Rational Inattention and Counter-Cyclical Lending Standards*

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Abstract

We develop a model of rational inattention to analyse the interaction between banks’ lending standards and aggregate economic conditions. Banks are constrained in their capacity to assess borrower quality and trade off the number of processed loan applications with the precision of their loan review. As aggregate economic conditions improve, the marginal return to additional scrutiny decreases and banks investigate applicants less carefully. As a result, they approve loans that are riskier ex-ante and generate lower expected returns, which can explain excessively lenient lending standards during market booms.

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1 Introduction

When banks process loan applications, they collect hard and soft information and apply quantitative and qualitative judgement to assess potential borrowers’ repayment probabilities. The decision-making process leaves considerable room for discretion, pertaining even to the use of hard information (Berg et al., 2016), and leads to heterogeneity in banks’ lending standards. Empirically, these standards have been shown to vary, for example, with loan officers’ compensation schemes (Agarwal and Wang, 2009; Cole et al., 2015) or the involvement of banks’ risk-management (Berg, 2015; Brown et al., 2015). Importantly, they also depend on aggregate conditions, with banks being more lenient towards risky borrowers during good economic times (Asea and Blomberg, 1998; Minnis et al., 2016; Rodano et al., 2016). Such counter-cyclical lending standards cause concern about amplified economic volatility and the accumulation of hidden risks. The subprime credit boom in the US, for example, was fuelled by permissive lending standards (Dell’Ariccia et al., 2012), which paid seemingly insufficient attention to available evidence of systemic fragility (e.g. Rajan, 2005).

To explain excessively lenient lending standards during market booms, we develop a model of rational inattention, in which banks are constrained in their capacity to assess borrower quality. For a given capacity (e.g. the length of a working day), they trade off the number of loan applications they can process with the precision of their loan review. When aggregate economic conditions deteriorate, the percentage return to additional attention per loan increases. Then, by choosing to evaluate fewer applications more thoroughly, banks can increase their overall profit. This happens because, with little attention paid to loan applications in bad states of the economy, banks are unlikely to invest - even if they receive seemingly good news about a borrower. Investment in such a case, can only be justified if the loan review was carefully conducted, i.e. with a sufficient amount of attention. As a result, banks pay more attention to projects during recessions, implying that funded projects are less risky ex-ante and, as we show, can actually generate higher expected returns. Thus, projects of lower quality would receive financing during a market boom, which could amplify

\[1\] For example, by decreasing the number of loan applications by 10% they can pay 10% more attention to each loan. If due to this attention the expected payoff from each project will increase by more than 10%, banks will find it profitable to reallocate their attention; this is more likely to occur in a recession.
the subsequent recession.

Consistent with our model, Brown et al. (2017) report that skill-adjusted processing times for loan applications at a Romanian bank are negatively correlated with GDP growth. In addition, our mechanism also aligns with time variation in banks’ propensity to request information from the Spanish Credit Registry, observed by Jimenez and Saurina (2006) and Jimenez et al. (2012). In line with our central modelling assumption, the authors hypothesize that this variation may be the result of non-pecuniary costs being particularly relevant “during economic expansions when capacity constraints at the bank become binding” (Jimenez et al., 2012, p. 2305).

Our paper is directly related to the costly screening literature (Ruckes 2004; Dell’Ariccia and Marquez 2006). In fact, attention allocation can be considered as a special case of costly screening, in which banks choose a certain capacity for attention (e.g. by hiring additional loan officers). Adding to this choice of attention capacity, we consider the allocation of a given capacity across projects. The contribution of our model, in other words, is that we deviate from the assumption of a representative project, which implies independent evaluation costs. Instead, we study the joint allocation of attention over a cross-section of projects. The important difference is that, unless the marginal cost of attention is constant, more attention paid to one project, now makes it more costly to evaluate others.

Considering the joint allocation of attention changes how the state of the economy affects the attention paid to individual projects. If applications are considered independently, it is optimal to pay most attention when the state of the economy is "not too good" or "not too bad", i.e. when it is unclear ex ante if a project is worthy of investment. With joint attention allocation the distribution of attention across projects also becomes important. As we show, irrespective of the overall amount of attention one chooses to allocate, it is optimal to pay more attention to each evaluated project when the state of the economy is bad.

We consider several extensions of the main model. In the presence of aggregate risk, when the pay-offs of different projects are correlated, banks divide their attention between idiosyncratic and common risk-factors. As we show, when aggregate economic conditions improve, banks could choose to pay less attention to aggregate risks, in order to evaluate a larger number of projects. This happens because projects
are likely to be profitable in a market boom, which implies that the opportunity cost of spending time to learn about aggregate risks can be high (banks are “too busy making money” to pay attention to risk). Thus, the optimal allocation of attention by capacity constrained banks can explain why risks may accumulate during economic upturns.

In another extension we consider multiple banks that share a common pool of potential borrowers. In this case banks’ attention allocation decisions are no longer socially efficient. Banks do not fully internalize how their decision to process a loan application reduces expected profits for other banks lending to the same borrower pool, and thus choose to evaluate more projects than socially optimal.

The remainder of this paper is organized as follows: Section 2 discusses the related literature, Section 3 introduces our model and derives results in the absence of aggregate uncertainty, Section 4 extends our model to include aggregate shocks, Section 5 considers cases in which banks (a) choose attention capacity and (b) share a common borrower pool. Section 6 concludes.

2 Related Literature

Our paper relates to different strands of the literature.

Methodologically, we rely on the work on rational inattention, which goes back to Sims (2003), and is summarized in Sims (2010) and Wiederholt (2010). Related applications include fund managers’ capacity to pick stocks (Kacperczyk et al., 2016) and, in the context of a dynamic stochastic general equilibrium model, the business cycle implications of rationally inattentive households and firms (Mackowiak and Wiederholt, 2015). Different from Kacperczyk et al. (2016), our results are driven solely by changes in aggregate payoffs, and do not require exogenous changes in aggregate volatility. Mackowiak and Wiederholt (2015), instead, focus on the economy’s sluggish response to macroeconomic shocks, while we are interested in the endogenous build-up of risk in banks’ portfolios. The information structure in this class of models is comparable to models of delegated portfolio management (e.g. Bhattacharya and Pfleiderer, 1985 or Stoughton, 1993), in which effort rather than attention is modelled as a linear increase in posterior signal precision. In this literature, risk-averse agents typically trade off signal precision with the disutility of effort. We,
instead, take overall attention as given and study how risk-neutral agents optimally allocate it across loan applications.\footnote{In Section 5 we study an extension of our model in which banks optimally choose their capacity for attention. This exercise is closer in nature to models of signal precision and, in the absence of aggregate risk, orthogonal to our analysis.}

Empirically, our model is motivated by evidence on time variation in banks’ lending standards. Using data from the European Central Bank’s **Bank Lending Survey**, we illustrate the counter-cyclicality of standards for loans to non-financial corporations (NFCs) and households in Figures 1 and 2.\footnote{See de Bondt et al. (2010) and van der Veer and Hoeberichts (2016) for evidence on the relevance of the Bank Lending Survey.} More systematic evidence is available, for example, in Asea and Blomberg (1998), who study commercial and industrial loans in the US between 1977 and 1993, or Cunningham and Rose (1994), who study variations in borrower characteristics. More recently Dell’Ariccia et al. (2012) document counter-cyclicality in the standards for US subprime loans, Rodano et al. (2016) for loans to Italian firms, and Minnis et al. (2016), for the construction industry in the US. The common message from this literature is that banks apply stricter lending standards during recessions, while they are more willing to lend in better aggregate conditions. As indicated earlier, this also ties our paper to the empirical evidence on discretion in banks’ lending choices: in addition to the papers mentioned in the introduction, lending standards have been shown to vary with loan officers’ skills (Brown et al., 2017) and gender (Beck et al., 2013), with loan officer rotation (Hertzberg et al., 2010), the degree of securitization the bank engages in (Keys et al., 2010), and with monetary policy (Maddaloni and Peydro, 2011). These papers collectively suggest that banks can and do adjust lending standards, and that this happens both at the level of the individual loan officer and at the organizational level. Berg et al. (2016), for instance, studies a change in the threshold of risk-management’s involvement (i.e. at the organizational level), and identifies loan officers’ response at the individual level. The wealth of evidence on discretion in lending serves to support our assumption that banks adjust their lending behaviour in response to external circumstances; the fact that our predictions align with the evidence on the cyclicality of lending standards suggest that our framework captures a relevant trade off. For completeness, it should be mentioned that some of the existing literature has also suggested that variation in lending standards is reflected in loan rates (Cerqueiro et al., 2011), and that interest rates provide a relevant mechanism for credit rationing during
Finally, our analysis also relates to the theoretical literature on cyclical lending patterns. Rajan (1994) rationalizes variations in lending standards with short managerial horizons and concerns for reputation and provides supportive evidence from real estate loans. Weinberg (1995) does not model banks explicitly, but offers a simple demand-driven rationalization for the expansion of credit, in which improving aggregate conditions allow increasingly risky borrowers to pay the required equilibrium return. Like us, Berger and Udell (2002) pay particular attention to the role of the loan officer, but focus on relationship lending and external determinants of loan officers’ propensity to grant relationship loans. Berger and Udell (2004) introduce the notion of institutional memory loss and ascribe the deterioration of lending standards during good times to the retirement of loan officers who experienced lending busts in the past. Different from our model, their mechanism predicts a history-dependence in the deterioration of standards, that is related to the frequency of the boom-bust cycle. Ruckes (2004) analyzes the interaction between banks’ incentives for costly screening over the business cycle, and price competition among banks. His results on lending standards are comparable to ours, for fixed attention levels, but do not take into account the intensive margin of information collection driving the trade off that is central in our model. Dell’Ariccia and Marquez (2006) consider changes in the demand for credit and, correspondingly, the fraction of new loan applicants. They keep the distribution of borrower types constant, but find that credit standards vary with the fraction of unknown borrowers in the economy. We see our model as complementary to these papers and intend to conduct formal tests of their respective empirical relevance in the future.

3 Model

3.1 Projects

There is a large pool of potential projects, which we think of as loan applications. Each project requires an investment of 1 unit of capital and generates a net return of $X = \mu + \epsilon$, where $\mu$ is the state of the economy and $\epsilon \sim \mathcal{N}(0, \sigma^2_\epsilon)$ is an idiosyncratic shock. The aggregate state is publicly known at all times and
characterised by $\mu < 0$. The idiosyncratic shock, instead, is \textit{ex-ante} unknown and commonly believed to be normally distributed with zero mean and variance $\sigma_\epsilon^2$. Without additional information about the idiosyncratic shock, projects are expected to be \textit{ex-ante} unprofitable ($E[X] = \mu < 0$).

3.2 Financial Intermediaries

Financial intermediaries ("banks") are risk-neutral and can freely borrow at the risk free rate, which we normalize to 0. By paying attention $I$ to a project, they can reduce the uncertainty about $\epsilon$ by $\sigma^2$ and, under the assumption that conditional beliefs remain normal, update their beliefs about $\epsilon$ to $\epsilon|I \sim \mathcal{N}(\epsilon_I, \sigma^2_{\epsilon} - \sigma^2)$. An alternative way to express this is to assume that attention $I$ generates a signal $S = \epsilon + u$, with $u \sim \mathcal{N}(0, \sigma^2_u)$, about $\epsilon$. Upon observing $S = s$, Bayesian updating of beliefs about $\epsilon$ would then imply $\epsilon_I = \frac{\sigma^2_{\epsilon}}{\sigma^2_{\epsilon} + \sigma^2_u} \cdot s$ and $\sigma^2 - \sigma^2 = \frac{\sigma^2_{\epsilon}}{\sigma^2_{\epsilon} + \sigma^2_u} \cdot \sigma^2_{\epsilon}$. We will assume that the relationship between the amount of attention $I$ and the reduction in uncertainty $\sigma^2$ is given by the known function $\sigma^2(I)$, which satisfies the following properties:

1. $\sigma^2(0) = 0$ and $\sigma^2(I) \to \sigma^2_\epsilon$ as $I \to \infty$

2. $0 < \frac{d\sigma^2(I)}{dI} < \infty$ and $\frac{d^2\sigma^2(I)}{dI^2} < 0$ for $I \geq 0$

Thus, paying no attention ($I = 0$) implies that there is no reduction in uncertainty and that the posterior beliefs are identical to the prior beliefs: $\epsilon|I \sim \mathcal{N}(0, \sigma^2_{\epsilon})$. Paying an infinite amount of attention, instead, implies learning the realization of $\epsilon$ perfectly, with no remaining uncertainty: $\epsilon|I \sim \mathcal{N}(\epsilon, 0)$. Finally, the reduction in uncertainty $\sigma^2$ increases with attention $I$, but at a decreasing rate.

These properties are consistent with the relationship between $\sigma^2$ and $I$ in models of rational inattention (e.g. Wiederholt (2010)) that express it as a reduction in entropy; i.e. through $\log\left(\frac{\sigma^2_{\epsilon}}{(\sigma^2_{\epsilon} - \sigma^2)}\right) = I$. We can rearrange this formulation to:

$$\sigma^2(I) = \sigma^2_{\epsilon} \cdot (1 - e^{-I}) \quad (1)$$

We will use this particular functional form for $\sigma^2(I)$ in our numerical examples.
Alternatively, we can define attention as the precision of the signal (as in Kacperczyk et al. (2016)), i.e.

\[ \frac{1}{\sigma_\varepsilon^2 - \sigma^2} = \frac{1}{\sigma_\varepsilon^2} + I \]

In this case

\[ \sigma^2(I) = \sigma_\varepsilon^2 \frac{I}{I + \sigma_\varepsilon^2} \] (2)

Our results are qualitatively the same as long as the above two properties are satisfied.

If banks have limited capacity for attention \( C \) and we assume symmetry across projects, they can vet a total of \( N = C/I \) loan applications each. \(^5\)

### 3.3 Payoffs and Maximization Problem

Banks trade off the precision of their information with the number of loan applications they can process. Conditional on attention \( I \), they approve all projects with a positive expected payoff, i.e. with:

\[ X_I \equiv E[X|I] = \mu + \varepsilon_I \geq 0 \]

Since \( X_I \sim \mathcal{N}(\mu, \sigma^2) \), the expected payoff of a funded project is given by

\[ w \equiv E[X_I|X_I \geq 0] = \mu + \sigma \frac{\phi \left( \frac{\mu}{\sigma} \right)}{\Phi \left( \frac{\mu}{\sigma} \right)} = \sigma \frac{F \left( \frac{\mu}{\sigma} \right)}{\Phi \left( \frac{\mu}{\sigma} \right)}, \]

where \( \phi \) and \( \Phi \) denote the pdf and the cdf of the standard normal distribution, and \( F(x) = \phi(x) + x \cdot \Phi(x) \).

Correspondingly, the expected payoff of an evaluated project (the \textit{ex-ante} expected payoff) is equal to:

\[ \pi \equiv w \cdot Pr\{X_I \geq 0\} = \sigma F \left( \frac{\mu}{\sigma} \right) \]

Banks’ problem is then to choose attention \( I \) to maximize total profit, \( N\pi \), subject to \( N = C/I \) and (1), \(^4\)

\(^4\) Notice that we could also express attention \( I \) as a function of effort \( \rho \) through \( I = \log (1 + \rho) \), if we assumed that effort generates a linear increase in posterior precision (as, for example, in Stoughton (1993)).

\(^5\) Notice that we assume perfect divisibility of projects, implying that \( N \) is a continuous variable.
or equivalently:

\[
\max_I \log \left( \sigma F \left( \frac{\mu}{\sigma} \right) \right) - \log(I) \tag{3}
\]

\text{s.t.: } \sigma^2 = \sigma^2(I)

The first order optimality condition from Problem (3) is

\[
\frac{d \log \left( \sigma F \left( \frac{\mu}{\sigma} \right) \right)}{d \log \sigma} \cdot \frac{d \log \sigma}{d \log I} - 1 = 0,
\]

which we can rewrite as

\[
G \left( \frac{\mu}{\sigma(I)} \right) \cdot \frac{d \log \sigma(I)}{d \log I} - 1 = 0, \tag{4}
\]

by defining \( G(x) \equiv 1 - \left( F'(x)/F(x) \right) \cdot x \).

Banks trade off two objectives: on the one hand, it is optimal to maximize the \textit{ex-ante} return per project (\( \pi \)); on the other hand expected profits increase in the number of vetted projects (\( N \)). The first term in the first order condition ((4)) is the percentage sensitivity to attention of the project return. The second term is the percentage sensitivity to attention of the number of projects \( \frac{d \log N}{d \log I} = -1 \). Thus, it is optimal to increase attention paid to each project if and only if if the first term is larger than 1.

3.4 Results. Level of Attention

Condition (4) characterizes a bank’s optimal attention level \( I \) as a function of the publicly known state of the economy \( \mu \). In Lemma 1 and Proposition 1 we summarize the relationship between \( I \) and \( \mu \).

Lemma 1. \( G(x) \) is a decreasing function of \( x \).

Proof. See Appendix.

According to Lemma 1 the first term of condition (4) is decreasing in \( \mu \). Thus, if \( \mu \) would increase the first order condition would become negative, implying that it would be optimal to reduce \( I \) and \( \sigma \) (see
Figure 3. This immediately implies Proposition 1 and allows us to also describe the relationship between the publicly known state of the economy ($\mu$) and the conditional payoff variance ($\text{Var}[X|I]$):

**Proposition 1.** The optimal amount of attention $I$, paid to each project, is a decreasing function of $\mu$.

**Proof.** See Appendix.

**Corollary 1.** $\text{Var}[X|I] = \sigma^2 - \sigma^2$ is an increasing function of $\mu$.

Together, Proposition 1 and Corollary 1 show that the amount of attention loan officers pay to individual projects is always lower when the state of the economy improves, and that projects become riskier as a result.

We can illustrate the intuition of this result in the following way. Suppose the loan officer decides to pay twice as much attention per project while investigating two times fewer projects. This translates into the corresponding increase in the reduction of uncertainty $\sigma$. Figure 4 shows how the ex-post conditional distribution of expected project return $X_I$ ($\sigma$ is also the standard deviation of this distribution) would change for the cases of high and low $\mu$. Since the ex-ante expected project return is $\pi \equiv E [X_I \cdot 1\{X_I \geq 0\}]$, we see from the figure that the percentage return to an increase in the attention level is higher for lower states of the economy. This happens because with relatively little attention in the downturn, it is unlikely that the conditional expected payoff $X_I$ will exceed zero. As a result, the expected profit is essentially zero as well, which translates into a very large percentage return to additional attention. As a result, the loan officer is more likely to increase the attention she pays to each project when the economy is in a recession.

### 3.5 Results. Expected Return

In addition to their riskiness, we also consider how the aggregate state affects expected project returns. We identify two effects: a direct effect, according to which average project returns increase with aggregate conditions, and an indirect effect, according to which profitability decreases with reduced attention. It turns out that the first effect dominates for very low levels of $\mu$, and thus for high levels of attention per project.
This happens when attention is already high and the bank already knows a lot about \( \varepsilon \) (which implies \( \sigma \approx \sigma_\varepsilon \) and therefore that the overall payoff is not very sensitive to changes in attention). In this case, we have

\[
\pi = \sigma \cdot F \left( \frac{\mu}{\sigma} \right) \approx \sigma_\varepsilon \cdot F \left( \frac{\mu}{\sigma_\varepsilon} \right),
\]

which increases in \( \mu \).

For very high \( \mu \), instead, i.e. when attention \( I \) is close to zero, the indirect effect dominates

\[
\sigma^2(I) \approx \frac{d\sigma^2}{dI} \bigg|_{I=0} \cdot I \quad \Rightarrow \quad \frac{d \log \sigma}{d \log I} \approx 2,
\]

and the first order condition becomes

\[
2 \cdot G \left( \frac{\mu}{\sigma} \right) - 1 = 0 \quad \Rightarrow \quad \frac{\mu}{\sigma} = G^{-1}(2) = \text{const}.
\]

As a result

\[
\pi = \sigma \cdot F \left( \frac{\mu}{\sigma} \right) \approx \sigma \cdot F \left( G^{-1}(2) \right),
\]

which is proportional to \( \sigma \), which is-in turn- decreasing with \( \mu \).

We can illustrate the intuition of the last result in the following way. The percentage return to an increase in \( \sigma \), or \( \frac{d \log \pi}{d \log \sigma} \), depends only on the ratio \( \frac{\mu}{\sigma} \), or on the probability of the project getting financing. Thus, for small \( I \) (when the percentage changes in \( \sigma \) are proportional to the percentages changes in \( I \)) it is optimal to keep the probability of financing constant when \( \mu \) changes. Figure 5 shows the distributions of \( X_I \) for high and low states of the economy \( \mu \) with optimally chosen attention levels. When the state of the economy is high, all projects receive very little attention, and the conditional expected payoffs of financed projects are just above zero. When the state of the economy is low, instead, each project receives a considerable amount of attention and the conditional expected payoff could be further away from zero.

The following condition is sufficient (but not necessary) for a unique global maximum of \( \pi^* \) or \( w^* \). It is

\footnote{If we have two conditional distributions of \( X_I \) that imply the same probability of project financing, then one is necessarily a scaled down version of the other. As a consequence, the percentage returns to an increase in \( \sigma \) must be identical.}
satisfied independent of whether we model attention as “entropy” (equation (1)) or “precision” (equation (2)).

Proposition 2. If \( \frac{d}{d \log \varphi} \left( \frac{d \log \sigma}{d \log \varphi} \right)^{-1} \) is an increasing function of \( \varphi \) then \( \pi^*(\mu) = \pi^*(\mu, I^*(\mu)) \) and \( w^*(\mu) = w^*(\mu, I^*(\mu)) \) are hump-shaped with global maxima at \( \bar{\mu}_\pi < 0 \) and \( \bar{\mu}_w < \bar{\mu}_\pi \).

Proof. See Appendix.

To illustrate our results further, we also solve our model numerically: Figure 6 depicts the optimal level of attention and the corresponding conditional payoff variance directly as functions of \( \mu \). Consistent with Proposition 1 and Corollary 1, we observe that \( I \) is strictly decreasing in \( \mu \), while \( \text{Var}[X|I] \) is strictly increasing. In Figure 7 we plot \( \pi^* \) and \( w^* \) as functions of \( \mu \). While the hump-shaped pattern is clearly visible, the figure also reveals that expected returns increase relatively slowly, for small \( \mu \), while they decrease more rapidly as \( \mu \) becomes large. For completeness, we also illustrate the link between overall expected profits, \( N \pi^* \), and \( \mu \) in Figure 8; as expected, we find that overall profits always increase in the state economy.

4 Model with Aggregate Shocks

In our baseline model, the payoffs of individual projects are independent. Assuming that \( N \) is sufficiently large, this implies that aggregate risk-taking is unaffected by the number of projects a bank invests in, and therefore irrelevant. In this section we deviate from this simplifying assumption and introduce correlation between individual asset returns. As a result, banks now have to trade off learning about the aggregate state of the economy with learning about idiosyncratic project risk. In this setting, we show that banks may find it optimal to reduce learning about aggregate conditions during booms, and that portfolio risk can thus increase in the state of the economy.

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7 We use the \( \sigma^2(I) \) from (1) for the numerical solution. In this case, the optimal attention level, \( I \), and the profitability measures, \( \pi/\sigma_I \) and \( w/\sigma_I \), can be uniquely described as functions of \( \mu/\sigma_I \).

8 Notice that one can also see how the sensitivity with respect to \( \mu \) increases mildly for moderate values of \( \mu \) and then decreases for very large values.
4.1 Setup

In addition to the known state of the economy, project returns now depend on an idiosyncratic shock $\epsilon \sim \mathcal{N}(0, \sigma^2_{\epsilon})$ and a common shock $\xi \sim \mathcal{N}(0, \sigma^2_{\xi})$ with publicly known distributions:

$$X = \mu + \xi + \epsilon$$

Intermediaries allocate attention $I$ per project, to learn about idiosyncratic risk-factors, and attention $J$ to learn about the common shock. We denote the corresponding reduction in uncertainty about $\epsilon$ and $\xi$ by $\sigma^2_I$ and $\sigma^2_J$, and assume that conditional beliefs remain normal, i.e. that $\epsilon|I \sim \mathcal{N}(\epsilon_I, \sigma^2_{\epsilon} - \sigma^2_I)$ and $\xi|J \sim \mathcal{N}(\xi_J, \sigma^2_{\xi} - \sigma^2_J)$. Again, we assume the relationships between the uncertainty reductions and the amounts of attention are described by known functions $\sigma^2_I(I)$ and $\sigma^2_J(J)$, which satisfy the properties from Section 3.2.

If $N$ projects are investigated, banks’ attention constraint is given by:

$$J + N \cdot I \leq C$$

Different from before, we now assume that the overall number of available projects in the economy is limited, i.e. that $N \leq \bar{N}$.\(^9\)

Similar to before, expected payoffs from individual projects are now denoted by

$$X_{I,J} \equiv E[X|I,J] = \mu + \epsilon_I + \xi_J,$$

\(^9\)In the numerical examples we use $\sigma^2_I = \sigma^2_{\epsilon} (1 - e^{-I})$ and $\sigma^2_J = \sigma^2_{\xi} (1 - e^{-J})$

\(^{10}\)We add this assumption here to prevent expected profits from converging to infinity. The results in the first part of the paper are unchanged for a sufficiently large $\bar{N}$; for smaller $\bar{N}$ they remain unchanged until $N = \bar{N}$. Afterwards all available projects are evaluated by the intermediary, and the available capacity for attention is distributed equally among them. Alternatively, we could have assumed a fixed attention cost per project.
and since $X_{I,J} \sim \mathcal{N}(\mu, \sigma^2)$, with
\[
\sigma^2 = \text{Var}[X_{I,J}] = \text{Var}[\epsilon_I] + \text{Var}[\xi_J] = \sigma_I^2 + \sigma_J^2, \tag{6}
\]
conditional expected returns of funded projects are equal to:
\[
w \equiv E[X_{I,J}|X_{I,J} \geq 0] = \mu + \sigma \frac{\phi(\frac{\mu}{\sigma})}{\Phi(\frac{\mu}{\sigma})} = \sigma \frac{F(\frac{\mu}{\sigma})}{\Phi(\frac{\mu}{\sigma})}
\]
For \emph{ex-ante} expected payoffs this implies:
\[
\pi \equiv w \cdot \Pr\{X_{I,J} \geq 0\} = \sigma \frac{F(\frac{\mu}{\sigma})}{\Phi(\frac{\mu}{\sigma})}
\]
Using this notation, the objective of the intermediary, when there is aggregate risk, is to maximize total profits, $N\pi$, by choosing $I$ and $J$ subject to (6), (5), and $N \leq \bar{N}$, or:
\[
\max_{I,J} \frac{C - J}{I} \cdot \pi(\mu, \sigma^2) \tag{7}
\]
\[
s.t. : \quad \sigma^2 = \sigma_I^2(I) + \sigma_J^2(J)
\]
\[
C - J \leq \bar{N}
\]

4.2 Portfolio Risk

If the number of financed projects in the portfolio ($\bar{N} \leq N$) is sufficiently large, the average actual payoff per project is
\[
\bar{r} = \frac{1}{\bar{N}} \sum_{i=1}^{\bar{N}} X_i \approx \mu + \xi + \frac{1}{\bar{N}} \sum_{i=1}^{\bar{N}} \epsilon_i,
\]
where $\sum_{i=1}^{N} \epsilon_i / \bar{N}$ is a known constant. This allows us to express the corresponding variance as
\[
\sigma_{\bar{r}}^2 = \text{Var}[\bar{r}|I,J] = \text{Var}[\xi|J] = \sigma_x^2 - \sigma_J^2(J), \tag{8}
\]
which implies that portfolio risk decreases in $J$, i.e. in attention to the common shock. To understand the implications of optimal attention allocation for aggregate portfolio risk, we can therefore focus on the optimal choice of $J$.

### 4.3 Results (fixed $I$)

For a better understanding of the involved trade-offs, we first keep the amount of attention allocated to an individual project $I$ constant. A bank’s only decision is then to determine the attention that is allocated to learning about the aggregate shock and its objective is to maximize $\mathcal{E}^{-1} \cdot \pi(\mu, \sigma^2)$ subject to $\sigma^2 = \sigma_I^2(I) + \sigma_\xi^2(J)$, which can be also expressed as:

$$\max_{J} \log \left( \sigma_F \left( \frac{\mu}{\sigma} \right) \right) + \log(C - J)$$

s.t.: $\sigma^2 = \sigma_I^2(I) + \sigma_\xi^2(J)$,

The first order condition of problem (9) is given by

$$G \left( \frac{\mu}{\sigma(J)} \right) \cdot \frac{d \log \sigma(J)}{d \log J} - \frac{1}{C - J} = 0. \tag{10}$$

Because the first term is a decreasing function $\mu$, the optimal amount of attention to the aggregate shock is decreasing with $\mu$ (the first order condition becomes negative as $\mu$ increases, which means it is optimal to reduce $\sigma$ and correspondingly $J$). In other words: keeping everything else constant, it is optimal to allocate less attention to aggregate risk when economic conditions improve. We summarize this finding in Proposition 3 and derive the implications for portfolio risk in Corollary 2.

**Proposition 3.** For a given $I$, the optimal amount of attention $J$ is a decreasing function of $\mu$.

**Corollary 2.** For a given $I$, $\sigma_\xi^2$ is an increasing function of $\mu$. 

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4.4 Results (general)

Next, we assume that the intermediary simultaneously chooses $I$ and $J$. To this end, we consider the highest reduction in uncertainty ($\sigma^2$) that can be achieved for a given number of evaluated projects ($N$):

$$\sigma^2(N) \equiv \max_{I,J} \left\{ \sigma^2(I) + \sigma^2(J) \right\}$$

s.t. \hspace{1cm} N = \frac{C - J}{I}

With this definition of $\sigma^2(N)$, we can rewrite the problem of the bank (7) as:

$$\max_N N \cdot \pi(\mu, \sigma^2(N)) \quad (11)$$

Next, we define $I(N)$ and $J(N)$ as the attention allocation that gives the highest $\sigma^2$ for a given $N$. Lemma 2 summarizes the properties of $\sigma^2(N), I(N)$ and $J(N)$.

**Lemma 2.**

1. If $N \leq \frac{\sigma^2_{\mu - a}}{\sigma^2_{\mu - c}}$, then $I(N)$ and $\sigma^2(N)$ are strictly decreasing functions of $N$

2. If $N \geq \frac{\sigma^2_{\mu - a}}{\sigma^2_{\mu - c}}$, then the optimal $I(N)$ is equal to zero and $\sigma^2(N) = \sigma^2_J(C)$

3. $J(N)$ is a decreasing function of $I \cdot \frac{d\sigma^2}{dI} |_{I=I(N)}$.

We know that $I \cdot \frac{d\sigma^2}{dI}$ is close to zero both for $I \approx 0$ and for $I \rightarrow \infty$. In what follows, to simplify the exposition, we will assume that $I \cdot \frac{d\sigma^2}{dI}$ has a unique global maximum $I^m$, and denote the corresponding attention to the aggregate shock and the number projects considered by $J^m$ and $N^m = \frac{C - J^m}{I^m}$ respectively.

We illustrate the results of Lemma 2 in Figure 9. Since $J = C - NI$, it follows that

$$\frac{d \sigma^2}{dI} = \frac{d \sigma^2_I}{dI} + \frac{d \sigma^2_J}{dJ} \cdot \frac{dJ}{dI} = \frac{d \sigma^2_I}{dI} - N \cdot \frac{d \sigma^2_J}{dJ}$$

11 since $\sigma^2_I \rightarrow \sigma^2_{\mu - a}$ then $I \cdot \frac{d\sigma^2_I}{dI} \rightarrow 0$ as $I \rightarrow \infty$. Also, since $\frac{d\sigma^2_I}{dI} < \infty$ then $I \cdot \frac{d\sigma^2_I}{dI} \approx 0$ for $I \approx 0$

12 Notice that $I \cdot \frac{d\sigma^2}{dI}$ has a global maximum, independent of whether we model attention as “entropy” or “precision”, i.e. as in equations (1) or (2).
\[
\frac{1}{I} \left( I \cdot \frac{d\sigma^2_I}{dI} - (C - J) \frac{d\sigma^2_J}{dJ} \right).
\]

For a given \(N\) the optimal \(I\) and \(J\) are therefore determined by the intersection of the line \(J = C - NI\) with the curve defined by \(I \cdot \frac{d\sigma^2_I}{dI} = (C - J) \frac{d\sigma^2_J}{dJ}\). As \(N\) increases, Figure 9 shows that \(I\) always declines. The direction for \(J\), instead, depends on the position on the curve. Since \((C - J) \frac{d\sigma^2_J}{dJ}\) is a decreasing function of \(J\), it follows that \(J\) is higher for lower \(I \cdot \frac{d\sigma^2_I}{dI}\).

The first order condition with respect to \(N\) for the optimization problem (11) is

\[
\frac{d \log \left( \sigma F \left( \frac{\mu}{\sigma} \right) \right)}{d \log \sigma} \cdot \frac{d \log \sigma}{d \log N} + 1 = 0,
\]

which we can express as:

\[
G \left( \frac{\mu}{\sigma(N)} \right) \cdot \frac{d \log \sigma(N)}{d \log N} + 1 = 0. \tag{12}
\]

The first term in condition (12) is a negative increasing function of \(\mu\). Thus, when \(\mu\) increases the first order condition turns positive, and it becomes optimal to increase the number of considered projects \(N\). This leads to Proposition 4.

**Proposition 4.**

1. The optimal number of vetted projects \((N)\) increases in \(\mu\).

2. The overall riskiness of each project

\[
Var[X \mid I, J] = Var[\xi + \epsilon \mid I, J] = \sigma^2_\epsilon + \sigma^2_\xi - \sigma^2(N)
\]

increases in \(\mu\).

3. Attention to the idiosyncratic shock \(I\) decreases in \(\mu\).

4. Attention to the common shock \(J\) decreases in \(\mu\) if \(N < N^m\).

That is, as the state of the economy improves, banks choose to finance more projects, and optimally learn less about each of them. As a result, funded projects become riskier. Together with condition (8), Proposition 4 also allows us to derive implications for aggregate portfolio risk.
Corollary 3. While $N < N^m$, $\sigma_f^2$ increases in $\mu$.

Corollary 4. For $\tilde{N} < N^m$, $\sigma_f^2$ always increases in $\mu$.

To understand the intuition behind the qualifications, consider first that $N$ increases in $\mu$. An increase in $N$, in turn, has two effects: first, $I$ and $J$ must be reduced so that each new project can be allocated $I$ units of attention (from the capacity constraint [5]); second, learning about the common shock becomes relatively more attractive (because knowledge about it applies to more projects). According to Proposition [4] the first effect is stronger when $I$ is large. This is the case, because a high $I$ means that $I$ and $J$ need to be reduced a lot in order to consider an additional project.

In Figure 10, we graphically illustrate the relationship between $N$, $\sigma^2$, $I$ and $J$, and the aggregate state, for $\tilde{N} = 100$ and $\tilde{N} = 200$, respectively. Consistent with Proposition [4] we observe an upward sloping function for $N$, and downward sloping functions for $\sigma^2$ and $I$. For $J$, instead, and thereby for aggregate portfolio variance, we observe that the relationship with $\mu$ depends on $\tilde{N}$. The reason is, that $N > N^m$ is not guaranteed for all $\mu$, if $\tilde{N}$ is sufficiently large.

5 Extensions

5.1 Flexible Attention Budget

So far, we assumed that banks’ attention budget is fixed. Especially if we adopt an institutional interpretation of our setup, however, it seems plausible that the overall capacity for attention can be adjusted, e.g. by hiring additional staff. In this section we extend our model to this case, and allow banks to choose the optimal $C$. We find that the returns to additional attention vary with the business cycle, and that it becomes profitable to increase the overall budget of attention during upturns. Our main results, however, remain unaffected. Even within a larger budget of attention, it still needs to be determined whether to allocate resources to analyzing more projects or to analyzing each project more carefully. This trade-off is orthogonal to the choice of $C$. As a result, the amount of attention allocated to each project is the same as in our baseline model, and decreasing with the state of the economy.
5.1.1 Setup

Suppose banks incur a cost $F(C)$ when choosing an amount of attention $C$. Assume, in addition, that costs are increasing in $C$ and convex: $F'(C) > 0, F''(C) > 0$. The objective function of the financial institution is then given by $N \cdot \pi(\sigma, \mu) - F(C)$, which, using $N \cdot I = C$, implies that we can write the maximization problem as:

$$\max_{I,C} C \cdot \frac{\pi(\sigma, \mu)}{I} - F(C)$$

$$\text{s.t. : } \sigma^2 = \sigma_e^2 \cdot (1 - e^{-I})$$

(13)

5.1.2 Results

From (13), it is clear that $C$ does not affect the amount of attention paid to each individual project. To see this, denote the highest amount of attention per project that a bank can reach by $\Pi^*$:

$$\Pi^* = \max_{I} \frac{\pi(\sigma, \mu)}{I}$$

$$\text{s.t. : } \sigma^2 = \sigma_e^2 \cdot (1 - e^{-I})$$

The optimal attention budget is then determined from

$$\max_C C \cdot \Pi^* - F(C),$$

and equal to $C^* = (F')^{-1}(\Pi^*)$. Since $\Pi^*$ is increasing in $\mu$, the following proposition follows immediately:\n
Proposition 5. When banks choose the attention budget $C$,

\[13\]

Because we include capacity choice primarily to distinguish our model from related work by [Ruckes 2004] and Dell’Ariccia and Marquez [2006], we focus on the case without aggregate risk. If we include aggregate risk in the model with capacity choice, the two margins of adjustment are no longer orthogonal and potentially interesting interactions arise. We leave this extension for future research.
1. optimal attention per project \((I^*)\) remains the same as in the baseline model, and decreases in the state of the economy.

2. the optimal choice \((C^*)\) increases in the state of the economy.

5.2 Model with Shared Borrower Pool

In this extension we study a setup in which two banks evaluate loan applications from the same pool of potential borrowers, and analyze implications for the optimal allocation of attention. Different from the case with a representative bank, where the outcome is always socially optimal, individual rationality now no longer guarantees efficiency: one bank’s choice to evaluate an additional project causes a negative externality on the other bank, as it increases the probability of loosing profits to the “competitor”\(^{14}\). In the decentralized equilibrium, banks do not fully internalize this externality, and process inefficiently many loan applications\(^{15}\).

5.2.1 Setup

As before, there is a large pool of potential projects with \(N \leq \bar{N}\). Each project requires an investment of one and generates a net return of \(X = \mu + \epsilon\), where \(\epsilon \sim N(0, \sigma^2_{\epsilon})\). Instead, there are now two banks, which can randomly select projects from the pool and learn, as before, about \(\epsilon\). By paying attention \(I_i\), bank \(i = 1, 2\) can reduce the uncertainty about \(\epsilon\) by \(\sigma^2_i = \sigma^2(I_i)\), where \(\sigma^2(I_i)\) is a known function satisfying the properties from Section 3.2. Similar to the original model, the new conditional beliefs about \(\epsilon\) are then given by:

\[
\epsilon | I_i \sim N(\epsilon_i, \sigma^2_{\epsilon} - \sigma^2(I_i))
\]

For simplicity, we assume that the information that banks receive is independent. This is equivalent to

\(^{14}\)Banks in our setup do not actively compete for borrowers. Instead, we assume that borrowers are randomly assigned to one bank if their application is approved by multiple lenders.

\(^{15}\)Notice that this extension links our framework to the literature on non-exclusive loan contracts. Parlour and Rajan (2001), for example, provide a model in which one lender’s loan offer imposes a negative externality on other lenders, because borrowers’ default depends on the overall loan volume.
assuming that both banks receive a signal \( S_i = \varepsilon + u_i \) where \( u_i \) are independent random variables. When choosing a project, banks do not observe the actions of the other bank, and decide whether to invest or not purely based on their own evaluation. Loan applicants accept any offer they receive, and randomize over the offers if both banks are willing to lend. As a result, banks expect to earn profits only half of the time, if the rival bank approves the same loan application.

We now proceed to consider an equilibrium in which it is optimal for a bank to invest, as long as its signal is higher than a certain cutoff \( y \). To this end, we denote the profit of bank \( i \) by \( \pi(\sigma_i, y_i) \), if it is the only one evaluating a project, and by \( \bar{\pi}(\sigma_i, y_i, \sigma_{-i}, y_{-i}) \) if both banks evaluate the same project. Both profits can be calculated as follows:

\[
\pi(\sigma_i, y_i) = E \left[ (\mu + \varepsilon) \cdot I \{ \varepsilon_i \geq y_i \} \right]
\]

\[
\bar{\pi}(\sigma_i, y_i, \sigma_{-i}, y_{-i}) = E \left[ (\mu + \varepsilon) \cdot I \{ \varepsilon_i \geq y_i \} \right] - \frac{1}{2} E \left[ (\mu + \varepsilon) \cdot I \{ \varepsilon_i \geq y_i \} \cdot I \{ \varepsilon_{-i} \geq y_{-i} \} \right]
\]

Next, we denote the number of projects evaluated by bank \( i \) by \( N_i \) and call \( \alpha_i = \frac{N_i}{\bar{N}} \) the probability that bank \( i \) evaluates a given project.

In equilibrium, each bank then maximizes its expected profits taking the choices of the other bank as given:

\[
\max_{\sigma_i, y_i, N_i} \Pi_i = N_i \cdot (\alpha_{-i} \cdot \bar{\pi}(\sigma_i, y_i, \sigma_{-i}, y_{-i}) + (1 - \alpha_{-i}) \cdot \pi(\sigma_i, y_i)))
\]

s.t. \( N_i \cdot I_i \leq C \)

\[
\sigma_i^2 = \sigma^2(I_i) \quad \sigma_{-i}^2 = \sigma^2(I_{-i})
\]

\( N_i \leq \bar{N} \)

The socially optimal outcome, instead, under which banks can coordinate on the number of processed
applications, is given by

$$\max_{\sigma,N} N \cdot (\alpha \cdot \bar{\pi}(\sigma, y_i, \sigma, y_{-i}) + (1 - \alpha) \cdot \pi(\sigma, y))$$

s.t.

$$N \cdot I \leq C$$
$$\sigma^2 = \sigma^2(I)$$
$$\alpha = 1 - \frac{N}{\bar{N}}$$
$$N \leq \bar{N}$$

$$y = \arg \max_{\bar{y}} \alpha \cdot \bar{\pi}(\sigma, y_i, \sigma, y) + (1 - y) \cdot \pi(\sigma, y_i)$$

5.2.2 Numerical Solution

We solve for the decentralized and the socially optimal outcome numerically, and illustrate the solution in Figure 11. The solution assumes $\bar{N} = 200$ and $C = \bar{N}/2 = 100$, which implies that the lowest possible amount of attention per project is equal to 0.5. Consistent with our earlier predictions, we observe that banks investigate loan applications less carefully in a decentralized equilibrium than under a social planner. By virtue of the binding capacity constraint, this implies that they process inefficiently many applications, and provide funding to inefficiently risky borrowers. As intuition would suggest, we also find the effect to be larger for higher $\mu$, i.e. when banks evaluate more projects and a duplicity of approvals occurs more frequently.

6 Conclusion

We develop a model of rational inattention that explains the excessively lenient lending standards and endogenous risk build-up during market booms. Banks optimally pay less attention to individual projects during booms, because they can make more profits by expanding their pool of borrowers. As a result, projects funded during booms are more risky and can have lower expected returns. Our results are consistent with

\[\sigma^2(I) = \sigma^2 \left(1 - e^{-I}\right)\]
the available evidence on the variation of lending standards over the business cycle (Asea and Blomberg [1998], and on the time-variation in loan officers’ processing times (Brown et al. [2017]) and the intensity of banks’ informational requests (Jimenez and Saurina [2006]; Jimenez et al. [2012]).

In our main model, with a representative bank, attention allocation decisions are necessarily socially optimal. This, however, might not be the case if banks are heterogeneous (for example, due to firesale externalities) or when risk preferences of regulators and loan officers are different (for example, due to convex compensations schemes). In addition, as we show in an extension of our model, attention allocation is also inefficient when banks provide credit to the same pool of borrowers. In this case, they do not internalize how their optimal choice of attention affects the depth of the pool of potential borrowers. As a result, they process too many loan applications with low scrutiny applied to each of them. In such environments, our model would have implications for the compensation of loan officers, as well as for the organizational structure and regulation of banks. Limiting the number of processed applications per loan officer or bank, for example, could be a mechanism to rein in excessive, and possibly hidden, risk-taking during economic booms. This could be achieved by reducing volume-based compensation, or by implementing time-varying bottlenecks in the organization and supervision of banks (e.g. by externally imposing a threshold, at which loan officers need to relay applications to a risk-management unit).

Alternatively, our framework also speaks to other domains in which time-varying certification quality is important; in the spirit of Bar-Isaac and Shapiro (2013), for instance, it could potentially contribute to the understanding of countercyclical ratings quality. In addition, it may also speak to the financing of start-ups by angel investors and venture capital funds. Lastly, it has also been argued that regulatory scrutiny itself is subject to fluctuations (e.g. Weinberg [1995]). In this context, an adaptation of our framework could help to study the consequences of attention-constrained supervisors.
References


Appendix

Figures

Figure 1: Lending Standards (NFC Loans)

Note: The figure illustrates the evolution of lending standards, according to the ECB’s Bank Lending Survey, and GDP growth in the Euro Area, between 2002Q3 and 2006Q2. The indicator for lending standards ranges from -100% (all respondents reply that criteria for the approval of loans have “eased considerably”) to +100% (all respondents reply that criteria for the approval of loans have “tightened considerably”). Source: NBB.
Figure 2: **Lending Standards (Mortgages)**

Note: The figure illustrates the evolution of lending standards, according to the ECB’s Bank Lending Survey, and GDP growth in the Euro Area, between 2002Q3 and 2006Q2. The indicator for lending standards ranges from -100% (all respondents reply that criteria for the approval of loans have “eased considerably”) to +100% (all respondents reply that criteria for the approval of loans have “tightened considerably”). Source: NBB.
Figure 3: Profit Maximization

Note: The figure shows how the percentage sensitivity of project return to attention $\frac{\partial \pi}{\partial \mu}$ depends on attention level $I$ for high and low values of $\mu$. The attention level is optimal when the this sensitivity is equal to 1 (which the sensitivity of the number of project to attention). The graphs are shown for $\mu = -1$ and for $\mu = -0.5$ with $\sigma^2[I] = 1 - e^{-I}$. 
Figure 4: Sensitivity of Profits to Increases in Attention

high $\mu$

low $\mu$

Note: Figures shows how the conditional distributions of expected project payoff $X_f$ changes with the amount of attention for high and low $\mu$. Since the expected project ex-ante project is $\pi = E[X_f | X_f \geq 0]$ the percentage return to additional attention is higher for the low state of the economy $\mu$. The graphs are shown for $\mu = -1$ and $\mu = -0.5$ and $\ell = 0.15$ and $\ell = 0.3$ with $\sigma^2(\ell) = 1 - e^{-\ell}$. 
Figure 5: Optimal Conditional Distributions of Returns

Note: The figure shows how the ex-post conditional distribution of expected project return $X_I$ with optimally chosen attention level $I$ depends on state of the economy. Since the ex-ante expected project return is $\pi = E[X_I \cdot \{X_I \geq 0\}]$, we can see it is actually lower for high $\mu$. The results are shown for $\mu = -0.2$ and $\mu = -0.1$ with $\sigma^2(I) = 1 - e^{-I}$. 
Figure 6: **Attention and Riskiness of a Project**

Note: The figures show how the optimal attention level $I$ and riskiness of the project $\text{Var}[X|I]$ depend on the state of the economy $\mu$. The results are shown for the “entropy” attention technology function $\sigma^2(I) = \sigma^2 \left(1 - e^{-I}\right)$.
Figure 7: Expected Returns

\[ \frac{\pi}{\sigma_e} \]

\[ w_{\sigma_e} \]

\( \mu/\sigma_e \)

Note: The figure shows how the ex-ante project return \( \pi \) and the average ex-post project return \( w \) depend on the state of the economy \( \mu \) for \( \sigma^2(t) = \sigma^2(1 - e^{-t}) \).

Figure 8: Expected Overall Profits

\[ (\Delta \pi)/(\xi_\sigma) \]

\( \mu/\sigma_e \)

Note: The figure shows how the overall expected profits \( N \cdot \pi \) depend on the state of the economy \( \mu \) for \( \sigma^2(t) = \sigma^2(1 - e^{-t}) \).
Figure 9: **With Aggregate Shocks** ($\bar{N} = 100$)

Note: For a given $N$ the optimal amounts of attention to idiosyncratic and aggregate shocks, $I$ and $J$, are determined by the intersections of the curves $J \frac{d\sigma_2^2}{dJ} = (C - J) \frac{d\sigma_1^2}{dJ}$ and $J = C - N \cdot I$. The figure plots these two curves for $\mu = 1$, $C = 20$, $N = 4$, $\sigma_1^2 = 1 - e^{-I}$ and $\sigma_2^2 = 1 - e^{-J}$. 
Figure 10: With Aggregate Shocks

\[
\bar{N} = 100
\]

\[
\bar{N} = 200
\]

Note: The figures show how the number of evaluated project \(\bar{N}\), uncertainty reduction \(\sigma^2\), optimal attention levels to the idiosyncratic and aggregate shocks, \(I\) and \(J\) depend on the state of the economy \(\mu\) for the cases when total number of potential projects either \(\bar{N} = 100\) or \(\bar{N} = 200\). The results are shown for \(\sigma^2(I) = \sigma^2 (1 - e^{-I})\) and \(\sigma^2(J) = 0.05 (1 - e^{-J})\) and \(C = 100\).
Figure 11: Attention and Number of Projects with Shared Borrower Pool

Note: The figures show how the optimal attention level $\ell$ and the optimal number of projects evaluated by each bank $N$ change with the state of the economy $\mu$. The results are shown for $\sigma^2(\ell) = \sigma^2(1 - e^{-\ell})$ and $N = 200$ and $C = N/2 = 100$. 

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Proofs.

Proof of Lemma [1].

By definition $G(x) = 1 - \frac{F'(x)}{F(x)}x$ where $F(x) = \phi(x) + x\Phi(x)$. Given that $\Phi'(x) = \phi(x)$ and $\phi'(x) = -x \cdot \phi(x)$ we have

$$F'(x) = -x \cdot \phi(x) + \Phi(x) + x \cdot \phi(x) = \Phi(x)$$

$$G = 1 - \frac{\phi(x) \cdot x}{\phi(x) + x\Phi(x)} = \frac{\phi}{\phi + x\Phi}$$

$$G'(x) = -\frac{x\phi \cdot (x\Phi) - \phi \cdot \Phi}{(\phi + x\Phi)^2} = -\phi \cdot \frac{(1 + x^2)\Phi + x\phi}{(\phi + x\Phi)^2}$$

It remains to show that $Q(x) = (1 + x^2)\Phi + x\phi > 0$. Since

$$Q'(x) = 2(\phi + x\Phi)$$

$$Q''(x) = 2\Phi(x) > 0$$

we get

$$\lim_{x \to -\infty} Q'(x) = 0 \quad \Rightarrow \quad Q'(x) > 0$$

$$Q''(x) > 0$$

$$\lim_{x \to -\infty} Q(x) = 0 \quad \Rightarrow \quad Q(x) > 0$$

Proof of Proposition [1].

Suppose $I$ and $\sigma$ and correspondingly $I + \Delta I$ and $\sigma + \Delta \sigma$ are are optimal economy $\mu$ and $\mu + \Delta \mu$ with $\Delta \mu > 0$. Then

$$\frac{\pi(\mu, \sigma)}{I} > \frac{\pi(\mu, \sigma + \Delta \sigma)}{I + \Delta I}$$

$$\frac{\pi(\mu + \Delta \mu, \sigma)}{I} < \frac{\pi(\mu + \Delta \mu, \sigma + \Delta \sigma)}{I + \Delta I}$$

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As a result
\[
\frac{\pi(\mu, \sigma + \Delta \sigma)}{\pi(\mu, \sigma)} < \frac{I + \Delta I}{I} < \frac{\pi(\mu + \Delta \mu, \sigma + \Delta \sigma)}{\pi(\mu + \Delta \mu, \sigma)}
\]

Now since \( \frac{d \log \pi}{d \log \sigma} = G\left(\frac{\mu}{\sigma}\right) \) is decreasing in \( \mu \) the above inequality can hold only if \( \Delta \sigma < 0 \).

**Proof of Proposition 2**

1. The first order condition (here \( x = \frac{\mu}{\sigma} \))
\[
G(x) = \frac{d \log I}{d \log \sigma}
\]
Differentiating the first order condition with respect to \( \mu \)
\[
G'(x) \cdot \frac{dx}{d\mu} = \frac{d}{d \log I} \left( \frac{d \log I}{d \log \sigma} \right) \cdot \frac{d \log \sigma}{d \mu}
\]
Note that since \( \frac{d}{d \log I} \left( \frac{d \log I}{d \log \sigma} \right) \geq 0 \) and \( G'(x) < 0 \) then \( \frac{dx}{d\mu} > 0 \). We can also rewrite the above equation as
\[
\frac{G'}{G} \cdot \frac{dx}{d\mu} = \frac{d}{d \log I} \left( \frac{d \log \sigma}{d \log I} \right) \cdot \frac{d \log \sigma}{d \mu}
\]

2. The expected payoff from the project is \( \pi = \sigma \cdot F(x) \) and the derivative with respect to \( \mu \)
\[
\frac{d \log(\pi)}{d \mu} = \frac{d \log \sigma}{d \mu} + \frac{F'}{F} \cdot \frac{dx}{d\mu} =
\]
\[
\frac{d \log \sigma}{d \mu} + \frac{G F'}{G' F} \cdot \frac{d}{d \log I} \left( \frac{d \log \sigma}{d \log I} \right) \cdot \frac{d \log \sigma^2}{d \mu} =
\]
\[
\left( -\frac{d \log \sigma}{d \mu} \right) \left( -1 + \left( -\frac{G F'}{G' F} \right) \cdot \frac{d}{d \log I} \left( \frac{d \log \sigma}{d \log I} \right) \right)
\]

3. We can show that
\[
H(x) = -\frac{G F'}{G' F} = -\frac{\Phi}{\Phi + x \phi} \cdot \frac{\phi}{\phi + x \Phi} \cdot \frac{\Phi}{\Phi + x \phi} = \frac{\Phi}{(1 + x^2) \Phi + x \phi}
\]
is a positive decreasing function of \( x \), and thus it is decreasing with \( \mu \). Also, since \( \frac{d}{d\log I} \left( \frac{d\log \sigma}{d\log I} \right) \) is increasing with \( I \), and thus decreasing with \( \mu \) we will have a unique global maximum.

4. The proof for the existence of the maximum for \( w \) is similar. Only in this case

\[
H(x) = -\frac{G}{G^2} \frac{F'}{F}
\]

where

\[
\bar{F} = \frac{F}{\Phi} \quad \bar{F}' = \frac{\Phi}{F} - \frac{\phi}{F}
\]

and thus

\[
H = -\frac{\frac{\phi}{\bar{F}' + x\bar{F}}}{-\phi \cdot \frac{(1+x^2)\Phi + x\Phi}{(\phi + x\Phi)^2}} \left( \frac{\Phi}{\phi + x\Phi} - \frac{\phi}{\Phi} \right) = \frac{\Phi^2 - \phi(\phi + x\Phi)}{\Phi((1+x^2)\Phi + x\phi)}
\]

Again, we can check that \( H(x) \) is a decreasing function of \( x \).

5. The maximum of \( w \) is below the maximum of \( \pi \) since \( \pi = w \cdot \Phi(x) \) and \( \Phi(x) \) is always decreasing with \( \mu \).

**Proof of Lemma 2**

1. For a given \( N \) the first order condition with respect to \( I \)

\[
\frac{d\sigma^2}{dI} = \frac{d\sigma^2}{dI} - N \cdot \frac{d\sigma^2}{dJ} = 0
\]

Moreover, notice that \( \sigma^2(I) \) is a convex function of \( I \)

\[
\frac{d^2\sigma^2}{dI^2} = \frac{d^2\sigma^2}{dI^2} + N^2 \cdot \frac{d^2\sigma^2}{dJ^2} < 0
\]

Thus if \( \frac{d\sigma^2}{dT} |_{t=0} < 0 \) then it is optimal to choose \( I = 0 \)
2. For a given $N$ consider $\sigma^2$ as function of $I$: $\sigma^2(I) = \sigma_I^2(I) + \sigma_J^2(C - NI)$. Then

$$\frac{d\sigma^2}{dN} = \frac{\partial \sigma^2}{\partial N} = \frac{d\sigma_I^2}{dI} \cdot \frac{\partial I}{\partial N} = -I \cdot \frac{d\sigma_I^2}{dJ} < 0$$

3. For the internal solution ($I > 0, J > 0$)

$$\frac{d\sigma_I^2}{dI} = N \cdot \frac{d\sigma_J^2}{dJ}$$
$$N \cdot I + J = C$$

If we differentiate the above equations we get

$$\frac{d^2\sigma_I^2}{dI^2} \cdot \frac{dI}{dN} = \frac{d\sigma_I^2}{dJ} \cdot \frac{dJ}{dN} + N \cdot \frac{d^2\sigma_I^2}{dJ^2} \cdot \frac{dJ}{dN}$$

$$I + N \frac{dI}{dN} + \frac{dJ}{dN} = 0$$

Solving for $\frac{dI}{dN}$ we get

$$\frac{dI}{dN} = \frac{d\sigma_I^2}{dJ} + N I \left( -\frac{d^2\sigma_I^2}{dI^2} \right) < 0$$

4. Finally, if $J = 0$ then $I = \frac{C}{N}$ and is also decreasing with $N$.

**Proof of Proposition**

1. Suppose $N$ and $\sigma$ are optimal for $\mu$ and $N + \Delta N$ and $\sigma + \Delta \sigma$ are optimal for $\mu + \Delta \mu$ with $\Delta \mu > 0$.

   Optimality conditions

   $$N \cdot \pi(\mu, \sigma) > (N + \Delta N) \cdot \pi(\mu, \sigma + \Delta \sigma)$$

   $$N \cdot \pi(\mu + \Delta \mu, \sigma) < (N + \Delta N) \cdot \pi(\mu + \Delta \mu, \sigma + \Delta \sigma)$$
as a result

\[
\frac{\pi(\mu, \sigma + \Delta \sigma)}{\pi(\mu, \sigma)} < \frac{\pi(\mu + \Delta \mu, \sigma + \Delta \sigma)}{\pi(\mu + \Delta \mu, \sigma)}
\]

Since \( \frac{d\log \pi}{d\log \sigma} = F \left( \frac{\mu}{\sigma} \right) \) is decreasing with \( \mu \) then the above inequality can hold only if \( \Delta \sigma < 0 \) and correspondingly \( \Delta N > 0 \).

2. The remaining results directly follow from lemma\(^2\) and the fact that \( \sigma^2 \) is decreasing with \( N \).