Learning Within or Outside Firms?  
Labor Market Frictions and Entrepreneurship*

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This Version: October 10, 2017.

Abstract

When labor mobility is imperfect, employers (firms) will invest in the discovery of their employees' talent at different tasks; in this case, agents become entrepreneurs only if they have a valuable business idea or cannot find employment. If instead employees can easily move to other firms, employers have little incentive to invest in talent discovery. In this case, an additional motive for entrepreneurship emerges: learning one's comparative advantage over tasks. We develop such a model and show a causal relationship between the degree of labor-market frictions and the level of entrepreneurial activity; the value of entrepreneurial failures; the payoff of entrepreneurs relative to workers; the wage of former entrepreneurs relative to former workers; the degree of firms' short-termism; the rate of within-firm talent discovery. The theoretical correlations between these variables are consistent with the evidence available for the US and continental Europe.


Keywords Entrepreneurship, labor-market frictions, entrepreneurial failures, organizational choice, learning, task allocation, career concerns.

*This paper supersedes an earlier version titled “The Value of Entrepreneurial Failures: Task Allocation and Career Concerns” (Canidio and Legros 2016). We are grateful to Carlos Caso Dominguez for excellent research assistance. We are also grateful to Chris Avery, Raul Baptista, Jing-Yuan Chiou, Roberta Dessi, Robert Gibbons, Denis Gromb, Thomas Hellmann, Andrea Mantovani, Massimo Riccaboni, Marko Terviö, Peter Thompson, John van Reenen, Timothy van Zandt, as well as participants at the CEPR Workshop in Entrepreneurship Economics (Calgliari 2015), INSEAD (2015), the Eighth Annual Scarle Center Conference on Innovation Economics (2015), the University of Montréal (2016), the Organizational lunch workshop at MIT (2017). Legros gratefully acknowledges the financial support of the European Research Council under the European Union’s Seventh Framework Programme (FP7/2007-2013 / ERC grant agreement n°339950.)

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1 Introduction

We propose a model of occupational choice in which, following MacDonald (1982a,b), an agent’s comparative advantage over different tasks is unknown and requires on the job learning. The rate of learning depends on the task an agent is allocated to, generating the possibility of a conflict between the task allocation maximizing present returns and the task allocation maximizing future returns. The novelty of our model is that the decision right over an agent’s task allocation depends on the agent’s occupation. An agent who chooses to be an entrepreneur acquires the control over his own task allocation, which he will choose to maximize the sum of his present and future payoffs. Instead, an agent who works for a firm cannot choose his own task allocation, which is chosen by his employer. If the employer does not fully internalize the benefit of learning its worker’s comparative advantage, it will favor short-term over long-term return in its task allocation. Hence, a novel determinant of an agent’s occupational choice emerges: learning his comparative advantage at different tasks.

In our model, the probability that an agent receives an external wage offer serves as a proxy for labor market frictions, and dictates whether firms can capture the benefit of learning their workers’ comparative advantage or whether this benefit will be competed away by other firms. When the labor market is frictionless, agents receive wage offers with high probability and employers have little incentives to invest in discovering their talent; as a consequence, employers allocate workers to the task that maximizes short-run profits, and some agents may become entrepreneurs in order to work on the task that is most informative regarding their comparative advantage. We call these entrepreneurs learning entrepreneurs. On the other hand, when labor mobility is subject to frictions, employers may be willing to allocate workers to tasks that do not maximize short-run profits since they can capture some of the benefits from learning. In this case, agents become entrepreneurs only when they have a very valuable idea—we call these entrepreneurs opportunity entrepreneurs—or when they are unable to find a job—we call these entrepreneurs necessity
entrepreneurs. Our analysis allows us to identify these different motives for entrepreneurship and how they respond to changes in labor market frictions.

Labor market frictions, therefore, co-determine firm’s organizational choice and agent’s career choice, and in turn whether learning occurs within or outside firms. The co-variations among different variables in our model are consistent with a number of empirical findings that may look, at first glance, unrelated with each other.

**Empirical evidence.** Because, by most estimates, labor-market frictions in continental Europe are significantly higher than in the USA, we expect the theoretical predictions of the low labor-market frictions case to be consistent with the evidence available for the US, while the predictions of the intermediate labor-market frictions case to be consistent with the evidence available for continental Europe.

In the model, because of the additional learning motive for entrepreneurship, the level of entrepreneurial activity and the proportion of serial entrepreneurs may be higher when labor market frictions are low (as in the US) rather than high (as in continental Europe). This is consistent with findings available in the Global Entrepreneurship Monitor 2015/16 Global Report. The model also suggests that European firms should be less short-termists than their US counterparts, which is well documented (e.g., Becht, Bolton, and Roëll, 2003). The usual explanations in the corporate finance literature are regulation or the dispersion of ownership. Our labor-market effect complements these explanations because the ability of firms to invest in long-term

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1 The crucial role that the labor market plays in determining the willingness of firms to increase their workers’ productivity, either by experimenting or by providing training, is clearly not novel (see for instance Acemoglu and Pischke, 1999). What is novel here is that the level of labor market frictions determines whether learning will be done within or outside firms.

2 Closest to our measure of labor frictions, Ridder and Berg (2003) estimate the rate of arrival of job offers to employed workers for the US, France, UK, Germany and Holland; they show that, with the exception of the UK, European countries have a rate of job arrival that is significantly lower than in the US; Layard, Nickell, and Jackman (2005) find a similar ranking among countries when looking at the arrival rate of job offers to unemployed workers.

projects that require highly specific and talented human capital depends on their ability to identify and retain talent in an incentive compatible way.

When labor market frictions are low, some agents will become learning entrepreneurs even if their instantaneous payoff is lower than the instantaneous payoff they would earn as workers. If the fraction of learning entrepreneurs is sufficiently large relative to that of opportunity entrepreneurs, the average instantaneous payoff of all entrepreneurs will be inferior to that of workers. However, because entrepreneurs learn more than workers, on average, the wage of a former entrepreneur will be greater than that of a former worker. These facts are consistent with Hamilton (2000), who shows that in the US, excluding the few superstars, entrepreneurs earn less than workers on average. For example, the median entrepreneur after 10 years in business earns 35% less than a similar individual who never left employment. At the same time, Hamilton (2000) shows that American entrepreneurs who leave entrepreneurship and re-enter the labor market after some years earn higher wages than comparable workers: the median entrepreneur returning to paid employment after 10 years as an entrepreneur earns a wage that is 15% higher than a comparable worker who never left employment. Our model suggests an opposite result for high labor market friction economies, that is that the wage of former entrepreneurs is lower than the wage of workers who have never left employment, which is consistent with the finding in Baptista, Lima, and Preto (2012) for Portugal.

In the US entrepreneurial failures seem to lead to entrepreneurial success. For example, Gompers, Kovner, Lerner, and Scharfstein (2010) show that entrepreneurs who previously failed are marginally more likely to succeed than first time entrepreneurs. Again the evidence available for Europe tells a very different story. Using German data, Gottschalk, Greene, Höwer, and Müller (2014) show that entrepreneurs who have previously failed are subsequently more likely to fail than first time entrepreneurs. Our model explains these dif-

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4 See Table 6 and the discussion on pages 625-626 of Hamilton (2000). Hamilton notes that this result is consistent with the findings of Evans and Leighton (1990). See also Daly (2015) for similar results.

Different values of failure if talent is horizontal: different agents have an absolute advantage at different tasks. In this case, depending on the task allocation, failures can reveal that the agent would be more productive if allocated to a different task, and therefore lead to future successes. Labor market frictions will determine the task allocation of entrepreneurs, and whether entrepreneurial failures carry a stigma or a reward. Instead, when talent is vertical (that is, the same agent has an absolute advantage at all tasks) failures are always bad news, independently of the level of labor market frictions, a finding which seems counterfactual.

Finally, one may be tempted to interpret the case of extremely high labor market frictions as illustrative of developing countries, and there is indeed ample evidence that many entrepreneurs in developing countries are necessity entrepreneurs. We refrain from this temptation however because while the US and the EU are relatively similar in their contracting abilities, the development of their financial markets, and their level of human capital, this is hardly the case for developing countries. These other dimensions are not part of our model but are likely to affect the type, frequency and market rewards of entrepreneurial ventures in developing countries.

The rest of this paper proceeds as follows. The reminder of this section discusses the relevant literature. In Section 2 we introduce the model. In Section 3 we derive conditions under which a trade off between learning and short-run profit maximization emerges. We assume that this tradeoff is present, and derive the equilibrium of the model in Section 4. We conclude in Section 5. Unless otherwise noted, all mathematical derivations are in Appendix A.

**Literature**

Our main contribution is to establish that agents’ occupational choices can be determined by the desire to learn their comparative advantage. We also contribute to the literatures on learning in the labor market, entrepreneurial failures and incentives for experimentation.
Occupational choices and learning comparative advantages. The literature on occupational choice started by Banerjee and Newman (1993) and Galor and Zeira (1993) has focused on the role played by the wealth distribution, financial market and enforcement frictions in determining the choice of individuals between self-employment, employment, or firm creation. In our paper the key determinant of the career and income paths of individuals is learning agents’ comparative advantages at different tasks. We therefore complement a literature that, going back to Vereshchagina and Hopenhayn (2009), has studied the choice between wage work and entrepreneurship under the assumption that the return on entrepreneurship is uncertain but can be learned. Within this literature, Manso (2016) and Dillon and Stanton (2017) show that, whenever agents who become entrepreneurs can go back to wage work, the instantaneous payoff of entrepreneurs may be lower than that of comparable workers (as in Hamilton, 2000). We also rationalize this empirical fact, but in our model agents learn their comparative advantage at different tasks, and by doing so increase their productivity at all possible occupations. Hence, agents do not learn their entrepreneurial ability, but rather they learn their ability tout court. This explains why in our model, in line with the additional empirical evidence in Hamilton (2000), a prior experience as an entrepreneur may be rewarded by the labor market in subsequent periods.

Talent discovery in the labor market. In pioneering papers, MacDonald (1982a,b) analyzes, in the context of a frictionless labor market and employment as the unique occupation, a task-assignment problem with symmetric uncertainty about talent. Gibbons and Waldman (1999) and Gibbons and Waldman (2004) develop within-firm task assignment models in which there is learning about an agent’s talent via task allocation, as well as human capital accumulation. Papageorgiou (2013) studies the link between labor-market frictions and talent discovery. His model assumes that firms are identified with one task, hence cannot choose their internal organization. In his framework, agents must move between firms to discover their comparative advantage. Hence, as labor-market frictions increase, mobility decreases and the rate of
talent discovery must decrease. This is not always true in our model because agents can learn within firms, and more severe labor-market frictions enhance learning in firms.

Pastorino (2013) structurally estimates a labor market model in which firms generate information about their workers via task assignment, and measures the importance of learning relative to human capital accumulation in explaining cumulative wage growth and wage dispersion. Antonovics and Golan (2012) address the issue of experimentation, defined as choosing a job where the expected probability of success is low, but where outcomes are strongly correlated with the agent’s type. Similarly, Terviö (2009) argues that cash constraints or the absence of long-term contracting prevent optimal talent discovery, in the sense that too few workers will be employed in jobs where their productivity can be revealed. In Canidio and Gall (2013) the rate of on-the-job talent discovery depends on the task allocation chosen within firms, and therefore the optimal rate of talent discovery may not be achieved because workers are allocated to tasks that are not informative.

While there are some important connections with all these papers, none of them allow agents to change occupation, that is, to become entrepreneurs.

**Value of failures.** It is a common assumption in the economic literature that failures provide bad news about the expected productivity of an agent. Prominent examples in the literature on entrepreneurship are Gromb and Scharfstein (2002) and Landier (2005), who build equilibrium models in which entrepreneurial failures always produce a stigma, which may be more or less pronounced depending on some features of the economy. In Gromb and Scharfstein (2002), failed entrepreneurs are hired by firms. Because of exogenous noise, failing in a start-up is not as bad a signal as being fired as a manager, and firms will replace failed managers with failed entrepreneurs. Landier (2005) shows that when failures are widespread, little information regarding the entrepreneur’s type is revealed by a failure and hence there is a high level of entrepreneurship. On the other hand, when failures are rare, they carry a
larger stigma and entrepreneurship is deterred.\footnote{See also \textit{Schumacher, Gerling, and Kowalik} (2015).}

By contrast, many business leaders and scholars share Henry Ford’s view that a failure “is only the opportunity to begin again more intelligently.” A recent issue of \textit{Harvard Business Review} (April 2011) collects several papers under the heading “Failure Chronicles,” each describing an example of failure, and how it ultimately led to business success. A recent book by the journalist Tim Harford, \textit{Adapt: Why Success Always Starts with Failure} well summarizes this positive attitude in the business world toward entrepreneurial failures.

Our contribution is to link the value of failures with the \textit{nature of talent}, horizontal or vertical. We show that talent can be good news or bad news depending on the level of labor market frictions only if talent is horizontal. If instead talent is vertical, failures are always bad news, independently on the level of labor market frictions. The model therefore provides evidence in favor of the horizontal view of talent.

\textbf{Experimentation and incentives.} The literature on experimentation and incentives (Jeitschko and Mirman, 2002; Manso, 2011; Drugov and Macchilavello, 2014; Gomes et al., 2016) focuses on how to design a contract that motivates an agent to experiment. By contrast, in our model the choice of task allocation (and therefore of whether to engage in learning) rests with the firm. We will therefore study how to design a contract that motivates a firm to learn.

Finally, at the core of our model there is a tradeoff between short-run profit maximization and learning. This tradeoff has been extensively studied by the literature on multi-arms bandit problems, and is therefore neither new nor specific to our model. However, this literature typically assumes that the arms are independent: success and failures at an arm is not informative with respect to the other arm. Hence, failures always reduce the probability of future success. This case is therefore equivalent to the vertical talent case. As already discussed, the horizontal talent case better matches the empirical evidence showing that entrepreneurial failures may lead to an increase of the
future probability of an entrepreneurial success.

2 The model

The economy is composed of a continuum of agents who live for two periods $t \in \{1, 2\}$. Each agent can be of type $\theta \in \{l, h\}$, where $l$ stands for low and $h$ for high. There is free entry of firms in both periods. Agents’ types are not observable by agents or firms. The common initial belief about a young agent’s type is $\Pr\{\theta = h\} = p_1$.

Production and returns. In period $t$ there is an “off-the-shelf” technology accessible to all firms. Depending on demand conditions or other aggregate shocks, this technology gives each firm a monetary return $K_t$ when there is success and 0 when there is failure. We assume that $K_t$ is drawn at the beginning of each period from the uniform distribution on $[0, 2]$, which implies that the aggregate shocks determining $K_t$ are uncorrelated across periods and hence the returns $K_1, K_2$ are independent.

In period $t$, each agent gets an idea about a project. If the agent becomes an entrepreneur, he can pursue this project and generate a monetary return $k_t$ in case of success. The aggregate shocks determining $K_t$ also affect an agent’s specific $k_t$, and we assume that each $k_t$ is drawn from the uniform distribution on $[0, \lambda K_t]$, where $\lambda \geq 1$. Hence, $k_t$ is drawn independently over time and across agents.

In each period $t$, an agent can work either in a firm or as an entrepreneur. In both cases, he can work either on an Advanced task ($\tau_t = A$) or a Basic task ($\tau_t = B$), and may fail ($s_t = 0$) or succeed ($s_t = 1$). The probability of success depends on the agent’s type and the task chosen:

<table>
<thead>
<tr>
<th>$\tau \setminus \theta$</th>
<th>$l$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$l_B$</td>
<td>$h_B$</td>
</tr>
<tr>
<td>$A$</td>
<td>$l_A$</td>
<td>$h_A$</td>
</tr>
</tbody>
</table>

When each type is assigned to the task at which he is the most likely to
succeed, high types have an advantage over low types:

\[ \max(h_A, h_B) \geq \max(l_A, l_B). \]  

(1)

To avoid trivialities, we assume that individuals have different comparative advantages, high types being better at the advanced task while low types being better at basic tasks:

\[ h_A - h_B > 0, \; l_B - l_A > 0. \]  

(2)

For instance, some agents may excel at finding creative solutions to a new problem but will be unproductive at following strict orders; others flourish and can be creative in a team environment but will be low performers in isolation. The environment described in (1)-(2) is a discrete version of MacDonald (1982a,b) and nests two standard visions of talent.

- **(Vertical talent)** If \( h_B \geq l_B \) the probabilities of success at both tasks are at least as large for type \( h \) than type \( l \). Hence types can be ranked in terms of productivity. High types have an absolute advantage over low types: they have higher “quality” independently of the task they are working on. This is the usual interpretation of talent as a vertical dimension.

- **(Horizontal talent)** When \( h_B < l_B \), high type agents have a larger probability of success only if assigned to the advanced task \( A \). Otherwise, if assigned to the basic task, a high type agent is in fact less successful than a low type agent. Talent is horizontal rather than vertical, and it is not possible to rank types in terms of productivity unless the task assignment is defined.

**Contract offers.** We will restrict attention to short-term contracts. In every period, a contract consists of a fixed payment \( f \) and a bonus payment \( b \)

\[ 7 \text{ If this is not the case, there is a task that maximizes the probability of success of each type, and no firm or entrepreneur will use the other task since learning has no value for task allocation.} \]
contingent on success. We make the following additional assumptions on the contracting environment.

**Assumption 1.**

(i) *Output is not fully contractible and the bonus is strictly bounded above by the monetary return of the firm, that is* \( b \leq \beta K_t \) *where* \( \beta < 1 \).

(ii) *Task allocations within firms are observable but not contractible.*

We interpret the parameter \( \beta \) in (i) as an index of contract completeness. Within a firm, the value of a success is \( K_t \), but contracts can be contingent only on \( \beta K_t \); this will imply that firms cannot be made indifferent between success or failures. For instance, if firm owners can “run away” and capture a proportion \( 1 - \beta \) of the monetary return, bonus payments with a share of monetary returns greater than \( \beta \) are not incentive compatible.

The second part (ii) of the assumption implies that contracts cannot be made contingent on task allocation. This is consistent with the modern literature on delegation which emphasizes that ownership restricts the ability not to interfere with other agents’ decisions, in particular in the context of the delegation of tasks (Aghion and Tirole 1997; Baker et al. 1999).

Our restrictions to short-term contracts and observable task allocations simplify the analysis but are not essential. In Appendix B we consider the case of unobservable task allocation, and show that our results hold in this case as well. In Appendix C we introduce the possibility of using long-term contracts. Not surprisingly, long-term contracts improve the value of entering in an employment relationship. However, they do not eliminate the probability that an agent becomes an entrepreneur to learn his type. It follows that our results hold qualitatively in that case as well.

**Labor-market frictions.** We introduce labor-market frictions by assuming that with probability \( 1 - \alpha \) an agent receives no offer from firms, and with probability \( \alpha \) he receives at least two offers. This would be the case for instance
if there is a central place where all vacancies are posted and an agent has access to an imperfect search technology.

**Timing** In each period $t$, the timing of events is as follows (see Figure 1):

1. Returns $K_t$ for firms and $k_t$ for each agent are realized. $K_t, k_t$ are observable.
2. Firms simultaneously offer contracts to all agents.
3. Agents who receive an employment offer choose between entrepreneurship and employment; Agents who do not receive an employment offer become entrepreneurs.
4. After a contract is signed, the firm chooses the worker’s task. Entrepreneurs choose their own task.
5. Outcomes are realized and observed by everybody. In the case of success, a firm’s output is $K_t$, while an entrepreneur’s output is $k_t$.

![Fig. 1: Timing within period $t$](image)

The above timing repeats identical each period. However, success and failures on a given task in period 1 are informative about an agent’s type, and can be used to decide period-2 task allocation. Hence, period-1 career choice and task allocation problem differ from period 2-career choice and task allocation problem because of learning.

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8 Hence there is a zero probability of receiving a single offer. If the probability of an agent’s receiving a single offer is positive, firms can design their contracts knowing that, with a small probability, they might have monopsony power over the agent. This significantly complicates the firm’s problem but does not modify our qualitative results.
3 When Learning Conflicts with Short-Term Return Maximization

In this section we derive conditions under which there is a conflict in period-1 between the task allocation maximizing the present probability of success and the task allocation maximizing the future expected probability of success. These conditions are necessary for a meaningful tradeoff between learning and short-run profit maximization to emerge. The main results from this section are to establish these conditions, and to show that failures may be valuable (in the sense of increasing the future probability of success) if talent is horizontal and are never valuable if talent is vertical. The reader interested in the equilibrium analysis for occupational choice and wage setting could go directly to section 4 below.

For any prior belief \( p_t \) that the individual is of type \( h \), the probability that there is a success in a given period is:

\[
\pi(\tau_t, p_t) \equiv \begin{cases} 
(1 - p_t) \cdot l_A + p_t \cdot h_A & \text{if } \tau_t = A \\
(1 - p_t) \cdot l_B + p_t \cdot h_B & \text{if } \tau_t = B.
\end{cases}
\]

It follows that the probability of success in the current period is maximized by assigning the agent to task \( B \) if and only if \( p_t \) is smaller than the cutoff value

\[
q^* \equiv \left(1 + \frac{h_A - h_B}{l_B - l_A}\right)^{-1}.
\]

Call \( \pi^M(p_t) \) the maximum probability of success in a given period, defined as

\[
\pi^M(p_t) \equiv \max_{\tau_t} \pi(\tau_t, p_t) = \begin{cases} 
(1 - p_t)l_B + p_t h_B & \text{if } p_t \leq q^* \\
(1 - p_t)l_A + p_t h_A & \text{if } p_t \geq q^*.
\end{cases}
\]

that is, the probability of instantaneous success assuming that the agent is allocated to the task with the largest probability of success. Because period 2 is the last period of the game, in that period both entrepreneurs and firms
choose the task allocation that maximizes the instantaneous probability of success, and therefore $\pi^M(p_2)$ is the equilibrium probability of success in period 2 for given $p_2$.

Before continuing with the analysis it is useful to observe that the shape of $\pi^M(p_t)$ depends on whether talent is vertical ($h_B \geq l_B$) or horizontal ($h_B < l_B$). The difference between the two cases is illustrated in Figure 2. When talent is horizontal, and the agent is assigned to task $B$, the success probability is decreasing in the prior since high types are less productive than low type on task $B$. By contrast, in the vertical case, the success probability is increasing in the prior for any task assignment. This explains why when $p_t < q^*$ the success probability $\pi^M(p_t)$ is increasing in $p_t$ if talent is vertical, but is decreasing in $p_t$ if talent is horizontal.

![Fig. 2: Maximum probability of success as a function of belief $p_t$.](image)

For ease of exposition, we define the period-1 probability of success as $\sigma_1(\tau_1) \equiv \pi(\tau_1, p_1)$, that is the probability of instantaneous success at the initial belief $p_1$. Without loss of generality, we assume that task $B$ is the short-term output maximizing task.

**Assumption 2.** $p_1 < q^*$: task $B$ maximizes the initial probability of success, that is $\sigma_1(B) > \sigma_1(A)$.

We are interested in establishing conditions under which $\tau_1 = A$ maximizes the period-2 expected probability of success. The posterior belief given a task
allocation $\tau_1$ in the first period and whether there is success ($s_1 = 1$) or failure ($s_1 = 0$) at the end of period 1 is:

$$p_2(\tau_1, s_1) \equiv \begin{cases} \left( \frac{1-p_1}{p_1} \frac{1}{h_{\tau_1}} + 1 \right)^{-1} & \text{if } s_1 = 1 \\ \left( \frac{1-p_1}{p_1} \frac{1}{1-h_{\tau_1}} + 1 \right)^{-1} & \text{if } s_1 = 0 \end{cases}$$

Comparing two posteriors is therefore equivalent to comparing the likelihood of facing a high type: for instance, $p_2(\tau_1, 1) > p_2(\tau_1, 0)$ if, and only if, the likelihood of facing a high type is greater after a success than after a failure: $h_{\tau_1} > 1 - h_{\tau_1}$.

From period 1’s point of view, choosing task $\tau_1$ yields in period 2 an expected probability of success equal to:

$$\sigma_2(\tau_1) \equiv \mathbb{E}_{s_1 \in \{0, 1\}} \pi_M(p_2(\tau_1, s_1)),$$

We are looking for conditions that insure that $A$ is (strictly) more informative than $B$, that is $\sigma_2(A) > \sigma_2(B)$. We present first a necessary condition and then a sufficient condition.

**Necessary condition for informativeness.** Independently of the task assignment in the first period, Bayesian updating implies that

$$\mathbb{E}_{s_1 \in \{0, 1\}} \pi(\tau_1, p_2(\tau_1, s_1))p_2(\tau_1, s_1) = p_1. \quad (4)$$

Because of Assumption 2 there is a realization of $s_1$ such that the posterior $p_2(\tau_1, s_1)$ is inferior to $q^*$, leading to task $B$ being adopted in period 2. Since the expected probability of success $\pi_M(p_t)$ is linear when $p \leq q^*$, a necessary condition for $A$ to be more informative than $B$ is that $\max_{s_1} p_2(A, s_1) > q^*$. Because in both the vertical and horizontal cases $\frac{h_A}{l_A} > \frac{1-h_A}{1-l_A}$, the maximum posterior following task $A$ is achieved following a success. More informative-
ness of }A\text{ therefore requires that }p_2(A,1) > q^*, \text{ that is }
\begin{equation}
p_1 > q_A \equiv \left(1 + \frac{h_A h_A - h_B}{l_A l_B - l_A}\right)^{-1}
\end{equation}

**Sufficient condition for (weak) informativeness.** Since the maximum probability of success is a convex function of the posterior, whenever the distribution of posteriors following }\tau_1 = A\text{ is a mean preserving spread of the distribution following }\tau_1 = B,\text{ we will have }\sigma_2(A) \geq \sigma_2(B).\text{ Using our previous remark that }\max_{s_1} p_2(A,s_1) = p_2(A,1),\text{ the distribution of posteriors following }A\text{ is a mean-preserving spread of the distribution following }B\text{ whenever: }
\begin{equation}
p_2(A,0) < \min_{s_1} p_2(B,s_1) < p_1 < \max_{s_1} p_2(B,s_1) < p_2(A,1).
\end{equation}
Under the above condition, }\sigma_2(A) = \sigma_2(B)\text{ if and only if }p_1 \leq q_A,\text{ that is if and only if no matter the task allocation and the realization of success and failure in period 1 the agent is always allocated to task }B\text{ in period 2. Hence, (MPS) and }p_1 > q_A\text{ are sufficient for }\sigma_2(A) > \sigma_2(B).

When talent is vertical, }h_A > h_B > l_B > l_A\text{, and the posteriors are ordered as }
\begin{equation}
p_2(A,0) < p_2(B,0) < p_1 < p_2(B,1) < p_2(A,1).
\end{equation}
and (MPS) is automatically satisfied.

**Proposition 1.** Under assumption 2, in the vertical talent case there is a conflict between maximizing today’s probability of success and tomorrow’s if and only if }p_1 > q_A.

When talent is horizontal, }l_B > h_B\text{ implies that }
\begin{equation}
p_2(B,1) < p_1 < p_2(B,0) \text{ and } p_2(A,0) < p_1 < p_2(A,1),
\end{equation}
but not necessarily (MPS). The distribution of posteriors following }A\text{ is a mean preserving spread of the distribution of posterior following }B\text{ whenever
p_2(A,1) > p_2(B,0) and p_2(A,0) < p_2(B,1). Simple algebra shows that these conditions are equivalent to h_A - l_A > l_B h_A - l_A h_B > l_B - h_B. Hence,

**Proposition 2.** Under assumption \([2]\) in the horizontal talent case there is a conflict between maximizing today’s probability of success and tomorrow’s if:

\[ p_1 > q_A \quad \text{and} \quad h_A - l_A > l_B h_A - l_A h_B > l_B - h_B. \]

When the \([\text{MPS}]\) condition is not satisfied in the horizontal talent case, there are other possibilities for having \(\sigma_2(A) > \sigma_2(B)\). We give in Proposition \([5]\) in the Appendix the necessary and sufficient conditions for \(\sigma_2(A) > \sigma_2(B)\) for the horizontal talent case.

Figure \([3]\) below illustrates a typical tradeoff between period-1 opportunity cost \(\sigma_1(B) - \sigma_1(A)\) and period-2 gain \(\sigma_2(A) - \sigma_2(B)\) from choosing \(\tau_1 = A\) instead of \(\tau_1 = B\)\(^9\).

![Fig. 3: Vertical talent example: \(h_A = 0.7\); \(h_B = 0.6\); \(l_B = 0.5\); \(l_A = 0.4\)](image)

What is learned from failures differs in the vertical and horizontal cases. When talent is vertical, we showed earlier that \(p_2(A,0) < p_2(B,0) < p_1\), and failures always reduce the probability of being a \(h\) type (more so when the failure is at task \(A\)) since \(h\) types are more likely to succeed than \(l\) types at any task. Because the function \(\pi^M(p_t)\) is monotonically increasing, we have the inequalities \(\pi^M(p_2(A,0)) < \pi^M(p_2(B,0)) < \pi^M(p_1)\) and failures are always

---

\(^9\) The units have been rescaled; the intersection of the two curves is at \(p < 5/11\), that is when task \(B\) is always chosen in the second period if task \(B\) is chosen in the first period. Horizontal talent leads to similar looking curves.
bad news because they decrease the probability of success in period 2 relative to the initial probability of success.

Instead, in the horizontal case low types are more likely to succeed at task B than high types and therefore $p_2(A,0) < p_1 < p_2(B,0)$ and a failure at task B increases the probability that the agent is of type h. Furthermore, the function $\pi^M(p_2)$ is decreasing for $p_2 < q^*$ and then increasing, implying that $\pi^M(p_2(A,0)) > \pi^M(p_1)$ and that failures at task A are good news because they lead to an increase of the future probability of success (relative to no history). Failures at task B may be good or bad news depending on $p_1$: if $p_1$ is sufficiently close to $q^*$ failures at task B are also good news; if instead $p_1$ is sufficiently low (for example, $p_1$ such that $\pi^M(p_2(B,0)) < q$), then failures at task B are bad news. The following Lemma summarizes these observations.

Lemma 1. (i) In the vertical-talent case failures are always bad news, that is, $\pi^M(p_2(\tau_1,0)) < \pi^M(p_1)$ for all $\tau_1 \in \{A,B\}$.

(ii) In the horizontal-talent case, failures at task A are always good news, that is, $\pi^M(p_2(A,0)) > \pi^M(p_1)$. There is a threshold $q_B$ such that failures at task B are bad news for $p_1 < q_B$ and good news for $p_1 > q_B$.

Having established the possibility of a conflict in period 1 between instantaneous success and learning, we now analyze how this conflict influences career choices and returns from these choices. We will take as given that $\sigma_1(A) < \sigma_1(B)$ and that $\sigma_2(A) > \sigma_2(B)$.

4 Equilibrium Analysis

We start the analysis by period 2, and contrast the choices of agents who receive and do not receive wage offers.

With probability $\alpha$, an agent receives at least two offers, and if she accepts one of them she earns the full expected return of the firms’ project $\sigma_2(\tau_1)K_2$[10].

[10] Because period 2 is the last period of the game, firms and workers have the same preferences over task allocation: they prefer the task allocation that maximizes period-2 output. Hence, the exact structure of a period-2 contract (that is, what part is paid as bonus $b$ and what part is paid as fixed wage $f$) is not relevant.
It follows that if an agent receives an employment offer the choice of becoming an entrepreneur or employee depends on what project is more valuable. Hence, from period-1 point of view, the expected period-2 payoff conditional on receiving an employment offer is\footnote{The calculations omitted from the text are in Appendix, page 40}

\[ \sigma_2(\tau_1) \cdot \mathbb{E}[\max\{k_2, K_2\}] = \sigma_2(\tau_1) \cdot \frac{1}{2} \left( \lambda + \frac{1}{\lambda} \right), \]

which depends on period-1 task allocation via the probability of period-2 success.

When an agent does not receive offers (with probability \(1 - \alpha\)), his period-2 payoff depends not only on his period-1 task allocation but also on his period-1 occupation. A period-1 entrepreneur remains an entrepreneur in period-2 whenever he does not receive a wage offer in period-2, and therefore earns \(k_2\) in case of success. Hence, the expected period-2 payoff of a period-1 entrepreneur is:

\[ \sigma_2(\tau_1) (\alpha \mathbb{E}[\max(k_2, K_2)] + (1 - \alpha) \mathbb{E}[k_2]) = \sigma_2(\tau_1) \left( \alpha \frac{1}{2} \left( \lambda + \frac{1}{\lambda} \right) + (1 - \alpha) \frac{\lambda}{2} \right), \]

which is increasing in \(\alpha\). Instead, a period-1 employee who does not receive wage offers can continue working for his period-1 employer. In this case, the agent and his period-1 employer need to split a surplus given by the difference between the value of continuing the employment relationship and the value of entrepreneurship, that is \(\sigma_2(\tau_1) \max\{K_2 - k_2, 0\}\), where \(\mathbb{E}[\max\{K_2 - k_2, 0\}] = \frac{1}{2\lambda}\). Without loss of generality we assume that this surplus is split by Nash bargaining. Hence, from period-1 point of view, each firm earns a period-2 expected profit equal to

\[ (1 - \alpha)\sigma_2(\tau_1) \mathbb{E} \left[ \frac{1}{2} \max\{K_2 - k_2, 0\} \right] = \sigma_2(\tau_1) \frac{1 - \alpha}{4\lambda}. \]

These profits are decreasing in \(\alpha\) and, crucially, for \(\alpha < 1\) are larger when \(\tau_1 = A\) than when \(\tau_1 = B\). That is, because of labor market frictions, in
period 2 firms may be able to earn part of the benefit of learning their workers’
talent. Similarly, from period 1 point of view, the expected period-2 payoff of
a period-1 worker is:

\[
\sigma_2(\tau_1) \left( \alpha \mathbb{E}[\max(k_2, K_2)] + (1 - \alpha) \mathbb{E} \left[ k_2 + \frac{1}{2} \max\{K_2 - k_2, 0\} \right] \right) \\
= \sigma_2(\tau_1) \left( \frac{\alpha}{2} \left( \lambda + \frac{1}{\lambda} \right) + \frac{5(1 - \alpha)}{4\lambda} \right)
\]

We now move to the analysis of period 1. We first derive the optimal task
allocation and the equilibrium two-period payoff of workers and entrepreneurs.
We then compare these two-period payoffs to determine the equilibrium career
choice in period 1.

**Period-1 entrepreneur.** The expected period-1 payoff of an entrepreneur is
\(\sigma_1(\tau_1) \cdot k_1\). Hence, task \(\tau_1\) generates an expected return over the two periods
equal to

\[
\sigma_1(\tau_1)k_1 + \sigma_2(\tau_1) \left( \frac{\alpha}{2} \left( \lambda + \frac{1}{\lambda} \right) + (1 - \alpha)\frac{\lambda}{2} \right)
\]

An entrepreneur chooses \(\tau_1 = A\) whenever

\[
k_1 \leq k^A(\alpha) \equiv \frac{\alpha + \lambda^2}{2\lambda} \times \frac{\sigma_2(A) - \sigma_2(B)}{\sigma_1(B) - \sigma_1(A)}
\]  \quad (6)

That is, the entrepreneur will favor learning over short-run profits whenever
the current value of a success is low relative to the future expected value of
a success. Higher labor market frictions (i.e., lower \(\alpha\)) reduce the probability
that the agent will receive a wage offer and that she will work for a firm when
\(K_2 > k_2\). Hence, from the point of view of period-1, as labor-market frictions
become more severe the value of a period-2 success decreases, learning becomes
less valuable, and the entrepreneur is more likely to choose task \(B\).
Finally, the two period payoff of a period-1 entrepreneur is:

\[
W^E(k_1, \alpha) = \begin{cases} 
\sigma_1(A)k_1 + \sigma_2(A)\frac{\alpha + \lambda^2}{2\lambda} & \text{if } k_1 \leq k^A(\alpha) \\
\sigma_1(B)k_1 + \sigma_2(B)\frac{\alpha + \lambda^2}{2\lambda} & \text{if } k_1 > k^A(\alpha),
\end{cases}
\]

which is continuous and strictly increasing in both arguments.

**Period-1 workers.** The total output generated within a firm is

\[
\sigma_1(\tau_1)K_1 + \sigma_2(\tau_1)\frac{\lambda^2 + 1}{2\lambda}.
\]

This expression is also equivalent to the total output generated by an entrepreneur with project \(k_1 = K_1\) in the absence of labor market frictions (that is, when \(\alpha = 1\)). It follows that the two period total output generated within firms is maximized by task \(A\) if, and only if, \(K_1 \leq k^A(1)\), where \(k^A(1)\) is defined in (6).

However, the output maximizing task allocation may not be incentive compatible. Remember that firms earn zero profits in equilibrium and therefore the period-2 profits that a period-1 employer expects to earn in case its employee does not receive an outside wage offer are factored into the period-1 contract offered to the worker. However these period-2 profits are relevant in deriving the period-1 task allocation. After a contract \((f, b)\) is signed in period 1, the fixed component \(f\) is sunk and the determinants of the optimal task choice are the bonus \(b\) and the expected period-2 profits. Choosing task \(A\) generates a period-1 opportunity cost equal to \((\sigma_1(B) - \sigma_1(A))(K - b)\), a decreasing function of \(b\). By contrast the future benefit of choosing task \(A\) in the first period is \((\sigma_2(A) - \sigma_2(B))\frac{1 - \alpha}{4\lambda}\), that is the expected value of the share of surplus accruing to the firm in case its worker does not receive a wage offer.

Because, by assumption, the largest possible bonus \(b\) is \(b = \beta K_1\), the firm can commit to implement task \(A\) in the first period if \((\sigma_1(B) - \sigma_1(A))(1 -
(β) $K_1 \leq (σ_2(A) - σ_2(B))\frac{1-α}{4λ}$, that is when

$$K_1 \leq K^A(α) \equiv \frac{1-α}{4λ(1-β)} \times \frac{σ_2(A) - σ_2(B)}{σ_1(B) - σ_1(A)}. \tag{7}$$

Since the smallest possible bonus is zero, the firm can commit to implement task $B$ if $(σ_1(B) - σ_1(A))K_1 \geq (σ_2(A) - σ_2(B))\frac{1-α}{4λ}$, that is when

$$K_1 \geq K^B(α) \equiv \frac{1-α}{4λ} \times \frac{σ_2(A) - σ_2(B)}{σ_1(B) - σ_1(A)}.$$

Clearly, for any $α$, $K^B(α) < k^A(1)$ and the firm can always implement task $B$ whenever it is output maximizing to do so.

A sufficiently large bonus $b$, therefore, serves as a commitment to implement the most informative task (task $A$). The observation that larger bonuses can generate more learning contrasts with that of [Manso (2011)] who argues that a principal may motivate a worker to experiment by paying a fixed wage initially and a large bonus for success far in the future. The reason for this contrast is that in [Manso (2011)] the choice of learning rests with the worker while in our model the choice of learning via task allocation rests with the firm. Hence if a large bonus is paid to the worker, the firm’s payoff is less sensitive to the realization of failures and success and therefore the firm is more likely to choose the learning-maximizing task allocation.

Competition among firms for workers allows us to reduce the firm’s problem to the choice of a task $τ_1$ that maximizes the two-period total output subject to the incentive compatibility constraints, that is:

$$W^F(K_1, α) \equiv \begin{cases} \max_{τ_1=A,B} σ_1(τ_1)K_1 + σ_2(τ_1)\frac{1+λ^2}{2λ} & \text{if } τ_1 = A \Rightarrow K_1 \leq K^A(α). \end{cases}$$

By observing that the incentive compatibility constraint is binding if and only if $K^A(α) < k^A(1)$ we arrive at the following lemma:

**Lemma 2.** (i) In a competitive equilibrium, firms choose contracts that implement task $τ_1 = A$ if $K_1 \leq \min \{K^A(α), k^A(1)\}$ and task $τ_1 = B$. 
otherwise.

(ii) Whenever $K_1 \geq k^A(1)$ or $K_1 \leq K^A(\alpha)$ the equilibrium task allocation maximizes the two-period total output. Whenever $K_1 \in (K^A(\alpha), k^A(1))$ the firm’s task allocation is inefficient: the two-period total output is maximized by $\tau_1 = A$ but firms implement $\tau_1 = B$.

Proof. In the text.  

Figure 4 provides a graphical illustration of the lemma. If $\alpha$ is sufficiently low, then for a given $K_1$ the firm allocates the worker to the task that maximizes the two-period total output. In particular, the firm sets $\tau_1 = A$ whenever $K_1$ is below $k^A(1)$ and $\tau_1 = B$ otherwise. If instead $\alpha$ is high, there is a range of $K_1$ for which it would be optimal to implement $\tau_1 = A$, but no contract can achieve it. In this case, for low $K_1$ the firm maximizes the two-period total output by setting $\tau_1 = A$, for high $K_1$ the firm maximizes two-period total output by setting $\tau_1 = B$, for intermediate $K_1$ the firm sets $\tau_1 = B$ despite the fact that $\tau_1 = A$ generates higher two-period total output. For every $\alpha$, the size of this last region depends on the degree of contract incompleteness $\beta$. In particular, for higher $\beta$ (i.e., a large fraction of output is contractible) the firm is able to pay larger bonuses, and is therefore more likely to maximize the two-period total output. The opposite holds for low $\beta$ (i.e., a large fraction of output not contractible).

Hence, the inability of firms to commit to a task allocation makes them short-termists when $K_1 \in (K^A(\alpha), k^A(1))$, which is more likely to happen when labor market frictions are low (i.e., $\alpha$ is high). When there are no labor market frictions ($\alpha = 1$), firms always implement the short-run output maximizing task allocation, and learning cannot occur within firms.

4.1 Equilibrium Occupational Choice – The Nature of Entrepreneurship

In period 1, a fraction $1 - \alpha$ of agents do not receive a wage offer and therefore become entrepreneurs. We call these agents necessity entrepreneurs. The
agents who receive a wage offer will choose their occupation by comparing the two period payoff earned as an entrepreneur with two period payoff earned as a worker. Figure 5 plots these payoffs as a function of $K_1$ and $k_1$ when $\alpha$ is small and large; the red arrows indicate the change in the curves when $\alpha$ increases.

The discontinuity in $W^F(K_1, \alpha)$ for high values of $\alpha$ illustrates the incentive problem faced by the firm in choosing task allocation. As $\alpha$ increases workers are more likely to receive competing offers, and firms will assign workers more often (that is for a large set of $K_1$) to the basic task. This assignment is inefficient for $K_1 < k^A(1)$ but maximizes short-run profits. The discontinuity arises at $K^A(\alpha)$, which is the value of $K_1$ at which the firm is indifferent between assigning the worker to either task. The reason is that when $K_1 = K^A(\alpha)$ the firm can credibly commit to assign the worker to the advanced one, leading to an upward jump in the value of working for a firm whenever the
advanced task is the efficient one at $K^A(\alpha)$. Because $K^A(\alpha)$ is decreasing in $\alpha$, the expected two-period payoff from employment decreases also decreases in $\alpha$. At the same time, $W^E(k_1, \alpha)$ increases with $\alpha$. Therefore, *ceteris paribus* as $\alpha$ increases agents are more likely to choose entrepreneurship when they obtain a wage offer.

Formally, an agent who receives a wage offer will become an entrepreneur whenever

$$W^E(k_1, \alpha) \geq W^F(K_1, \alpha)$$

We call $k^E(K_1, \alpha)$ the project value $k_1$ leaving an agent indifferent between becoming an entrepreneur and working for a firm:

$$k^E(K_1, \alpha) \equiv k_1 \text{ solution to } W^E(k_1, \alpha) = W^F(K_1, \alpha).$$

Note that the payoff earned from working for a firm can be rewritten as:

$$W^F(K_1, \alpha) = \max_{\tau_1 \in \{A, B\}} \left\{ \sigma_1(\tau_1)K_1 + \sigma_2(\tau_1) \left( \frac{1}{2} \left( \lambda + \frac{1}{\lambda} \right) \right) \right\}$$

$$-I \left\{ K_1 \in [K^A(\alpha), k^A(1)] \right\} (\sigma_1(A) - \sigma(B))K_1 + (\sigma_2(A) - \sigma_2(B)) \frac{1 + \lambda^2}{2\lambda}.$$
that is, total output assuming that the task allocation implemented within firms is optimal, minus a loss whenever learning cannot occur within firms, which is realized whenever $K_1 \in [K^A(\alpha), k^A(1)]$. Given this, we can categorize the mass of agents who become entrepreneurs after receiving a wage offer into two groups:

- **Opportunity entrepreneurs**: These agents prefer entrepreneurship to working for a firm for any task they may be allocated to within the firm. In other words, these agents have a project value $k_1$ larger than $k_O(K_1, \alpha) \equiv k_1: \max_{\tau_1 \in \{A, B\}} \left\{ \sigma_1(\tau_1)K_1 + \sigma_2(\tau_1)\frac{1+\lambda^2}{2\lambda} \right\}$, where the RHS is the *maximum* two period output generated within a firm. Note that, by definition, $k^O(K_1, \alpha)$ is decreasing in $\alpha$ and increasing in $K_1$. Furthermore $k^O(\alpha, K_1) > K_1$ for $\alpha < 1$ and $k^O(\alpha, K_1) = K_1$ for $\alpha = 1$. Hence, opportunity entrepreneurs always work on projects of value higher of that of firms.

- **Learning entrepreneurs**: those for which $k_1 \in [k^E(K_1, \alpha), k^O(K_1, \alpha)]$. These agents become entrepreneurs because firms implement task $\tau_1 = B$ despite the fact that task $\tau_1 = A$ maximizes the two period output. These entrepreneurs will implement task $\tau_1 = A$. In other words, these agents become entrepreneurs to choose the learning-maximizing task whenever this cannot happen within firms.

We can therefore decompose the probability of becoming an entrepreneur in period 1 into three elements corresponding to the three motives:

$$P^E_1(\alpha) \equiv \left(1 - \alpha\right) + \alpha \cdot \text{pr}\{k_1 > k^O(K_1, \alpha)\} + \alpha \cdot \text{pr}\{k^E(K_1, \alpha) < k_1 < k^O(K_1, \alpha)\}$$

$$= 1 - \alpha + \alpha \cdot \text{pr}\{k_1 > k^E(K_1, \alpha)\}.$$

Whenever $\alpha$ is sufficiently low $K^{A_1}(\alpha) > k^A(1)$ and the task allocation within firms is optimal; therefore there are no learning entrepreneurs. This is
apparent from the left panel in Figure 5 since for any value of $K_1$, individuals who become entrepreneurs while receiving offers are more likely to use the basic task. Instead, whenever $\alpha$ is sufficiently high $K^A(\alpha) < k^A(1)$ and the task allocation within firms may not be optimal; in this case, there is a positive probability of being a learning entrepreneur. For instance, in the right panel of Figure 5 at the intersection of $W^F$ and $W^E$, that is when the agent gets the same project as the firm, he will choose task $A$ as an entrepreneur while the firm would choose task $B$. Similarly, the probability of becoming an opportunity entrepreneur is zero whenever $k^O(K_1, \alpha) > \lambda K_1$ which happens whenever either $\lambda$ or $\alpha$ is sufficiently low, and is strictly positive otherwise.

The level of labor market frictions therefore affects both the probability of becoming an entrepreneur in period 1, and the importance of the different motives for entrepreneurship. This is illustrated by Figure 6, in which we report a numerical simulation. Note that the learning motive becomes relatively more important with respect to the other motives when $\alpha$ is large, generating a U-shape relationship between $\alpha$ and the probability of becoming an entrepreneur in period 1. The following lemma confirms that this is a general property, coming from the fact that as $\alpha$ increases, there are fewer necessity entrepreneurs but more learning and opportunity entrepreneurs.

**Lemma 3.** The probability of first period entrepreneurs $P_1^E(\alpha)$ is decreasing for $\alpha$ close to 0 and increasing for $\alpha$ close to 1.

In period 2, instead there is no value of learning and hence there are no “learning entrepreneurs”. All those who previously worked for a firm become entrepreneurs if $k_2 > K_2$ and continue working for a firm otherwise. Similarly, all former entrepreneurs who receive a wage offer choose entrepreneurship if and only if $k_2 > K_2$. Instead, former entrepreneurs who do not receive a wage offer are again entrepreneurs. The probability of becoming an entrepreneur in period 2 is therefore:

$$P_2^E(\alpha) \equiv (1 - \alpha)P_1^E(\alpha) + (1 - (1 - \alpha)P_1^E(\alpha))\Pr\{k_2 > K_2\}$$

$$= (1 - \alpha)P_1^E(\alpha)(1 - \Pr\{k_2 > K_2\}) + \Pr\{k_2 > K_2\}$$
Having derived $P_E^1(\alpha)$ and $P_E^2(\alpha)$, we can now compute two commonly used measures of aggregate entrepreneurial activity: the probability of being a serial entrepreneur and the average probability of becoming an entrepreneur across periods. The probability of being a serial entrepreneur (that is, an entrepreneur in both periods) is

$$P_{\text{serial}}(\alpha) \equiv P_E^1(\alpha) \cdot (1 - \alpha + \alpha \cdot \Pr\{k_2 > K_2\}),$$

and the average probability of becoming an entrepreneur across periods:

$$P_{(1/2)}(\alpha) \equiv \frac{1}{2} \left( P_E^1(\alpha) + P_E^2(\alpha) \right)$$

$$= \frac{1}{2} \left( P_E^1(\alpha)(1 + (1 - \alpha)(1 - \Pr\{k_2 > K_2\})) + \Pr\{k_2 > K_2\} \right).$$

The next proposition shows results that, if the learning motive for entrepreneurship is sufficiently strong, countries with lower labor market frictions
can have higher entrepreneurial activity than countries with higher labor market frictions, which is consistent with evidence from the US and continental Europe.

**Proposition 3.** $P_{\text{serial}}^E(\alpha)$ and $P_{(1/2)}^E(\alpha)$ are decreasing for $\alpha$ close to 0, and increasing for $\alpha$ close to 1 if

$$\frac{\sigma_2(A) - \sigma_2(B)}{\sigma_1(B) - \sigma_1(A)} > \frac{4\lambda^2(1 - \beta)}{\lambda - 1}$$

The LHS of the above expression measures the value of learning one comparative advantage relative to its cost. On the RHS, $\beta$ measures the firm’s ability to internalize the benefit of learning. When either $\frac{\sigma_2(A) - \sigma_2(B)}{\sigma_1(B) - \sigma_1(A)}$ or $\beta$ is large, a small amount of labor market frictions allow firms to internalize the benefit of learning. However, as labor market frictions disappear, learning cannot happen within firms. It follows that under (8), the fraction of learning entrepreneurs reacts very rapidly to changes in $\alpha$, therefore determining the shape of $P_{\text{serial}}^E(\alpha)$, and $P_{(1/2)}^E(\alpha)$.

Using the same parameter values as in Figure 6, a simulation shows indeed a non-monotonic relationship between $\alpha$ and these two aggregate measures of entrepreneurship (see Figure 7).

### 4.2 Output

In period 1 a fraction $1 - \alpha$ of the population will not receive a wage offer and is forced into entrepreneurship, while a fraction $\alpha$ of the population chooses entrepreneurship or wage work depending on the two period output generated by these two options. Hence, the two-period total expected output in the economy is

$$(1 - \alpha) \cdot E[W^E(k_1, \alpha)] + \alpha \cdot E[\max \{W^E(k_1, \alpha), W^F(K_1, \alpha)\}]$$

Therefore, for a fixed $W^F(k_1, \alpha)$, total expected output increases with $\alpha$ both because fewer agents become involuntary entrepreneurs, and because $E[W^E(k_1, \alpha)]$ increases with $\alpha$. At the same time, $W^F(k_1, \alpha)$ is decreasing with $\alpha$, because as
4 Equilibrium Analysis

Fig. 7: Serial and Time Average Probabilities of Entrepreneurship ($\beta = 0.9$, $\lambda = 1.5$, $p = 0.45$, $h_A = 0.9$, $l_A = 0.1$, $h_B = 0.6$, $l_B = 0.4$)

$\alpha$ approaches 1 firms are unable to implement the two-period output maximizing task allocation. It is theoretically possible that total output is decreasing over some range of $\alpha$. However, our numerical simulations suggest that output is an increasing function of $\alpha$ for most values of $\beta$, $\lambda$, $p$, $h_A$, $l_A$, $h_B$ and $l_B$.

We can also use the model to understand how a specific realization of the aggregate shock $K_1$ affects the incentive of firms to learn, the number and motives of entrepreneurs, and total output. The key observation here is that for $\alpha$ sufficiently large, the value of working for a firm $WF(K_1, \alpha)$ is discontinuous at $K_1 = K^A(\alpha)$. Hence, it is possible that an arbitrarily small drop in $K_1$ from above $K^A(\alpha)$ to below $K^A(\alpha)$ will cause a reallocation of workers from task $B$ (the inefficient task allocation) to task $A$ (the efficient task allocation), generating several consequences:

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13 For example, if $\beta$ is large, a small amount of labor market frictions is sufficient to induce the efficient task allocation within firms. In this case, for $\alpha$ close to 1 the task allocation within firms is efficient while for $\alpha = 1$ it is not, and therefore output increases in $\alpha$ for $\alpha$ close to 1.
• Period-1 output decreases by more than the change in $K_1$. The reason is the change in period-1 allocation from $\tau_1 = B$ to $\tau_1 = A$, that is from the task with the highest probability of success in period 1 to the task with the lowest probability of success in period 1.

• The number of period-1 entrepreneurs decreases, because the learning motive for entrepreneurship disappears.

• Period-2 output generated within firms increases because period-1 workers are now allocated to the learning-maximizing task allocation.

• Total two-period expected output also increases. There are two causes for this increase. The drop in $K_1$ from above $K^A(\alpha)$ to below $K^A(\alpha)$ generates an upward jump in $W^F(K_1, \alpha)$, that is, in the sum of the two-period outputs produced within firms. At the same time, more agents in period 1 will choose to work for a firm rather than becoming an entrepreneur.

Therefore, despite the fact that $K_1$ and $k_1$ and independent from $K_2$ and $k_2$, the realization of period-1 aggregate shock has long term implications for future output because it determines firms’ incentives to learn.

Instead, for $\alpha$ sufficiently low $W^F(K_1, \alpha)$ is continuous and the task allocation implemented within firms maximizes two-period output. In this case, total two-period output is monotonic in $K_1$.

### 4.3 Additional Results and Links to the Empirical Literature

The model can generate other results that are consistent with the empirical evidence discussed in the Introduction.

**Payoffs of Entrepreneurs and Workers**

When both $\alpha$ and $\lambda$ are close to 1, the majority of entrepreneurs are learning entrepreneurs, who work on task $A$ and are therefore more likely to fail
than workers. Furthermore, \( k^E(K_1, \alpha) \) will be less than \( K_1 \), that is, these entrepreneurs will work on projects that have smaller value of that of firms. It follows that, in period 1, the instantaneous payoff of an entrepreneur is lower than that of workers. This is consistent with the findings in Hamilton (2000), who shows that, in the US, the majority of entrepreneurs earn less than workers of comparable type. Our model therefore suggests that some people value entrepreneurship because they have control over their task even if this comes at the cost of a lower initial payoff than workers.

Wages of Past Workers and Past Entrepreneurs

We showed that as \( \alpha \) changes, the task allocations of workers and of entrepreneurs change in opposite directions. As \( \alpha \) increases, workers are more likely to be allocated to task \( \tau = B \) while entrepreneurs are more likely to choose task \( \tau = A \). In the limit case of \( \alpha = 1 \) all workers are allocated to \( \tau = B \) and a positive mass of entrepreneurs (the learning entrepreneurs) chooses instead task \( A \). It follows, therefore, that for \( \alpha \) sufficiently large the period 2 wage of a former entrepreneur is greater than the period-2 wage of a former worker. This is consistent with another result in Hamilton (2000), who finds that in the US former entrepreneurs receive higher wages compared to former workers of equivalent characteristics.

On the other hand, the next lemma shows that when \( \alpha \) is sufficiently low, workers will learn more than entrepreneurs.

Lemma 4. For \( \alpha \leq 1 - 2(1 + \lambda^2)(1 - \beta) \), workers are more likely than entrepreneurs to work on task \( A \) in period 1.

The lemma shows that when \( \alpha \) is sufficiently low such that \( K^A(\alpha) \geq k^A(1) \) (i.e., the task allocation within firms is efficient), workers learn more than

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14 This holds by continuity and because at \( \alpha = \lambda = 1 \) there are only learning entrepreneurs.
15 On the other hand, for lower \( \alpha \) the instantaneous payoff of an entrepreneur may be higher than that of a worker. For \( \alpha \) sufficiently low there are only opportunity entrepreneurs and necessity entrepreneurs, who are less likely than workers to choose task \( A \) and therefore less likely than workers to fail. These entrepreneurs work on projects of higher value of that of firms, and therefore earn an instantaneous payoff that is larger than that of workers. See below, the proof of Lemma 4 for a formal argument.
entrepreneurs in period 1. Hence, the period-2 wage of former workers who receive a wage offer is larger than that of former entrepreneurs. Of course, the average wage of former workers also depends on the payoff of former workers who did not receive an outside offer. However, if $\beta$ is large (i.e. degree of contract incompleteness is low) even a small degree of labor market frictions can induce the efficient task allocation within firms. In this case, there exist values of $\alpha$ such that workers are more likely than entrepreneurs to work on task $A$, and at the same time the fraction of period-1 workers who do not receive a wage offer is low. On average, former entrepreneurs receive lower wages compared to former workers of equivalent characteristics, which is consistent with the available evidence for continental Europe (see Baptista et al., 2012).

The Value of Failures

A last result follows from the observation that, for a given entrepreneurial project $k_1$, fewer entrepreneurs choose $\tau_1 = A$ and learn their type as $\alpha$ decreases. Furthermore, as $\alpha$ decreases, fewer agents become entrepreneurs when receiving a wage offer. Therefore as $\alpha$ decreases, entrepreneurs pursue only projects with large returns, those for which the entrepreneur is more likely to set $\tau_1 = B$.

Proposition 4. For a serial entrepreneur, the probability of succeeding as an entrepreneur in period 2 is increasing in $\alpha$. Furthermore

(i) If talent is vertical, failures are always “bad news”. That is, the probability of succeeding in period 2 as an entrepreneur following an entrepreneurial failure in period 1 is below the initial probability of success $\sigma_1(B)$ for all $\alpha$.

(ii) If talent is horizontal, there exist parameter values such that failures are good news for $\alpha$ sufficiently high, and bad news for $\alpha$ sufficiently low.

Therefore, the vertical view of talent delivers predictions that contradict the empirical evidence on the value of entrepreneurial failures discusses in the
introduction. Whenever talent is horizontal, instead, failures can be “good news” whenever labor market frictions are low (i.e., high $\alpha$) and the majority of entrepreneurs are learning entrepreneurs (i.e., $\lambda$ low), because these entrepreneurs will choose $\tau_1 = A$ and a failure at this task leads to an increase in the future probability of success, which is consistent with the evidence for the US. When $\alpha$ is low, instead, most entrepreneurs choose task $B$, because agents become entrepreneurs only if they have valuable projects. Lemma 1 shows that when talent is horizontal, for $p_1 < q_B$ failures at $B$ are “bad news.” Hence, if the proportion of entrepreneurs choosing task $B$ in period 1 is sufficiently large, then entrepreneurial failures will breed more failures, which is consistent with the evidence for Europe.

## 5 Conclusion

We show that, ceteris-paribus, the intensity of labor-market frictions determines the proportion of different types of entrepreneurs in the economy, the relative wages of former entrepreneurs and former workers, and the probability of becoming an entrepreneur. By focusing on labor market frictions, our model provides a set of results that are consistent with evidence both for the US and for the EU, which are examples of low and high labor market frictions.

One of the prediction of the model is that on-the-job talent discovery via task allocation should increase with the severity of labor market frictions. Because there is a conflict between a firm and its workers only when the firm is short-termist, for sufficiently low labor market frictions firms must impose a task on the worker. For higher labor market frictions instead, such conflict will not arise, and therefore the firm may as well give discretion over the task choice to the worker; learning within firms could be achieved via task discretion. According to a 2013 study by OECD, the US has lower “task discretion” than several European countries, and is below the OECD average, consistent with our model’s results.\(^{16}\)

\(^{16}\) The US ranks 14th out of 22, below most European countries. In the OECD database, the variable “task discretion” is defined as “Choosing or changing the sequence of job tasks, the speed of work, working hours; choosing how to do the job.” See [OECD (2013)](http://www.oecd.org) (avail-
In order to focus on the learning motive for entrepreneurship, we have ignored other important determinants of entrepreneurial activity, such as financial constraints, skill acquisition, learning by doing, or differential ability of agents to become entrepreneurs.

When entrepreneurs face financial constraints, the effect of labor market frictions on entrepreneurial activity will be stronger. Indeed, if the labor market is frictionless, firms’ competition insures that workers are able to appropriate the full benefit of learning. Hence firms adopt a less informative task allocation independently of the importance of financial constraints. However, when there are labor-market frictions, financial constraints limit the exit of workers into entrepreneurship and therefore increase the ability of firms to appropriate the benefit of learning. Hence, labor-market frictions and financial constraints are complementary since they increase the likelihood that learning will occur within firms.

There is an element of learning by doing in our model because agents acquire information about their comparative advantage, are better able to match their talent to a task, and therefore increase their productivity over time. We do not however allow agents to increase their productivity on a given task by simply working on that task, that is, there is no task-specific human capital (see Gibbons and Waldman, 1999 and Gibbons and Waldman, 2004). Our results stand as long as this increase in productivity is small compared to the benefit of learning one’s comparative advantage.

Finally, we have assumed that the production process involves only one task. By contrast, Lazear (2004) assumes that workers work at a single task while entrepreneurs work at multiple tasks and he shows, both theoretically and empirically, that people with a more balanced skill set enter entrepreneurship. It would be interesting to add uncertainty about talent to Lazear’s model, able at http://skills.oecd.org/documents/SkillsOutlook_2013_Chapter4.pdf, especially, chapter 4, Figures 4.2 and 4.3.

On the role of financial constraints, see Hellmann (2007), who shows that cash constraints shape the way ideas are financed, within or outside the firm, and Tervio (2009), who argues that, absent long-term contracts, financial constrains may prevent optimal talent discovery in firms.
and study if and how learning determines agents’ occupational choices. This extension is left for future work.

A Mathematical appendix

Generalization of Proposition

The following proposition provides necessary and sufficient conditions for \( \sigma_1(A) < \sigma_1(B) \) but \( \sigma_2(A) > \sigma_2(B) \) in the horizontal case.

**Proposition 5.** Under assumption in the horizontal talent case there is a conflict between today’s probability of success and tomorrow’s if and only if \( q_A < p_1 \) and one of the following conditions hold:

- \( \frac{l_B}{h_B} < \frac{1-l_A}{1-h_A} \) and \( \frac{l_A}{h_A} < \frac{1-l_B}{1-h_B} \),
- \( \frac{l_B}{h_B} > \frac{1-l_A}{1-h_A} \), \( \frac{l_A}{h_A} < \frac{1-l_B}{1-h_B} \), and \( l_A + l_B < h_A + h_B < 1 \),
- \( \frac{l_B}{h_B} > \frac{1-l_A}{1-h_A} \), \( l_A + h_B < l_A + l_B < 1 \), and \( p_1 < q^{ooo} \),
- \( \frac{l_B}{h_B} < \frac{1-l_A}{1-h_A} \), \( l_A + h_B < h_A + h_B \) and \( p_1 > q^{ooo} \),

where

\[
q^{ooo} \equiv \left( 1 + \frac{h_A - h_B}{l_B - l_A} \frac{1 - (h_A + h_B)}{1 - (l_A + l_B)} \right)^{-1}.
\]

**Proof.** We already argued in the body of the text that

\[
p_1 > q_A \equiv \left( 1 + \frac{h_A h_B - h_B}{l_A l_B - l_A} \right)^{-1}, \tag{9}
\]

is a necessary condition for \( \sigma_2(A) > \sigma_2(B) \). Note that this condition is consistent with \( p_1 < p^* \) because \( h_A > l_A \) and therefore \( q_A < q^* \).

Using the expression for the posterior probability \( p_2(\tau_1, s_1) \), we have that whenever

\[
\frac{l_B}{h_B} < \frac{1 - l_A}{1 - h_A}, \tag{10}
\]
then \( p_2(A, 0) < p_2(B, 1) < p_1 \) in the horizontal talent case, and \( p_2(A, 0) < p_2(B, 0) < p_1 \) in the vertical talent case. Whenever

\[
\frac{l_A}{h_A} < \frac{1 - l_B}{1 - h_B},
\]

then \( p_1 < p_2(B, 0) < p_2(A, 1) \) in the horizontal talent case, and \( p_1 < p_2(B, 1) < p_2(A, 1) \) in the vertical talent case. Hence, whenever (10) and (11) hold the distribution of posteriors if \( \tau_1 = A \) is a mean-preserving spread of the distribution of posteriors if \( \tau_1 = B \), and we are in the case considered in the body of the text. Conditions (10) and (11) always hold when talent is vertical (i.e., \( l_B \geq h_B \)), but may not hold when talent is horizontal. For the horizontal talent case, therefore, we need to consider few additional cases.

**Horizontal talent, both (10) and (11) are violated.** In this case \( p_2(B, 1) < p_2(A, 0) < p_1 < p_2(A, 1) < p_2(B, 0) \). The distribution of posteriors if \( \tau_1 = B \) is a mean-preserving spread of the distribution of posteriors if \( \tau_1 = A \), and \( \tau_1 = A \) generates less learning and a lower probability of success in period 2 than \( \tau_1 = B \).

**Horizontal talent, (11) holds but (10) is violated.** In this case \( p_2(B, 1) < p_2(A, 0) < p_1 < p_2(B, 0) < p_2(A, 1) \). Hence, if \( p_2(B, 0) < q^* \), task \( B \) is not informative, because both in case of success and failures the period-2 task allocation will again be \( B \). Hence, condition (9) is sufficient for learning to be beneficial. Simple algebra shows that \( p_2(B, 0) < q^* \) whenever

\[
p_1 < q^{oo} \equiv \left( 1 + \frac{1 - h_B}{1 - l_B} \cdot \frac{h_A - h_B}{l_B - l_A} \right)^{-1}.
\]

Note that, by (11), \( q_A < q^{oo} < q^* \). Hence, \( \tau_1 = A \) generates more learning and a higher probability of success in period 2 if \( q_A < p_1 < q^{oo} \).

For \( q^{oo} < p_1 < q^* \), \( p_2(B, 0) > q^* \). That is, following a failure at task \( B \) the
agent is allocated to task $A$ in period 2. It follows that

$$\sigma_2(A) = p_1 h_A^2 + (1 - p_1) l_A^2 + p_1 (1 - h_A) h_B + (1 - p_1)(1 - l_A) l_B$$
if $p_1 > q^{oo}$

$$\sigma_2(B) = p_1 h_B^2 + (1 - p_1) l_B^2 + p_1 (1 - h_B) h_A + (1 - p_1)(1 - l_B) l_A$$
if $p_1 > q^{oo}$

and therefore:

$$\sigma_2(A) - \sigma_2(B) = (1-p_1)(l_B-l_A)(1-(l_B+l_A)) - p_1(h_A-h_B)(1-(h_A+h_B))$$
if $p_1 > q^{oo}$.

(12)

There are few subcases to consider:

- Whenever $h_A + h_B > 1$ and $l_B + l_A < 1$, then (12) is always positive. However, this case is incompatible with (10) being violated.

- Whenever $h_A + h_B < 1$ and $l_B + l_A > 1$, (12) is always negative. However, $h_A + h_B < 1$ and $l_B + l_A > 1$ is incompatible with (11).

- Whenever $h_A + h_B < 1$ and $l_B + l_A < 1$, then (12) is positive whenever

$$p_1 < q^{ooo} \equiv \left(1 + \frac{h_A - h_B}{l_B - l_A} \frac{1 - (h_A + h_B)}{1 - (l_A + l_B)}\right)^{-1}.$$ 

In this case condition (11) imply $\frac{1-(h_A+h_B)}{1-(h_A+l_B)} < \frac{1-h_B}{1-l_B}$, so that $q^{ooo} > q^{oo}$.

Furthermore, $q^{ooo} < q^*$ if and only if $l_A + l_B > h_A + h_B$. Hence, when (11) holds, (10) is violated, $h_A + h_B < l_B + l_A < 1$, we have that $\sigma_2(A) > \sigma_2(B)$ if and only if $q_A < p_1 < q^{ooo}$. When (11) holds, (10) is violated, $l_B + l_A < h_A + h_B < 1$, we have that $\sigma_2(A) > \sigma_2(B)$ if and only if $q_A < p_1$.

- Whenever $h_A + h_B > 1$ and $l_B + l_A > 1$, (12) is positive whenever

$$p_1 > q^{ooo} \equiv \left(1 + \frac{h_A - h_B}{l_B - l_A} \frac{1 - (h_A + h_B)}{1 - (l_A + l_B)}\right)^{-1},$$

In this case condition (11) imply $\frac{1-(h_A+h_B)}{1-(h_A+l_B)} > \frac{1-h_B}{1-l_B}$, so that $q^{ooo} < q^{oo}$.

Hence, when (11) holds, (10) is violated, $l_B + l_A > 1$ and $h_A + h_B > 1$, we have that $\sigma_2(A) > \sigma_2(B)$ if and only if $p_1 > q_A$. 


Horizontal talent, (11) is violated but (10) holds. In this case \( p_2(A,0) < p_2(B,1) < p_1 < p_2(A,1) < p_2(B,0) \). Hence, if \( p_2(A,1) > q^* \), then also \( p_2(B,0) > q^* \). For any \( p_1 > q \), the difference between \( \sigma_2(A) \) and \( \sigma_2(B) \) is again given by (12). Going through the same subcases, we get

- Whenever \( h_A + h_B > 1 \) and \( l_B + l_A < 1 \), then (12) is always positive. However, this case is incompatible with (11) being violated.

- Whenever \( h_A + h_B < 1 \) and \( l_B + l_A > 1 \), (12) is always negative. However, \( h_A + h_B < 1 \) and \( l_B + l_A > 1 \) is incompatible with (10).

- Whenever \( h_A + h_B < 1 \) and \( l_B + l_A < 1 \), then (12) is positive whenever

\[
p_1 < q^{ooo} \equiv \left( 1 + \frac{h_A - h_B}{l_B - l_A} \frac{1 - (h_A + h_B)}{1 - (l_A + l_B)} \right)^{-1},
\]

In this case, the fact that (11) is violated imply \( q^{ooo} < q_A \). Hence this condition never leads to \( \sigma_2(A) > \sigma_2(B) \).

- Whenever \( h_A + h_B > 1 \) and \( l_B + l_A > 1 \), (12) is positive whenever

\[
p_1 > q^{ooo} \equiv \left( 1 + \frac{h_A - h_B}{l_B - l_A} \frac{1 - (h_A + h_B)}{1 - (l_A + l_B)} \right)^{-1},
\]

In this case, the fact that (11) is violated imply \( q^{ooo} > q_A \). Note also that \( q^{ooo} < q^* \) if and only if \( l_A + l_B < h_A + h_B \). Hence, when (10) holds, (11) is violated, \( 1 < l_A + l_B < h_A + h_B > 1 \) we have that \( \sigma_2(A) > \sigma_2(B) \) if and only if \( p_1 > q^{ooo} \).

\[\Box\]

Proof of Lemma 1

What is left to show is that, in the horizontal talent case, there is a threshold value of \( p_1 \) below which \( \pi^M(p_2(B,0)) < \pi^M(p_1) \) (failures at \( B \) are bad news) and above which \( \pi^M(p_2(B,0)) > \pi^M(p_1) \) (failures at \( B \) are good news). If \( p_1 \) is so low that \( p_1 < p_2(B,0) < q^* \), then quite immediately failures are bad news.
Whenever instead \( p_1 < q^* < p_2(B, 0) \) we have that \( \pi^M(p_1) \) is monotonically decreasing in \( p_1 < q^* \), but \( \pi^M(p_2(B, 0)) \) is monotonically increasing in \( p_1 \). The statement therefore follows by continuity.

**Omitted calculations relative to Section 4**

For the reader’s convenience, we report here all the calculations relative to Section 4:

\[
E[k_2] = E[E[k_2|K_2]] = E[\frac{\lambda K_2}{2}] = \frac{\lambda}{2},
\]
equivalently:

\[
E[k_2] = \int_0^2 \left[ \int_0^{\lambda K_2} k_2 \frac{d k_2}{\lambda K_2} \right] \frac{d K_2}{2} = \int_0^2 \frac{\lambda K_2 d K_2}{2} = \frac{\lambda}{2}
\]

\[
E[\max\{K_2 - k_2, 0\}] = E[E[\max\{K_2 - k_2, 0\}|K_2]] = E[E[\text{pr}\{k_2 < K_2\}E[K_2 - k_2]|K_2]
\]
\[
= E[\frac{1}{\lambda} K_2]
\]
\[
= \frac{1}{2\lambda},
\]
equivalently:

\[
E[\max\{K_2 - k_2, 0\}] = \int_0^2 \left[ \int_0^{K_2} (k_2 - k_2) \frac{d k_2}{\lambda K_2} \right] \frac{d K_2}{2} = \int_0^2 \left( \frac{K_2^2 - K_2^2}{2} \right) \frac{1}{\lambda K_2} \frac{d K_2}{2} = \int_0^2 \left( \frac{K_2^2}{2\lambda} \right) d K_2 = \frac{1}{2\lambda}.
\]

\[
E[\max\{k_2, K_2\}] = E[k_2 + \max\{K_2 - k_2, 0\}] = \frac{\lambda}{2} + \frac{1}{2\lambda} = \frac{1}{2} \left( \lambda + \frac{1}{\lambda} \right),
\]
equivalently:

\[
\mathbb{E}[\max\{k, K\}] = \int_0^2 \left[ \int_0^K K f(k|K) dk + \int_K^{\lambda K} k f(k|K) dk \right] \frac{dK}{2}
\]

\[
= \int_0^2 \left[ \int_0^K \frac{1}{\lambda} dk + \int_K^{\lambda K} \frac{k}{\lambda K} dk \right] \frac{dK}{2}
\]

\[
= \int_0^2 \left[ \frac{K}{\lambda} + \frac{(\lambda K)^2 - K^2}{2\lambda K} \right] \frac{dK}{2}
\]

\[
= \int_0^2 \left[ \frac{K}{\lambda} + \frac{K(\lambda^2 - 1)}{2\lambda} \right] \frac{dK}{2}
\]

\[
= \frac{1}{\lambda} + \frac{\lambda^2 - 1}{2\lambda} = \frac{1}{2} \left( \lambda + \frac{1}{\lambda} \right).
\]

**Proof of Lemma 3**

Because \( W^F(K_1, \alpha) \) is discontinuous at \( K_1 = K^A(\alpha) \) whenever \( K^A(\alpha) < k^A(1) \), by definition \( k^E(K_1, \alpha) \) is also discontinuous at \( K_1 = K^A(\alpha) < k^A(1) \).

Note also that for \( K_1 \leq K^A(\alpha) \leq k^A(1) \) the task allocation within firm is efficient, while for \( K_1 \in (K^A(\alpha), k^A(1)) \) it is not. It follows that, whenever \( K^A(\alpha) < k^A(1) \) we have

\[
W^F(K^A(\alpha), \alpha) > \lim_{K_1 \to K^A(\alpha)^+} W^F(K_1, \alpha),
\]

and therefore

\[
k^E(K^A(\alpha), \alpha) > \lim_{K_1 \to K^A(\alpha)^+} k^E(K_1, \alpha)
\]

At every point at which \( k^E(K_1, \alpha) \) is differentiable with respect to \( K_1 \), by the implicit function theorem

\[
\frac{\partial k^E(K_1, \alpha)}{\partial K_1} = \begin{cases} 
1 & \text{if } (K_1 - K^A(\alpha))(k^E(K_1, \alpha) - k^A(\alpha)) \geq 0 \\
\frac{\sigma_1(A)}{\sigma_1(B)} & \text{if } K_1 - K^A(\alpha) < 0 \& k^E(K_1, \alpha) - k^A(\alpha) > 0 \\
\frac{\sigma_1(B)}{\sigma_1(A)} & \text{if } K_1 - K^A(\alpha) > 0 \& k^E(K_1, \alpha) - k^A(\alpha) < 0.
\end{cases}
\]

Similarly, at every point at which \( k^E(K_1, \alpha) \) is differentiable with respect to
α, by the implicit function theorem

\[
\frac{\partial k^E(K_1, \alpha)}{\partial \alpha} = -\frac{1}{2\lambda} \begin{cases} 
\frac{\sigma_2(A)}{\sigma_1(A)} & \text{if } k^E(K_1, \alpha) \leq k^A(\alpha) \\
\frac{\sigma_2(B)}{\sigma_1(B)} & \text{otherwise.}
\end{cases}
\]

We can therefore write

\[
k^E(K_1, \alpha) = \begin{cases} 
Z(K_1, \alpha) & \text{if } K_1 \leq K^A(\alpha) \\
Y(K_1, \alpha) & \text{otherwise,}
\end{cases}
\]

where \(Z(K_1, \alpha)\) and \(Y(K_1, \alpha)\) are two continuous functions, increasing in \(K_1\) and decreasing in \(\alpha\), with \(Z(K_1, \alpha) = Y(K_1, \alpha)\) for \(K_1 \geq k^A(1)\) and \(Z(K_1, \alpha) > Y(K_1, \alpha)\) for \(K_1 < k^A(1)\).

We can now compute

\[
\begin{align*}
\Pr\{k_1 > k^E(K_1, \alpha)\} &= \int_{0}^{2} \frac{1}{2} \min \left\{ \max \left\{ 1 - \frac{k^E(K_1, \alpha)}{\lambda K_1}, 0 \right\}, 1 \right\} dK_1 \\
&= \frac{1}{2} \left( \int_{0}^{K^A(\alpha)} \min \left\{ \max \left\{ 1 - \frac{Z(K_1, \alpha)}{\lambda K_1}, 0 \right\}, 1 \right\} dK_1 + \int_{K^A(\alpha)}^{2} \min \left\{ \max \left\{ 1 - \frac{Y(K_1, \alpha)}{\lambda K_1}, 0 \right\}, 1 \right\} dK_1 \right)
\end{align*}
\]

which is continuous because the integrand has only finitely many discontinuities.

It follows that

\[
\frac{\partial \Pr\{k_1 > k^E(K_1, \alpha)\}}{\partial \alpha} \Bigg|_{18} = \frac{1}{2} \int_{0}^{2} \mathbb{1}\left\{ 0 \leq k^E(K_1, \alpha) \leq \lambda K_1 \right\} \left( 1 - \frac{1}{\lambda K_1} \frac{\partial k^E(K_1, \alpha)}{\partial \alpha} \right) dK_1 \\
+ \frac{\partial K^A(\alpha)}{\partial \alpha} \left( \min \left\{ \max \left\{ 1 - \frac{Z(K^A(\alpha), \alpha)}{\lambda K^A(\alpha)}, 0 \right\}, 1 \right\} - \min \left\{ \max \left\{ 1 - \frac{Y(K^A(\alpha), \alpha)}{\lambda K^A(\alpha)}, 0 \right\}, 1 \right\} \right)
\]

which is positive because \(\frac{\partial k^E(K_1, \alpha)}{\partial \alpha} < 0\), \(\frac{\partial K^A(\alpha)}{\partial \alpha} < 0\), and \(Y(K^A(\alpha), \alpha) < Z(K^A(\alpha), \alpha)\) (so that the last terms in brackets is negative), and continuous because, again, \(\frac{\partial k^E(K_1, \alpha)}{\partial \alpha}\) has only finitely many discontinuities.

It follows that

\[
\frac{\partial P^E(\alpha)}{\partial \alpha} = \alpha \frac{\partial \Pr\{k_1 > k^E(K_1, \alpha)\}}{\partial \alpha} - \left(1 - \Pr\{k_1 > k^E(K_1, \alpha)\}\right)
\]

\footnote{Remember that, by definition, \(k^E(K_1, \alpha)\) can be greater than \(\lambda K_1\) or smaller that zero.}
is negative at $\alpha = 0$ because

$$\frac{\partial \Pr \{k_1 > k^E(K_1, \alpha)\}}{\partial \alpha} \bigg|_{\alpha = 0} = \frac{1}{2} \int_0^2 \mathbb{1} \{0 \leq k^E(K_1, \alpha = 0) \leq \lambda K_1\} \left(1 - \frac{1}{\lambda K_1} \frac{\partial k^E(K_1, \alpha = 0)}{\partial \alpha}\right) dK_1$$

is finite.

At $\alpha = 1$ instead we have

$$\frac{\partial P^E_1(\alpha)}{\partial \alpha} \bigg|_{\alpha = 1} = \frac{1}{2} \int_0^2 \mathbb{1} \{k^E(K_1, \alpha = 1) > 0\} \left(1 - \frac{1}{\lambda K_1} \frac{\partial k^E(K_1, \alpha = 1)}{\partial \alpha}\right) dK_1$$

$$- \frac{\partial K^A(\alpha)}{\partial \alpha} \bigg|_{\alpha = 1} - (1 - \Pr \{k_1 > k^E(K_1, \alpha = 1)\})$$

$$= \Pr \{k_1 > k^E(K_1, \alpha = 1)\} + \Pr \{k^E(K_1, \alpha = 1) > 0\} - 1$$

$$- \frac{1}{2} \int_0^2 \mathbb{1} \{k^E(K_1, \alpha) > 0\} \frac{1}{\lambda K_1} \frac{\partial k^E(K_1, \alpha = 1)}{\partial \alpha} \bigg|_{\alpha = 1} dK_1 - \frac{\partial K^A(\alpha)}{\partial \alpha} \bigg|_{\alpha = 1} \frac{1}{\lambda}$$

where we used the fact that at $\alpha = 1$, $Z(K_1, \alpha = 1) = K_1$, that is, assuming that the task allocation is efficient and that there are no labor market frictions, agents become entrepreneurs only if they have a project that is more valuable than that of firms. Furthermore $K^A(\alpha = 1) = 0$, and hence $k^E(K_1, \alpha = 1) \leq K_1$, and $Y(K^A(\alpha = 1), \alpha = 1) < 0$.

Define $\tilde{K} \equiv K_1 : k^E(K_1, \alpha = 1) = 0$. We can write

$$\Pr \{k_1 > k^E(K_1, \alpha = 1)\} = \Pr \{K_1 < \tilde{K}_1\}$$

$$+ (1 - \Pr \{K_1 < \tilde{K}_1\}) \cdot \Pr \{k_1 > k^E(K_1, \alpha = 1) | k^E(K_1, \alpha = 1) > 0\}$$

and

$$\Pr \{k^E(K_1, \alpha = 1) > 0\} = 1 - \Pr \{K_1 < \tilde{K}_1\}$$

Using this, we can rewrite

$$\frac{\partial P^E_1(\alpha)}{\partial \alpha} \bigg|_{\alpha = 1} = -\frac{1}{2} \int_0^2 \mathbb{1} \{k^E(K_1, \alpha) > 0\} \frac{1}{\lambda K_1} \frac{\partial k^E(K_1, \alpha = 1)}{\partial \alpha} \bigg|_{\alpha = 1} dK_1 - \frac{\partial K^A(\alpha)}{\partial \alpha} \bigg|_{\alpha = 1} \frac{1}{\lambda}$$

$$+ (1 - \Pr \{K_1 < \tilde{K}_1\}) \cdot \Pr \{k_1 > k^E(K_1, \alpha = 1) | k^E(K_1, \alpha = 1) > 0\}$$

which is positive because, as we already saw, both $\frac{\partial k^E(K_1, \alpha = 1)}{\partial \alpha}$ and $\frac{\partial K^A(\alpha)}{\partial \alpha}$ are negative. By continuity, $\frac{\partial P^E_1(\alpha)}{\partial \alpha}$ is decreasing for $\alpha$ sufficiently close to 0, and
increasing for $\alpha$ sufficiently close to 1.

**Proof of Proposition 3**

We can compute

$$\frac{\partial P^E_{\text{serial}}(\alpha)}{\partial \alpha} = \frac{\partial P^E_1(\alpha)}{\partial \alpha} \left(1 - \frac{\alpha}{\lambda}\right) - P^E_1(\alpha) \frac{1}{\lambda},$$

$$\frac{\partial P^E_{1/2}(\alpha)}{\partial \alpha} = \frac{1}{2} \left(\frac{\partial P^E_1(\alpha)}{\partial \alpha} \left(1 + \frac{1 - \alpha}{\lambda}\right) - P^E_1(\alpha) \frac{1}{\lambda}\right),$$

which are both negative at $\alpha = 0$ since, by Lemma 3, $P^E_1(\alpha)$ is decreasing at $\alpha = 0$. They are both positive at $\alpha = 1$ if and only if

$$\frac{\partial P^E_1(\alpha = 1)}{\partial \alpha} (\lambda - 1) > P^E_1(\alpha = 1).$$

By the derivations in the proof of Lemma 3, the above expression is satisfied whenever

$$-\frac{\partial K^A(\alpha)}{\partial \alpha}|_{\alpha = 1}\frac{1}{\lambda}(\lambda - 1) > 1,$$

$$\frac{\lambda - 1}{4\lambda^2(1 - \beta)} \left(\frac{\sigma_2(A) - \sigma_2(B)}{\sigma_1(B) - \sigma_1(A)}\right) > 1.$$

**Proof of Lemma 4**

Assume that $\alpha$ is sufficiently low so that $K^A(\alpha) \geq k^A(1)$. In this case, the task allocation within firms maximizes the two-period output. That is, if an agent works for a firm, she is allocated to task $\tau_1 = A$ if and only if

$$K_1 \leq k^A(1) := \frac{1}{2} \left(1 + \frac{1}{\lambda}\right) \left(\frac{\sigma_2(A) - \sigma_2(B)}{\sigma_1(B) - \sigma_1(A)}\right).$$

At the same time, entrepreneurs set $\tau_1 = A$ if and only if

$$k_1 \leq k^A(\alpha) := \frac{\alpha + \lambda^2}{2\lambda} \left(\frac{\sigma_2(A) - \sigma_2(B)}{\sigma_1(B) - \sigma_1(A)}\right) \leq k^A(1).$$
Hence, for any $\alpha$ such that $K^A(\alpha) \geq k^A(1)$, if all agents who receive a wage offer become workers — so that there is no selection into different professions based on $k_1$ — the probability that a worker is allocated to $\tau = A$ is greater than the probability that an entrepreneur is allocated to $\tau = A$.

To conclude the proof, we need to address the issue of selection into entrepreneurship based on $k_1$. We use the fact that agents become entrepreneurs if they have a sufficiently valuable project, which makes them less likely to choose the task allocation that maximizes learning. Note that an agent who receives a wage offer chooses to be an entrepreneur rather than working for a firm if and only if

$$\max_{\tau_1 \in \{A,B\}} \left\{ \sigma_1(\tau_1)k_1 + \sigma_2(\tau_1)\frac{\alpha + \lambda^2}{2\lambda} \right\} \geq \max_{\tau_1 \in \{A,B\}} \left\{ \sigma_1(\tau_1)K_1 + \sigma_2(\tau_1)\frac{1}{2} \left( \frac{1}{\lambda} + \lambda \right) \right\}$$

Therefore, for every $K_1$, there is a threshold $k(K_1) > K_1$ such that for every $k_1 \geq k(K_1)$ the agent becomes an entrepreneur, and for every $k_1 \leq k(K_1)$ the agent becomes a worker. Suppose that $K_1 \leq k^A(1)$, so that all workers are allocated to $\tau = A$. It is easy to see that entrepreneurs are allocated to task $\tau = B$ with positive probability. Suppose instead that $K_1 \geq k^A(1)$, so that workers are allocated to task $\tau = B$. Again, because $k(K_1) > K_1 > k^A(1)$ all agents who become entrepreneurs also set $\tau = B$. It follows that, among agents who receive an offer, the unconditional probability (i.e., for any $K_1$, $k_1$) of being allocated to task $A$ is greater for workers than for entrepreneurs.

**Proof of Proposition 4**

For given project value $k_1$ the probability that an entrepreneur sets $\tau_1 = A$ increases with $\alpha$. At the same time $\alpha$ determines the set of $k_1$ that will be pursued by agents who receive a wage offer and become entrepreneurs. For these agents, as $\alpha$ increases, the set of projects that are pursued enlarges: smaller $k_1$ are pursued by entrepreneurs. These projects are the ones for which the entrepreneurs are more likely to choose $\tau_1 = A$. Overall, the probability of setting $\tau_1 = A$ increases with $\alpha$, which implies that the probability of
succeeding in period 2, also increase with $\alpha$.

The second part of the Proposition follows by Lemma 1. In the vertical talent case the probability of period-2 success following a failure is always below the initial probability of success $\pi(p_1) \equiv \sigma_1(B)$. In the horizontal talent case failures at task $A$ are always good news, while if $p_1 < q_B$ failures at task $B$ are bad news. Hence, if talent is horizontal, $\alpha = 1$ and $\lambda$ low, for any $p_1 < q_B$ the majority of entrepreneurs are motivated by learning and set $\tau_1 = A$. In this case, failures are good news. As $\alpha$ decreases, the majority of entrepreneurs are opportunity entrepreneurs or necessity entrepreneurs who set $\tau_1 = A$ whenever

$$k_1 \leq k^A(\alpha) := \frac{\alpha + \lambda^2}{2\lambda} \left( \frac{\sigma_2(A) - \sigma_2(B)}{\sigma_1(B) - \sigma_1(A)} \right) \leq k^A(1).$$

and $B$ otherwise. Hence, as $\alpha$ decreases, entrepreneurs are more likely to choose task $B$. Also, for given $\alpha$, in the limit case $p_1 \to 0$, we have $\sigma_2(A) \to \sigma_2(B)$ and all entrepreneur choose task $B$ and entrepreneurial failures are bad news. By continuity, there exists a $p_1$ and $\alpha < 1$ such that entrepreneurial failures are bad news.

### B Unobservable Task Allocation

When past task allocation is not observable outside of the firm, at the beginning of period 2 there may be asymmetry of information between firms and any agent who did not work for the same firm previously. We restrict our analysis of this problem to the case $\alpha = 1$. Our goal is to show that, also here, the learning motive for entrepreneurship may emerge. It is quite immediate to see that, also here, as $\alpha$ decreases the learning motive for entrepreneurship disappears.

**Screening equilibria.** Suppose that, for every observable history, in period 2 firms offer a contract for every possible type, where a contract has the form \{b, f, $\tau_2$\} i.e., a bonus, fixed payment, and a task allocation. Clearly, if the
agent produced a success in the previous period, a menu of contracts \( \{ b, f, \tau_2 = A \} \) and \( \{ b', f', \tau_2 = B \} \) such that \( f + qb = f' + qb' = K_2 \) is an equilibrium screening menu of contracts, because each firm makes zero profits, agents of different types prefer different contracts (strictly so if \( b, b' > 0 \)), and the firm has no incentive to implement a task allocation that is different from that specified in the contract.\(^{19}\)

However, in order to use such contracts, it must be the case that conditional on a given outcome \( s_1 \), those who worked at different period-1 task maximize period-2 probability of success by working at different period-2 tasks. In other words, after observing a failure, screening is possible if those who worked at task \( \tau_1 = A \) should work in period 2 on task \( \tau_2 = B \), and vice versa. Similarly, after observing a success, screening is possible if those who worked at task \( \tau_1 = A \) should work in period 2 again on task \( \tau_2 = A \), and the same for those who worked on task \( \tau_1 = B \).

This condition is never satisfied when talent is vertical. In this case, successes (whether at task \( \tau_1 = A \) or task \( \tau_1 = B \)) increase the probability that the agent is of type \( h \) and that he should be allocated to task \( \tau_2 = A \). Similarly, failures (whether at task \( \tau_1 = A \) or task \( \tau_1 = B \)) increase the probability that the agent is of type \( l \) and that he should be allocated to task \( B \). Hence, in general, screening is not possible when talent is vertical.\(^{20}\)

Screening is possible whenever talent is horizontal and \( p_1 \) is sufficiently close to \( p^* \). In this case, successes at task \( \tau_1 = A \) or task \( \tau_1 = B \) make it more likely that the agent should work at that task in period 2 as well. Similarly, failures at task \( \tau_1 = A \) or task \( \tau_1 = B \) make it more likely that the agent should work at the other task in period 2. If the initial prior is sufficiently uncertain, conditional on the outcome \( s_1 \) there is a one-to-one correspondence between \( \tau_1 \) and \( \tau_2 \) maximizing period-2 probability of success.

\(^{19}\) Note that this contract amounts to delegating task allocation to the worker. Delegation is possible because, in period 2, workers and firms have aligned preferences regarding task allocation.

\(^{20}\) It may still, however, be possible to screen conditional on a given period-1 outcome, but not on the other outcome.
No screening equilibrium. If workers past task allocation is not observable and screening is not possible, then the contract offered by firms to former workers depends on the market belief over the workers previous task allocation. In particular, if the market expects the worker to be allocated to task $\tau_1 = B$, period-1 employer may have the incentive switch to $\tau_1 = A$. Doing so, period-1 employer gains the difference between the employer period-2 probability of success and the market beliefs about it.

It is easy to see, however, that there is no equilibrium in which firms set $\tau_1 = A$. If the market expects $\tau_1 = A$, then the period-1 employer always makes zero profits in period 2. Hence, he is better off by maximizing period-1 output and setting $\tau_1 = B$. Of course, it is possible that, in equilibrium firms set $\tau_1 = A$ with positive probability (but less than 1). However, remember that workers prefer to work on task $A$ (with probability 1) if $K_1 \leq k^A(1)$. Also here, if $k_1 < K_1$ but $K_1 - k_1$ sufficiently small, some agents will become entrepreneurs despite the fact that their project value is lower of that of firms.

C Long-Term Contracts

In the text we assume that long-term contracts are not available. In this section, we relax this assumption by introducing the possibility that, in period 1, firms and workers can sign a contract specifying a wage for period 2.

To start, note that if firms can shutdown at no cost, long-term contracting cannot improve our sequence of short-term contracts. Long-term contracting may be valuable when it induces the firm to choose, in period-1, a task allocation that is not short-term profit maximizing. As long as workers can freely leave a firm, competition requires that firms’ make zero profits in period 2. But then, a firm is better off implementing the short-term profit maximization task allocation in the first period and shutting down the firm.

If instead firms can commit not to shut down, we show here that long-term contracting does not affect our main qualitative result as long as workers are

\[21\] This is the relevant case since we already show that when there are significant market frictions, short-term contracting is efficient.
free to move across firms and occupations. Our argument rests on the fact that a period-1 worker may become an entrepreneur in period 2, which limits the period-2 profits a firm may expect to make from learning its worker’s talent in period 1. Below, we limit our attention to the case $\alpha = 1$ (no labor-market frictions). We want to show that the learning motive for entrepreneurship emerges with long-term contracts as well. It is quite evident that, similarly to what discussed in the body of the paper, also here as $\alpha$ decreases the learning motive for entrepreneurship disappears.

If workers are free to leave, any long-term contract signed in period-1 should pay in period 2 at least the period-2 market wage. Therefore, a long-term contract may pay the worker in period 2 a wage — contingent on success or failure in period 1 and on period 2 project $K_2$ — equal to the market value of this worker in case she was allocated to task $A$ in period 1.

Assume that such a contract is signed. We argue here that the firm may deviate and set $\tau_1 = B$. For given $K_2$, this deviation delivers an expected loss in period-2 equal to $K_2(\sigma_2(A) - \sigma_2(B))$, because the employee will have to be paid as if he had worked on task $A$ while instead he worked on task $B$. However, this loss is realized only if the agent does not become an entrepreneur and continue working for the firm, and hence it is discounted by the probability that $k_2 > K_2$, which is monotonically increasing in $\lambda$. At the same time, such deviation increases the probability of success in period 1 and therefore delivers a period-1 gain equal to $(\sigma_1(B) - \sigma_1(A))(K_1 - b)$.

It is easy to see that for $\lambda$ sufficiently large, the probability that the worker will continue working for the same firm vanishes to zero and the firm will deviate to $\tau_1 = B$. Hence, for $\lambda$ large, long-term contracts do not always implement the worker-preferred task allocation and therefore the learning motive for entrepreneurship survives.

References


