Bank Runs and Asset Prices: The Role of Coordination Failures for Determinacy and Welfare *

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November 17, 2017

Abstract

We study competitive equilibria in which banks provide liquidity insurance, make productive investments and interact on asset markets. All banks are subject to extrinsic risk, but a bank's portfolio choice determines whether it is prone to a bank run caused by coordination failures in one of the extrinsic states. Asset price volatility and the share of run-prone banks are equilibrium outcomes. There is indeterminacy when no bank is run-prone. Equilibria with at least some run-prone banks are determinate. Multiple equilibria with different shares of run-prone banks can exist, causing a trade-off between bank stability and determinacy. In an example welfare is higher with than without extrinsic risk, but only if no bank is run-prone.

Keywords Liquidity Insurance · Extrinsic Risk · Financial Crisis · General Equilibrium

JEL Classification G01 · G21 · D53

*The paper has benefited from discussions with Kartik Anand, Falko Fecht, Thomas Gehrig, Jochen Mankart, Pascal Mossay, Maria Näther, Thilo Pausch, Nicola Persico and Sergey Zhuk, for which we are grateful. Diemo Dietrich thanks Deutsche Bundesbank Research Centre for its hospitality. The paper represents the authors' personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank or its staff.

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1 Introduction

From time to time, a significant number of banks fail and asset markets crash. These financial crises can be caused by exogenous shocks to fundamentals or they can be properties of financial systems. In the latter case, what are the economic consequences of financial crises? Answers to this question are important for three reasons. First, they contribute to understand how the stability of a financial system, rather than its propensity to amplify shocks, affects equilibrium outcomes. Second, they offer valuable clues on whether maintaining financial stability is desirable and at which cost. Third, they help regulators and supervisors in their assessment of banks' liquidity conditions, and guide them in designing regulation.

In the present paper we consider economies where a continuum of ex-ante identical consumers lives for two or three dates. At the first date banks offer simple deposit contracts and allocate deposits between productive investments and storage. Productive investments fully mature only at the third date but banks can buy and sell productive investments on interbank asset markets at the second date. The banking sector is perfectly competitive. There is no intrinsic risk as there are no exogenous shocks to the fundamentals. However, there is extrinsic risk as there can be endogenous shocks in form of bank runs at the second date caused by sunspot-driven coordination failures. In a bank run, a bank's assets can be either unwound and liquidated or sold to another bank. Depositors of the failed bank equally share the respective revenues. Whether banks are immune to coordination failures depends on whether the value of their portfolio of stored goods and productive investments allows them to meet the demands of depositors independently of whether depositors run. Banks that are able to meet the withdrawal demands of depositors even in case of a coordination failure are *safe*, otherwise banks are *risky*. In equilibrium consumers deposit in the banks they prefer, banks allocate deposits between productive investments and storage in order to maximize their profits, and asset markets clear.

Our results can be summarized as follows. Equilibria exist (Theorem 1). There is a threshold such that there is an equilibrium with solely safe banks provided the sunspot probability is above the threshold; the consumers' expected utility is bounded away from the first-best for equilibria with safe banking sectors (Theorem 2). Moreover, there is indeterminacy of equilibria provided there is an equilibrium where depositors strictly prefer safe banks over risky banks (Theorem 3). Since asset price volatility and portfolios of banks vary across such equilibria, welfare varies too, making the indeterminacy real. There is a second threshold such that an equilibrium with solely risky banks exists provided the sunspot probability is below this threshold (Theorem 4). A coordination failure, when it occurs, brings down all banks in those equilibria. No bank survives and all productive investments are physically liquidated. However, the consumers' expected utility converges to the first-best if the probability of the extrinsic state with coordination failures approaches zero. Conditions under which equilibria with both, risky and safe banks are generally difficult to identify. There is a third threshold, though, such that there is no equilibrium in which risky banks exist provided the sunspot probability is above this threshold (Theorem 5). There is also a fourth threshold such that there is no equilibrium in which safe banks exist provided the sunspot probability is below this threshold (Theorem 6). In a set of examples multiple equilibria can exist, with some equilibria featuring a safe banking sector, others a mixed or even a risky banking sector. These results suggest that coordination failures can not only cause a trade-off between bank stability and the value of the financial services banks provide, but also between bank stability and determinacy of the real economic outcome.

To illustrate the real effects of coordination failures we embed our financial sector in overlapping generations economies with production and endogenous growth. Since there is no liquidation of productive investments provided that at least some safe banks exist, the actual occurrence of bank runs may have no consequences. However the allocation of deposits between productive investments and storage depends on which equilibrium the economy is in. We provide an example in which welfare is maximized if asset prices are volatile, solely safe banks exist and, quite remarkably, the sunspot probability is significantly different from zero. This implies that the possibility of coordination failures is not necessarily bad and that the welfare-maximizing equilibrium can be indeterminate.

The idea of sunspots affecting real outcomes goes back to Cass and Shell (1983). That bank runs can be triggered by sunspots has been first suggested by Diamond and Dybvig (1983) and then scrutinized in-depth from a mechanism design perspective (e.g. Jacklin, 1987; Green and Lin, 2003; Peck and Shell, 2003; Sultanum, 2014). Recent experimental studies have looked for evidence that bank runs can be caused by coordination failures. Arifovic et al. (2013) provide such evidence in the absence of sunspot variables. Arifovic and Jiang (2014) introduce sunspot variables. They show that sunspots are important determinants for agents' behavior when strategic uncertainty is high, i.e. if resources available to a bank relative to the amount promised to agents are neither too large nor too small. Interestingly, in their experiment resources are split pro-rata among all agents if the bank does not have enough to pay everyone the promised amount. This implies that a first-come-first-served principle seems not essential for the existence of multiple equilibria and sunspot effects. Chakravarty et al. (2014) find that coordination failures leading to bank runs can affect several banks simultaneously even if they are neither linked to each other nor exposed to the same fundamental risks.

Allen and Gale (1998) and Cooper and Ross (1998) consider banks which make productive investments and provide liquidity insurance using simple deposit contracts as we do. However, these papers do not consider interactions among banks. In Allen and Gale (1998) there is no coordination failure but intrinsic risk, such that bank runs are caused by shocks to the fundamentals. Cooper and Ross (1998) consider extrinsic risk. They show that there is a unique threshold such that all banks are safe if and only if the sunspot probability is above this threshold; otherwise all banks are risky. In the present paper there are several threshold levels. Hence, for a given sunspot probability, there can be multiple equilibria, one in which all banks are safe and another in which all are risky. Moreover, there can also be equilibria in which risky and safe banks coexist.

In the literature on the interactions of asset markets and liquidity-providing banks, crises are often treated as zero probability events (e.g. Fecht, 2004) or the amount of stored goods (reserves) available to banks is exogenous (e.g. Diamond and Rajan, 2005). Allen and Gale (2004a,b) analyze economies with positive crisis probabilities and endogenous storage by banks. There, shocks to the fundamentals can have disproportionately large effects on banks and asset prices. However, in the limit economy where fundamentals become asymptotically deterministic, the equilibrium converges to a trivial sunspot equilibrium in which asset prices are volatile, banks never default, and consumers enjoy unconstrained efficient liquidity insurance. In the present paper, the possibility of coordination failures imposes a tighter constraint on banks to be safe than in Allen and Gale (2004a). This changes the characteristics of equilibria. First, it prevents safe banks from ever providing optimal liquidity insurance. Accordingly, any financial system with at least some banks being safe deviates from optimum liquidity insurance. Second, as risky banks choose not to be subject to such additional constraint, if a banking sector can offer liquidity insurance close to the first-best, it is one with solely risky banks, provided the probability of coordination failures is close to zero. Third, the possibility of coordination failures can lead to multiple equilibria and even indeterminacy in real terms. Indeed, default is a necessary condition for determinacy of the real outcome.

Bencivenga and Smith (1991), Ennis and Keister (2003) and Fecht et al. (2008) analyze the role of banks providing liquidity insurance for capital formation and growth. Bencivenga and Smith (1991) were the first to explore the conditions under which liquidity provision by banks results in stronger growth. However, there are no bank runs and no asset markets. In Ennis and Keister (2003) and Fecht et al. (2008) there is a market for existing capital. Ennis and Keister (2003) allow for coordination failures, but only symmetric equilibria are considered. Therefore, bank runs inevitably lead to the physical liquidation of productive investments held in the banking sector. This implies that the actual occurrence of a crisis does affect growth, which in the present paper holds only for equilibria in which solely risky banks exist. In Fecht et al. (2008) there are neither extrinsic nor intrinsic risks. Their focus is on the trade-off between capital formation and insurance of idiosyncratic liquidity risks and how it depends on the participation of depositors in asset markets. We find that such trade-off crucially depends on which asset prices prevail in equilibrium.

The paper has the following structure. In section 2 we lay out the model. In section 3 we show that equilibria exist, examine the characteristics of banking sectors in equilibrium and look into the role of coordination failures for determinacy of equilibria. In section 4 we study the consequences of coordination failures for the real economy. In section 5 we discuss indeterminacy and multiplicity of equilibria from a policy perspective. Section 6 concludes.

2 The model

2.1 Setup

There are three dates $t \in \{0,1,2\}$ with a single good at every date and extrinsic risk at the second date. At this date there are two possible states $s \in \mathbb{S} = \{1,2\}$. With probability $p \in]0,1[$ the state is $s = 1$ and with probability $1-p$ the state is $s = 2$.

There is a continuum of identical consumers with mass one. A consumer is described by her endowment $(1,0,0)$ and her consumption set $X = \mathbb{R}^2_+$. A consumer is either impatient and values consumption at date $t = 1$ or patient and values consumption at date $t = 2$. At date $t = 1$ consumers learn their type, which is private information. Patience among consumers is uncorrelated and the share of impatient consumers $\lambda \in]0,1[$ is given and common knowledge. Let $x_{t,s}$ denote what a consumer gets at date t in state s. Then, her expected utility is

$$\lambda(pu(x_{1,1})+(1-p)u(x_{1,2}))+(1-\lambda)(pu(x_{2,1})+(1-p)u(x_{2,2}). \tag{1}$$

The Bernoulli utility function u is twice differentiable with $u' > 0$, $u'' < 0$, and $\lim_{x\to 0} u'(x) = \infty$. Like in many varieties of the Diamond and Dybvig (1983) model, relative risk aversion $k(x) = -\frac{u''(x)}{u'(x)}x$ is supposed to be larger than one. At each date $t \in \{0,1\}$, consumers have access to storage with a gross return of one at date $t+1$.

There is a continuum of identical banks. A bank has access to two technologies, storage and production. Storage of the good is a short asset and can be thought of as reserve holdings. It can be used at dates $t \in \{0,1\}$. Production of the good is a long asset and can be thought of as investment. It can be initiated at the first date $t = 0$ and physically liquidated for some positive but arbitrarily small gross return ε at the second date $t = 1$. If it is not liquidated, it yields a gross return of $R > 1$ at the final date $t = 2$. At date $t = 1$, there is also a perfectly competitive interbank market for production. The price in state s is P_s.

A bank offers deposit contracts in exchange for consumer endowments at the first date $t = 0$. Such contracts specify the face value d which a consumer can withdraw at date $t = 1$. It is not possible to write state-contingent contracts. The market for deposits is perfectly competitive. A consumer chooses in which bank to deposit her endowment, but she has to put all her endowments in the same bank. A bank stores a share $y \in [0,1]$ of these endowments and invests $1-y$ in production. Depositors observe how the bank uses their endowments at $t = 0$.

Impatient consumers always withdraw at date $t = 1$. A patient consumer contemplates to withdraw at this date if the first state $s = 1$ materializes: she compares what she gets by withdrawing at $t = 1$ with the payoff associated with holding on until $t = 2$, assuming that all other patient consumers withdraw at $t = 1$. If the former is higher, everyone withdraws at $t = 1$. If the market value of the bank's assets is lower than what the bank owes to its consumers, the value of assets is split pro-rata among them and the bank ceases to exist. In state $s = 2$, a patient consumer assumes that all other patient consumers do not withdraw at date $t = 1$, hence there is no such coordination failure. Consumers who do not withdraw at $t = 1$ will equally share the residual value of their bank's assets at $t = 2$.

As standard, first-best consumption for impatient and patient consumers is y^*/λ and $R(1-y^*)/(1-\lambda)$, respectively, and optimum storage y^* satisfies

$$u'(y^*/\lambda) = Ru'\left(\tfrac{R(1-y^*)}{1-\lambda}\right). \tag{2}$$

2.2 Bank behavior

For a given probability distribution of the extrinsic state, banks can either take their chances, or they make provisions to prevent a possible bank run. Accordingly, banks are either risky or safe. Let $x = (x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2})$ denote the bundle of consumption $x_{t,s}$ at date t in state s. A bank's objective then is to maximize expected utility

$$\max_{(y,d,x)} \quad \lambda\left(pu(x_{1,1}) + (1-p)u(x_{1,2})\right) + (1-\lambda)\left(pu(x_{2,1}) + (1-p)\,u(x_{2,2})\right) \tag{3}$$

subject to its constraints. These constraints are different for safe and risky banks. For a bank to be safe, the market value of its assets must at least cover all outstanding deposits as of $t = 1$. It is not necessary that the reserves of a safe bank cover all outstanding deposits. As long as the bank can signal to depositors that by selling its assets it will always be able to satisfy everyone's withdrawal demand at once and in full, patient consumers do not have an incentive to run. Hence, a bank is safe if

$$d \leq y + P_s(1-y) \quad \forall s \in \mathbb{S}. \tag{4}$$

Note, since $k(x) > 1$, a safe bank does not hold more reserves than needed to deter consumers from running.[1] Hence

$$\begin{aligned} d &= y + P_s(1-y) \quad \text{if} \quad 1 - P_s > 0, \\ d &< y + P_s(1-y) \quad \text{if} \quad 1 - P_s < 0. \end{aligned} \tag{5}$$

[1]Consumers are simply too risk averse to be interested in speculating on fire-sales, as this would only benefit patient consumers at the expense of impatient consumers. See Appendix A.

The resource constraints on consumption with a safe bank are

$$x_{1,s} \leq d, \tag{6a}$$

$$x_{2,s} \leq \frac{R}{P_s} \frac{y + P_s(1-y) - \lambda d}{1-\lambda}. \tag{6b}$$

The first constraint reflects that a safe bank can always repay its deposits at date $t = 1$. The second requires that consumption of patient consumers equals the pro-rata share of the future value of the bank's assets net of its liabilities to impatient consumers.

As for a risky bank, there is a run in state $s = 1$ if the market value of the bank's assets is not sufficient to fully pay all depositors the promised amount, i.e. if

$$y + P_1(1-y) \leq d. \tag{7}$$

Two remarks are due. First, if condition (7) holds with equality, a safe bank strictly dominates a risky bank. Second, a risky bank cannot fail in both states. Otherwise the marginal rate of substitution between early and late consumption would be 1, regardless in which state s the economy is, while the ex-ante marginal rate of transformation is R^{-1}. This cannot be optimal.

The resource constraints on consumption with a risky bank read

$$x_{1,s} \leq \begin{cases} y + P_1(1-y) & \text{if} \quad s = 1, \\ d & \text{if} \quad s = 2, \end{cases} \tag{8a}$$

$$x_{2,s} \leq \begin{cases} y + P_1(1-y) & \text{if} \quad s = 1, \\ \dfrac{R}{P_2} \dfrac{y + P_2(1-y) - \lambda d}{1-\lambda} & \text{if} \quad s = 2. \end{cases} \tag{8b}$$

The first lines in these budget constraints reflect that in a bank run everyone gets a pro-rata share of the bank's liquidation value. The second lines state that impatient consumers get what the deposit contract entitles them to, while consumption of patient consumers equals the pro-rata share of the future value of the bank's assets net of its liabilities to impatient consumers.

Let superscript $\mathscr{R}$ denote the solution to a risky bank's problem and superscript $\mathscr{S}$ the solution to a safe bank's problem. At date $t=1$, risky banks either liquidate or sell all their assets $(1-y^{\mathscr{R}})$ in state $s=1$. In state $s=2$ they possess reserves of $y^{\mathscr{R}}$ and pay $\lambda d^{\mathscr{R}}$ to impatient consumers. Hence, at date $t=1$ they sell assets if necessary to pay the promised amounts to their impatient consumers. Otherwise they either buy or sell assets provided the promised payments can be made. Assets are bought if the rate of return as of date $t=1$ on storage is smaller than the respective return on the long asset, and sold if it is larger. Regarding safe banks, since patient consumers have no incentive to ever withdraw early, the actual outflow in both states is only $\lambda d^{\mathscr{S}}$. Moreover, since the bank's decision about $y^{\mathscr{S}}$ and $d^{\mathscr{S}}$ is made at $t=0$, i.e. before the extrinsic risk is resolved, net reserves at date $t=1$, $y^{\mathscr{S}}-\lambda d^{\mathscr{S}}$, are state-independent. In principle, they can be positive or negative.

Provided $P_1 \geq \varepsilon$, and abusing terminology slightly, liquidity demand q^D of a single risky bank (supply of investments) and liquidity supply q^S of a single safe bank (demand for investments) can be written as

$$q_s^D = \begin{cases} P_1(1-y^{\mathscr{R}}) & \text{if} \quad s=1, \\ \lambda d^{\mathscr{R}} - y^{\mathscr{R}} & \text{if} \quad s=2, \end{cases} \tag{9a}$$

$$q^S = y^{\mathscr{S}} - \lambda\left(y^{\mathscr{S}} + P_1(1-y^{\mathscr{S}})\right). \tag{9b}$$

On the interbank market, banks trade reserves for investments. Let ρ be the share of consumers who have put their endowments in risky banks, or the share of risky banks for short. Then,

$$Q_s^D = \rho q_s^D, \tag{10a}$$

$$Q^S = (1-\rho)q^S, \tag{10b}$$

denote aggregate liquidity demand and aggregate liquidity supply, respectively.

3 Coordination failures and determinacy of equilibrium

3.1 Equilibrium concept and existence

It is convenient to simplify some notation. A consumption plan (x^τ, d^τ, y^τ) for a consumer who deposits her endowments with a bank of type $\tau \in \{\mathscr{S}, \mathscr{R}\}$ is a consumption bundle x^τ and a bank portfolio (d^τ, y^τ) satisfying the constraints (5), (6a) and (6b) for $\tau = \mathscr{S}$, or (7), (8a) and (8b) for $\tau = \mathscr{R}$. Moreover, for given prices $\mathbf{P} = (P_1, P_2)$, let $V^\tau(\mathbf{P})$ denote the indirect utility offered to consumers by a bank of type τ.

Definition 1 *For a given probability distribution of the extrinsic state, an* ***equilibrium*** *is a set of consumption plans, asset prices and the share of risky banks*

$$\left((y^{\mathscr{S}}, d^{\mathscr{S}}, x^{\mathscr{S}}), (y^{\mathscr{R}}, d^{\mathscr{R}}, x^{\mathscr{R}}), \mathbf{P}, \rho\right)$$

with the following properties:

- *Banks maximize expected utility:* $(y^{\mathscr{S}}, d^{\mathscr{S}}, x^{\mathscr{S}})$ *is a solution to the consumer problem for safe banks, and* $(y^{\mathscr{R}}, d^{\mathscr{R}}, x^{\mathscr{R}})$ *is a solution to the consumer problem for risky banks.*

- *The interbank market clears:*

$$\rho P_1 \left(1 - y^{\mathscr{R}}\right) + (1-\rho)\left(\lambda \left(y^{\mathscr{S}} + P_1\left(1 - y^{\mathscr{S}}\right)\right) - y^{\mathscr{S}}\right) = 0 \quad \text{if} \quad s = 1,$$
$$\rho \left(\lambda d^{\mathscr{R}} - y^{\mathscr{R}}\right) + (1-\rho)\left(\lambda \left(y^{\mathscr{S}} + P_1\left(1 - y^{\mathscr{S}}\right)\right) - y^{\mathscr{S}}\right) = 0 \quad \text{if} \quad s = 2.$$

- *Consumers are not better off by going to another operating bank:*

$$\begin{aligned} V^{\mathscr{S}}(\mathbf{P}) &= V^{\mathscr{R}}(\mathbf{P}) \quad \text{if} \quad \rho \in]0,1[, \\ V^{\mathscr{S}}(\mathbf{P}) &\geq V^{\mathscr{R}}(\mathbf{P}) \quad \text{if} \quad \rho = 0, \\ V^{\mathscr{S}}(\mathbf{P}) &\leq V^{\mathscr{R}}(\mathbf{P}) \quad \text{if} \quad \rho = 1. \end{aligned}$$

In any equilibrium, prices are such that arbitrage opportunities do not exist. At date $t = 0$ banks have access to two assets with identical costs: the long asset with values (P_1, P_2) and the short asset with values $(1,1)$, both at date $t=1$. If $P_1, P_2 \geq 1$ with $P_1+P_2>2$, then all banks would solely invest in the long asset. However consumers are better off with a mix of long and short assets. If $P_1, P_2 \leq 1$ with $P_1+P_2<2$, then all banks would solely invest in the short asset. Consumers can do so on their own without using banks, hence banks have a mix of long and short assets. Therefore, prices satisfy $P_1<1<P_2$, $P_2<1<P_1$ or $P_1=P_2=1$. For markets to clear, prices must also satisfy $P_1, P_2 \geq \varepsilon$, $P_1, P_2 \leq R$ and $P_1 \leq P_2$. If $P_s<\varepsilon$, then banks could make risk-free profits by buying long assets in state s and physically liquidate them, hence there is no bank willing to sell. If $P_s>R$ banks could make profits by selling all long assets in state s, hence there is no bank willing to buy. Finally, $P_1 \leq P_2$ because risky banks sell all their long assets in state $s=1$ and (weakly) fewer assets in state $s=2$ whereas the supply of liquidity from safe banks is identical in both states.

Theorem 1 *There is an equilibrium for every probability distribution.*

Proof: See Appendix B.1. □

An equilibrium always exists, although solving for it is difficult. However, several interesting insights, particularly about the structure of the banking sector and the allocation of funds across storage and production, can be obtained from the solutions to the banks' problems. Non-satiation implies that the budget constraints for safe banks, (6a) and (6b), and for risky banks, (8a) and (8b), hold with equality. For a safe bank, replacing d by $y+P_1(1-y)$, the objective function can be expressed solely in terms of y. As the problem is convex, its solution is unique and, if interior, solves the first-order condition

$$\begin{aligned}\left(\tfrac{1}{R}\tfrac{\lambda}{1-\lambda}u'\left(y+P_1(1-y)\right)+\tfrac{p}{P_1}u'\left((y+P_1(1-y))R/P_1\right)\right)(1-P_1)\\ -\tfrac{1-p}{P_2}u'\left(\tfrac{R}{P_2}\tfrac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{1-\lambda}\right)\left(P_2-1+\tfrac{\lambda}{1-\lambda}(P_2-P_1)\right)=0.\end{aligned} \tag{11}$$

As for a risky bank, we can replace $x_{t,s}$ accordingly in the objective function, which is then expressed in terms of y and d. Again, the problem is convex. Hence, the solution $(d^{\mathscr{R}}, y^{\mathscr{R}})$ is unique and satisfies the first-order conditions

$$\frac{u'(d)}{u'\left(\frac{R}{P_2}\frac{y+P_2(1-y)-\lambda d}{1-\lambda}\right)}-\frac{R}{P_2}=0, \tag{12}$$

and

$$\frac{u'\left(y+P_1(1-y)\right)}{u'\left(\frac{R}{P_2}\frac{y+P_2(1-y)-\lambda d}{1-\lambda}\right)}-\frac{1-p}{p}\frac{P_2-1}{1-P_1}\frac{R}{P_2}\leq 0. \tag{13}$$

with strict inequality only if $y^{\mathscr{R}}=0$.

As regards the structure of the banking sector, there are potentially three types of equilibria.

Definition 2 *Suppose* $\left((y^{\mathscr{S}},d^{\mathscr{S}},x^{\mathscr{S}}),(y^{\mathscr{R}},d^{\mathscr{R}},x^{\mathscr{R}}),\mathbf{P},\rho\right)$ *is an equilibrium for a given probability distribution. It is an equilibrium with a* ***safe banking sector*** *if* $\rho=0$*; with a* ***risky banking sector*** *if* $\rho=1$*; and with a* ***mixed banking sector*** *if* $\rho\in]0,1[$.

In the remainder of this section we examine the role of the probability distribution of the extrinsic state. We describe the characteristics of the banking sector implied by the probability of coordination failures and its implications for equilibrium determinacy.

3.2 Safe banking sectors

We begin with equilibria with a safe banking sector and stable asset prices.

Theorem 2 *There is a* $\check{p}<1$ *such that an equilibrium with a safe banking sector and stable asset prices exists if and only if* $p\geq\check{p}$*. Consumers' expected utility in those equilibria is bounded away from the first-best expected utility.*

Proof: See Appendix B.2. □

Intuitively, neither do banks go bust nor do asset prices fluctuate as a result of extrinsic risk in such an equilibrium. Asset prices are equal across states only if $P_1=P_2=1$, which has two major implications. First, an individual safe bank's reserves are indeterminate, as structuring its portfolio at $t=0$ is as good as structuring it at $t=1$. A bank can simply buy and sell the long asset at $t=1$ as a unit of reserves is as much worth as a unit of the long asset at both dates. In aggregate, however, the banking sector makes provisions against bank runs driven by coordination failures. There are just sufficient reserves in the banking sector to allow all banks to pay out all depositors at $t=1$, i.e. $\lambda d^{\mathscr{S}}=y^{\mathscr{S}}$. Second, while there may be a trade of assets at $t=1$, it does not affect the consumption for impatient or patient consumers. The market value of a bank's total assets at $t=1$ is always one regardless how it is structured. Hence, impatient consumers always get one unit of consumption and patient consumers always get R units.

If there were any risky bank for $\mathbf{P} = (1,1)$, its individual reserve holdings would also be indeterminate. For the extrinsic state in which no coordination failure occurs, a risky bank would provide the first-best liquidity insurance, which is better than what safe banks offer. However, there is a cost. In the extrinsic state in which consumers coordinate to run on the risky bank, a safe bank pays R units to patient consumers while a risky bank pays only one unit. Hence, if the probability of the state in which coordination failures can occur is sufficiently large, this disadvantage of risky banks is too large and safe banks are strictly preferred by all consumers.

Safe banking sectors may not only exist for $\mathbf{P} = (1,1)$ but also if asset prices differ across states. In any equilibrium without risky banks there is no liquidity demand. Hence, $q^S = 0$ must hold for $\rho = 0$. According to Equation (9b), a necessary and sufficient condition thus is $y^{\mathscr{S}} = \lambda P_1/(\lambda P_1 + 1 - \lambda)$, implying $d^{\mathscr{S}} = P_1/(\lambda P_1 + 1 - \lambda)$. With $\left(y^{\mathscr{S}}, d^{\mathscr{S}}\right)$ being set at date $t = 0$ and no trade with risky banks taking place at date $t = 1$, consumption does not depend on the extrinsic state. Consumption depends, however, on asset prices. Impatient consumers get $P_1/(\lambda P_1 + 1 - \lambda)$ and patient consumers get $R/(\lambda P_1 + 1 - \lambda)$.

A safe bank will find it optimal to hold reserves equal to $\lambda P_1/(\lambda P_1 + 1 - \lambda)$ and promise impatient consumers to pay $P_1/(\lambda P_1 + 1 - \lambda)$ if and only if prices are such that they are a solution to its first-order condition (11). Let h be a correspondence such that for $P_1 \in [\varepsilon, 1]$

$$h(P_1) = \left\{ P_2 \in [1, R] \,\middle|\, P_2 \text{ satisfy (11) and } y^{\mathscr{S}} = \lambda P_1/(\lambda P_1 + 1 - \lambda) \right\}. \tag{14}$$

Accordingly, the solution to a safe bank's optimization problem implies a liquidity supply of zero provided $P_2 = h(P_1)$. If $h(P_1) = \emptyset$ then P_1 is incompatible with a zero-liquidity supply. For $h(P_1) \neq \emptyset$, the correspondence h satisfies

$$h(P_1) = \frac{\lambda P_1 + (1-\lambda)}{1 - \frac{1}{1-p}\frac{1-P_1}{P_1}\left(\lambda \frac{u'\left(\frac{P_1}{\lambda P_1 + 1 - \lambda}\right)}{u'\left(\frac{R}{\lambda P_1 + 1 - \lambda}\right)}\frac{P_1}{R} + p(1-\lambda)\right)}. \tag{15}$$

One characteristic of h is $h(1) = 1$. Recall that for $P_1 = P_2 = 1$, banks are indifferent between buying and selling the long asset at $t = 1$ regardless which state the economy is in. An individual bank's supply of liquidity is thus indeterminate, as any reserve holdings are consistent with maximizing consumers' expected utility. This includes the amount of reserves for which liquidity supply is zero.

Another characteristic of h is that it is a continuous and monotonically decreasing function for $P_1 \in [h^{-1}(R), 1]$ with $\lim_{p \to 1} h^{-1}(R) = 1$. Suppose prices are such that safe banks do not hold any excess reserves. With $y^{\mathscr{S}} = \lambda P_1 / (\lambda P_1 + 1 - \lambda)$, the budget constraints (6a) and (6b) thus require that consumption for patient as well as impatient consumers is state-independent. Everything else equal, a marginal increase in the asset price in state $s = 1$ then implies that a safe bank holds not enough reserves to cover its promised payments. Accordingly, to maintain a zero liquidity supply, safe banks have to find it optimal to increase their reserve holdings. As consumption is lower for patient consumers and higher for impatient consumers, this is the case only if the asset price in state $s = 2$ is lower.

As a third characteristic, liquidity supply is positive for all $P_1 < h^{-1}(P_2)$ and negative for all $P_1 > h^{-1}(P_2)$. This is because the first-order condition (11) implicitly defines $y^{\mathscr{S}}$ as a function of P_2 for any given P_1. Evaluated at $y^{\mathscr{S}} = \lambda P_1 / (\lambda P_1 + 1 - \lambda)$, this function satisfies $dy^{\mathscr{S}}/dP_2 < 0$. Since for every P_1 there is a unique P_2 such that $q^S = 0$. Therefore, we conclude for all $P_1 < h^{-1}(P_2)$ that $y^{\mathscr{S}} > \lambda P_1 / (\lambda P_1 + 1 - \lambda)$ and thus $q^S > 0$ (and vice versa).

These consideration lead to our next result.

Theorem 3 *Suppose* $\left((y^{\mathscr{S}}, d^{\mathscr{S}}, x^{\mathscr{S}}), (y^{\mathscr{R}}, d^{\mathscr{R}}, x^{\mathscr{R}}), \mathbf{P}, \rho\right)$ *is an equilibrium with a safe banking sector and stable asset prices. If* $V^{\mathscr{S}}(\mathbf{P}) > V^{\mathscr{R}}(\mathbf{P})$ *for* $\mathbf{P} = (1,1)$*, there are other equilibria with a safe banking sector in which asset prices and consumption are indeterminate. Expected utility in such equilibria is the larger the higher is the asset price* P_1*.*

Proof: See Appendix B.3. □

Clearly, a sufficient condition for $V^{\mathscr{S}}(1,1) > V^{\mathscr{R}}(1,1)$ is $p > \check{p}$. Asset prices are indeterminate because if safe banks offer a strictly better expected utility than risky banks for $\mathbf{P} = (1,1)$, asset prices can deviate somewhat from $\mathbf{P} = (1,1)$ and safe banks are still the better choice. This holds also true for any combination of asset prices in some neighborhood of $\mathbf{P} = (1,1)$ that satisfy the zero-liquidity supply condition (14).

This implies that there is indeterminacy of asset prices. In Allen and Gale (2004a) equilibria with only safe banks and indeterminacy of asset prices also exist if the difference between the fundamentals in the intrinsic states goes to zero. There, indeterminacy has no real implications though. For economies with coordination failures, however, the indeterminacy has real implications. The possibility of coordination failures requires safe banks to hold more reserves relative to the payments promised to consumers who withdraw early. The more volatile asset prices are, the lower is the asset price in the state with possible coordination failures and the tighter is this constraint on safe banks. Therefore, banks have to adjust their portfolio and the consumption bundles they offer to consumers.

3.3 Risky banking sectors

To understand equilibria with a risky banking sector and the circumstances in which they may exist, we start with the following observation.

Lemma 1 *Suppose* $\left((y^{\mathscr{S}},d^{\mathscr{S}},x^{\mathscr{S}}),(y^{\mathscr{R}},d^{\mathscr{R}},x^{\mathscr{R}}),\mathbf{P},\rho\right)$ *is an equilibrium and let*

$$\hat{p} := \frac{R-1}{R-1+u'\left(\frac{\lambda R}{\lambda R+1-\lambda}\right)/u'\left(\frac{R}{\lambda R+1-\lambda}\right)}.$$

Then the banking sector cannot be risky in equilibrium if $p > \hat{p}$*.*

Proof: See Appendix B.4. □

This result has the following intuition. Without safe banks, there is no supply of reserves at the interim date regardless which state the economy is in. Hence, a necessary condition for banking sectors to be risky is that liquidity demand is zero in both states. In the state with coordination failure, liquidity demand is zero if and only if the asset price is not larger than the physical liquidation value of assets: banks prefer to liquidate production over selling. Things are more delicate in the state without coordination failure. There, liquidity demand is zero if and only if reserves held by risky banks exactly cover its total payout to impatient consumers. Provided that reserves equal payouts to impatient consumers, the budget constraint requires that patient consumers get less the more reserves these banks holds. But there is a lower limit to the optimal consumption of patient consumers. This is because the optimal consumption plan requires that the marginal rate of substitution between consumption when patient and when impatient needs to be equal to the rate of return on holding the long asset between date 1 and date 2; see Equation (12). This rate of return is the lower the higher is the asset price. However, there is a lower bound for the rate of return since R, the return on production as of date $t = 0$, forms an upper bound for the asset price. Hence there is an upper bound on the consumption of patient consumers. This in turn implies that there is a threshold for reserves above which it is better for consumers to have some payout in excess of the bank's reserves when becoming impatient. Liquidity demand would not be zero in the state in which no coordination failures happen. Since optimal reserves are the larger the higher is the probability of coordination failures, liquidity demand can be zero in both states if and only if the probability of coordination failures is below some threshold $\hat{p}$.

The upper bound $\hat{p}$ on the probability of coordination failures is smaller than $(R-1)/R < 1$ and depends on the characteristics of the economy. It is the lower the smaller the share of early consumers λ is. Provided liquidity demand is zero, fewer impatient consumers implies that the maximum payoff to consumers in the state without a bank run is larger while the maximum payoff in case of a bank run is smaller. Consumers will find this consumption profile efficient only if they are less likely to experience a bank run. The effects of the return on the long asset R on $\hat{p}$ are generally not clear-cut, for there are two effects possibly working in opposite directions. On the one hand, a larger R eases the upper bound on P_2. This allows patient consumers to get more for any given reserve holdings, and in order to re-balance their optimal consumption profile consumers want to consume more when impatient too. Banks can offer this even without resorting to the asset market in the state without coordination failure by holding more reserves. Hence, the probability of coordination failures, which determines optimal reserve holdings, can be higher. On the other hand, a larger R also changes the optimum consumption profile for consumers in case of a run compared to what they get as late consumers in case there is no run. If u exhibits constant relative risk aversion with $k(x) = \kappa$, however, increasing reserves according to the first effect is more than enough to re-balance the marginal utilities across those states. Then the threshold for the probability of coordination failures is $\hat{p} = (R-1)/\left(R-1+\lambda^{-\kappa}\right)$ with $d\hat{p}/dR > 0$ and $d\hat{p}/d\kappa < 0$.

For risky banking sectors to exist, zero liquidity demand in both states is necessary but not sufficient. It must also be true that risky banks offer deposit contracts which generate a higher expected utility than deposit contracts offered by safe banks. This leads to our next main result.

Theorem 4 *There is a $\bar{p} > 0$ with $\bar{p} \leq \hat{p}$ such that for all $p \leq \bar{p}$ an equilibrium with a risky banking sector exists. Such equilibrium is locally isolated. A consumer's expected utility approaches the first-best expected utility if the probability of coordination failures converges to zero.*

Proof: See Appendix B.5. □

In an equilibrium with a risky banking sector, all banks survive in one state and none survives in the other state. If the extrinsic state with coordination failure materializes, all banks are forced to exchange their long assets for reserves simultaneously. As there is no bank supplying any reserves, all banks have to physically liquidate their assets. This is an equilibrium if coordination failures are sufficiently unlikely. The reason is that with a low probability of coordination failures, it does not pay for any bank to be safe. The prospects of buying assets at fire sale prices are slim while fending off a bank run to be able to buy assets from distressed banks is costly. It requires a bank to hold large reserves relative to what it promises to impatient consumers. If the probability of profiting from buying assets at fire sale prices is very low, there is thus no scope for safe banks to sufficiently compensate their consumers for the efficiency loss associated with the more liquid portfolio.

With a risky banking sector, the real outcome is well defined in equilibrium. This is because the equilibrium is locally isolated. Local comparative statics reveal that, if the probability of coordination failures approaches zero, reserves held by risky banks converge to the first-best reserve holdings. Therefore, expected utility also converges to its first-best level. While the asset price in the crisis state is always equal to the physical liquidation value, the asset price in the no-crisis state converges to one.

The finding that all banks can be risky while asset prices are determinate is worthy of comparison to Allen and Gale (2004a). They consider economies which differ from ours only in that there is no extrinsic risk but intrinsic risk. Accordingly, banks only fail for fundamental reasons there. They show that for any probability distribution of instrinsic states, all banks are safe and asset prices are indeterminate in equilibrium if the difference between the fundamentals in the intrinsic states converge to zero. This indeterminacy has no real effects though. For the sake of comparison, their intrinsic states can be considered as our extrinsic states once the difference in fundamentals is reduced to zero. In our model, these states determine whether there is a coordination failure or not. Letting the probability of coordination failures approach zero does not change the main insights: Banks will always be prone to bank runs driven by coordination failures in our model and, although without real implications, asset prices will always remain indeterminate in Allen and Gale (2004a).

3.4 Mixed banking sectors

If risky banks can sell their productive investments in a bank run, no productive investment made by them will ever go to waste. If safe banks can buy additional productive investments, excess reserves they hold are not idle but available to risky banks without jeopardizing the stability of safe banks. There are thus potentially gains from trading the extrinsic risk with each other. In an equilibrium with a mixed banking sector, these gains from trade are realized and shared.

A mixed banking sector is the result of an equilibrium in mixed strategies. With probability ρ a consumer goes to a risky bank and with probability $1-\rho$ to a safe bank. Whether such an equilibrium exists depends on whether there are feasible asset prices for which both types of banks are equally good to consumers while liquidity supply is positive and liquidity demand is positive and state-independent. The latter is required because liquidity supply is state-independent and markets have to clear in all states.

According to the Envelope theorem, indirect utilities $V^{\mathscr{R}}(\mathbf{P})$ and $V^{\mathscr{S}}(\mathbf{P})$ are characterized by

$$\frac{dV^{\mathscr{R}}(\mathbf{P})}{dP_2} = \quad (1-p)\,u'\left(x_{2,2}^{\mathscr{R}}\right)\frac{R}{P_2}\frac{q_2^D}{P_2} \in \begin{cases} \mathbb{R}_{++} & \text{if} \quad q_2^D > 0, \\ \{0\} & \text{if} \quad q_2^D = 0, \\ \mathbb{R}_{-} & \text{if} \quad q_2^D < 0, \end{cases} \tag{16a}$$

$$\frac{dV^{\mathscr{S}}(\mathbf{P})}{dP_2} = -(1-p)\,u'\left(x_{2,2}^{\mathscr{S}}\right)\frac{R}{P_2}\frac{q^S}{P_2} \in \begin{cases} \mathbb{R}_{-} & \text{if} \quad q^S > 0, \\ \{0\} & \text{if} \quad q^S = 0, \\ \mathbb{R}_{++} & \text{if} \quad q^S < 0, \end{cases} \tag{16b}$$

$$\frac{dV^{\mathscr{R}}(\mathbf{P})}{dP_1} = p(1-y^{\mathscr{R}})u'\left(x_{1,1}^{\mathscr{R}}\right) > 0. \tag{16c}$$

The sign of $dV^{\mathscr{S}}(\mathbf{P})/dP_1$ is not clear. Let g be a correspondence such that for $P_1 \in [\varepsilon, 1]$

$$g(P_1) = \left\{ P_2 \in [1,R] \,\middle|\, q_2^D \geq 0, q^s \geq 0 \text{ and } V^{\mathscr{R}}(\mathbf{P}) - V^{\mathscr{S}}(\mathbf{P}) = 0 \right\}. \tag{17}$$

If $P_2 = g(P_1)$, a consumer is indifferent between safe and risky banks, which is one necessary condition for a mixed banking sector. Provided $g(P_1) = \emptyset$ for a given P_1, there is no P_2 such that risky and safe banks are equally good from a consumers perspective. Either risky banks are strictly better than safe banks or safe banks are strictly better than risky banks for this P_1 regardless P_2.

Provided $g(P_1) \neq \emptyset$, the above characteristics of the indirect utilities thus imply that the correspondence g is an injective function and a consumer strictly prefers a risky bank over a safe bank if and only if $P_2 > g(P_1)$. A higher asset price in state $s = 2$ makes a risky bank more attractive because it can offer more consumption to patient consumers while holding fewer reserves. It makes a safe bank less attractive because its patient consumers get less if the bank cannot buy as many long assets in state $s = 2$ in exchange for a given amount of excess reserves.

The other necessary condition for mixed banking sectors, as for any equilibrium with $\rho > 0$, is that liquidity demand is state-independent. According to Equation (9a), $q_1^D = q_2^D$ requires $d^{\mathscr{R}} = \left(y^{\mathscr{R}} + P_1(1 - y^{\mathscr{R}})\right)/\lambda$. To derive feasible prices $\mathbf{P}$ that induce risky banks to find it optimal to set $y^{\mathscr{R}}$ and $d^{\mathscr{R}}$ such that liquidity demand is state-independent, we define a correspondence f such that for $P_1 \in [\varepsilon, 1]$

$$f(P_1) = \begin{cases} \left\{ (y^{\mathscr{R}}, P_2) \in \{0\} \times [1,R] \,\middle|\, \left(y^{\mathscr{R}}, d^{\mathscr{R}}\right) \text{ satisfy (12) and } d^{\mathscr{R}} = P_1/\lambda \right\}, \\ \left\{ (y^{\mathscr{R}}, P_2) \in]0,1] \times [1,R] \,\middle|\, \left(y^{\mathscr{R}}, d^{\mathscr{R}}\right) \text{ satisfy (12), (13) and } d^{\mathscr{R}} = \left(y^{\mathscr{R}} + P_1(1 - y^{\mathscr{R}})\right)/\lambda \right\}. \end{cases} \tag{18}$$

If $f(P_1) = \emptyset$, then P_1 is incompatible with state-independent liquidity demand. For $f(P_1) \neq \emptyset$, let $(\boldsymbol{y}^{\mathscr{R}}, \boldsymbol{P}_2)$ denote a solution to Equation (18). Then, $(y^{\mathscr{R}}, d^{\mathscr{R}})$ is a solution to a risky bank's optimization problem and the implied liquidity demand is state-independent provided $y^{\mathscr{R}} = \boldsymbol{y}^{\mathscr{R}}$ and $d^{\mathscr{R}} = \left(P_1(1 - \boldsymbol{y}^{\mathscr{R}}) + \boldsymbol{y}^{\mathscr{R}}\right)/\lambda$. In principle, there can be many solutions for a given P_1. Let ϕ be the projection of f, as defined in (18), on the P_2-coordinate. Then, a mixed banking sector is characterized by an asset price $P_1 \in [\varepsilon, 1]$ for which $\phi(P_1) = g(P_1) \neq \emptyset$. Except perhaps for some pathological cases, these equilibria are locally isolated and thus determinate.

Unfortunately, it is difficult to explicitly state under which circumstances a mixed banking sector exists. However, we can specify two conditions that are sufficient to rule out a mixed banking sector. Recall Theorem 2 which has established $p \geq \check{p}$ as the condition that is necessary and sufficient for an equilibrium with safe banking sectors and stable asset prices to exist. Satisfying this condition does not exclude though that other equilibria in which risky banks operate may also exist.

Theorem 5 *There is a $\tilde{p} \in [\check{p}, 1[$ such that for all $p > \tilde{p}$, risky banks cannot exist in equilibrium.*

Proof: See Appendix B.6. □

Intuitively, suppose there is scope for risky banks to exist for some $p > \check{p}$. A sufficient condition that there is some larger probability $\tilde{p}$ above which no risky bank operates is that risky banks never exist if the sunspot probability approaches one. For $p \to 1$, risky banking sectors do not exist (see Lemma 1). Moreover, market clearing in both states implies that the asset price in state $s = 1$ converges to one regardless the asset price in the other state. The reason is that risky banks, which are highly unlikely to survive, are willing to give up a lot in terms of consumption for patient consumers in case of survival. However, in any equilibrium with risky banks, their demand for liquidity has to be the same across states. Therefore, a large supply of investments in the state without coordination failure has to be matched with a large liquidity demand in the state with coordination failure. The latter is maximal for $P_1 = 1$. Given this price and the (almost) certainty of coordination failures, even if risky banks make productive investments, their returns are (almost) never collected and the total asset value of risky banks is (almost) always equal to 1. Hence risky banks do not provide any meaningful liquidity insurance and the best they can do for consumers is just about as good as storage. Safe banks, however, always collect the returns on the productive investments they make. They also offer at least some liquidity insurance. Hence, only safe banks will exist in equilibrium.

Similarly, Lemma 1 and Theorem 4 have established $p \leq \hat{p}$ as a necessary and $p \leq \bar{p}$ as a sufficient condition for the existence of risky banking sectors. Satisfying these conditions alone does not rule out other equilibria in which safe banks exist though.

Theorem 6 *There is a $\check{p} \in]0, \bar{p}]$ such that for all $p < \check{p}$, safe banks cannot exist in equilibrium.*

Proof: See Appendix B.7. □

The intuition is as follows. Suppose there is scope for safe banks to exist for some $p \leq \bar{p}$. A sufficient condition that there is some smaller probability $\breve{p}$ below which no safe bank operates is that safe banks never exist if the sunspot probability approaches zero. Then, a risky banking sector exists (see Theorem 4). Moreover, market clearing in both states requires that the asset price in state $s = 2$ converges to one, not just for $P_1 = \varepsilon$ but for all P_1 for which the risky bank finds it optimal to hold some positive amount of reserves on its own. With the price P_2 converging to one, the first-order condition (12) implies that consumption by impatient and patient consumers in the state without coordination failures converge to their respective first-best levels. Bearing in mind that state-independent liquidity demand requires $d^{\mathscr{R}} = \left(y^{\mathscr{R}} + P_1(1 - y^{\mathscr{R}})\right)/\lambda$ for any p, reserves held by risky banks thus converge to $(y^* - P_1)/(1 - P_1)$.

Only if $P_1 = \varepsilon$, a risky bank does not tap into the asset market. For prices P_1 higher than ε the risky bank holds fewer reserves than in the first-best y^*. To be still able to provide impatient consumers with the first-best consumption, risky banks would have to exchange some of their productive investments for stored goods at date $t = 1$ in state $s = 2$. These stored goods would have to come from safe banks. Safe banks, however, will not exist in equilibrium for $P_1 \in [\varepsilon, y^*]$. Provided the sunspot probability approaches zero, and the asset price in state $s = 2$ thus converges to one, the optimal storage by a safe bank $y^{\mathscr{S}}$ is one. Notwithstanding a probability of a coordination failure of almost zero, a safe bank has to structure its portfolio such that it is run-proof even in the highly unlikely state with possible coordination failures. Therefore, safe banks would not be able to match the expected utility offered by a risky bank.

For $P_1 > y^*$, reserves held by risky banks are zero. It then follows from Equation (18) that state-independence of liquidity demand requires that P_2 is larger than one and increasing in P_1. According to Equations (16a) and (16c) higher prices in both states imply that risky banks would offer an expected utility that is even larger than the first-best. An equilibrium in which safe banks would co-exist at those prices would thus imply that all banks offer more than the first-best expected utility. This is not feasible.

Finally, a safe banking sector cannot be an equilibrium either. For any P_2, to ensure a zero liquidity supply, the associated P_1 would be even higher than with state-independent liquidity demand because $\phi^{-1}(P_2) \leq h^{-1}(P_2)$. A higher P_1 would make risky banks even more attractive because $dV^{\mathscr{R}}(\mathbf{P})/dP_1 > 0$ (see Equation (16c)), while the maximum utility a safe banking sector can offer is strictly lower than the first-best expected utility (see Theorems 2 and 3). Therefore, only risky banks exist in equilibrium.

To sum up, mixed banking sectors require that risky and safe banks co-exist in equilibrium. Therefore, mixed banking sectors are feasible only for probability distributions of the extrinsic state for which neither risky banks nor safe banks are ruled out, i.e. for $p \in]\breve{p}, \tilde{p}[$.

3.5 Graphical illustration and numerical examples

The three curves in Figure 1 are possible graphs of the three conditions derived above. The blue graph $h(P_1)$ depicts all combinations of prices P_1 and P_2, as implicitly defined in Equation (14), for which liquidity supply of safe banks is zero. It is downward-sloping, goes through $(1,1)$ and does not intersect the vertical axis within the interval $[1,R]$. For any price combination to the northeast of this graph, the liquidity supply is negative which cannot be in any equilibrium. Hence, only price combinations directly on or to southwest of that line can hold in equilibrium. The slope of the graph depends on the sunspot probability. The higher it is, the steeper is the graph and for $p \to 1$ the graph converges to a vertical line through $(1,1)$.

The red graph $g(P_1)$ represents all price combinations such that consumers are indifferent between depositing their endowments with a safe or a risky bank, as defined in Equation (17). Little is known about this graph, but if it exists within the relevant range of prices it is the graph of an injective, continuous function. For price combinations below the red graph, consumers strictly prefer a safe bank, and for those above they strictly prefer a risky bank. Hence, equilibria in which both safe and risky banks exist lie on the red graph, equilibria with solely safe banks lie below, and those with solely risky banks lie above.

The orange graph $\phi(P_1)$ represents the condition for state-independent liquidity demand, with ϕ being the projection of f, as defined in (18), on the P_2-coordinate. Equilibria in which risky banks operate lie on this line. This graph has two branches. Consider first the right branch in Figure 1, where prices are such that risky banks do not hold any reserves and their demand for reserves is state-independent. An increase in the asset price in state $s = 1$ then implies that the banks' demand for reserves increases in this state, see Equation (9a). The demand for liquidity thus remains state-independent only if a risky bank's demand for reserves increases also in the other state, i.e. if it offers

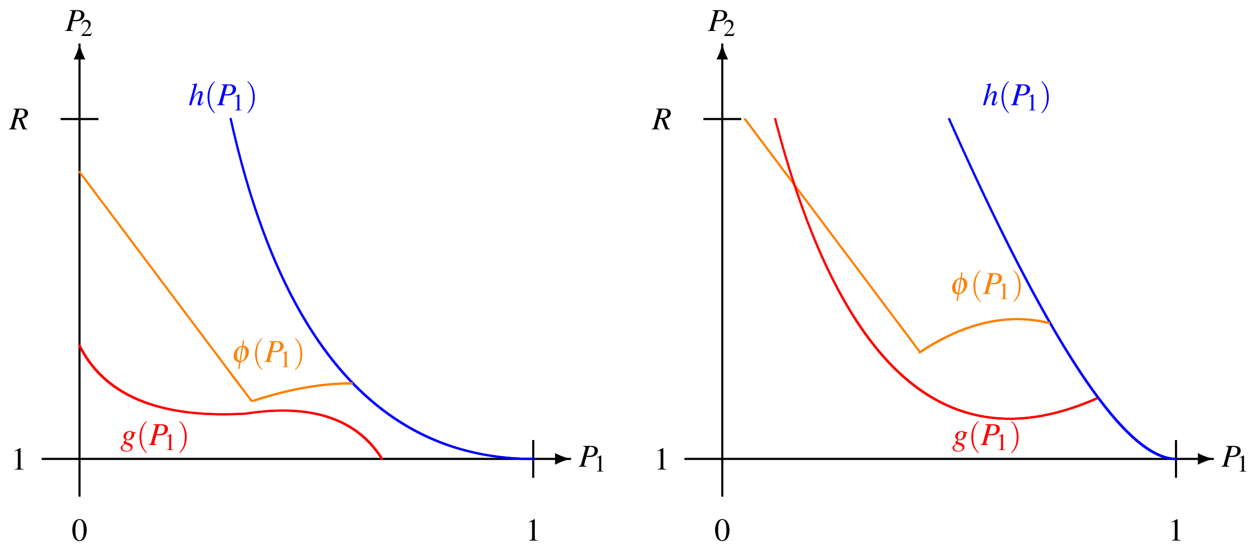

Figure 1: Equilibrium conditions.

a higher face value of deposits. According to the first-order condition (12), this is optimal only if the asset price in state $s = 2$ is also higher. Then, the bank would have to sell less projects to pay impatient consumers a given amount, leaving more for both, patient and impatient consumers.

The situation is more complex when prices are such that risky banks' demand for reserves is state-independent but they hold some positive amount of reserves on their own. This situation refers to the left branch of the graph in Figure 1. There, the bank will in general also adjust its reserve holding when the price in state $s = 1$ changes, which renders the shape of the graph rather difficult to determine. However, a sufficient condition for the asset price in state $s = 2$ and the reserve holdings $y^{\mathscr{R}}$ to be a continuous, monotone and decreasing function of the asset price in state $s = 1$ is relative risk aversion to be non-increasing.[2]

Non-increasing relative risk aversion has a straightforward intuition which makes it a reasonable case to consider. The possibility of a coordination failure creates additional volatility in consumption for both, patient and impatient consumers. If relative risk aversion is non-increasing, this risk tends

[2]For a formal derivation see Appendix C. Non-increasing relative risk aversion is a common assumption made in models of bank runs (e.g. Fecht, 2004) or in macro models with banks (e.g. Gertler and Kiyotaki, 2015) where risk aversion is often even constant.

to be more harmful to impatient consumers. To see this, consider the solutions $(\boldsymbol{y}^{\mathscr{R}}, \boldsymbol{P}_2)$ to (18) for $y^{\mathscr{R}} > 0$. Combining (12) and (13) implies that these solutions also satisfy

$$\frac{u'\left(y^{\mathscr{R}} + P_1(1-y^{\mathscr{R}})\right)}{u'\left(\frac{y^{\mathscr{R}}+P_1(1-y^{\mathscr{R}})}{\lambda}\right)} - \frac{1-p}{p}\frac{P_2-1}{1-P_1} = 0. \tag{19}$$

For a higher probability of coordination failures, it is optimal for consumers that the bank holds more reserves $y^{\mathscr{R}}$ if and only if doing so lowers the marginal rate of substitution between consumption in the state with coordination failures and consumption of impatient consumers in the other state. For given prices, a consumer then asks for more consumption in the state with coordination failures and is willing to forfeit some of the liquidity insurance the bank offers in the other state. This is equivalent to saying that risk aversion is decreasing, which is a necessary and sufficient condition for the marginal rate of substitution to be decreasing in $y^{\mathscr{R}}$.

In the left panel in Figure 1, for all prices for which liquidity supply is zero, risky banks offer a strictly better expected utility than safe banks, because the blue graph is above the red. Hence, there cannot be an equilibrium with solely safe banks; at least some risky banks will operate in equilibrium. However, liquidity demand then has to be state-independent. For those price combinations, i.e. on the orange graph, consumers are also strictly better off with a risky bank than with a safe bank, because the red graph is below the orange graph. Hence there cannot be any safe bank in equilibrium. Therefore, constellations as in the left panel represent an equilibrium with solely risky banks. As shown in Theorem 4, such equilibrium requires $P_1 = \varepsilon$. Hence the isolated and unique equilibrium lies where the ϕ graph (almost) intersects the vertical axis.

In the right panel, there are price combinations for which safe banks do not supply any liquidity and are better than risky banks from a consumer's perspective. Those are thus equilibrium combinations and lie on the bottom part of the blue graph, below its intersection with the red graph. There is another equilibrium at the intersection of the orange with the red graph. There, liquidity demand is state-independent, liquidity supply is positive, and safe and risky banks are equally good from a consumer's perspective.

The following numerical examples illustrate our results and show that equilibria with mixed banking sectors can exist and that there can be multiple equilibria. In all examples, the Bernoulli utility function is $u(x) = -x^{-1}$, implying constant relative risk aversion of $k(x) = 2$. Moreover, the physical liquidation value is set to $\varepsilon = 10^{-29}$ which we approximate below by zero.

Recall that the condition for state-independent liquidity demand is $(y^{\mathscr{R}}, P_2) = f(P_1)$ where f is defined in Equation (18). For our examples, the projection ϕ of f on the P_2-coordinate then satisfies

$$\phi^{-1}(P_2) = \begin{cases} 1 - \lambda^2 \frac{1-p}{p} (P_2 - 1) & \text{if} \quad y^{\mathscr{R}} \in]0,1[\\ \\ P_2 \left(1 + \frac{1-\lambda}{\lambda} \left(\frac{P_2}{R}\right)^{0.5}\right)^{-1} & \text{if} \quad y^{\mathscr{R}} = 0. \end{cases} \tag{20}$$

The condition for zero liquidity supply is $P_2 = h(P_1)$ where h is defined in Equation (14). For our examples this function reads

$$h(P_1) = \frac{1 - \lambda + \lambda P_1}{1 - \frac{1}{1-p} \frac{1-P_1}{P_1} (\lambda R / P_1 + (1-\lambda) p)}. \tag{21}$$

The condition under which consumers are indifferent between bank types is $P_2 = g(P_1)$ with g being defined in (17). Deriving g is somewhat difficult because the set of price combinations satisfying this condition can often be empty for some subsets of its domain. Therefore, we calculate and compare indirect utilities with safe and risky banks, respectively, for prices satisfying either $P_1 = \phi^{-1}(P_2)$ or $P_1 = h^{-1}(P_2)$ or both. Price combinations for which indirect utilities are equal for safe and risky banks constitute an equilibrium with a mixed banking sector. With these price combinations we calculate d^{τ} and y^{τ} for $\tau \in \{\mathscr{R}, \mathscr{S}\}$, and the implied individual liquidity demand and supply determine the share ρ of risky banks according to

$$\rho = \frac{y^{\mathscr{S}} - \lambda \left(y^{\mathscr{S}} + P_1(1 - y^{\mathscr{S}})\right)}{y^{\mathscr{S}} - \lambda \left(y^{\mathscr{S}} + P_1(1 - y^{\mathscr{S}})\right) + d^{\mathscr{R}} - y^{\mathscr{R}}}. \tag{22}$$

Example 1 For $R = 5$, $\lambda = 0.4$ and $p = 0.02$ there is exactly one equilibrium.

$$\rho = 1, \quad P_1 \approx 0, \quad P_2 = 1.127551.$$

This is an example for an equilibrium characterized in Theorem 4 and illustrated in the left panel of Figure 1. Solely risky banks exist and when a bank run occurs, the asset price drops to the liquidation value.

Example 2 For $R = 5$, $\lambda = 0.4$ and $p = 0.13275$ there are exactly two equilibria.

1. $\rho = 0, \quad P_1 = 1, \quad P_2 = 1;$
2. $\rho = 1, \quad P_1 \approx 0, \quad P_2 = 1.956688.$

In this example, an equilibrium characterized in Theorem 2 exists. The example also demonstrates that there can be multiple equilibria. The existence of multiple equilibria has the following intuition. If a consumer expects that all other consumers will deposit their endowments with safe banks, the consumer will also expect that there will be no demand for liquidity at the interim date. If asset prices are furthermore expected to be certain, a safe bank may indeed be the better choice for herself. If she expects, however, that all other consumers will deposit their endowments with risky banks, she also expects that the price of the long asset will drop to its physical liquidation value in the event of a bank run. Provided this price drop implies a very tight constraint on a safe bank's ability to provide liquidity insurance, the consumer may be better off to place her endowment also with a risky bank rather than with a safe bank.

Example 3 For $R = 2.5$, $\lambda = 0.4$ and $p = 0.15$ there are two types of equilibria.

1. $\rho = 0.995991, \quad P_1 = 0.005137, \quad P_2 = 2.097275;$
2. $\rho = 0, \qquad P_1 \geq 0.915570, \quad P_2 = \frac{0.6+0.4P_1}{1-\frac{1}{0.85}\frac{1-P_1}{P_1}\left(\frac{1}{P_1}+0.09\right)} \leq 1.108388.$

In this example, there are not only multiple equilibria in that a mixed banking sector can exist as well as a safe banking sector. There is also indeterminacy as there are infinitely many safe banking sectors which differ with respect to asset prices. This situation is like in the right panel of Figure 1.

Example 4 For $R = 5$, $\lambda = 0.7$ and $p = 0.17$ there is only one equilibrium.

$$\rho = 0.836239, \quad P_1 = 0.306249, \quad P_2 = 1.289987.$$

This example shows that neither a safe nor a risky banking sector but a mixed banking sector can characterize an equilibrium. In an equilibrium with only risky banks, all consumers accept the risk of coordination failures. In an equilibrium with only safe banks, no consumer takes this risk and accepts that the need to hold a liquid portfolio imposes a constraint on the financial service the banks provide. In an equilibrium with a mixed banking sector there is scope for insurance against the extrinsic risk, but it requires that asset prices differ across states.

4 Coordination failures and welfare

Our results so far indicate that there is real indeterminacy of equilibria if the probability of coordination failures is above some threshold $\check{p}$. If the probability is below this threshold there is always asset price volatility and possibly bank failure. This suggests a trade-off between bank stability and determinacy of the real economic outcome. Moreover, multiple equilibria can exist with some featuring a safe banking sector while others imply a mixed or even a risky banking sector. If these equilibria are associated with different expected utility, there may be another trade-off between the stability of the banking sector and the value of banks' financial services to consumers. Therefore, the welfare implications of coordination failures are not trivial.

In this section we sketch some welfare effects. We compare equilibria for a given probability distribution of the extrinsic state and across probability distributions. We also take a dynamic perspective, taking into account the effects of coordination failures on capital accumulation and growth.

4.1 Comparing equilibria

For a given probability distribution, comparing equilibria from a set of multiple equilibria is equivalent to comparing economies which are identical except for the endogenous characteristics of their financial sectors. Accordingly, any differences in the real outcomes can be attributed exclusively to the difference in financial stability.

In equilibria in which risky banks operate, expected utility for depositors of a safe bank is equal to the expected utility for depositors of a risky bank for $\rho \in]0,1[$ or strictly smaller for $\rho = 1$. Hence it suffices to look at the indirect expected utility for depositors of a risky bank at prices for which liquidity demand is state-independent. For $(\boldsymbol{y}^{\mathscr{R}}, \boldsymbol{P}_2) = f(P_1)$, indirect utility is

$$V^{\mathscr{R}}(\mathbf{P}) = pu\left(\boldsymbol{y}^{\mathscr{R}} + P_1(1-\boldsymbol{y}^{\mathscr{R}})\right) + (1-p)\,\lambda u\left(\tfrac{\boldsymbol{y}^{\mathscr{R}}+P_1(1-\boldsymbol{y}^{\mathscr{R}})}{\lambda}\right) + (1-p)\,(1-\lambda)\,u\left(\tfrac{R}{\boldsymbol{P}_2}\tfrac{\boldsymbol{P}_2-P_1}{1-\lambda}(1-\boldsymbol{y}^{\mathscr{R}})\right). \tag{23}$$

A higher P_1 increases the amount which all consumers get in a bank run as well as the amount an impatient consumer gets when there is no bank run. However, the amount a patient consumer gets is lower. A sufficient condition for the sign of $dV^{\mathscr{R}}(\mathbf{P})/dP_1$ being always strictly positive is that relative risk aversion is non-increasing.[3] Then, comparing any two equilibria in which risky banks exist, expected utility is higher in the equilibrium in which the asset price P_1 is higher.

Similarly, comparing any two equilibria in which no risky banks operate, it suffices to consider the expected indirect utility for price combinations for which liquidity supply is zero. For $P_2 = h(P_1)$, this indirect utility is

$$V^{\mathscr{S}}(\mathbf{P}) = \lambda u\left(\frac{P_1}{\lambda P_1 + 1 - \lambda}\right) + (1-\lambda)\,u\left(\frac{R}{\lambda P_1 + 1 - \lambda}\right), \tag{24}$$

which implies that comparing any two equilibria in which no risky banks exist, expected utility is higher in the equilibrium with a higher asset price P_1 (see Theorem 3).

Comparing expected utility between equilibria with risky banks and equilibria without risky banks can be difficult as it depends on the structural parameters of the economy. In Example 2 above, two equilibria exist, one with a safe and another with a risky banking sector. In the equilibrium with a safe banking sector expected utility is $V(\mathbf{P}) = -0.52$ and in the equilibrium with an risky banking sector it is $V(\mathbf{P}) = -0.594364$. In this example, the safe banking sector makes consumers better off.

The probability distribution of the extrinsic state has some further interesting implications. Consider two economies which differ only in the probability of coordination failures and suppose that their fundamentals are such that $\check{p} < \bar{p}$. For one economy, the probability of state $s = 1$ is equal to $\check{p}$. For the other economy the probability is just a little below $\check{p}$. Then, according to Theorems 2 and 4, at least two equilibria are possible for the first economy: one with a safe banking sector and

[3] See Appendix D.

state-independent asset prices, and another with a risky banking sector. For the second economy, an equilibrium with a safe banking sector and state-independent asset prices is not feasible. Hence, consumers are strictly better off in the economy with the higher probability provided its safe banking sector is associated with a higher expected utility than the risky banking sector. To illustrate this point consider again Example 2, and compare this economy with another one which has the same fundamentals but the probability of coordination failures is not $p = 0.13275$ but only $p = 0.132$. There, the only equilibrium is one with a risky banking sector, offering an expected utility of $V(\mathbf{P}) = -0.593598$. If the first economy is in the equilibrium with a safe banking sector, then a sunspot probability which is smaller by only 7.5 basis points comes with a significantly lower expected utility for the second economy.

Although an intramarginal drop in the probability for state $s = 1$ can make consumers worse off, it seems that consumers are best off if the probability for state $s = 1$ approaches zero. According to Theorems 4 and 6, all banks would then find it optimal to expose themselves to the risk of coordination failures and consumers enjoy the first-best expected utility. However, a zero probability for coordination failures may not be socially optimal. The reason is that the accumulation of capital, and thus growth, also depend on the equilibrium which the economy is in.

4.2 Dynamics

To illustrate a mechanism through which coordination failures can affect welfare in a dynamic context, we embed our financial sector in simple overlapping generations economies with production and growth.

Time is discrete and extends from $-\infty$ to $+\infty$. Let $\mathbb{T} = \{2T : T \in \mathbb{Z}\}$ be the set of even dates. At every even date $t \in \mathbb{T}$ a continuum of identical consumers of mass one is born with an endowment of labor $\ell = 1$ which is inelastically supplied at date t. A consumer lives for at most two subsequent dates $t+1$ and $t+2$ and is described by her consumption set $X \in \mathbb{R}^3_+$ and her lifetime utility

$$\Omega_t = u(x_t) + \lambda\,(pu(x_{t+1,1}) + (1-p)u(x_{t+1,2})) + (1-\lambda)\,(pu(x_{t+2,1}) + (1-p)u(x_{t+2,2}))\,. \qquad (25)$$

There is a continuum of firms of mass one. At even dates $t \in \mathbb{T}$, firms transform capital K_t and labor L_t into the consumption good. Capital used at date t is formed from savings of the previous generation of consumers born at $t-2$. It fully depreciates either due to physical liquidation at the interim date $t-1$ or after completion of the production cycle at t. As in the preceding analysis, liquidation yields an infinitesimal return ε. Firms are described by a Cobb-Douglas production function

$$Y_t = \bar{A} K_t^{\alpha} L_t^{1-\alpha}, \tag{26}$$

with output Y_t, output elasticity of capital $\alpha \in]0,1[$ and an individual firm's total factor productivity $\bar{A}$. The latter depends on the aggregate capital stock according to $\bar{A} = A K_t^{1-\alpha}$. Clearing of the labor market, i.e. $L_t = \ell$, implies that aggregate production is given by a standard AK function

$$Y_t = A K_t. \tag{27}$$

Perfect competition among firms ensures that capital and labor are paid according to their marginal product, i.e. $W_t = (1-\alpha) A K_t$ and $R = \alpha A$. To maintain consistency with the previous analysis, capital is more productive than storing goods, i.e. $A > \alpha^{-1}$.

At date $t \in \mathbb{T}$, in the first stage of her life, a consumer saves $W_t - x_t$ for the second stage of her life, which starts at $t+1$. Saving can be either storing consumption goods or holding deposits in banks. Deposit contracts and the bank's asset allocation choice are as described in section 2. Banks can store goods and invest in capital K_{t+2} which they rent to firms for the rental price R.

To simplify matters we assume that the Bernoulli utility function is $u(x) = -x^{1-\kappa}$ with $\kappa > 1$. This assumption has two implications. First, because relative risk aversion κ is constant, the equilibrium portfolio structures of banks and asset prices are independent of the amount consumers put into the bank. Second, because a consumer's lifetime utility is homothetic, optimal consumption when young and savings are linear functions of a consumer's wage income. Expected utility of a consumer born at t can thus be expressed as

$$\Omega_t = -x_t^{1-\kappa} + V(\mathbf{P})(W_t - x_t)^{1-\kappa}, \tag{28}$$

with $V(\mathbf{P})$ being the indirect utility consumers get by saving one unit using a financial sector as described in section 3. Let $\Theta = (-V(\mathbf{P}))^{\frac{1}{\kappa}}$ and $\bar{y} = \rho y^{\mathscr{R}} + (1-\rho) y^{\mathscr{S}}$. Optimal consumption when young and savings are thus $x_t = W_t/(1+\Theta)$ and $W_t - x_t = W_t\Theta/(1+\Theta)$, respectively. Aggregate savings are $S_t = (1-\alpha) A K_t \Theta/(1+\Theta)$ and aggregate investment in capital is $K_{t+2} = (1-\bar{y}) S_t$. Hence, capital evolves according to

$$\frac{K_{t+2}}{K_t} = (1-\bar{y})(1-\alpha)A\frac{\Theta}{1+\Theta}. \tag{29}$$

Equation (29) shows that coordination failures affect growth in two ways. The first refers to how much consumers will save. The lower is the expected utility associated with the financial services a banking sector provides, the less they consume when young and the more they save. This is because consumers want to smooth any change in the value of financial service over their lifetime. The second refers to the allocation of savings between storage and capital investment.

In general, the overall effect the banking sector has on growth can be ambivalent though. Consider for example an economy which can have either a safe banking sector with stable asset prices or a risky banking sector. Provided the safe banking sector offers better services to consumers than the risky banking sector, savings will be smaller with a safe banking sector. However, with a safe banking sector liquidity supply is zero, implying $y^{\mathscr{S}} = \lambda$ which is below the first-best. With a risky banking sector, aggregate liquidity holdings are higher than the first-best.[4] Therefore, of any unit deposited with banks, the share used to form capital will be lower with a risky banking sector.

Consider next an economy which can have only equilibria with safe banking sectors. Although risky banks exist in neither of these equilibria, these safe banking sectors differ in the asset prices. A lower asset price in state $s = 1$ induces consumers to save more as the indirect utility consumers get from investing one unit in the banking sector is increasing in P_1 (see Theorem 3). Moreover, safe banks make more productive investments if the asset price P_1 is lower. This is because market clearing requires that these banks hold reserves of $\lambda P_1/(\lambda P_1 + 1 - \lambda)$ which are increasing in P_1. Therefore, the economy grows stronger the lower the asset price P_1 is in a safe banking sector.

With their implications for capital accumulation and growth, different equilibria can give rise to a discrepancy between how one generation and its following generation would rank equilibria. The reason is an externality: When making their savings and investment decision, consumers do not take into account the effects their decision has on the future generation's labor income. There are three important implications. First, if the financial sector provides first-best liquidity insurance, savings are relatively small and storage is large. With extrinsic risk, however, consumers do not get the optimum liquidity insurance from their banks. Consumers consume less when young and save more.

[4] For details see proof for Lemma 1.

Hence, extrinsic risk harms the current generation but may benefit the next generation. Second, in economies with extrinsic risk and in which only safe banking sectors exist, the asset price in the sunspot state can be indeterminate. The lower the price, the lower is the expected utility for the current generation. However, safe banks also make more productive investments which benefits the next generation. Third, in economies in which risky banking sectors constitute an equilibrium, a bank run will cause the physical liquidation of the capital stock. Labor income of the next generation is thus state-dependent. Future consumers will earn much less when current consumers coordinate on a bank run.[5] No capital is ever liquidated if there are at least some safe banks in equilibrium. In a banking crisis, all capital originated by risky banks will be sold and thus fully available for the next generation.

4.3 Numerical example

To illustrate the welfare implications of coordination failures, consider the following example. Let $R = 2.5$, $\lambda = 0.4$ and $A = 10$, such that $\alpha = R/A = 0.25$. Suppose that at some even date $t \in \mathbb{T}$ the capital stock is $K_t = \frac{100}{(1-\alpha)A}$, such that consumers born at this date earn wages of $W_t = 100$. We compare five different economies. They all share the same fundamentals at t and start with the same capital stock.

Economies A, B and C have the same probability distribution of the extrinsic state with $p = 0.15$. Economy A is in an equilibrium with a mixed banking sector. Economy B is in an equilibrium with a safe banking sector but volatile asset prices. Economy C is in an equilibrium with a safe banking sector and stable asset prices. Note, economies B and C are in equilibia which belong to a set of indeterminate equilibria, where the asset price in the sunspot state is the lowest in economy B and the highest in economy C.

Economies D, E and F have different probability distributions. In economy D the sunspot probability approaches one. According to Theorem 5 only safe banks can exist, and while the asset price in state $s = 1$ is one, the asset price in the other state is indeterminate. There is no real indeterminacy though. For the sake of comparison, we chose the same price as in economy A. Economy E has a

[5] Although not explicitly considered here, one could think of consumers to start working with a different, much less productive technology that requires only labor. This would then kick-start the economy after a the financial crash and hence capital starts growing again.

	p	ρ	P_1	P_2	S_t	$\bar{y}$	K_{t+2}/K_t	Ω_t	Ω_{t+2}
A	0.150	0.996	0.005	2.097	46.927	0.604	1.395	−0.0355	−0.0255
B	0.150	0.000	0.916	1.108	44.712	0.379	2.082	−0.0327	−0.0157
C	0.150	0.000	1.000	1.000	44.444	0.400	2.000	−0.0324	−0.0162
D	1.000	0.000	1.000	2.097	44.444	0.400	2.000	−0.0324	−0.0162
E	0.075	1.000	ε	1.507	45.569	0.564	1.490	−0.0338	−0.0227
F	0.000	1.000	ε	1.000	43.804	0.513	1.599	−0.0317	−0.0198
G	0.150	0.000	0.900	1.020	43.804	0.513	1.599	−0.0317	−0.0198

Table 1: Coordination failures, growth and welfare.
Parametrization: $\kappa = 2$, $R = 2.5$, $\lambda = 0.4$, $A = 10$, $K_t = 13\frac{1}{3}$.

probability for state $s = 1$ of $p = 0.075$. It has only one equilibrium which is with a risky banking sector. In economy F the sunspot probability approaches zero. According to Theorem 6 the only equilibrium is a risky banking sector.

Finally, while for economy G the probability for state $s = 1$ is 0.150 as in economies A, B and C, coordination failures are ruled out in all states. This economy is what Allen and Gale (2004a) refer to as the trivial sunspot equilibrium in which the allocation is the same as in an economy without extrinsic risk. Although asset prices are indeterminate, they will satisfy a no-arbitrage condition which requires that holding a unit of capital from date t must generate the same expected return as the storage of one unit and using it to buy capital at date $t+1$.

Key characteristics for these economies are summarized in Table 1. Comparing economies A, B and C reveals that consumers born at t have a higher expected lifetime utility Ω_t with a safe banking sector than with a risky banking sector. Comparing with the other economies reveals that it would be best for them if the probability of the extrinsic state $s = 1$ approaches zero or coordination failures can be ruled out altogether (economies F and G). However, for the next generation of consumers, born at $t+2$, expected lifetime utility $\Omega_t + 2$ is not the highest if coordination failures are zero-probability events or ruled out entirely. Instead, it is maximal if coordination failures are possible although not certain, all banks make themselves immune against runs and asset prices are volatile. These examples indicate that while the highest welfare is associated with an equilibrium with a safe banking sector, it matters why there is no bank run.

Equilibria with risky banks are poor outcomes for our example economies. This is not only evident from comparing economies A, B and C, which share the same probability distribution, but also from inspecting economy E, which has a smaller probability for state $s = 1$. Interestingly, economy E performs worse for both generations than economies B and C which have a higher probability for state $s = 1$. Economy E performs also worse than economies D, F or G. The reason is that, although savings are higher in economy E, a larger share of them goes into storage to shield consumers against losses in the event of a bank run.

5 Policy implications

In the present section we discusses some policy implications of our preceding analysis. To this end, we refer to the expected utility of the second generation, which includes possible growth implications of the banking sector, as the steady-state welfare, or welfare for short.

5.1 Determinacy and multiplicity of equilibria

Policy makers find it often attractive when economic analysis derives at a determinate and if possible unique solution. This is understandable for two reasons. First, the culprits for bad economic outcomes are easier to identify. Second, one can use comparative static analysis to find out how the economy can be steered towards a desired outcome.

In economies considered here, there is uniqueness if there is no extrinsic risk. The examples used above indicate, however, that one may not arrive at the best possible outcome without a sufficiently large extrinsic risk. Taking into account possible differences in economic growth between economies with identical fundamentals we have demonstrated that without coordination failures, or with certainty about the extrinsic state, the steady-state welfare is not necessarily maximal because capital accumulation is rather slow. Uniqueness of equilibrium may also arise with extrinsic risk. Again, the examples have demonstrated that economies with a unique equilibrium can perform worse than fundamentally identical economies with a probability distribution that allows for multiple equilibria.

	ρ	Ω_{t+2}	$\bar{y}$	$\bar{y}/\bar{d}$	$\dfrac{(1-\lambda)\bar{d}}{1-\bar{y}}$
A	0.996	−0.0255	0.6038	0.4000	2.2858
B	0.000	−0.0157	0.3790	0.4000	0.9156
C	0.000	−0.0162	0.4000	0.4000	1.0000
G	0.000	−0.0198	0.5132	0.4000	1.5811

Table 2: Aggregate indicators of bank liquidity .
Parametrization: $\kappa = 2$, $R = 2.5$, $\lambda = 0.4$, $p = 0.15$, $A = 10$, $K_t = 13\frac{1}{3}$.

In any case, the examples show that safe banking sectors seem key for high welfare. Since a banking sector in which risky banks exist in the presence of extrinsic risk is typically a locally isolated equilibrium, while a safe banking sector can be associated with real indeterminacy, there can be a trade-off between determinacy of economic outcomes and welfare.

5.2 Monitoring aggregate bank liquidity

For policy makers it could be interesting to identify the equilibrium in which the economy is in. Simple indicators, derived from aggregate figures, are particularly helpful if they can be observed before a financial crisis unfolds, i.e. before the extrinsic state materializes. Table 2 contains a selection of such indicators for those four economies from Table 1 for which the probability of state $s = 1$ is $p = 0.15$ (economies A, B, C and G). The fourth column shows the aggregate reserve holdings in the banking sector, $\bar{y} = \rho y^{\mathscr{R}} + (1-\rho)y^{\mathscr{S}}$. The fifth column presents the ratio of aggregate reserves and aggregate repayment obligations of banks to depositors, $\bar{d} = \rho d^{\mathscr{R}} + (1-\rho)d^{\mathscr{S}}$. The sixth column shows the value of deposits that are not going to be withdrawn from the banking sector unless there is a crisis, relative to banks' investments in the long-term asset.

The indicator in the fourth column is a simple aggregate reserve ratio which measures total reserves relative to what banks raise from consumers. The highest reserve ratio exists in economy A, the lowest in economy B. While economy A is the worst performing economy in terms of bank stability, growth and welfare, economy B is the best in all three categories. This suggest that a low rather than a high aggregate reserve ratio indicates better economic outcomes. It may appear counterintuitive that a safe banking sector holds fewer reserves and invests more in production than a banking sector dominated by risky banks. However, asset prices are endogenous. A safe banking sector coincides with asset prices that are higher in the sunspot state than those associated with a mixed banking sector. This is why a safe banking sector invests a lot in production. By contrast, if risky banks dominate the banking sector, market clearing requires that the asset price is very low when there are bank runs. To be still able to pay consumers some meaningful amounts, risky banks store a lot. Safe banks in a mixed banking sector also hold more reserves than in a safe banking sector. This is because the value of their total assets has to be sufficient to prevent depositors from running for asset prices that are much lower than in a safe banking sector.

Another reserve ratio is presented in the fifth column. It measures aggregate reserves relative to the total amount banks have promised to pay depositors at $t = 1$. This indicator has the same value for all economies. Therefore it does not contain any information and is thus useless. This ratio is the same across all economies because safe banks do not speculate on fire sale prices. Therefore, in every economy the banking sector as a whole has just enough reserves to satisfy all impatient consumers in case there is no bank run. This holds regardless which asset prices prevail and how many risky banks exist.

The indicator in the last column is made of two components. Its numerator measures the amount of funds which depositors are entitled to withdraw at $t = 1$ but, provided there is no crisis, do not withdraw from the banking sector. The denominator measures the market value of the productive, long-term investments at the time of their origination. This indicator is helpful in ranking possible equilibria. The lower its value, the better the outcome. The explanation is similar to the one given above. Only in a safe banking sector do safe banks store relatively little because only then is the asset price relatively high in the sunspot state. Moreover, safe banks make relatively modest promises to depositors in case they wish to withdraw early. This implies that patient consumers will leave (or re-deposit) comparatively little in the bank. Note that economy B has the best outcome. It is also the only economy among those four in which the value of the indicator is below one.

This measure can be put into perspective of recent changes to the regulatory framework for banks. In order to mitigate the risk of future funding stress, the new Basel Framework for Liquidity Risk Measurement, Standards and Monitoring demands that banks fund their activities with sufficient stable funding. Starting in 2018, for each bank the available stable funding has to be larger than the required stable funding. In our example, which looks at aggregates, the Net Stable Funding Ratio is much lower in economies with only safe banks than in the economy with both safe and risky banks, and it is below one in the economy which has a stable banking sector, grows fastest and delivers the highest expected utility.

One may argue that the Net Stable Funding Ratio has not been designed for the analysis of aggregates. However, even at the individual bank level can the Net Stable Funding Ratio be misleading. Consider economy A where a mixed banking sector exists. There, as one may expect, safe banks have a higher Net Stable Funding Ratio than risky banks (3.2871 compared to 2.2842) but both are significantly larger than one. The regulator may thus wrongly infer that this banking sector is not prone to bank runs at a large scale for there is plenty stable funding for every bank. By contrast, in economies B and C all banks have much lower individual Net Stable Funding Ratios of at most one. Large Net Stable Funding Ratios at the bank level may thus not be an indicator for a safe banking system but for an economy that braces itself for a rather wide-spread banking crisis. This suggests that a measure which helps with microprudential objectives can be counterproductive from a macroprudential perspective.

5.3 Monitoring asset markets

Asset prices and the share of risky banks are joint equilibrium outcomes. Their interactions form a key element of the economies considered in this paper, with important implications for welfare. We conclude this section with a discussion of three aspects.

First, interbank asset markets allow consumers to share extrinsic risks. At first sight, this appears to be good. However, trading assets among banks is a feature of a banking sector in which some banks will fail with some probability. Because asset prices will drop relatively sharply when these banks fail, such a banking sector invests relatively much in reserves. This is not a good outcome for any generation of consumers, even though the occurrence of a banking crisis affects neither growth nor welfare. Note that only in a mixed banking sector will the demand for liquidity be always positive. It thus appears that some banks invest in long-term production only to sell it later. Therefore, some banks pursuing such a business model is an indicator for an equilibrium with a mixed banking sector.

Second, equilibrium asset prices are key to determine whether or not a bank is safe or risky. However, any assessment of a bank being able to survive some extreme price movements depends on how extreme they should be assumed to be. Suppose an economy is in an equilibrium with a safe banking sector. If regulators would stress-test a bank assuming that prices may drop to the liquidation value, these tests would falsely indicate that the banks are not safe. Conversely, suppose an economy is in an equilibrium with a risky banking sector. Regulators may then falsely conclude that banks are safe if the simulation does not consider a total market freeze. This implies that stress-tests can function properly only if the regulator already knows in which equilibrium the economy is.

The third, and related aspect refers to the newly created Liquidity Coverage Ratio. It requires that banks hold enough unencumbered high-quality liquid assets which can easily and immediately be converted into cash to fully cover their liquidity needs for a 30 day liquidity stress scenario. In our model, reserves and assets are both high-quality liquid assets because they are risk-free and can be immediately transformed into goods. A stress scenario is equivalent to all depositors fully withdrawing from their bank in the state with possible coordination failures. The analysis suggests, however, that the liquidity needs in such a stress episode and the value of high-quality liquid assets are simultaneously determined in equilibrium. Therefore, one cannot easily compare one with the other without knowing in which equilibrium the economy is in.

6 Concluding remarks

Simultaneous asset market crashes and bank failures may be the result of coordination failures among bank depositors triggered by sunspots. In equilibrium, risky banks which expose themselves to such bank runs may well exist even in the absence of fundamental risks. There are other types of equilibria in which safe banks exist. These banks hold portfolios that take away the incentives for consumers to coordinate on bank runs. Consumption by at least some patient and impatient consumers is stochastic if risky banks exist and the financial sector provides too little liquidity insurance when safe banks exist. In any case, consumption deviates from the first-best.

For policy makers, the possibility of coordination failures makes it difficult to decide which equilibrium should be considered better. Two problems arise from coordination failures. One is that the possibility of coordination failures can cause trade-off between instability of the banking sector and determinacy of equilibria. The other problem is related to the welfare implications of coordination failures, which can be difficult to assess when their effects on capital accumulation and growth are taken into account.

We have considered a rather limited set of options for consumers to interact with banks. A key feature in the world financial crisis has been that funds withdrawn from one bank were re-deposited in another bank. This migration of deposits when banks get into distress is a channel through which the available aggregate liquidity is distributed in times of systemic crises. As this channel would work parallel to and possibly interacts with asset markets, the implications of deposit migration on asset prices and the risk-taking behavior of banks in equilibrium remains to be explored.

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A Supersafe banks

This appendix shows that there can be no supersafe banks if $-\frac{u''(x)}{u'(x)}x > 1$. Suppose (4) would never be binding such that the associated FOC are

$$\begin{aligned} u'(d) &= R\left(u'\left(\tfrac{R}{P_1}\tfrac{y+P_1(1-y)-\lambda d}{(1-\lambda)}\right)\tfrac{p}{P_1}+u'\left(\tfrac{R}{P_2}\tfrac{y+P_2(1-y)-\lambda d}{(1-\lambda)}\right)\tfrac{1-p}{P_2}\right), \\ u'(x_{2,1}) &= -\tfrac{1-p}{p}\tfrac{P_1}{1-P_1}\tfrac{1-P_2}{P_2}u'(x_{2,2}). \end{aligned}$$

There is a d which maximizes expected utility and satisfies $d < y + P_1(1-y)$ if

$$u'(y+P_1(1-y)) < p\tfrac{R}{P_1}u'\left(\tfrac{R}{P_1}(y+P_1(1-y))\right)+(1-p)\tfrac{R}{P_2}u'\left(\tfrac{R}{P_2}\tfrac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)}\right).$$

To show that this cannot be, we argue that

$$\tfrac{R}{P_1}u'\left(\tfrac{R}{P_1}(y+P_1(1-y))\right) > u'(y+P_1(1-y)), \tag{A1}$$

and

$$\tfrac{R}{P_2}u'\left(\tfrac{R}{P_2}\tfrac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)}\right) > u'(y+P_1(1-y)), \tag{A2}$$

cannot be true. Condition (A1) cannot hold for $-\frac{u''(x)}{u'(x)}x > 1$ since

$$\tfrac{R}{P_1}u'\left(\tfrac{R}{P_1}(y+P_1(1-y))\right) = u'(y+P_1(1-y)) + \tfrac{1}{y+P_1(1-y)}\int_{y+P_1(1-y)}^{\frac{R}{P_1}(y+P_1(1-y))}\left[u'(x)+xu''(x)\right]dx.$$

As regards condition (A2), consider first the differential equation

$$\begin{aligned} u'(y+P_1(1-y)) &= \tfrac{R}{P_2}\tfrac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)(y+P_1(1-y))}u'\left(\tfrac{R}{P_2}\tfrac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)}\right) \\ &\quad -\tfrac{1}{y+P_1(1-y)}\int_{y+P_1(1-y)}^{\frac{R}{P_2}\frac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)}}\left[u'(x)+xu''(x)\right]dx. \end{aligned}$$

Condition (A2) would hold if

$$u'\left(\frac{R}{P_2}\frac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)}\right)\frac{R}{P_2} \quad > \quad \frac{R}{P_2}\frac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)(y+P_1(1-y))}u'\left(\frac{R}{P_2}\frac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)}\right)$$
$$-\frac{1}{y+P_1(1-y)}\int_{y+P_1(1-y)}^{\frac{R}{P_2}\frac{(1-\lambda)y+\left(P_2-\lambda P_1\right)(1-y)}{(1-\lambda)}}\left[u'(x)+xu''(x)\right]dx.$$

Rearranging terms gives

$$\frac{R}{P_2}u'\left(\frac{R}{P_2}\frac{(1-\lambda)y+(P_2-\lambda P_1)(1-y)}{(1-\lambda)}\right)\left(\frac{(P_2-P_1)(1-y)}{(1-\lambda)}\right) < \int_{y+P_1(1-y)}^{\frac{R}{P_2}\frac{(1-\lambda)y+\left(P_2-\lambda P_1\right)(1-y)}{(1-\lambda)}}\left[u'(x)+xu''(x)\right]dx.$$

However, this cannot be if $-\frac{u''(x)}{u'(x)}x > 1$ because $P_1 \leq P_2$.

B Proofs

This appendix contains the formal proofs of our main results.

B.1 Theorem 1

For the functions $M,N:\mathbb{R}_{++}\to\mathbb{R}_{++}$ defined by $M(P)=\max\{R/P,1\}$ and $N(P)=\max\{P,\varepsilon\}$ consider the consumer problem

$$\max_{(y,d,x)} \quad \lambda(pu(x_{1,1})+(1-p)u(x_{1,2}))+(1-\lambda)(pu(x_{2,1})+(1-p)u(x_{2,2}))$$

$$\text{s.t.}\quad \begin{cases} \left.\begin{array}{lcl} x_{1,1} & \leq & d \\ x_{1,2} & \leq & d \\ x_{2,1} & \leq & M(P_1)(y+N(P_1)(1-y)-\lambda d) \\ x_{2,2} & \leq & M(P_2)(y+N(P_2)(1-y)-\lambda d) \end{array}\right\} \text{for} \begin{array}{lcl} y+N(P_1)(1-y) & \geq & d \\ y+N(P_2)(1-y) & \geq & \lambda d \end{array} \\ \left.\begin{array}{lcl} x_{1,1} & \leq & y+N(P_1)(1-y) \\ x_{1,2} & \leq & d \\ x_{2,1} & \leq & y+N(P_1)(1-y) \\ x_{2,2} & \leq & M(P_2)(y+N(P_2)(1-y)-\lambda d) \end{array}\right\} \text{for} \begin{array}{lcl} y+N(P_1)(1-y) & < & d \\ y+N(P_2)(1-y) & \geq & \lambda d \end{array} \\ \quad y \in [0,1] \end{cases}$$

For all $(P_1,P_2)\in\mathbb{R}^2_{++}$ there is a solution because the set of alternatives is compact. According to Berge's maximum theorem the solution correspondence $F:\mathbb{R}^2_{++}\to[0,1]\times\mathbb{R}_+\times\mathbb{R}^4_+$ is upper hemi-continuous because expected utility is a continuous function and the set of alternatives is a continuous correspondence.

Let the correspondence $G : \mathbb{R}^2_{++} \to \mathbb{R}^2$ be defined by: in case $(y,d,x) \in F(P_1,P_2)$ satisfies the first four budget constraints,

$$G_s(P_1,P_2) \;=\; \begin{cases} \dfrac{y+\varepsilon(1-y)-\lambda d}{P_s} & \text{for } P_s < \varepsilon, \\ \left[\dfrac{y+P_s(1-y)-\lambda d}{P_s}, \dfrac{y-\lambda d}{P_s}\right] & \text{for } P_s = \varepsilon, \\ \dfrac{y-\lambda d}{P_s} & \text{for } \varepsilon < P_s < R, \\ \left[\dfrac{y-\lambda d}{P_s}, -(1-y)\right] & \text{for } P_s = R, \\ -(1-y) & \text{for } P_s > R, \end{cases}$$

for both s; or, in case (y,d,x) satisfies the second four budget constraints,

$$G_1(P_1,P_2) \;=\; \begin{cases} 0 & \text{for } P_1 < \varepsilon, \\ [-(1-y), 1-y] & \text{for } P_1 = \varepsilon, \\ -(1-y) & \text{for } P_1 > \varepsilon, \end{cases}$$

and $G_2(P_1,P_2)$ as in case (y,d,x) satisfies the first four budget constraints. Then G is upper hemicontinuous.

For $(P_1,P_2) \in \mathbb{R}^2_{++}$ and $(y,d,x) \in F(P_1,P_2)$, if $P_s < \varepsilon$ and $(z_1,z_2) \in G(P_1,P_2)$, then $z_s \geq 0$. For $(P_1,P_2) \in \mathbb{R}^2_{++}$ and $(y,d,x) \in F(P_1,P_2)$, if $P_s > R$ and $(z_1,z_2) \in G(P_1,P_2)$, then $z_s \leq 0$. Therefore prices are bounded from below by $\varepsilon - \delta$ and from above by $R + \delta$ for some $\delta \in]0,\varepsilon[$, $(P_1,P_2) \in [\varepsilon - \delta, R + \delta]^2$.

For $A \subset \mathbb{R}^2$ being the convex hull of the range of G with prices restricted to the set $[\varepsilon - \delta, \mathbb{R} + \delta]^2$,

$$A \;=\; \operatorname{co}\{\, (z_1,z_2) \in \mathbb{R}^2 \mid \exists (P_1,P_2) \in [\varepsilon - \delta, R + \delta]^2 : (z_1,z_2) \in G(P_1,P_2) \,\}$$

let the correspondence $H : A \to [\varepsilon - \delta, R + \delta]^2$ be defined by

$$H(z_1,z_2) \;=\; \{\, (P_1,P_2) \in [\varepsilon - \delta, R + \delta] \mid \forall (P_1', P_2') \in [\varepsilon - \delta, R + \delta] : P_1 z_1 + P_2 z_2 \geq P_1' z_1 + P_2' z_2 \,\}.$$

Then H is upper hemi-continuous.

The correspondence $(\operatorname{co} G, H) : [\delta, R+\delta]^2 \times A \to [\delta, R+\delta]^2 \times A$ has a fixed point according to Kakutani's fixed point theorem, because $[\varepsilon-\delta, R+\delta]^2 \times A$ is convex and compact and $(\operatorname{co} G, H)$ is convex valued and upper hemi-continuous. Suppose $(P_1, P_2, z_1, z_2) \in [\varepsilon-\delta, R+\delta]^2 \times A$ is a fixed point, so $(z_1, z_2) \in \operatorname{co} G(P_1, P_2)$ and $(P_1, P_2) \in H(z_1, z_2)$. Suppose $z_s \neq 0$, then $H_s(z_1, z_2) = \varepsilon - \delta$ in case $z_s < 0$ and $H_s(z_1, z_2) = R + \delta$ in case $z_s > 0$. Suppose $P_s = \varepsilon - \delta$, then either $z_s = 0$ or $z_s > 0$ contradicting $P_s = \varepsilon - \delta$, so $z_s = 0$. If $P_s = R + \delta$, then either $z_s = 0$ or $z_s < 0$ contradicting $P_1 = R + \delta$, so $z_s = 0$. Therefore $z_s = 0$ for both s.

For every $(z_1, z_2) \in \operatorname{co} G(P_1, P_2)$ there are at most three points $(z_1^i, z_2^i)_i$ with $(z_1^i, z_2^i) \in G(P_1, P_2)$ for every i and at most three weights $(w^i)_i$ with $w^i > 0$ for every i and $\sum_i w^i = 1$ such that $(z_1, z_2) = \sum_i w^i (z_1^i, z_2^i)$ according to Caratheodory's theorem. Hence (P_1, P_2, z_1, z_2) is an equilibrium.

B.2 Theorem 2

$\rho = 0$ requires $q^S = 0$. Absence of asset price volatility requires $P_1 = P_2 = 1$. For safe banks, the budget constraints (6a) and (6b) then imply $d^{\mathscr{S}} = 1$, $x_{2,1}^{\mathscr{S}} = R$ and $x_{2,2}^{\mathscr{S}} = R$. For risky banks, $d^{\mathscr{R}}$ solves

$$u'(d^{\mathscr{R}}) = Ru'\left(R\tfrac{1-\lambda d^{\mathscr{R}}}{1-\lambda}\right),$$

implying $x_{1,2}^{\mathscr{R}} = d^{\mathscr{R}} = y^*/\lambda$ and $x_{2,2}^{\mathscr{R}} = R(1-\lambda d^{\mathscr{R}})/(1-\lambda) = R(1-y^*)/(1-\lambda)$. Since $k(x) > 1$, it follows $x_{1,2}^{\mathscr{R}} \in \left(1, \lambda^{-1}\right)$ and $x_{2,2}^{\mathscr{R}} \in (1, R)$ such that $x_{1,1}^{\mathscr{R}} = 1 = x_{1,1}^{\mathscr{S}}$, $x_{1,2}^{\mathscr{R}} > 1 = x_{1,2}^{\mathscr{S}}$, $x_{2,1}^{\mathscr{R}} = 1 < R = x_{2,1}^{\mathscr{S}}$ and $x_{2,2}^{\mathscr{R}} < R = x_{2,1}^{\mathscr{S}}$. Let

$$X(p) = (1-p)\,\lambda u\left(x_{1,2}^{\mathscr{R}}\right) + (1-p)\,(1-\lambda)\,u\left(x_{2,2}^{\mathscr{R}}\right) + pu\,(1),$$

and $\check{p}$ be a solution to

$$\lambda u\,(1) + (1-\lambda)\,u\,(R) = X\,(p).$$

Since $\lambda u\left(x_{1,2}^{\mathscr{R}}\right) + (1-\lambda)\,u\left(x_{2,2}^{\mathscr{R}}\right) > \lambda u\,(1) + (1-\lambda)\,u\,(R)$ for $k(x) > 1$, $u\,(1) < \lambda u\,(1) + (1-\lambda)\,u\,(R)$ and $X' < 0$, there is a unique $\check{p} < 1$ such that $V^{\mathscr{S}}(\mathbf{P}) \geq V^{\mathscr{R}}(\mathbf{P})$ for $\mathbf{P} = (1,1)$ if and only if $p \geq \check{p}$. Since $k > 1$, expected utility satisfies $\lambda u\,(1) + (1-\lambda)u\,(R) < \lambda u\,(y^*/\lambda) + (1-\lambda)u\,(R(1-y^*)/(1-\lambda))$.

B.3 Theorem 3

In any equilibrium with a safe banking sector, $P_2 = h(P_1)$ must hold. Continuity of h implies there exists a continuum of equilibrium prices which support equilibria with safe banking sectors provided $V^{\mathscr{S}}(1,1) > V^{\mathscr{R}}(1,1)$, i.e. if $p > \check{p}$. Different asset prices are associated with different consumption bundles according to Equation (11). Since arbitrage-free equilibrium requires $1 \leq P_2 \leq R$ and because $h^{-1}(R) > 0$, P_1 is strictly bounded away from ε.

Indirect utility is given by

$$V^{\mathscr{S}}(\mathbf{P}) = \lambda u\left(\frac{P_1}{\lambda P_1 + 1 - \lambda}\right) + (1-\lambda)\, u\left(\frac{R}{\lambda P_1 + 1 - \lambda}\right),$$

With $P_2 = h(P_1)$, applying the Envelope theorem yields

$$\begin{aligned} \frac{dV^{\mathscr{S}}(\mathbf{P})}{dP_1} = {} & \lambda u'\left(\frac{P_1}{\lambda P_1 + 1 - \lambda}\right) \frac{1-\lambda}{(\lambda P_1 + 1 - \lambda)^2} \\ & - (1-\lambda)\, u'\left(\frac{R}{\lambda P_1 + 1 - \lambda}\right) \frac{\lambda R}{(\lambda P_1 + 1 - \lambda)^2}. \end{aligned}$$

Its sign is always positive for all $P_1 \in [h^{-1}(R), 1]$. This is because $u'(1) \geq Ru'(R)$ (since $k(x) > 1$) and $\frac{d}{dP_1}\left(u'\left(\frac{P_1}{\lambda P_1+1-\lambda}\right) - Ru'\left(\frac{R}{\lambda P_1+1-\lambda}\right)\right) < 0$ (since $u'' < 0$) together imply $u'\left(\frac{P_1}{\lambda P_1+1-\lambda}\right) \geq Ru'\left(\frac{R}{\lambda P_1+1-\lambda}\right)$.

B.4 Lemma 1

$\rho = 1$ implies $Q^S = 0$. Accordingly, for equilibria with $\rho = 1$ it requires $\lambda d^{\mathscr{R}} - y^{\mathscr{R}} = 0$ and either $1 - y^{\mathscr{R}} = 0$ or $P_1 \leq \varepsilon$. We can rule out $1 - y^{\mathscr{R}} = 0$ because state-independence of liquidity demand requires $y^{\mathscr{R}}$ to solve

$$\frac{u'\left(\frac{y^{\mathscr{R}} + P_1(1-y^{\mathscr{R}})}{\lambda}\right)}{u'\left(\frac{R}{P_2}\frac{P_2 - P_1}{1-\lambda}(1-y^{\mathscr{R}})\right)} - \frac{R}{P_2} = 0,$$

and concavity of u implies an upper bound on $y^{\mathscr{R}}$ given by $y^{\mathscr{R}} \leq \lambda R/(\lambda R + 1 - \lambda) < 1$. Hence, an equilibrium exists only if $P_1 \leq \varepsilon$ and $f(\varepsilon) \neq \emptyset$, i.e. there is some $(y^{\mathscr{R}}, P_2) \in [0, \lambda R/(\lambda R + 1 - \lambda)] \times$

$[1,R]$ satisfying

$$\frac{u'\left(y^{\mathscr{R}}/\lambda\right)}{u'\left(\frac{R(1-y^{\mathscr{R}})}{1-\lambda}\right)} = \frac{R}{P_2},$$

$$\frac{u'\left(y^{\mathscr{R}}\right)}{u'\left(\frac{R(1-y^{\mathscr{R}})}{1-\lambda}\right)} = \frac{R}{P_2}\frac{1-p}{p}\left(P_2-1\right).$$

Let Y_1 be the solution to the first Equation for a given P_2. Then, $\lim_{P_2\to 1} Y_1 = y^*$, $\lim_{P_2\to R} Y_1 = \lambda R/\left(\lambda R+(1-\lambda)\right)$ and $dY_1/dP_2 > 0$. Let Y_2 be the solution to the second Equation for a given P_2. Then, $\lim_{P_2\to 1} Y_2 = 1$, $\lim_{P_2\to R} Y_2 = \tilde{y} \in (0,1)$ and $dY_2/dP_2 < 0$ where $\tilde{y}$ is implicitly defined by

$$\frac{u'\left(\tilde{y}\right)}{u'\left(\frac{R(1-\tilde{y})}{1-\lambda}\right)} = \frac{1-p}{p}\left(R-1\right).$$

Since $y^* < 1$, there is no $f(\varepsilon) \in [0,\lambda R/\left(\lambda R+1-\lambda\right)] \times [1,R]$ if

$$\frac{u'\left(\frac{\lambda R}{\lambda R+(1-\lambda)}\right)}{u'\left(\frac{R}{\lambda R+(1-\lambda)}\right)} > \frac{1-p}{p}\left(R-1\right),$$

or, equivalently, if $p > \hat{p}$.

B.5 Theorem 4

According to Lemma 1, provided $p \leq \hat{p}$ there is some $(y^{\mathscr{R}}, P_2) \in [0,\lambda R/\left(\lambda R+1-\lambda\right)] \times [1,R]$ for which liquidity demand in either state is zero. By the implicit function theorem, (12) and (13) imply for $P_1 = \varepsilon$ that $\lim_{p\to 0} P_2 = 1$ and $\lim_{p\to 0} y^{\mathscr{R}} = y^*$. Therefore, for $P_1 = \varepsilon$,

$$\lim_{p\to 0} V^{\mathscr{R}}(\mathbf{P}) = \lambda u\left(\frac{y^*}{\lambda}\right) + (1-\lambda)u\left(R\frac{1-y^*}{1-\lambda}\right).$$

For $P_1 = \varepsilon$ and $p \to 0$ the first-order condition for safe banks becomes

$$u'(\bar{y}) \leq Ru'\left(R\frac{1-\lambda\bar{y}}{1-\lambda}\right)$$

which would hold with equality only if there would be some $\bar{y} \in (0,1)$ to solve the equation. However, since relative risk aversion satisfies $k(x) > 1$, there is no $\bar{y} \in (0,1)$ to meet the first-order condition with equality. Hence, $\bar{y} = 1$ which implies

$$\lim_{p \to 0} V^{\mathscr{S}}(\mathbf{P}) = \lambda u(1) + (1-\lambda) u(R).$$

Relative risk aversion of $k(x) > 1$ then further implies that $\lim_{p \to 0} V^{\mathscr{R}}(\mathbf{P}) > \lim_{p \to 0} V^{\mathscr{S}}(\mathbf{P})$. Therefore, provided $P_1 = \varepsilon$ and liquidity demand be zero in either state, either is $V^{\mathscr{R}}(\mathbf{P}) > V^{\mathscr{S}}(\mathbf{P})$ for all $p \leq \hat{p}$, or by the intermediate value theorem there is a $\bar{p} \leq \hat{p}$ such that $V^{\mathscr{R}}(\mathbf{P}) > V^{\mathscr{S}}(\mathbf{P})$ for all $p < \bar{p}$. The equilibrium is locally isolated because for $p < \bar{p}$ the solution to (18), satisfying

$$\begin{aligned} pu'\left(y^{\mathscr{R}}\right) - (1-p)\left(u'\left(y^{\mathscr{R}}/\lambda\right) - Ru'\left(\frac{R\left(1-y^{\mathscr{R}}\right)}{1-\lambda}\right)\right) &= 0, \\ \frac{Ru'\left(\frac{R\left(1-y^{\mathscr{R}}\right)}{1-\lambda}\right)}{u'\left(y^{\mathscr{R}}/\lambda\right)} &= P_2, \end{aligned}$$

is unique.

B.6 Theorem 5

As for liquidity demand, $q_1^D = q_2^D \geq 0$ and thus $d = \left(y + P_1(1-y)\right)/\lambda$ holds in any equilibrium with $\rho \in]0,1]$. For a given $P_2 \in [1,R]$, a necessary condition is that there is a $(P_1, y) \in [\varepsilon, 1] \times [0,1]$ such that condition (12) is met. If there is such a pair, it satisfies $dy/dP_1 < 0$. Note, if $R < \lambda^{-1}$ there is no $P_2 \in [1,R]$ such that liquidity demand can be state-independent for $P_1 = 1$. Condition (13) can be rewritten as

$$(1-P_1)\frac{u'\left(y + P_1(1-y)\right)}{u'\left(\frac{R}{P_2}\frac{y + P_2(1-y) - \lambda d}{1-\lambda}\right)} \leq (P_2 - 1)\frac{1-p}{p}\frac{R}{P_2}.$$

The ride side converges to 0 if $p \to 1$. The marginal rate of substitution in condition (13) converges to $u'(1)/u'\left(\frac{R}{P_2}\frac{(P_2-1)(1-y)}{1-\lambda}\right) > 0$ if $P_1 \to 1$, where y is either zero or satisfies

$$\frac{u'(1/\lambda)}{u'\left(\frac{R}{P_2}\frac{(P_2-P_1)(1-y)}{1-\lambda}\right)} = \frac{R}{P_2}.$$

Therefore, if $p \to 1$ then either P_1 converges to 1 for a given $P_2 \in [1,R]$, or liquidity demand cannot be state-independent.

As for liquidity supply, note that $\lim_{p\to 1} h^{-1}(P_2) = 1$ for all $P_2 \in [1,R]$. Therefore, if $p \to 1$ and $P_1 \to 1$, $q^S \geq 0$ for all $P_2 \in [1,R]$. Provided $q_1^D = q_2^D \geq 0$ for $p \to 1$ and $P_1 \to 1$, $V^{\mathscr{R}}(\mathbf{P})$ converges to $u(1)$ while $V^{\mathscr{S}}(\mathbf{P})$ converges to $\lambda u(1) + (1-\lambda)u(R) > u(1)$. However, if liquidity demand cannot be state-independent, risky banks cannot exist anyway whilst $q^S = 0$.

Therefore, either there is no $\mathbf{P} \in [\varepsilon, 1] \times [1,R]$ for which $q^S \geq 0$, $q_1^D = q_2^D \geq 0$ and $V^{\mathscr{S}}(\mathbf{P}) \leq V^{\mathscr{R}}(\mathbf{P})$ for all $p \geq \check{p}$. Or, if there is some $p > \check{p}$ for which some $\mathbf{P} \in [\varepsilon, 1] \times [1,R]$ exists such that $q^S \geq 0$, $q_1^D = q_2^D \geq 0$ and $V^{\mathscr{S}}(\mathbf{P}) \leq V^{\mathscr{R}}(\mathbf{P})$, then there is some $\tilde{p} \in]\check{p}, 1[$ such that for all $p > \tilde{p}$ there is no $\mathbf{P}$ for which $q_1^D = q_2^D \geq 0$ and $V^{\mathscr{S}}(\mathbf{P}) \leq V^{\mathscr{R}}(\mathbf{P})$ according to the intermediate value theorem.

B.7 Theorem 6

Again, $q_1^D = q_2^D \geq 0$ and thus $d = (y + P_1(1-y))/\lambda$ holds in any equilibrium with $\rho \in]0,1]$. Condition (13) can be rewritten as

$$p \frac{u'(y + P_1(1-y))}{u'\left(\frac{R}{P_2}\frac{y+P_2(1-y)-\lambda d}{1-\lambda}\right)} \leq (1-p)\frac{P_2-1}{1-P_1}\frac{R}{P_2}.$$

with strict inequality only if $y = 0$. The left hand side converges to zero for $p \to 0$, whereas the right hand side converges to $\frac{P_2-1}{1-P_1}\frac{R}{P_2} > 0$. Hence, as long as $y^{\mathscr{R}} > 0$ such that above condition holds with equality, it follows for a given P_1 that $P_2 \to 1$.

Provided $P_2 \to 1$ and $P_1 \in [\varepsilon, y^*[$, condition (12) implies $x_{1,2} = y^*/\lambda$, $x_{2,2} = R(1-y^*)/(1-\lambda)$, $y^{\mathscr{R}} = (y^* - P_1)/(1-P_1) > 0$, and $V^{\mathscr{R}}(\mathbf{P}) = \lambda u(y^*/\lambda) + (1-\lambda)u(R(1-y^*)/(1-\lambda))$. For $P_2 = 1$ and $P_1 \in [0, y^*]$, safe banks optimally store $y^{\mathscr{S}} = \max\{1, (y^*/\lambda - P_1)/(1-P_1)\} = 1$ such that $V^{\mathscr{S}}(\mathbf{P}) = \lambda u(1) + (1-\lambda)u(R) < \lambda u(y^*/\lambda) + (1-\lambda)u(R(1-y^*)/(1-\lambda))$.

Concavity of u together with the budget constraints (8a) and (8b) imply that the left side in (12) is a continuous, monotone and decreasing function of $y^{\mathscr{R}}$ and continuous, monotone and increasing in P_2. Hence, for $y^{\mathscr{R}} = 0$, there is at most one P_2 satisfying (18). The projection ϕ_1 of f on the P_2-coordinate provided $y^{\mathscr{R}} = 0$ is a bijective function $\phi_1 : [\phi_1^{-1}(1), \min\{1, \lambda R\}] \times [1, \min\{R, \phi_1(1)\}]$ with

$$\frac{dP_2}{dP_1} = \frac{k_{2,2} + \left(\frac{P_2}{P_1} - 1\right) k_{1,1}}{k_{2,2} + \left(\frac{P_2}{P_1} - 1\right)} \frac{P_2}{P_1} > 0$$

where $k_{t,s} = k(x^{\mathscr{R}}_{t,s})$ is relative risk aversion at $x^{\mathscr{R}}_{t,s}$. For $\rho \in]0,1[$ it must be that $V^{\mathscr{R}}(\mathbf{P}) = V^{\mathscr{S}}(\mathbf{P})$. However, according to (16a) and (16c), $V^{\mathscr{R}}(\mathbf{P}) > \lambda u(y^*/\lambda) + (1-\lambda)u(R(1-y^*)/(1-\lambda))$. Hence, $V^{\mathscr{R}}(\mathbf{P}) > V^{\mathscr{S}}(\mathbf{P})$. Hence, $\rho \in]0,1[$ cannot be an equilibrium.

Finally, according to Theorem 3, $V^{\mathscr{S}}(\mathbf{P}) \leq \lambda u(1) + (1-\lambda)u(R) < \lambda u(y^*/\lambda) + (1-\lambda)u(R(1-y^*)/(1-\lambda))$ for all $P_2 = h(P_1)$. Since (i) $\phi^{-1}(P_2) \leq h^{-1}(P_2)$ for $\phi^{-1}(P_2) \neq \emptyset$, (ii) $V^{\mathscr{R}}(\mathbf{P}) \geq \lambda u(y^*/\lambda) + (1-\lambda)u(R(1-y^*)/(1-\lambda))$ for $P_1 = \phi^{-1}(P_2)$, and (iii) $dV^{\mathscr{R}}(\mathbf{P})/dP_1 > 0$ we have $V^{\mathscr{R}}(\mathbf{P}) > \lambda u(y^*/\lambda) + (1-\lambda)u(R(1-y^*)/(1-\lambda))$. Hence, $\rho = 0$ cannot be an equilibrium.

C State-independent liquidity demand

This appendix shows that non-increasing relative risk aversion is a sufficient condition that all combinations of asset prices for which liquidity demand is state-independent can be described by a continuous function that maps P_1 onto P_2.

For any $(\boldsymbol{y}^{\mathscr{R}}, \boldsymbol{P_2})$, Equation (18) defines P_2 and $y^{\mathscr{R}}$ as implicit functions of P_1 in some neighborhood of $(\boldsymbol{y}^{\mathscr{R}}, \boldsymbol{P_2})$ according to the general implicit function theorem. Provided $y^{\mathscr{R}} \in]0,1[$, each of these solutions satisfy

$$\frac{dP_2}{dP_1} = -\frac{(k_{1,1}-k_{1,2})\,k_{2,2}\frac{P_2-1}{P_2-P_1} + k_{1,2} + k_{2,2}\frac{y^{\mathscr{R}}+P_1(1-y^{\mathscr{R}})}{(1-P_1)(1-y^{\mathscr{R}})}}{(k_{1,1}-k_{1,2})\,k_{2,2}\frac{P_1}{P_2-P_1} + k_{1,2}\frac{1}{P_2-1} + k_{2,2}\frac{y^{\mathscr{R}}+P_1(1-y^{\mathscr{R}})}{(1-P_1)(1-y^{\mathscr{R}})}\frac{P_2}{P_2-1} + k_{1,1}}\frac{P_2}{1-P_1}$$

and

$$\frac{dy^{\mathscr{R}}}{dP_1} = -\frac{(k_{1,1}-k_{1,2})\,k_{2,2}\frac{P_1}{P_2-P_1} + k_{1,2}\frac{1}{P_2-1} + k_{2,2}\frac{y^{\mathscr{R}}+P_1(1-y^{\mathscr{R}})}{(1-P_1)(1-y^{\mathscr{R}})}\frac{1}{P_2-1} + k_{1,1} + \frac{1}{1-P_1}}{(k_{1,1}-k_{1,2})\,k_{2,2}\frac{P_1}{P_2-P_1} + k_{1,2}\frac{1}{P_2-1} + k_{2,2}\frac{y^{\mathscr{R}}+P_1(1-y^{\mathscr{R}})}{(1-P_1)(1-y^{\mathscr{R}})}\frac{P_2}{P_2-1} + k_{1,1}}\frac{1-y^{\mathscr{R}}}{1-P_1}$$

For any P_1, Equation (12) defines P_2 as a monotone and increasing function of $y^{\mathscr{R}}$. Then, a sufficient condition that there is at most one $(y^{\mathscr{R}}, P_2)$ satisfying (18) and $y^{\mathscr{R}} > 0$ is that the left side in (13) is strictly monotone in $y^{\mathscr{R}}$ while taking into account the relation between $y^{\mathscr{R}}$ and P_2 according to (12). Let

$$\Phi_1 := \left(\frac{k_{1,2}}{k_{2,2}}\frac{1}{P_1} + \left(\frac{y^{\mathscr{R}}}{1-y^{\mathscr{R}}} + P_1\right)\frac{1}{1-P_1}\frac{P_2}{P_1} + \frac{k_{1,1}}{k_{2,2}}\frac{P_2-1}{P_1}\right)\frac{P_2-P_1}{P_2-1},$$

$$\Phi_2 := \left(\frac{k_{1,2}}{k_{2,2}} + \left(\frac{y^{\mathscr{R}}}{1-y^{\mathscr{R}}} + P_1\right)\frac{1}{1-P_1}\right)\frac{P_2-P_1}{P_2-1}.$$

This monotonicity holds if for all P_1 either $\Phi_1 > k_{1,2}-k_{1,1}$ or $\Phi_1 < k_{1,2}-k_{1,1}$. The sign of dP_2/dP_1 is positive if and only if $\Phi_1 > k_{1,2}-k_{1,1} > \Phi_2$. Hence, with non-increasing risk aversion, i.e. $k_{1,1} \geq k_{1,2}$, the projection ϕ_2 of f on the P_2-coordinate provided $y^{\mathscr{R}} \in]0,1[$ is a bijective function $\phi_2 : [\max\{\varepsilon, \phi_2^{-1}(R)\}, \min\{\phi_1^{-1}(1), \phi_2^{-1}(1)\}] \times [1,R]$ satisfying $d\phi_2(P_1)/dP_1 < 0$. Hence, for $P_2 = \phi_2(P_1)$ we have $q_1^D = q_2^D$ and $y^{\mathscr{R}} > 0$. Similarly, the projection of f on $y^{\mathscr{R}}$ satisfies $dy^{\mathscr{R}}/dP_1 < 0$ for $k_{1,1} \geq k_{1,2}$.

Continuity of the projection of f on P_2 holds because (12) implies that $\phi_1(P_1) = 1$ for some $P_1 \in]0,1[$, where ϕ_1 is the projection of f on the P_2-coordinate provided $y^{\mathscr{R}} = 0$ as defined in proof

B.7. Moreover, (12) and (13) imply that $\phi_2(P_1) > 1$ for all $P_1 \in]0,1[$. Hence, there is a unique $P_1 \in]0,1[$ such that $\phi_1(P_1) = \phi_2(P_1)$ and $\phi_1(P_1) \in]1,R]$.

D Indirect utility and asset prices

This appendix derives the condition under which the indirect utility consumers get in equilibria in which risky banks exist is strictly increasing in P_1. Consider indirect utility as given in Equation (23). With $(\boldsymbol{y}^{\mathscr{R}}, \boldsymbol{P}_2) = f(P_1)$, applying the Envelope theorem yields

$$\frac{dV^{\mathscr{R}}(\mathbf{P})}{dP_1} = \begin{cases} \dfrac{\left(\frac{k_{2,2}}{1-P_1} + \frac{k_{1,1}}{P_1} + \frac{\boldsymbol{P}_2-1}{1-P_1}\frac{1}{P_1}\right)(1-p)\left(1-\boldsymbol{y}^{\mathscr{R}}\right)(\boldsymbol{P}_2-P_1)u'\left(x^{\mathscr{R}}_{2,2}\right)}{k_{2,2}+\frac{\boldsymbol{P}_2-P_1}{P_1}} & \text{for} \quad \boldsymbol{y}^{\mathscr{R}} = 0, \\ \\ \dfrac{(1-p)\left(1-\boldsymbol{y}^{\mathscr{R}}\right)\left(k_{1,2}+\left(k_{2,2}\left(\frac{\boldsymbol{y}^{\mathscr{R}}}{1-\boldsymbol{y}^{\mathscr{R}}}+P_1\right)\frac{1}{1-P_1}+k_{1,1}\right)\frac{\boldsymbol{P}_2-1}{1-P_1}\right)u'\left(x^{\mathscr{R}}_{2,2}\right)}{(k_{1,1}-k_{1,2})k_{2,2}\frac{P_1}{\boldsymbol{P}_2-P_1}+k_{1,2}\frac{1}{\boldsymbol{P}_2-1}+k_{2,2}\left(\frac{\boldsymbol{y}^{\mathscr{R}}}{1-\boldsymbol{y}^{\mathscr{R}}}+P_1\right)\frac{1}{1-P_1}\frac{\boldsymbol{P}_2}{\boldsymbol{P}_2-1}+k_{1,1}} & \text{for} \quad \boldsymbol{y}^{\mathscr{R}} > 0. \end{cases}$$

For $\boldsymbol{y}^{\mathscr{R}} = 0$ we have $dV^{\mathscr{R}}(\mathbf{P})/dP_1 > 0$. For $\boldsymbol{y}^{\mathscr{R}} > 0$ it is positive if and only if

$$\left(\frac{k_{1,2}}{k_{2,2}}\frac{1}{P_1} + \left(\frac{\boldsymbol{y}^{\mathscr{R}}}{1-y^{\mathscr{R}}} + P_1\right)\frac{1}{1-P_1}\frac{\boldsymbol{P}_2}{P_1} + \frac{k_{1,1}}{k_{2,2}}\frac{\boldsymbol{P}_2-1}{P_1}\right)\frac{\boldsymbol{P}_2-P_1}{\boldsymbol{P}_2-1} > k_{1,2}-k_{1,1},$$

for which a sufficient condition is non-increasing relative risk aversion.