + \frac{\pi}{1 - \pi} \alpha e^{-\alpha (\overline{C} - C(d))}}{\alpha Q},$$

seller $i$’s supply curve can be locally decreasing in $P$. This completes the proof.

**Proof of Proposition 2.** Here I prove the case if the ambiguity-averse agent $i$, i.e., $\alpha_i > 1/2$, is a seller. In the case of a buyer, the analogous reasoning applies. Relying on the identities in Eq. (1), any agent $i$’s utility from consumption according to the $\alpha$-maxmin in Eq. (2) can be rewritten as (neglecting the subscript $i$)

$$U(C(\omega)) = \alpha \min_{\pi \in \mathcal{C}} (E[U(\pi)]) + (1 - \alpha) \max_{\pi \in \mathcal{C}} (E[U(\pi)])
\begin{align*}
&= \alpha \left( \pi U \left( \mu + \sqrt{\frac{1 - \pi}{\pi}} \sigma \right) + (1 - \pi) U \left( \mu - \sqrt{\frac{\pi}{1 - \pi}} \sigma \right) \right) \\
&\quad + (1 - \alpha) \left( \pi U \left( \mu + \sqrt{\frac{1 - \pi}{\pi}} \sigma \right) + (1 - \pi) U \left( \mu - \sqrt{\frac{\pi}{1 - \pi}} \sigma \right) \right),
\end{align*}$$

where $\pi = \arg \min_{\pi \in \mathcal{C}} E[U(\pi)]$ ($\pi = \arg \max_{\pi \in \mathcal{C}} E[U(\pi)]$) and $\mu$ ($\overline{\mu}$) and $\sigma$ ($\overline{\sigma}$) denote expected consumption and standard deviation of consumption according to $\overline{\pi}$ ($\overline{\pi}$). For $Q \neq \bar{Q}$, i.e., for

53
strictly positive \( \sigma \) and \( \pi \), it directly follows from \( U \)'s strict concavity that

\[
U(C) < \alpha U(\mu) + (1 - \alpha) U(\pi) < U(\alpha \mu + (1 - \alpha) \pi).
\]

Eq. (A.3) and Eq. (A.4) imply

\[
\alpha \mu + (1 - \alpha) \pi = \alpha \left( \bar{\pi} \left( (S - Q) X(u) + (B + Q P) + I(u) \right) + (1 - \bar{\pi}) \left( (S - Q) X(d) + (B + Q P) + I(d) \right) \right) + (1 - \alpha) \left( \pi \left( (S - Q) X(u) + (B + Q P) + I(u) \right) + (1 - \pi) \left( (S - Q) X(d) + (B + Q P) + I(d) \right) \right)
\]

\[
= \text{... terms indep. from } Q \ldots + Q \left( P - \left( \alpha E[\bar{\pi}[X] + (1 - \alpha) E[\pi][X] \right) \right).
\]

Hence, if \( P = \alpha E[\bar{\pi}[X] + (1 - \alpha) E[\pi][X] \), denoted by \( \tilde{P} \) hereafter, the linear combination of expected consumption (for constant \( \bar{\pi} \) and \( \pi \)) does not change for different quantities of shares sold. Therefore, for \( \tilde{P} \), it is optimal for the seller to exactly sell \( \hat{Q} \) share and get the constant utility \( U(\alpha \mu + (1 - \alpha) \pi) = U(\mu) = U((1 - \alpha) \pi) \).

In general, it holds that

\[
U(C) = \alpha \left( \bar{\pi} U(C(u)) + (1 - \bar{\pi}) U(C(d)) \right) + (1 - \alpha) \left( \pi U(C(u)) + (1 - \pi) U(C(d)) \right),
\]

and, for any given price, the corresponding FOC reads

\[
\frac{\delta U}{\delta Q} = \alpha \left( \bar{\pi} U'(C(u))(P - X(u)) + (1 - \bar{\pi}) U'(C(d))(P - X(d)) \right) + (1 - \alpha) \left( \pi U'(C(u))(P - X(u)) + (1 - \pi) U'(C(d))(P - X(d)) \right) = 0. \tag{A.10}
\]

As shown, for \( \tilde{P} \), it is optimal to sell \( \hat{Q} \) shares. Hence, the question now is, for what prices it is optimal to sell less (more) than \( \hat{Q} \)? Or, put differently, starting from \( \tilde{P} \) per share, below (above) which price does it become beneficial to sell less (more) than \( \hat{Q} \) shares?

Since when selling \( \hat{Q} \) shares \( C(u) = C(d) \), Eq. (A.10) yields

\[
\left. \frac{\delta U}{\delta Q} \right|_{Q=\hat{Q}} = 0 \iff P = \tilde{P}.
\]

I denote by \( \tilde{P}(\hat{Q} \downarrow) = L \) (\( \tilde{P}(\hat{Q} \uparrow) = U \)) the lowest (highest) price for which the seller prefers to sell \( \hat{Q} \) shares. Because \( \bar{\pi} < \pi \) whenever the seller considers to sell less than \( \hat{Q} \), and \( \bar{\pi} > \pi \) whenever she thinks about selling more than \( \hat{Q} \), it follows that \( L < U \).
Therefore, in summary, seller $i$’s supply curve is constant over the closed subset $[L, U] \subset P$ and the difference $U - L$ becomes larger as her $C$ becomes wider and/or as $\alpha \to 1$. This completes the proof.

Proof of Proposition 3. Whenever there is a nonzero mass of ambiguity-averse agent whose supply (demand) curves do not go through the benchmark equilibrium $(E[X], \hat{Q})$, they draw average supply (demand) away from the latter. Given the result in Proposition 2, this clearly occurs if either

$$L > E[X] \quad \text{or} \quad U < E[X]. \quad \hspace{1cm} (A.11)$$

For ambiguity-averse agents, $L$ is always strictly smaller than $U$, hence, the two cases in Eq. (A.11) are mutually exclusive.

I begin with the first inequality in Eq. (A.11). For any ambiguity-averse seller $i$ it holds that (neglecting the subscript $i$)

$$L = \alpha E^\pi[X] + (1 - \alpha)E^\bar{\pi}[X],$$

where $\alpha > 1/2$ and $\bar{\pi} < \pi$ since $L$ denotes the lower price limit below which she prefers to sell less than $\hat{Q}$ shares. Thus, denoting by $\pi_M$ the midpoint and by $2\Delta$ the length of the seller’s set of priors, the inequality $L > E[X]$ can be written as

$$E[X] < L$$

$$E[X] < \alpha((\pi_M - \Delta)X(u) + (1 - (\pi_M - \Delta))X(d))$$

$$+ (1 - \alpha)((\pi_M + \Delta)X(u) + (1 - (\pi_M + \Delta))X(d))$$

$$\pi X(u) + (1 - \pi)X(d) < \pi'X(u) + (1 - \pi')X(d), \quad \hspace{1cm} (A.12)$$

where $\pi' := \pi_M - \Delta(2\alpha - 1)$. By the analogous argument and relying on the same notation, it follows that the second inequality in Eq. (A.11) is equivalent to

$$E[X] > U$$

$$\pi X(u) + (1 - \pi)X(d) > \pi''X(u) + (1 - \pi'')X(d), \quad \hspace{1cm} (A.13)$$

whereas now $\pi'' := \pi_M + \Delta(2\alpha - 1)$.

Together, Eq. (A.12) and Eq. (A.13) imply that

$$L < E[X] < U \iff \pi' < \pi < \pi'', \quad \hspace{1cm} (A.14)$$

Alternatively, for a discrete set of priors, $2\Delta$ refers to the difference $\max(C) - \min(C)$. 

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where \( \pi' < \pi'' \) because \( \alpha > \frac{1}{2} \). Hence, whenever \( \pi \notin \mathcal{B} \) as defined in Eq. (4), the seller’s supply curve draws average supply away from the benchmark equilibrium. Because of perfect symmetry, the same condition simultaneously holds for any ambiguity-averse buyer. This completes the proof.

\[\textit{Proof of Proposition 4.}\] I first prove (ii). Eq. (5) can be written as (neglecting the subscript \( i \))

\[ U(C(\omega)) = \frac{1}{\pi - \frac{\pi}{2}} \int_{\mathcal{\bar{\pi}}}^{\mathcal{\bar{\pi}}} \phi(E[U(\bar{\pi})]) \, d\bar{\pi}. \]

For any given price, the FOC with respect to \( Q \) reads

\[ \frac{\delta U}{\delta Q} = \frac{1}{\pi - \frac{\pi}{2}} \int_{\mathcal{\bar{\pi}}}^{\mathcal{\bar{\pi}}} \phi'(E[U(\bar{\pi})]) \left( \bar{\pi} \frac{\partial}{\partial Q} U(C(u)) + (1 - \bar{\pi}) \frac{\partial}{\partial Q} U(C(d)) \right) \, d\bar{\pi} = 0. \]  

(A.15)

Eq. (A.3) and Eq. (A.4) imply

\[ \frac{1}{\pi - \frac{\pi}{2}} \int_{\mathcal{\bar{\pi}}}^{\mathcal{\bar{\pi}}} \phi'(E[U(\bar{\pi})]) \left( \bar{\pi} U'(C(u))(P - X(u)) + (1 - \bar{\pi}) U'(C(d))(P - X(d)) \right) \, d\bar{\pi} = 0. \]

For \( Q = \hat{Q} \) the agent bears no consumption risk, i.e., \( C(u) = C(d) \forall \bar{\pi} \) and \( E[U] \perp \bar{\pi} \). At \( Q = \hat{Q} \), Eq. (A.15) therefore becomes

\[ \frac{1}{\pi - \frac{\pi}{2}} \int_{\mathcal{\bar{\pi}}}^{\mathcal{\bar{\pi}}} (\bar{\pi} P - X(u)) + (1 - \bar{\pi})(P - X(d)) \, d\bar{\pi} = 0 \quad \Leftrightarrow \]

\[ \frac{1}{\pi - \frac{\pi}{2}} \int_{\mathcal{\bar{\pi}}}^{\mathcal{\bar{\pi}}} Pd\bar{\pi} = \frac{1}{\pi - \frac{\pi}{2}} \int_{\mathcal{\bar{\pi}}}^{\mathcal{\bar{\pi}}} \bar{\pi} X(u) + (1 - \bar{\pi}) X(d) \, d\bar{\pi} \quad \Leftrightarrow \]

\[ P = \frac{1}{2} \frac{\pi^2 - \pi^2}{\pi - \frac{\pi}{2}} X(u) + \left( 1 - \frac{1}{2} \frac{\pi^2 - \pi^2}{\pi - \frac{\pi}{2}} \right) X(d) \quad \Leftrightarrow \]

\[ P = \frac{\pi - \pi}{2} X(u) + \left( 1 - \frac{\pi - \pi}{2} \right) X(d). \]  

(A.16)

Hence, any seller’s (buyer’s) supply (demand) curve only goes through the benchmark equilibrium \( (\hat{Q}, E[X]) \), if the RHS of Eq. (A.16) equals the stock’s expected dividend, i.e.,

\[ \frac{\pi - \pi}{2} X(u) + \left( 1 - \frac{\pi - \pi}{2} \right) X(d) = E[X] \quad \Leftrightarrow \quad \pi = \frac{\pi + \pi}{2}. \]

Thus, whenever \( \pi \) does not correspond to the midpoint of her set of priors \([\bar{\pi}, \pi] \), she induces mispricing and suboptimal risk sharing of complex risks.

I hereafter prove (i) for the case where the considered nonzero mass of agents are sellers. In the case of buyers, the analogous reasoning applies. For a given seller \( i \) and price \( P \) per share, let \( Q_i^*(P) \) denote the number of shares satisfying Eq. (A.15). Taking the first order derivative

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of the second integrand in Eq. (A.15) with respect to $\hat{\pi}$ yields (neglecting again the subscript $i$)

$$
\frac{\partial}{\partial \hat{\pi}} \left( \hat{\pi}U'(C(u))(P - X(u)) + (1 - \hat{\pi})U'(C(d))(P - X(d)) \right) =
\frac{U'(C(u))(P - X(u)) - U'(C(d))(P - X(d))}{>0} < 0,
$$

(A.17)
i.e., is always strictly negative for $X(d) \leq P \leq X(u)$.

Regarding the first integrand in Eq. (A.15), there are three different cases. First, if seller $i$ is ambiguity-neutral, i.e., if $\phi'(\cdot)$ is a positive constant, only the second integrand is relevant for determining the optimal number of shares to be sold at $P$, hereafter denoted by $Q^*_N(P)$. Second, if seller $i$ is ambiguity-averse, i.e., if $\phi'(\cdot)$ is a decreasing function, then the first integrand becomes relevant for determining $Q^*_A(P)$. Third, if she is ambiguity-loving, her increasing function $\phi'(\cdot)$ conversely affects $Q^*_L(P)$.

Because Eq. (A.17) strictly decreases at a constant rate over $[\underline{\pi}, \overline{\pi}]$, Eq. (A.15) can only hold for $Q^*_N(P)$, if the second integrand changes its sign between $\underline{\pi}$ and $\overline{\pi}$. For $Q < \hat{Q}$, it holds that

$$
\frac{\partial}{\partial \hat{\pi}} E[U(\hat{\pi})] = U(C(u)) - U(C(d)) > 0 \ \forall Q < \hat{Q},
$$
i.e., whenever seller $i$ is ambiguity-averse, the first integrand in Eq. (A.15) is a strictly decreasing function over $[\underline{\pi}, \overline{\pi}]$. Hence, for $Q^*_A(P)$ the second integrand in Eq. (A.15) needs to switch its sign for a smaller $\hat{\pi} \in [\underline{\pi}, \overline{\pi}]$, relative to $Q^*_N(P)$, in order to satisfy the first order condition.

Taking the first order derivative of the second integrand in Eq. (A.15) with respect to $Q$ yields

$$
\frac{\partial}{\partial Q} \left( \hat{\pi}U'(C(u))(P - X(u)) + (1 - \hat{\pi})U'(C(d))(P - X(d)) \right) =
\hat{\pi} \frac{U''(C(u))(P - X(u))^2}{<0} + (1 - \hat{\pi}) \frac{U''(C(d))(P - X(d))^2}{<0} < 0,
$$
i.e., is always strictly negative for any risk-averse seller. It therefore follows that $Q^*_A(P) > Q^*_N(P)$, i.e., that $Q^*_A(P)$ is closer to $\hat{Q}$ than $Q^*_N(P)$. Since, for any ambiguity-loving seller, the first integrand in Eq. (A.15) then is a strictly increasing function over $[\underline{\pi}, \overline{\pi}]$, the analogous reasoning implies $Q^*_L(P) < Q^*_N(P)$. Thus, the distance between $\hat{Q}$ and $Q^*_L(P)$ is larger than between $Q^*_N(P)$ and $\hat{Q}$. Finally, the symmetric argument for $Q^*(P) > \hat{Q}$ yields $Q^*_A(P) < Q^*_N(P) < Q^*_L(P)$. This completes the proof.
Appendix B: Determining $\pi$ in the Presence of Complex Risks

Starting point is the SDE of the geometric Brownian motion in Figure 5, i.e.,

$$dS_t = 10\%S_t dt + 32\%S_t dW_t,$$

where $W_t$ is a standard Brownian motion. Applying Itô to $f := \ln(S_t)$ yields

$$S_2 = \exp \left\{ \left( 10\% - \frac{32\%^2}{2} \right) + 32\%(W_2 - W_1) \right\}.$$

Hence,

$$\mathbb{P}(S_2 \geq 1.05) = \mathbb{P} \left( W_2 - W_1 \leq \frac{\ln(1.05) - 10\% + \frac{32\%^2}{2}}{32\%} \right).$$

Recalling that the increment $W_2 - W_1$ has a standard normal distribution,\textsuperscript{30} it follows that $\mathbb{P}(S_2 \geq 1.05)$ corresponds to $\frac{1}{2}.\textsuperscript{31}$

Appendix C: Adjustment of average supply and demand curves according to subjective beliefs

For a given case, I denote by $\bar{E}_S[X]$ sellers’ average point estimate of the risky asset’s expected payoff under complex risks. In order to account for deviations of $\bar{E}_S[X]$ from $E[X]$, the following linear transformation is applied to the predefined price vector used to elicit sellers’ supply functions:

$$adj(P) = \begin{cases} 
P - (\bar{E}_S[X] - E[X]) \frac{P - X(d)}{E_S[X] - X(d)}, & \text{for } X(d) \leq P < \bar{E}_S[X] \\
\bar{E}_S[X] - E[X] \frac{X(u) - P}{X(u) - \bar{E}_S[X]}, & \text{for } \bar{E}_S[X] \leq P \leq X(u).
\end{cases}$$

Furthermore, let $\bar{Q}_S$ denote the linearly interpolated average supply curve and $\bar{Q}_{S,adj}$ the corresponding curve plotted against $adj(P)$ instead of $P$. It then still holds that $\bar{Q}_{S,adj}$ spans from $X(d)$ to $X(u)$, but simultaneously that $\bar{Q}_{S,adj}(E[X]) = \bar{Q}_S(\bar{E}_S[X])$. The exact same linear transformation with $\bar{E}_B[X]$ instead of $\bar{E}_S[X]$, with $B$ for buyers, is also used to adjust average demand curves under complex risks.

\textsuperscript{30} This information was provided as part of the instructions.

\textsuperscript{31} Strictly speaking, it holds that $\mathbb{P}(S_2 \geq 1.05) = 0.49999$. Linearly approximating $\ln(1.05)$ by 0.05 implies $\mathbb{P}(S_2 \geq 1.05) = 0.50150$. 

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Appendix D: Additional Tables and Figures

Table D.1. Regression analysis for nonnegative ambiguity aversion

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_1$</th>
<th>$M_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>3.954$^a$</td>
<td>0.736$^a$</td>
<td>2.943</td>
<td>0.939$^b$</td>
</tr>
<tr>
<td></td>
<td>(0.563)</td>
<td>(0.118)</td>
<td>(2.190)</td>
<td>(0.392)</td>
</tr>
<tr>
<td>Complexity (dummy)</td>
<td>0.238</td>
<td>-0.199$^b$</td>
<td>-0.452</td>
<td>-0.232$^b$</td>
</tr>
<tr>
<td></td>
<td>(0.770)</td>
<td>(0.084)</td>
<td>(0.702)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>Ambiguity (dummy)</td>
<td>0.517</td>
<td>0.213</td>
<td>0.926</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>(1.208)</td>
<td>(0.185)</td>
<td>(1.225)</td>
<td>(0.196)</td>
</tr>
<tr>
<td>RA (risk aversion)</td>
<td>7.721$^a$</td>
<td>-0.091</td>
<td>6.931$^a$</td>
<td>-0.137</td>
</tr>
<tr>
<td></td>
<td>(2.224)</td>
<td>(0.347)</td>
<td>(2.230)</td>
<td>(0.344)</td>
</tr>
<tr>
<td>AA (ambig. aversion) × Complexity</td>
<td>3.384</td>
<td>-0.542$^b$</td>
<td>0.096</td>
<td>-0.345</td>
</tr>
<tr>
<td></td>
<td>(2.442)</td>
<td>(0.259)</td>
<td>(2.371)</td>
<td>(0.231)</td>
</tr>
<tr>
<td>AA × Ambiguity</td>
<td>9.792$^b$</td>
<td>-1.233$^a$</td>
<td>7.473</td>
<td>-1.041$^b$</td>
</tr>
<tr>
<td></td>
<td>(4.669)</td>
<td>(0.472)</td>
<td>(4.729)</td>
<td>(0.522)</td>
</tr>
<tr>
<td>Order × Complexity</td>
<td>-</td>
<td>-</td>
<td>2.243$^b$</td>
<td>0.000</td>
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<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>(1.132)</td>
<td>(0.114)</td>
</tr>
<tr>
<td>Gender</td>
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<td>-</td>
<td>1.187</td>
<td>-0.298$^a$</td>
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<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>(0.908)</td>
<td>(0.113)</td>
</tr>
<tr>
<td>Controls</td>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>373</td>
<td>518</td>
<td>373</td>
<td>518</td>
</tr>
</tbody>
</table>

Notes: This table reports OLS coefficient estimates for subjects with nonnegative ambiguity aversion (as indicated by the variable ‘Ambiguity aversion’—see below). The dependent variables are unconditional measures of local price sensitivity. $M_1$ denotes the cardinality of consecutive prices for which subjects adopt the perfect hedging strategy, i.e., aiming to trade $Q$ shares. $M_2$ measures the average slope of subjects’ supply and demand curves at their individual point estimates of the risky asset’s expected payoff. ‘Complexity’ and ‘Ambiguity’ are dummy variables indicating trading rounds with complex and ambiguous risks, respectively. ‘Risk aversion’ measures the normalized difference between the simple lottery’s expected payoff and subjects’ respective certainty equivalents. The first two interaction terms control for different effects of ‘Ambiguity aversion’ across trading rounds with simple and complex risks, where ambiguity aversion is measured as the difference between subjects’ certainty equivalents for the simple and the ambiguous lottery. The term ‘Order × Complexity’ interacts the dummy variable ‘Order’, indicating sessions where complex risks were proceeded by simple risks, with complex trading rounds. ‘Gender’ is a dummy variable indicating female subjects. ‘Controls’ comprise subjects’ age, their attended university, and number of completed semesters. Furthermore, ‘Controls’ contain subjects’ self-evaluated understanding and difficulty level of the assigned task (measured by integers from one to five) and two additional dummy variables controlling for their familiarity and knowledge regarding the Brownian motion. Numbers in parenthesis denote robust standard errors clustered at the subject level. Superscripts $^a$, $^b$, and $^c$ indicate statistical significance at the 1%, 5%, and 10%-level, respectively.
Figure D.1. Example of nonmonotonic supply curve

Notes: Supply curve for seller $i$ with utility function as defined in Remark 1. Parameters:
$X(u) = 1.5, X(d) = 0, \pi = 1/10, \epsilon = 0, \alpha = 1, C = 3 + 2\pi X(u), E_i = 4, I_i(u) = 0, I_i(d) = 3.$

Figure D.2. Lottery based on urn with simple risks

Notes: This figure shows the lottery based on the urn with simple risks. Whenever the randomly drawn ball is green, the lottery pays ECU 600 (experimental currency units) and ECU 300 if it is red. Subjects’ respective certainty equivalents were elicited via Abdellaoui et al. (2011)’s iterative choice list method.
Figure D.3. Testing for differences in price sensitivity

Notes: This figure reports the $p$-values of a Wilcoxon signed-rank test of the differences between average supply (demand) curves for simple and complex risks. Averages are computed across subjects and trading rounds. Average curves for complex risks are adjusted as described in Appendix C and linearly interpolated to allow for a direct comparison with simple risks. In the top (bottom) row, average supply (demand) curves are computed across all sessions. In the left (right) column, averages are computed across trading rounds where $\pi$ is equal to $1/2$ ($1/3$). The dotted line indicates a $p$-value equal to 10\%. 
Figure D.4. Testing for price-taking behavior under complex risks

Notes: This figure reports the p-values of a Wilcoxon signed-rank test of the differences between average supply (demand) curves for complex risks under market clearing and random price draws. Averages are computed across subjects and complex trading rounds. Average curves are adjusted as described in Appendix C and linearly interpolated to allow for a direct comparison with simple risks. In the top (bottom) row, average supply (demand) curves are computed across all sessions. In the left (right) column, averages are computed across complex trading rounds where $\pi$ is equal to $1/2$ ($1/3$). The dotted line indicates a p-value equal to 10%.
Figure D.5. Demand distribution for prices equal to estimated payoffs

Notes: This figure shows the number of shares demanded by buyers for prices equal to (estimated) expected payoffs. The empirical distributions are computed across subjects and sessions. The left (right) plot contrasts average distributions between simple and complex trading rounds with \( \pi \) equal to \( \frac{1}{2} \) (\( \frac{1}{3} \)). If, under complex risks, buyers’ point estimate \( E_i[X] \) lies between two elements of the predefined price vector, linearly interpolated quantities are reported.
Figure D.6. Demand distribution for prices different from expected payoffs

Notes: This figure shows the number of shares demanded by buyers for prices different from expected payoffs. The empirical distributions between simple and complex risks are computed across subjects and sessions. In the top (bottom) row, total demands for prices below (above) $E_i[X]$ are reported. The left (right) column shows average demand distributions across trading rounds with $\pi$ equal to $1/2$ ($1/3$).
Figure D.7. Learning under complex risks

Notes: This figure shows the evolution of the average percentage of dominated trading strategies (see Figure 14) over the four trading rounds with complex risks (see Table III). Error bars indicate standard errors of the mean.
Figure D.8. Distribution for prices equal to expected payoff (reference point)

Notes: This figure shows empirical distributions of supplied and demanded shares at a fixed price of ECU 75. Percentages are computed across subjects and sessions. For simple risks, only the trading round with \( \pi \) equal to \( \frac{1}{2} \) is considered. For ambiguous risks, a price of ECU 75 corresponds to the natural reference point, assuming that subjects believe in a fifty-fifty likelihood under pure ambiguity.
**Figure D.9. Equilibration Variability**

*Notes:* This figure shows bootstrapped standard deviation estimates of market-clearing prices and quantities for simple and complex risks. Average supply and demand curves are determined for different resample sizes. For each pair of averaged supply and demand, linearly interpolated market-clearing prices and quantities are computed. Repeating this procedure ten thousand times yields the depicted standard deviation estimates of equilibrium prices (top row) and quantities (bottom row). The left (right) column shows bootstrapped moment estimates for trading rounds with $\pi$ equal to $1/2$ ($1/3$). Error bars indicate 99%-confidence intervals.
Appendix E: Experimental Instructions

Instructions I/II

Welcome to this experiment at the Department of Banking and Finance, University of Zurich. This is the first out of 2 instruction sheets. Please read each sheet very carefully. Fully understanding the instructions will allow you to perform better on the task, thereby earning more money. Raise your hand if you have any questions or as soon as you have read everything and are ready to continue.

1 Situation

The experiment consists of a sequence of 7 trading rounds. In each trading round the same number of buyers and sellers are present. You are a seller. Your role will not change throughout the experiment.

At the beginning of every round, you will receive a fresh supply of 4 shares of a given security. During each round you can sell between 0 and 4 of these shares. The security either pays a dividend per share equal to 150 or 0. Besides this dividend per share, the security does not pay anything else (no capital gains). Additionally, you are provided with some non-tradable income: whenever the security happens to pay a dividend of 150 per share, you receive 0, and if it does not pay anything (dividend of 0), you receive 300. This additional income does not depend on how many shares you are selling. The following graph summarizes your holdings at the beginning of every round:

 Shares: 4
   \[\text{Dividend} = 150\]  \[\text{Income} = 0\]
   +
   \[\text{Dividend} = 0\]  \[\text{Income} = 300\]

You can sell up to 4 shares

Non-tradable

Your wealth at the end of each round is the sum of received proceeds from trading, collected dividends, and additional income. It is not carried over to the subsequent round, this means you always start out with 4 shares. At the end of every round, the trading outcome, realized dividends, and your respective wealth are displayed.

2 Trading

Trading happens in 2 phases. First, you have to select how many shares you want to sell in case the price equals 0, 25, 50, 75, 100, 125, or 150. The computer then linearly fills up your selling quantities for the remaining 5-unit steps between 0 and 150 (5, 10, 15, ...). Second, you are asked to make further adjustments until you end up with the exact quantities you want to sell for any given price. Note, quantities can be entered with up to 2 decimal places of precision.

The price determination method of the current round is always displayed in the upper right corner of your screen. There are two ways how prices are determined. If there is market clearing, the computer sets the price such that the number of traded shares is maximized. Alternatively, the computer will choose the price randomly (random price) with equal probabilities across the full list of given prices.

You will now go through a first practice round. This practice round will not impact your payment.
Instructions II/II

Please read this sheet very carefully. Raise your hand if you have any questions or as soon as you have read everything and answered the comprehension questions at the end.

3 How Dividends Are Determined

The computer randomly determines whether the security is going to pay a dividend or not. However, the information about the structure that governs the computer’s random choices varies between trading rounds. There are 2 different cases:

1. Urn.—The computer draws 1 ball out of an urn with 30 balls. The balls are either green or red, the respective composition is revealed at the beginning of the trading round. Whenever the color of the drawn ball is green, the security pays a dividend equal to 150 per share (and 0 if red).

2. Simulated reference path.—The computer simulates the evolution of a reference path over 2 time periods, but only the first period will be displayed. Whenever the path ends up above a certain limit, the security pays a dividend equal to 150 per share (and 0 if the path ends up below this limit). The only purpose of this path is to determine whether the security pays a dividend or not.

What you will see.—You are provided with a formal description of the reference path $S_t$, where the random component is denoted by $W_t$. $W_t$ follows a normal distribution with mean equal to 0 and variance equal to the corresponding change in time. For example, the full description of the path $S_t$ could look like this

$$dS_t = 5\%S_t dt + 10\%S_t dW_t,$$

where $dS_t$ denotes the change of $S_t$ over the very small (infinitesimal) time change of length $dt$. Additionally, you will see a video of the path $S_t$ between time 0 and the end of period 1:

(a) Beginning of period 1  
(b) Middle of period 1  
(c) End of period 1

The difference of the random component $W_t$ between the ends of period 1 and 2, $W_2 - W_1$, follows a normal distribution with mean 0 and variance 1. For simplicity, every path is scaled such that $S_1 = 1$.

Based on this information you can assess the probability of the dividend being equal to 150 ($\rightarrow$ path ends up in the green region at time 2).

4 List of Lotteries, Questionnaire, and Payment

After the 7 trading rounds, you have to repeatedly choose 1 out of 2 options for 2 lists of lotteries. For both lists, the computer randomly selects and plays 1 of your chosen options. Finally, you will be asked
to fill in a short questionnaire.

Your final payment will be determined as follows:

1. The computer randomly picks 1 out of the 7 trading rounds or 1 of the 2 lottery outcomes with equal probability (\( \frac{1}{7} \) for each). It is therefore critical that you concentrate on every round. You will be paid either your wealth at the end of the selected trading round or the outcome of the selected lottery, both in CHF divided by 12.

2. In all rounds with simulated reference paths, you are asked to submit your best guess regarding the probability of the dividend being equal to 150. If your guess is correct (within +/- 3%), you earn an additional 3 CHF whenever this round is selected for payment.

5 Comprehension Questions

(1) Assume you have sold 4 shares at a price of 50 per share, what is your wealth in the 2 scenarios?

\[
\begin{align*}
\text{per share} & \quad \text{Dividend} = 150 \rightarrow \text{Wealth} = \\
\text{per share} & \quad \text{Dividend} = 0 \rightarrow \text{Wealth} = 
\end{align*}
\]

(2) Assume you have sold 4 shares at a price of 150 per share, what is your wealth in the 2 scenarios?

\[
\begin{align*}
\text{per share} & \quad \text{Dividend} = 150 \rightarrow \text{Wealth} = \\
\text{per share} & \quad \text{Dividend} = 0 \rightarrow \text{Wealth} = 
\end{align*}
\]

(3) Assume you have sold 2 shares at a price of 50 per share, what is your wealth in the 2 scenarios?

\[
\begin{align*}
\text{per share} & \quad \text{Dividend} = 150 \rightarrow \text{Wealth} = \\
\text{per share} & \quad \text{Dividend} = 0 \rightarrow \text{Wealth} = 
\end{align*}
\]

(4) Does the difference between your wealth in the green and the red scenario depend on...?

\( \square \) The paid price \( \square \) The number of sold shares

(5) What is the difference between your wealth in the 2 scenarios, if you exactly sell 2 shares?

\[
\text{Difference} = 
\]

********************************************************************************************************************
Raise your hand after you have answered the comprehension questions. After double-checking, you will go through 2 last practice rounds. These practice rounds will not impact your payment.