GLOBAL COLLATERAL:

How Financial Innovation Drives Capital Flows and Increases Financial Instability

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Abstract

Cross-border financial flows arise when countries differ in their abilities to use assets as collateral to back financial contracts. Foreign demand for collateral and for collateral-backed financial promises increases the collateral value of domestic assets, and cheap foreign assets provide attractive returns to Home investors who do not demand collateral to issue promises. Global flows export and amplify financial volatility: both Home and Foreign asset prices become more volatile following financial integration. Gross flows driven by collateral differences can collapse following bad news about fundamentals. Our results explain financial flows among rich, similarly-developed countries, and why these flows increase volatility.

Keywords: Collateral, financial innovation, asset prices, capital flows, securitized markets, asset-backed securities, global imbalances.

JEL classification: D52, D53, E32, E44, F34, F36, G01, G11, G12.

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1 Introduction

Recent decades have exhibited a proliferation of financial innovation and dramatic increases in gross international financial flows among financially developed countries. Economists have tended to focus on flows between developed and emerging economies, but gross flows among developed economies are substantial.\footnote{1} If these flows were primarily driven by diversification motives, then capital flows would tend to dampen shocks and decrease volatility. However, there is ample evidence that financial integration increases volatility and amplifies shocks, suggesting that the nature of these flows are at least in part driven by other motives.

We show that a collateral-based definition of financial innovation can rationalize the gross flows observed among developed countries and the recent increases in global volatility following the U.S. financial crisis. Following \cite{FostelGeanakoplos2012b}, we define financial innovation as the use of new assets as collateral or new kinds of financial promises that can be backed by collateral. We show that cross-border differences in the ability to collateralize financial promises are enough to generate international capital flows because international financial trade is a way of sharing scarce collateral. Critically, these flows both amplify and export volatility in asset prices and lead to collapses in flows during crises.

Motivating evidence

The following empirical observations motivate our analysis.

\textit{Observation 1: There are substantial gross financial flows between rich countries with similar levels of financial development. These gross global financial flows (countries simultaneously buying and selling foreign and domestic assets) are an order of magnitude larger than net trade in assets. In some cases these offsetting flows are heavily concentrated in financial assets and in particular in securitized mortgage securities.} First, among developed countries there are substantial foreign holdings of government bonds, and for many countries non-residents make up the largest investor base \cite{Andritzky2012}. Second, there are substantial gross flows between the U.S. and Europe \cite{Shin2012, Bertaut2012}. These flows were most striking pre-crisis,\footnote{1See for example Bruno and Shin (2014), who discuss implications for intermediation costs.}
and while they have partially reversed post-crisis (as did all gross flows), the patterns remain (see BIS locational banking statistics). Finally, as documented by Hale and Obstfeld (2016), there are substantial gross flows within Europe, and these flows are concentrated in just a few countries: Germany, Belgium, and France stand out among the core countries, and crucially Ireland behaves much like the core countries, intermediating funds to the other peripheral countries as well as to the core.

Observation 2: Financial integration tends to increase co-movement and volatility, particularly in response to financial shocks, and particularly through banking flows and securitized markets. Many empirical papers find that financial integration increases co-movement across countries (see Imbs, 2006; Kose, Otrok and Whiteman, 2008; Davis, 2014). Furthermore, financial crises are particularly likely to propagate: Santis and Gerard (1997) find that severe U.S. market declines are internationally contagious, with domestic crashes propagating through (even diversified) international portfolios; Kalemli-Ozcan, Papaioannou and Perri (2013) find that financial crises induce co-movements among financially integrated countries; Loutskina and Strahan (2015) find that the housing boom was exported via financial integration in securitized markets, thus raising the local effect of collateral shocks and increasing volatility. Andritzky (2012) finds that government bond yield volatility increases in the presence of non-resident investors. Additionally, Acharya and Schnabl (2010) find that the geography of financial crises is determined by global banks and securitized markets, not by “global imbalances” and net flows.

These observations suggest a need for a theory of capital flows with the following characteristics: (i) flows, especially among similarly developed, rich countries, are not necessarily driven by diversification and liquidity motives, and (ii) these capital flows amplify volatility. One possible explanation is that gross capital flows are driven in part by cross-country differences in the ability to use assets as collateral, and therefore driven in part by cross-country differences in demand for collateralizable assets.

However, to test the hypothesis that capital flows are driven by cross-country differences in

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2From 2000–2007 core European banks increased their balance sheets, intermediating funds from the rest of the world, to finance the net current accounts of the periphery countries—Greece, Italy, Ireland, Portugal, and Spain (“GIIPS”). As a result, core banks borrowed from abroad to invest in GIIPS.
collateralizability, one would need data on security-level margins offered in funding markets across countries. Data on the loan terms for U.S. securities used as collateral in domestic funding markets is only beginning to be collected and used in limited samples (see Copeland, Martin and Walker (2014) and Baklanova et al. (2017) for evidence). In the absence of direct evidence, one could instead consider proxies for cross-country differences in demand for and supply of collateral. In countries with limited abilities to use assets as collateral, we would expect to see two patterns. First, domestic investors that employ leverage should be more likely to invest in the limited set of assets that serve as collateral, which typically means government bonds. Second, domestic investors should be more inclined to purchase from abroad financial assets that can serve as collateral. In contrast, countries that have greater abilities to use assets as collateral should disproportionately supply such assets to the rest of the world. Indeed, the following evidence suggests that there may be underlying differences in how countries treat assets as collateral in funding markets.

Several demand proxies suggest countries have differential needs for collateral. First, there is substantial variation in the degree to which financial institutions (leveraged investors) invest in domestic government debt. Specifically, financial institutions in deep financial centers hold much smaller portfolios of government bonds compared to financial institutions in countries with less-deep financial centers (Andritzky, 2012). Second, there is heterogeneous foreign demand for US long-term debt (Treasury bonds, corporate bonds, and asset-backed securities (ABS)). Developed countries, and especially those with deep financial centers, hold large portfolios of US corporate debt, but less developed G20 countries primarily hold Treasuries, and Asian countries hold comparably much higher portfolios of ABS[3]. Importantly, there is variation even among rich countries. As an example, the portfolio holdings of Japan look much more like the holdings of China than the holdings of the UK. Compared to the UK, Japan holds lots of Treasuries and ABS[4].

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[3] According to the US Department of the Treasury TIC data, as of June 2016 roughly 50% of the US long-term portfolio holdings of countries in Europe, Canada, and Australia were in corporate non-ABS, while the portfolios of Africa, Asia, and Latin America were less than 10% corporate non-ABS. Similarly, Europe, Canada, and Australia hold fewer than 40% of US long-term securities in Treasuries, whereas the holdings for Africa, Asia, and Latin America were 75–87 percent.

[4] Japan has 74% in Treasuries, 13% in Agency ABS, and 12% in corporate non-ABS. The UK has 31% in Treasuries and 65% in corporate non-ABS. China looks more like Japan, with 15% in Agency ABS and the rest in Treasuries. Finally, South Korea holds 50% Treasuries, 23% Agency ABS, 11% Agency non-ABS, and 16% corporate non-ABS.
These patterns could reflect cross-country differences in supply of collateral. In countries that do not produce collateral, government bonds (typically good collateral) are more valuable for leveraged buyers.\textsuperscript{5} Similarly, countries that do not produce collateral have greater demand for U.S. Treasuries and ABS, which can be used as collateral either directly in funding markets (Treasuries) or indirectly in how they are sold as tranches.\textsuperscript{6,7}

Supply proxies also suggest that countries have different ability to produce collateral. First, global flows are increasingly characterized by demand for “safe assets” and “negative-beta assets,” many of which are created through financial innovations in securitized markets. Safe assets originate from the U.S., and to a lesser extent from Europe. According to the External Wealth of Nations dataset from Lane and Milesi-Ferretti (2007), in 2011, the U.S. net supply of safe assets accounted for roughly two-thirds of the global net supply of safe assets, and the Eurozone accounted for another fifth.\textsuperscript{8} Furthermore, from 1980–1990, the global net supply of safe assets was between 2 and 3% of world GDP, and has risen to 14% in 2011. The U.S. share was 5% in 2000 and rose to 9% in 2011. Many have noted that the increase in safe asset supply has been driven primarily by financial innovations in how the U.S. securitizes assets. Securitization also creates negative-beta assets because some safe assets tend to increase in value in bad states of the world.\textsuperscript{9}

\textsuperscript{5}As evidence of this argument, Wang (2016) documents substantial heterogeneous portfolio adjustments among US following quantitative easing (QE). After QE, financial institutions held more foreign long-term bonds, while non-financial organizations reduced holdings of foreign long-term bonds. These patterns are exactly what one would expect when the supply of collateral shrinks (as a result of QE), given heterogeneous demand for leverage and variations in the ability of assets to serve as collateral.

\textsuperscript{6}Senior-subordinated tranches implicitly provide investors the ability to use the underlying bonds as collateral. See Gong and Phelan (2016b) for more detail.

\textsuperscript{7}As additional supporting evidence, Maggiori, Neiman and Schreger (2017) show that foreign mutual funds demonstrate a “home currency bias,” holding low levels of corporate bonds denominated in foreign currency—except when the currency is the dollar. In that case, foreign and U.S. investors hold nearly identical bond portfolios. One potential explanation for this currency preference could be that dollar assets are more readily accepted as collateral in funding markets.

\textsuperscript{8}The U.S. net supply of safe assets accounted for 9% of world GDP, and the Eurozone accounted for 3%: total global net supply was roughly 14% of world GDP.

\textsuperscript{9}For example, long-maturity bonds increase in price in bad states because long-term interest rates decline, even as the face value of the promised payoffs remain the same. Trade in these securities have important consequences for gross flows. Shin (2012) documents how European banks greatly expanded their balance sheets by increasing both U.S. assets and liabilities (European banks borrowed from U.S. markets and purchased U.S. assets). Similarly, Bertaut et al. (2012) show that during the 2000s European investors purchased U.S. asset-backed securities and similar securities. Additionally, Bertaut et al. (2012) provide consistent evidence of differential abilities to supply securitized assets.
these flows are suggestive evidence of differential abilities to use assets as collateral.

**Overview of the model and results**

Given this motivating evidence, we analyze how collateral-based financial innovations can produce the observed flows and increased levels of financial volatility. We consider a two-country model with incomplete markets and collateralized financial markets, which provides an international extension of the models in Geanakoplos (2003); Fostel and Geanakoplos (2012). The two countries, Home and Foreign, are identical in every way except for the sophistication with which their financial systems can use collateral. The Home country has an advanced financial system that enables investors to use a risky domestic asset as collateral to issue state-contingent financial promises that can be sold domestically or abroad. In contrast, in the Foreign country investors can use collateral to produce only non-contingent promises. The differential ability to collateralize financial promises gives rise to differential abilities to create risk-free and negative-beta financial securities (only Home can tranche the asset into negative-beta securities). Our model provides precise predictions on the directions of capital flows and their effects on asset prices and financial instability.

We first conduct a static analysis to understand how financial innovation drives capital flows. Financial integration allows Foreign investors to buy Home collateral and to make financial promises unavailable before, and allows Home investors to buy cheap Foreign assets. Flows arise even though there are no interest rate differentials, nor hedging or risk-sharing motives to trade assets (agents are risk-neutral and assets have identical payoffs). Additionally, financial flows increase the price of Home asset but decrease the price of the Foreign asset. Home asset prices tend to increase given their attractiveness to international agents to serve as collateral to issue contingent promises (that is, they have a higher collateral value). In contrast, Foreign asset prices decrease by virtue of being priced relative to a higher-valued alternative with identical payoffs but which is better collateral. Importantly, Home runs a current account deficit financed by the sale of its

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10In Appendix A we also consider when Foreign has a very low level of financial development and cannot use assets as collateral at all. In that scenario, we consider two cases: (i) Home has a medium level of financial development that can issue non-contingent debt against the risky asset (Home can leverage the asset), (ii) Home has a high level of financial development that can issue contingent promises against the risky asset.
more-expensive assets.

We then consider a dynamic setting to study the effects of financial integration on asset price volatility and capital flow dynamics. We find that financial integration with differences in collateral generally increases asset crashes and can lead to collapses in gross and net flows after bad news. Asset markets in Foreign become much more volatile due to fluctuations in the attractiveness of alternative assets (Home assets that can be tranched). Furthermore, Foreign demand for collateral-backed financial promises (negative-beta securities) increases the collateral value of Home assets, amplifying pricing fluctuations driven by financial innovations. Hence, asset prices in both countries become more volatile as a result of financial integration. Furthermore, global flows collapse following bad news.

Our mechanism, that financial flows arise as a way to share collateral and contingent financial contracts, has several attractive features and important implications. First, trades driven by global demands to share collateral lead to gross international flows even among countries that are otherwise identical. We propose that differences in financial innovation between the U.S. and Europe contributed to the expansion of European banks’ balance sheets. In particular, the ability of the U.S. financial system to leverage and tranche U.S. assets (especially mortgages) created securities in demand by European banks. Our story provides an attractive hypothesis to explain some of the differences between Ireland, Germany, and the rest of Europe. Given our definition of financial innovation, the ability of a country to use assets as collateral is the feature that differentiates countries that are otherwise similarly financially developed. If collateral-based financial innovations drive gross flows, then a logical conjecture is that the wave of securitization beginning in the late 1990s could possibly explain the recent divergence between gross and net global flows.

\[^{11}\text{This is a complementary result to Fostel and Geanakoplos (2008), which shows that when the same marginal buyer uses leverage to buy two independent assets, the condition of one market spills over to the other. Furthermore, in a model in which Foreign assets cannot be used as collateral at all, even assets which cannot be used as collateral behave similarly to collateralizable assets because they are priced relative to each other (see Appendix C). Furthermore, when assets have low levels of correlation, financial integration induces investors to issue risky debt that defaults following very bad news, an outcome that does not occur in autarky (endogenously, all debt is risk-free in autarky).}^{12}\]

\[^{12}\text{European banks may also have an advantage at intermediation (one explanation for the pre-crisis expansion), but our results imply that gross flows would arise even if they do not.}^{13}\]

\[^{13}\text{Shin argues that the regulatory environment in Europe and the advent of the Euro enabled banks to easily expand their balance sheets. We argue that the question remains: why did European banks expand by intermediating U.S. assets and liabilities as much as they did?}^{11}\]
Second, our story has important implications for asset prices, global financial stability, and crisis transmission. Financial integration tends to increase asset prices, export volatility across borders, and lead to collapses in flows. While Shin (2012) emphasizes how expanded intermediation by European banks depresses credit spreads in the U.S., we document how global banking flows and financial linkages of the type seen between the U.S. and Europe can create serious spillovers, exporting U.S. volatility to European markets and greatly increasing the fragility of the global financial system.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the basic general equilibrium model with collateral. We then study the effects of financial integration in a static model in which countries have different levels of financial innovation. Section 4 considers when the Home country can tranche assets, while Foreign can only leverage. Section 5 uses a three-period model to consider the dynamic consequences of financial integration when Home can tranche assets, with the main results that price volatility increases and flows collapse following bad news. Section 6 concludes.

2 Related Literature

Our paper follows the model of collateral equilibrium developed in Geanakoplos (1997, 2003), Fostel and Geanakoplos (2008, 2012a,b, 2015, 2016), Geanakoplos and Zame (2014), Phelan (2015), and Gong and Phelan (2016a,b). Our paper adds to this work by studying the effect of collateral and financial innovation on global flows and international asset prices.

Our paper is related to a large literature on how differences in financial systems drive capital flows. This “global imbalances” literature has tended to focus on how net capital flows arise between developed and developing countries. The literature has broadly considered differences in (i) state-completeness, (ii) the ability to supply financial assets, (iii) sharing idiosyncratic risk, and (iv) funding costs. In this literature, financial flows are driven primarily by interest rate (or investment return) differentials that manifest in different savings across countries. Financial integration leads to a convergence in savings levels and interest rates, and current account deficits can

\[^{14}\text{Bekaert and Harvey (2000) provide empirical evidence that financial integration increases domestic asset prices.}\]
be financed indefinitely because the financially “deep” country earns intermediation rents. Willen (2004) shows that market incompleteness across countries causes trade imbalances because superior risk-sharing in one country leads to a lower precautionary demand for saving. Caballero, Farhi and Gourinchas (2008) emphasize the role of heterogeneous domestic financial systems in explaining global imbalances in which financial imperfections are captured by a country’s ability to supply assets in a deterministic model. Their paper assumes that “Home” can supply more financial assets from real assets, which affects autarkic savings and interest rates, and the model can explain capital flows, current account deficits, and low interest rates. Mendoza, Quadrini and Rios-Rull (2009) and Angeletos and Panousi (2011) have emphasized how net capital flows arise when the developed country can better insure idiosyncratic risk. Poor risk sharing increases buffer-stock savings and decreases autarkic interest rates. Within this literature, Phelan and Toda (2016) study how the risk-sharing qualities of securitized markets affect international capital flows, growth, and welfare, showing that capital flows from the high- to low-margin country. Maggiori (2017) provides a model in which Home financiers can take on greater financial risk as a result of funding advantages. This leads Home to run persistent current account deficits financed by the risk-premium earned by its financial sector, which can better absorb aggregate shocks.

While these interest rate and risk-sharing mechanisms are clearly important for understanding global flows and imbalances, we instead emphasize the role of collateral to facilitate gross flows, especially among developed countries. In our model, agents are risk-neutral, assets are identical, and interest rates do not change with financial integration (they are always zero). Flows are not driven by different savings demand. In our model all agents have the same savings demand (there is no precautionary motive), but agents have different portfolio demands. Instead, leverage and tranching create contingent securities from underlying collateral, and international trade allows investors to buy securities that are not available domestically. In our model, flows emerge because agents trade in underlying assets and not simply in a risk-free bond. Furthermore, our earlier observations suggest that focusing on net flows alone is insufficient as the differentiation between gross inflows and outflows has become more important (Forbes and Warnock, 2012). While, in the early and mid 1990s net and gross flows used to move together, more recently the size and
volatility of gross flows have increased while net capital flows have been more stable.

Finally, our focus on how financial integration leads to propagation and comovement is related to Caballero and Simsek (2016), who consider a model in which gross flows are driven by demands for liquidity (diversification) and the “fickle” reversal of capital flows creates instability. Flow reversals are assumed and the focus of the paper is on the implications for policy ex ante taking fickle flows as given. Theoretical work by Mendoza and Quadrini (2010), Devereux and Yetman (2010), and Ueda (2012) present models, with financial intermediaries or with leverage constraints, in which financial integration affects spillovers, propagation through interdependent portfolios, and business-cycle synchronization.

3 General Equilibrium Model with Collateral

This section first presents the one-country $C$-model following Geanakoplos (2003), in which heterogeneous beliefs are a reduced-form modeling device for investor heterogeneity. Section 4 uses this architecture for the two-country model. Section 5 extends to a dynamic setting.

Time and Assets

The model is a two-period general equilibrium model, with time $t = 0, 1$. Uncertainty is represented by a tree $S = \{0, U, D\}$ with a root $s = 0$ at time 0 and two states of nature $s = U, D$ at time 1.

There is one consumption good and one asset in the economy, which produces dividends of the consumption good $c$ at time 1. The risky asset $Y$ produces $d^Y_U$ units in state $U$ and $0 < d^Y_D < d^Y_U$ units of the consumption good in state $D$. The consumption good $X$ produces $d^X_U = d^X_D = d^X$ units of the consumption good in each state. Thus, $X$ is a risk-free durable good; it can also be thought of as a risk-free asset with dividends $d^X$. Thus, we will refer to $X$ interchangeably as “risk-free asset” or as “goods.” Figure 1 shows asset payoffs.

We can always normalize one price in each state, so we take the price of $X$ in state 0 and the price of consumption in each state $U$ and $D$ to be one. Thus $X$ is both risk-free and the numeraire (analogous to a durable consumption good like gold, or to money, in our one commodity model).
The price of the risky asset $Y$ at time 0 is denoted by $p$.

**Agents**

Agents are uniformly distributed in the continuum $(0, 1)$ described by the Lebesgue measure. Each investor $i \in (0, 1)$ is risk-neutral and characterized by a linear utility for consumption of the single consumption good $c$ at time 1, with subjective probabilities $(\gamma_i^U, \gamma_i^D) = (\gamma(i), 1 - \gamma(i))$. The von-Neumann-Morgenstern expected utility to agent $i$ is

$$U^i(c_U, c_D) = \gamma_i^U c_U + \gamma_i^D c_D. \tag{1}$$

We suppose that $\gamma(i)$ is strictly increasing and continuous in $i$. Since only the output of $Y$ depends on the state and $0 < d_D^Y < d_U^Y$, higher $i$ denotes more optimism. Heterogeneity among the agents stems entirely from the dependence of $\gamma(i)$ on $i$.

Each investor $i \in (0, 1)$ has endowments $e^X$ and $e^Y$ of $X$ and $Y$ respectively at time 0 and nothing else. Hence, consumption at time 1 is entirely derived from asset dividends and holdings of $X$. 
Financial Contracts and Collateral

The heart of our analysis involves contracts and collateral. We explicitly incorporate repayment enforceability problems. Agents cannot be coerced into honoring their promises except by seizing collateral agreed upon by contract in advance. Agents need to post collateral in the form of durable assets in order to issue promises.

At time 0 agents can trade financial contracts. A financial contract \( j = ((j_U, j_D), Y) \) consists of a promise \((j_U, j_D)\) of repayment in terms of the consumption good at each state, collateralized by one unit of \( Y \). We denote the total set of contracts by \( J \).

The lender has the right to seize as much of the collateral as will make him whole once the promise comes due, but no more. Hence, the delivery of the contract is

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(\min(j_U, d_U^Y), \min(j_D, d_D^Y)).
\]

We denote the sale of promise \( j \) by \( \varphi_j > 0 \) and the purchase of the same contract by \( \varphi_j < 0 \). The sale of a contract corresponds to borrowing the sale price, and the purchase of a promise is tantamount to lending the price in return for the promise. The sale of \( \varphi_j > 0 \) units of contract type \( j \in J \) requires the ownership of \( \varphi_j \) units of \( Y \), whereas the purchase of the same number of contracts does not require any ownership of \( Y \).

Each contract \( j \in J \) trades at price \( \pi^j \). An investor can borrow \( \pi^j \) today by selling contract \( j \) in exchange for a promise of \((j_U, j_D)\) tomorrow, provided she owns \( Y \).

Budget Set

Given asset and contract prices at time 0, \((p, (\pi^j)_{j \in J})\), each agent \( i \in I \) chooses asset holdings \( x \) of \( X \) and \( y \) of \( Y \), contract trades \( \varphi_j \) in state 0, and consumption in final states \( c_U, c_D \), in order to maximize utility \((1)\) subject to the budget set defined by

\[15\text{We exclude cash flow problems. The value of the collateral in each future state does not depend on the size of the promise, or on what other choices the seller makes, or on who owns the asset at the very end. This eliminates any issues associated with hidden effort or unobservability. For an extensive analysis on the of the implications on asset prices, leverage and production arising from the distinction see Fostel and Geanakoplos (2016, 2015).}

\[16\text{Restricting contracts to be collateralized by one unit of } Y \text{ is without loss of generality. A contract promising } (0.4, 0.4) \text{ backed by two units of } Y \text{ is identical to two units of a contract promising } (0.2, 0.2) \text{ backed by one unit of } Y.\]
At time 0, expenditures on the assets purchased (or sold) cannot exceed the value borrowed selling contracts using the assets as collateral. The second constraint is the collateral constraint, which requires that in order to sell contract \( j \) investors must hold the assets to put up as collateral. Consumption in the final states equals dividends coming from asset holds net any debt repayment.

**Collateral Equilibrium**

A Collateral Equilibrium in this economy is a price of asset \( Y \), contract prices, asset purchases, contract trades and consumption decisions by all the agents \( ((p, \pi), (x^i, y^i, \varphi^i, c^i_U, c^i_D)_{i \in I}) \in (R_+ \times R_+^I \times R_+ \times R_+)^I \), such that

1. \( \int_0^1 x^i \, di = e^X \)
2. \( \int_0^1 y^i \, di = e^Y \)
3. \( \int_0^1 \varphi^i_j \, di = 0 \quad \forall j \in J \)
4. \( (x^i, y^i, \varphi^i, c^i_U, c^i_D) \in B^i(p, \pi), \forall i \) and \( (x, y, \varphi, c_U, c_D) \in B(p, \pi) \Rightarrow U^i(x) \leq U^i(x^i), \forall i \)

In equilibrium, all markets clear and agents optimize their utility in their budget sets. Geanakopoulos and Zame (2014) show that, given our assumptions, equilibrium in this model always exists.

**Collateral and Financial Innovation**

A vitally important source of financial innovation involves the possibility of using assets as collateral to back promises. Following Fostel and Geanakoplos (2012a), we define financial innovation as the use of new kinds of collateral, or new kinds of promises that can be backed by the same
collateral. Financial innovation in our model is described by a different set \( J \).

Financial systems differ in a myriad of both subtle and complex ways—for example, the level of insurance and risk sharing—but the salient features that we are focusing on are the ability to leverage, securitize and tranche assets, which is reflected in the financial structures we assume. We model investors as directly borrowing against assets, but these trades could also capture the role of financial intermediaries in producing the financial assets that correspond to these cash flows.

4 A Static Model of Global Flows

We now consider a model with two countries, Home and Foreign (denoted by \( * \)), each defined as in Section 3. Both countries are identical in every way except for the feasible contracts available in each country, \( J \) and \( J^* \). In each country there is a risky asset, \( Y \) and \( Y^* \), and a risk-free asset, \( X \) and \( X^* \), with identical state-payoffs as defined in Figure 1. In each country there is an identical set of investors, a unit continuum of investors as described in Section 3. Prices for risky assets \( Y \) and \( Y^* \) are denoted by \( p \) and \( p^* \).

The key—and only—difference between the two countries is that Home has a more advanced financial system than Foreign. In our baseline model, we assume that Foreign assets can be used as collateral to issue non-contingent financial promises (i.e., debt). In other words, Foreign assets can be leveraged. In contrast, we consider a more sophisticated use of \( Y \) as collateral by the Home financial sector: the Home financial sector can create state-contingent tranches using the Home risky asset as collateral. In other words, the risky Home asset can be used as collateral and tranched to issue contingent promises. One salient feature captured by this model is the advanced ability of the U.S. financial system to securitize and tranche mortgages and other financial assets, which then get traded with other countries with less sophisticated markets for securitized assets.

Without loss of generality, we suppose that \( J \) consists of the single promise \( j^T = ((0, d_Y^D), 1_Y) \), which implies agents can completely tranche \( Y \)

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\[ ^{17} \text{Allowing this single promise is sufficient to allow for completely tranching } Y. \text{ An investor can create any consumption profile } (y, z) \text{ by buying } y \text{ units of } Y \text{ and selling off } y \text{ units of the tranche } j^T = (0, d_Y^D), \text{ and also buying } z/d_Y^D \text{ units of the down tranche (perhaps created by somebody else). Thus, any other tranches backed by } Y \text{ would be redundant, and since } j^T = (0, d_Y^D) \text{ already embodies Arrow tranching, there is no reason to consider more complicated} \]
assets and financial promises\textsuperscript{18}.

The main insight from the static analysis is that differences in the ability to collateralize financial promises across borders are enough to generate international capital flows. In our model, there are no interest rate differentials, nor hedging or risk-sharing motives to trade assets (agents are risk-neutral and assets have identical payoffs). International financial trade is a way of internationally sharing scarce collateral. Accordingly, financial integration has implications for asset prices, raising Home asset prices and decreasing Foreign asset prices.

We first describe autarkic equilibria in each country and then describe the equilibrium with financial integration. In the remainder of the section we will solve for equilibrium for the following set of parameters: agents’ utilities and endowments in both countries are given by $\gamma(i) = 1 - (1 - i)^2$, $e^Y = e^X = 1$, and asset payoffs in both countries are given by $d^X = d^Y_U = 1$ and $d^Y_D = 0.2$. However, our results are robust to the choice of parameters (we discuss parameter sensitivity and comparative statics; also see Fostel and Geanakoplos, 2012a).

### 4.1 Foreign Autarky: Leverage

In autarky, the Foreign financial sector can issue non-contingent promises using the risky asset as collateral. In this case $J = \{ j : j = ((j, j), 1_Y) \}$. Investors can promise to repay $j$ at $t = 1$, which is collateralized by the risky asset, to borrow $\pi^j$ at $t = 0$. By purchasing $Y^*$ and promising $j$, Foreign investors can leverage their purchases of risky assets.

Importantly, leverage is endogenously determined in equilibrium: although all contracts will be priced in equilibrium, only one contract is actively traded. The only contract actively traded is the “max min” contract $\min_{s=U,D}\{d^Y_s\} = d^Y_D$, ruling out default in equilibrium (Fostel and Geanakoplos, 2012b). Given the normalizations on goods prices across time, the risk-free interest rate is zero and so the equilibrium contract has price $\pi^* = d^Y_D$.

In equilibrium there is a marginal buyer $i^{Y^*}$, who is indifferent between holding $X^*$ and buying tranching schemes.

\textsuperscript{18}In Appendix A we also consider when Foreign assets cannot be used as collateral at all, and when Home can either leverage or tranche assets. This model with no leverage in Foreign presents many similar intuitions, mechanisms, and insights that are be present in the richer model with leverage and tranching. The model when Home can leverage provides insights corresponding to trade between developed and developing countries.
$Y^*$ with leverage. All agents $i > i^Y$ buy $Y^*$ with leverage: they sell their endowment of $X^*$ and borrow $d^Y_D$ using $Y^*$ as collateral. Notice that when agents buy the asset $Y^*$ with leverage, they are effectively buying the Arrow $U$ security and they end up consuming only at state $U$. Agents $i < i^Y$ sell their endowment of $Y^*$ and lend to the more optimistic investors, holding $X^*$ and risk-free financial promises. Equilibrium is described by a system of two equations in two unknowns:

$$1 = (1 - i^Y)(1 + p^*) \frac{1}{p^* - d^Y_D},$$

(3)

$$p^* = \gamma(i^Y)1 + (1 - \gamma(i^Y))d^Y_D.$$

(4)

Equation (3) is market clearing. The top $1 - i^Y$ agents are buying the asset and issuing debt to finance their purchases, thus borrowing $d^Y_D$ and paying only the downpayment $p^* - d^Y_D$. As a result, borrowing allows a relatively small fraction of agents to buy all the asset in the economy—certainly fewer than would be required if no borrowing were allowed. Equation (30) states that the asset is priced according to the marginal buyer. Notice that equation (30) can also be written as

$$1 = \frac{\gamma(i^Y)(1 - d^Y_D)}{p^* - d^Y_D},$$

where $1 - d^Y_D$ is what the borrower gets from $Y^*$ in $U$ after paying back the loan, so that the leveraged return equals 1 for the marginal buyer. For the given parameters, equilibrium is given by $i^Y = 0.63$ and $p^* = 0.89$.

### 4.2 Home Autarky: Tranching

The autarkic equilibrium at Home features two marginal buyers, $i^Y$ and $i^T < i^Y$. Agents $i > i^Y$ buy $Y$ and sell the down tranche $j^T = (0, d^Y_D)$, effectively holding an Arrow $U$ security. Agents $i \in [i^T, i^Y]$ sell their endowment of $Y$ and purchase $X$. Finally, agents $i < i^T$ sell their assets $Y$ and $X$ and buy the down tranche $j^T = (0, d^Y_D)$ from the most optimistic investors, effectively buying an Arrow $D$ security.

Equilibrium is described by a system of four equations and four unknowns: the price of the asset, $p$, the price of the down tranche $\pi^T$, and the two marginal buyers, $i^Y$ and $i^T$. Market clearing
for the asset $Y$ and the tranche $j^T$ are

$$1 = (1 - i^Y) \frac{(1 + p)}{p - \pi^T}, \quad (5)$$

$$i^T (1 + p) = \pi^T. \quad (6)$$

The top $1 - i^Y$ agents are buying the asset and selling off the down tranche: they each have wealth $1 + p$ plus the revenue from the tranche sale $\pi^T$. Their expenditure $(1 - i^Y)(1 + p) + \pi^T$ must equal the revenue from the sale of one unit of $Y$. The bottom $i^T$ agents spend their endowments to buy the tranche. There is 1 unit of total supply of the asset, implying there is one unit supply of the tranche. Optimality conditions are given by

$$\gamma(i^Y) = p - \pi^T, \quad (7)$$

$$(1 - \gamma(i^T))d_D^Y = \pi^T. \quad (8)$$

The marginal buyer $i^Y$ is indifferent between the riskless asset $X$, which has a return of 1, and buying $Y$ while selling the tranche, which delivers an Arrow $U$ payoff. The marginal buyer $i^T$ is indifferent between $X$ and buying the down tranche. Hence, each marginal buyer sets the return of the $U$ or $D$ payoff equal to 1, and so the value of the Arrow $U$ and $D$ securities created through tranching are correctly priced by the marginal agent\textsuperscript{19}

The equilibrium regime is shown in Figure 2. For the same parameter values, in equilibrium we get $i^Y = 0.58$, $i^T = 0.08$, $p = 1$, and $\pi^T = 0.17$. Notice that the marginal buyer $i^Y = 0.58$ is lower than was the case with leverage ($i^{Y*} = 0.63$). This is because optimists buying the asset sell the tranche, which allows them to only borrow $\pi^T = 0.17$, whereas in the leverage economy investors could borrow $\pi^* = 0.2$.

The asset price is much higher even than it was with leverage. Tranching $Y$ creates contingent promises that are highly valued by pessimistic investors. This more sophisticated scheme of using $Y$ to issue financial promises further increases its value as collateral, increasing its price. Indeed,

\textsuperscript{19}Despite the fact that both Arrow securities can be created through tranching the asset, markets are not complete because Arrow securities are created through the risky asset only.
the price of one is higher than any agent $i < 1$ believes can be justified by the fundamentals (if $d_Y^i > 0.2$ then the price $p$ would exceed 1, implying no agent believes fundamentals justify the price)\textsuperscript{20}

\[ i = 1 \quad \text{Optimists: buy } Y \]
\[ \quad \text{sell down tranche } j^T \text{ (hold Arrow U)} \]
\[ i^Y = 0.58 \quad \text{Marginal } Y \text{ buyer} \]
\[ i^T = 0.08 \quad \text{Marginal tranche buyer} \]
\[ i = 0 \quad \text{Pessimists: buy down tranche } j^T \]

Figure 2: Equilibrium Regime with Tranching in Autarky.

4.3 Financial Integration

With financial integration, Home and Foreign agents can trade assets and financial promises—any agent can use the Home asset $Y$ as collateral in order to issue tranches, and any agent can use the Foreign asset $Y^*$ as collateral to issue debt. We denote equilibrium variables after financial integration by a ‘hat’ (\(\hat{\cdot}\)) to distinguish them from their autarkic counterparts.

In equilibrium, in each country there are three common marginal investors: $\hat{i}^M > \hat{i}^Y > \hat{i}^T$. The marginal investors across countries are the same because the assets have identical payoffs and agents have the same endowments and preferences. Investor $\hat{i}^Y$ is indifferent between holding risky assets $Y$ or $Y^*$ and holding risk-free assets. Investors in both countries with $i \geq \hat{i}^Y$ buy risky assets. The most optimistic of these investors buy $Y$ and issue the down tranche using the asset as

\textsuperscript{20}Even though the marginal buyer is less optimistic, the ability to tranche the asset, selling a down security to very pessimistic agents, increases the collateral value enough so that the price increases. \textit{Postel and Geanakoplos (2012a)} show that the asset price is always higher with tranching whenever the beliefs $\gamma(i)$ are concave.
collateral (hence holding Arrow $U$ securities), and the remaining investors buy $Y^*$ with leverage (also holding Arrow $U$ securities). Investor $i^T < i^Y$ is indifferent between holding the down tranche and holding risk-free assets. Investors with $i \in (i^T, i^Y)$ hold goods (risk-free assets) $X$ and $X^*$, as well as debt issued by holders of $Y^*$. Investors $i \leq i^T$ buy the down tranche (hence holding an Arrow $D$).

Determining the portfolio holdings of investors $i > i^Y$ requires some care. First, note that buying $Y$ and issuing a down tranche, and buying $Y^*$ with leverage (issuing debt), both create Arrow $U$ securities. By a no-arbitrage requirement, in equilibrium the returns to investing in Home and Foreign risky assets are equal. Second, because the returns to risky assets $Y$ and $Y^*$ are identical in equilibrium, investors are indifferent and so portfolio allocations for $Y$ or $Y^*$ are indeterminate. However, the smallest friction affecting the Foreign financial sector—perhaps coming from agency problems, informational asymmetries, or simply regulation—would lead to determinate portfolios, so that investors strictly prefer either $Y$ or $Y^*$ (we show this formally in Appendix B). Accordingly, we choose portfolio allocations consistent with the allocations that would arise from a minor perturbation of the model: the most optimistic agents $i > i^M$ purchase $Y$ and issue tranches, and the remaining agents $i \in (i^M, i^Y)$ purchase $Y^*$ with leverage. Hence, $i^M$ determines market clearing, and so unlike the other marginal buyers, does not also determine asset pricing. Figure 3 illustrates the equilibrium regime.

Equilibrium is described by a system of six equations in six unknowns: the prices of the assets, $\hat{p}^*$ and $\hat{p}$, the tranche, $\hat{\pi}^T$, the marginal buyers, $i^Y$ and $i^T$, and the investor $i^M$ determining which investors buy Home assets. The returns to optimally investing in $Y$ and $Y^*$ are equal:

$$\frac{1}{\hat{p} - \hat{\pi}} = \frac{1 - d_Y^{Y^*}}{\hat{p}^* - d_D^{Y^*}}.$$  \hfill (9)

The marginal buyer $i^Y$ is indifferent between safe and risky assets (receiving either the leveraged return on $Y^*$ or the return from buying $Y$ and selling a tranche).

$$\gamma(i^Y) = \hat{p} - \hat{\pi}^T.$$  \hfill (10)
The marginal buyer $\hat{i}^T$ is indifferent between the tranche and risk-free assets:

$$\hat{\pi}^T = (1 - \gamma(\hat{i}^T))d_Y. \quad (11)$$

Market clearing conditions for $Y$ and $Y^*$ are given by

$$(1 - \hat{\gamma}^M)\frac{2 + \hat{\rho} + \hat{\rho}^*}{\hat{\rho} - \hat{\pi}^T} = 1, \quad (12)$$

$$(\hat{\gamma}^M - \hat{\gamma}^Y)\frac{2 + \hat{\rho} + \hat{\rho}^*}{\hat{\rho}^* - d_Y} = 1, \quad (13)$$

where the most optimistic fraction of investors $i > \hat{i}^Y$ in both countries invest in Home assets $Y$, and the remaining fraction of investors $i > \hat{i}^Y$ in both countries invest in Foreign assets $Y^*$. Compared to the autarkic market clearing equations (5) and (6), the market clearing conditions (12) and (13) include wealth from both countries: Home investors have wealth $1 + \hat{\rho}$ and Foreign investors have wealth $1 + \hat{\rho}^*$. Market clearing for the tranche is

$$\hat{i}^T (2 + \hat{\rho} + \hat{\rho}^*) = \hat{\pi}^T. \quad (14)$$
Compared to equation (6), which was market clearing for the tranche in autarky, the market clearing condition (14) includes wealth from both countries. Therefore, the marginal buyer $\hat{i}^T$ decreases as a result of trade and so the tranche price must increase.

4.3.1 Financial Integration and Equilibrium Asset Prices

Table 1 shows the equilibrium prices and marginal buyers. The Foreign asset price decreases from 0.893 to 0.878, while the Home asset price increases from 1 to 1.029, and the Home tranche price increases from 0.168 to 0.182.

Table 1: Equilibrium when Home Tranches Assets and Foreign Leverages Assets.

<table>
<thead>
<tr>
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<th>Autarky</th>
<th>Financial Integration (↑)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>1</td>
<td>1.029↑</td>
</tr>
<tr>
<td>$p^*$</td>
<td>0.893</td>
<td>0.878↓</td>
</tr>
<tr>
<td>$\pi^T$</td>
<td>0.168</td>
<td>0.182↑</td>
</tr>
<tr>
<td>$i^Y$</td>
<td>0.58</td>
<td>0.61↑</td>
</tr>
<tr>
<td>$i^Y^*$</td>
<td>0.63</td>
<td>0.61↓</td>
</tr>
<tr>
<td>$i^T$</td>
<td>0.084</td>
<td>0.047↓</td>
</tr>
</tbody>
</table>

Financial integration affects equilibrium prices and portfolio holdings even though assets in both countries have identical payoffs and agents are identical. This is because $Y$ is better collateral: $Y$ can be tranched into state-contingent promises whereas $Y^*$ can only be used to issue debt. This has a number of important consequences. First, financial integration increases the price of the tranche. Pessimistic Foreign investors demand down tranches (contingent promises that were previously unavailable), which pushes up their price, and as a result the marginal tranche buyer is always more pessimistic with financial integration. Second, because buyers of $Y$ can now issue a

\[ \int_0^{\hat{i}^T} \left( 1 + \hat{\rho} \right) di + \int_0^{\hat{i}^T} \left( 1 + \hat{\rho}^* \right) di = \hat{i}^T \left( 2 + \hat{\rho} + \hat{\rho}^* \right). \]

In contrast, Appendix A considers when trade occurs when Home can leverage and Foreign cannot. In this case,
more expensive tranche, fewer optimists are required to buy up all of \( Y \), and so the marginal buyer of the Home asset increases with financial integration, rising from 0.58 to 0.61. Both of these effects cause the price of \( Y \) to increase, since the collateral value has greatly increased. Notice that the asset price is given by \( \hat{p} = \gamma(\hat{T})1 + (1 - \gamma(\hat{T}))d_{Y}^{\hat{Y}} \), which we get from combining equations (7) and (8). Foreign demand for the asset increases \( \hat{T} \), and foreign demand for the tranche decreases \( \hat{T} \).

However, financial integration decreases the price of \( Y^* \) (notice that the marginal buyer of Foreign assets has decreased from 0.63 to 0.61). This is because the attractiveness of alternative investments increases. With financial integration, investors in the Foreign asset compare investing in the asset to a tranched return in the Home asset, rather than simply cash, as was the case in autarky. In autarky, the Foreign asset price includes a collateral value because it can be used to issue debt (i.e., the price exceeds the fundamental payoffs). But financial integration decreases this collateral value as investors prefer the Home asset, which is better collateral (the price of debt backed by \( Y^* \) has not changed), and so \( p^* \) decreases.\(^{23}\)

Finally, let us define the price gap between the Home and Foreign asset prices \( \Delta \), so that we have \( p = p^* + \Delta \). Since both Home and Foreign risky assets have collateral value (i.e., their price exceeds the marginal valuation of future dividends), the price gap \( \Delta \) is a measure of the collateral value gap between the two assets.\(^{24}\) As just discussed, the collateral gap \( \Delta \) is positive because the Home asset is better collateral than the Foreign asset. Furthermore, the collateral gap \( \Delta \) always increases with financial integration (\( p \) increases and \( p^* \) decreases) since Foreign demand for Home tranches increases the collateral value of the Home asset.

**Proposition 1.** With financial integration, it is always the case that \( \hat{p} > \hat{p}^* \) and hence the collateral gap \( \Delta \) increases.

---

\(^{23}\) In contrast, when \( Y^* \) is imperfect collateral to issue debt, financial integration can increase the price of \( Y^* \). Appendix A shows that when \( Y^* \) cannot be leveraged at all, financial integration increases the Foreign asset price, and Appendix B shows how the asset pricing implications vary with the Foreign borrowing friction.

\(^{24}\) In other words, \( \Delta \) is not formally the “collateral value” as defined by Fostel and Geanakoplos (2008), which is an investor-specific measure depending on the marginal utility of alternative investments and the marginal utility associated with being able to borrow using the asset. With a continuum of investors, the economy features a continuum of investor-specific collateral values.
gap $\Delta$ is strictly positive. Furthermore, $\Delta = d^Y_D (\gamma(\hat{i}^Y) - \gamma(\hat{i}^T))$.

Proof of Proposition 1. From equations (9), (10), and (11), we can write asset prices as

$$\hat{p} = \gamma(\hat{i}^Y) + \hat{\pi} = \gamma(\hat{i}^Y) + (1 - \gamma(\hat{i}^T)) d^Y_D,$$

$$\hat{p}^* = \gamma(\hat{i}^Y) (1 - d^Y_D) + d^Y_D.$$

Hence we have $\Delta = d^Y_D (\gamma(\hat{i}^Y) - \gamma(\hat{i}^T))$, which is positive since $\hat{i}^Y > \hat{i}^T$ and beliefs are monotonic.

Proposition 2. With financial integration, $\hat{p}^* < p^*$ and $\hat{p} > p$—financial integration increases the Home asset price and decreases the Foreign asset price.

Proof of Proposition 2. Since the prices of $Y$ and $Y^*$ are determined by the marginal buyer $\hat{i}^Y$, it suffices to show that $i^Y < \hat{i}^Y < i^Y^*$ (we already know that the tranche investor is lower with financial integration, i.e., $i^T > i^T$).

With financial integration, since we have common marginal investors across countries, the economy is isomorphic to a single economy with an asset $Z$ which is the composite of $Y$ and $Y^*$: delivering 1 or $d^Y_D$ and that can be tranched into a down payoff promising $d^Y_D - zd^Y_D$ and into a debt contract promising $zd^Y_D$, with $z = 0.5$. This economy has marginal buyers $i^Y_z = \hat{i}^Y$ and $i^T_z = \hat{i}^T$. Furthermore, the asset price $p_z = \frac{\hat{p} + \hat{p}^*}{2}$. To prove the proposition, we show that $i^Y^* > i^Y > i^Y_z$.

We first show that $i^Y^* > i^Y_z$. Note that we can write the equilibrium prices as

$$p^* = d^Y_D + \gamma(i^Y^*) (1 - d^Y_D),$$

$$p_z = \gamma(i^Y_z) (1 - zd^Y_D) + zd^Y_D + \gamma(i^T_z) (d^Y_D - zd^Y_D)$$

$$> d^Y_D + \gamma(i^Y_z) (1 - d^Y_D).$$

First, suppose that $p^* > p_z$. If $p^* > p_z$ then $\gamma(i^Y^*) > \gamma(i^Y_z)$ and so $i^Y^* > i^Y_z$ and we are done. Suppose instead that $p^* < p_z$. Then market clearing yields

$$i^Y^* = \frac{1 + d^Y_D}{1 + p^*}, \quad \text{and} \quad i^Y_z = \frac{1 + zd^Y_D + \pi_z}{1 + p_z}.$$
Note however that \( \pi_z < d_Y^d - z d_Y^d \) and hence we have

\[
i_z^Y < \frac{1 + d_Y^d}{1 + p_z} < \frac{1 + d_Y^d}{1 + p^*} = i^*\]

where the final inequality uses the assumption that \( p^* < p_z \). And thus we are done.

Now we show that \( i_Y > i_z^Y \). Note that we can write the asset prices as

\[
p = \gamma(i_Y^Y) + \pi = \gamma(i_Y^Y) + (1 - \gamma(i_T^T)) d_Y^d
\]

\[
= \gamma(i_Y^Y) + d_Y^d - \gamma(i_Y^T) d_Y^d,
\]

\[
p_z = \gamma(i_z^Y) (1 - z d_Y^d) + \pi_z + zd_Y^d = \gamma(i_z^Y) (1 - z d_Y^d) + (1 - \gamma(i_T^T)) (d_Y^d - zd_Y^d) + zd_Y^d
\]

\[
= \gamma(i_z^Y) + d_Y^d - \gamma(i_Y^T) d_Y^d - zd_Y^d (\gamma(i_z^Y) - \gamma(i_Y^T)).
\]

First, suppose that \( p < p_z \). Assume also that \( i_Y > i_z^Y \) (the opposite of what we want to prove).

From market clearing for the risky assets and the pricing equations, we have

\[
(1 - i_Y^Y) (1 + p) = p - \pi = \gamma(i_Y^Y)
\]

\[
(1 - i_z^Y) (1 + p_z) = p_z - \pi_z - zd_Y^d = \gamma(i_z^Y) (1 - zd_Y^d).
\]

But if \( p < p_z \) and \( 1 - i_Y^Y < 1 - i_z^Y \), then \( \gamma(i_Y^Y) < \gamma(i_z^Y) (1 - zd_Y^d) \), \( \gamma(i_Y^Y) \), and so \( i_Y < i_z^Y \).

Now suppose that \( p > p_z \). If \( \pi_z + zd_Y^d < \pi \), then \( i_T^T < i_z^T \) from the tranche pricing equation.

From market clearing for the tranche we have

\[
i_T = \pi \frac{\pi_z}{1 + p} > \frac{\pi_z}{1 + p} > \frac{\pi_z}{1 + p_z} = i_z^T
\]

a contradiction, so this condition can’t occur. If instead \( \pi_z + zd_Y^d > \pi \), then from market clearing for the risky assets we have

\[
i_Y^Y = \frac{1 + \pi}{1 + p} < \frac{1 + \pi_z + zd_Y^d}{1 + p} < \frac{1 + \pi_z + zd_Y^d}{1 + p_z} = i_z^T,
\]

and so we have \( i_Y^Y < i_z^Y \), and we are done.
4.3.2 Financial Integration and Equilibrium Financial Flows

Figure 4 illustrates the effects of financial integration on marginal investors and financial flows. Given the equilibrium regime, our model predicts the following global financial flows. First, optimistic Foreign investors buy 0.495 worth of Home assets and sell 0.09 worth of the down tranche (capital flows to Home and assets flow to Foreign). Second, moderately optimistic Home investors buy 0.456 worth of Foreign assets with leverage (capital flows to Foreign and assets flow to Home). Third, moderate Home investors buy 0.04 worth of $X^*$ (goods, or the safe Foreign asset). Since the “safe assets” $X$ and $X^*$ can be interpreted as durable goods (our preferred interpretation), we interpret Home’s net purchase of $X^*$ as a net trade/current account deficit. The flows in goods $X, X^*$ finance the difference in the value of flows in risky assets. As we discuss below, Home will always buy $X^*$ from Foreign.

Finally, Home collateral is used for Foreign domestic borrowing. First, pessimistic Foreign investors buy 0.09 worth of tranches backed by Home assets. Importantly, the quantity of tranches issued by Foreign optimists is exactly the quantity of tranches held by Foreign pessimists, and so we can interpret the Foreign tranche holdings as being issued directly by Foreign optimists. Second, because Foreign demand for $X^*$ and debt is less than the endowment of $X^*$ and the issuance of debt by Foreign optimists, we can interpret Foreign domestic borrowing (promises issued by Foreign optimists using $Y$ or $Y^*$ as collateral) as financed entirely by Foreign pessimists holding financial promises. In other words, once the Foreign economy can use the Home asset as collateral, no cross-border borrowing needs to occur. This clarifies the role of collateral in driving global flows: Foreign buys $Y$ to use as collateral domestically.

While the down tranche is a state-contingent security (and therefore very risky), we interpret

\[\text{flows} = pQ\]

\[\text{where } p \text{ is the asset price and } Q \text{ is the quantity purchased.}\]
Figure 4: Equilibrium Flows with Financial Integration when Home can Tranche and Foreign can Leverage.

the creation of down tranches as related to the the creation of “safe assets,” which as it turns out tend to increase in value in bad states of the world: negative-beta assets are truly safe assets. First, long-maturity bonds increase in price in bad states because long-term interest rates decline (even as the face value of the promised payoffs remain the same). Second, because the US Dollar tends to appreciate during crises, dollar-denominated bonds provide a natural hedge for foreign buyers. This is a point made in Maggiori (2017), which, in a different financial context, describes the rest of the world buying “down state” Arrow securities from Home in order to achieve safer portfolios. Thus, as Foreign investors sell $X^*$ following integration, they are exchanging risk-free assets for better-than-risk-free assets.

Thus, our model offers the following implications for global flows.

**Proposition 3.** *Home runs a current account deficit, financed by the sale of risky assets. Since the value of the Home assets sold to Foreign always exceeds the value of the Foreign assets bought by Home, finances a deficit in goods through asset sales.*
Proof of Proposition \[\text{[3]}\] Let \(V_{Y^*}\) denote the value of Home purchases of \(Y^*\) and let \(V_Y^*\) denote the value of Foreign purchases of \(Y\). Let \(Q_{jT}\) and \(Q_{jT}^*\) denote the quantities of Home and Foreign holdings of tranches \(jT\), and let \(Q_X\) and \(Q_X^*\) denote the Home and Foreign holdings of goods (or risk-free assets, \(X, X^*\), and debt). Then from market clearing and some simple accounting, we can calculate the holdings in each asset as follows:\[\text{[27]}\]

\[
V_{Y^*} = \frac{(1 + \hat{\rho})\hat{\rho}^*}{2 + \hat{\rho} + \hat{\rho}^*}, \quad V_Y^* = \frac{(1 + \hat{\rho}^*)\hat{\rho}}{2 + \hat{\rho} + \hat{\rho}^*},
\]

\[
Q_X^* = (2 + d_Y^*)\frac{1 + \hat{\rho}^*}{2 + \hat{\rho} + \hat{\rho}^*}, \quad Q_X = (2 + d_Y^*)\frac{1 + \hat{\rho}}{2 + \hat{\rho} + \hat{\rho}^*},
\]

\[
Q_{jT}^* = \frac{1 + p^*}{2 + \hat{\rho} + \hat{\rho}^*}, \quad Q_{jT} = \frac{1 + \hat{\rho}}{2 + \hat{\rho} + \hat{\rho}^*}.
\]

Indeed, the quantities of \(Y\) and \(Y^*\) held by Home and Foreign investors equal \(Q_{jT}\) and \(Q_{jT}^*\) respectively. By market clearing, \(Q_X - Q_X^* = V_Y^* - V_{Y^*}\), implying that the net trade in goods \(X\) is the difference between the values of the gross flows in risky assets. In particular, we have

\[
Q_X - Q_X^* = \frac{\hat{\rho} - \hat{\rho}^*}{2 + \hat{\rho} + \hat{\rho}^*} = \frac{\Delta}{2 + \hat{\rho} + \hat{\rho}^*},
\]

so trade in \(X\) is directly proportional to the collateral gap. Because \(\Delta > 0\), Home always purchases \(X^*\) from Foreign, and this net flow is larger when the value of the Home asset increases relative to the Foreign asset.

Comparative Statics Flows are generally greater when the borrowing capacity of the Home asset increases. The values of gross flows in risky assets are increasing in the overall level of asset prices and also in the collateral gap \(\Delta = \hat{\rho} - \hat{\rho}^*\). Holding fixed the Foreign asset price \(\hat{\rho}^*\) (partial equilibrium), then an increase in the Home asset price increases gross flows in both \(Y\) and \(Y^*\). Thus, a change in the economic environment that increases the Home collateral value, without decreasing the Foreign asset price, would increase global flows.

\[\text{[27]}\] Rearranging equation \[\text{[12]}\], we have \(\frac{1}{\hat{\rho} - \hat{\rho}^*} = \frac{1}{\hat{\rho} - \hat{\rho}^*} \frac{1}{\hat{\rho} - \hat{\rho}^*} \frac{1}{\hat{\rho} - \hat{\rho}^*}\), and thus the quantity of Home assets held by Foreign investors is \(\frac{1}{\hat{\rho} - \hat{\rho}^*} (1 + \hat{\rho}^*) = \frac{1}{\hat{\rho} - \hat{\rho}^*} \frac{1}{\hat{\rho} - \hat{\rho}^*} \frac{1}{\hat{\rho} - \hat{\rho}^*}\). Multiplying by \(\hat{\rho}\) gives the value of the flows. The value of Home purchases of Foreign assets follows from rearranging equation \[\text{[13]}\] in the same way.
Since a key driver of flows is the collateral gap $\hat{p} - \hat{p}^*$, which from Proposition 1 can be written $\Delta = d_D^Y (\gamma(i^Y) - \gamma(i^T))$, flows are generally greater when the payoff in the down state $d_D^Y$ is greater, reflecting a greater borrowing capacity of the Home asset and a greater distinction between debt and down tranches. Our model implies that gross and net flows increase as borrowing capacity increases. Figure 5 plots asset prices and flow values as a function of the downside risk $d_D^Y$. As is clear, both $\hat{p}$ and $\hat{p}^*$ are increasing in $d_D^Y$, and so is the collateral gap. Accordingly, gross flow values are larger the higher is $d_D^Y$, as are net flows.\footnote{These results are robust to varying beliefs, including allowing beliefs to be substantially convex. Since varying the down payoff mechanically increases the expected payoff of both assets, we have also considered the same comparative static varying $d_D^Y$ while also varying beliefs so that the expected payoff to each agent remains the same even as the down payoff varies. To do so, we set a base down payoff of $\bar{R} = 0.2$ and define modified beliefs by $\tilde{\gamma}(i; d_D^Y) = \bar{y}(i) - d_D^Y$, where $\bar{y}(i)$ is the expected asset payoff for person $i$ when $d_D^Y = \bar{R}$. Correcting in this way, the results are in fact even stronger.}

Figure 5: Financial Integration: Prices, gross flows, and net flows increase as borrowing capacity increases ($d_D^Y$ increases).

5 A Dynamic Model of Global Flows

The static models illustrate how financial integration affects prices and marginal investors and creates cross-border flows. In this section we examine how collateral-driven flows in financial
integration affects volatility of asset prices and flows following bad news about fundamentals. We consider a dynamic model with bad news arriving at an interim date. Our dynamic analysis with financial integration yields the following main sets of results regarding price crashes and financial flows. First, financial integration exports volatility from Home to Foreign. Second, Home volatility is amplified with financial integration (and thus global volatility everywhere is higher). Third, financial flows collapse following bad news, with greater decreases in flow values when downside risk is greater.

In light of Observation 2 in the introduction regarding co-movement and propagation, our results highlight that financial integration amplifies financial shocks and exports volatility across countries. Thus, in the presence of other shocks (trade, import prices, productivity, etc.), financial shocks become more significant given the higher volatility, and so financial shocks will tend to transmit through financial integration.

The increase in volatility in Foreign is perhaps surprising since, as we saw in the static model, the Foreign asset price actually decreases with financial integration. Since high prices (propped up by leverage) are often symptomatic of financial fragility, one might be inclined to think that the economy is more stable for Foreign following financial integration. After all, prices are not propped up as much and perhaps have less far to fall. However, as we will see, considering this effect alone masks the dynamic consequences.

5.1 Dynamic Model

We consider a dynamic variation of the model from Section 4 with three periods, $t = 0, 1, 2$.

**Time and Assets** The assets in each country pay identical dividends in each state, which come only in $t = 2$. The asset payoffs for $Y$ and $X$ are represented by a tree $S = \{0, U, D, UU, DU, DD\}$ illustrated in Figure 6 (these are also the payoffs for $Y^*$ and $X^*$). Thus, at $t = 1$ investors receive news about the asset payoffs at $t = 2$. In state $U$ the news is good and both assets will pay for sure; in state $D$ the news is bad and there is uncertainty about what the assets will pay. To simplify the
analysis, the news about the asset payoffs is perfectly correlated. In the main examples we set $d^X = d^Y_{UU} = d^Y_{DU} = 1$ (normalizations) and $d^Y_{DD} = 0.2$.

Figure 6: Asset Payoffs with Three Periods.

Agents Agents consume only at the terminal nodes. The von-Neumann-Morgenstern expected utility to agent $i$ is

$$U^i(c_{UU}, c_{DU}, c_{DD}) = ic_{UU} + i(1-i)c_{DU} + (1-i)^2c_{DD},$$

which corresponds to agent $i$ having belief $i$ of receiving good news at $t = 1$, and having belief $i$ that, conditional on receiving bad news, assets will pay high dividends at $t = 2$ (thus, with probability $(1-i)^2$ the economy receives bad news twice and the asset pays the low dividend $d^Y_{DD}$).

All agents in both countries receive endowments of $(e^Y, e^X) = (1, 1)$ both at $t = 0$ and at $t = 1$. All assets, whether endowed at $t = 0$ or $t = 1$ pay identical dividends in $t = 2$ (the endowment of assets at time 1 are a separate vintage of risky assets). We denote the asset prices in states 0 and $D$ by $p_0$, $p_0^*$, $p_D$, and $p_D^*$.

Appendix C considers extensions regarding the nature of uncertainty with partial correlation when (as a simplification) the Foreign asset cannot be leveraged. Our analysis shows that partial correlation reinforces the effects of collateral present in the model with perfect correlation.
Contracts  At $t = 0$ and $t = 1$ agents can trade one-period financial contracts collateralized by $Y$ or $Y^*$ (contract sellers promise to make a payment the following period). Investors can trade non-contingent, one-period debt contracts backed by $Y^*$. Agents can also trade one-period tranche contracts backed by $Y$. As before, without loss of generality, we can restrict our attention to a single tranche traded each period, promising the minimum payoff next period.

Budget sets and equilibrium are analogous to the static model. We first characterize the autarkic equilibrium in Foreign and Home to demonstrate how leverage and tranching affect dynamics, and then we consider the equilibrium with financial integration. For expositional ease, all equations for this section are provided in Appendix D.

5.2 Foreign Autarky: Leverage Cycle

Agents choose portfolios of $Y^*$ and $X^*$ at $s = 0$ and $s = D$, taking into account that in $D$ they can rebalance their portfolio with an asset price $p_D^D$. When agents can use leverage, the dynamic equilibrium is essentially different from the static equilibrium, though the equilibrium regimes in each state resemble the equilibrium regime in the static economy of Section 4.1. Denote the asset price in each state by $p_0^*$ and $p_D^*$. As before, in equilibrium the only contracts traded each period are the max min contracts: at $s = 0$ the equilibrium debt contract promises to pay $p_D^*$ at $t = 1$, and at $s = D$ the equilibrium debt contract promises to pay $d_D^D$ at $t = 2$. These contracts have prices $\hat{p}_D^*$ and $d_D^D$ respectively.

In $s = 0$ there is a marginal buyer $\hat{i}_0^{Y^*}$ such that all investors with $i > \hat{i}_0^{Y^*}$ buy $Y^*$ with leverage promising $p_D$ due at $t = 1$. Crucially, in state $D$ all investors that bought the risky asset with leverage lose their initial investments (the debt they owe is the value of their entire asset holdings) and their only wealth comes from the new endowments. This means that the asset must be bought by more pessimistic investors $i \in [0, \hat{i}_0^{Y^*}]$. In state $D$ there is a marginal buyer $\hat{i}_D^{Y^*}$, such that all remaining investors with $i > \hat{i}_D^{Y^*}$ buy $Y^*$ with leverage by borrowing $d_D^Y$ against each unit of the asset. For the same parameters the equilibrium is given by: $p_0^* = 0.957$, $p_D^* = 0.722$, $\hat{i}_0^{Y^*} = 0.88$, $\hat{i}_D^{Y^*} = 0.65$.

\[30\text{As shown by Geanakoplos [2003], investors endogenously choose one-period contracts even when multi-period contracts are available. The reason is that one-period contracts provide investors the ability to use the most leverage.}\]
The price crashes by 24.55%, greater than would occur if agents could not borrow (which would be roughly 21%). The economy exhibits what Geanakoplos (2003) called the “Leverage Cycle,” in which the use of an asset as collateral to facilitate borrowing creates excess volatility. Specifically, after bad news in \( s = D \) the asset price falls for three reasons. First, fundamentals are worse (a bad payoff is more likely). The marginal buyer \( i_0^{Y^*} \) thinks the expected payoff of \( Y^* \) is 98.5 at 0 and only 90 at \( D \), a drop of 9%. Second, the equilibrium margin increases in state \( D \), and so investors can use less leverage (borrow less) and the marginal buyer is therefore less optimistic. Third, investors who used leverage in the first period have less wealth after bad news as a result of the margin call to repay their debts. These optimistic buyers—the most optimistic buyers—after repaying their debt have no wealth besides their new endowments, and as a result less optimistic investors are required to buy the asset. The change in the marginal buyer from 0.88 to 0.65, together with the change in margins, almost triples the price drop (24%), since the change from fundamentals alone accounts for 9%. Leverage creates excess volatility.\(^{31}\)

### 5.3 Home Autarky: Tranching Cycle

In the dynamic economy with tranching, the equilibrium regimes in each state resemble the equilibrium regime in the static economy of Section 4.2. However, the dynamic equilibrium is essentially different from the static equilibrium because of how the marginal investors will change from \( s = 0 \) to \( s = D \) as a result of wealth distributions among agents. As in the static model, without loss of generality, we can restrict our attention to a single tranche traded each period: at \( s = 0 \) agents can trade a tranche promising \((0, p_0)\) at \( t = 1 \), and at \( s = D \) agents can trade a tranche promising \((0, d^{Y^{DD}})\) at \( t = 2 \).

In equilibrium there are two marginal buyers in each state. In \( s = 0 \), investors \( i > i_0^{Y^*} \) buy the risky asset and issue a down tranche promising \( p_D \) at \( t = 1 \), investors with \( i \in [i_0^{Y^*}, i_0^X] \) hold \( X \), and...\(^{31}\) Compared to the original Geanakoplos (2003) model, the difference between the crashes with and without leverage are smaller in our example because of the interim endowments (the original model only has endowments at \( t = 0 \).) As a result, the wealth effect following bad news is smaller in our model. See also Fostel and Geanakoplos (2014) for an equivalent model with two agents with heterogeneous risk-aversion and endowments. In that model, the marginal buyer is the same but the wealth of the marginal buyer decreases in the bad state, increasing the marginal utility of consumption and causing the investor to discount future payments by more.
investors with $i < i^T_Y$ buy the down tranche, which has a price $\pi^T_Y$. In $s = D$, investors $i > i^Y_Y$ lose all their initial wealth after repaying their debt from issuing tranches, and they use their new wealth to purchase $Y$. There is a marginal buyer $i^Y_D$ such that the new endowments of $Y$ are bought by the investors $i > i^Y_D$, and the investors $i \in [i^T_D, i^Y_D]$ buy all the old $Y$. All investors buying $Y$ issue a down tranche promising $d^Y_D$ at $t = 2$ (note that $d^Y_D < p_D$ and so the value of tranche issuance falls). There is a marginal buyer $i^T_D$ such that all investors $i < i^T_D$ buy the down tranche, which has a price $\pi^T_D$. The remaining investors hold $X$. For the same parameter values we get the following equilibrium $p_0 = 1.131$, $p_D = 0.844$, $i^Y_Y = 0.88$, $i^T_Y = 0.65$, $\pi^Y_0 = 0.508$, $\pi_D = 0.184$, $i^T_0 = 0.24$, $i^T_D = 0.08$. The asset price with tranching starts out significantly higher (1.131 compared to 0.957 in Foreign), due to its higher collateral value (notice a bubble emerges, as was also the case in the static model with tranching).

The price crash is now 41%, much larger than in the leverage economy. The model with Home tranching exhibits an amplified “Leverage Cycle.” Because tranching greatly increases the initial collateral value of the asset, which decreases following bad news, tranching creates excess volatility even compared to when the asset could be used to issue debt.

### 5.4 Financial Integration in the Dynamic Model

We now suppose that Home and Foreign agents can trade assets and tranches, and any agent can issue tranches by using the Home asset $Y$ as collateral, and any agent can issue debt by using the Foreign asset $Y^*$ as collateral. In equilibrium in period 0 there are three marginal buyers: $\hat{i}^M_0$, $\hat{i}^Y_0$, and $\hat{i}^T_0$ in both countries. The most optimistic investors, with $i \geq \hat{i}^M_0$, buy risky assets and borrow to finance their purchases: they buy $Y$, selling the down tranche due at $t = 1$ (which pays $\hat{p}_0$ in $s = D$). The next most optimistic with $i \in (\hat{i}^M_0, \hat{i}^Y_0)$ buy $Y^*$ selling debt due at $t = 1$ (promising $\hat{p}^*_D$). Intermediate buyers with $i \in [\hat{i}^T_0, \hat{i}^Y_0)$ hold risk-free $X$, $X^*$, and debt backed by $Y^*$. The most pessimistic investors, with $i < \hat{i}^T_Y$, buy down tranches.

After bad news, important wealth distributions takes place: the optimists holding $Y$ and $Y^*$ have limited wealth after debt repayment/margin calls, and the pessimists holding tranches have increased wealth. In $D$ there are three marginal buyers, $\hat{i}^M_D$, $\hat{i}^Y_D$, and $\hat{i}^T_D$ in both countries. The
original investors in risky assets use their new endowments to purchase risky assets, but they have no other wealth to use after selling their initial asset holdings in order to repay their debts. The assets sold are purchased by new, but less optimistic, buyers with \( i \in [\bar{i}_Y, \bar{Y}_0] \), which are the most optimistic investors who held safe assets in 0. The most pessimistic investors, with \( i < \bar{Y}_D \), buy down tranches.

Table 2 presents asset prices and crashes both in autarky and with financial integration. Consistent with the static analysis, financial integration increases asset prices at \( s = 0 \): the Home price increases from 1.131 to 1.396 (a substantial bubble), the Foreign price decreases from 0.957 to 0.913, and the tranche price increases from 0.508 to 0.668.

Table 2: Prices and Crashes when Home can Tranche and Foreign can Leverage.

<table>
<thead>
<tr>
<th>Time-0</th>
<th>State-D</th>
<th>Crashes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aut.</td>
<td>Fin Int</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>1.131</td>
<td>1.396↑</td>
</tr>
<tr>
<td>( p^*_0 )</td>
<td>0.957</td>
<td>0.913↓</td>
</tr>
<tr>
<td>( \pi_0 )</td>
<td>0.508</td>
<td>0.668↑</td>
</tr>
</tbody>
</table>

Financial integration with tranching increases the price crashes for both Home and Foreign: the Home price crash increases from 41.04% to 43.41%, and the Foreign crash increases from 24.55% to 25.60%. The “Tranching Cycle” mechanisms amplifying volatility in the Home autarkic equilibrium affect both Home and Foreign asset prices with financial integration. First, because the Foreign asset is priced relative to the Home asset price, the excess volatility of the Home asset transfers to the Foreign asset. Second, financial integration also amplifies volatility at Home because financial integration has increased the value of tranching, which increases the collateral value of the Home asset and therefore increases the price volatility of the Home asset. Because the Foreign asset is priced relative to the Home asset, the Foreign asset is much more volatile as well. Thus, even though in the static model the Foreign asset price decreases with financial

\[32\text{Gong and Phelan (2016b)}\] derive a similar result in a closed-economy setting by studying how equilibrium changes when debt contracts can be used as collateral to make financial promises (“debt collateralization”). They show that debt collateralization increases the collateral value of the risky asset and increases the volatility of asset prices.

\[33\text{To isolate the effect of the increase in the collateral value, in Appendix A.5 we consider when countries can trade only in tranches but not in the risky assets. Because there is no trade in } Y^*, \text{ the volatility of the Foreign asset does not}\]
integration, which might suggest that the economy is more stable, the Foreign price following bad news is even lower with financial integration compared to autarky because of the additional amplifying mechanisms absorbed through financial integration. This is because the large price crash in the volatile Home asset also affects the price crash in the Foreign assets, which is priced relative to the Home asset.

Furthermore, gross and net flows decrease following bad news. Table 3 shows flows in $Y$, $Y^*$, and goods $X^*$ at $s = 0$ and following bad news at $s = D$. The value of Foreign purchases of Home assets decreases by 36.8% and the value of Home purchases of Foreign assets decreases by 32.5%, and net flows in $X^*$ decrease by over 56%. These results are consistent with intuition from the comparative statics varying the down payoff. In the initial period, the subsequent down payoffs next period in $t = 1$ (prices of 0.79 and 0.679) are not nearly as severe as the possible down payoffs in $t = 2$ having received bad news at $t = 1$ ($d_{DD}^Y = 0.2$). Accordingly, the debt capacity of the Home asset dramatically decreases after bad news: the tranche price decreases from 0.668 at $s = 0$ to 0.191 at $s = D$, implying investors can borrow less than one-third as much. Thus, as the possible down payoff next period decreases from $t = 0$ to $t = 1$, flows decrease, which is consistent with the comparative static of varying $d_D^Y$.

Table 3: Dynamic Global Flows with Financial Integration when Home can Tranche and Foreign can Leverage.

<table>
<thead>
<tr>
<th></th>
<th>$s = 0$</th>
<th>$s = D$</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign purchases of $Y$</td>
<td>0.62</td>
<td>0.39</td>
<td>36.81%</td>
</tr>
<tr>
<td>Home purchases of $Y^*$</td>
<td>0.51</td>
<td>0.34</td>
<td>32.52%</td>
</tr>
<tr>
<td>Home purchases of $X^*$</td>
<td>0.11</td>
<td>0.05</td>
<td>56.27%</td>
</tr>
</tbody>
</table>

**Comparative Dynamics**

Our results are robust to varying the degree of downside risk. We provide comparative dynamics by varying the down payoff $d_{DD}^Y$ and consider the price crashes and collapse in flows. First, the collapse in flows is greatest when downside risk is larger (lower $d_{DD}^Y$). Figure 7 plots the ratio increase relative to autarky. However, the volatility of the Home asset increases because of dynamic trade in tranches.
of flows at $s = D$ to the value at $s = 0$. Flows collapse dramatically after bad news, with greater collapse the greater the severity (lower $d_{DD}^H$).

Figure 7: Ratio of flows following bad news to initial flows. Sensitivity of Flows to Downside Risk when Home can Tranche and Foreign can Leverage.

Second, price crashes are always bigger with financial integration regardless of the crash risk. Figure 8 plots Home and Foreign price crashes in autarky and with financial integration. Crashes in both countries are always larger with financial integration. Furthermore, our results continue to hold when news about the Home and Foreign asset are only partially correlated, which would provide diversification motives for trade. Indeed, the results in Appendix C.3 show that diversification mechanisms reinforce collateral-based mechanisms rather than undoing them. Collateral-driven trades continue to amplify volatility, and the effects with partial correlation can be even larger.

6 Conclusion

We presented a two-country general equilibrium model with collateralized lending and tranching in which global capital flows are driven by different levels of financial innovation across countries. We define financial innovation as the use of new assets as collateral or new kinds of financial promises that can be issued by collateral. Financial integration provides Foreign access to attractive Home financial assets, and cross-border flows arise in both directions as a result of general
equilibrium changes in the prices of currently available assets. These flows arise as a way for
countries to share scarce collateral and to trade contingent claims, including safe and negative beta
assets. Differences in the ability to use collateral are enough to generate global financial flows.

Our results imply that collateral-driven flows increase asset price volatility globally and lead to
collapses in flows following bad news about fundamentals. Financial integration leads to portfolio
rebalancing and cross-country asset purchases, especially among investors with the highest demand
for leverage. The resulting flows have important consequences for financial stability, exporting
volatility abroad and amplifying volatility globally. Thus, our results can explain flows among
similarly developed countries that increase volatility rather than dampen shocks.

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Appendices for Online Publication

A Static Model of Global Flows when Foreign cannot Leverage

In this section we consider that the Foreign financial sector has a low level of development so that the risky asset $Y^*$ cannot be used as collateral at all, i.e., $J^* = \emptyset$. As a result, global flows can arise when Home can leverage the asset but Foreign cannot. We first consider when the Home financial sector can issue non-contingent promises using the asset as collateral. In this case $J = \{ j : j =}$
where \( ((j, j), 1_Y) \). We later suppose that the Home asset can be used to issue tranches. In this case \( J = j^T = ((0, d^1_D), 1_Y) \).

In what follows, we first describe the autarkic equilibrium in Foreign (we have already described the autarkic equilibria with leverage and tranching) and then describe the equilibria with financial integration.

The results of this analysis provide implications for trade among developed and developing countries. These environments both produce patterns of financial flows consistent with those described in our benchmark model. However, this analysis provides slightly different results with regards to asset pricing implications. Most notably, when the Foreign asset cannot be used as collateral, financial integration increases the price of the Foreign asset. Additionally, when the Home asset can be leveraged but not tranched, financial integration can decrease the Home asset price (though not always). This occurs because the alternative investment for the Home asset is more attractive than it was in autarky. Furthermore, we also consider when Home can tranche assets and countries can trade tranches but not the risky assets. This is a natural case to consider because in many cases the underlying collateral cannot be traded internationally, as is the case for houses.

### A.1 Foreign Autarky: No Leverage

In autarky, Foreign agents can only trade assets \( Y^* \) and \( X^* \); they cannot borrow using the assets as collateral. Given the strict monotonicity and continuity of \( \gamma(i) \) in \( i \), linear utilities, and the connectedness of the set of agents \((0, 1)\), in equilibrium at state \( s = 0 \) there is a unique marginal buyer, \( i^{Y^*} \), who is indifferent between holding \( Y^* \) and \( X^* \). Agents \( i > i^{Y^*} \) sell their endowment of \( X^* \) to buy \( Y^* \). Agents \( i < i^{Y^*} \) sell all their endowment of \( Y^* \) to buy \( X^* \). Equilibrium is described by a system of two equations in two unknowns: the price of the asset, \( p^* \), and the marginal buyer, \( i^{Y^*} \):

\[
1 = (1 - i^{Y^*}) \left(1 + \frac{p^*}{p^*}\right), \tag{20}
\]

\[
p^* = \gamma(i^{Y^*}) 1 + (1 - \gamma(i^{Y^*})) d_Y. \tag{21}
\]
Equation (20) is market clearing for the risky asset; agents have $1 + p^*$ in wealth, with which they can buy $\frac{1 + p^*}{p^*}$ risky assets, and then there is a mass of $1 - i^{Y^*}$ agents who wish to do so. Equation (21) states that the marginal buyer is indifferent between buying the risky asset and holding cash. For the given parameters, equilibrium is given by $i^{Y^*} = 0.54$ and $p^* = 0.83$.

### A.2 Home Autarky: Leverage

Home agents can borrow by issuing non-contingent promises using the risky asset as collateral. Equilibrium is just as defined in Section 4.1. Having described the equilibrium with no borrowing, it is now easier to see how leverage affects equilibrium. Equation (3) differs from equation (20) because Home optimists can borrow $d^Y_D$, paying only the downpayment $p - d^Y_D$, implying that equilibrium requires a fewer number of optimists to buy all the asset in the economy. Hence, the marginal buyer in the Home economy will be more optimistic than the marginal buyer in the Foreign economy. Both the marginal buyer and the asset price are higher at Home than at Foreign in autarky because $Y$ has a collateral value.\(^\text{34}\)

### A.3 Financial Integration with Home Leverage and Foreign No Leverage

With financial integration, Home and Foreign agents can trade assets and bonds, and any agent can use the Home asset $Y$ as collateral in order to borrow. In equilibrium, in each country there are two common marginal investors $i^Y > i^{Y^*}$. The marginal investors across countries are the same because the assets have identical payoffs and agents have the same endowments and preferences. Investors $i \geq i^Y$ in both countries buy $Y$ with leverage (hence holding the Arrow $U$ security); investors $i \in (i^{Y^*}, i^Y)$ in both countries buy $Y^*$ with cash; and the remaining investors hold $X$ and $X^*$, and risk-free debt. Figure 9 shows the equilibrium regime.

Given the equilibrium regime, our model predicts the following global financial flows. First, optimistic Foreign investors buy Home assets using leverage (capital flows to Home and assets flow to Foreign). Second, moderate Home investors buy Foreign assets (capital flows to Foreign

and assets flow to Home). Third, the difference in the value of flows in risky assets is financed by
flows in goods $X, X^*$; as we discuss below, Home will always buy $X^*$ from Foreign.

Equilibrium is described by a system of four equations in four unknowns: the prices of the
assets, $\hat{p}^*$ and $\hat{p}$, and the marginal buyers, $\hat{i}^*$ and $\hat{i}$. The marginal buyer $\hat{i}$ is indifferent between
the leveraged return on $Y$ and the un-leveraged return $Y^*$:

$$
\frac{\gamma(\hat{i}) (1 - d^Y_D)}{\hat{p} - d^Y_D} = \frac{\gamma(\hat{i}) + (1 - \gamma(\hat{i})) d^Y_D}{\hat{p}^*}.
$$

(22)

The marginal buyer $\hat{i}^*$ is indifferent between the un-leveraged return on $Y^*$ and holding the risk-
free asset (and bond):

$$
\hat{p}^* = \gamma(\hat{i}^*) 1 + (1 - \gamma(\hat{i}^*)) d^Y_D.
$$

(23)

Market clearing conditions for $Y$ and $Y^*$ are given by

$$
1 = (1 - \hat{i}) \frac{(2 + \hat{p} + \hat{p}^*)}{\hat{p} - d^Y_D},
$$

(24)

$$
1 = (\hat{i}^* - \hat{i}^**) \frac{(2 + \hat{p} + \hat{p}^*)}{\hat{p}^*}.
$$

(25)
Compared to (3), the market clearing condition (24) includes wealth from both countries. Therefore, the marginal buyer $\hat{b}_i$ increases as a result of trade. Compared to equation (30), in equation (22) the alternative return to leveraging $Y$ is not a risk-free return but rather the unleveraged return on $Y^*$. For investor $\hat{b}_i$ the unleveraged return on $Y^*$ is greater than 1—it is a more attractive alternative than cash. This available opportunity will tend to increase the required return to leveraging $Y$, which will decrease the price $\hat{p}$ compared to its autarkic level, even though the marginal investor will increase.

Table 4 shows the equilibrium prices and Figure 10 illustrates the effects of financial integration on marginal investors and financial flows. The Foreign asset price increases from 0.834 to 0.864, while the Home asset price decreases slightly from 0.893 to 0.886. We first analyze finance flows and then discuss the forces affecting asset prices.

| Table 4: Equilibrium Prices when Home can Leverage, Autarky and Financial Integration. |
|---------------------------------|----------|---------------|
|                                  | Autarky  | Integration (°) |
| Home Asset Price: $p$            | 0.893    | 0.886 ↓       |
| Foreign Asset Price: $p^*$       | 0.834    | 0.864 ↑       |
| Debt Price: $\pi^{0.2}$          | 0.2      | 0.2           |

**Flows** Our model provides precise predictions on the direction of capital flows and their effect on asset prices. First, optimistic Foreign investors buy 0.44 worth of Home assets (capital flows to Home and assets flow to Foreign). Second, moderate Home investors buy 0.43 worth of Foreign assets (capital flows to Foreign). Third, Home investors buy 0.01 worth of safe assets $X^*$ from Foreign. Finally, pessimistic Foreign investors hold 0.09 worth of risk-free debt beyond their  

\[ \int^{\hat{b}_i} (1 + \hat{p}) - d_B^i \, di + \int^{\hat{b}_i} (1 + \hat{b}_i^*) - d_B^i \, di = (1 - \hat{b}_i) \left( \frac{2 + \hat{p} + \hat{b}_i^*}{\hat{b}_i - d_B^i} \right). \]
initial endowment, where 0.10 is debt issued by Foreign optimists, and 0.01 is sold to Home.\footnote{Notice that Foreign wealth has increased \((1 + p^* < 1 + \hat{p}^*)\) and more investors hold risk-free assets \((\hat{p}^* < \bar{p}^*)\), implying a greater demand for risk-free assets. Since the supply of \(X^*\) has not changed, the demand is being met by debt backed by the Home asset.}

Since the “safe assets” \(X\) and \(X^*\) can be interpreted as durable goods (our preferred interpretation), we interpret Home’s net purchase of \(X^*\) as a net trade/current account deficit.

\[\hat{Y} = 0.82\]

\[\hat{Y}^* = 0.59\]

\[i = 1\]

\[\hat{Y} = 0.63\]

\[\hat{Y}^* = 0.59\]

\[i = 0\]

Figure 10: Equilibrium flows with Financial Integration when Home can Leverage.

Home runs a current account deficit, and flows are generally greater when the borrowing capacity of the Home asset increases. Figure 11 plots gross and net flows as a function of the down payoff. Gross flows increase as \(d^Y_D\) increases.\footnote{Since varying the down payoff mechanically increases the expected payoff of both assets, we also consider the same comparative static varying \(d^Y_D\) while also varying beliefs so that the expected payoff to each agent remains the same even as the down payoff varies. To do so, we set a base down payoff of \(\bar{R} = 0.2\) and define modified beliefs by \(\tilde{y}(i; d^Y_D) = \frac{\bar{y}(i) - d^Y_D}{1 - d^Y_D}\), where \(\bar{y}(i)\) is the expected asset payoff for person \(i\) when \(d^Y_D = \bar{R}\).}

**Asset Prices** Financial integration affects equilibrium prices and portfolio holdings even though assets in both countries have identical payoffs and agents are identical. This is because \(Y\) can serve...
as collateral. Investors that buy $Y$ are able to borrow against their asset purchases and leverage their positions. Optimistic Foreign investors believe that $U$ is very likely: they prefer to buy $Y$ with leverage rather than $Y^*$. At the same time, moderate Home investors are unwilling to buy $Y$ with leverage (because the price is high) and prefer to buy $Y^*$ at a lower price without leverage. Thus, cheap Foreign assets are attractive to Home investors who are otherwise priced out of the Home market by optimistic, leveraged investors.

The effect of financial integration on asset valuation is subtle in this economy. The Foreign asset price increases, while the Home asset price can actually decrease despite increased foreign demand for collateral. To understand these effects better, consider the effect of financial trade on marginal buyers. As a result of financial integration, the marginal buyers of both assets increase. First, moderate Home investors (though more optimistic than the autarkic marginal buyer of $Y^*$) seek to buy the Foreign asset, which increases both the marginal buyer and the asset price. Second, optimistic Foreign investors desire to buy the Home asset with leverage, and this increases the marginal buyer. Investors in both countries achieve the same amount of leverage, but investors in Home, who are endowed with the more expensive asset, buy more units of the asset because they start with more wealth ($\hat{\rho} > \hat{\rho}^*$).

While the marginal buyer of the Home asset increases, the effect on the price of the Home asset
is complicated by the fact that the alternative investment (the Foreign asset) is more attractive than the risk-free return. Leveraged investors are now comparing the leveraged return to a (very attractive to them) un-leveraged return in the Foreign asset. Thus, even though the marginal investor is more optimistic, the required return is higher and so the asset price decreases. There are two opposing forces: greater optimism and a more attractive investment alternative. The overall effect on the Home price is ambiguous. The “alternative return” force tends to dominate when the function $\gamma(i)$ is concave. The Home price can increase if $\gamma(i)$ is sufficiently convex. (See Appendix ?? for parameter robustness.) After financial integration there is still a gap in prices between Home and Foreign assets due to the Home asset role as collateral. However, this gap is smaller than the one observed in autarky because the required return has increased.

### A.3.1 Parameter Robustness

Since the static model when Foreign cannot leverage provides some ambiguous implications for asset prices, we provide numerical results to illustrate when each of the described effects tends to dominate. We provide comparative statics by varying the shape of the beliefs function $\gamma(i)$ and the down payoff $d^Y_D$. We parametrize beliefs by $\gamma(i) = i^\zeta$, where $\zeta$ determines the relative frequency of optimism/pessimism, which is equivalent to the relative demand for positive/negative-beta assets. When $\zeta > 1$ ($\gamma(i)$ convex), the economy features high demand for negative-beta assets (many pessimists), and the reverse is true when $\zeta < 1$. We consider $\zeta \in [.35, 5]$ and vary $d^Y_D \in [.01, .75]$.

Figure [12] shows the percent changes in asset prices from financial integration with leverage for Home and Foreign asset prices given parameters $(\zeta, d^Y_D)$. As noted, financial integration always increases the Foreign asset price, with the largest increases ($\sim 12\%$) occurring for high demand for negative-beta assets (high $\zeta$) and moderate downside risk ($d^Y_D$ around .4). Financial integration increases the Home asset price when $\zeta$ is sufficiently high (the economy features high demand for negative-beta assets). However, price increases are at best very modest.
A.4 Financial Integration with Home Tranching and Foreign No Leverage

We now suppose that the Home financial sector can create tranches using the risky asset as collateral. We suppose that Home and Foreign agents can trade assets and tranches, and any agent can issue tranches by using the Home asset $Y$ as collateral. In equilibrium in each country there are three marginal buyers: $\hat{i}^Y$, who is indifferent between buying $Y$ against a tranche and buying $Y^*$ with cash, $\hat{i}^{Y^*}$, who is indifferent between buying $Y^*$ with cash and holding risk-free assets, and $\hat{i}^T$ who is indifferent between buying the down tranche and holding risk-free assets. Investors in both countries with $i \geq \hat{i}^Y$ buy $Y$ and issue the down tranche using the asset as collateral (hence holding the Arrow $U$ security). Investors $i \in (\hat{i}^{Y^*}, \hat{i}^Y)$ buy $Y^*$ with cash. Investors with $i \in (\hat{i}^T, \hat{i}^{Y^*})$ hold the risk-free assets $X$ and $X^*$. Finally, investors $i \leq \hat{i}^T$ buy the down tranche (hence holding the Arrow $D$). Figure 13 illustrates the equilibrium regime.

Given the equilibrium regime, our model predicts the following global financial flows. First, optimistic Foreign investors buy Home assets by issuing tranches (capital flows to Home and assets flow to Foreign). Second, moderate Home investors buy Foreign assets (capital flows to Foreign and assets flow to Home). Third, Home investors buy $X^*$ from Foreign. Fourth, pessimistic Foreign investors buy tranches, which as we discuss below, can be interpreted as being issued by Foreign optimists holding $Y$ as collateral.
Equilibrium is described by a system of six equations in six unknowns: the prices of the assets and the tranche, \( \hat{\rho}, \hat{\rho}^*, \) and \( \hat{\pi}^T, \) and the marginal buyers, \( \hat{i}^Y, \hat{i}^{Y*}, \) and \( \hat{i}^T. \) The equations for the marginal valuation and market clearing for the Foreign asset and for the tranche are given by equations (23), (25), (8), and (26).

\[
\hat{i}^T (2 + \hat{\rho} + \hat{\rho}^*) = \hat{\pi}^T \tag{26}
\]

The marginal buyer \( \hat{i}^Y \) is indifferent between the leveraged return on \( Y \) (by issuing a down tranche), and the un-leveraged return \( Y^*:\)

\[
\frac{\gamma(\hat{i}^Y) \times 1}{\hat{\rho} - \hat{\pi}^T} = \frac{\gamma(\hat{i}^Y) + (1 - \gamma(\hat{i}^Y))d_Y^T}{\hat{\rho}^*} \tag{27}
\]

The market clearing condition for the Home asset is

\[
1 = (1 - \hat{i}^Y) \frac{(2 + \hat{\rho} + \hat{\rho}^*)}{\hat{\rho} - \hat{\pi}^T}. \tag{28}
\]
Table 5 presents the equilibrium prices and Figure 14 illustrates the effects of financial integration on marginal investors and financial flows. With tranching, all asset prices increase: the Home asset price increases from 1 to 1.021, the Foreign asset price increases from 0.834 to 0.848, and, importantly, the tranche price increases from 0.168 to 0.182. We analyzing flows and then discuss the asset-pricing forces.

Table 5: Equilibrium Prices when Home can Tranche, Autarky and Financial Integration.

<table>
<thead>
<tr>
<th></th>
<th>Autarky</th>
<th>Integration (↑)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Asset Price: ( p )</td>
<td>1</td>
<td>1.021</td>
</tr>
<tr>
<td>Foreign Asset Price: ( p^* )</td>
<td>0.834</td>
<td>0.848</td>
</tr>
<tr>
<td>Down Tranche Price: ( \pi^T )</td>
<td>0.168</td>
<td>0.182</td>
</tr>
</tbody>
</table>

**Flows** As before, our model provides precise predictions for the direction of capital flows. First, optimistic Foreign investors buy 0.49 worth of Home assets and sell 0.09 worth of the down tranche (capital flows to Home and assets flow to Foreign). Second, moderate Home investors buy 0.44 worth of Foreign assets (capital flows to Foreign) and 0.05 worth of the safe Foreign asset. Third, pessimistic Foreign investors buy 0.09 worth of tranches backed by Home assets. As we did in the model with leverage, we can interpret the Foreign tranche holdings as being issued directly by Foreign optimists (the quantity held of tranches is exactly the quantity of \( Y \) held by Foreign investors). However, given the way that financial sectors create tranches from collateral (e.g., mortgage-backed securities, asset-backed securities, collateralized-loan obligations), one might want to consider the tranche flows (tranches sold by optimists and tranches bought by pessimists) as going through Home, thus generating additional gross flows.

The static model with tranching thus extends the results about global flows we saw in the model with leverage. As was the case with leverage, our model with tranching implies that gross and net flows increase as the Home collateral value increases. Notice that with tranching, the net flows in goods \( X \) are much larger than was true with only leverage. This is because tranching greatly increases the Home collateral value, and so Home can finance an even larger deficit through asset
Figure 14: Equilibrium Flows with Financial Integration when Home can Tranche.

sales. Furthermore, Figure 15 shows that gross flows in risky assets \( Y \) and \( Y^* \) and net flows in goods \( X \) all increase as \( d_{Y_D} \) increases.\(^{38}\)

**Asset Prices**  With tranching, both risky asset prices increase after financial integration. Consider again the effect of financial trade on marginal buyers. As before, the marginal buyers of both risky assets increase, and the marginal buyer of the tranche decreases, which increases the price of the tranche and increases the collateral value of the Home asset. Notice that the asset price is given by
\[
\hat{p} = \gamma(\hat{p}^T)1 + (1 - \gamma(\hat{p}^T))d_{Y_D}^T,
\]
which we get from combining equations (7) and (8). Foreign demand for the asset increases \( \hat{p}^T \), and foreign demand for the tranche decreases \( \hat{p}^T \). The increase in the collateral value of the Home asset due to financial integration outweighs the “alternative investment return” effect we discussed before. As a result of financial integration, pessimistic Foreign investors demand down tranches, which pushes up their price (with leverage, the demand

\(^{38}\)As we describe in greater detail just below, with tranching, financial integration increases the Home asset price because the increase in the collateral value (from selling tranches to Foreign investors) outweighs the alternative return effect. Thus, the price gap between Home and Foreign assets following financial integration is larger the higher is \( d_{Y_D}^T \), even holding beliefs fixed.
Figure 15: Gross Flows, price gap, and downside risk, correcting for optimism, with Home Tranching. Dashed lines update beliefs as $d^T_H$ increases to maintain same expected payoff.

for risk-free assets did not change the risk-free price). Now, more-expensive tranches increase the value of the Home asset. In other words, financial integration increases the value of the promise backed by collateral when the promise is contingent, which is otherwise not available to Foreign agents in autarky.

A.4.1 Parameter Robustness

Figure 16 shows the percent changes in asset prices from financial integration with tranching. In this case, the increase in the Home asset price following financial integration is largest when $d^T_H$ is largest, which is when the ability to tranche the Home asset is most valuable (there is more of a down tranche to sell).

A.5 Financial Integration with Trade in Tranches but Not Assets

To illustrate the separate role played by trade in risky assets and in tranches, we now suppose that countries can trade financial promises backed by collateral but not the asset itself. In this case, only Home investors can buy the Home asset, but there is Foreign demand for the tranches backed by the asset, and so there will be trade in tranches. There will be trade in the down tranche, so we
Figure 16: Percent Changes in Asset Prices with Financial Integration when Home can Tranche.

refer to this case as “T-Trade.” This will have no effect on the price of the Foreign asset, which is purchased without leverage, but it will increase the price of the Home asset. The Home asset is bought by issuing a down tranche against the collateral. Trade in the tranche will increase the price of the tranche and thus increase the amount of assets that Home investors can purchase using tranching.

In equilibrium, there is a marginal Home investor for the Home asset, \( i^Y \), and all Home investors with \( i \geq i^Y \) buy \( Y \) by issuing a down tranche in order to borrow. There is a marginal investor for the Foreign asset \( i^{Y*} \) and all Foreign investors with \( i \geq i^{Y*} \) buy \( Y^* \) without leverage. There is a marginal investor \( i^T \) such that all Home and Foreign investors with \( i \leq i^T \) buy the down tranche. Intermediate (moderate) Home investors with \( i \in (i^T, i^Y) \) and Foreign investors \( i \in (i^T, i^{Y*}) \) hold the risk-free assets \( X, X^* \). Optimists buy assets, moderates hold debt, pessimists hold tranches.

Because there is no trade in assets, the marginal valuations and market clearing conditions for \( Y, Y^* \) are as they were in the respective situations in autarky, i.e., given by equations (20), (21), (5), and (7). The conditions for the tranche are given by equations (8) and (26). The static equilibrium with \( \gamma(i) = 1 - (1 - i)^2 \) and \( d_B^I = 0.2 \) is summarized in Table 6.
Table 6: Static Equilibrium prices with trade in tranches but not assets.

<table>
<thead>
<tr>
<th></th>
<th>Autarky</th>
<th>T-Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>1</td>
<td>1.011</td>
</tr>
<tr>
<td>$p^*$</td>
<td>0.834</td>
<td>0.83</td>
</tr>
<tr>
<td>$\pi^T$</td>
<td>0.168</td>
<td>0.182</td>
</tr>
</tbody>
</table>

B Static Model of Global Flows with Foreign Leverage Limits

We now consider when the Foreign financial sector can leverage $Y^*$, but promises are limited to some maximum promise $j^{\text{max}} < d_Y^D$. Foreign promises may be limited to some $j^{\text{max}} < d_Y^D$ because of agency problems, information issues, or regulation. In this case $J^* = \{ j \in [0, j^{\text{max}}] : j = ((j, j), 1_Y) \}$. That is, investors can promise to repay $j \in [0, j^{\text{max}}]$, collateralized by the risky asset. As before, leverage is endogenously determined in equilibrium: although all contracts $j \in [0, \bar{j}]$ will be priced in equilibrium, the only contract actively traded is $j^{\text{max}}$, which is still risk-free. Thus, it has equilibrium price $\pi^* = j^{\text{max}}$.

B.1 Foreign Autarky: Leverage Limits

As was the case without leverage limits (see Section 4.1), in equilibrium there is a marginal buyer $i^{Y^*}$, who is indifferent between holding $X^*$ and buying $Y^*$ with leverage promising $j^{\text{max}}$. All agents $i > i^{Y^*}$ buy $Y^*$ with leverage; however, since $j^{\text{max}}$ is strictly less than $d_Y^D$, these agents will consume $d_Y^D - j^{\text{max}} > 0$ in state $D$, and so their consumption profile will differ from an Arrow $U$. Agents $i < i^{Y^*}$ sell their endowment of $Y^*$ and lend to the more optimistic investors, holding $X^*$ and risk-free financial promises. Equilibrium is described by a system of two equations in two unknowns:

$$1 = \frac{(1 - i^{Y^*})(1 + p^*)}{p^* - j^{\text{max}}}, \quad (29)$$

$$p^* = \gamma(i^{Y^*}) 1 + (1 - \gamma(i^{Y^*}))d_Y^D. \quad (30)$$
Equation (29) is market clearing, which now reflects that agents borrow $j_{\text{max}}$. The top $1 - \bar{i_Y}$ agents are buying the asset and issuing debt to finance their purchases, thus borrowing $d_Y$ and paying only the downpayment $p^* - d_Y$. As a result, borrowing allows a relatively small fraction of agents to buy all the asset in the economy—certainly fewer than would be required if no borrowing were allowed. Equation (30) states that the asset is priced according to the marginal buyer. Notice that equation (30) can also be written as

$$1 = \frac{\gamma(p^*)(1-j_{\text{max}})+(1-\gamma(p^*))(d_Y-j_{\text{max}})}{p^*-d_Y}$$

so that the leveraged return equals 1 for the marginal buyer.

### B.2 Financial Integration with Foreign Leverage Limits

We now consider financial integration. Since $j_{\text{max}} < d_Y$, in equilibrium, in each country there are three common marginal investors: $\bar{i_Y}^*$, who is indifferent between buying $Y$ against a tranche and buying $Y^*$ with leverage promising $j_{\text{max}}$; $\bar{i_Y}^{**}$, who is indifferent between buying $Y^*$ with leverage and holding risk-free assets; and $\bar{i_T}$, who is indifferent between buying the down tranche and holding risk-free assets. Accordingly, investors in both countries with $i \geq \bar{i_Y}$ buy $Y$ and issue the down tranche using the asset as collateral (hence holding an Arrow U security). Investors $i \in (\bar{i_Y}^{**}, \bar{i_Y})$ buy $Y^*$ with leverage promising $j_{\text{max}}$ (hence holding Arrow U and Arrow D securities). Investors with $i \in (\bar{i_T}, \bar{i_Y}^{**})$ hold the risk-free assets $X$ and $X^*$, as well as debt issued by holders of $Y^*$. Finally, investors $i \leq \bar{i_T}$ buy the down tranche (hence holding an Arrow D).

Given the equilibrium regime, our model predicts the following global financial flows. First, optimistic Foreign investors buy Home assets by issuing tranches (capital flows to Home and assets flow to Foreign). Second, moderate Home investors buy Foreign assets with leverage (capital flows to Foreign and assets flow to Home). Third, the difference in the value of flows in risky assets is financed by flows in goods $X, X^*$; as we discuss below, Home will always buy $X^*$ from Foreign. Fourth, pessimistic Foreign investors buy tranches, which as we discuss below, can be interpreted as being issued by Foreign optimists holding $Y$ as collateral.

The equilibrium equations are as in Appendix A.4, except that now market clearing and marginal valuations reflect that $Y^*$ can be used to promise $j_{\text{max}}$. In particular, the marginal buyer $\bar{i_Y}$ is indifferent between the return on $Y$ (by issuing a down tranche), and the leveraged return $Y^*$ promising
\[ \frac{\gamma(\tilde{r}^Y) \times 1}{\tilde{p} - \tilde{p}^T} = \frac{\gamma(\tilde{r}^Y)(1 - j^{\max}) + (1 - \gamma(\tilde{r}^Y)(d_D^Y - j^{\max}))}{\tilde{p}^* - j^{\max}}. \] 

(31)

Notice that as \( j^{\max} \to d_Y^D \), this equation converges to \( \frac{\gamma(\tilde{r}^Y) \times 1}{\tilde{p} - \tilde{p}^T} = \frac{\gamma(\tilde{r}^Y)(1 - d_Y^D)}{\tilde{p}^* - d_Y^D} \), which was the return condition in equation (9) from Section 4. In this way, the model with Foreign leverage limits converges to the baseline model as \( j^{\max} \to d_Y^D \), thus supporting our portfolio allocations in the baseline model. Furthermore, as \( j^{\max} \to 0 \), the model converges to the model with no leverage in Foreign. Thus, it is instructive to solve the model with financial integration varying \( j^{\max} \). Figure 17 plots prices and flows varying \( j^{\max} \in (0, d_Y^D) \) for the baseline parameters.

There are three main conclusions from Figure 17. First, gross flows increase the looser are Foreign leverage limits (as \( j^{\max} \) increases). Second, financial integration always increases the Home asset price regardless of the Foreign leverage limit, but the increase is greater the higher is the Foreign leverage limit. Third, financial integration increases the Foreign asset price only if Foreign leverage limits are sufficiently tight (i.e., \( j^{\max} \) is low). When \( j^{\max} \) is very low, the Foreign collateral value is low and the value of the Foreign asset is derived almost entirely from fundamental payoffs. In this case, Foreign assets are an attractive alternative to expensive Home assets that have collateral value, and so Home demand for Foreign assets (with no collateral value)
increases their price. Alternatively, when $j_{\text{max}}$ is very large, the Foreign price already includes a collateral value, and financial integration decreases this collateral value as investors prefer the Home asset, which is better collateral.

C Dynamic Models when Foreign Cannot Leverage

In this section we provide the dynamic analysis when the Foreign asset cannot be used as collateral. We consider when Home can leverage and tranche, and we also consider the models with leverage and with tranche when news about the Home and Foreign assets are partially correlated instead of perfectly correlated as in the main text.

C.1 Financial Integration with Home Leverage and Foreign No Leverage

We first consider the case when the Home economy can leverage the risky asset $Y$ to issue non-contingent promises. All equations for this section are provided in Appendix D.

C.1.1 Foreign Autarky: No Leverage

Agents choose portfolios of $Y^*$ and $X^*$ at $s = 0$ and $s = D$, taking into account that in $D$ they can rebalance their portfolio with an asset price $p_D^*$. In both states there is a single marginal agent, $i_Y^*$ and $i_D^* > i_Y^*$. At time 0, all agents $i > i_Y^*$ buy the risky asset, and those with $i > i_D^*$ hold to maturity. The agents with $i \in (i_Y^*, i_D^*)$ buy the risky asset at 0, but in state $D$ they sell the asset to more optimistic agents (who have just received new endowments) and hold safe assets. The remaining agents always hold safe assets. The marginal buyer $i_D^*$ can get 1 util per dollar in state $D$, and so values a dollar at $DU$ and $DD$ at his probabilities $i_D^*$ and $1 - i_D^*$, respectively. Because $i_D^* > i_Y^*$, the marginal buyer $i_Y^*$ at 0 can also get 1 util per dollar in every state, and hence values every security equal to its expected payoff at her probabilities. The asset is thus priced by the marginal agent according to the payoff distribution at each state. For the parameter values considered before, in equilibrium we get $i_Y^* = 0.542$, $i_D^* = 0.575$, $p_0^* = 0.844$, and $p_D^* = 0.66$. The price falls in $D$ because the bad payoff is more likely; however, there are no amplifying
mechanisms. The price crashes by 21.86%.

C.1.2 Home Autarky: Leverage

With leverage, the dynamic equilibrium is the same as described in Section 5.2.

C.1.3 Financial Integration with Home Leverage and Foreign No Leverage

We now suppose that Home and Foreign agents can trade assets and bonds, and any agent can use the Home asset \( Y \) as collateral in order to borrow. As in the static model, some investors in both countries will choose to hold \( Y \) with leverage, while some will choose to hold \( Y^* \) without leverage. In \( s = D \) the amplifying mechanisms from leverage will cause the marginal buyer of \( Y \) to fall as leveraged agents lose wealth and margins increase. Importantly, a decrease in the marginal investor in \( Y \) will decrease the marginal investor for \( Y^* \), thus decreasing \( \hat{p}_D^* \), even though the Foreign asset is not leveraged. The amplification in the volatility of the Foreign asset is entirely driven by global flows.

In period 0 there are the same two marginal buyers \( \hat{i}_Y^0 \) and \( \hat{i}_Y^* \) in both countries. The most optimistic investors, with \( i \geq \hat{i}_Y^0 \), buy \( Y \) with leverage using debt due at \( t = 1 \). Intermediate buyers with \( i \in [\hat{i}_Y^* \hat{i}_Y^0) \) hold \( Y^* \) un-leveraged; some of these investors hold \( Y^* \) to maturity, while some will sell \( Y^* \) to buy \( Y \) in state \( D \). The most pessimistic investors, with \( i < \hat{i}_Y^* \), hold risk-free claims, either \( X \) or risk-free debt. In \( D \) there are two marginal buyers \( \hat{i}_Y^D \) and \( \hat{i}_Y^* \) in both countries. The optimists holding \( Y \) lose wealth because of the margin call, and will not be able to add much more at \( D \) after paying their debt. The new buyers of \( Y \), with \( i \in [\hat{i}_Y^D, \hat{i}_Y^0) \), are the most optimistic investors who held \( Y^* \) in 0., and dump the asset in \( D \) to buy \( Y \). The rest of the original \( Y^* \) investors, \( i \in [\hat{i}_Y^*, \hat{i}_Y^D) \), add more \( Y^* \), and the remaining \( Y^* \) is bought by investors, with \( i \in [\hat{i}_Y^D, \hat{i}_Y^* \) who held risk-free claims at \( s = 0 \).

Figure 18 displays marginal investors across states in autarky and with financial integration. In autarky, the Foreign marginal buyer hardly changes across states—indeed, the marginal buyer increases after bad news—while the Home marginal buyer decreases following bad news. However, with financial integration the Foreign marginal buyer decreases after bad news, because the
Leverage Cycle mechanisms also affect the market for the Foreign asset. Accordingly, the Foreign assets inherits some of the Home amplification mechanisms.

\begin{align*}
  i = 1 & \quad \hat{\rho}_0 = 0.93 \quad \hat{\rho}_D = 0.81 \quad \hat{\rho}_0^* = 0.87 \quad \hat{\rho}_D^* = 0.65 \\
  i = 1 & \quad \hat{\rho}_0 = 0.7 \quad \hat{\rho}_D = 0.47 \quad \hat{\rho}_0^* = 0.55 \quad \hat{\rho}_D^* = 0.57
\end{align*}

Figure 18: Marginal Investors when Home can Leverage, Autarky and Financial Integration.

Table 7 shows the equilibrium asset prices in autarky and with financial integration, where the asset prices in each state are denoted by $\hat{\rho}_0$, $\hat{\rho}_0^*$, $\hat{\rho}_D$, $\hat{\rho}_D^*$. The crash is slightly moderated in the Home country, but it is exacerbated in the Foreign country because the marginal buyer decreases following bad news. The total volatility in autarky is 21.86 + 24.55 = 46.41 and the total volatility with trade is slightly higher, 23.92 + 24.51 = 48.43. Even though the Foreign asset is not leveraged, the price crash is about as large as it is in the domestic asset in autarky. The Leverage Cycle becomes global. As the price of $Y$ fluctuates, the relative attractiveness of investing in $Y^*$ fluctuates as well. Since $Y$ has excess volatility because it is bought with leverage, $Y^*$ has excess volatility.

Notice that the state-0 Home asset price is lower with financial integration than in autarky, as was also true in the static model. However, the state-$D$ Home asset price is higher with financial integration. The relative attractiveness of the Foreign asset is lower in $D$ than it was at 0, or in other words, collateral becomes more valuable when margins become tougher.
Table 7: Prices and Crashes when Home can Leverage, Autarky and Financial Integration.

<table>
<thead>
<tr>
<th></th>
<th>Autarky</th>
<th>Integration (\textsuperscript{*})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$</td>
<td>0.957</td>
<td>0.957 $\downarrow$</td>
</tr>
<tr>
<td>$p_0^*$</td>
<td>0.844</td>
<td>0.900 $\uparrow$</td>
</tr>
<tr>
<td>$p_D$</td>
<td>0.722</td>
<td>0.723 $\uparrow$</td>
</tr>
<tr>
<td>$p_D^*$</td>
<td>0.660</td>
<td>0.685 $\uparrow$</td>
</tr>
<tr>
<td>$Y$ crash</td>
<td>24.55%</td>
<td>24.51% $\downarrow$</td>
</tr>
<tr>
<td>$Y^*$ crash</td>
<td>21.86%</td>
<td>23.92% $\uparrow$</td>
</tr>
<tr>
<td>total</td>
<td>46.41%</td>
<td>48.43% $\uparrow$</td>
</tr>
</tbody>
</table>

In addition to amplifying volatility, our model predicts that gross and net flows decrease after bad news. Table 8 lists the value of asset purchases by each country in each state. Flows in both assets decrease after bad news by roughly 24\%, a change that can be attributed almost entirely to the price decrease (the crash is nearly 24\%).

Table 8: Dynamic Global Flows with Financial Integration when Home can Leverage.

<table>
<thead>
<tr>
<th></th>
<th>$s = 0$</th>
<th>$s = D$</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign purchases of $Y$</td>
<td>0.47</td>
<td>0.36 $\downarrow$</td>
<td>24.15%</td>
</tr>
<tr>
<td>Home purchases of $Y^*$</td>
<td>0.46</td>
<td>0.35 $\downarrow$</td>
<td>24.20%</td>
</tr>
</tbody>
</table>

C.1.4 Comparative Dynamics with Downside Crash Risk

We provide comparative dynamics by varying the down payoff $d_{DD}$ and consider the price crashes and changes in financial flows, as shown in Figure 19. When $d_{DD}$ is low, crash risk is severe following bad news. The left frame plots the changes in global volatility by plotting the Home and Foreign crashes. The right frame plots the ratio of the asset flows following bad news to the initial flows. When the ratios are less than one, gross flows decrease after bad news.

All crashes are larger when crash risk is more severe, but Home volatility is essentially the same in autarky and with financial integration. However, Foreign volatility with financial integration exceeds volatility in autarky when $d_{DD}$ is low. Notice that for low $d_{DD}$ (severe crash risk), the
autarkic percent crash is larger with leverage than without. This is because when $d_{DD}^{Y}$ is high, leverage actually cushions the crash after bad news because, even after bad news, investors can borrow money to buy the asset, which dampens the amplifying mechanisms. But when $d_{DD}^{Y}$ is low, the amplifying mechanisms from leverage thus increase volatility for Foreign. As a result, global volatility is higher when crash risk is severe. Furthermore, the collapse in financial flows is also greater when crash risk is greater. Furthermore, net flows in $X$ can decrease substantially following bad news in the dynamic model, with greater decreases the greater the crash risk. Figure 20 plots the percent decrease in net flows as a function of $d_{DD}^{Y}$ for both leverage and tranching. Percent decreases are largest with tranching, and in both cases percent decreases in flows are decreasing in $d_{DD}^{Y}$. With leverage, if $d_{DD}^{Y}$ is large, net flows can actually increase following bad news. Recalling the comparative statics results, this is because with leverage the price gap $\Delta$ is hump-shaped in the down payoff.

C.2 Financial Integration with Home Tranching and Foreign No Leverage

We now suppose that Home and Foreign agents can trade assets and tranches, and any agent can issue tranches by using the Home asset $Y$ as collateral. In equilibrium in period 0 there are three
marginal buyers \( \hat{i}_Y, \hat{i}_Y^*, \) and \( \hat{i}_T \) in both countries. The most optimistic investors, with \( i \geq \hat{i}_Y \), buy \( Y \) selling the down tranche due at \( t = 1 \). Intermediate buyers with \( i \in [\hat{i}_Y^*, \hat{i}_Y^T] \) hold \( Y^* \) outright. The most pessimistic investors, with \( i < \hat{i}_Y^T \), buy down tranches, and the rest hold risk-free \( X \) and \( X^* \).

After bad news, important wealth distributions takes place: the optimists holding \( Y \) have limited wealth after debt repayment/margin calls, the moderates holding \( Y^* \) have impaired wealth because \( p^* \) decreases, and the pessimists holding tranches have increased wealth. In \( D \) there are three marginal buyers \( \hat{i}_Y, \hat{i}_Y^*, \) and \( \hat{i}_T \) in both countries. The new buyers of \( Y \), with \( i \in [\hat{i}_Y^*, \hat{i}_Y^T] \), are the most optimistic investors who held \( Y^* \) in \( s = 0 \). The rest of the \( Y^* \) investors continue to hold their investment, and add more using their new endowments. The remaining \( Y^* \) is bought by investors, with \( i \in [\hat{i}_Y^*, \hat{i}_Y^T] \) who held risk-free assets at \( s = 0 \). The most pessimistic investors, with \( i < \hat{i}_T \), buy down tranches.

Table 9 presents asset prices and crashes both in autarky and with financial integration. Consistent with the static analysis, financial integration increases asset prices at \( s = 0 \): the Home price increases from 1.131 to 1.394 (a substantial bubble), the Foreign price increases from 0.844 to 0.860, and the tranche price increases from 0.508 to 0.667.
Financial integration with tranching increases the price crashes for both Home and Foreign: the Home price crash increases from 41.04% to 43.23%, and the Foreign crash increases from 21.86% to 25.18%. The Leverage Cycle mechanisms amplifying volatility in the Home autarkic equilibrium affect both Home and Foreign asset prices with financial integration. First, while the Foreign asset does not exhibit excess volatility in autarky, with financial integration the Foreign asset price also exhibits excess volatility. Second, financial integration also amplifies volatility at Home.

The reason for these changes is that financial integration has increased the value of tranching, which increased the collateral value of the Home asset and therefore increased the price volatility of the Home asset. Because the Foreign asset is priced relative to the Home asset, the Foreign asset is much more volatile as well. Thus, even though in the static model the Foreign asset price increases with financial integration, the Foreign price following bad news is lower with financial integration compared to autarky. This is because the large price crash in the volatile Home asset also affects the price crash in the Foreign assets, which is priced relative to the Home asset.

Furthermore, gross and net flows decrease following bad news. Table 10 shows flows in $Y$, $Y^*$, and goods $X^*$ at $s = 0$ and following bad news at $s = D$. The value of Foreign purchases of Home assets decreases by 35% and the value of Home purchases of Foreign assets decreases by 30%, and net flows in $X^*$ decrease by almost 54%. These results are consistent with intuition from the comparative statics varying the down payoff. In the initial period, the subsequent down payoffs

---

39 Gong and Phelan (2016b) derive a similar result in a closed-economy setting by studying how equilibrium changes when debt contracts can be used as collateral to make financial promises (“debt collateralization”). They show that debt collateralization increases the collateral value of the risky asset and increases the volatility of asset prices.

40 To isolate the effect of the increase in the collateral value, in Appendix A.5 we consider when countries can trade only in tranches but not in the risky assets. Because there is no trade in $Y^*$, the volatility of the Foreign asset does not increase relative to autarky. However, the volatility of the Home asset increases because of dynamic trade in tranches.
next period in $t = 1$ (prices of 0.791 and 0.643) are not nearly as severe as the possible down payoffs in $t = 2$ having received bad news at $t = 1$ ($d'_{DD} = 0.2$). Accordingly, the debt capacity of the Home asset dramatically decreases after bad news: the tranche price decreases from 0.667 at $s = 0$ to 0.191 at $s = D$, implying investors can borrow less than one-third as much. Thus, as the possible down payoff next period decreases from $t = 0$ to $t = 1$, flows decrease, which is consistent with the comparative static of varying $d'_{D}$.

Table 10: Dynamic Global Flows with Financial Integration when Home can Tranche.

<table>
<thead>
<tr>
<th></th>
<th>$s = 0$</th>
<th>$s = D$</th>
<th>Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign purchases of $Y$</td>
<td>0.61</td>
<td>0.40</td>
<td>35.16%</td>
</tr>
<tr>
<td>Home purchases of $Y^*$</td>
<td>0.48</td>
<td>0.34</td>
<td>30.53%</td>
</tr>
<tr>
<td>Home purchases of $X^*$</td>
<td>0.13</td>
<td>0.06</td>
<td>53.85%</td>
</tr>
</tbody>
</table>

C.2.1 Comparative Dynamics

Our results are robust to varying the degree of downside risk. We provide comparative dynamics by varying the down payoff $d'_{DD}$ and consider the price crashes and collapse in flows. First, the collapse in flows is greatest when downside risk is larger (lower $d'_{DD}$). Figure 21 plots the ratio of flows at $s = D$ to the value at $s = 0$. Flows collapse dramatically after bad news, with greater collapse the greater the severity (lower $d'_{DD}$).

Second, price crashes are always bigger with financial integration regardless of the crash risk. Figure 22 plots Home and Foreign price crashes in autarky and with financial integration. Crashes in both countries are always larger with financial integration. Furthermore, our results continue to hold when news about the Home and Foreign asset are only partially correlated, which would provide diversification motives for trade. Indeed, the results in Appendix C.3 show that diversification mechanisms reinforce collateral-based mechanisms rather than undoing them. Collateral-driven trades continue to amplify volatility, and the effects with partial correlation can be even larger.
C.2.2 Trade in Tranches but not assets

With trade in tranches but not assets (T-trade), excess volatility is being driven by the change in the marginal buyer of the down security. In state $D$, the tranche gets bid up because pessimists believe the ultimate bad state is more likely and pessimists are wealthy because they bought a down security last period, which just paid off big. The equilibrium allocations are as in the 2-period static economy, but the marginal buyers for $Y$ for the tranche change in state $D$. Because there is no trade in assets, there is no effect on the marginal buyer or the price of the Foreign asset. The marginal buyer of the Home asset changes because of the Leverage Cycle. How much the marginal leveraged investor changes depends on trade because the price of the tranche changes, reflecting foreign demand for the tranche. The dynamic equilibrium is summarized in Table 11.

<table>
<thead>
<tr>
<th>Time-0</th>
<th>Aut.</th>
<th>TY-Trade</th>
<th>TT-Trade</th>
<th>State-D</th>
<th>Aut.</th>
<th>TY-Trade</th>
<th>TT-Trade</th>
<th>Crash</th>
<th>Aut.</th>
<th>TY-Trade</th>
<th>T-Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$</td>
<td>1.131</td>
<td>1.394</td>
<td>1.4</td>
<td>$\rho_D$</td>
<td>0.667</td>
<td>0.791</td>
<td>0.845</td>
<td>$Y$</td>
<td>41.04%</td>
<td>43.23%</td>
<td>39.54%</td>
</tr>
<tr>
<td>$\rho_0^*$</td>
<td>0.844</td>
<td>0.860</td>
<td>0.834</td>
<td>$\rho_D^*$</td>
<td>0.660</td>
<td>0.643</td>
<td>0.636</td>
<td>$Y^*$</td>
<td>21.86%</td>
<td>25.18%</td>
<td>23.77%</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>0.508</td>
<td>0.667</td>
<td>0.704</td>
<td>$\pi_D$</td>
<td>0.184</td>
<td>0.191</td>
<td>0.191</td>
<td>total</td>
<td>62.90%</td>
<td>68.41%</td>
<td>63.31%</td>
</tr>
</tbody>
</table>

Figure 21: Ratio of flows following bad news to initial flows. Sensitivity of Flows to Downside Risk when Home can Tranche.
Figure 22: Price Crashes in Autarky and with Financial Integration. Sensitivity of Price Volatility to Downside Risk when Home can Tranche.

C.3 Dynamic Model with Partial Correlation

The previous results highlight how collateral-driven trades can amplify volatility and lead to collapses in flows when the only purpose of trade is motivated by collateral. However, when the assets are not perfectly correlated, then there are typically gains from “diversification” owing to portfolio rebalancing that occurs when only one asset receives bad news. One may rightly wonder if diversification mechanisms may potentially undo the effects driven by collateral-motivated trades. On the contrary, diversification mechanisms reinforce collateral-based mechanisms rather than undoing them. We show that collateral-driven trades continue to amplify volatility, and the effects with partial correlation can be even larger.

In this section we relax the assumption of perfect correlation of interim news for assets $Y$ and $Y^*$ so that bad news can be about one or both assets. We now suppose that at time 1 agents may receive news about one or both assets. As before, good news is that the asset pays 1 for sure, but bad news is that the asset payoff is uncertain and may pay 1 or $d_{YY}^Y$. Since agents can receive good or bad news about either asset, we now have 4 states of nature at time 1. The structure of
uncertainty is illustrated in Figure 23. In state $s_1$ agents receive good news about both assets. In $s_2$ agents receive good news about $Y$, but bad news about $Y^*$ (there is risk). In $s_3$ agents receive good news about $Y^*$, but bad news about $Y$. In $s_4$ agents receive bad news about both assets.

![Diagram showing payoffs in the dynamic model with partial correlation.]

Figure 23: Payoffs in the dynamic model with partial correlation.

Agent $i$ now holds beliefs $\gamma_i$ that state $s_n$ occurs. We continue to maintain that the probability of good news about either asset equals $i$, hence $\gamma_1(i) + \gamma_2(i) = i$ and $\gamma_1(i) + \gamma_3(i) = i$, but now correlation in states (via dependence in beliefs) is given by the coefficient $\phi = \frac{\gamma_1(i) - h^2}{h(1-h)}$, which we allow to vary between zero (independence) and one (perfect correlation). Notice that the correlation of payoffs in the final state is irrelevant since agents are risk neutral and agent $i$ believes at time 1 that assets will pay with probability $i$.

Because the qualitative results with leverage are most interesting, in this section we first consider when Home can leverage its assets. The results with tranching and partial correlation reinforce the results with perfect correlation.
C.3.1 Equilibrium regime with partial correlation and Home Leverage

Our equilibrium regime with partial correlation now has an important qualitative difference. First, the price of risky assets in states $s_2$ and $s_3$ are higher than in state $s_4$, i.e., $p_3 > p_4$ and $p_2^* > p_4^*$. To see this, consider how investors would choose to invest following bad news about one asset. In state $s_2$, any investors who bought $Y$ at $t = 0$ receive a huge payoff (the asset is worth 1). Since these investors are also optimistic about $Y^*$, they use this wealth to buy $Y^*$, thus increasing its price. Similarly, any investors who bought $Y^*$ at $t = 0$ are wealthy in $s_3$ (the Foreign asset is worth 1) and these investors can buy Home assets $Y$ that are sold as a result of margin calls.

Second, as a result, two contracts are typically traded at time zero: a contract promising $p_4$ (safe), and a contract promising $p_3$, which is risky (defaulting in $s_4$). With perfect correlation the economy was binomial and so only the safe contract was traded. In equilibrium, all agents buying $Y$ issued a safe promise backed by enough collateral so that even after bad news borrowers could repay their debt. With multiple states of uncertainty, equilibrium can feature multiple traded contracts, which crucially includes risky contracts that may default after sufficiently bad news. Accordingly, equilibrium with financial integration and partially correlated assets now includes agents who buy the risky contract promising $p_3$, which delivers only $p_4$ when both assets receive bad news. Thus, at $t = 0$ there are four marginal buyers with investors (i) buying $Y$ and promising $p_3$ (high leverage), (ii) buying $Y$ and promising $p_4$ (low leverage), (iii) buying $Y^*$, (iv) buying the risky contract promising $p_3$, and (v) holding safe assets. Importantly, in state $s_4$ with bad news about both assets, investors that bought the risky promise have significant losses because the risky promise defaults and delivers only $p_4$.

C.3.2 Crashes with partial correlation

The left frame of Figure 24 plots the price crashes varying the correlation coefficient $\phi \in [0, 1)$ with financial integration and compared to autarky (dotted). The reddish lines are crashes in Home and the blue lines are crashes in Foreign. There are three sets of lines for each country. The dotted line shows the autarkic crash following bad news. The solid lines are crashes with financial integration when both assets receive bad news, which is state $s_4$. The dashed lines are the crashes
with financial integration when only one asset receives bad news, which are states $s_3$ (bad news for Home) and $s_2$ (bad news for Foreign). The right frame of Figure 24 plots the prices of the Home asset in the initial and interim states as well as the price of the risky contract (promising $p_3$).

There is an important regime change that occurs with partially correlated assets. For very high levels of correlation, the intermediate states are very unlikely, and so all investors buying the Home asset do so by issuing the safe promise (using low leverage), just as occurs when the assets are perfectly correlated. However, when states are less correlated so that the intermediate state with good news about one asset is more likely, investors buy the Home asset by making the risky promise (use high leverage). Indeed, only the risky promise is traded in this regime; no agents trade the safe promise (see Figure 26 and discussion below). Accordingly, the economy experiences substantial defaults when both assets receive bad news.

Within this high-leverage regime, crashes are larger the lower is the level of correlation. Foreign price crashes in the worst state ($s_4$) are everywhere greater than the crash in autarky. When risky promises are traded, the Home asset becomes even less volatile than the unleveraged Foreign asset. It is perhaps surprising that the result of this regime shift is that Home price crashes are smaller than when no defaults occurred. Investigating the right panel, it is evident that crashes are smaller because the initial price is lower (the prices in the crash states are hardly changed).
In addition, the price crashes in states $s_2$ and $s_3$ are much lower than would occur in autarky because of the portfolio rebalancing that occurs when good news about one asset provides a wealth transfer to agents who want to invest in the other asset. This does not occur in autarky because there are no “diversification” gains from rebalancing. To isolate this mechanism, Figure 25 plots the crashes with financial integration when the assets are partially correlated but have the same collateral treatment (in black). In the left panel, black lines indicate crashes when both assets cannot be leveraged but there are diversification gains because assets are partially correlated. In the right panel, black lines indicate crashes when both assets can be leveraged.

When neither asset can be leveraged, the portfolio rebalancing mechanism is so strong that when an asset receives good news, the price of the risky asset receiving bad news is almost completely unchanged (black dotted line). Thus, the small crashes in states $s_2$ and $s_3$ with financial integration are substantially larger than the small crash that would occur if neither asset could be leveraged (collateral amplifies crashes). The right panel shows crashes when both assets can be leveraged. In this case, the crash after double-bad news is nearly the same, though slightly larger, as the Foreign crash in $s_4$. The crash after bad news in just one asset is similarly slightly larger.

Figure 25: Comparing to diversification gains with otherwise identical assets.

There are two effects that explain why, with low correlation, initial prices and crashes are lower with financial integration. First, consider Figure 26 which plots marginal buyers. The left panel
plots the economy with financial integration between Home and Foreign, and the right panel plots an economy with financial integration with two assets that can be leveraged, thus isolating the diversification mechanism. In the right panel, in the two-leveraged asset economy, the fraction of agents using high leverage gradually increases as correlation decreases ($i^Y_R$ gradually decreases) while the total fraction of agents buying risky assets is roughly constant ($i^Y_S$ hardly changes). In contrast, in the left panel, above $\phi = 0.8$ only the safe contract is traded, and below $\phi = 0.8$ only the risky contract is traded ($i^Y_R = i^Y_S$ indicates that there are no investors using the safe contract). Accordingly, a much smaller set of agents are buying the Home asset at all. While agents switch to making the risky promise precisely because they can borrow more than when they make the safe promise ($\pi_0 > p_4$), significantly fewer optimistic investors are buying the asset. Even though these investors use more borrowed money, because there are fewer of them the Home asset price must fall.

However, there is a second effect that explains why investors endogenously in equilibrium do not use the low-leveraged contract to buy the Home asset. The second effect is the alternative investment effect we discussed earlier. Investors who would consider using low leverage to buy the Home asset have an attractive alternative to invest in the Foreign asset without any leverage. Whereas in our static analysis this “alternative investment return” force decreased the Home price, in this dynamic analysis with partial correlation, this effect endogenously changes the contracts traded in equilibrium. As a result, the required return on leveraged investments increases and only high-leverage contracts are traded in equilibrium. Fewer investors buying risky assets decreases the initial prices of those assets. Accordingly the price crash following bad news is smaller.

### C.3.3 Leverage Regulation: Risk-Free Promises

When the level of correlation is sufficiently low, investors switch from making safe promises (promising $p_4$) to issuing risky promises (promising $p_3$) that would default after bad news about both assets. In other words, with partial correlation, collateral-driven trades will lead investors to make high-leverage, risky promises that default after bad news about both assets. With perfect correlation, only default-free debt is traded in equilibrium.
Regulators may understandably worry about (unmodeled) adverse consequences of widespread default. If regulators wish to avoid default after very bad news (an issue that might not arise in autarky), then with financial integration regulators would have to impose that investors can only issue safe contracts. This would indeed avoid default, but we show that this would make price crashes worse. Restricting agents to only making safe promises leads to even larger price crashes in both countries, with larger crashes the lower is the degree of correlation.

Figure 27 illustrates the price crashes in Home and Foreign when only safe contracts can be traded. The left panel compares crashes in autarky and with financial integration. Unlike before, there is no regime shift when agents switch to trading risky contracts. Now, lower correlation increases all the crashes in financial integration. The right panel presents the Home asset prices. As we saw with perfect correlation, the Foreign asset essentially imports nearly all the volatility from Home while Home remains volatile itself. Constraining agents to only trade risk-free contracts increases crashes because initial prices remain high. Agents do not switch to using only high-leverage contracts, and as a result a large fraction of agents buy risky assets. Accordingly, the price is high and so price crashes are larger.
C.3.4 Dynamic Model with Partial Correlation and Home Tranching

We consider the dynamic tranching economy with partial correlation. We allows agents to tranche Y into a security that pays the value of Y whenever Y receives bad news. Hence, the tranche pays $(0, 0, p_3, p_4)$, since there are now 2 interim states in which the Home asset can pay below 1. In equilibrium, at $t = 0$ there are three marginal buyers with investors (i) buying Y and issuing the tranche, (ii) buying $Y^*$, (iii) holding safe assets, and (iv) buying down tranches. Figure 28 plots crashes against crashes in autarky. As noted with perfect correlation, crashes are substantially larger than in autarky. Unlike in the leverage case, agents do not switch from low-leverage to high-leverage contracts, but always issue the down tranche. Thus, there is no countervailing “default mechanism” that decreases initial leverage.

Finally, comparing to what would occur if both assets were identical but partially correlated reveals two interesting features (Figure 29). First, when only the Foreign asset receives bad news, the portfolio effect from the good news on the Home asset is so large that the Foreign crash is smaller than it would be if neither asset could be leveraged. Indeed, if endowments at $t = 1$ were smaller, the price of the Foreign asset could even increase, even though it received bad news about future payoffs. Furthermore, the crashes for the Home asset are much larger with financial integration.
when the Foreign asset cannot be tranched than when it can. Additionally, lower correlation amplifies our results so that the increased volatility coming from financial integration is even greater.

Figure 28: Price crashes with partially correlated news, with financial integration when Home can Tranche.

D Equilibrium Equations for Dynamic Economies

D.1 Autarky Leverage

The equilibrium conditions with leverage and no trade are

\[
(1 - i_0^\gamma) \frac{1 + p_0}{p_0 - p_D} = 1 \quad (32)
\]

\[
(i_0^\gamma - i_D^\gamma) \frac{1 + p_0}{p_D - d_D^\gamma} + (1 - i_D^\gamma) \frac{1 + p_D}{p_D - d_D^\gamma} = 2 \quad (33)
\]

\[
i_0^\gamma (1 - p_D) = i_0^\gamma + (1 - i_0^\gamma) i_0^\gamma (1 - d_D^\gamma) \quad (34)
\]

\[
i_D^\gamma + (1 - i_D^\gamma) d_D^\gamma = p_D, \quad (35)
\]

where the market clearing condition at \( s = D \) reflects the new endowments at \( t = 1 \).
Figure 29: Comparing to diversification gains with otherwise identical assets.

D.2 Financial Integration with Leverage and No Leverage

With trade, the marginal buyers in period 0 are \((\hat{i}_{0}, \hat{i}_{0}^*)\) and in D are \((\hat{i}_{D}, \hat{i}_{D}^*)\):

\[
(1 - \hat{\nu}_0) = \frac{\hat{p}_0 + \hat{p}_0^*}{\hat{p}_0 - \hat{p}_0^*} = 1
\]  
(36)

\[
(\hat{i}_{0} - \hat{i}_{0}^*) = \frac{\hat{p}_0 + \hat{p}_0^*}{\hat{p}_0^*} = 1
\]  
(37)

\[
\hat{i}_{0}^* (1 - \hat{p}_D) = \frac{\hat{i}_{0}^*}{\hat{p}_0} + \frac{(1 - \hat{\nu}_0)\hat{p}_D^* \hat{i}_{0}^*(1 - d_D^*)}{\hat{p}_0 - \hat{p}_D^*}
\]  
(38)

\[
1 - (1 - \hat{\nu}_0)^2 (1 - d_D^*) = \hat{i}_{0}^* + (1 - \hat{\nu}_0)^* \hat{i}_{0}^* + (1 - \hat{\nu}_0^* d_D^*)
\]  
(39)

\[
\frac{\hat{i}_{0}^* - \hat{i}_{D}^*}{\hat{p}_0^* - \hat{p}_D^*} + (1 - \hat{\nu}_0^*) \frac{\hat{p}_D^*}{\hat{p}_D - d_D^*} = 2
\]  
(40)

\[
\frac{\hat{i}_{0}^* - \hat{i}_{0}^*}{\hat{p}_0^* - \hat{p}_D^*} + \frac{(1 - \hat{\nu}_0) \hat{p}_D^* (2 + \hat{p}_D + \hat{p}_0^*)}{\hat{p}_0^*} = 2
\]  
(41)

\[
\frac{\hat{i}_{D}^* (1 - d_D^*)}{\hat{p}_D^*} = \frac{\hat{i}_{D}^* + (1 - \hat{\nu}_D) d_D^*}{\hat{p}_D^*}
\]  
(42)

\[
\hat{i}_{D}^* + (1 - \hat{\nu}_D) d_D^* = \hat{p}_D^*
\]  
(43)
D.3 Autarky Tranching

Denote by $i^Y_0, i^T_0, i^Y_D, i^T_D$ the marginal buyers in state $0$ and $D$:

\[
(1 - i^Y_0) \frac{(1 + p_0)}{p_0 - p_D p_0^{AD}} = 1 \quad (44)
\]

\[
i^T_0 (1 + p_0) = p_D p_0^{AD} \quad (45)
\]

\[
\frac{i^Y_0}{p_0 - p_D p_0^{AD}} = i^Y_0 + (1 - i^Y_0) \frac{i^Y_0}{p_0 - d_D^Y p_0^{AD}} \quad (46)
\]

\[
1 - i^T_0 = p_0^{AD} \quad (47)
\]

\[
\frac{(i^Y_0 - i^Y_D)(1 + p_0)}{p_D - d_D^Y p_D^{AD}} + \frac{(1 - i^Y_D)(1 + p_D)}{p_D - d_D^Y p_D^{AD}} = 2 \quad (48)
\]

\[
i^T_D p_D + i^T_D (1 + p_D) = 2 d_D^Y p_D^{AD} \quad (49)
\]

\[
\frac{i^T_D}{p_D - d_D^Y p_D^{AD}} = 1 \quad (50)
\]

\[
1 - i^T_D = p_D^{AD} \quad (51)
\]

D.4 Financial Integration with Tranching and No Leverage and trade in Tranches (T-Trade)

Eleven variables: $i^Y_0, i^Y_D, i^T_0, i^T_D, p_0, p_0^{AD}, p_D, p_D^{AD}, \hat{p}_0, \hat{p}_0^{AD}, \hat{p}_D, \hat{p}_D^{AD}$. Foreign market clearing is similar to the autarkic case. In addition, 3 Market clearing in $0$, 3 valuation in $0$, 2 market clearing in $D$, 3 valuation in $D$.

\[
(1 - i^Y_0) \frac{(1 + \hat{p}_0)}{\hat{p}_0 - \hat{p}_D \hat{p}_0^{AD}} = 1 \quad (52)
\]

\[
p_0^* = (1 - \hat{i}^Y_0^*)(1 + \hat{p}_0^*) \quad (53)
\]

\[
i^T_0 (2 + \hat{p}_0 + \hat{p}_0^*) = \hat{p}_D \hat{p}_0^{AD} \quad (54)
\]
\[
\frac{\tilde{\gamma}_0}{\tilde{\rho}_0 - \tilde{\rho}_D \hat{\rho}_0^{AD}} = \tilde{\gamma}_0 + (1 - \tilde{\gamma}_0) \frac{\tilde{\gamma}_0}{\tilde{\rho}_D - d_D \hat{\rho}_D^{AD}}
\]  
(55)

\[
\hat{\rho}_0^* = 1 - (1 - \tilde{\gamma}^*)^2 d_D^\gamma
\]  
(56)

\[
1 - \tilde{\gamma}_0 = \hat{\rho}_0^{AD}
\]  
(57)

\[
2(\hat{\rho}_D - d_D^\gamma \hat{\rho}_D^{AD}) = (\tilde{\gamma}_0^* - \tilde{\gamma}_0)(1 + \hat{\rho}_0) + (1 - \tilde{\gamma}_0^*)(1 + \hat{\rho}_D)
\]  
(58)

\[
\frac{\tilde{\gamma}_0}{\hat{\rho}_D}(1 + \hat{\rho}_D) = 2 d_D^\gamma \hat{\rho}_D^{AD}
\]  
(59)

\[
\frac{\tilde{\gamma}_0^*}{\hat{\rho}_D - d_D^\gamma \hat{\rho}_D^{AD}} = 1
\]  
(60)

\[
\hat{\rho}_D = \tilde{\gamma}^* + (1 - \tilde{\gamma}^*) d_D^\gamma
\]  
(61)

\[
1 - \tilde{\gamma}_D = \hat{\rho}_D^{AD};
\]  
(62)

D.5 Financial Integration with Tranching and No Leverage and trade in Tranches and Assets (TY-Trade)

Twelve variables: \(i_0^*, i_D^*, i_0^*, i_D^*, \hat{i}_0^*, \hat{i}_D^*, \hat{\rho}_0, \hat{\rho}_D, \hat{\rho}_0^{AD}, \hat{\rho}_D, \hat{\rho}_D^{AD}, \hat{\rho}_D^{AD} \). 3 Market clearing in 0, 3 valuation in 0, 3 market clearing in D, 3 valuation in D.

\[
(1 - \tilde{\gamma}_0^*) \frac{2 + \hat{\rho}_0 + \hat{\rho}_0^*}{\hat{\rho}_0 - \hat{\rho}_D \hat{\rho}_0^{AD}} = 1
\]  
(63)

\[
\hat{\rho}_0^* = (\tilde{\gamma}_0^* - \tilde{\gamma}_0^*)(2 + \hat{\rho}_0 + \hat{\rho}_0^*)
\]  
(64)

\[
\tilde{\gamma}_0^*(2 + \hat{\rho}_0 + \hat{\rho}_0^*) = \hat{\rho}_D \hat{\rho}_0^{AD}
\]  
(65)

\[
\frac{\tilde{\gamma}_0^*}{\hat{\rho}_0 - \hat{\rho}_D \hat{\rho}_0^{AD}} = \frac{\tilde{\gamma}_0^*}{\hat{\rho}_0^*} + \frac{1 - \tilde{\gamma}_0^*}{\hat{\rho}_D - d_D^\gamma \hat{\rho}_D^{AD}}
\]  
(66)

\[
1 - (1 - \tilde{\gamma}^*)^2 d_D^\gamma = \tilde{\gamma}^* + (1 - \tilde{\gamma}_0^*) \frac{\tilde{\gamma}_0^*}{\hat{\rho}_D^*} + (1 - \tilde{\gamma}_0^*) d_D^\gamma
\]  
(67)

\[
1 - \tilde{\gamma}_0^* = \hat{\rho}_0^{AD}
\]  
(68)

\[
\frac{\tilde{\gamma}_0^* - \tilde{\gamma}_D^*}{\tilde{\gamma}_0^* - \tilde{\gamma}_0^*} = \frac{\hat{\rho}_D}{\hat{\rho}_D - d_D^\gamma \hat{\rho}_D^{AD}} + (1 - \tilde{\gamma}_D^*) \frac{2 + \hat{\rho}_D + \hat{\rho}_D^*}{\hat{\rho}_D - d_D^\gamma \hat{\rho}_D^{AD}} = 2
\]  
(69)
Financial Integration with Foreign Leverage and Home Tranching

10 variables: $\hat{\nu}_0, \hat{\nu}_D, \hat{\nu}_T, \hat{i}_D, \hat{\phi}_0, \hat{\phi}_D, \hat{\phi}_T, \hat{\phi}_D^A, \hat{\phi}_D^D$. 2 Market clearing in 0, 3 valuation in 0, 2 market clearing in D, 3 valuation in D. With $\phi_0 = \frac{1}{2 - \hat{\nu}_D}$ and $\phi_D = \frac{1}{2 - d_D}$.

\[
\begin{align*}
(1 - \hat{\nu}_D) & \frac{2 + \hat{\phi}_D + \hat{\phi}_D^*}{\hat{\phi}_D - \hat{\phi}_D^D} = 1 \\
\hat{\nu}_0 & = \hat{\nu}_D \hat{\phi}_D^D \\
\hat{i}_0 & = \hat{\nu}_D \hat{\phi}_D^D \\
\hat{\phi}_D & = \hat{\nu}_D (1 - \hat{\nu}_D) \\
1 - \hat{i}_D & = \hat{\phi}_D^D \; ;
\end{align*}
\]

\[(70)\]  
\[(71)\]  
\[(72)\]  
\[(73)\]  
\[(74)\]