# Son Preference and the Demographic Transition 

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#### Abstract

In a general equilibrium model of fertility with higher economic returns to sons relative to daughters, parents choose overall fertility and the gender composition of their children. Son preference is partially endogenized to reflect how relative scarcity of females raises their value even while social norms and lack of economic opportunities lessen their value. These competing factors lead to an oscillating sex ratio. Model simulations demonstrate that son preference increases fertility, but that sex selection reduces fertility in the presence of son preference. The results suggest that effectively banning sex-selective abortions in places such as India, which has struggled to enforce a ban on the practice, could actually hinder demographic transition, reduce quality investment in girls, and slow human capital accumulation and economic growth. Instead, improving economic opportunities for women will increase the value placed on daughters, thereby improving the sex ratio and human capital investment in all children.


JEL Codes: J11, J13, J16
Keywords: son preference; demographic transition; sex selection; quantity-quality tradeoff

[^0]
## 1 Introduction

Since 1980 India's child mortality rate (CMR) fell by $71 \%$ (from 166 deaths per 1,000 live births to 47.7 in 2015) while the total fertility rate (TFR) declined from 4.8 to 2.4 children per woman. In China the CMR fell $82 \%$ (from 61 deaths per 1,000 live births to 10.7 in 2015) while the TFR declined from 2.6 to 1.6 (World Development Indicators). Yet this apparent progress is obscured by a severe sex ratio imbalance in both countries. The most recent Indian census data of 2011 indicate that the current sex ratio among children under the age of six is 1.09 males per female, up from 1.08 in 2001 (India Census, 2011). The sex ratio at birth (SRB) reaches as high as 1.3-1.5 in smaller areas within the states of Punjab and Haryana (Guilmoto 2009). In China the SRB rose from 1.09 in 1982 to 1.18 in 2012 (UNICEF). Much of these imbalances are attributed to the technologically-aided sex-selective termination of pregnancies, even though the practice has been outlawed in both countries. We theoretically examine the potential consequences for the well-being of both male and female children, the demographic transition, and economic growth.

A preference for sons arises from economic considerations and cultural norms. Patrilocal and patrilineal traditions are stronger in India's northern states, coinciding with a higher sex ratio in the north, and this relationship is observed in other parts of Asia, the Middle East and North Africa (Jayachandran 2015). Where social security programs and an efficient financial system by which to save are lacking, parents rely on their children to care for them in old age. In India, China and South Korea, for instance, traditionally the son supports his parents when they age, inherits the property, and continues the family line (Jayachandran 2015; Chung and Das Gupta 2007). For Hindus, a son is deemed essential since he must light the funeral pyre (Bhaskar 2011). According to Confucianism only sons can care for parents in their life and their afterlife (Jayachandran 2015; Chung and Das Gupta 2007). In contrast, raising a dauther is considered akin to "watering your neighbor's garden" since she eventually moves in with her husband's family and cares for his parents (Guilmoto 2009). Moreover, dowries in South Asia have increased in real value over time and are a financial burden to the daughter's family. In India widows traditionally do not inherit their husbands' ancestral property and thus rely on their sons as the conduit to holding on to the land (Jayachandran 2015).

The practice of sex-selective abortion magnifies the demographic effects of more traditional methods of gender control, specifically stopping behavior whereby parents have children until the desired number of sons is born, and even infanticide or neglect of newborn daughters. Rapid increases in the SRB have followed geographical patterns of ultrasound technology diffusion, particularly throughout the 1980s in India, South Korea, and China. While South Korea's sex ratio has returned to normal since its peak of 1.15 in the early 1990s, India remains less economically developed and its SRB continues to worsen (Guilmoto 2009). As India modernizes and its demographic transition proceeds, in many regions parents prefer to have fewer children,
potentially exasperating the sex ratio imbalance (Dharmalingam, Rajan and Morgan 2014; Das Gupta and Bhat 1997). Parents can effectively choose to have more sons and fewer daughters. China's experience has been similar although some of its fertility decline can be attributed to state-mandated restrictions on family size via the so-called "one-child policy". Falling fertility combined with son preference led to rapid increases in SRBs during the 1990s in all newly independent countries of the South Caucasus, reaching 1.17 in Azerbaijan in 2002, 1.19 in Georgia in 1998, and 1.16 in Armenia in 2001 (Guilmoto 2009). Some argue that falling desired fertility could improve sex ratios. For example, among Hindus there is a general desire to have one daughter because it is considered sacramental to give away one daughter in marriage (Bhat and Zavier 2003). Sons, on the other hand, are perceived as productive assets. Thus as ideal family size declines, the ideal number of daughters changes little, while the ideal number of sons changes more. If fertility decline in fact lowers son preference, that the sex ratio is nevertheless worsening in India, for instance, might be explained by a decline in unwanted fertility, of which $60 \%$ are females, made possible by sex-selective technologies (Bhat and Zavier 2003). While the impact of falling fertility on sex ratios in unclear, similarly, the introduction of sex-selective technology into a society exhibiting strong son preference may act to reduce fertility if fewer unwanted daughters are born in pursuit of the desired number of sons, or it may increase fertility if parents can have more of the preferred type of child.

We develop a general equilibrium model of fertility in which parents choose overall fertility as well as the gender composition of their children. The economic returns to quality investment in females is lower than that in males, generating a preference for sons. Model implications are examined when the level of relative returns is exogenous versus when it is endogenized to the sex ratio itself. Lower labor productivity in the female labor sector relative to the male labor sector generates higher economic returns to investing in sons over daughters, but the relative scarcity of women in the labor force increases their marginal productivity of labor thereby raising the value of females. This economic characterization of a more culturally complex phenomenon represents how, for instance, strains in the marriage market may contribute to a self-correcting of the sex ratio imbalance (Diamond-Smith and Bishai 2015). Model simulations demonstrate the behavior of son preference, the sex ratio, fertility rates, and human capital accumulation.

The model predicts higher fertility with son preference, but less so when sex selection is used. Without sex selection, parents invest more in the quality of sons over daughters, but with sex selection the sex ratio rises in favor of sons while parents invest more in the quality of daughters. When sex selection is used, fertility is lower and human capital accumulation and economic growth are higher. Moreover, as child survival improves, the demographic transition is stronger where sex selection is permitted, although weaker in the presence of son preference relative to no gender preference.

Our results reflect those of Lagerlöf (2003) and Galor and Weil (1996) where narrowing gaps
in the human capital of women relative to men promote demographic transition and economic growth. However, these models analyze historical patterns prior to the availability of effective sex-selective technologies and thus do not allow for sex selection, only discrimination by parents in the human capital investment of sons and daughters.

A number of papers examine different components of the modern son preference phenomenon. For example, Leung (1994) demonstrates how theoretically sex selection can either increase or decrease fertility, but he does not explore the sex ratio itself. Davies and Zhang (1997) theoretically demonstrate how sex selection may increase fertility, in contrast to the results here, but improve well-being of the daughters that are born, in line with the results here. In their model, the addition of a pure son preference parameter in the utility function means that the ability to sex select raises fertility because the average cost of children declines when parents can substitute towards the higher valued sex. In our paper the ability to sex select lowers fertility by increasing the average cost of children; quality investment in all children increases but it is higher for daughters. Sex selection strengthens the quantity-quality tradeoff for children that typically accompanies improvements in child survival in previous models of demographic transition (Kalemli-Ozcan 2002; Strulik 2003; Soares 2005; Cervellati and Sunde 2007).

This paper is unique in that it considers several features of the sex ratio imbalance problem at once within a general equilibrium framework. By endogenizing the sex ratio and partially endogenizing son preference itself, we examine the interaction of the demographic transition with son preference and the implications for economic growth. Previous models of sex selection are limited to partial equilibrium effects (Leung 1994; Davies and Zhang 1997; Bhaskar 2011; Edlund and Lee 2013). Fertility in India and elsewhere has declined in response to economic growth and improving child survival rates. This model suggests that the spread of ultrasound technology that facilitates sex selection among parents may have contributed to that fertility decline.

The paper proceeds as follows. In Section 2 we develop the model, first with exogenous and then with endogenous son preference. Simulations are presented in Section 3 demonstrating the impact of son preference and sex selection on the sex ratio, fertility rates, and human capital accumulation as child survival improves. Section 4 concludes with some policy implications.

## 2 The Model

In the model adults choose own consumption $c$, how many children to have $n$, how many of them boys $\pi \in[0,1]$, how many of them girls $1-\pi$, and how much to invest in the quality of each surviving son $e_{s} \in[0,1]$ and each surviving daughter $e_{d} \in[0,1]$ in order to maximize their own utility:

$$
\begin{equation*}
U=\ln \left(c_{t}\right)+\gamma \theta \ln \left(s_{t} n_{t}\right)+\gamma(1-\theta) s_{t}\left(\ln \left(\pi_{t} x_{t+1}^{s}\right)+\ln \left(\left(1-\pi_{t}\right) x_{t+1}^{d}\right)\right) \tag{1}
\end{equation*}
$$

subject to the budget constraint $c_{t}$ and the human capital production functions for males $x_{t+1}^{s}$ and females $x_{t+1}^{d}$ :

$$
\begin{align*}
c_{t} & =\left[1-\left(\pi_{t}\left(\tau+s_{t} e_{s, t}\right)+\left(1-\pi_{t}\right)\left(\tau+s_{t} e_{d, t}\right)\right) n_{t}\right] w_{t} x_{t}  \tag{2}\\
x_{t+1}^{s} & =\lambda\left(\epsilon+e_{s, t}\right)^{v} \bar{x}_{t}  \tag{3}\\
x_{t+1}^{d} & =\lambda\left(\epsilon+\delta e_{d, t}\right)^{v} \bar{x}_{t} \tag{4}
\end{align*}
$$

It is assumed that the sex composition of children $\pi$ can be perfectly and costlessly controlled. Adults take as given the wage rate $w$ per effective unit of labor $x$, the child survival rate $s$, and the child rearing cost $\tau$. We assume that rearing costs are the same for sons and daughters. This does not affect results importantly, and Appendix Ademonstrates the implications on the analysis that follows. $\gamma \in[0,1]$ is the relative value parents place on children versus their own consumption, and $\theta$ is parental valuation of child quantity versus quality. There are increasing returns to quality investment $v>1$, the productivity parameter $\lambda$ is positive, $\epsilon>0$ ensures positive human capital in the absence of parental investment, and $\bar{x}_{t}$ is average human capital across working adults. The role of increasing returns to quality investment is highlighted later in Section $2{ }^{11}$

Parents' motivation for investing in the quality of their children is modeled as altruism here, but this could also be thought of as representing the utility parents gain by having higher quality children who can better care for them financially in old age or are more likely to marry well. The value of daughters relative to sons is captured by $\delta \in[0,1]$, where $\delta<1$ indicates lower returns to educating females. This is an adaptation of the fertility model of Aksan and Chakraborty (2014) which examines the role of childhood morbidity on the demographic transition and economic growth $\sqrt{2}$ Daughters with lower human capital may be less likely to be paired with a high quality husband, may require a higher dowry payment, and may provide less financial support in old age through lower earnings (Bhaskar 2011).

[^1]Ignoring time subscripts for now, the first-order conditions with respect to $\pi, n, e_{s}, e_{d}$ are:

$$
\begin{align*}
\pi & : \frac{n s\left(e_{s}-e_{d}\right)}{1-\chi n}=\gamma(1-\theta) s\left(\frac{1}{\pi}-\frac{1}{1-\pi}\right)  \tag{5}\\
n & : \frac{\chi}{1-\chi n}=\frac{\gamma \theta}{n} \rightarrow n=\left(\frac{\gamma \theta}{1+\gamma \theta}\right) \frac{1}{\chi}  \tag{6}\\
e_{s} & : \frac{n \pi s}{1-\chi n}=\frac{\gamma(1-\theta) s v\left(\epsilon+e_{s}\right)^{v-1}}{\left(\epsilon+e_{s}\right)^{v}} \rightarrow \pi\left(\epsilon+e_{s}\right)=\frac{(1-\theta) v \chi}{\theta}  \tag{7}\\
e_{d} & : \frac{n(1-\pi) s}{1-\chi n}=\frac{\gamma(1-\theta) s v \delta\left(\epsilon+\delta e_{d}\right)^{v-1}}{\left(\epsilon+\delta e_{d}\right)^{v}} \rightarrow \frac{(1-\pi)\left(\epsilon+\delta e_{d}\right)}{\delta}=\frac{(1-\theta) v \chi}{\theta} \tag{8}
\end{align*}
$$

where $\chi=\pi\left(\tau+s e_{s}\right)+(1-\pi)\left(\tau+s e_{d}\right)=\tau+s\left(\pi e_{s}+(1-\pi) e_{d}\right)$ represents the average cost per childbirth. Using (7) and (8),

$$
\begin{equation*}
e_{d}=\left(\frac{\pi}{1-\pi}\right) e_{s}-\left(\frac{1-(1+\delta) \pi}{\delta(1-\pi)}\right) \epsilon=\underbrace{\left(\frac{\pi}{1-\pi}\right)\left(\epsilon+e_{s}\right)}_{\text {Quantity-quality tradeoff }}-\underbrace{\frac{\epsilon}{\delta}}_{\text {Lower returns for daughters }} \tag{9}
\end{equation*}
$$

where $e_{d} \geq 0$ if $e_{s} \geq\left(\frac{1-(1+\delta) \pi}{\delta \pi}\right) \epsilon$. The first component on the right-hand-side of $\sqrt{9 p}$ reflects a quantity-quality tradeoff whereby having more sons than daughters leads parents to invest more in the quality of daughters. The second component reflects that quality investment in daughters is lower the lower is their economic return. Thus it is possible that more is invested in daughters than in sons, or vice versa. Using (7) and (9),

$$
\begin{equation*}
e_{s}=\frac{(1-\theta) v}{\pi(\theta-2 s(1-\theta) v)}\left(\tau-\frac{\theta \pi \epsilon}{(1-\theta) v}-\left(\frac{1-(1+\delta) \pi}{\delta}\right) s \epsilon\right) \tag{10}
\end{equation*}
$$

and $e_{s}>0$ if

$$
\begin{equation*}
v<\frac{\theta}{2 s(1-\theta)} \tag{A1}
\end{equation*}
$$

and

$$
\begin{equation*}
\tau>\frac{\theta \pi \epsilon}{(1-\theta) v}+\left(\frac{1-(1+\delta) \pi}{\delta}\right) s \epsilon \tag{A2}
\end{equation*}
$$

For a given value of $\pi$, quality investment $e_{s}$ is increasing in $\delta$; all else equal, a higher $\delta$ increases the average return to quality investment across all children, so investment rises for both sons and daughters. For sufficiently low $\delta$ there is no quality investment $\left(e_{s}=e_{d}=0\right) \cdot{ }^{3} \mathrm{We}$ focus on

$$
\begin{align*}
& { }^{3} \text { Rewritting (A22), } e_{s}>0 \text { if } \\
& \qquad \delta>\frac{(1-\pi) s \epsilon}{\tau-\frac{\theta \pi \epsilon}{(1-\theta) v}+\pi s \epsilon} \equiv Z_{s} \in(0,1) \tag{11}
\end{align*}
$$

where the denominator is positive as implied by (A2), and $Z_{s}<1$ if

$$
\begin{equation*}
\tau>(1-2 \pi) s \epsilon+\theta \pi \epsilon /(1-\theta) v \tag{12}
\end{equation*}
$$

an interior solution such that $e_{d}, e_{s}>0$ ( $\delta$ is sufficiently high). The corner solution $e_{s}=e_{d}=0$ is analyzed in Appendix $B$.

Using (10) $\chi$ now simplifies to

$$
\begin{equation*}
\chi=\frac{\theta}{(\theta-2 s(1-\theta) v)}\left(\tau-\left(\frac{1-(1-\delta) \pi}{\delta}\right) s \epsilon\right) \tag{14}
\end{equation*}
$$

Substituting (6), (9) and (10) into (5),

$$
\begin{equation*}
\frac{\theta s\left(\left(\frac{1-2 \pi}{1-\pi}\right) e_{s}+\left(\frac{1-(1+\delta) \pi}{\delta(1-\pi)}\right) \epsilon\right)}{\chi}=(1-\theta) s\left(\frac{1-2 \pi}{\pi(1-\pi)}\right) \tag{15}
\end{equation*}
$$

and using $e_{s}=\frac{(1-\theta) v \chi}{\theta \pi}-\epsilon$

$$
\begin{equation*}
G \equiv \frac{(1-\theta) s(1-2 \pi)}{\pi(1-\pi)}(v-1) \chi+\left(\frac{1-\delta}{\delta}\right) \theta s \epsilon=0 \tag{16}
\end{equation*}
$$

which defines $\pi$ implicitly. Recall that $v>1$. If $\delta=1$, $\chi$ is positive while the third component in $G$ becomes 0 , so $G=0$ requires $1-2 \pi=0$, or $\pi=1 / 2$. If $\delta<1$, the third component in $G$ is now positive, so $G=0$ requires $1-2 \pi<1 / 2$, or $\pi>1 / 2$. ${ }^{4}$

Proposition 1. Iffemales are valued less than males ( $\delta<1$ ), son preference manifests in a skewed sex ratio at birth ( $\pi>1 / 2$ ).

Note that if there are instead decreasing returns to quality investment ( $v<1$ ), then parents will compensate for a lower value of females $(\delta<1)$ by having more daughters than sons, not fewer. This is counter to what has been observed in India and China, for instance, where the sex ratios are heavily skewed towards males.
which is implied by (A2). Similarly, $e_{d}>0$ if

$$
\begin{equation*}
\delta>\frac{(1-\pi)[\theta-s(1-\theta) v] \epsilon}{(1-\theta) v(\tau-\pi s \epsilon)} \equiv Z_{d} \tag{13}
\end{equation*}
$$

where the positive denominator in (11) implies a positive denominator in (13) by A1. When there is no sex selection ( $\pi=1 / 2$ ), $Z_{s}<Z_{d}$, so for $\delta<Z_{s}$ we have $e_{s}=e_{d}=0$, for $Z_{s}<\delta<Z_{d}$ we have $e_{s}>e_{d}=0$, and for $\delta>Z_{d}$ we have $e_{s}>0$ and $e_{d}>0$. If there is sex selection $(\pi \neq 1 / 2)$, this is not clearly the case, and it could be that $e_{s}>e_{d}=0$.
${ }^{4}$ Looking at it differently, 16 can be re-written as

$$
\begin{equation*}
(v-1)(1-\theta)(1-2 \pi) \tau-\left(\frac{[\theta-2 s(1-\theta)](1-\delta) \pi^{2}-[\theta(1-\delta)-(3-v-\delta(1+v)) s(1-\theta)] \pi-(1-v) s(1-\theta)}{\delta}\right) \epsilon=0 \tag{17}
\end{equation*}
$$

When $\delta=1$, this solves to $\pi=1 / 2$. When $\delta=0$, this simplifies to

$$
\begin{equation*}
[\theta-2 s(1-\theta)] \pi^{2}-[\theta-(3-v) s(1-\theta)] \pi+(v-1) s(1-\theta)=0 \tag{18}
\end{equation*}
$$

which is satisfied by $\pi=1$. That is, if women have no value, parents choose to have only sons (although this case does not apply since we are assuming an interior solution so $\delta>0$ ).

### 2.1 Quality investment

Using (91, when $\delta=1, \pi=1 / 2$ and $e_{d}=e_{s} .{ }^{5}$ When $\delta<1, \pi>1 / 2$ and quality investment in sons may or may not be higher than investment in daughters.

$$
\begin{equation*}
e_{d}=\frac{\pi}{1-\pi} e_{s}-\left(\frac{1-(1+\delta) \pi}{\delta(1-\pi)}\right) \epsilon<e_{s} \quad \text { if } e_{s}<\left(\frac{1-(1+\delta) \pi}{\delta(2 \pi-1)}\right) \epsilon, \tag{21}
\end{equation*}
$$

or

$$
\begin{equation*}
\delta<\frac{(1-\pi)[\theta \pi-s(1-\theta) v] \epsilon}{(2 \pi-1)(1-\theta) v \tau+[(1-\pi) \theta-s(1-\theta) v] \pi \epsilon} \equiv Z_{h} \tag{22}
\end{equation*}
$$

where $\left.Z_{s}, Z_{d}<Z_{h}<1\right]^{6}$ Emprical evidence supports both effects: a positive relationship between scarcity and quality investment, and a negative relationship between son preference and quality investment in girls. Rosenblum (2013) uses household data across India to demonstrate that mortality improves for girls but worsens for boys when the first born is male and thus the total sex composition of the household is less female. Altindag's (2016) analysis of son preference in Turkey yields similar conclusions. Mishra, Roy and Retherford (2004) find evidence of resource discrimination against boys in families where girls are scarce and against girls where boys are scarce. Chamarbagwala (2011) similarly finds less discrimination against girls when they have older brothers rather than older sisters.

Proposition 2. If sons and daughters are valued equally $(\delta=1)$, quality investment is equal for sons and daughters. When daughters are valued less than sons, investment may be higher in sons ( $\delta<Z_{h}$ ), or investment may be higher in daughters if quality compensation for scarcity of daughters dominates lower returns to that investment in daughters ( $\delta>Z_{h}$ ).

Simulations in Section 3 demonstrate higher quality investment in girls relative to boys when sex selection occurs, but the opposite when sex selection is prohibited (i.e. when $\pi$ is fixed at $1 / 2$ even though $\delta<1$ ).

$$
\begin{array}{r}
{ }^{5} \quad \begin{array}{r}
e_{d}
\end{array}=\left(\frac{\pi}{1-\pi}\right) e_{s}-\left(\frac{1-(1+\delta) \pi}{\delta(1-\pi)}\right) \epsilon<e_{s} \\
\underbrace{\frac{2 \pi-1}{1-\pi}}_{\leq 0 \text { if } \pi \leq 1 / 2} e_{s}-\underbrace{\left(\frac{1-(1+\delta) \pi}{\delta(1-\pi)}\right)}_{\geq 0 \text { if } \pi \leq \frac{1}{1+\delta} \in\left[\frac{1}{2}, 1\right]} \epsilon<0
\end{array}
$$

so in fact $e_{d} \leq e_{s}$ if $\pi \leq 1 / 2$.
${ }^{6} Z_{h}>Z_{s}$ is implied by A1, and $Z_{h}>Z_{d}$ is implied by 12. To see that $Z_{h}<1$, rewrite $Z_{h}$ as

$$
\begin{equation*}
\frac{(1-\pi) \theta \pi \epsilon-(1-\pi) s(1-\theta) v \epsilon}{(1-\pi) \theta \pi \epsilon-\pi s(1-\theta) v \epsilon+(2 \pi-1)(1-\theta) v \tau}<1 \text { if }(1-\pi) s(1-\theta) v \epsilon>\pi s(1-\theta) v \epsilon+(1-2 \pi)(1-\theta) v \tau \tag{23}
\end{equation*}
$$

which simplifies to $\tau>s \epsilon$, which is implied by 12 because of A1.

### 2.2 The sex ratio

Although we cannot solve for a closed-form solution of $\pi$, we examine its comparative statics. Using $G$ and invoking the implicit function theorem,

$$
\begin{equation*}
\frac{d \pi}{d \delta}=-\frac{G_{\delta}}{G_{\pi}}<0 \text { and } \frac{d \pi}{d s}=-\frac{G_{s}}{G_{\pi}}<0 \text { if } \pi>\frac{1}{2} \tag{24}
\end{equation*}
$$

See Appendix Cor proofs.
Proposition 3. An increase in the economic returns to quality investment in daughters towards that of sons ( $\delta \uparrow$ towards 1) reduces the sex ratio ( $\pi \downarrow$ towards $1 / 2$ ), that is, $d \pi / d \delta<0$. An increase in the child survival rate reduces the sex ratio ( $\pi \downarrow$ towards $1 / 2$ ), that is, $d \pi / d s<0$.

The result that $d \pi / d s<0$ does not necessarily contradict the observation that sex ratio imbalances have worsened in son-preferring societies as child survival has improved. It could be that child survival rebalances sex ratios but that sex-selective technologies become more available at the same time. However, Das Gupta and Bhat (1997) and others argue that falling desired family size, the typical response to improved child survival, exasperates sex-ratio imbalances. Model simulations will demonstrate that once $\delta$ is endogenized in Section 2.4. $\delta$ declines as child survival improves, thereby increasing $\pi$, although other factors act on $\pi$ as well.

### 2.3 Demographic transition

It is expected that the total fertility rate (TFR) falls as child survival improves and that eventually the net fertility rate (NFR) follows. The TFR is represented by $n$, and

$$
\begin{equation*}
\frac{d n}{d s}=-\left(\frac{\gamma \theta}{1+\gamma \theta}\right) \frac{d \chi / d s}{\chi^{2}}<0 \tag{25}
\end{equation*}
$$

if

$$
\begin{equation*}
\frac{d \chi}{d s}=2 s e_{s} \frac{d \pi}{d s}+2 \pi e_{s}+2 \pi s \frac{d e_{s}}{d s}+\left(\frac{1+\delta}{\delta}\right) s \epsilon \frac{d \pi}{d s}-\left(\frac{1-(1+\delta) \pi}{\delta}\right) \epsilon>0 \tag{26}
\end{equation*}
$$

The NFR is represented by $s n$, and

$$
\begin{equation*}
\frac{d N F R}{d s}=\left(\frac{\gamma \theta}{1+\gamma \theta}\right)\left(\frac{\chi-s\left(\frac{d \chi}{d s}\right)}{\chi^{2}}\right)<0 \text { if } \chi<s\left(\frac{d \chi}{d s}\right) \tag{27}
\end{equation*}
$$

that is, if the elasticity of $\chi$ with respect to $s$ is greater than one. Whether the inequality holds is not immediately clear due to the endogeneity of $\pi$. Simulations in Section 3 demonstrate the demographic transition under various model conditions.

### 2.4 Endogenizing the value of women

We next endogenize $\delta$, the value of females relative to males, and examine the potential for a self-correcting of the sex ratio, a phenomemon recently observed in the parts of India with the greatest sex ratio imbalance (Diamond-Smith and Bishai 2015). Increased economic opportunities improve the bargaining power of women, giving them more autonomy over their lives and bodies, and may reverse the traditional role of daughters as a net financial burden on parents. One consequence of a severe sex ratio imbalance is a lack of potential brides and thus an increased number of single adult males, which has negative ramifications for economic growth and social stability and has even led to bride abductions (Kaur 2013). If men cannot marry, then they do not have children, which may lead to destitution in old age if they have no one to care for them. The "bare branches" phenomenon, the label given to unmarried men in China, can reduce the economic returns, so to speak, of having a son (Guilmoto 2009). A shortage of women on the marriage market may increase their bargaining power (Anukriti 2014). In theory, the scarcity of brides could lead to a reversed dowry, whereby males instead pay the bride and her family (Bhaskar 2011; Park and Cho 1995).

We model $\delta$ such that the value of women is increasing in their relative economic returns and in their scarcity. We assume that men and women are employed in two separate sectors of the economy, and without labor mobility across the two sectors, wages are not necessarily equalized. The production functions for the male and female employment sectors are, respectively,

$$
\begin{align*}
Y_{s} & =A_{s}\left(L_{s} x^{s}\right)^{\alpha}=A_{s}\left(L_{s} \lambda\left(\epsilon+e_{s}\right)^{v} \bar{x}\right)^{\alpha}  \tag{28}\\
Y_{d} & =A_{d}\left(L_{d} x^{d}\right)^{\beta}=A_{d}\left(L_{d} \lambda\left(\epsilon+\delta e_{d}\right)^{v} \bar{x}\right)^{\beta} \tag{29}
\end{align*}
$$

where $A_{s} \geq A_{d}$ so that total factor productivity in the female sector may be lower than in the male sector, and $\beta \leq \alpha<1$ so that diminishing returns to labor may be greater in the female sector. The wage per male worker is $w_{s}=\alpha A_{s}\left(\lambda\left(\epsilon+e_{s}\right)^{v} \bar{x}\right)^{\alpha} L_{s}^{\alpha-1}$ and the wage per female worker is $w_{d}=\beta A_{d}\left(\lambda\left(\epsilon+\delta e_{d}\right)^{v} \bar{x}\right)^{\beta} L_{d}^{\beta-1}$, their respective marginal productivities of labor.

Define $\Delta$ as the ratio of female to male wages:

$$
\begin{equation*}
\Delta=\frac{w_{d}}{w_{s}}=\frac{\beta A_{d}\left(\lambda\left(\epsilon+\delta e_{d}\right)^{v} \bar{x}\right)^{\beta} L_{d}^{\beta-1}}{\alpha A_{s}\left(\lambda\left(\epsilon+e_{s}\right)^{v} \bar{x}\right)^{\alpha} L_{s}^{\alpha-1}} \tag{30}
\end{equation*}
$$

For simplicity, set $\alpha=\beta \equiv a$ such that

$$
\begin{equation*}
\Delta=\frac{A_{d}}{A_{s}}\left(\frac{\epsilon+\delta e_{d}}{\epsilon+e_{s}}\right)^{v a}\left(\frac{L_{d}}{L_{s}}\right)^{a-1}=\frac{A_{d}}{A_{s}}\left(\frac{\delta \pi}{1-\pi}\right)^{v a}\left(\frac{1-\pi}{\pi}\right)^{a-1}=\frac{\delta^{v a} A_{d}}{A_{s}}\left(\frac{\pi}{1-\pi}\right)^{(v-1) a+1} \tag{31}
\end{equation*}
$$

Since $\Delta$ exceeds 1 for $\pi>\frac{\left(A_{s} /\left(\delta^{v a} A_{d}\right)\right)^{1 /(v-1) a+1)}}{1+\left(A_{s} /\left(\delta^{v a} A_{d}\right)\right)^{1 /(v-1) a+1)}} \in[1 / 2,1]$, define

$$
\begin{equation*}
\delta=g(\delta) \equiv \min \{\Delta, 1\} \tag{32}
\end{equation*}
$$

$g(\delta)<1$ is more feasible when $A_{d}<A_{s}$; lower labor productivity in the female sector of the economy is an exogenous source of son preference. Structural features of the economy may affect the wage returns to labor for men versus women. For example, Bhaskar (2011) describes how women have superior status in Sub-Saharan Africa relative to Asia because of their greater utility in hoe-cultivation as compared to plough-cultivation. Women in the rice-growing south of India enjoy higher status relative to those in the wheat-growing north, because rice has greater use for female labor than does wheat. Similarly, Chinese economic reforms raised the returns to female labor in tea growing regions, and to male labor in regions with orchard fruits (Bhaskar 2011).
$\Delta$ is increasing in the relative economic returns to females versus males ( $\delta$ and $A_{d} / A_{s}$ ) and in the scarcity of females $(\pi /(1-\pi))$. A large enough sex ratio imbalance will raise the value of women: fewer workers in the female labor market raises their marginal product, so then quality investment in daughters rises ( $\partial e_{d} / \partial \delta>0$ ) further raising the value of women. But since $\pi$ then also declines, women become less scarce, so their value declines. Because $\pi$ is a function of $\delta$, we cannot solve for a closed-form solution of $\delta$, and we turn to numerical simulations to demonstrate the potential of an oscillating sex ratio.

## 3 Simulations

We solve the model numerically for the cases where $\delta$ is exogenous and endogenous to examine the behavior of $\delta$, the sex ratio $\pi /(1-\pi)$, and fertility as child survival $s$ improves. The parameter values are set with $v=1.15$ and $\theta=0.7$ such that (A1) is compatible with $v>1$ at the most binding value of $s=1$. Also, $\gamma=10, \epsilon=0.1, \tau=0.15, \alpha=\beta=a=0.9, A_{s}=1$, and $A_{d}=0.7$ or 1 . Appendix $D$ describes the simulation process in more detail.

### 3.1 Impact of son preference on the demographic transition

Figure 1 presents simulation results for various exogenous values of $\delta$, when parents are not permitted to sex select ( $\pi=1 / 2$ ) versus when parents can choose $\pi$.

When there is no sex selection, at very low $\delta$ we are in a corner solution with investment in daughters nil and even so for investment in sons (see Figure 2). The TFR does not respond to rising child survival as there is no quantity-quality tradeoff, so the NFR rises. For higher values of $\delta$ we are in an interior solution. Now investment in all children is increasing in $s$, although investment is higher for sons than for daughters ( $e_{s}>e_{d}$ ) when $\delta<1$. As child survival
improves, investment in daughters approaches that in sons. For higher values of $\delta$ (interior solution), the TFR is decreasing in $s$, and the NFR is decreasing in $s$ for moderate to high survival rates. This is consistent with historical demographic transitions where population growth first rises and then slows (Kalemli-Ozan 2008; Strulik and Weisdorf 2014).

When sex selection is permitted we are in an interior solution even at very low $\delta$; since parents can substitute towards having more sons when females are less valued, quality investment is positive for all children. Moreover, investment is higher for daughters now ( $e_{s}<e_{d}$ ) when $\delta<1$. As child survival improves, so do the sex ratio and quality investment in all children, and investment in sons approaches that in daughters. For all values of $\delta$, the TFR is decreasing in $s$, and the NFR is decreasing in $s$ for moderate to high survival rates.

In the absence of sex selection ( $\pi=1 / 2$ for all values of $\delta$ ), parents invest more in the quality of sons relative to daughters $\left(e_{s}>e_{d}\right)$ as they cannot adjust quantity to the more highly valued sex. In contrast, when sex selection occurs, parents invest more in the scarcer sex (daughters). For example, Jayachandran (2015) notes that if son-biased stopping behavior is practiced (families stop having children once a boy is born), then daughters will tend to grow up in larger families than sons thereby giving them limited resources. Also, mothers may stop breastfeeding daughters sooner in order to try for a son sooner. With the ability to sex select, fewer unwanted daughters are born and resources are less limited and discrimination in their distribution is reduced (Rosenblum 2013).

When the sex ratio is fixed, the demographic transition is weaker and population growth may even increase with improving child survival. When females are valued less than males, sex selection facilitates demographic transition; the NFR falls as $s$ improves if $\pi$ can respond to $\delta<1$, while the NFR remains higher, and may even rise with $s$, if $\pi$ is fixed at $1 / 2 \cdot 7$ This confirms findings by Dharmalingam, Rajan and Morgan (2014) that son preference has contributed to fertility decline in India; son preference will increase fertility if women have additional births in pursuit of a son, but will reduce fertility if sex selection is used instead. With stopping behavior, families that have a son early on may end up with fewer children while families that have many daughters before achieving a son end up with more children. With sex selection, fertility falls if now the larger families can avoid those (unwanted) female births.

Overall, however, the simulations in Figure 1 demonstrate that son preference (lower $\delta$ ) contributes to population growth even if sex selection is used. For instance, even at $s=1$ the NFR is higher for lower values of $\delta$ in the bottom panel of Figure 1 .

Simulation results when $\delta$ is endogenous support these conclusions. Figure 3 presents results when $A_{d}=A_{s}=1$ and so $\delta=1$ versus when $A_{d}<A_{s}=1$ and so $\delta<1$. For the latter, $\delta$ decreases as child survival improves, while the sex ratio oscillates without a clear upward or downward trend. The drop in $\delta$ occurs in response to a decrease in the relative human capital

[^2]



Figure 2: On the left $\pi=1 / 2$, while on the right $\pi$ is solved for each value of $\delta$.

Sex ratio(malefemale) ( -


Total Fertility Rate



$0_{0}^{2}$
3 , Net Fertility Rate d survival rate
Figure 3: $\delta$ and $\pi$ are both endogenous with $A_{d}=A_{s}$ on the left and $A_{d}<A_{s}$ on the right.


Figure 4: Average cost per childbirth $\chi$ : On the top row, the left $\pi=1 / 2$, while on the right $\pi$ is solved for each value of $\delta$. On the bottom row, $\delta$ and $\pi$ are both endogenous with $A_{d}=A_{s}$ on the left and $A_{d}<A_{s}$ on the right.
of women; $e_{d} / e_{b}$ declines with $s$, and in equation (31) this lowers $\Delta$. The oscillation in $\pi$ is due to two counteracting forces. According to Proposition 3, $\pi$ is decreasing in $\delta$ and in $s$. Since here $\delta$ is decreasing in $s$, this pushes $\pi$ up, while the latter effect $(d \pi / d s<0)$ pushes $\pi$ down. Theoretically, fertility decline associated with rising survival rates could exasperate sex ratio imbalances through a "fertility squeeze" or "intensification effect" such that parents use sex selection to ensure they do not end up sonless, a more probable outcome at low fertility levels (Guilmoto 2009). On the other hand, if the ideal number of daughters remains low while the ideal number of sons drops with desired fertility, then falling fertility may improve sex ratio imbalances (Bhat and Zavier 2003). In Figure 3 the sex ratio imbalance persists as child survival improves.

Results in Figure 3 also confirm those when $\delta$ is exogenous in that the demographic transition is weaker for $\delta<1$. Because $e_{s}<e_{d}$, the manifestation of son preference through sex selection dampens the quantity-quality tradeoff for children. Figure 4 illustrates how $\chi$, the average cost per childbirth, increases with $s$ more for high $\delta$ because $e_{s}$ and $e_{d}$ are increasing in $\delta$; when $\delta$ is considerably lower than one, lower investment in all children combined with lower investment in sons relative to daughters reduce $\chi$.

### 3.2 Impact of son preference on economic growth

Since quality investment in daughters increases with the sex ratio, it is conceptually possible that aggregate human capital accumulation may be stronger or weaker under sex selection. We next examine how economic growth, via human capital accumulation, is affected by son preference and sex selection over time as the child survival rate improves. Thus $s$ is now time dependent as are all variables that respond to $s$. Assuming as before that $\alpha=\beta=a$, output per worker in the economy is

$$
\begin{align*}
y & =\frac{Y_{s}+Y_{d}}{L_{s}+L_{d}}  \tag{33}\\
& =\frac{A_{s}\left(L_{s} \bar{x} \lambda\left(\epsilon+e_{s}\right)^{v}\right)^{a}+A_{d}\left(L_{d} \bar{x} \lambda\left(\frac{\delta \pi}{1-\pi}\right)^{v}\left(\epsilon+e_{s}\right)^{v}\right)^{a}}{L}  \tag{34}\\
& =\frac{\left(\bar{x} \lambda\left(\epsilon+e_{s}\right)^{v}\right)^{a}\left(A_{s} \pi^{a}+A_{d}(1-\pi)^{a}\left(\frac{\delta \pi}{1-\pi}\right)^{v}\right)}{L^{1-a}} \tag{35}
\end{align*}
$$

where $L$ is the total labor force (all adult men and women). Then the growth in output per worker between generations $t$ and $t+1$ is

$$
\begin{align*}
g_{y} & =\frac{y_{t+1}}{y_{t}}-1  \tag{36}\\
& =\left(\frac{L_{t}}{L_{t+1}}\right)^{1-a}\left(\frac{\left(\bar{x}_{t+1} \lambda\left(\epsilon+e_{s, t+1}\right)^{v}\right)^{a}\left(A_{s} \pi_{t+1}^{a}+A_{d}\left(1-\pi_{t+1}\right)^{a}\left(\frac{\delta_{t+1} \pi_{t+1}}{1-\pi_{t+1}}\right)^{v}\right)}{\left(\bar{x}_{t} \lambda\left(\epsilon+e_{s, t}\right)^{v}\right)^{a}\left(A_{s} \pi_{t}^{a}+A_{d}\left(1-\pi_{t}\right)^{a}\left(\frac{\delta_{t} \pi_{t}}{1-\pi_{t}}\right)^{v}\right)}\right)-1  \tag{37}\\
& =\left(\frac{1}{\left(1-\pi_{t}\right) s_{t} n_{t}}\right)^{1-a}\left(\frac{\bar{x}_{t+1}\left(\epsilon+e_{s, t+1}\right)^{v}}{\bar{x}_{t}\left(\epsilon+e_{s, t}\right)^{v}}\right)^{a}\left(\frac{A_{s} \pi_{t+1}^{a}+A_{d}\left(1-\pi_{t+1}\right)^{a}\left(\frac{\delta_{t+1} \pi_{t+1}}{1-\pi_{t+1}}\right)^{v}}{A_{s} \pi_{t}^{a}+A_{d}\left(1-\pi_{t}\right)^{a}\left(\frac{\delta_{t} \pi_{t}}{1-\pi_{t}}\right)^{v}}\right)-1 \tag{38}
\end{align*}
$$

where $L_{t+1}=\left(1-\pi_{t}\right) n_{t} s_{t} L_{t}$ because childbearing is limited to women. Average human capital per worker $\bar{x}$ depends on the investment in sons and daughters in the previous period:

$$
\begin{align*}
\bar{x}_{t+1} & =\pi_{t} x_{s, t}+\left(1-\pi_{t}\right) x_{d, t}  \tag{39}\\
& =\pi_{t} \lambda\left(\epsilon+e_{s, t}\right)^{v} \bar{x}_{t}+\left(1-\pi_{t}\right) \lambda\left(\epsilon+\delta_{t} e_{d, t}\right)^{v} \bar{x}_{t}  \tag{40}\\
& =\lambda \bar{x}_{t}\left(\pi_{t}\left(\epsilon+e_{s, t}\right)^{v}+\left(1-\pi_{t}\right)\left(\epsilon+e_{s, t}\right)^{v}\left(\frac{\delta_{t} \pi_{t}}{1-\pi_{t}}\right)^{v}\right)  \tag{41}\\
& =\lambda \bar{x}_{t}\left(\epsilon+e_{s, t}\right)^{v}\left(\pi_{t}+\left(1-\pi_{t}\right)\left(\frac{\delta_{t} \pi_{t}}{1-\pi_{t}}\right)^{v}\right) \tag{42}
\end{align*}
$$

Incorporating this into (38),

$$
\begin{equation*}
g_{y}=\left(\frac{1}{\left(1-\pi_{t}\right) s_{t} n_{t}}\right)^{1-a}\left(\lambda\left(\epsilon+e_{s, t+1}\right)^{v}\left(\pi_{t}+\left(1-\pi_{t}\right)\left(\frac{\delta_{t} \pi_{t}}{1-\pi_{t}}\right)^{v}\right)\right)^{a}\left(\frac{A_{s} \pi_{t+1}^{a}+A_{d}\left(1-\pi_{t+1}\right)^{a}\left(\frac{\delta_{t+1} \pi_{t+1}}{1-\pi_{t+1}}\right)^{v}}{A_{s} \pi_{t}^{a}+A_{d}\left(1-\pi_{t}\right)^{a}\left(\frac{\delta_{t} \pi_{t} t}{1-\pi_{t}}\right)^{v}}\right)-1( \tag{43}
\end{equation*}
$$

If $\delta=1$ and $\pi=1 / 2\left(\right.$ and $\left.A_{d}=A_{s}\right)$, this simplifies to

$$
\begin{equation*}
g_{y}(\delta=1, \pi=1 / 2)=\left(\frac{2}{s_{t} n_{t}}\right)^{1-a}\left(\lambda\left(\epsilon+e_{s, t+1}\right)^{v}\right)^{a}-1 \tag{44}
\end{equation*}
$$

Whether son preference reduces or raises $g_{y}$ relative to 44) depends on how much the increased quality investment in daughters $\left(e_{d}>e_{s}\right)$ compensates for their scarcity ( $\pi>1 / 2$ ), i.e. whether $\left(\frac{\delta \pi}{1-\pi}\right)^{v}$ is sufficiently greater than one. Regardless, because quality investment is increasing in $\delta$, lower investment in both sons and daughters when $\delta<1$ hinders growth. Moreover, $g_{y}$ is decreasing in the NFR (the first set of parentheses in (43)), and since the demographic transition is weaker with son preference (i.e. when $\delta<1$ ), economic growth is also weaker through this channel.

Focusing only on growth in human capital per worker $\lambda \bar{x}\left[L_{s}\left(\epsilon+e_{s}\right)^{v}+L_{d}\left(\epsilon+\delta e_{d}\right)^{v}\right] /\left(L_{s}+L_{d}\right)$ yields the simpler

$$
\begin{equation*}
g_{\bar{x}}=\lambda\left(\epsilon+e_{s, t+1}\right)^{v}\left(\pi_{t+1}+\left(1-\pi_{t+1}\right)\left(\frac{\delta_{t+1} \pi_{t+1}}{1-\pi_{t+1}}\right)^{v}\right)-1 \tag{45}
\end{equation*}
$$

To examine the net effect on human capital accumulation, we again turn to numerical simulations. To ensure positive economic growth when $\pi=1 / 2$ and $\delta=1, \lambda$ is set to 10 . Comparing the four scenarios in Figure5, growth in human capital per worker is lower when $\delta<1$, but when $\delta<1$ sex selection increases human capital accumulation relative to a fixed sex ratio ( $\pi=1 / 2$ ).

## 4 Conclusion

Families and communities are caught up in the nexus of competing forces of traditional values, urbanization, globalization, and modernity. Economic development and demographic transition contribute to smaller desired family size and combine with cultural preference for sons to exasperate sex ratio imbalances. The model developed here provides insight on the potential ramifications of different policies aimed at reversing son-biased discrimination. When there is a preference for sons, effectively banning gender control methods such as sex-selective abortion may lead to lower investment in daughters while slowing demographic transition, with negative consequences for human capital accumulation and economic growth.

While a ban on sex-selective abortion may be socially desired, improving the sex ratio requires addressing the demand for sex selection. Increasing the value of females will lead parents to have more daughters but fewer children overall and invest more in all of their children $\left(\partial e_{s}, e_{d} / \partial \delta>0\right)$. This can be achieved by increasing economic opportunities for women, ensuring equal earnings among men and women (akin to raising $A_{d}$ in the model), and enforcing laws that protect the right of women to inherit property. Economic development and its contribution to changing social norms has been credited with restabilizing South Korea's sex ratio. Increased female labor force participation and movement towards nonfarm employment, in


Figure 5: Growth rate of human capital per worker: On the top row, the left $\pi=1 / 2$, while on the right $\pi$ is solved for each value of $\delta$. On the bottom row, $\delta$ and $\pi$ are both endogenous with $A_{d}=A_{s}$ on the left and $A_{d}<A_{s}$ on the right.
which women have a relative advantage and which diversifies sources of livelihood, make people more independent of familial pressures and traditions (Chung and Das Gupta 2007). In contrast, labor force participation among women in India is low at 33\%, compared to an average of $63 \%$ for East Asia and $50 \%$ globally, and is on a downward trend (Das et al. 2015). An extension of our model could explicitly consider labor force participation.

Increased economic opportunities for women are likely to improve their bargaining power within the husband's household (Majilesi 2014). While an expecting mother in India herself may want to keep a female child, cultural norms dictate that her senior, female in-laws often make the decision on her behalf (Robitaille and Chatterjee, 2013). Postponenement of marriage and increased singledom by economically empowered women challenges longstanding patriarchal institutions (Guilmoto 2009).

India's Pre-Natal Diagnostic Techniques Act of 1994 bans sex-selective abortions, but a black market in the practice is thriving and sex ratios continue to worsen (Basu 1999). Similar bans exist in most countries experiencing skewed sex ratios. This paper suggests that a ban, even if it were effective, would actually hinder demographic transition.

Nevertheless, the state can play a role in promoting the status of women. The Hindu Succes-
sion Act of 1956 was amended in the 1980s and 1990s to allow daughters to inherit their fathers' land in India (Jayachandran 2015). After decades of policies reinforcing patriarchal traditions, in 1987 South Korea passed the Sexual Equality Employmnet Act, and in 1989 it reformed the components of its "Family Law" that discriminated against women's rights to child custody, inheritance, and property rights, and in 2005 the biased family-head system was finally abolished (Guilmoto 2009; Yang 2008; Chung and Das Gupta 2007) $]^{8}$ However, caution must be exercised when implementing policies. For instance, programs such as Laadii in India that subsidize girls through conditional cash transfers, pension benefits to their parents, and scholarships may have unintended consequences. If such programs disproportionately affect the fertility behavior of low-income parents, then girls will be more likely to grow up in low-income families, perpetuating economic gender disparities (Anukriti 2014). Media can be a powerful tool in changing perceptions (MacPherson 2007). For example, telenovelas, or soap operas, depicting small families have been credited with reducing fertility in parts of Latin America (La Ferrara, Chong and Duryea 2012). Changing discriminatory perceptions against females will require the complementary influences of legal, social, and economic change.

## Compliance with Ethical Standards:

The authors declare that they have no conflict of interest.

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## A Different rearing costs for sons and daughters $\left(\tau_{s} \neq \tau_{d}\right)$

When rearing costs of sons ( $\tau_{s}$ ) and daughters ( $\tau_{d}$ ) differ, the utility maximization problem becomes:

$$
\begin{equation*}
\max _{c, n, \pi, e_{s}, e_{d}} U=\ln \left(c_{t}\right)+\gamma \theta \ln \left(s_{t} n_{t}\right)+\gamma(1-\theta) s_{t}\left(\ln \left(\pi_{t} x_{t+1}^{s}\right)+\ln \left(\left(1-\pi_{t}\right) x_{t+1}^{d}\right)\right) \tag{46}
\end{equation*}
$$

subject to the budget constraint $c_{t}$ and the human capital production functions for males $x_{t+1}^{s}$ and females $x_{t+1}^{d}$ :

$$
\begin{align*}
c_{t} & =\left[1-\left(\pi_{t}\left(\tau_{s}+s_{t} e_{s, t}\right)+\left(1-\pi_{t}\right)\left(\tau_{d}+s_{t} e_{d, t}\right)\right) n_{t}\right] w_{t} x_{t}  \tag{47}\\
x_{t+1}^{s} & =\lambda\left(\epsilon+e_{s, t}\right)^{v} \bar{x}_{t}  \tag{48}\\
x_{t+1}^{d} & =\lambda\left(\epsilon+\delta e_{d, t}\right)^{v} \bar{x}_{t} \tag{49}
\end{align*}
$$

and the first-order conditions are:

$$
\begin{align*}
\pi & :  \tag{50}\\
n & : \frac{n\left(\tau_{s}+s e_{s}-\tau_{d}-s e_{d}\right)}{1-\chi n}=\gamma(1-\theta) s\left(\frac{1}{\pi}-\frac{1}{1-\pi}\right)  \tag{51}\\
e_{s} & \left.: \pi\left(\epsilon+e_{s}\right)=\frac{(1-\theta) v \chi}{\theta}\right) \frac{1}{\chi}  \tag{52}\\
e_{d} & : \frac{(1-\pi)\left(\epsilon+\delta e_{d}\right)}{\delta}=\frac{(1-\theta) v \chi}{\theta} \tag{53}
\end{align*}
$$

where now $\chi=\pi\left(\tau_{s}+s e_{s}\right)+(1-\pi)\left(\tau_{d}+s e_{d}\right)$ represents the average cost per childbirth. As before,

$$
\begin{equation*}
e_{d}=\left(\frac{\pi}{1-\pi}\right) e_{s}-\left(\frac{1-(1+\delta) \pi}{\delta(1-\pi)}\right) \epsilon=\left(\frac{\pi}{1-\pi}\right)\left(\epsilon+e_{s}\right)-\frac{\epsilon}{\delta} \tag{54}
\end{equation*}
$$

and

$$
\begin{equation*}
e_{s}=\frac{(1-\theta) v}{\pi(\theta-2 s(1-\theta) v)}\left(\tau-\frac{\theta \pi \epsilon}{(1-\theta) v}-\left(\frac{1-(1+\delta) \pi}{\delta}\right) s \epsilon\right) \tag{55}
\end{equation*}
$$

where now $\tau=\pi \tau_{s}+(1-\pi) \tau_{d}$. Equation (16) becomes

$$
\begin{equation*}
G \equiv \frac{(1-\theta) s(1-2 \pi)}{\pi(1-\pi)}(v-1) \chi+\theta\left(\tau_{s}-\tau_{d}\right)+\left(\frac{1-\delta}{\delta}\right) \theta s \epsilon=0 \tag{56}
\end{equation*}
$$

Unlike the $\tau_{s}=\tau_{d}$ case, now $\pi \neq 1 / 2$ is possible even when $\delta=1$. When $\delta=1, \chi$ is positive while the third component in $G$ becomes 0 , so if $\tau_{s}>\tau_{d}, G=0$ requires $1-2 \pi<0$, or $\pi>1 / 2$; if $\tau_{s}=\tau_{d}$, $G=0$ requires $1-2 \pi=0$, or $\pi=1 / 2$; and if $\tau_{s}<\tau_{d}, G=0$ requires $1-2 \pi>0$, or $\pi<1 / 2$. If $\delta<1$, the third component in $G$ is now positive, so if $\tau_{s}=\tau_{d}, G=0$ requires $1-2 \pi<0$, or $\pi>1 / 2$; if $\tau_{s}>\tau_{d}$, then the first component in $G$ needs to be even more negative, so $\pi$ will be further above $1 / 2$; and if $\tau_{s}<\tau_{d}$, the first component in $G$ need not be as negative (or possibly not negative at
all), so $\pi$ need not exceed $1 / 2$ by as much (or not at all). Proposition 1 is amended accordingly:
Proposition 1A. If males and females are equally valued $(\delta=1)$, then parents exhibit preference for the sex with the higher rearing cost, otherwise a sex preference does not manifest $(\pi=1 / 2)$. If females are valued less than males ( $\delta<1$ ), son preference manifests ( $\pi>1 / 2$ ) under the sufficient condition that $\tau_{s} \geq \tau_{d}$ (or if $\tau_{s}<\tau_{d}$ that the rearing cost of daughters not be too much greater than that of sons).

## A. 1 Quality investment

Similarly, unlike the $\tau_{s}=\tau_{d}$ case, now $e_{d} \neq e_{s}$ is possible even when $\delta=1$. Using (54), consider the case $\delta=1$. If $\tau_{s}=\tau_{d}$, then $\pi=1 / 2$, so $e_{d}=e_{s}$. If $\tau_{s}<\tau_{d}$, then $\pi<1 / 2$, and

$$
\begin{equation*}
e_{d}=\underbrace{\frac{\pi}{1-\pi}}_{<1} e_{s}-\left(\frac{1-(1+\delta) \pi}{\delta(1-\pi)}\right) \epsilon<e_{s} \tag{57}
\end{equation*}
$$

under the sufficient condition $1-(1+\delta) \pi>0$, or $\pi<1 /(1+\delta)=1 / 2$, which is satisfied. If $\tau_{s}>\tau_{d}$, then $\pi>1 / 2$, and

$$
\begin{equation*}
e_{d}=\underbrace{\frac{\pi}{1-\pi}}_{>1} e_{s}-\left(\frac{1-(1+\delta) \pi}{\delta(1-\pi)}\right) \epsilon>e_{s} \tag{58}
\end{equation*}
$$

under the sufficient condition $1-(1+\delta) \pi<0$, or $\pi>1 /(1+\delta)=1 / 2$, which is satisfied. Thus all else equal $(\delta=1)$, there is a quantity-quality tradeoff for children whereby parents invest more in the scarcer sex. However, this can be overshadowed by $\delta<1$ as discussed in Section 2.1. Proposition 2 is amended accordingly:

Proposition 2A. If sons and daughters are valued equally $(\delta=1$, quality investment is higher for the scarcer sex, that with the lower rearing cost. When daughters are valued less than sons, investment may be higher in sons ( $\delta<Z_{h}$ ), or investment may be higher in daughters if quality compensation for scarcity of daughters dominates lower returns to that investment in daughters ( $\delta>Z_{h}$ ).

All else equal $(\delta=1)$, when $\tau_{s} \neq \tau_{d}$, parents balance the cost of children by investing less in the quality of the more expensive child ( $e_{s}<e_{d}$ if $\tau_{s}>\tau_{d}$ ) and having less of the high $e$ child ( $\pi>1 / 2$ ). Thus compared to the $\tau_{s}=\tau_{d}$ case, the case $\tau_{s}>\tau_{d}$ amplifies the results when $\delta<1$ ( $\pi>1 / 2$ and $e_{s}<e_{d}$ by more), while the case $\tau_{s}<\tau_{d}$ weakens the results ( $\pi>1 / 2$ and $e_{s}<e_{d}$ by less).

## A. 2 The sex ratio

The comparative statics in Section 2.2 are unaffected by assuming $\tau_{s} \neq \tau_{d}$. The proof of Proposition 3 in Appendix Cpresents the generalized case where rearing costs may or may not be equal.

## A. 3 The demographic transition

The NFR declines as child survival improves if

$$
\begin{equation*}
\frac{d N F R}{d s}=\left(\frac{\gamma \theta}{1+\gamma \theta}\right)\left(\frac{\chi-s\left(\frac{d \chi}{d s}\right)}{\chi^{2}}\right)<0 \text { if } \chi<s\left(\frac{d \chi}{d s}\right) \tag{59}
\end{equation*}
$$

where now

$$
\begin{equation*}
\frac{d \chi}{d s}=\left(\tau_{s}-\tau_{d}\right) \frac{d \pi}{d s}+2 s e_{s} \frac{d \pi}{d s}+2 \pi e_{s}+2 \pi s \frac{d e_{s}}{d s}+\left(\frac{1+\delta}{\delta}\right) s \epsilon \frac{d \pi}{d s}-\left(\frac{1-(1+\delta) \pi}{\delta}\right) \epsilon>0 \tag{60}
\end{equation*}
$$

has the additional term $\left(\tau_{s}-\tau_{d}\right) \frac{d \pi}{d s}$, which is positive if $\tau_{s}<\tau_{d}$ and negative otherwise.

## B Corner solution ( $e_{s}=e_{d}=0$ )

We present the general case where rearing costs may or may not differ for sons ( $\tau_{s}$ ) versus daughters $\left(\tau_{d}\right)$. Under a corner solution where $e_{s}=e_{d}=0$, average cost per childbirth becomes $\chi=\pi \tau_{s}+(1-\pi) \tau_{d}$. The first-order condition with respect to $\pi$ becomes

$$
\begin{equation*}
\frac{n\left(\tau_{s}-\tau_{d}\right)}{1-\chi n}=\gamma(1-\theta) s\left(\frac{1-2 \pi}{\pi(1-\pi)}\right) \tag{61}
\end{equation*}
$$

which simplifies to the following equation which we define as $G^{c}$ and which defines $\pi$ implicitly:

$$
\begin{equation*}
G^{c} \equiv\left(\tau_{d}-\tau_{s}\right)(1-2 s(1-\theta)) \pi^{2}+\left[\tau_{s}(1-s(1-\theta))-\tau_{d}(1-3 s(1-\theta))\right] \pi-s(1-\theta) \tau_{d}=0 \tag{62}
\end{equation*}
$$

If $\tau_{s}=\tau_{d}$, this simplifies to $\pi=1 / 2$. If $\tau_{s}<\tau_{d}, \pi>1 / 2$, and if $\tau_{s}>\tau_{d}, \pi<1 / 2$, i.e. there is preference for the sex with the lower rearing cost, in contrast to the interior solution where
parents can balance, let's say $\tau_{s}>\tau_{d}$ with $e_{s}<e_{d} \cdot 9$
Next we establish some comparative statics using $G^{c}$, where by the implicit function theorem 10

$$
\begin{equation*}
\frac{d \pi}{d \delta}=-\frac{G_{\delta}^{c}}{G_{\pi}^{c}}=0 \text { and } \frac{d \pi}{d s}=-\frac{G_{s}^{c}}{G_{\pi}^{c}}<0 \text { if } \pi>\frac{1}{2} \tag{74}
\end{equation*}
$$

Proposition 4. When there is no quality investment in children, then a change in the value of females ( $\delta$ ) does not affect son preference $(\pi)$. If there is preference for sons or daughters, as child survival improves that preference decreases ( $\pi \rightarrow 1 / 2$ ).

When child survival improves, the TFR falls if $d \chi / d s>0$. In the corner solution

$$
\begin{equation*}
\frac{d \chi}{d s}=\left(\tau_{s}-\tau_{d}\right) \frac{d \pi}{d s}=0 \text { if } \tau_{s}=\tau_{d} \tag{75}
\end{equation*}
$$

so then net fertility rises as survival improves. Since $\pi>1 / 2$ when $\tau_{s}<\tau_{d}$ and thus $d \pi / d s<0$, and since $\pi<1 / 2$ when $\tau_{s}>\tau_{d}$ and thus $d \pi / d s>0$,

$$
\begin{equation*}
\frac{d \chi}{d s}=\left(\tau_{s}-\tau_{d}\right) \frac{d \pi}{d s}>0 \text { if } \tau_{s} \neq \tau_{d} \tag{76}
\end{equation*}
$$

$$
\begin{align*}
& { }^{9} \text { Rewriting 62], } \\
& \qquad \begin{aligned}
G^{c} & =\tau_{d}\left[\pi^{2}-2 \pi^{2} s(1-\theta)-\pi+3 \pi s(1-\theta)-s(1-\theta)\right]-\tau_{s}\left[\pi^{2}-2 \pi^{2} s(1-\theta)-\pi+\pi s(1-\theta)\right] \\
& =\tau_{d}\left[\pi^{2}-2 \pi^{2} s(1-\theta)-\pi+\pi s(1-\theta)\right]+\tau_{d}(2 \pi-1) s(1-\theta)-\tau_{s}\left[\pi^{2}-2 \pi^{2} s(1-\theta)-\pi+\pi s(1-\theta)\right] \\
& =\left(\tau_{d}-\tau_{s}\right)\left[\pi^{2}-2 \pi^{2} s(1-\theta)-\pi+\pi s(1-\theta)\right]+\tau_{d}(2 \pi-1) s(1-\theta) \\
& =\pi \underbrace{[\pi(1-2 s(1-\theta))-(1-s(1-\theta))]}_{<0}\left(\tau_{d}-\tau_{s}\right)+(2 \pi-1) s(1-\theta) \tau_{d}=0
\end{aligned} \tag{63}
\end{align*}
$$

If $\tau_{s}=\tau_{d}$, the first component is zero, so $G^{c}=0$ requires that $2 \pi-1=0$, or $\pi=1 / 2$. If $\tau_{s}>\tau_{d}$, the first component is positive, so $G^{c}=0$ requires that $2 \pi-1<0$, or $\pi<1 / 2$. If $\tau_{s}<\tau_{d}$, the first component is negative, so $G^{c}=0$ requires that $2 \pi-1>0$, or $\pi>1 / 2$.
${ }^{10}$ The partial derivative of $G^{c}$ with respect to $\pi$ is

$$
\begin{align*}
G_{\pi}^{c} & =2\left(\tau_{d}-\tau_{s}\right)(1-2 s(1-\theta)) \pi+\tau_{s}(1-s(1-\theta))-\tau_{d}(1-3 s(1-\theta))  \tag{67}\\
& =\tau_{d}(2 \pi-1-4 \pi s(1-\theta)+3 s(1-\theta))+\tau_{s}(1-2 \pi+4 \pi s(1-\theta)-s(1-\theta))  \tag{68}\\
& =\tau_{d}(2 \pi(1-2 s(1-\theta))+3 s(1-\theta)-1)-\tau_{s}(2 \pi(1-2 s(1-\theta))+s(1-\theta)-1)  \tag{69}\\
& =\left(\tau_{d}-\tau_{s}\right) \underbrace{(2 \pi(1-2 s(1-\theta))+s(1-\theta)-1)}_{>0 \text { if } \pi>1 / 2}+2 s(1-\theta) \tau_{d}>0 \tag{70}
\end{align*}
$$

If $\tau_{s}=\tau_{d}, G_{\pi}^{c}=2 s(1-\theta) \tau_{d}>0$. If $\tau_{s}<\tau_{d}$, we have $\pi>1 / 2$ and therefore the term attached to $\left(\tau_{d}-\tau_{s}\right)>0$ is positive, thus $G_{\pi}^{c}>0$. If $\tau_{s}>\tau_{d}$, we have $\pi<1 / 2$ so the term attached to $\left(\tau_{d}-\tau_{s}\right)<0$ is negative, thus $G_{\pi}^{c}>0$.
$\delta$ only matters when there is quality investment in children, that is, $G_{\delta}^{c}=0$, so $d \pi / d \delta=0$.
The partial derivative of $G^{c}$ with respect to $s$ is

$$
\begin{align*}
G_{s}^{c} & =-2(1-\theta)\left(\tau_{d}-\tau_{s}\right) \pi^{2}-(1-\theta) \pi\left(\tau_{s}-3 \tau_{d}\right)-(1-\theta) \tau_{d}  \tag{71}\\
& =-2\left(\tau_{d}-\tau_{s}\right) \pi^{2}+\left(3 \tau_{d}-\tau_{s}\right) \pi-\tau_{d}  \tag{72}\\
& =-\underbrace{\left(2 \pi^{2}-3 \pi+1\right)}_{>0 \text { if } \pi<1 / 2(\text { or } \pi>1)} \tau_{d}+\pi \underbrace{(2 \pi-1)}_{<0 \text { if } \pi<1 / 2} \tau_{s} \tag{73}
\end{align*}
$$

Thus $G_{s}^{c}>0$ when $\pi>1 / 2$, and $G_{s}^{c}<0$ when $\pi<1 / 2$.

The TFR falls as improvements in child survival shift parents towards the more expensive sex, thereby increasing $\chi$. However, even when the TFR falls, the NFR may not.

$$
\begin{equation*}
\frac{d N F R}{d s}=\left(\frac{\gamma \theta}{1+\gamma \theta}\right)\left(\frac{\chi-s\left(\frac{d \chi}{d s}\right)}{\chi^{2}}\right)<0 \text { if } \chi<s\left(\frac{d \chi}{d s}\right) \tag{77}
\end{equation*}
$$

or

$$
\begin{equation*}
\chi<s\left(\tau_{s}-\tau_{d}\right) \frac{d \pi}{d s}>0 \text { if } \tau_{s} \neq \tau_{d} \tag{78}
\end{equation*}
$$

In order for the NFR to fall, the shift towards the more expensive sex ( $d \pi / d s$ ) must be strong enough and/or the difference in rearing costs $\left(\left|\tau_{s}-\tau_{d}\right|\right)$ must be large enough. Without an increase in quality investment when child survival improves, child rearing costs ( $\chi$ ) might not rise sufficiently to reduce the NFR.

## C Proof of Proposition 3

We present the general case where rearing costs may or may not differ for sons ( $\tau_{s}$ ) versus daughters $\left(\tau_{d}\right)$. The partial derivative of $G$ with respect to $\pi$ is

$$
\begin{equation*}
G_{\pi}=\frac{(1-\theta) s(v-1)}{\pi^{2}(1-\pi)^{2}}\left(\left(-2 \chi+(1-2 \pi) \frac{\partial \chi}{\partial \pi}\right) \pi(1-\pi)-(1-2 \pi)^{2} \chi\right) \tag{79}
\end{equation*}
$$

When $\pi \geq 1 / 2, G_{\pi}<0$ if $v>1$ under the sufficient condition $\partial \chi / \partial \pi>0$, for which a sufficient condition is ${ }^{[1]} \tau_{s} \geq \tau_{d}$. The partial derivative of $G$ with respect to $\delta$ is

$$
\begin{equation*}
G_{\delta}=\frac{(1-\theta) s(1-2 \pi)(v-1)}{\pi(1-\pi)} \frac{\partial \chi}{\partial \delta}-\frac{\theta s \epsilon}{\delta^{2}} \tag{80}
\end{equation*}
$$

where $\partial \chi / \partial \delta>0$ (using (14)). When $\pi=1 / 2, G_{\delta}<0$. When $\pi>1 / 2, G_{\delta}<0$ under the sufficient condition $v>1$. Thus when $v>1, d \pi / d \delta<0$ for $\pi \geq 1 / 2$.

The partial derivative of $G$ with respect to $s$ is

$$
\begin{equation*}
G_{s}=\frac{(1-\theta)(1-2 \pi)(v-1)}{\pi(1-\pi)}\left(\chi+s \frac{\partial \chi}{\partial s}\right)+\left(\frac{1-\delta}{\delta}\right) \theta \epsilon \tag{81}
\end{equation*}
$$

where $\partial \chi / \partial s>0{ }^{12}$ When $\pi=1 / 2$ and $\delta=1, G_{s}=0$. When $\pi<1 / 2, G_{s}>0$ if $v>1$. When $\pi>1 / 2$,

[^4]using (17),
\[

$$
\begin{equation*}
G_{s}=-\left(\frac{(1-\theta) \epsilon}{\delta}\right)\left(-2(1-\delta) \pi^{2}+[3-v-\delta(1+v)] \pi-(1-v)\right)<0 \tag{83}
\end{equation*}
$$

\]

if

$$
\begin{equation*}
-2(1-\delta) \pi^{2}+(3-v-\delta(1+v)) \pi-(1-v)>0 \tag{84}
\end{equation*}
$$

whose left-hand-side is decreasing in $\pi \cdot{ }^{[13}$ Thus (84) is most binding at the maximum value of $\pi=1$ and becomes $\delta(1-v) \leq 0$. However, since $\pi=1$ occurs when $\delta=0, \delta(1-v)=0$. The minimum value of (84) is zero, and for $\pi<1$ the condition in (84) holds. Thus $G_{s} \leq 0$ for $\pi \geq 1 / 2$.

## D Simulation process

The model is solved numerically using Matlab for the cases where $\delta$ is exogenous and endogenous following these steps:

1. For each possible value of $\delta \in[0.1,1]$, solve $G(\pi)$ in 17$)$ for $\pi^{*}(\delta)$, and then calculate $g(\delta)$ in (32) as a function of $\delta$ and $\pi^{*}(\delta)$.
2. Solve for $\delta^{*}$ by finding where $g\left(\delta^{*}\right)=\delta^{*}$. (Skip this step if $\delta$ is exogenous.)
3. Find $\pi^{*}\left(\delta^{*}\right)$. (Skip this step if $\delta$ is exogenous.)
4. Solve for all other variables of the model: $n\left(\delta^{*}, \pi^{*}\left(\delta^{*}\right)\right), e_{s}\left(\delta^{*}, \pi^{*}\left(\delta^{*}\right)\right), e_{d}\left(\delta^{*}, \pi^{*}\left(\delta^{*}\right)\right.$ ). (Or for each possible value of $\delta$ and $\pi^{*}(\delta)$ if $\delta$ is exogenous.)
because $\partial e_{s} / \partial s>0$ by the condition that $e_{d} \geq 0$ if $e_{s}>\left(\frac{1-(1+\delta) \pi}{\delta \pi}\right) \epsilon$, and $2 \pi e_{s}-\left(\frac{1-(1+\delta) \pi}{\delta \pi}\right) \epsilon>0$ also by the condition for $e_{d} \geq 0$.
${ }^{13} \partial 84 / \partial \pi=-4(1-\delta) \pi+3-v-\delta(1+v)<0$ if $\pi>\frac{3-v-\delta(1+v)}{4(1-\delta)}$ which is most binding when $\pi=1 / 2$, its minimum value if $\tau_{s}=\tau_{d}$, and this inequality holds as long as $v>1$.

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[^1]:    ${ }^{1}$ Agrawal (2012) estimates increasing returns to education for India. Carnoy, Loyalka, Androushchak and Proudnikova (2012) demonstrate higher returns to education at higher levels of education in the BRIC countries, and Peet, Fink and Fawzi (2015) confirm this for a broader sample of developing countries. There are, of course, studies with less clear or contradicting conclusions, the latter consistent with the theoretical concept of diminishing returns to capital. Trostel (2005) finds a nonlinear relationship with increasing returns at low levels of education and decreasing returns at high levels. Patrinos, Ridao-Cano and Sakellariou (2006) find decreasing returns among low income countries.
    ${ }^{2}$ In that model $\delta$ symbolizes lower returns to human capital investment in unhealthy children, those who suffered but survived disease in early childhood, relative to healthy children, those who avoided disease in childhood.

[^2]:    ${ }^{7}$ The optimal number of births in the model $n$ is decreasing in $\chi$, the average cost per childbirth, and when $\delta<1$ $\chi$ is greater when $\pi>1 / 2$ versus when $\pi=1 / 2$ (see equation $\sqrt{14}$ ).

[^3]:    ${ }^{8}$ It is possible that such reforms were responses to changing social norms (the rising prevalence of divorce, female economic autonomy, and the feminist movement more generally) rather than the impetus for social change themselves (Yang 2008).

[^4]:    ${ }^{11}$ Using $14, \partial \chi / \partial \pi>0$ if $\tau_{d}-\tau_{s}<\left(\frac{1-\delta}{\delta}\right) s \epsilon$, which holds for sure if $\tau_{s}>\tau_{d}$, or if $\tau_{s}<\tau_{d}$ requires that the difference not be too great; if $\tau_{s}=\tau_{d}$, then $\partial \chi / \partial \pi \geq 0$ for $\delta \leq 1$.
    12

    $$
    \begin{equation*}
    \frac{\partial \chi}{\partial s}=2 \pi s\left(\frac{\partial e_{s}}{\partial s}\right)+2 \pi e_{s}-\left(\frac{1-(1+\delta) \pi}{\delta}\right) \epsilon>0 \tag{82}
    \end{equation*}
    $$

