

# Voluntary Disclosure in Bilateral Transactions

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## Abstract

We characterize optimal voluntary disclosures by a privately informed agent facing a counterparty endowed with market power in a bilateral transaction. Although disclosures reveal some of the agent's private information, they may increase his information rents by mitigating the counterparty's incentives to resort to inefficient screening. We show that when disclosures are restricted to be ex post verifiable, the informed agent optimally designs a disclosure plan that is partial and that implements socially efficient trade in equilibrium. Our results shed light on the conditions necessary for asymmetric information to impede trade and the determinants of disclosures' coarseness.

Keywords: Information Design; Disclosure; Asymmetric Information; Market Power; Persuasion.

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# 1 Introduction

Asymmetric information can make agents worse off by disrupting efficient trade (e.g., Akerlof 1970, Myerson and Satterthwaite 1983, Glosten and Milgrom 1985). But why would an agent *allow* his private information to be an impediment to trade in the first place? In this paper, we study the incentives of a privately informed agent to share his information with a counterparty endowed with market power prior to a bilateral transaction.

We examine an environment with both private- and common-value uncertainty and consider voluntary disclosures that are ex post verifiable, as in Grossman (1981), Milgrom (1981), and Shin (2003). Ex post verifiability is a common restriction in the literature that is imposed to ensure that disclosures are not subject to commitment and incentive problems, even in one-shot interactions.<sup>1</sup> If erroneous disclosures can be verified with probability one and are penalized — perhaps by regulators or courts — the sender optimally designs signals that are always truthful. Moreover, the sharing of verifiable information is relevant in many important economic contexts with hard information, such as the trading of financial securities, corporate takeovers, and supply chain transactions.<sup>2</sup>

In this environment, we obtain a set of sharp predictions. First, the informed agent always finds it privately optimal to design a partial disclosure plan that yields socially efficient trade in equilibrium. Whereas possessing superior information allows the informed agent to extract information rents, sharing information reduces the extent to which the agent is being inefficiently screened by his counterparty. We show that the agent is always willing and able to design ex post verifiable signals such that he privately benefits from giving up part of his private information in order to preempt inefficient screening. Yet, he finds it privately suboptimal to disclose all information as doing so completely eliminates his information rents. Compared to a no-disclosure policy, the optimal

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<sup>1</sup>The early literature analyzing these types of “persuasion games” is surveyed by Milgrom (2008). Since these games focus on ex post verifiable disclosures, they significantly differ from “cheap talk games” popularized by Crawford and Sobel (1982).

<sup>2</sup>See Boyarchenko, Lucca, and Veldkamp (2016) and Di Maggio, Franzoni, Kermani, and Somnavilla (2016) for empirical evidence consistent with broker-dealers sharing private (deal-flow) information among themselves and with clients, Hong, Kubik, and Stein (2005) and Pool, Stoffman, and Yonkers (2015) for empirical evidence consistent with information sharing among socially connected portfolio managers, Heimer and Simon (2012) for empirical evidence of information sharing among foreign exchange traders, Eckbo and Langohr (1989) and Brennan (1999) for empirical evidence of information sharing among bidders and target companies in corporate takeovers, and Zhou and Benton (2007) for empirical evidence of information sharing among firms part of the same supply chain.

disclosure plan improves both the informed agent’s surplus and that of his counterparty with market power.

We characterize general properties of privately optimal disclosure plan(s) and analyze the trade-offs the informed agent faces. Moreover, we derive solutions for the shapes of optimal disclosure plans when disclosure functions are restricted to be monotone, an assumption that is related to a commonly imposed restriction in the security design literature. While we initially consider an environment where the disclosure plan is designed before any uncertainty is realized — as is common in models of information design (e.g., Rayo and Segal 2010, Kamenica and Gentzkow 2011, Goldstein and Leitner 2015, Ely 2017) — we also consider the case where disclosure is chosen after the agent obtains private information (as in, e.g., Grossman 1981, Milgrom 1981, Verrecchia 1983, Shin 2003). We show that, in line with our earlier analysis, partial disclosure leading to socially efficient trade also characterizes all equilibria satisfying two standard refinements of this interim disclosure game.

Our results have relevant implications from both a positive and a normative perspective. First, the economic mechanisms underlying optimal disclosure plans in our model shed light on existing disclosure practices in financial markets, in particular on variation in disclosures’ *coarseness*. For example, the mechanisms may contribute to the intriguing fact that credit rating agencies like Moody’s Analytics publish ratings on a *discrete* scale when they are solicited and paid for by issuers, but at the same time provide *continuous* credit scores when investors subscribe and pay for credit information. Second, given the standard set of assumptions we consider and the strong predictions we obtain — in particular, trade is always efficient in equilibrium — our paper also sheds light on the economically relevant conditions that must be *violated* for asymmetric information and market power to impede the efficiency of trade. Our paper thus speaks to the regulation of information disclosure in bilateral transactions with imperfect competition and asymmetric information problems, such as corporate takeovers, real estate transactions, and over-the-counter trading.<sup>3</sup> In

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<sup>3</sup>For empirical evidence that these types of bilateral transactions often feature imperfect competition, see Ambrose, Highfield, and Linneman (2005), Glaeser, Gyourko, and Saks (2005), Boone and Mulherin (2007), King, Osler, and Rime (2012), Atkeson, Eisfeldt, and Weill (2013), Li and Schürhoff (2014), Begenau, Piazzesi, and Schneider (2015), Hendershott et al. (2015), Di Maggio, Kermani, and Song (2016), Li, Taylor, and Wang (2016), and Siriwardane (2016). For empirical evidence that these types of bilateral transactions often involve heterogeneously informed traders, see

an environment like ours, a regulator does not need to mandate what information agents should disclose nor does he need to produce additional information for uninformed market participants. All the regulator needs to do is to enforce the truthfulness of verifiable disclosures by disciplining agents who send signals that ex post prove to violate their own disclosure standards.

**Related Literature.** Our paper contributes to the theoretical literature that studies optimal information sharing among traders. An important result in this literature goes back to Grossman (1981) and Milgrom (1981) who show that, when disclosures are restricted to be ex post verifiable, an agent may find it optimal to fully reveal his private information to his counterparties. However, unlike in our model, the agent making the disclosure decision in these papers is not being screened by counterparties with market power — either all traders take the price as given (as in Milgrom 1981) or it is the informed agent who sets the price (as in Grossman 1981). The lack of counterparties’ market power implies that the agent who discloses information is concerned with his counterparties’ conditional beliefs about the *mean* asset payoff. The agent then optimally discloses all his private information, since any information he withholds is interpreted to be unfavorable, lowering his expected payoff (a result commonly referred to as “unraveling”; see also Grossman and Hart 1980, Milgrom and Roberts 1986). In contrast, in our environment, the disclosing agent is concerned with the full conditional *distributions* of payoffs resulting from disclosures (rather than just the mean payoff), since his counterparty has market power and decides to screen the agent based on the shapes of these conditional distributions.

Verrecchia (1983) modifies a setting akin to Grossman (1981) and Milgrom (1981) by adding disclosure costs whereas Fishman and Hagherty (1990) assume that a subset of private information cannot be disclosed. In both cases, maximal disclosure may not be optimal for the informed party. Admati and Pfleiderer (2000), however, show that a firm may pick a socially optimal disclosure plan despite disclosure costs if that firm is a monopolist that captures all gains to trade. Matthews and Postlewaite (1985), Okuno-Fujiwara, Postlewaite, and Suzumura (1990), Fishman and Hagherty (2003), Shin (2003), Acharya, DeMarzo, and Kremer (2011), and Guttman, Kremer, and Skrzypacz

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Eckbo, Giammarino, and Heinkel (1990), Garmaise and Moskowitz (2004), Green, Hollifield, and Schürhoff (2007), Hollifield, Neklyudov, and Spatt (2014), Jiang and Sun (2015), Menkhoff et al. (2016), and Stroebel (2016).

(2014) show, in different environments, that full disclosure also becomes suboptimal once there is uncertainty about the existence of private information or its content.

Unlike in these settings, the information designer in our model is the responder to an ultimatum offer, and his private information is thus his only source of profits. As a result, his optimal disclosure plan must be partial, despite the existence of his private information being common knowledge and disclosure being costless. Yet, we also show that it must yield socially efficient trade in equilibrium. Our framework therefore speaks to how voluntary information sharing can eliminate inefficient rationing in classic monopolistic screening problems, where private information and bargaining power are separated. Examples of situations where such separation, whether full or partial, applies abound, and include a real estate transaction in which a buyer submits a bid without knowing the seller's reservation value for the property, a financial transaction in which a dealer quotes a price to a hedge fund possessing proprietary data and valuation models, and a supply chain transaction in which a producer must price merchandise before offering it to a retailer informed about local demand conditions.<sup>4</sup>

Monopoly pricing is also studied by Bergemann, Brooks, and Morris (2015) who analyze how signals providing monopolists with additional information for price discrimination affect total surplus and its allocation. Their analysis shows that, in a setting with private-value uncertainty, general information structures (including randomization) exist such that total surplus can be increased to any level less than or equal to the one from efficient trade, and any allocation of the incremental surplus is attainable. Information available for price discrimination thus critically determines efficiency and the allocation of surplus, raising the question of what part of a buyer's private information a monopolist should be expected to endogenously gain access to. Our analysis shows that when information disclosure by the informed agent is (a) voluntary and (b) ex post verifiable (with randomization not being possible), precise predictions for both total surplus and its allocation obtain: (i) total surplus is unique and equal to the surplus generated by efficient trade, and (ii) both agents benefit from the optimal disclosure plan. These results hold for both private- and common-value uncertainty,

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<sup>4</sup>While our framework assumes full separation of bargaining power and private information, as in classic models of monopolistic screening, the main economic insights we develop will also speak to the multitude of scenarios where these two sources of rents are *partially* separated and the full-disclosure/unraveling reasoning from Grossman (1981) and Milgrom (1981) fails to apply.

whether the disclosure functions are restricted to be monotone or not, and when disclosure plans are designed ex ante or at an interim stage.

More broadly, our focus on market power also relates our paper to Gal-Or (1985) who models oligopolistic firms that can commit ex ante to sharing noisy signals of their private information about the uncertain demand for their products. Since sharing information increases the correlation of firms' output decisions, thereby lowering their expected profits, the unique symmetric pure-strategy equilibrium is characterized by no information sharing among firms. Lewis and Sappington (1994) investigate in a setting without disclosure whether an uninformed seller with market power would like to help his prospective buyer(s) acquire private information about the value of the asset (see also Esó and Szentes 2007, who assume that trading occurs through an auction). Under general conditions, the seller in Lewis and Sappington (1994) either wants his prospective buyer(s) to be fully informed or completely uninformed about how much the asset is worth to them. Finally, Roesler and Szentes (2017) solve for a buyer's optimal information acquisition in a monopoly setting without disclosure and show that the buyer finds it optimal to limit his information acquisition and avoid that the monopolist seller inefficiently screens him (see also Glode, Green, and Lowery 2012).

The next section presents the classic problem of a monopolist who inefficiently screens a privately informed agent. In Section 3, we study the agent's incentives to share some of his private information with the monopolist and how the resulting disclosure plan affects the efficiency of trade. Section 4 shows that our main insights survive when the agent designs his disclosure plan after obtaining private information rather than before. The last section concludes. Proofs omitted from the text can be found in the Appendix.

## **2 The Bilateral Transaction**

The monopolist seller of an asset (or good) chooses the price he will quote to a prospective buyer (or customer) in a take-it-or-leave-it offer.<sup>5</sup> The seller is uncertain about how much the buyer is willing to pay for the asset but knows that the buyer's valuation of the asset, which we denote by

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<sup>5</sup>The buyer/seller roles could be reversed without affecting our main results, as long as market power and private information are still allocated to distinct agents.

$v$ , has a cumulative distribution function (CDF) denoted by  $F(v)$ . The buyer only accepts to pay a price  $p$  in exchange for the asset if  $v \geq p$ ; otherwise, the seller must retain the asset, which is worth  $c(v)$  to him. The CDF  $F(v)$  is continuous and differentiable and the probability density function (PDF), denoted by  $f(v)$ , takes strictly positive values everywhere on the support  $[v_L, v_H]$ .<sup>6</sup> The function  $c(v)$  is assumed to be weakly increasing and continuous. Both agents are risk neutral and the functions  $F(v)$  and  $c(v)$  are common knowledge.<sup>7</sup>

Whenever the buyer's valuation is greater than the seller's — perhaps due to heterogeneity in preferences, inventories, or liquidity needs — trade would create a social surplus. However, the seller may find it privately optimal to use his market power and inefficiently screen the informed buyer, thereby jeopardizing the gains to trade  $[v - c(v)]$ . We assume that whenever indifferent between two strategies, an agent picks the one that maximizes the social surplus in the resulting subgame-perfect Nash equilibrium.

The seller's expected payoff from quoting a price  $p$  is given by:

$$\begin{aligned}\Pi(p) &\equiv [1 - F(p)]p + F(p)\mathbb{E}[c(v)|v < p] \\ &= \int_p^{v_H} pf(v)dv + \int_{v_L}^p c(v)f(v)dv.\end{aligned}\tag{1}$$

When picking a price, the seller considers the tradeoff between the probability that a sale occurs and the profit he obtains if a sale occurs. The seller's marginal profit of increasing the price  $p$  is:

$$\Pi'(p) = \int_p^{v_H} f(v)dv - pf(p) + c(p)f(p),\tag{2}$$

which can be rewritten as:

$$\Pi'(p) = [1 - F(p)] - f(p)[p - c(p)].\tag{3}$$

The first term on the right-hand side of equation (3) is the seller's marginal expected benefit from

<sup>6</sup>Our results would also hold if the support of  $v$  was unbounded from above. If the support of  $v$  was unbounded from below, our results would hold whenever  $\lim_{v \downarrow -\infty} (v - c(v)) < 0$ .

<sup>7</sup>See, e.g., Hirshleifer (1971), Diamond (1985), and Kurlat and Veldkamp (2015) for the costs and benefits of disclosure in the presence of risk aversion.

collecting a higher price when trade occurs. The second term is the marginal expected cost from reducing the probability of trade and destroying gains to trade obtained from trading with the marginal buyer type. We impose the following condition on the surplus from trade  $[v - c(v)]$ :

**Assumption 1.** *The surplus from trade  $[v - c(v)]$  crosses zero at most once (from below).*

This condition is fairly benign as it is guaranteed to hold under any of the following assumptions common in the literature: (i) the seller's valuation for the asset is a constant; (ii) the seller's valuation for the asset is a fraction of  $v$ ; (iii) the surplus from trade  $[v - c(v)]$  is a constant; (iv) the ratio of the above-mentioned cost and benefit of marginally increasing the price, i.e.,  $\frac{f(v)}{1-F(v)}[v - c(v)]$ , is strictly increasing in  $v$ .<sup>8</sup>

Assumption 1 implies that we can designate a cutoff  $\hat{v} \in [v_L, v_H]$  such that trade is socially efficient if it occurs when  $v \geq \hat{v}$ , and fails when  $v < \hat{v}$ . Since it is possible under Assumption 1 that  $[v - c(v)]$  remains at zero for a positive-measure subset of  $[v_L, v_H]$  and then becomes positive for higher values of  $v$ , we define the relevant cutoff as  $\hat{v} \equiv \inf\{v \in [v_L, v_H] : v > c(v)\}$ . Since  $f(v)$  is strictly positive everywhere on the support  $[v_L, v_H]$ , the maximum price the seller can quote and still maintain socially efficient trade is  $p = \hat{v}$ . As a result, trade can be efficient only if:

$$\Pi'(\hat{v}) \leq 0. \tag{4}$$

This necessary condition for efficient trade to occur in equilibrium can be interpreted as follows. Efficient trade requires that  $\hat{v} - c(\hat{v}) \geq \frac{1-F(\hat{v})}{f(\hat{v})}$ , which means that either the gains to trade are large, or that the seller's beliefs about  $v$  are concentrated (i.e., the density  $f(v)$  is high enough) when the surplus from trade becomes positive. If instead  $\Pi'(\hat{v}) > 0$ , the seller inefficiently screens the buyer, jeopardizing gains to trade. Moreover, the seller never optimally quotes a price  $p < \hat{v}$ , because quoting a price  $p = \hat{v}$  yields strictly higher profits.<sup>9</sup>

<sup>8</sup>See, e.g., Glode and Opp (2016) and Glode, Opp, and Zhang (2017) who specifically impose this condition, Fuchs and Skrzypacz (2015) who define a "strictly regular environment" in a similar way, and Myerson (1981) who similarly assumes that bidders' virtual valuation functions are strictly increasing.

<sup>9</sup> Specifically, if the seller quotes a price  $p < \hat{v}$ , his expected payoff can be written as:

$$\Pr(v \geq \hat{v})p + \Pr(p \leq v < \hat{v})p + \Pr(v < p)\mathbb{E}[c(v)|v < p].$$



Since we assume that whenever indifferent, an agent picks the strategy that maximizes social surplus, we can rule out any equilibrium where the seller inefficiently mixes between quoting multiple prices  $p_n \in [v_L, v_H]$ . If he were to mix over several prices, the seller would have to be indifferent between mixing and quoting any of these prices with probability one (taking into account the buyer's best response to each price). The tie-breaking rule implies that the seller instead plays the pure strategy of quoting the price that socially dominates all other prices. Similarly, we can rule out equilibria where the buyer inefficiently mixes between accepting and not accepting a price quote. A tie-breaking rule based on social optimality thus ensures that we can restrict our attention to pure-strategy subgame-perfect Nash equilibria.

We further illustrate the seller's incentives to set inefficient prices through a simple parameterized example that we will revisit later.

**Example 1.** *Suppose the buyer values the asset at  $v \sim U[1, 2]$  and the seller values it at a constant  $\bar{c} \leq 1$ . The surplus from trade is then always positive (i.e.,  $\hat{v} = 1$ ) and trade is efficient if and only if it occurs with probability 1. The seller's optimization problem when picking a price can be written as:*

$$\max_{p \in [1, 2]} \Pi(p) = \Pr(v \geq p)p + \Pr(v < p)\bar{c} = (2 - p)p + (p - 1)\bar{c}. \quad (5)$$

*Since  $\Pi'(1) = \bar{c}$ , the seller quotes a price  $p = 1$  whenever  $\bar{c} \leq 0$  and the buyer always accepts, implying that trade is efficient. However, when  $\bar{c} \in (0, 1]$  the seller finds it optimal to quote a price  $p = 1 + \frac{\bar{c}}{2}$ , which destroys the surplus from trade with probability  $\frac{\bar{c}}{2}$ .*

The example above shows a simple case where  $\hat{v} = v_L$ , that is, the surplus from trade is positive for any realization of  $v$ . In such cases, sustaining efficient trade requires that  $v_L - c(v_L) \geq \frac{1}{f(v_L)}$ . For cases where  $\hat{v} \in (v_L, v_H)$  however, efficient trade can never be sustained in equilibrium since  $\hat{v} - c(\hat{v}) = 0 < \frac{1 - F(\hat{v})}{f(\hat{v})}$ , implying that the seller always finds it optimal to quote a price that is at least marginally higher than the efficient price  $p = \hat{v}$ . This situation arises, for example, whenever the seller values the asset at a constant  $\bar{c} \in (v_L, v_H)$ .

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In contrast, if the seller quoted a price  $\hat{v}$ , his payoff would increase by  $\hat{v} - p > 0$  when  $v \geq \hat{v}$ , by  $c(v) - p \geq 0$  when  $p \leq v < \hat{v}$  and would remain the same when  $v < p$ .

Although we model a bilateral trading interaction where only one offer can be made, similar inefficiencies associated with screening can arise in dynamic environments. First, if the seller could commit to following any dynamic pricing strategy, he would still *optimally* choose to make the buyer a take-it-or-leave-it offer at the start of the dynamic game and no further offer later on, just as in our setup here (see, e.g., Stokey 1979, Harris and Raviv 1981, Riley and Zeckhauser 1983). Second, in environments where such commitment is not feasible or credible, equilibria with “sequential skimming” would typically obtain, where the seller would gradually decrease his price quotes trading off higher prices with an increased probability of trade delays (see, e.g., Fudenberg, Levine, and Tirole 1985). Private information and market power would then again impede efficiency, in this case through socially costly delays. Given the presence of inefficient screening in either environment, the central forces of our analysis below would thus carry over even if multiple bargaining rounds were allowed.

### **3 Information Disclosure prior to Trading**

In this section, we analyze the buyer’s decision to share a subset of his information with the seller before trade occurs. If trade is already socially efficient without disclosure, additional disclosures are suboptimal for the buyer — in this case, the seller already quotes the lowest possible price,  $\hat{v}$ , and additional information can only cause him to increase his quote. Thus, for the remainder of the paper we focus on situations where trade would be socially inefficient if the buyer did not disclose any of his private information. Sharing information might hurt the buyer since possessing private information yields information rents, but it might also reduce the seller’s incentives to charge inefficient mark-ups that reduce the expected gains from trade.

For now, we assume that the agent must design his disclosure plan prior to acquiring private information, and that he can commit to not manipulating signals specified by this plan later, as is common in models of information design. Assuming that the buyer is uninformed at the time of the information design increases the tractability of the analysis, as it eliminates the existence of signaling concerns. We will relax this assumption in Section 4. We also restrict our attention to

ex post verifiable disclosures or signals, as in Grossman (1981), Milgrom (1981), and Shin (2003). In practice, the ex ante design of such disclosure plans is likely relevant in economic contexts with hard information. In a variety of industries, information is shared automatically between firms via information technology (IT) systems according to pre-determined algorithms. For example, firms in the same supply chain are typically connected to a common IT system that automatically shares information about inventories and production problems. Similarly, in the context of financial markets, hedge funds systematically share financial data (e.g., holdings and performance data) with broker-dealers and clients, which reduces information asymmetries about trading motives, such as liquidity needs (see also footnote 2). We formally define *ex post verifiability* in the context of our model as follows.

**Definition 1.** *A signal whose realization  $s$  belongs to a set  $S$  is called “ex post verifiable” if it can be represented by a function  $D : [v_L, v_H] \rightarrow S$  such that  $D^{-1}(s) \equiv \{v : D(v) = s\} \in \mathcal{B}([v_L, v_H])$   $\forall s \in S$ , where  $\mathcal{B}([v_L, v_H])$  denotes the Borel algebra on  $[v_L, v_H]$ .*

This definition implies that for any signal realization  $s \in S$ ,  $D^{-1}(s)$  is a Borel set in  $[v_L, v_H]$ . Since a Borel set of  $[v_L, v_H]$  must be characterized by unions of intervals, designing a disclosure plan implies combining partitions to inform the seller about possible realizations of  $v$ . If the buyer sends a signal, the seller must be able to confirm ex post that the true realization of  $v$  was indeed possible given the signal sent. Signals that are subject to additional random shocks (due to “noise” components or randomization) are thus ruled out by ex post verifiability. This restriction is common in the literature on disclosure (see Verrecchia 2001, Milgrom 2008, Beyer, Cohen, Lys, and Walther 2010, for related surveys) and strikes us as natural given the assumption that the “sender” of the information does not manipulate his signal, as is commonly assumed in persuasion games.<sup>10</sup> In the presence of ex post verifiability, erroneous disclosures could be penalized heavily, providing the sender with the incentives to indeed send signals that are truthful, even when manipulation is a

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<sup>10</sup>Ex post verifiability implies that, for different signals that can be sent in equilibrium, posterior beliefs about  $v$  do not have overlapping support. Since Bayesian persuasion only requires that the distribution of posteriors is such that the expected posterior probability equals the prior probability, it allows for randomization and generally does not satisfy the criterion of ex post verifiability (see Kamenica and Gentzkow 2011).

feasible action.<sup>11</sup>

Before proceeding, we summarize the timeline of our baseline model. First, the buyer designs a disclosure plan. Then the buyer observes the realization of  $v$  and the seller receives a signal consistent with the chosen disclosure plan. Finally, the seller quotes a price and the buyer decides whether to pay this price in exchange for the asset.

### 3.1 Disclosure and Efficiency of Trade

We now study how the buyer's choice of a disclosure plan affects the efficiency of trade. We first revisit and extend our parameterized Example 1 to provide some initial intuition for the logic that will underlie our first main result below.

**Example 2.** *As in Example 1, we assume the buyer values the asset at  $v \sim U[1, 2]$  and the seller values it at a constant  $\bar{c} \leq 1$ . We already know from Example 1 that absent disclosure, the seller quotes a price  $p = 1 + \frac{\bar{c}}{2}$  when  $\bar{c} \in (0, 1]$ , which destroys the gains to trade with probability  $\frac{\bar{c}}{2}$ . The buyer then acquires the asset whenever  $v \geq p$  and he collects an expected profit equal to:*

$$\Pr\left(v \geq 1 + \frac{\bar{c}}{2}\right) \left[ \mathbb{E}\left(v | v \geq 1 + \frac{\bar{c}}{2}\right) - \left(1 + \frac{\bar{c}}{2}\right) \right] = \frac{(2 - \bar{c})^2}{8}. \quad (6)$$

*Now consider what happens if the buyer promises to share some of his information with the seller, in particular, by disclosing whether  $v \in [1, 1 + \frac{\bar{c}}{2})$  or  $v \in [1 + \frac{\bar{c}}{2}, 2]$ . The seller's optimization problem when quoting a price to the buyer now depends on the realization of the signal. If the seller learns that  $v \geq 1 + \frac{\bar{c}}{2}$ , his optimization problem becomes:*

$$\max_{p \in [1 + \frac{\bar{c}}{2}, 2]} \Pr\left(v \geq p | v \geq 1 + \frac{\bar{c}}{2}\right) p + \Pr\left(v < p | v \geq 1 + \frac{\bar{c}}{2}\right) \bar{c} = \left(\frac{2 - p}{1 - \frac{\bar{c}}{2}}\right) p + \left(\frac{p - (1 + \frac{\bar{c}}{2})}{1 - \frac{\bar{c}}{2}}\right) \bar{c}, \quad (7)$$

*and if instead he learns that  $v < 1 + \frac{\bar{c}}{2}$ , the problem takes the form:*

$$\max_{p \in [1, 1 + \frac{\bar{c}}{2})} \Pr\left(v \geq p | v < 1 + \frac{\bar{c}}{2}\right) p + \Pr\left(v < p | v < 1 + \frac{\bar{c}}{2}\right) \bar{c} = \left(\frac{1 + \frac{\bar{c}}{2} - p}{\frac{\bar{c}}{2}}\right) p + \left(\frac{p - 1}{\frac{\bar{c}}{2}}\right) \bar{c}. \quad (8)$$

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<sup>11</sup>Due to the absence of noise, penalties would then remain off-equilibrium — penalties would only be triggered if the sender intentionally violated the standards set by his own disclosure plan.

In the first case, it is easy to verify that the seller finds it optimal to quote  $p_h = 1 + \frac{\bar{c}}{2}$ , just as he did without disclosure. However, in the second case, the seller finds it optimal to quote the price  $p_l = \max\{\frac{1}{2} + \frac{3}{4}\bar{c}, 1\}$ .

Under this disclosure plan, the buyer collects an expected profit of:

$$\begin{aligned} & \Pr\left(v \geq 1 + \frac{\bar{c}}{2}\right) \left[\mathbb{E}\left(v \mid v \geq 1 + \frac{\bar{c}}{2}\right) - \left(1 + \frac{\bar{c}}{2}\right)\right] \\ & + \Pr\left(p_l \leq v < 1 + \frac{\bar{c}}{2}\right) \left[\mathbb{E}\left(v \mid p_l \leq v < 1 + \frac{\bar{c}}{2}\right) - p_l\right]. \end{aligned} \quad (9)$$

The first term in equation (9) is equal to the expected profit the buyer would collect absent disclosure. The second term is the additional profit the buyer is able to collect due to the introduced disclosure plan. This additional profit is strictly positive whenever  $\bar{c} > 0$ . Thus, the buyer is strictly better off under this disclosure plan than without any disclosure. Moreover, if  $\bar{c} \leq \frac{2}{3}$  the seller quotes  $p_l = 1$  when  $v < 1 + \frac{\bar{c}}{2}$ , which implies that trade is efficient regardless of the signal realization.

If  $\bar{c} > \frac{2}{3}$  however, the seller quotes  $p_l = \frac{1}{2} + \frac{3}{4}\bar{c}$  when  $v < 1 + \frac{\bar{c}}{2}$ , which leads to a higher efficiency of trade than without disclosure but still causes trade to break down with positive probability. A similar reasoning can then be applied again to construct an alternative disclosure plan that splits the region of inefficient trade  $[1, 1 + \frac{\bar{c}}{2})$  into  $[1, \frac{1}{2} + \frac{3}{4}\bar{c})$  and  $[\frac{1}{2} + \frac{3}{4}\bar{c}, 1 + \frac{\bar{c}}{2})$ , such that the buyer is strictly better off and trade is more efficient than under the first disclosure plan. Note, however, that even though these alternative disclosure plans represent profitable deviations for the buyer, they do not represent the buyer's optimal plan. We will derive properties of optimal disclosure plans below.

The example above shows that, if trade is inefficient without disclosure, it is possible to construct a disclosure plan that improves the social efficiency of trade and makes the buyer strictly better off. Below, we extend this reasoning to establish a stronger result: the buyer will in fact design a disclosure plan that leads to efficient trade in order to maximize his expected profit in equilibrium. In particular, we will show that any disclosure plan that leads to the destruction of trade surplus cannot be part of an equilibrium, as it can be replaced by a more efficient disclosure plan that strictly dominates from the buyer's perspective. In order to show this result, it is useful to introduce Lemma 1, which is proved in Appendix A.

**Lemma 1.** *Suppose that the seller would quote a price  $\tilde{p}$  if the buyer's valuation was drawn from a distribution with CDF  $F(v)$  on  $[v_L, v_H]$ . Then the seller would also quote a price  $\tilde{p}$  if the buyer's valuation was drawn from this distribution truncated from below at  $\tilde{p}$ , i.e.,  $F(v|v \geq \tilde{p})$ .*

Lemma 1 is both simple and powerful. If a seller finds it optimal to quote a price  $\tilde{p}$  under a given distribution of  $v$ , then truncating this distribution from below at  $\tilde{p}$  will not impact his pricing decision. In other words, by eliminating the possibility that  $v < \tilde{p}$ , we do not change the fact that the seller is better off quoting  $\tilde{p}$  than any other  $p \in (\tilde{p}, v_H]$ . As a result, a buyer expecting the seller to quote an inefficient price  $\tilde{p} > \hat{v}$  under a given disclosure plan can design a simple alternative disclosure plan that increases his expected profit while also increasing the social surplus from trade. In particular, if the buyer were to create a new signal that is triggered only if  $v \in [v_L, \tilde{p})$ , the seller would optimally respond to receiving this signal by quoting a price below  $\tilde{p}$ . This response, in turn, would imply that efficient trade occurs with a higher probability, with both the buyer and the seller extracting a fraction of the incremental social surplus. Moreover, since the seller's behavior when  $v \geq \tilde{p}$  is unaffected by this new signal, this alternative disclosure plan strictly improves both the buyer's expected profit and the efficiency of trade.

We now state our first main result, and include the associated proof in the main text to highlight the underlying logic.

**Proposition 1.** *If the buyer can commit to any disclosure plan that sends ex post verifiable signals to the seller, he designs in equilibrium a partial disclosure plan that yields socially efficient trade.*

*Proof.* By contradiction, suppose that the buyer's optimal disclosure plan is represented by  $D(\cdot)$ , which does not implement efficient trade. As in the case without disclosure, it cannot happen in equilibrium that trade occurs for some  $v < \hat{v}$ , where  $c(v) > v$  (see footnote 9). Thus, in equilibrium the seller always quotes prices weakly greater than  $\hat{v}$ , independently of the signal sent by a disclosure plan.

Now suppose that trade is inefficient, given the disclosure plan, because trade fails for some  $v > \hat{v}$  where  $c(v) < v$ . We can show that there exists another disclosure plan that yields a higher

profit for the buyer. Going forward, we denote by  $p_s$  the price the seller would quote after receiving a signal  $s \in S$  generated by a disclosure function  $D(\cdot)$ . More generally, we will use the subscript  $s$  on a given function to indicate the conditional version of that function after receiving a signal  $s$ , e.g.,  $F_s(v) \equiv F(v|s)$ . There exists an  $s_0 \in S$ , such that upon receiving signal  $s_0$ , the seller quotes a price  $p_{s_0} > \hat{v}$  and  $p_{s_0} > \inf\{v \in [v_L, v_H] : D(v) = s_0\}$ . (Since singletons have zero measure, we assume, without loss of generality, that  $D^{-1}(s)$  does not admit any singletons.) A buyer whose valuation belongs to  $\{v : D(v) = s_0\} \cap (\hat{v}, p_{s_0})$  would refuse to pay the seller's quoted price  $p_{s_0}$ , leading to inefficient trade. Consider the following alternative disclosure plan where  $S' = S \cup \{s'\}$  for some  $s' \notin S$  and

$$\tilde{D}(v) \equiv \begin{cases} D(v) & \text{if } D(v) \neq s_0 \\ s_0 & \text{else if } D(v) = s_0, v \geq p_{s_0} \\ s' & \text{otherwise.} \end{cases} \quad (10)$$

By definition, the disclosure plan  $\tilde{D}(\cdot)$  also satisfies ex post verifiability.

We now show that  $\tilde{D}(\cdot)$  would give the buyer a strictly higher ex ante expected profit. First, note that when  $s \neq s_0$  and  $s \neq s'$ , trading behavior is unaltered and the seller still quotes a price  $p_s$ . Second, Lemma 1 guarantees that the seller still quotes  $p_{s_0}$  under the alternative disclosure plan  $\tilde{D}(\cdot)$  when he receives a signal  $s_0$ . Finally, suppose the seller quotes a price  $z$  when he receives a signal  $s'$ . Since quoting  $p_{s_0}$  yields zero profit in this case, it must be that  $z \in [\inf D^{-1}(s_0), p_{s_0})$ . As a result, the buyer's ex ante expected profit under the alternative disclosure plan  $\tilde{D}(\cdot)$  is given by:

$$\sum_{s \in S} \underbrace{\int_{D^{-1}(s) \cap [p_s, v_H]} (v - p_s) dF(v)}_{\text{Profit from } s \in S} + \underbrace{\int_{D^{-1}(s_0) \cap [z, p_{s_0})} (v - z) dF(v)}_{\text{Profit from } s'} \quad (11)$$

whereas the profit under the disclosure plan  $D(\cdot)$  is equal to only the first term. Since  $p_{s_0} > z$ , the second term is strictly positive and the buyer earns a strictly higher profit under the disclosure plan  $\tilde{D}(\cdot)$  than he does under the plan  $D(\cdot)$ , thereby contradicting the optimality of  $D(\cdot)$ . We have thus shown that in equilibrium the buyer's optimal disclosure plan must result in socially efficient trade.

We can also show that the optimal disclosure plan must reveal the buyer's information only partially. Otherwise, the seller quotes the buyer a price  $p = v$  for all realizations of  $v$  and the buyer obtains no surplus. A full disclosure plan is therefore weakly dominated by a no-disclosure plan that leads to inefficient trade, which is then strictly dominated by a partial disclosure plan that leads to efficient trade, consistent with the arguments above.  $\square$

Proposition 1 states two key characteristics of an optimal disclosure plan. First, the privately informed buyer's incentives to disclose verifiable information are aligned with social surplus maximization. By sharing a subset of his information with the seller, the buyer ensures that he will be quoted prices that avoid inefficient rationing, thereby yielding incremental social surplus.<sup>12</sup> A key insight is that, even though the buyer does not have any bargaining power, he can always ensure that he obtains a fraction of this incremental surplus (in the form of an information rent). As a result, as long as a given disclosure plan does not lead to socially efficient trade, the buyer can always construct an alternative plan that strictly improves his expected payoff while also increasing social surplus.

Second, the proposition reveals that it is never optimal for the buyer to share all his information with the seller, as such a disclosure plan would drive the buyer's rents to zero. Unlike in Grossman (1981) where full disclosure is optimal and unraveling obtains, the informed trader in our model does not have market power and can only extract rents if he conceals some information from his counterparty. Our results thus highlight that the extent to which market power and private information are *separated* (that is, allocated to different agents) will greatly affect whether information revelation is perfect (as in Grossman's case) or partial (as in our case). As we show in the next subsection, such partial revelation will imply "coarseness" in disclosures, a feature that is commonly observed in financial and goods markets.

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<sup>12</sup>The seller always weakly benefits from verifiable disclosures, since he can disregard the information provided and quote the same price as he would absent disclosures.



## 3.2 Optimal Disclosure Plans

In the general environment we have considered so far, solving for the optimal disclosure function  $D(\cdot)$  involves functional optimization, which is equivalent to choosing an infinitely dimensional vector of choice variables indexed by  $v \in [v_L, v_H]$ . Moreover, this problem is generally non-convex. We have already shown that the buyer's optimal disclosure plan in this general environment must satisfy two properties in equilibrium: (i) disclosure is partial, and (ii) leads to efficient trade. In this subsection, we go several steps further in characterizing optimal disclosure plans. First, we recast the buyer's information design problem in a simplified form by relying on Proposition 1, derive additional properties of optimal disclosure plans, and provide intuition for the tradeoffs the buyer faces in our general environment. Second, we provide a full characterization of optimal disclosure plans when disclosure plan functions  $D(v)$  are restricted to be *monotone*, an assumption that is related to a commonly imposed restriction in the security design literature. Finally, we show how optimal disclosure plans can be determined under *discrete* distributions  $F(v)$ , without imposing monotonicity.

### 3.2.1 General Properties of an Optimal Disclosure Plan

We now recast the buyer's information design problem in a simplified form and discuss the tradeoffs the buyer faces. We can write the buyer's expected payoff from a disclosure plan  $D(\cdot)$  as follows:

$$W(D) = \sum_{s \in S} \int_{\{v \in D^{-1}(s) : v \geq p_s\}} (v - p_s) dF(v). \quad (12)$$

The proof of Proposition 1 implies that an optimal disclosure plan must yield efficient trade, that is, trade occurs when  $v > \hat{v}$ , and does not occur when  $v < \hat{v}$ . As a result, *maximizing* the buyer's expected payoff is equivalent to finding a disclosure function  $D(v)$  that *minimizes* the expected price paid for all  $v \geq \hat{v}$ :

$$\sum_{\{s \in S : \inf[D^{-1}(s)] \geq \hat{v}\}} \inf[D^{-1}(s)] \int_{D^{-1}(s)} dF(v), \quad (13)$$

subject to the efficiency constraints that conditional on  $v > \hat{v}$ , the prices the seller optimally quotes are all accepted by the buyer:

$$p_s \leq \inf[D^{-1}(s)] \quad \forall s \in S : \inf[D^{-1}(s)] \geq \hat{v}, \quad (14)$$

where we assume, without loss of generality, that the disclosure function  $D(v)$  separates all realizations of  $v$  based on whether trade is strictly beneficial or not, that is,<sup>13</sup>

$$\{v \in D^{-1}(s) : v \leq c(v)\} \cap \{v \in D^{-1}(s) : v > c(v)\} = \emptyset \quad \forall s \in S. \quad (15)$$

From the set of efficient disclosure plans — which satisfy the efficiency constraints (14) — the buyer optimally chooses a plan that pools realizations of  $v$  so as to minimize the expected price he pays. Ideally, the buyer would like to pool all realizations where  $v \geq \hat{v}$  in one signal, if this did not violate the efficiency constraint (14), as doing so would imply that the expected price he pays is just  $\hat{v}$ . Yet, whenever condition (4) is violated, the efficiency constraint (14) would also be violated under such a plan, and would lead to inefficient rationing. As a result, the buyer's optimal disclosure plan has to provide some separation of realizations where  $v \geq \hat{v}$ , enough to ensure that the seller is not tempted to resort to inefficient screening.

Disclosure plans that provide more separation lower the seller's incentives to screen in the following sense. Consider the decision of a buyer to pool or separate two generic intervals in a disclosure plan  $D(\cdot)$ . Let  $A \equiv [a_L, a_H)$  and  $B \equiv [b_L, b_H)$  denote these two intervals, where  $b_L \geq a_H$  and  $a_L \geq \hat{v}$ . When a pooling signal is generated by the disclosure plan, the necessary condition for efficient trade to occur in equilibrium is:

$$a_L - c(a_L) \geq \frac{\Pr(v \in A \cup B)}{f(a_L)}. \quad (16)$$

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<sup>13</sup>Under Proposition 1, it is still possible to have a signal  $s$  where  $\hat{v} \in (\inf\{D^{-1}(s)\}, \sup\{D^{-1}(s)\})$  as long as  $p_s = \hat{v}$ . However, it was shown as part of the proof of Proposition 1 that the seller never finds it optimal to quote a price below  $\hat{v}$ , regardless of the buyer's disclosure. Hence, a disclosure plan that includes this particular signal  $s$  would yield the same trading outcomes as an alternative disclosure plan where the signal  $s$  is split into two new signals, based on whether  $v \leq \hat{v}$  and  $v > \hat{v}$ . To simplify the exposition of our results, we assume that whenever relevant, it is this refined, yet equivalent, disclosure plan that is chosen by the buyer.

This condition is strictly more restrictive than the corresponding condition when  $v \in A$ , and a separating signal is generated:

$$a_L - c(a_L) \geq \frac{\Pr(v \in A)}{f(a_L)}. \quad (17)$$

As a result, the set of the functions  $F(\cdot)$  and  $c(\cdot)$  for which the seller inefficiently screens the buyer is strictly larger when a disclosure plan  $D(\cdot)$  pools the regions  $A$  and  $B$ . Yet, if the functions  $F(\cdot)$  and  $c(\cdot)$  are such that the seller does not resort to inefficient screening after receiving the pooling signal, then the buyer strictly prefers a disclosure plan that sends this pooling signal, as it implies that he pays a strictly lower expected price.

Related to this intuition, we will derive additional properties of optimal disclosure plans that rely on the following definition.

**Definition 2.** *The constraint that trade has to be efficient conditional on a signal  $s \in S$  generated by a disclosure function  $D(\cdot)$  is said to be “binding” if*

$$\Pi'_s(\inf\{D^{-1}(s)\}) = 0, \quad (18)$$

or if there exists a price  $p' \in D^{-1}(s)$  such that  $p' \neq \inf\{D^{-1}(s)\}$ , and

$$\Pi_s(\inf\{D^{-1}(s)\}) = \Pi_s(p'), \quad (19)$$

where  $\Pi_s(p)$  denotes the seller's expected payoff from quoting a price  $p$  conditional on receiving a signal  $s$ .

Recall that  $p_s$  denotes the price the seller quotes to maximize his conditional expected payoff  $\Pi_s$ . Suppose an optimal disclosure plan involves  $n$  signal realizations  $s \in \{1, \dots, n\}$  for which trade creates a surplus (i.e.,  $v > c(v)$ ), and the corresponding quoted prices are denoted as  $p_1, \dots, p_n$ . Without loss of generality, assume  $p_1 < p_2 < \dots < p_n$ . We will refer to  $s \in \{1, \dots, (n-1)\}$  as the  $(n-1)$  lowest signal realizations, since the associated quoted prices  $\{p_1, \dots, p_{n-1}\}$  are lower than

that associated with  $s = n$ . We show in Appendix A that a buyer's optimal disclosure plan satisfies the following property.

**Proposition 2.** *Under an optimal disclosure plan with  $n$  possible signal realizations for which  $v > c(v)$ , the efficiency constraints associated with the  $(n - 1)$  lowest signal realizations are binding.*

Proposition 2 shows that under an optimal disclosure plan, the seller is indifferent between quoting an efficient price and at least one other higher, inefficient price after receiving the  $(n - 1)$  lowest signals for which  $v > c(v)$ . Starting from an optimal disclosure plan, any attempts to reassign a positive measure of realizations of  $v$  that pay the highest price  $p_n$  under this plan to any of the lower signals would violate the efficiency constraint associated with that lower signal (see condition (14)). Otherwise, the buyer could construct an alternative disclosure plan that lowers the expected price he pays for the asset.

To see this, consider a simple situation where trade would always create a surplus, and where the buyer designs a disclosure plan that sends one of two signals: a “low” signal (associated with a low price) and a “high” signal (associated with a high price). If the seller were to *strictly* prefer quoting the low price over quoting any other possible price (including a marginally higher price) after receiving the low signal, the buyer could improve his disclosure plan as follows. He could move a small measure of the highest realizations of  $v$  (those close to  $v_H$ ) that were initially assigned to the high signal, and reassign them to the low signal. By doing so, the buyer would increase the probability of the low signal being sent, without impacting the prices the seller quotes under either signal. This deviation would decrease the average price the buyer pays for the asset, making him strictly better off. Such a profitable deviation remains possible until the efficiency constraint of a buyer's optimal disclosure plan is binding for each of the  $(n - 1)$  lowest signal(s).

### 3.2.2 Monotone Disclosure Functions

In this subsection we characterize the buyer's optimal disclosure plan when disclosure functions  $D(\cdot)$  are restricted to be monotone. Monotonicity assumptions are commonly imposed in general

environments like ours. For example, they are a common restriction in the security design literature,<sup>14</sup> and some of the arguments used in that literature to justify monotonicity also carry over to the context of our model.<sup>15</sup> Formally, we make the following assumption in this subsection:

**Assumption 2.** *Disclosure plans  $D(v)$  are restricted to be monotone in  $v$ .*

Assumption 2 maintains the property that the function  $D(\cdot)$  is an infinite-dimensional object and that solving for the optimal plan is a non-convex problem. Nonetheless, it yields additional tractability, allowing us to characterize optimal disclosure plans under fairly mild technical conditions further detailed below. Given Assumption 2, an optimal disclosure plan  $D(v)$  can be represented in the following form:<sup>16</sup>

$$D(v) = \begin{cases} 0, & \text{for } v \in [v_L, \hat{v}], \text{ if } v_L \leq c(v_L) \\ 1, & \text{for } v \in (\hat{v}, v_2), \text{ if } v_L \leq c(v_L), \text{ and for } v \in [\hat{v}, v_2] \text{ if } v_L > c(v_L) \\ 2, & \text{for } v \in [v_2, v_3), \\ \dots & \\ n, & \text{for } v \in [v_n, v_H]. \end{cases} \quad (20)$$

where  $(n - 1)$  partition cutoffs  $v_2, \dots, v_n$ , with  $v_1 \equiv \hat{v} < v_2 < \dots < v_n < v_{n+1} \equiv v_H$ , separate realizations where  $v > \hat{v}$  into  $n$  subsets. Moreover, this function satisfies condition (15) by generating a separate signal  $s = 0$  for all realizations of  $v$  for which trade is not strictly beneficial, that is, for all  $v \in [v_L, v_H] : v \leq c(v)$ .

<sup>14</sup>See, e.g., Innes (1990), Nachman and Noe (1994), and DeMarzo and Duffie (1999).

<sup>15</sup>For example, if disclosure functions  $D(\cdot)$  were not monotone, then the buyer could benefit by contributing additional funds to the asset before  $v$  is verified. Analogously to the argument in footnote 28 of DeMarzo and Duffie (1999), suppose that there are two buyer types  $v'$  and  $v''$  with  $v' < v''$ , where under the equilibrium disclosure plan  $v'$  is associated with a “higher signal”  $s''$ , and  $v''$  with a “lower signal”  $s'$ , in the sense that the buyer is charged a higher price conditional on the signal  $s''$ , that is  $p_{s''} > p_{s'}$ . Given a realization  $v'$ , suppose the buyer can inject  $(v'' - v')$  into the asset before the final payoff  $v$  is verified. Then doing so would allow the buyer to pay the lower price  $p_{s'}$ , while still collecting  $v'$  and his own contribution  $(v'' - v')$ . Thus, whenever there exists a  $\tilde{v} > v$  that is associated with a price  $p_{D(\tilde{v})}$  that is lower than  $p_{D(v)}$ , the buyer would have an incentive to inject  $(\tilde{v} - v)$  into the asset and pay the lower price. If such contributions cannot be prevented, then only monotone disclosure functions are observed in equilibrium, and the monotonicity assumption is without loss of generality.

<sup>16</sup>Note that the function  $D(v)$  that achieves the optimum is clearly not unique. For instance, instead of having signals  $S = \{1, 2, 3, \dots, n\}$ ,  $D(v)$  could produce the signals  $\{2, 4, 6, \dots, 2n\}$ .

To further increase analytical tractability, we impose the following assumption that is closely related to the standard assumption in auction theory that bidders virtual valuation functions are strictly increasing (e.g., Milgrom 1981).

**Assumption 3.** *The functions  $h(v) \equiv \frac{f(v)}{1-F(v)}$  and  $H(v) \equiv \frac{f(v)(v-c(v))}{1-F(v)}$  are strictly increasing in  $v$  for  $v \in [v_L, v_H]$ .*

If Assumption 3 is satisfied under some distribution  $F(\cdot)$ , then it is also satisfied under any truncated version of that distribution.<sup>17</sup> As a result, Assumption 3 guarantees that the seller's marginal profit function conditional on each signal, which we denote by  $\Pi'_s(\cdot)$ , crosses zero from above at most in one point. The seller then quotes an efficient price  $p_s = v_s$  for any  $v_s > c(v_s)$  whenever the following condition is satisfied:

$$H_s(v_s) \equiv \frac{f(v_s)(v_s - c(v_s))}{F(v_{s+1}) - F(v_s)} \geq 1. \quad (21)$$

The proof of Proposition 1 implies that even when disclosure plans are restricted to be monotone in  $v$ , an *optimal* plan from the buyer's perspective has to ensure efficient trade. Thus, the buyer's problem of finding an optimal monotone  $D(\cdot)$  function can be recast as the problem of finding partition cutoffs  $(v_2, \dots, v_n)$  that minimize the expected price the buyer pays for all  $v > c(v)$ , which is given by:

$$\sum_{s=1}^n (F(v_{s+1}) - F(v_s)) v_s, \quad (22)$$

subject to the efficiency constraints:

$$\begin{aligned} H_s(v_s) &\geq 1 \quad \forall s \in \{1, \dots, n\} && \text{if } v_L > c(v_L), \\ \lim_{v \downarrow v_1} H_1(v) &\geq 1 \text{ and } H_s(v_s) \geq 1 \quad \forall s \in \{2, \dots, n\} && \text{if } v_L \leq c(v_L). \end{aligned} \quad (23)$$

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<sup>17</sup>See Lemma 1 in Glode and Opp (2016).

It is useful to introduce the following two functions that are related to these efficiency constraints. We define the *left efficiency bound* function  $lb : (\hat{v}, v_H] \rightarrow \mathbb{R}$  and the *right efficiency bound* function  $rb : [\hat{v}, v_H) \rightarrow \mathbb{R}$  as follows:

$$lb(v) \equiv \begin{cases} x \in [\hat{v}, v) : \frac{f(x)(x-c(x))}{F(v)-F(x)} = 1, & \text{for } \frac{f(\hat{v})(\hat{v}-c(\hat{v}))}{F(v)-F(\hat{v})} < 1, \\ \hat{v}, & \text{for } \frac{f(\hat{v})(\hat{v}-c(\hat{v}))}{F(v)-F(\hat{v})} \geq 1, \end{cases} \quad (24)$$

$$rb(v) \equiv \begin{cases} x \in [v, v_H) : \frac{f(x)(x-c(x))}{F(v)-F(x)} = 1, & \text{for } \frac{f(v)(v-c(v))}{F(v_H)-F(v)} < 1, \\ v_H, & \text{for } \frac{f(v)(v-c(v))}{F(v_H)-F(v)} \geq 1. \end{cases} \quad (25)$$

Given the above definition of  $lb(\cdot)$ , the efficiency constraints (23) imply the following inequality constraints:

$$\begin{aligned} v_s &\geq lb(v_{s+1}) & \forall s \in \{1, 2, \dots, n\} & \text{ if } v_L > c(v_L), \\ v_s &\geq lb(v_{s+1}) & \forall s \in \{2, 3, \dots, n\} & \text{ if } v_L \leq c(v_L). \end{aligned} \quad (26)$$

Moreover, the functions  $lb(\cdot)$  and  $rb(\cdot)$  allow us to derive the following useful property of optimal partition cutoffs (its proof can be found in Appendix A).

**Lemma 2.** *Given any two partition cutoffs  $v_{s-1}$  and  $v_{s+1}$  with  $s \geq 2$  that satisfy the efficiency constraint  $v_{s-1} \geq lb(lb(v_{s+1}))$  implied by (26), the interior cutoff  $v_s$  that maximizes the buyer's expected payoff is either equal to  $rb(v_{s-1})$  or equal to  $lb(v_{s+1})$ .*

Lemma 2 dramatically reduces the set of potentially optimal partition cutoffs: given any two neighboring partition cutoffs  $v_{s-1}$  and  $v_{s+1}$  that do not immediately violate the efficiency constraints (26), the optimal interior cutoff  $v_s$  takes one of two possible candidate values:  $rb(v_{s-1})$  or  $lb(v_{s+1})$ . We now derive a condition that ensures that one of these two candidates consistently dominates on the relevant part of the domain, that is, for  $v > c(v)$ . To do so, we specify the function

$\Phi : \Omega \rightarrow \mathbb{R}$  as:

$$\Phi(u, w) \equiv u \cdot [F(rb(u)) - F(lb(w))] + rb(u)[F(w) - F(rb(u))] - lb(w)[F(w) - F(lb(w))], \quad (27)$$

where the domain is defined as follows:

$$\Omega \equiv \begin{cases} \{(u, w) : u \in (lb(lb(w)), lb(w)] \text{ and } w \in (rb(v_L), v_H]\}, & \text{for } c(v_L) \leq v_L, \\ \{(u, w) : u \in (lb(lb(w)), lb(w)] \text{ and } w \in (\hat{v}, v_H]\}, & \text{for } c(v_L) > v_L. \end{cases} \quad (28)$$

The  $\Phi$ -function represents the buyer's expected net benefit of choosing, given some generic neighboring partition cutoffs  $u$  and  $w$ , an interior cutoff  $\tilde{v}$  that is equal to  $lb(w)$  rather than equal to  $rb(u)$ . The  $\Phi$ -function encodes non-local properties of the functions  $F(\cdot)$  and  $c(\cdot)$  affecting the optimal disclosure plan. In particular, we can show that even if Assumption 3 is satisfied, the sign of  $\Phi$  is not uniquely determined on its domain (see Appendix B). Yet, under commonly used specifications involving distributions  $F(\cdot)$  such as the uniform distribution, and (truncated) parameterizations of the Normal distribution and the gamma distribution, and  $c(\cdot)$ -functions such as  $c(v) = \bar{c}$ ,  $c(v) = v - \Delta$ , and  $c(v) = \beta v$  (with  $\beta < 1$ ), the  $\Phi$ -function consistently takes weakly positive values. In cases like this, the characterization of an optimal disclosure plan becomes highly tractable. In light of this fact, we specify the following technical condition.

**Assumption 4.** *The functions  $F(\cdot)$  and  $c(\cdot)$  imply that  $\inf\{\Phi(u, w) : (u, w) \in \Omega\} \geq 0$ .*

We can now proceed to fully characterizing optimal disclosure plans under Assumptions 1-4 (see proof in Appendix A).

**Proposition 3.** *The partition cutoffs of an optimal disclosure plan function (20) are given by a descending sequence starting with  $v_{n+1} = v_H$  and where:*

$$v_s = lb(v_{s+1}), \quad \text{for } s = n, (n-1), \dots, 3, 2. \quad (29)$$



When  $v_L > c(v_L)$ , this sequence  $\{v_n, v_{n-1}, v_{n-2}, \dots, v_2\}$  is finite. When  $v_L \leq c(v_L)$ , this sequence is infinite (that is,  $n \rightarrow \infty$ ) and converges to  $\hat{v}$ .

Recall that the  $\Phi$ -function quantifies the net benefit of setting the partition cutoff  $v_s$  equal to  $lb(v_{s+1})$  rather than equal to  $rb(v_{s-1})$ , taking as given generic neighboring partition cutoffs  $v_{s-1}$  and  $v_{s+1}$ . Naturally, if this net benefit is consistently positive then the buyer consistently optimally chooses  $v_s = lb(v_{s+1})$ , giving rise to the optimal sequence described in Proposition 3. While under many standard specifications of the functions  $F(\cdot)$  and  $c(\cdot)$  the sign of the  $\Phi$ -function is positive everywhere on its domain  $\Omega$ , we also provide in Appendix B an example of functions  $F(\cdot)$  and  $c(\cdot)$  satisfying Assumption 3 where the sign is consistently negative, thereby violating Assumption 4. In Appendix B, we also characterize the optimal disclosure plan in such environments. To streamline the exposition, we focus here on the case where Assumption 4 is satisfied, which is also the relevant case under the standard parameterizations we consider below.

Consistent with Definition 2, an efficiency constraint (23) of a monotone disclosure function  $D(\cdot)$  is binding for a signal  $s$  if it holds with equality.<sup>18</sup> Note that the earlier result that the  $(n-1)$  lowest efficiency constraints must be binding in equilibrium (Proposition 2) does not hold once we restrict disclosure functions to be monotone – the proof of Proposition 2 relies on a deviation that is not feasible when disclosure plans are restricted to be monotone. In fact, it is straightforward to see that the optimal plan characterized in Proposition 3 implies that the efficiency constraints with the  $(n-1)$  highest signal realizations are binding, but generally not the lowest efficiency constraint. For example, when  $v_L > c(v_L)$ , the optimal plan in Proposition 3 will generically imply that conditional on the signal  $s = 1$  being sent, the efficiency constraint is non-binding, that is,  $H_1(v_1) > 1$ . Yet, as highlighted above, monotonicity may be viewed as a plausible restriction for disclosure plans (see in particular footnote 15).

Equipped with these results, we now turn to several examples that illustrate how the fundamentals of our environment, as described by the functions  $F(\cdot)$  and  $c(\cdot)$ , affect the buyer's optimal disclosure plan  $D(\cdot)$ . In all these examples Assumptions 3 and 4 are satisfied, implying that an

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<sup>18</sup>Note that  $H_s(v_s) = 1$  is equivalent to  $\Pi'_s(v_s) = 0$ .

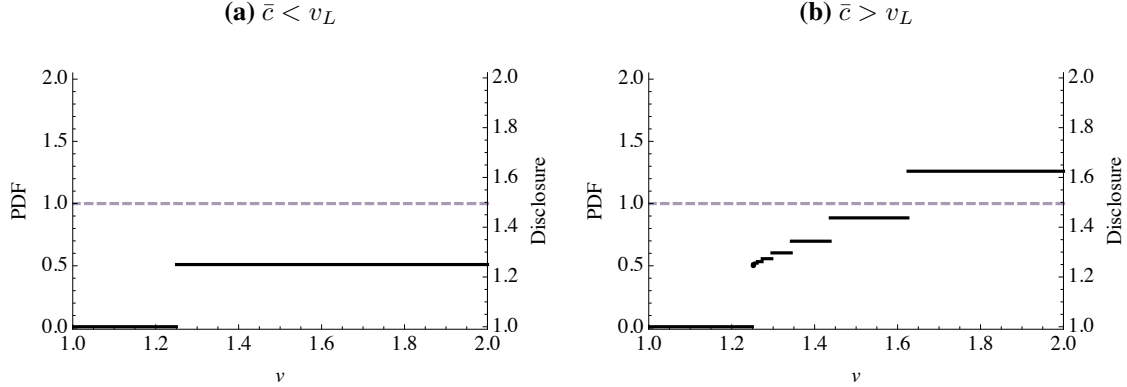
optimal disclosure plan’s partition cutoffs are characterized by the descending sequence defined in Proposition 3. Thus, in all these disclosure plans, the highest partition cutoff  $v_n$  is set equal to  $lb(v_H)$ , which also coincides with the price the seller would quote absent disclosures. If additional separating information is needed to avoid inefficient screening by the seller, the buyer specifies another partition cutoff  $v_{n-1} = lb(v_n) < v_n$ , which coincides with the price the seller would charge conditional on knowing that the CDF of  $v$  is  $F(v|v < v_n)$ . These steps are repeated until the disclosure plan yields full efficiency.

**Varying the seller’s valuation.** First, we return to the environment considered in Examples 1 and 2, where  $v \sim U[1, 2]$  and where the seller values the asset at a constant  $\bar{c}$ . The panels in Figure I illustrate two separate cases. In Panel (a), we set  $\bar{c} = 0.5 < 1$ , implying that trade should occur for all  $v \in [1, 2]$  in order to be efficient. In Panel (b), we set  $\bar{c} = 1.25 > 1$ , implying that trade should occur only for  $v \geq 1.25$ . In the first case, illustrated by Panel (a), the buyer finds it optimal to disclose relatively little information, as graphically represented by a largely “flat” disclosure function that pools large regions of  $v$ . He releases only two signals that split the domain of  $v$  into two subintervals that are associated with the prices  $p_1 = v_L = 1$  and  $p_2 = lb(v_H) = 1.25$ . Doing so suffices to ensure efficient trade. In contrast, absent disclosure, gains to trade would be destroyed with probability 0.25. Both agents benefit from the buyer’s optimal disclosure plan, and disclosure increases the surplus from trade by 19%, relative to the case without disclosure.<sup>19</sup>

In the second case, illustrated in Panel (b), the optimal disclosure plan consists of an infinite number of signals. As  $v$  approaches  $\hat{v} = \bar{c} = 1.25$  from above, the pooling regions become smaller and smaller. Now the buyer optimally discloses relatively more information, as shown by a disclosure function that is more sensitive to the underlying value of  $v$ . The higher reservation value of the seller naturally increases the prices that the buyer has to pay, and increasing incentives

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<sup>19</sup>As discussed above, the monotonicity restriction imposed in this subsection affects the design of the optimal plan, which is evident from the fact that the lowest efficiency constraint is non-binding, contrary to the result in Proposition 2. Absent the requirement that disclosure plans be monotone, the buyer can design a non-monotone plan that yields him a higher expected payoff. For example, a plan informed by the optimal solution to the case of the discrete uniform distribution analyzed in Figure V of Subsection 3.2.3 below yields a higher surplus. Specifically, choosing a disclosure function that generates two signals  $s \in \{1, 2\}$ , with  $D(v) = 1$  for all  $v \in \{[1, 1.2) \cup [1.5, 1.7)\}$  and  $D(v) = 2$  for all  $v \in \{[1.2, 1.5) \cup [1.7, 2]\}$  increases the buyer’s expected surplus by 22%, relative to the optimal monotone disclosure plan shown in Figure Ia.



**FIGURE I**

**Changing the seller's value  $c$ .** The graphs illustrate the PDF and the optimal disclosure plan for  $v \sim U[1, 2]$ , where  $c(v) = \bar{c} = 0.5$  (Panel (a)) and  $c(v) = \bar{c} = 1.25$  (Panel (b)). The vertical axis on the left of each graph corresponds to the dashed line that plots the PDF  $f(v)$ , and the vertical axis on the right corresponds to the solid line that plots the partition cutoff  $v_s$  (with  $s = 1, \dots, n$ ) of the optimal disclosure function for all  $v \in [v_s, v_{s+1})$ . Moreover, in Panel (b), the graph plots  $v_L$  for  $v \in [v_L, \hat{v}]$ , where the generated signal is  $s = 0$ .

to resort to inefficient screening imply that the buyer has to provide more and more separating information as  $v$  approaches  $\bar{c} = 1.25$  from above. Disclosure leads again to efficient trade and benefits both agents, increasing the total surplus from trade by 33%, relative to the scenario without disclosure.

The reasons why optimal disclosure plans often provide varying degrees of precision in different parts of the support of  $v$  are intimately linked to the seller's screening incentives. Recall that the seller, after receiving a signal  $s$ , has a strict incentive to marginally increase the price relative to the efficient price  $p = v_s$  unless:

$$f(v_s)(v_s - c(v_s)) \geq F(v_{s+1}) - F(v_s). \quad (30)$$

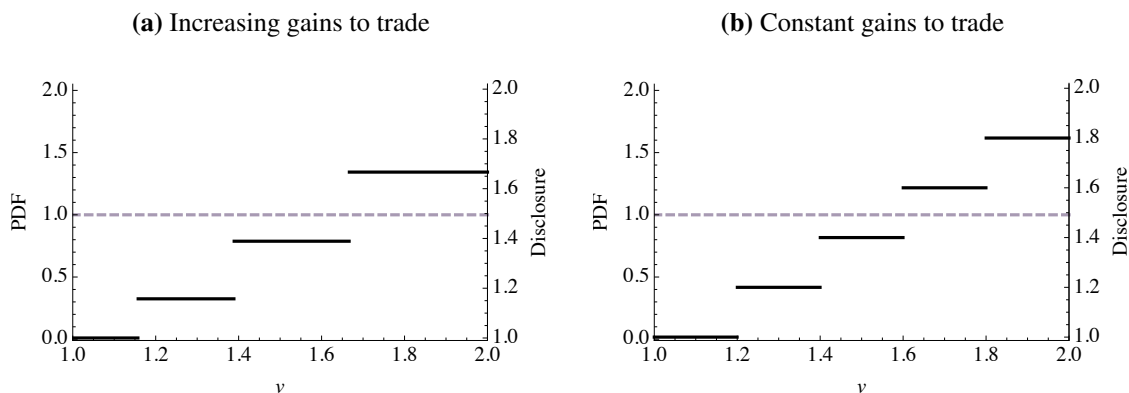
To ensure efficient trade, the partitions of a disclosure plan thus have to be *finer* whenever  $v_s$  must be chosen to be closer to a given  $v_{s+1}$  in order to ensure that condition (30) holds. Thus, the left-hand side of condition (30) indicates two central determinants of the precision of a disclosure plan in a given area of the support of  $v$ : the magnitude of the marginal type's gains to trade  $[v - c(v)]$

and the density  $f(v)$ . In the considered case where  $c(v)$  is a constant, the gains to trade  $(v - \bar{c})$  are increasing in  $v$ , necessitating finer partitions for realizations of  $v$  that are lower (and thus closer to  $\bar{c}$ ) while still generating positive gains to trade ( $v - \bar{c} > 0$ ). For this reason, the disclosure plan in Panel (b) of Figure I becomes infinitely precise as  $v$  approaches  $\bar{c}$  from above. In the uniform case, the density does not vary on the support, implying that any variation in disclosure precision must come from the magnitude of the gains to trade. We will revisit the role played by the density function in later examples.

In Figure II we introduce common value uncertainty, that is, the seller's valuation becomes dependent on  $v$ . In Panel (a), the specification maintains the property that gains to trade are increasing in  $v$ , implying that partitions are finer for lower values of  $v$ . In contrast, in Panel (b) we consider constant gains to trade  $c(v) = v - 0.2$ , where the partitions of the optimal disclosure plan are of equal size. Interestingly, in this case, the  $\Phi$ -function takes the value zero everywhere on the domain  $\Omega$ , indicating that it is irrelevant whether the optimal plan is constructed by the descending sequence stated in Proposition 3, or by an ascending sequence where  $v_s = rb(v_{s-1})$ . This result obtains due to the knife-edge case that both the gains to trade  $[v - c(v)]$  and the density  $f(v)$  are constant. As discussed in Appendix B, when the gains to trade are *decreasing* and the  $\Phi$ -function takes consistently negative values, the optimal plan is constructed by an ascending sequence of partition cutoffs. In this case, more separating information is provided for *higher* realizations of  $v$ . Yet, the economics underlying the optimal disclosure plan still follow the same principles — the buyer's optimal plan pools as many realizations of  $v$  as possible subject to the efficiency constraints (26). Since generically not all constraints can be binding (when constructing an ascending or descending sequence either the top constraint or the bottom constraint will not bind), the buyer chooses the  $(n - 1)$  lowest constraints to be binding when the gains to trade are higher for low realizations of  $v$ .

Taken together, these results highlight that more precise disclosures naturally occur in parts of the domain of  $v$  where the gains to trade are small but positive. Thus, when gains to trade are increasing in the fundamental  $v$ , disclosures will tend to be more precise in the left tail of the distribution. Relatedly, disclosure plans vary based on the presence of private and common value uncertainty. In the private value case where  $c(v) = \bar{c}$  for all  $v$  (see Figures Ia, Ib, and IIa),

disclosures become more precise for lower values of  $v$ , as long as gains to trade are positive. In the pure common value case (see Figure IIB), constant gains to trade generate no variation in screening incentives, and thus no need to vary precision on the support of  $v$ . While the examples thus far have abstracted from any variation in the PDF of the buyer's valuation (in all examples  $v$  followed a uniform distribution), we now turn to examples exploring this channel.



**FIGURE II**

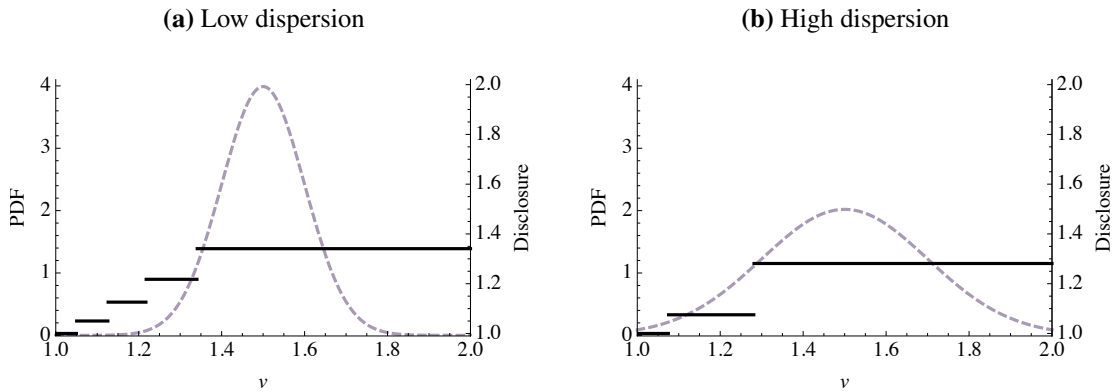
**Constant and proportional gains to trade.** The graphs plot the PDF and the optimal disclosure plan for  $v \sim U[1, 2]$  when gains to trade are constant,  $c(v) = v - 0.20$  (Panel (a)), and when gains to trade are increasing,  $c(v) = 0.8 \cdot v$  (Panel (b)). The vertical axis on the left of each graph corresponds to the dashed line that plots the PDF  $f(v)$ , and the vertical axis on the right corresponds to the solid line that plots the partition cutoff  $v_s$  (with  $s = 1, \dots, n$ ) of the optimal disclosure function for  $v \in [v_s, v_{s+1})$ .

**Varying the distribution of the buyer's valuation.** As highlighted above, our results in Proposition 3 apply to a variety of standard specifications for the distribution of  $v$ . In Figures III and IV we consider several examples where  $v$  follows a truncated normal distribution. For simplicity, these examples assume that  $c(v) = \bar{c} < v_L$ , implying that the optimal disclosure plans create a finite number of partitions.

First, in the two panels of Figure III, we consider normal distributions that are centered on the support  $[v_L, v_H]$ . The two panels vary the dispersion of the distribution. In Panel (a) dispersion is lower, implying that the density  $f(v)$  takes lower values in the tails of the distribution. Following our earlier discussion in relation to condition (30), two channels now increase the incentives for the

seller to screen the buyer in the left tail of the distribution. First, the gains to trade  $(v - \bar{c})$  are smaller for lower realizations of  $v$ . Second, the density  $f(\cdot)$  takes lower values in the tails, again reducing the left-hand side of the inequality (30) for low realizations of  $v$ . As a result of these two effects, the optimal disclosure plan provides relatively more information for low realizations of  $v$ .

In contrast, in Panel (b) the dispersion of the  $v$ -distribution is higher, implying that the density  $f(\cdot)$  takes larger values in the tails of the distribution. Thus, the seller's incentives to screen are lower, allowing the buyer to design an optimal disclosure plan that reveals less information. The high dispersion case is more profitable for the buyer, as it implies that the seller has less precise information about  $v$  ex ante. As a result, the buyer can extract larger information rents in equilibrium — the buyer's surplus is 21% higher in Panel (b) than it is in Panel (a), even though the mean asset value and the total gains to trade are identical across the two panels.

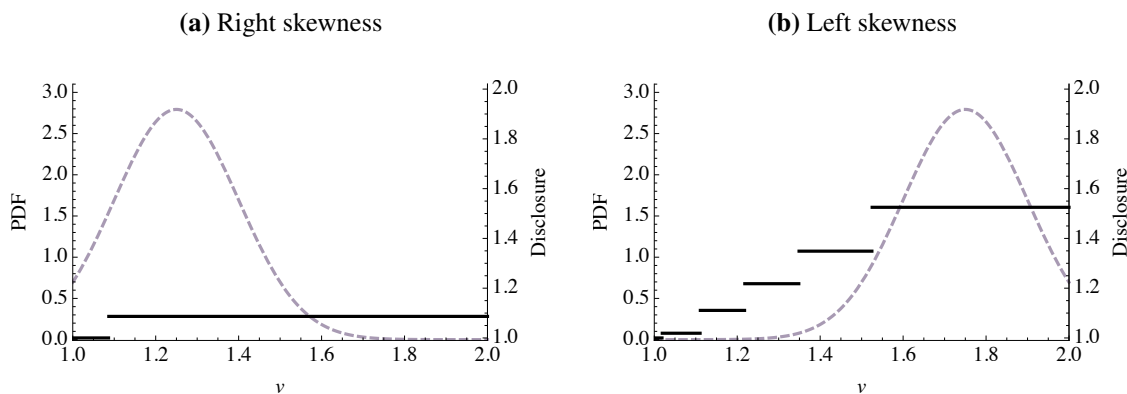


**FIGURE III**

**Mean-preserving spread in  $v$ .** The graphs plot the PDF and the optimal disclosure plan for  $v \sim N(1.5, 0.1)$  (left panel) and for  $v \sim N(1.5, 0.2)$ , where the normal distributions are each truncated by the boundaries  $v_L = 1$  and  $v_H = 2$ . In both panels  $c(v) = \bar{c} = 0.5$ . The vertical axis on the left of each graph corresponds to the dashed line that plots the PDF  $f(v)$ , and the vertical axis on the right corresponds to the solid line that plots the partition cutoff  $v_s$  (with  $s = 1, \dots, n$ ) of the optimal disclosure function for  $v \in [v_s, v_{s+1})$ .

Finally, in the two panels of Figure IV, we also consider cases where the buyer's valuation  $v$  follows truncated normal distributions, but here we vary the mean of the distribution, implying variation in skewness. In Panel (a) the distribution is right skewed. Following our arguments above, as the density  $f(\cdot)$  takes relatively high values for low realizations of  $v$ , this distribution discourages

inefficient screening. As a result, the buyer discloses little information. In contrast, under the left-skewed distribution in Panel (b), the seller’s incentives to screen are larger, implying that the buyer’s optimal disclosure plan has to provide relatively more information about the underlying value  $v$ .



**FIGURE IV**

**Changing the skewness of  $v$ .** The graphs plot the PDF and the optimal disclosure plan for  $v \sim N(1.25, 0.15)$  (left panel) and for  $v \sim N(1.75, 0.15)$ , where the normal distributions are each truncated by the boundaries  $v_L = 1$  and  $v_H = 2$ . In both panels  $c(v) = \bar{c} = 0.5$ . The vertical axis on the left of each graph corresponds to the dashed line that plots the PDF  $f(v)$ , and the vertical axis on the right corresponds to the solid line that plots the partition cutoff  $v_s$  (with  $s = 1, \dots, n$ ) of the optimal disclosure function for  $v \in [v_s, v_{s+1})$ .

The above examples illustrate how our theory consistently produces *coarse* information disclosures. This feature mirrors the coarseness of credit ratings in practice, which also pool creditors with ranges of credit risks. In Section 5 we discuss how key features of our environment, and its predictions, might shed light on the prevalence of ratings in debt markets (rather than in equity markets), and on the fact that these ratings are discrete (and coarse) when purchased by the issuer (issuer-paid ratings), whereas continuous information tends to be provided when investors pay for similar services.

### 3.2.3 Discrete Distributions

In the previous subsection we restricted disclosure plans to be monotone, which greatly increased the tractability of the analysis. An alternative setting that simplifies the analysis of optimal disclosure plans without imposing monotonicity is an environment where the distribution  $F(\cdot)$  is discrete.

Analyses of discrete environments are useful, as any continuous distribution can be approximated arbitrarily well by a discrete counterpart. Yet, in the presence of a discrete distribution, standard optimization procedures can be used to determine an optimal disclosure plan. In particular, since the choice variables are integers and the system to be solved is linear, an integer linear programming problem obtains. Moreover, as we will discuss next, the analysis is again facilitated by our key result that we can restrict our attention to efficient disclosure plans when solving for a plan that maximizes the buyer’s expected payoff. As a result, the set of candidates for the optimal disclosure plan is drastically reduced.

We show in Appendix B how the proof of Proposition 1 can be adapted to discrete distributions of  $v$ . A slight difference is that with a discrete distribution of  $v$ , there may exist cases where the alternative, more efficient disclosure plan makes the buyer only weakly better off, rather than strictly better off as in our baseline model with a continuous distribution. Under the tie-breaking rule introduced in Section 2 (i.e., whenever indifferent an agent takes the action that maximizes social surplus), the buyer’s optimal disclosure plan then still always leads to socially efficient trade. For similar reasons, the optimal disclosure plan may be fully revealing under certain parameterizations of discrete distributions (for example, when  $v$  can only take one of two values and trade would be inefficient without disclosure).

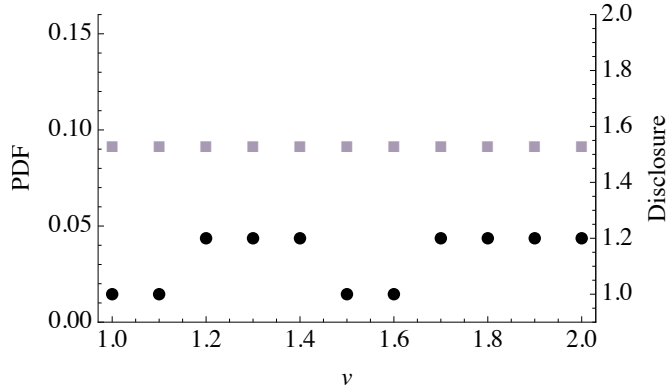
In the following, we provide a concrete example that is related to our earlier Examples 1 and 2 where  $v$  is uniformly distributed. In the discrete environment considered in Figure V, all possible values  $v$  have equal probability mass and the seller’s value is constant and equal to  $\bar{c} = 0.5$ . Again, the buyer’s optimization problem effectively aims to pool possible sets of  $v$  to minimize the expected transaction price while ensuring that trade remains socially efficient.

The buyer finds it optimal to split the set of realizations of  $v$  into two subsets associated with the signals  $s \in \{1, 2\}$ . Figure V shows that the signal structure involves gaps between these subsets. Our restriction that disclosure plans must be ex post verifiable still allows for the design of signals that pool multiple disjoint subsets,<sup>20</sup> and in this example, a non-monotone plan allows the buyer to

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<sup>20</sup>Non-monotonicity is also a property of the signal functions associated with the optimal disclosure plans in Goldstein and Leitner (2015) and Inostroza and Pavan (2017) who study the information design problem of a regulator in the context of bank stress tests.





**FIGURE V**

**Discrete distribution for  $v$ .** The graphs plot the PDF and an optimal disclosure plan when  $v$  follows a discrete uniform distribution on the support  $\{1, 1.1, \dots, 1.9, 2\}$ , where each possible outcome has probability  $1/11$  and where  $c(v) = \bar{c} = 0.5$ . The vertical axis on the left of each graph corresponds to the squares that identify the PDF  $f(v)$ , and the vertical axis on the right corresponds to the circles that plot  $\min[D^{-1}(s)]$  for  $v \in D^{-1}(s)$  with  $s \in \{1, 2\}$ .

minimize the average price paid while preventing the seller from quoting prices that cause inefficient rationing.

When the seller receives a signal that  $v$  belongs to the lower combination of circles, he responds by quoting a price  $p = 1$ . When the seller instead receives a signal that  $v$  belongs to the higher combination of circles, quoting a price  $p = 1.2$  maximizes his conditional expected payoff. In both cases, these price quotes are equal to the lowest possible realizations of  $v$ , given the signal, and as a result the buyer always accepts them. Note that there exist alternative disclosure plans that deliver identical payoffs to all agents, implying that the equilibrium disclosure plan is not unique, even though the allocation of the surplus is.

## 4 Interim Disclosure

In the previous section, we assumed that the buyer designs his disclosure plan prior to obtaining private information. We now discuss the robustness of our results to “interim” disclosure, that is, the scenario where the buyer chooses the disclosure plan after obtaining private information, but

before the realization of  $v$  becomes publicly observable. Specifically, the timeline of the sequential game we now study is as follows. First, the buyer privately observes  $v$ . Second, he designs an ex post verifiable signal that he sends to the seller. Finally, the seller quotes a price and the buyer decides whether to accept or reject. Below we show that equilibria satisfying sensible and well-known refinements must feature partial information disclosure leading to socially efficient trade, just as in our baseline setting (see Proposition 1).

We start by describing each agent's strategy. Consistent with our earlier notation, let  $D(v)$  denote the signal that the buyer sends when his valuation for the asset is  $v$ . In the context of interim disclosure (where the buyer does not commit ex ante to a mapping between realizations of  $v$  and signals), ex post verifiability requires that any signal  $s = D(v)$  is itself a Borel set in  $[v_L, v_H]$  and that  $v \in D(v)$  for any  $v$ . Since  $D(v)$  is now designed by the buyer after he observes  $v$ , we can interpret  $D(v)$  as the pure-strategy message that the buyer sends in this signaling game (see also Bertomeu and Cianciaruso 2017). Upon receiving a signal  $s$ , the seller forms beliefs about the buyer's type  $v$ , which we denote by the distribution function  $\mu(s) \in \Delta([v_L, v_H])$ .<sup>21</sup> Then the seller quotes a price, which we now denote as  $p(s)$ , to maximize his expected profit, and the buyer decides whether to accept. A buyer's optimal strategy in that last stage is simply to accept the offer if and only if the quoted price is weakly less than his true valuation  $v$ . For ease of exposition, we do not introduce extra notation for this final stage and directly impose that the buyer follows this dominant strategy.

To summarize, we now consider a signaling game where the buyer sends a message and the seller chooses an action based on that message. We dub this signaling game as the *interim disclosure game*. We can now state the definition of an equilibrium in this setting.

**Definition 3.** A  $(D(\cdot), \mu(\cdot), p(\cdot))$  profile forms a *perfect Bayesian equilibrium of the interim disclosure game* if:

1. For every possible signal  $s$ ,  $p(s)$  solves  $\max_p \{\Pi_s(p)\}$ , where  $\Pi_s(p)$  denotes the seller's expected profit if he quotes a price  $p$  and the buyer's valuation is drawn from  $\mu(s)$ .

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<sup>21</sup>We use  $\Delta([v_L, v_H])$  to denote the set of all possible probability distributions on  $[v_L, v_H]$ .

2. For every  $v \in [v_L, v_H]$ ,  $D(v)$  solves  $\max_s \{\max[v - p(s), 0]\}$ , where  $v \in D(v)$ .
3. For every  $s$  in the range of  $D$  (i.e., every Borel set  $s$  that can be disclosed in equilibrium), the seller's belief  $\mu(s)$  is obtained by applying Bayes' rule given the particular signal  $s$ .

Since beliefs are unrestricted following off-equilibrium deviations, there exist beliefs such that the seller (who has market power) drives the buyer's information rents to zero following any off-equilibrium deviation in disclosure. This leads to the existence of multiple perfect Bayesian equilibria with various degrees of information revelation, as opposed to a unique equilibrium with full revelation as in Grossman (1981) and Milgrom (1981) (see Perez-Richet 2014, for a broader discussion of equilibrium multiplicity when the information designer picks a signal structure after acquiring private information). For instance, either full disclosure, partial disclosure, or no disclosure can be supported in equilibrium if the seller has the following beliefs: if for any  $s$  not in the range of  $D$  (that is, whenever  $s$  is an off-equilibrium signal), the belief  $\mu(s)$  assigns probability 1 to type  $\bar{v}(s)$ , where  $\bar{v}(s) \equiv \sup s$  (recall that  $s$  is a Borel set).<sup>22</sup>

Given the multiplicity of equilibria, we now focus on “buyer-preferred” equilibria. An important insight from the earlier analysis was that the buyer's preference in terms of disclosure plan coincides with the social planner's (see Proposition 1). Thus, selecting equilibria that the buyer “prefers” provides us with a natural way to capture the spirit of our earlier setting where the buyer moved first, before any private information was obtained. What it means for the buyer to “prefer” an equilibrium here is, however, complicated by the fact that he can be of many types when designing the disclosure plan. We define as buyer-preferred equilibria the set of equilibria that are not dominated among buyer types — in the Pareto sense — by another equilibrium based on their interim payoffs. Consistent with Riley (1975) and Riley (1979), we treat different types as distinct players. We now state our first main result for the interim disclosure game, which is proved in Appendix A.

**Proposition 4.** *In any buyer-preferred equilibrium of the interim disclosure game, the buyer's optimal disclosure is partial and yields socially efficient trade.*

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<sup>22</sup>An equilibrium is said to feature full disclosure if  $\mu(D(v))$  assigns probability 1 to type  $v$ , whereas it is said to feature no disclosure if  $D(v) = [v_L, v_H]$  for all  $v \in [v_L, v_H]$ , and thus  $\mu([v_L, v_H])$  is equal to  $F(v)$ , the prior distribution of  $v$ .

This proposition shows, using a logic similar to the one used in Proposition 1, that any equilibrium in which trade is socially inefficient is dominated from the buyer’s perspective by an equilibrium that features socially efficient trade due to more informative disclosures. Lemma 1 implies that for any equilibrium featuring inefficient trade we can construct an alternative equilibrium featuring more informative disclosures and Pareto improvements among buyer types — some previously excluded buyer types are not rationed anymore and extract some surplus from trade while all other types are equally well off as before. Also consistent with the logic of Proposition 1, full disclosure is also not attractive to the buyer since it yields zero profit in all states of the world. As a result, buyer-preferred equilibria cannot feature full disclosure.

For robustness, we show that these equilibrium properties also apply under an alternative equilibrium refinement known as Grossman-Perry-Farrell, based on the perfect sequential equilibrium of Grossman and Perry (1986) and the neologism-proof equilibrium of Farrell (1993).<sup>23</sup> This refinement is commonly used in models of verifiable disclosure (see, e.g., Bertomeu and Cianciaruso 2017, and the references therein). Instead of comparing equilibria in a Pareto sense as above, this refinement eliminates equilibria with off-equilibrium beliefs that are deemed unreasonable given agents’ incentives to deviate from their equilibrium strategies.

Denote by  $U(v, s, \mu(s))$  the buyer’s utility if his valuation is  $v$ , he sends a message  $s$ , and the seller quotes an optimal price given the belief function  $\mu(s)$ . For any signal  $s$  that is a Borel set in  $[v_L, v_H]$  (including off-equilibrium messages), denote by  $\mu_s$  the actual distribution of  $v$  conditional on  $v \in s$ . (Recall that we restrict the sets of signals to be Borel sets in the interim disclosure game, to be consistent with ex post verifiability.) As in Bertomeu and Cianciaruso (2017), we define a Grossman-Perry-Farrell equilibrium by ruling out the existence of self-signaling sets.

**Definition 4.** *A pure-strategy perfect Bayesian equilibrium of the interim disclosure game  $(D(\cdot), \mu(\cdot), p(\cdot))$  is called a “Grossman-Perry-Farrell equilibrium” if there does not exist a self-signaling set, which*

<sup>23</sup>We adopt the terminology “Grossman-Perry-Farrell” from Gertner, Gibbons, and Scharfstein (1988), Lutz (1989), and Bertomeu and Cianciaruso (2017).

is defined as a non-empty Borel set  $s \subset [v_L, v_H]$  such that:

$$s = \{v \in s : U(v, s, \mu_s) > U(v, D(v), \mu(D(v)))\}. \quad (31)$$

Note that only buyers whose valuation  $v \in s$  can send the signal  $s$  because ex post verifiability requires that the true valuation belongs to the chosen signal. A self-signaling set  $s$  contains all buyer types who could be strictly better off by sending the signal  $s$  rather than playing according to the considered perfect Bayesian equilibrium. A deviation from an equilibrium consists of a message announcing “my type is in  $s$ .”<sup>24</sup> The deviation is credible if  $s$  is self-signaling. An equilibrium survives the refinement if it does not admit any credible deviation.

We now state our main results regarding the properties of equilibria satisfying the Grossman-Perry-Farrell refinement (the proof is provided in Appendix A).

**Proposition 5.** *In any Grossman-Perry-Farrell equilibrium of the interim disclosure game, the buyer’s optimal disclosure is partial and yields socially efficient trade.*

The logic underlying Proposition 5 is again related to the insights from Proposition 1. In a perfect Bayesian equilibrium where trade breaks down and surplus is destroyed, excluded buyer types would like to form a self-signaling set and warn the seller that he is about to quote a price that will be rejected. Since this deviation is credible, the seller would then adjust his beliefs and lower his price quote, thereby improving the efficiency of trade and making some of these excluded buyer types strictly better off.

Finally, the following proposition further highlights how the economics underlying the interim disclosure game resemble those of the ex ante disclosure game.

**Proposition 6.** *An equilibrium disclosure plan of the ex ante disclosure game can be sustained both in a buyer-preferred equilibrium and in a Grossman-Perry-Farrell equilibrium of the interim disclosure game.*

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<sup>24</sup>Unlike Farrell (1993) who allows for the possibility of any type of senders announcing “my type is in  $s$ ”, we assume only buyer types whose true valuation  $v \in s$  can do so, consistent with our restriction of ex post verifiability.

Although the proposition above shows that an equilibrium of the ex ante disclosure game can be sustained in the interim disclosure game under either equilibrium refinement, it is straightforward to construct examples showing that the converse is not true. Specifically, while the equilibrium allocation of surplus in the ex ante game can be shown to be unique, multiple surplus allocations can be sustained in the refined equilibria of the interim game (although all these refined equilibria must feature partial disclosure and efficient trade, as stated in Propositions 4 and 5).<sup>25</sup>

## 5 Concluding Remarks

We characterize optimal voluntary disclosures by a privately informed agent who faces a counterparty endowed with market power in a bilateral transaction. Although disclosures reduce the agent's private information, they may increase his information rents by mitigating the counterparty's incentives to charge prices that lead to inefficient rationing. We show that when disclosures are restricted to be ex post verifiable, the privately informed agent always finds it optimal to design a partial disclosure plan that implements socially efficient trade in equilibrium.

Our paper speaks to the fundamental forces determining whether asymmetric information impedes trade in the presence of imperfect competition. We show that in a relevant class of settings, efficient trade should not be impeded, in particular, when information is ex post verifiable, truthfulness is enforced, and private information pertains only to the bilateral transaction considered. By the same token, our results highlight conditions that would need to be violated in practice in order for inefficiencies to arise. Only in the presence of such violations might improving efficiency require the involvement of informed intermediaries,<sup>26</sup> signaling through trade delays,<sup>27</sup> or an external

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<sup>25</sup>For example, we can construct a buyer-preferred equilibrium in the parameterization with  $v \sim U[1, 2]$  and  $\bar{c} = 0.5$  where the buyer only discloses whether  $v \in [1, 1.5]$  or  $v \in [1.5, 2]$ . In this equilibrium of the interim disclosure game, the seller collects an expected surplus of 0.75 whereas the buyer collects an expected surplus of 0.25. This allocation of surplus is not the equilibrium outcome of the ex ante disclosure game.

<sup>26</sup>As in, e.g., Biglaiser (1993), Li (1998), Glode and Opp (2016), and Zhang (2016).

<sup>27</sup>See, e.g., Fudenberg, Levine, and Tirole (1985). In a dynamic environment, an informed agent can, under specific conditions, signal his low value to a counterparty with market power by rejecting high early offers, at the cost of delaying gains from trade. In particular, such behavior may occur in practice when ex post verification of disclosures is infeasible or too costly, relative to the inefficiencies from delay, and the conditions for the possibility of separating signaling equilibria are satisfied.

regulatory intervention.<sup>28</sup> Our insights thus have relevant implications for regulating information disclosure in bilateral transactions. Under the conditions we lay out, a regulator does not need to mandate what information agents must disclose, nor does he need to produce additional information for uninformed market participants. The regulator should instead focus on enforcing the truthfulness of disclosures by disciplining agents who send signals that ex post prove to violate their own disclosure standards. Agents would then have incentives to share their private information in ways that maximize the social efficiency of trade.

Our analysis also provides relevant predictions related to existing disclosure practices in financial markets. For instance, the two distinguishing features of our environment — (1) the separation of market power and private information and (2) ex post verifiability — may shed light on the reasons why *coarse* ratings play a particularly prevalent role in *credit* markets, whereas such ratings are effectively absent in equity markets.<sup>29</sup> Feature (1) is likely present in debt markets as debt issuers naturally have private information about their assets, and since debt investors are likely to have some bargaining power given that the debt market is relatively concentrated (e.g., large insurance companies are key players in this market). Feature (2) is also more likely present in debt markets, as debt investors are concerned with a firm’s assets in place, which can be subjected to verification. In contrast, these two features might be less typical for equity markets. First, empirical evidence suggests that competition among investors in the equity market is stronger than it is in debt markets (see, e.g., Biais and Green 2007), such that private information and market power might *not* be separated. Second, equity investors are interested in growth expectations, which depend on information that is more difficult to verify. Taken together, these distinguishing features of bond and equity markets might, in turn, contribute to the fact that firms in practice disclose information by soliciting coarse ratings for their debt, whereas no equivalent phenomenon exists for equity.

In addition, our model can shed light on the fact that credit ratings that are solicited and paid for by the issuer are on a discrete scale,<sup>30</sup> whereas investor-paid information about credit quality tends

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<sup>28</sup>As in, e.g., Tirole (2012), Goldstein and Leitner (2015), and Faria-e-Castro, Martinez, and Philippon (2017).

<sup>29</sup>It is useful to recall that we can reverse the roles of the buyer and seller in our environment without affecting the key predictions of our theory: the issuer of a security can have private information about  $v$ , and the buyer can be the one making the take-it-or-leave-it offer. Moreover, a rating agency, which in practice is hired by the issuer, could be interpreted as a device for the seller to commit to truthful disclosure.

<sup>30</sup>For example, the leading rating agencies Moody’s, Fitch, and S&P issue ratings that are on a discrete scale when

to be continuous. For example, even though Moody's issuer-solicited ratings business and Moody's investor-paid ratings business are owned by the same parent company, one subsidiary issues coarse ratings (and the issuer pays for the disclosure) whereas the other one provides continuous credit scores (and investors pay for the information). The coarseness of issuer-solicited ratings thus appears to be a deliberate choice that, according to our theory, can be in the interest of issuers when facing investors that have market power.<sup>31</sup>

Overall, these results suggest that the economic forces highlighted by our model yield relevant insights on existing disclosure practices in financial markets, and might help gauge the benefits of regulatory interventions. Extensions of our framework that provide more concrete applications to specific contexts, such as credit ratings, are a promising endeavor that we leave for future research.

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ratings are paid for by the issuer.

<sup>31</sup>The regulatory use of issuer-paid ratings may also affect rating agencies' disclosure policies (see, e.g., Opp, Opp, and Harris 2013).



## Appendix A: Proofs Omitted from the Text

**Proof of Lemma 1:** Let  $F$ ,  $\mathbb{E}$ , and  $\Pi$  denote the CDF, the expectation operator, and the profit function under the initial distribution of  $v$ . We use a subscript 0 to indicate the counterparts of these functions under the truncated distribution of  $v$ . The seller's expected payoff from quoting  $p$  under  $F_0(v)$  can be written as:

$$\begin{aligned}
\Pi_0(p) &= (1 - F_0(p))p + F_0(p)\mathbb{E}_0[c(v)|v < p] \\
&= \left(\frac{1 - F(p)}{1 - F(\tilde{p})}\right)p + \left(\frac{F(p) - F(\tilde{p})}{1 - F(\tilde{p})}\right)\mathbb{E}[c(v)|\tilde{p} \leq v < p] \\
&= \frac{1}{1 - F(\tilde{p})} \left[ (1 - F(p))p + \int_{\tilde{p}}^p c(v)dF(v) \right] \\
&= \frac{1}{1 - F(\tilde{p})} \left[ \int_p^{v_H} v dF(v) + \int_{v_L}^p c(v)dF(v) - \int_{v_L}^{\tilde{p}} c(v)dF(v) \right] \\
&= \frac{1}{1 - F(\tilde{p})} \left[ \Pi(p) - \int_{v_L}^{\tilde{p}} c(v)dF(v) \right]. \tag{A1}
\end{aligned}$$

The seller's expected payoff from quoting  $p$  under  $F_0(v)$  is thus a positive linear transformation of  $\Pi(p)$ . Since by definition quoting  $p = \tilde{p}$  maximizes  $\Pi(p)$  among all  $p \in [v_L, v_H]$ , it must also maximize the seller's expected payoff under  $F_0(v)$  among all  $p \in [\tilde{p}, v_H]$ .  $\square$

**Proof of Proposition 2:** We argue by contradiction. Suppose the efficiency constraint is not binding for a signal realization  $s = j$ , despite it being one of the  $(n - 1)$  lowest realizations in  $S$  where trade is efficient, implying that:

$$\Pi'_j(p_j) \neq 0, \text{ and } \Pi_j(p_j) > \Pi_j(p), \forall p \neq p_j. \tag{A2}$$

Since  $p_j$  is the optimal price conditional on receiving a signal  $s = j$ , it follows that  $\Pi'_j(p_j) < 0$ .

Take the highest signal  $s = n$  and denote  $\bar{v}_n \equiv \sup\{D^{-1}(n)\}$ . Without loss of generality, assume that  $\bar{v}_n$  is not a singleton of  $D^{-1}(n)$ .<sup>32</sup> For a small  $\epsilon > 0$ , the interval  $(\bar{v}_n - \epsilon, \bar{v}_n) \subset D^{-1}(n)$ .

<sup>32</sup>Otherwise, we can let  $\bar{v}_n = \sup\{D^{-1}(n) \setminus \{\max\{D^{-1}(n)\}\}\}$ .

Now, consider the following alternative disclosure plan:

$$\tilde{D}(v) \equiv \begin{cases} D(v) & \text{if } D(v) \neq n \\ j & \text{else if } D(v) = n, v \in (\bar{v}_n - \epsilon, \bar{v}_n) \\ n & \text{otherwise.} \end{cases} \quad (\text{A3})$$

Let  $\tilde{\Pi}_s(p)$  denote the seller's payoff from quoting a price  $p$  conditional on receiving  $s$  under the disclosure plan  $\tilde{D}$ .

We first show that we can pick  $\epsilon$  small enough such that the optimal quoted price conditional on receiving  $s = j$  under the disclosure plan  $\tilde{D}$  is the same as under the disclosure plan  $D$ . By contradiction, suppose that for any  $\epsilon_m \rightarrow 0$ , there exists a price candidate  $y_m \in \tilde{D}^{-1}(j)$  such that:

$$\tilde{\Pi}_j(y_m) > \tilde{\Pi}_j(p_j). \quad (\text{A4})$$

Since  $\{y_m : m = 1, 2, \dots\}$  is a bounded sequence, there exists a convergence subsequence  $\{y_{m_k} : k = 1, 2, \dots\}$  (Bolzano-Weierstrass theorem). Suppose  $y_{m_k} \rightarrow y$ . If  $y \neq p_j$ , using  $\epsilon_m \rightarrow 0$  in equation (A4) with  $y_{m_k}$  implies that  $\tilde{\Pi}_j(y) \geq \tilde{\Pi}_j(p_j)$ , which is contradicted by the assumption that the constraint is not binding conditional on the signal realization  $s = j$ . Thus, it must be that  $y = p_j$ . From equation (A4) and Lagrange's Mean Value Theorem, it follows that there exists  $z_{m_k} \in (p_j, y_{m_k})$  such that  $\tilde{\Pi}'_j(z_{m_k}) > 0$ . Since  $y_{m_k} \rightarrow p_j$ , we also have  $z_{m_k} \rightarrow p_j$ . Taking limit on  $\tilde{\Pi}'_j(z_{m_k}) > 0$  yields  $\tilde{\Pi}'_j(p_j) \geq 0$ , which is a contradiction since the efficiency constraint after a signal  $s = j$  is not binding.

We next show that the optimal quoted price conditional on receiving  $s = n$  under the disclosure plan  $\tilde{D}$  is the same as under disclosure plan  $D$ . To do so, we need to show that:

$$\tilde{\Pi}_n(\inf\{D^{-1}(n)\}) \geq \tilde{\Pi}_n(p), \forall p. \quad (\text{A5})$$

Again, we show this by contradiction. Suppose there exists  $p_0 < \bar{v}_n$  such that  $\tilde{\Pi}_n(p_0) > \tilde{\Pi}_n(\inf\{D^{-1}(n)\})$ .

Let  $\xi \equiv \Pr(v \in D^{-1}(n))$  and  $\eta \equiv \Pr(v \in (\bar{v}_n - \epsilon, \bar{v}_n))$ . In other words,  $\xi$  is the probability that

the signal realization is  $s = n$  under the disclosure plan  $D$  whereas  $\eta$  is the probability that  $v$  falls in the region that used to be associated with  $s = n$  under the disclosure plan  $D$ , but is now associated with  $s = j$  under the disclosure plan  $\tilde{D}$ . Consider the seller's profit by quoting  $p_0$  conditional on receiving  $s = n$  under the disclosure plan  $D$ . We can pick  $\epsilon$  small enough such that all buyer types in  $(\bar{v}_n - \epsilon, \bar{v}_n)$  would accept the quoted price  $p_0$ , implying that:

$$\Pi_n(p_0) = \left(1 - \frac{\eta}{\xi}\right) \tilde{\Pi}_n(p_0) + \frac{\eta}{\xi} p_0, \quad (\text{A6})$$

or equivalently,

$$\xi \Pi_n(p_0) = (\xi - \eta) \tilde{\Pi}_n(p_0) + \eta p_0. \quad (\text{A7})$$

Since  $p_0 > \inf\{D^{-1}(n)\}$ , then

$$\begin{aligned} \xi \Pi_n(p_0) &> (\xi - \eta) \Pi_n(\inf\{D^{-1}(n)\}) + \eta \inf\{D^{-1}(n)\} \\ &= \xi \inf\{D^{-1}(n)\} \\ &= \xi \Pi_n(\inf\{D^{-1}(n)\}). \end{aligned} \quad (\text{A8})$$

This inequality is contradicted by the fact that  $\inf\{D^{-1}(n)\}$  is the optimal price to quote when the seller receives the signal  $s = n$  under the disclosure plan  $D$ .

Lastly, we know that buyer types whose valuation  $v$  belongs to  $(\bar{v}_n - \epsilon, \bar{v}_n)$  pay a lower price under the new disclosure plan. The buyer's payoff under the disclosure plan  $\tilde{D}$  is thus strictly higher than under  $D$ . Thus, if the efficiency constraint is not binding after the signal  $s = j$ , we can construct an alternative disclosure plan that strictly improves the buyer's payoff, contradicting the conjectured optimality of  $D$ .  $\square$

**Proof of Lemma 2:** For given partition cutoffs  $v_{s-1}$  and  $v_{s+1}$  the buyer's marginal cost of increasing  $v_s$  (if feasible given efficiency constraints) is given by the partial derivative of the expected price

paid by the buyer (22) with respect to  $v_s$ :

$$\frac{\partial (\sum_{s=1}^n (F(v_{s+1}) - F(v_s)) v_s)}{\partial v_s} = (F(v_{s+1}) - F(v_s)) - f(v_s)(v_s - v_{s-1}). \quad (\text{A9})$$

We first show that for any two partition cutoffs  $v_{s-1}$  and  $v_{s+1}$  with  $s \geq 2$  that satisfy the efficiency constraint  $v_{s-1} \geq lb(lb(v_{s+1}))$  implied by (26), the marginal cost (A9) as a function of  $v_s$  crosses zero at most once. Setting equation (A9) equal to zero and rearranging, we obtain:

$$\frac{f(v_s)(v_s - v_{s-1})}{F(v_{s+1}) - F(v_s)} = 1. \quad (\text{A10})$$

Note that  $h_s(v) = \frac{f(v)}{F(v_{s+1}) - F(v)}$ , and thus,  $\frac{\partial(h_s(v)(v - v_{s-1}))}{\partial v} = h'_s(v)(v - v_{s-1}) + h_s(v) > 0$  for  $v \in (v_{s-1}, v_{s+1})$ , implying that the left-hand-side of equation (A10) is an increasing function of  $v_s$ . Thus, the marginal cost (A9) crosses zero at most once from above. Then the expected price paid by the buyer (22) is first increasing and then decreasing in  $v_s$  for  $v_s \in [lb(v_{s+1}), rb(v_{s-1})]$ . Consequently, the buyer's expected payment (22) must reach its minimum when  $v_s$  is equal to either  $rb(v_{s-1})$  or equal to  $lb(v_{s+1})$ .  $\square$

**Proof of Proposition 3:** Lemma 2 implies that if the buyer's optimal disclosure plan includes the cutoffs  $v_{s-1}$  and  $v_{s+1}$ , then it must be that  $v_s$  is either equal to  $rb(v_{s-1})$  or equal to  $lb(v_{s+1})$ . To evaluate whether  $v_s = lb(v_{s+1})$  dominates  $v_s = rb(v_{s-1})$  for all possible values that  $v_{s-1}$  and  $v_{s+1}$  can take (as defined by the domain  $\Omega$ ), we define the difference in the expected prices paid by the buyer when choosing  $v_s = rb(v_{s-1})$  instead of  $v_s = lb(v_{s+1})$ :

$$\begin{aligned} \Phi(v_{s-1}, v_{s+1}) = & [v_{s-1} \cdot (F(rb(v_{s-1})) - F(v_{s-1})) + rb(v_{s-1}) \cdot (F(v_{s+1}) - F(rb(v_{s-1}))) \\ & - [v_{s-1} \cdot (F(lb(v_{s+1})) - F(v_{s-1})) + lb(v_{s+1}) \cdot (F(v_{s+1}) - F(lb(v_{s+1})))]. \end{aligned} \quad (\text{A11})$$

If  $\Phi(v_{s-1}, v_{s+1}) \geq 0$  everywhere on the domain  $\Omega$  then choosing  $v_s = lb(v_{s+1})$  always (weakly) dominates and is the optimal solution, implying that the proposed descending sequences determine

the optimal partition cutoffs. □

**Proof of Proposition 4:** To show that trade is socially efficient in any buyer-preferred equilibrium  $(D(\cdot), \mu(\cdot), p(\cdot))$ , we argue by contradiction. Suppose there exists a signal  $s_0 = D(v)$  for some  $v \in [v_L, v_H]$  such that  $p(s_0) > \hat{v}$  and  $p(s_0) > \inf\{v \in [v_L, v_H] : D(v) = s_0\}$ . A buyer whose valuation belongs to  $\{v : D(v) = s_0\} \cap (\hat{v}, p(s_0))$  would refuse to pay the seller's quoted price  $p(s_0)$ , leading to inefficient trade. Let  $s' \equiv \{v \in s_0 : \hat{v} \leq v < p(s_0)\}$ . Consider the following candidate equilibrium  $(\tilde{D}(\cdot), \tilde{\mu}(\cdot), \tilde{p}(\cdot))$ , where:

$$\tilde{D}(v) \equiv \begin{cases} D(v) & \text{if } v \notin s_0 \\ s' & \text{else if } v \in s' \\ s_0 \setminus s' & \text{otherwise .} \end{cases} \quad (\text{A12})$$

We obtain  $\tilde{\mu}(s')$  and  $\tilde{\mu}(s_0 \setminus s')$  using Bayes' rule at  $s'$  and  $s_0 \setminus s'$ , respectively. For any off-equilibrium signal  $s$ ,  $\tilde{\mu}(s)$  assigns probability 1 to  $\bar{v}(s) \equiv \sup s$ . For any equilibrium signal outside the range of  $s_0$ ,  $\tilde{\mu}(s) = \mu(s)$ . Let  $\tilde{p}(s)$  maximize the seller's expected profit if he quotes a price  $p$  and the buyer's valuation is drawn from  $\tilde{\mu}(s)$ . It is clear that we are indeed in an equilibrium, since deviating to any other disclosure yields a profit of 0 for the buyer. Now consider the buyer's interim payoffs in this alternative equilibrium. Buyers whose type either satisfies  $v \notin s_0$  or  $v \in s_0 \setminus s'$  receive payoffs identical to those from the original equilibrium  $(D(\cdot), \mu(\cdot), p(\cdot))$ . However, buyer types in  $s'$  receive weakly higher payoffs. Moreover, a buyer type  $v = (p(s_0) - \epsilon)$ , where  $\epsilon$  is a small positive number, receives a strictly higher payoff, since he made zero profit in the original equilibrium. Overall, if trade is not efficient in an equilibrium, then it is Pareto dominated among buyer types by a more efficient equilibrium. Consequently, in any buyer-preferred equilibrium of the interim disclosure game, trade must be socially efficient.

To show that a buyer-preferred equilibrium does not feature full disclosure, where each buyer type is quoted  $p = v$  and makes zero profit, it is sufficient to construct an equilibrium where some buyer types receive positive payoffs (as no buyer type can do worse than zero profit given their right to reject a price quote). Consider the equilibrium induced by the ex ante disclosure plan we

solved for in Proposition 1 of Section 3. Formally, suppose  $D(\cdot)$  is the verifiable disclosure plan chosen by the buyer in the ex ante disclosure game and let the interim disclosure follow  $D(v)$  for all  $v \in [v_L, v_H]$ . Now, let  $\mu(\cdot)$  be a belief function obtained using Bayes' rule on the equilibrium path and that assigns probability 1 to the highest type for any signal off the equilibrium path. Lastly,  $p(s)$  maximizes the seller's profit based on the conditional distribution  $\mu(s)$ . The profile  $(D(\cdot), \mu(\cdot), p(\cdot))$  is clearly an equilibrium of the interim disclosure game. In this equilibrium, the buyer receives profits identical to those obtained in the ex ante disclosure game. Thus, this equilibrium featuring partial disclosure Pareto dominates among buyer types any equilibrium with full disclosure.  $\square$

**Proof of Proposition 5:** To show that trade is socially efficient in any Grossman-Perry-Farrell equilibrium  $(D(\cdot), \mu(\cdot), p(\cdot))$ , we argue by contradiction. Suppose there exists a signal  $s_0 = D(v)$  for some  $v \in [v_L, v_H]$  such that  $p(s_0) > \hat{v}$  and  $p(s_0) > \inf\{v \in [v_L, v_H] : D(v) = s_0\}$ . A buyer whose valuation belongs to  $\{v : D(v) = s_0\} \cap (\hat{v}, p(s_0))$  would refuse to pay the seller's quoted price  $p(s_0)$ , leading to inefficient trade. Let  $s_1 \equiv \{v \in s_0 : \hat{v} \leq v < p(s_0)\}$  and suppose the seller would quote a price  $p_1$  under beliefs characterized by the conditional distribution  $\mu_{s_1}$ .

Now consider the following set:  $s_2 \equiv \{v \in s_0 : p_1 < v < p(s_0)\}$ . From Lemma 1, we know that the seller would also quote a price  $p_1$  under the belief characterized by the conditional distribution  $\mu_{s_2}$ . Thus,  $U(v, s_2, \mu_{s_2}) > 0, \forall v \in s_2$ . Recall that all types of buyers in  $s_2$  do not trade in the equilibrium  $(D(\cdot), \mu(\cdot), p(\cdot))$ , thus  $U(v, D(v), \mu(D(v))) = 0, \forall v \in s_2$ . As a result, all types of buyers in  $s_2$  are strictly better off by announcing "my type is in  $s_2$ ," and  $s_2$  is a self-signaling set, contradicting the conjecture that a Grossman-Perry-Farrell equilibrium can feature inefficient trade.

To show that full disclosure cannot be a feature of a Grossman-Perry-Farrell equilibrium, it is sufficient to construct a self-signaling set. Suppose the seller would quote a price  $p'$  under the prior belief. Then, it is clear that  $(p', v_H]$  is a self-signaling set, since these buyer types would be strictly better off being quoted a price  $p'$  than a price equal to their respective valuation  $v$ .  $\square$

**Proof of Proposition 6:** We first show how to construct a strategy profile that supports the optimal disclosure plan of the ex ante game in the interim game. Suppose the optimal disclosure plan of the ex ante game is denoted by  $D : [v_L, v_H] \rightarrow S$ . Without loss of generality, assume  $S$  is itself a

collection of Borel sets such that  $D^{-1}(s) = s, \forall s \in S$ .<sup>33</sup> In the interim game, consider a candidate equilibrium in which:

1. Disclosure follows the ex ante disclosure  $D(v)$ .
2. For  $s \in S$ , the belief  $\mu(s)$  is given by  $F$  conditional on  $s$ . For any  $s \notin S$  (that is, whenever  $s$  is an off-equilibrium signal), the belief  $\mu(s)$  assigns probability 1 to type  $\bar{v}(s)$ .
3. For every possible signal  $s$ ,  $p(s)$  solves  $\max_p \Pi_s(p)$ , where  $\Pi_s(p)$  denotes the seller's expected profit if he quotes a price  $p$  and the buyer's valuation is drawn from  $\mu(s)$ .

We can show that the constructed strategy profile forms a buyer-preferred equilibrium. By contradiction, suppose  $(D_1(\cdot), \mu_1(\cdot), p_1(\cdot))$  is an equilibrium that dominates  $(D(\cdot), \mu(\cdot), p(\cdot))$  among buyer types in the Pareto sense based on their interim payoffs. Let  $S_1 = \{D_1(v) : v \in [v_L, v_H]\}$  and consider an ex ante disclosure  $D_1 : [v_L, v_H] \rightarrow S_1$ . Since  $\mu_1(\cdot)$  is obtained by applying Bayes' rule on the equilibrium path, the buyer's expected payoff under the disclosure  $D_1$  of the ex ante game is equal to his expected payoff under the  $(D_1(\cdot), \mu_1(\cdot), p_1(\cdot))$  equilibrium of the interim game. It then follows that the buyer's expected payoff under the disclosure plan  $D_1(\cdot)$  is strictly higher than the one under the disclosure plan  $D(\cdot)$  in the ex ante game, contradicting the fact that  $D(\cdot)$  is an optimal disclosure plan of the ex ante game.

We now show that the constructed strategy profile above also forms a Grossman-Perry-Farrell equilibrium of the interim game. We argue by contradiction, that is, suppose there exists a self-signaling set  $s_0$  in that case. Let  $S = \{D(v) : v \in [v_L, v_H]\}$  be the set of signals that are on the equilibrium path. We first show that  $s_0 \notin S$ . Otherwise, the signal  $s_0$  is on the equilibrium path:  $s_0 \in S$ , implying that for some  $v_0 \in s_0$ ,  $D(v_0) = s_0$ . Then,  $U(v_0, s_0, \mu_{s_0}) = U(v_0, D(v_0), \mu(D(v_0)))$ ,<sup>34</sup> contradicting the fact that  $s_0$  is a self-signaling set.

Now we turn to the ex ante game. Let  $S_2 = S \cup \{s_0\}$ . Consider the following ex ante disclosure

<sup>33</sup>If  $S$  is not a collection of Borel sets, then we can define  $S' = \{D^{-1}(s) : s \in S\}$ , which is a collection of Borel sets. Consider an ex ante disclosure  $D'(v) = D^{-1}(D(v)) : [v_L, v_H] \rightarrow S'$ . Under this disclosure, a buyer whose type belongs to  $s'$  sends the signal  $s'$ , i.e.,  $s' \in S'$ ,  $(D')^{-1}(s') = s'$ .

<sup>34</sup>Recall that  $U(v, s, \mu(s))$  is the buyer's utility if his valuation is  $v$ , he sends a message  $s$ , and the seller quotes an optimal price given the belief  $\mu(s)$ . Recall also that  $\mu_s$  is the distribution of  $v$  conditional on  $v \in s$ .

plan  $D_2 : [v_L, v_H] \rightarrow S_2$ :

$$D_2(v) \equiv \begin{cases} D(v) & \text{if } v \notin s_0, \\ s_0 & \text{otherwise.} \end{cases}$$

For buyers whose types are not in  $s_0$ , their payoffs are equal to their payoffs under the  $(D(\cdot), \mu(\cdot), p(\cdot))$  equilibrium of the interim game. For a buyer whose type is  $v \in s_0$ , his payoff is now given by  $U(v, s_0, \mu_{s_0})$ , which is strictly greater than  $U(v, D(v), \mu(D(v)))$ . Thus, the ex ante disclosure plan  $D_2(\cdot)$  yields a strictly higher expected buyer payoff than the disclosure plan  $D(\cdot)$ , contradicting the fact that  $D(\cdot)$  is an optimal disclosure plan of the ex ante game.  $\square$

## Appendix B: Additional Characterizations of Optimal Disclosure Plans

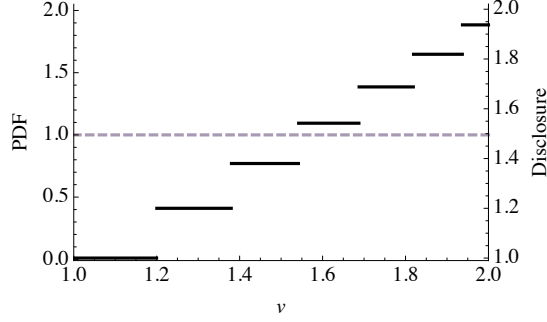
### (i) Monotone Disclosure Functions

As pointed out in Subsection 3.2.2, many standard parameterizations of our model lead the  $\Phi$ -function to take positive values everywhere on its domain  $\Omega$ . For this reason, we focused our derivations in the main text on this particular case. Here, we relax this assumption and briefly explain how to solve for the optimal monotone disclosure plan when  $\Phi$  takes negative values everywhere on its domain.

First note that Assumption 3 does not determine the sign of the  $\Phi$ -function. Below is an example where  $\Phi$  takes negative values everywhere on the domain  $\Omega$  but Assumption 3 is satisfied. The caption of Figure VI provides details.

The  $\Phi$ -function taking negative values everywhere on its domain implies that for any two partition cutoffs  $v_{s-1}$  and  $v_{s+1}$  with  $s \geq 2$  that satisfy the efficiency constraint  $v_{s-1} \geq lb(lb(v_{s+1}))$  implied by (26), the optimal interior cutoff is given by  $rb(v_{s-1})$ . Instead of obtaining a descending sequence of partition cutoffs as in Proposition 3, we then obtain an ascending sequence of cutoffs. Formally, when  $c(v_L) < v_L$  and  $\Phi(v_{s-1}, v_{s+1}) < 0$  for all  $(v_{s-1}, v_{s+1}) \in \Omega$ , an optimal disclosure plan is constructed as follows: the partition cutoffs are determined by an ascending sequence





**FIGURE VI**

**Example where  $\Phi$  takes negative values.** The graphs plot the PDF and the optimal disclosure plan for  $v \sim U[1, 2]$  and  $c(v) = 1.1 \cdot v - 0.3$ . The vertical axis on the left corresponds to the dashed line that plots the PDF  $f(v)$ , and the vertical axis on the right corresponds to the solid line that plots the partition cutoff  $v_s$  (with  $s = 1, \dots, n$ ) of the optimal disclosure function for  $v \in [v_s, v_{s+1})$ . Since the  $\Phi$ -function takes negative values everywhere on its domain  $\Omega$ , the optimal disclosure plan's partition cutoffs are constructed via an ascending sequence  $v_s = rb(v_{s-1})$ .

starting with  $v_1 = v_L$  and

$$v_s = rb(v_{s-1}), \quad (\text{B1})$$

until  $v_{n+1} = v_H = rb(v_n)$ .

Furthermore, we can show the existence of an optimal disclosure plan as follows. Disclosure plans are restricted to connected intervals (partitions), thus we must have  $i = \{1, 2, \dots, n\}$  signals associated with the partitions  $(v_1, v_2), [v_2, v_3), \dots, [v_n, v_{n+1}]$ , where  $v_1 = \hat{v}$  and  $v_{n+1} = v_H$  and  $n$  could be  $\infty$ . This notation already embeds the insight that the agent will disclose whenever  $v \leq \hat{v}$  with a separate signal ( $i = 0$ ). Our disclosure plan analysis thus focuses on the region where  $v > \hat{v}$ .

Suppose that  $V \equiv \{v_i : i = 1, 2, \dots\}$  gives the cutoffs for the optimal disclosure plan associated with connected intervals. Let  $\bar{V}$  be the closure of  $V$ .

**Lemma 3.** *There is at most one point,  $\hat{v}$ , that is in the closure of  $V$  but not in  $V$ , i.e.,  $\bar{V} \setminus V \subseteq \{\hat{v}\}$ .*

*Proof.* From the efficient trade constraints:  $v_i \in (lb(v_{i+1}), rb(v_{i-1}))$ ,  $\forall i \geq 1$ . There is no redundant interval in the sense that  $v_{i+2} > rb(v_i)$  because otherwise removing  $v_{i+1}$  strictly increases the buyer's payoff. Now since  $\forall v' \neq \hat{v}$ , we know that  $rb(v') > v'$  and  $lb(v') < v'$ . Then there cannot

exist a sub-sequence in  $V$  that approaches to  $v', \forall v' \neq \hat{v}$ , implying  $\bar{V} \setminus V \subseteq \{\hat{v}\}$ .  $\square$

We can now show that there exists  $\{v_i : i = 1, 2, \dots\}$  such that the optimal monotone disclosure plan can be characterized by cutoffs  $v_i$ . Given Lemma 3, we construct the cutoffs starting from the top: suppose  $k_0 = v_H$ , and  $k_1 > k_2 > \dots$ . Let  $k = (k_1, k_2, \dots)$ . Recall that the buyer chooses a disclosure plan to minimize the expected price subject to the cutoffs yielding efficient trade, that is:

$$\min_k EP(k) \equiv \sum_{i=1}^{\infty} k_i (F(k_{i-1}) - F(k_i)),$$

subject to

$$k_i \geq lb(k_{i-1}), \forall i.$$

Define the space  $\mathcal{S} = \{(k_1, k_2, \dots) : k_1 \geq k_2 \geq \dots, \text{ and } k_i \geq 0, \forall i\}$ , i.e.,  $\mathcal{S}$  consists of decreasing sequences. Define a metric on  $\mathcal{S}$  as

$$d(k^1, k^2) = \sum_{i=1}^{\infty} 2^{-i} |k_i^1 - k_i^2|,$$

where  $k^1 = (k_1^1, k_2^1, \dots)$  and  $k^2 = (k_1^2, k_2^2, \dots)$ . The metric  $d$  naturally induces a topology on  $\mathcal{S}$ .

Define the feasible set  $\mathcal{S}_1 = \{k \in \mathcal{S} : k_i \in [v_L, v_H], \text{ and } k_i \geq lb(k_{i-1}), \forall i \geq 1\}$ . The feasible set is a bounded set because  $|k| \leq \sum_{i=1}^{\infty} 2^{-i} |k_i| \leq \sum_{i=1}^{\infty} 2^{-i} |v_H| = v_H$ . We show that the feasible set is a closed set. Suppose  $k^j \in \mathcal{S}_1$  and  $k^j \rightarrow k^0$ . Since  $|k_i^j - k_i^0| \leq 2^i d(k^j, k^0) \rightarrow 0$ , we have  $k_i^j \rightarrow k_i^0$ . Since  $lb(\cdot)$  is a continuous function, it follows that  $k_i^0 \geq lb(k_{i-1}^0)$ , i.e.  $k^0 \in \mathcal{S}_1$ . Overall the feasible set is a compact set.

Let  $EP^m(k) \equiv \sum_{i=1}^m k_i (F(k_{i-1}) - F(k_i))$ . We claim that  $EP^m(k)$  uniformly converges to  $EP(k)$  on  $\mathcal{S}_1$ . For any  $\epsilon > 0$ , there exists  $\underline{k} > \hat{v}$  such that  $F(\underline{k}) - F(\hat{v}) < \frac{\epsilon}{v_H}$ . Since  $lb(k_i) < k_i$  for all  $k_i \neq \hat{v}$  and Lemma 1, there exists a sufficiently large  $m$ , such that applying  $lb(\cdot)$  for  $m$  times leads to a lower value than  $\underline{k}$ , i.e.,  $lb(\dots(lb(v_H))\dots) < \underline{k}$ . For this  $m$ , we know that  $EP(k) - EP^m(k) = \sum_{i=m+1}^{\infty} k_i (F(k_{i-1}) - F(k_i)) \leq \underline{k} \sum_{i=m+1}^{\infty} (F(k_{i-1}) - F(k_i)) \leq \underline{k} (F(\underline{k}) - F(\hat{v})) < \underline{k} \frac{\epsilon}{v_H} < \epsilon$ . So  $EP^m(\cdot)$  uniformly converges to  $EP(\cdot)$ .

We next show that the objective function  $EP(\cdot)$  is a continuous function on  $\mathcal{S}_1$ . Suppose

given any sequence  $\{k^j : j = 1, 2, \dots\}$  in  $\mathcal{S}_1$  that  $k^j \rightarrow k^0$ . We have  $\lim_{j \rightarrow \infty} EP(k^j) = \lim_{j \rightarrow \infty} \lim_{m \rightarrow \infty} EP^m(k^j) = \lim_{m \rightarrow \infty} \lim_{j \rightarrow \infty} EP^m(k^j)$ , where the exchange of  $\lim$  is due to the uniformly convergent property. Since  $\lim_{m \rightarrow \infty} \lim_{j \rightarrow \infty} EP^m(k^j) = \lim_{m \rightarrow \infty} EP^m(k^0) = EP(k^0)$ , it follows that  $\lim_{j \rightarrow \infty} EP(k^j) = EP(k^0)$  for any  $k^j \rightarrow k^0$ .

Since a continuous function maps a compact set into a compact set, we know that  $EP(\mathcal{S}_1)$  is a compact set. So, a minimum must be attainable, which in turns implies the existence of an optimal monotone disclosure plan.

## (ii) Discrete Distributions

As stated in Subsection 3.2.3, our results on optimal disclosure hold also under a discrete distribution. The main difference is that there may now exist cases where the alternative, more efficient disclosure plan being considered makes the buyer only weakly better off, rather than strictly better off as in our baseline model. Specifically, while the second term of equation (11) in the proof of Proposition 1 is strictly positive when  $v$  is continuously distributed with strictly positive density everywhere on the support, this term may occasionally take a value of 0 with a discrete distribution. If we apply the tie-breaking rule that assumes that whenever indifferent between disclosure plans the buyer chooses the one that maximizes social surplus, our result that the buyer's optimal disclosure plan always leads to socially efficient trade also holds with discrete distributions.

Formally, denote the possible realization of the distribution of  $v$  by the set  $\{a_i : 1 \leq i \leq n\}$ , where  $a_1 < a_2 < \dots < a_n$ . Suppose  $P(v = a_i) = q_i, \forall 1 \leq i \leq n$  and let  $c_i \equiv c(a_i), \forall 1 \leq i \leq n$ . Suppose  $k$  is the smallest index such that  $a_i \geq c_i$  and denote  $\hat{v} = a_k$ . As in our baseline model, in equilibrium the seller always quotes prices weakly greater than  $\hat{v}$ .

Denote by  $D(\cdot)$  the buyer's optimal disclosure plan given the tie-breaking rule and suppose that trade is inefficient under a signal  $s_0$  of  $D(\cdot)$ . Just as in the proof of Proposition 1, we can show that there exists another disclosure plan that yields a strictly higher social surplus and a weakly higher profit for the buyer.

Recall that  $p_s$  denotes the price the seller quotes conditional on receiving a signal  $s$ . A buyer whose valuation belongs to  $\{v : D(v) = s_0, \hat{v} \leq v < p_{s_0}\}$  would refuse to pay the seller's

quoted price  $p_{s_0}$ , making trade inefficient. Consider the following alternative disclosure plan where  $S' = S \cup \{s'\}$  for some  $s' \notin S$  and

$$\tilde{D}(v) \equiv \begin{cases} D(v) & \text{if } D(v) \neq s_0, \\ s_0 & \text{else if } D(v) = s_0, v \geq p_{s_0}, \\ s' & \text{otherwise.} \end{cases} \quad (\text{B2})$$

First, note that when  $s \neq s_0$ , nothing changes and the seller still quotes a price  $p_s$ . Second, Lemma 1 guarantees that the seller still quotes  $p_{s_0}$  under the alternative disclosure plan  $\tilde{D}(\cdot)$  when he receives a signal  $s_0$ . Finally, suppose the seller quotes a price  $z$  when he receives a signal  $s'$ . Since quoting  $p_{s_0}$  yields zero profit in this case, it must be that  $z \in [\min D^{-1}(s_0), p_{s_0}]$ . As a result, the buyer's ex ante expected profit under the alternative disclosure plan  $\tilde{D}(\cdot)$  is given by:

$$\underbrace{\sum_{s \in S} \sum_{a_i \in D^{-1}(s) \cap [p_s, v_H]} (a_i - p_s) q_i}_{\text{Profit from } s \in S} + \underbrace{\sum_{a_i \in D^{-1}(s_0) \cap [z, p_{s_0}]} (a_i - z) q_i}_{\text{Profit from } s'} \quad (\text{B3})$$

where the profit under the disclosure plan  $D(\cdot)$  is given by only the first term. If the second term is strictly positive, then the buyer earns a strictly higher profit under the disclosure plan  $\tilde{D}(\cdot)$  than under the plan  $D(\cdot)$ , contradicting the optimality of  $D(\cdot)$ . If the second term is zero, the price must equal the highest possible realization in  $\tilde{D}^{-1}(s')$ , i.e.,  $z = \max\{v : D(v) = s_0, v < p_{s_0}\}$ . Note that gains to trade must be positive when the buyer's valuation is  $z : z > c(z)$ , otherwise trade is efficient under the signal  $s_0$ . Then, the buyer with type  $z$  gets the asset under the disclosure plan  $\tilde{D}(\cdot)$  but not under the plan  $D(\cdot)$ , contradicting the tie-breaking rule. We have thus shown that the buyer's optimal disclosure plan must result in socially efficient trade.

Furthermore, it is easy to show that an optimal disclosure plan always exists when the distribution of  $v$  is discrete. A disclosure plan basically divides the set  $\{a_i : 1 \leq i \leq n\}$  into subgroups. Denote  $\mathcal{D}$  the set of all possible disclosure plans. Since there are  $n$  realizations, there is a finite number of possible combinations of subgroups, implying that the cardinality of  $\mathcal{D}$  is finite (in fact,  $\#\mathcal{D} \leq n!$ ). Now, the buyer chooses a disclosure plan in  $\mathcal{D}$  to maximize his expected payoff. Since

there are finitely many choices, there exists a disclosure plan that gives the buyer his maximum expected payoff, i.e., an optimal disclosure plan always exists.

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