Bayesian Belief Update and Mispricing: Theory and Experiment

Munenori Nakasato (Aoyama Gakuin Univ., Tokyo, nakasato@gsim.aoyama.ac.jp)
Tomoki Kitamura (NLI Research Institute, Tokyo, kitamura@nli-research.co.jp)
Hirotaka Fushiya (Aoyama Gakuin Univ., Tokyo, fushiya@gmail.com)

Abstract

- We assume an asymmetric information environment.
- There are two uncertainties: uncertainty of the asset value and the existence of informed traders.
- When a market maker does not know the existence of informed traders and informed traders do not exist, the asset price systematically deviates from the fair value, causing asset mispricing.
- This situation is close to the information mirage that Camerer and Weigelt (1991) found in their asset market experiments.
- After the market maker sufficiently updates his belief, he adequately finds the non-existence of informed traders. Then asset mispricing shrinks, and the market becomes efficient.

Market

1) Risky Asset

Asset value: \( \theta = \begin{cases} 1 \ (\text{prob.} = 1/2) \\ 0 \ (\text{prob.} = 1/2) \end{cases} \)

2) Sequential Trading

- Market Maker
- Noise Trader
- Informed Trader

3) Market Type (Given \( \theta = 1 \))

- Informed Market (M=I)
  - (No informed Trader)
  - \( \mu = \text{Prob}(\theta = 1 | M = I, \mathcal{H}_n) \)
  - \( \xi = \text{Prob}(M = I | \mathcal{H}_n) \)

Belief Update

1) Market Maker’s belief

\( \mu_n = \text{Prob}(\theta = 1 | M = I, \mathcal{H}_n) \)

\( \mathcal{H}_n = \omega_1 \omega_2 \cdots \omega_n : \text{prob. history} \)

Example \( \mathcal{H}_n = BSBBB \)

2) Bayesian Update

\( \xi_{n+1} = \begin{cases} (1 + \phi)\mu_n / (1 - \phi + 2\phi\mu_n) & (\omega_{n+1} = B) \\ (1 - \phi)\mu_n / (1 + \phi - 2\phi\mu_n) & (\omega_{n+1} = S) \end{cases} \)

3) Quasi-Bayesian Update

\( \xi_{n+1} = \begin{cases} 0.5 + \phi(\mu_n - 0.5) & (\omega_{n+1} = B) \\ 0.5 - \phi(\mu_n - 0.5) & (\omega_{n+1} = S) \end{cases} \)

\( \xi_{n+1} = \begin{cases} 0.5 + \phi(\xi_n - 0.5) & (\omega_{n+1} = B) \\ 0.5 - \phi(\xi_n - 0.5) & (\omega_{n+1} = S) \end{cases} \)

We can observe that the transition of the belief \( \xi \) occurs in the Experiment.

Theorem

1) \( \xi_n \rightarrow \text{True Value} \) Bayesian markets become efficient.
2) \( \xi_n \rightarrow \text{True Value} \) Quasi-Bayesian markets become efficient.
3) \( \xi_n < \xi_{n+1} \) for all \( n \) (and all \( \mathcal{H}_n \))
4) \( E[\xi_n] \) has Local Maximum Temporal Miss price (Bubble) occurs.

Figure 1. Transition of \( \xi \). Figure 2. A typical pass of miss pricing.

Figure 3. Transition of \( \xi \) and \( \xi \).

Figure 4. \( D_i = \sum_{n=1}^{10}(p_{i,n} - \xi_n) \)

Conclusion

Our results suggest that mispricing may occur when investors believe that private information exists in stock markets and trade on the basis of their own belief even if private information does not exist. For stock markets to be efficient, controlling the information flow is important for policy makers and regulators.

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