

Financial Fragility with SAM?

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Abstract

Shared Appreciation Mortgages (SAMs) feature mortgage payments that adjust with house prices. These mortgage contracts are designed to stave off home owner default by providing payment relief in the wake of a large house price shock. SAMs have been hailed as an innovative solution that could prevent the next foreclosure crisis, act as a work-out tool during a crisis, and alleviate fiscal pressure during a downturn. They have inspired fintech companies to offer home equity contracts. However, the home owner's gains are the mortgage lender's losses. A general equilibrium model with financial intermediaries who channel savings from saver households to borrower households shows that indexation of mortgage payments to aggregate house prices increases financial fragility, reduces risk sharing, and leads to expensive financial sector bailouts. In contrast, indexation to local house prices reduces financial fragility and improves risk-sharing. The two types of indexation have opposite implications for wealth inequality.

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1 Introduction

The \$10 trillion market in U.S. mortgage debt is the world's largest consumer debt market and its second largest fixed income market. Mortgages are not only the largest liability for U.S. households, they are also the largest asset of the U.S. financial sector. Banks and credit unions hold \$3 trillion in mortgage loans directly on their balance sheets in the form of whole loans, and an additional \$2.2 trillion in the form of mortgage-backed securities.¹ Given the exposure of the financial sector to mortgages, large house price declines and the default wave that accompanies them can severely hurt the solvency of the U.S. financial system. This became painfully clear during the Great Financial Crisis of 2008-2011. Moreover, exposure to interest rate risk could represent an important source of financial fragility going forward if mortgage rates rise from historic lows.

In this paper we study the allocation of house price and interest rate risk in the mortgage market between mortgage borrowers, financial intermediaries, and savers. The standard 30-year fixed-rate mortgage (FRM) dictates a particular distribution of these risks: borrower home equity absorbs the initial house price declines, until a sufficiently high loan-to-value ratio, perhaps coupled with an adverse income shock, leads the homeowner to default, inflicting losses on the lender. As a result, lenders only bear the risk of large house price declines.

During the recent housing crash, U.S. house prices fell 30% nationwide, and by much more in some regions. The financial sector had written out-of-the-money put options on aggregate house prices with more than \$5 trillion in face value, and the downside risk materialized. About 25% of U.S. home owners were underwater by 2010 and seven million forecloses ensued. Charge-off rates of residential real estate loans at U.S. banks went from 0.1% in mid-2006 to 2.8% in mid-2009, and remained above 1% until the end of 2012. Only by mid-2016 did they return to their level from a decade earlier. The stress on banks' balance sheets caused lenders to dramatically tighten mortgage lending standards, precluding many home owners from refinancing their mortgage and take advantage of the low interest rates. Homeowners' reduced ability to tap into their housing wealth short-circuited the stimulative consumption response from lower mortgage rates that policy makers hoped for.

This crisis led many to ask whether a fundamentally different mortgage finance sys-

¹Including insurance companies, money market mutual funds, broker-dealers, and mortgage REITs in the definition of the financial sector adds another \$1.5 trillion to the financial sector's agency MBS holdings. Adding the Federal Reserve Bank and the GSE portfolios adds a further \$2 trillion and increases the share of the financial sector's holdings of agency MBS to nearly 80%.

tem could lead to a better risk sharing arrangement between borrowers and lenders.² While contracts offering alternative allocations of interest rate risk are already widely available — most notably, the adjustable rate mortgage (ARM), which offers nearly perfect pass-through of interest rates — contracts offering alternative divisions of *house price* risk are essentially unavailable to the typical household. To fill this gap, researchers have begun to design and analyze such contracts.

The most well known proposal is the shared appreciation mortgage (SAM). The SAM indexes mortgage payments to house price changes. In the fully symmetric version, payments are linked to house prices — increasing when they rise and decreasing when they fall — making the contract more equity-like. Such a contract ensures that the borrower receives payment relief in bad states of the world, potentially reducing mortgage defaults and the associated deadweight losses to society. On the other hand, SAMs impose losses on mortgage lenders in these adverse aggregate states, which may increase financial fragility at inopportune times. We argue for a shift in focus in the mortgage design debate from a *household risk management* focus to a *system-wide risk management* focus. The main goal of this paper is to quantitatively assess whether SAMs present a better arrangement to the overall economy than FRMs.

We model the interplay between mortgage borrowers, mortgage lenders, and savers. All agents face aggregate labor income risk. Borrowers also face idiosyncratic house valuation shocks, which affect their optimal mortgage default decision. Uncertainty shocks, shocks to the cross-sectional dispersion of the house valuation shocks, affect the economy-wide mortgage default rate and are the second source of aggregate risk in the economy. Mortgage lenders make long-term, defaultable, prepayable mortgage loans to impatient borrowers, funded by deposits raised from patient savers. Borrowers face a maximum loan-to-value constraint, but only at loan origination, while banks face their own leverage constraint, capturing macro-prudential bank equity capital requirements.

We contrast this economy to an economy with SAMs. We study SAMs whose payments are indexed to aggregate house prices, as well as SAMs whose payments are partially indexed to idiosyncratic house price risk. We interpret the partial insurance against idiosyncratic house price risk as indexation to local price fluctuations, which is often used in place of direct indexation to individual house values to reduce moral hazard.

Surprisingly, aggregate indexation reduces borrower welfare even though it (slightly) reduces mortgage defaults, because it amplifies financial fragility. Intermediary wealth

²The New York Federal Reserve Bank organized a two-day conference on this topic in May 2015 with participants from academia and policy circles.

falls substantially in crises as mortgage lenders absorb house price declines. The bank failure rate increases, triggering bailouts that must ultimately be funded by taxpayers, including the borrowers. Equilibrium house prices are lower and fall more in crises with aggregate indexation. Ironically, intermediary welfare increases as they reap the profits from selling foreclosed houses back to borrowers, as well as from the larger mortgage spreads lenders are able to charge in a riskier financial system.

In contrast, by partially indexing mortgage payments and principal to individual house valuation shocks, SAMs can eliminate most mortgage defaults. By extension, local indexation reduces bank failures and fluctuations in intermediary net worth substantially. Banking becomes safer, but also less profitable, due to a fall in mortgage spreads. Lower bank failure rates generate fewer deadweight costs and lower maintenance expenses from houses in foreclosure, so that more resources are available for consumption. Welfare of borrowers and savers rises, at the expense of that of bank owners.

Section 2 discusses the related literature. Section 3 presents the theoretical model. Section 4 characterizes the solution. Section 5 discusses its calibration. The main results are in section 6. Section 7 concludes. Model derivations are relegated to the appendix.

2 Related Literature

This paper contributes to the literature that studies innovative mortgage contracts. While an extensive body of work studies designs to mitigate an array of interest rate indexation and amortization schemes, we focus on mortgage contracts that are indexed to house prices.³

In early work, [Shiller and Weiss \(1999\)](#) discuss the idea of home equity insurance policies. The idea of SAMs was discussed in a series of papers by [Caplin, Chan, Freeman, and Tracy \(1997\)](#); [Caplin, Carr, Pollock, and Tong \(2007\)](#); [Caplin, Cunningham, Engler, and Pollock \(2008\)](#). They envision a SAM as a second mortgage in addition to a conventional FRM with a smaller principal balance. The SAM has no interest payments and its principal needs to be repaid upon termination (e.g., sale of the house). At that point the borrower shares a fraction of the house value appreciation with the lender, but only if the

³Related work on contract schemes other than house price indexation include [Piskorski and Tchisty \(2011\)](#), who study optimal mortgage contract design in a partial equilibrium model with stochastic house prices and show that option-ARM implements the optimal contract; [Kalotay \(2015\)](#), who considers automatically refinancing mortgages or ratchet mortgages (whose interest rate only adjusts down); and [Eberly and Krishnamurthy \(2014\)](#), who propose a mortgage contract that automatically refinances from a FRM into an ARM, even when the loan is underwater.

house has appreciated in value. The result is lower monthly mortgage payments throughout the life of the loan, which enhances affordability, and a better sharing of housing risk. They emphasize that SAMs are not only a valuable work-out tool after a default has taken place, but are also useful to prevent a mortgage crisis in the first place.⁴

Recently, Mian (2013) and Mian and Sufi (2014) introduced a version of the SAM, which they call the Shared Responsibility Mortgage (ARM). The SRM replaces a FRM rather than being an additional mortgage. It features mortgage payments that adjust down when the local house price index goes down, and back up when house prices bounce back, but never above the initial FRM payment. To compensate the lender for the lost payments upon house price declines, the lender receives 5% of the home value appreciation. They argue that foreclosure avoidance raises house prices in a SRM world and shares wealth losses more equitably between borrowers and lenders. When borrowers have higher marginal propensities to consume out of wealth than lenders, this more equitable sharing increases aggregate consumption and reduces job losses that would be associated with low aggregate demand. The authors argue that SRMs would reduce the need for counter-cyclical fiscal policy and give lenders an incentive to “lean against the wind” by charging higher mortgage rates when house price appreciation seems excessive.

Shared appreciation mortgages have graduated from the realm of the hypothetical. They have been offered to faculty at Stanford University for leasehold purchases for fifteen years (Landvoigt, Piazzesi, and Schneider, 2014). More recently, several fintech companies such as FirstREX and EquityKey have been offering home equity products where they offer cash today for a share in the future home value appreciation.⁵ These products

⁴Among the implementation challenges are (i) the uncertain holding period of SAMs, (ii) returns on investment that decline with the holding period, and (iii) the tax treatment of SAM lenders/investors. The first issue could be solved by a maximum maturity provision of say 15 years. The second issue can be solved by replacing the lender’s fixed appreciation share by a shared-equity rate. For example, instead of 40% of the total appreciation, the investor would have a 4% shared-equity rate. If the holding period of the SAM is 10 years and the original SAM principal represented 20% of the home value, the lender is entitled to the maximum of the SAM principal and $20\% \times (1.04)^{10} = 29.6\%$ of the terminal home *value*. This scheme delivers an annual rate of return to the lender that is constant rather than declining in the holding period. The authors refer to this variant as SAMANTHA, a SAM with A New Treatment of Housing Appreciation.

⁵EquityKey started issuing such shared equity contracts in the early 2000s. It was bought by a Belgian retail bank in 2006. The founders bought the business back from the Belgian bank after the housing crisis and resumed its activities. In 2016, the company closed its doors after the hedge fund that funded the operations lost interest. FirstREX changed its name to Unison Home Ownership Investors in December 2016. It has been making home ownership investments since March 2004. Its main product offers up to half of the down payment in exchange for a share of the future appreciation. The larger down payment eliminates the need for mortgage insurance. Its product is used alongside a traditional mortgage, just like the original SAM contract. Unison is active in 13 U.S. states and plans to add 8 more states in 2017. It is funded by 8 lenders.

are presented as an alternative to home equity lines of credit, closed-end second mortgages, reverse mortgages for older home owners, or to help finance the borrower's down payment at the time of home purchase. They allow the home owner to tap into her home equity without taking on a new debt contract. Essentially, the home owner writes a call option on the local house price index (to avoid moral hazard issues) with strike price equal to the current house price value and receives the upfront option premium in exchange. Our work sheds new light on the equilibrium implications of introducing home equity products.

[Kung \(2015\)](#) studies the effect of the disappearance of non-agency mortgages for house prices, mortgage rates and default rates in an industrial organization model of the Los Angeles housing market. He also evaluates the hypothetical introduction of shared appreciation mortgages in the 2003-07 housing boom. He finds that symmetric SAMs would have enjoyed substantial uptake, partially supplanting non-agency loans, and would have further exacerbated the boom. They would not have mitigated the bust. Our model is an equilibrium model of the entire U.S. market with an endogenous risk-free rate rather than of a single city where households face an exogenously specified outside option of moving elsewhere and constant interest rates. Our lenders are not risk neutral, and charge an endogenously determined risk premium on mortgages. When lenders are risk neutral, they are assumed to be better able to bear house price risk than risk averse households. That seems like a fine assumption when all house price risk is idiosyncratic. However, banks may be severely negatively affected by aggregate house price declines and SAMs may exacerbate that financial fragility.

[Hull \(2015\)](#) studies house price-indexed mortgage contracts in a simple incomplete markets equilibrium model. He finds that such mortgages are associated with lower mortgage default rates and higher mortgage interest rates than standard mortgages. Our analysis features aggregate risk, long-term prepayable mortgage debt, and an intermediary sector that is risk averse.

Finally, two contemporaneous papers study mortgage design questions in general equilibrium. [Piskorski and Tchisty \(2017\)](#) study mortgage design from first principles in a tractable, risk neutral environment, emphasizing asymmetric information about home values between borrowers and unconstrained lenders. This setting yields closed-form solutions for the optimal contract, which takes the form of a Home Equity Insurance Mortgage that eliminates the strategic default option and insures borrower's home equity. They study the implications of this equilibrium contract for welfare relative to a

fixed-rate mortgage benchmark. Our setup features risk averse borrowers and lenders, and focuses on the levered financial sector, bringing issues relating to risk sharing and financial fragility front and center.

Next, [Guren, Krishnamurthy, and McQuade \(2017\)](#) investigate the interaction of ARM and FRM contracts with monetary policy. They study an FRM that costlessly converts to an ARM in a crisis so as to provide concentrated payment relief in a crisis. These authors focus on interest rate risk, which is relatively easy for banks to hedge, and hence focus on characterizing the borrower risk profile, incorporating a life cycle and uninsurable idiosyncratic income risk. Our framework considers house price risk that is difficult for banks to hedge, and emphasizes the role of the intermediation sector. We see both of these approaches as highly complementary to our own.

[Elenev, Landvoigt, and Van Nieuwerburgh \(2016b\)](#) studies the role the default insurance provided by the government-sponsored enterprises, Fannie Mae and Freddie Mac. They consider an increase in the price of insurance that restores the absorption of mortgage default risk by the private sector and show it leads to an allocation that is a Pareto improvement. This paper introduces SAMs, REO housing stock dynamics, and long-term mortgages whose rate does not automatically readjusts every period. [Greenwald \(2016\)](#) studies the interaction between the payment-to-income and the loan-to-value constraint in a model of monetary shock transmission through the mortgage market but without default. [Favilukis, Ludvigson, and Van Nieuwerburgh \(2017\)](#) study the role of relaxed down payment constraints in explaining the house price boom. [Corbae and Quintin \(2014\)](#) investigate the effect of mortgage product innovation in a general equilibrium model with default. [Guren and McQuade \(2016\)](#) study the interaction of foreclosures and house prices in a model with search.

Our paper also relates to the literature that studies the amplification of business cycle shocks provided by credit frictions. E.g., [Bernanke and Gertler \(1989\)](#), [Bernanke, Gertler, and Gilchrist \(1996\)](#), [Kiyotaki and Moore \(1997\)](#), and [Gertler and Karadi \(2011\)](#). A second generation of models has added nonlinear dynamics and a richer financial sector. E.g., [Brunnermeier and Sannikov \(2014\)](#), [He and Krishnamurthy \(2012\)](#), [He and Krishnamurthy \(2013\)](#), [He and Krishnamurthy \(2014\)](#), [Gârleanu and Pedersen \(2011\)](#), [Adrian and Boyarchenko \(2012\)](#), [Maggiore \(2013\)](#), [Moreira and Savov \(2016\)](#), and [Elenev, Landvoigt, and Van Nieuwerburgh \(2016a\)](#). Recent work has begun to empirically test these intermediary-based asset pricing models ([Adrian, Etula, and Muir, 2014](#); [He, Kelly, and Manela, 2017](#)). Our solution uses a state-of-the-art global non-linear solution technique

of a problem with occasionally binding constraints.

Finally, we connect to a recent empirical work has found strong consumption responses and lower default rates (Fuster and Willen, 2015) to exogenously lowered mortgage interest rates Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao (2017) and to higher house prices (Mian and Sufi, 2009; Mian, Rao, and Sufi, 2013).

3 Model

3.1 Demographics

The economy is populated by a continuum of agents of three types: borrowers (denoted B), depositors (denoted D), and intermediaries (denoted I). The measure of type j in the population is denoted χ_j , with $\chi_B + \chi_D + \chi_I = 1$.

3.2 Endowments

The two consumption goods in the economy — nondurable consumption and housing services — are provided by two Lucas trees. The overall endowment grows at a deterministic rate g , and is subject to temporary but persistent shocks \tilde{y}_t :

$$Y_t = Y_{t-1} \exp(g + \tilde{y}_t),$$

where $\mathbb{E}(\exp(\tilde{y}_t)) = 1$ and

$$\tilde{y}_t = (1 - \rho_y)\mu_y + \rho_y\tilde{y}_{t-1} + \sigma_y\varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim N(0, 1). \quad (1)$$

The $\varepsilon_{y,t}$ can be interpreted as transitory shocks to the level of aggregate labor income. For nondurable consumption, each agent type j receives a fixed share s_j of the overall endowment Y_t , which cannot be traded.

Shares of the housing tree are in fixed supply. Shares of the tree produce housing services proportional to the stock, growing at the same rate g as the nondurable endowment. Housing also requires a maintenance cost proportional to its value, ν^K . Housing capital is divided among the three types of households in constant shares, $\bar{K} = \bar{K}^B + \bar{K}^I + \bar{K}^D$. Households can only trade housing capital with members of their own type.

3.3 Preferences

Each agent of type $j \in \{B, D, I\}$ has preferences following [Epstein and Zin \(1989\)](#), so that lifetime utility is given by

$$U_t^j = \left\{ (1 - \beta_j) (u_t^j)^{1-1/\psi} + \beta_j \left(\mathbb{E}_t \left[(U_{t+1}^j)^{1-\gamma_j} \right] \right)^{\frac{1-1/\psi}{1-\gamma_j}} \right\}^{\frac{1}{1-1/\psi}} \quad (2)$$

$$u_t^j = (C_t^j)^{1-\zeta_t} (H_t^j)^{\zeta_t} \quad (3)$$

where C_t^j is nondurable consumption and H_t^j is housing services, and the preference parameter ζ_t is allowed to vary with the state of the economy. Housing capital produces housing services with a linear technology. We denote by Λ^j the intratemporal marginal rate of substitution (or stochastic discount factor) of agent j .

3.4 Financial Technology

There are two financial assets in the economy: mortgages that can be traded between the borrower and the intermediary, and deposits that can be traded between the depositor and the intermediary.⁶

Mortgage Contracts. Mortgage contracts are modeled as nominal perpetuities with payments that decline geometrically, so that one unit of debt yields the payment stream $1, \delta, \delta^2, \dots$ until prepayment or default. The interest portion of mortgage payments can be deducted from taxes. New mortgages face a loan-to-value constraint (shown below in (7)) that is applied at origination only, so that borrowers do not have to delever if they violate the constraint later on.

Borrower Refinancing. Non-defaulting borrowers can choose at any time to obtain a new mortgage loan and simultaneously re-optimize their housing position. If a refinancing borrower previously held a mortgage, she must first prepay the principal balance on the existing loan before taking on a new loan.

The transaction cost of obtaining a new loan is proportional to the balance on the new loan M_t^* , given by $\kappa_{i,t} M_t^*$, where $\kappa_{i,t}$ is drawn i.i.d. across borrowers and time from a

⁶Equivalently, households are able to trade a complete set of state-dependent securities with households of their own type, providing perfect insurance against idiosyncratic consumption risk, but cannot trade these securities with members of the other types.

distribution with c.d.f. Γ_κ . Since these costs largely stand in for non-monetary frictions such as inertia, these costs are rebated to borrowers and do not impose an aggregate resource cost. We assume that borrowers must commit in advance to a refinancing policy that can depend in an unrestricted way on $\kappa_{i,t}$ and all aggregate variables, but cannot depend on the borrower's individual loan characteristics. This setup keeps the problem tractable by removing the distribution of loans as a state variable while maintaining the realistic feature that a fraction of borrowers choose to refinance in each period and that this fraction responds endogenously to the state of the economy.

We guess and verify that the optimal plan for the borrower is to refinance whenever $\kappa_{i,t} \leq \bar{\kappa}_t$, where $\bar{\kappa}_t$ is a threshold cost that makes the borrower indifferent between refinancing and not refinancing. The fraction of non-defaulting borrowers who choose to refinance is therefore

$$Z_{R,t} = \Gamma_\kappa(\bar{\kappa}_t).$$

Once the threshold cost (equivalently, refinancing rate) is known, the total transaction cost per unit of debt is defined by

$$\Psi_t(Z_{R,t}) = \int^{\bar{\kappa}_t} \kappa d\Gamma_\kappa = \int^{\Gamma_\kappa^{-1}(Z_{R,t})} \kappa d\Gamma_\kappa.$$

Borrower Default and Mortgage Indexation. Before deciding whether or not to refinance a loan, borrowers decide whether to or not to default on the loan. Upon default, the housing collateral used to back the loan is seized by the intermediary. To allow for an aggregated model in which the default rate responds endogenously to macroeconomic conditions, we introduce shocks $\omega_{i,t}$ to the quality of borrowers' houses, drawn i.i.d. across borrowers and time from a distribution with c.d.f. $\Gamma_{\omega,t}$, with $\mathbb{E}_t(\omega_{i,t}) = 1$ and $\text{Var}_t(\omega_{i,t}) = \sigma_{\omega,t}^2$.

In addition to the standard mortgage contracts defined above, we introduce Shared Appreciation Mortgages whose payments are indexed to house prices. We allow SAM contracts to insure households in two ways. First, mortgage payments can be indexed to the aggregate house price p_t . Specifically, each period, the principal and payment on each existing mortgage loan is multiplied by:

$$\zeta_{p,t} = \iota_p \left(\frac{p_t}{p_{t-1}} \right) + (1 - \iota_p). \quad (4)$$

The special cases $\iota_p = 0$ and $\iota_p = 1$ correspond to the cases of no insurance and complete

insurance against aggregate house price risk.

Second, mortgage contracts can be indexed to individual movements in house prices $\omega_{i,t}$. Specifically, each period, the principal and payment on a loan backed by a house that receives shock ω_t are multiplied by:

$$\zeta_{\omega,t}(\omega) = \iota_\omega \omega_t + (1 - \iota_\omega).$$

The special cases $\iota_\omega = 0$ and $\iota_\omega = 1$ correspond to the cases of no insurance and complete insurance against idiosyncratic house price risk. Since the model does not distinguish between shocks to local house prices and “basis risk” to an individual house, indexation to local house prices can be captured by partial indexation: $0 < \iota_\omega < 1$.

Borrowers must commit to a default plan that can depend in an unrestricted way on $\omega_{i,t}$ and the aggregate states, but not on a borrower’s individual loan conditions. We guess and verify that the optimal plan for the borrower is to default whenever $\omega_{i,t} \leq \bar{\omega}_t$, where $\bar{\omega}_t$ is the threshold shock that makes the borrower indifferent between defaulting and not defaulting. The level of the default threshold depends on the aggregate state and, importantly, also on the level of mortgage payment indexation.

Given $\bar{\omega}_t$, the fraction of non-defaulting borrowers is:

$$Z_{N,t} = 1 - \Gamma_{\omega,t}(\bar{\omega}_t).$$

Since non-defaulting borrowers are those who receive relatively good shocks, the share of borrower housing kept by non-defaulting households is:

$$Z_{K,t} = \int_{\bar{\omega}_t} \omega d\Gamma_{\omega,t},$$

and the outstanding borrower debt by non-defaulting borrowers is:

$$Z_{A,t} = \int_{\bar{\omega}_t} \zeta_\omega(\omega) d\Gamma_{\omega,t} = \iota_\omega \int_{\bar{\omega}_t} \omega d\Gamma_{\omega,t} + (1 - \iota_\omega) \Gamma_{\omega,t}(\bar{\omega}_t) = \iota_\omega Z_{K,t} + (1 - \iota_\omega) Z_{N,t}. \quad (5)$$

Intuitively, with zero indexation to idiosyncratic shocks, defaulting is attractive for borrowers if the value of the houses lost in foreclosure (fraction $1 - Z_{K,t}$) is smaller than the value of debt shed in default (fraction $1 - Z_{A,t}$). Equation (5) shows that increasing indexation shrinks this difference and therefore makes defaulting less attractive for borrowers. It is easy to show that for the case of full indexation to idiosyncratic shocks, $\iota_\omega = 1$, one gets $Z_{N,t} = Z_{A,t} = Z_{K,t} = 1$, i.e. borrowers optimally do not default on any

payments in that case.

REO Sector. The housing collateral backing defaulted loans is seized by the intermediary and rented out as REO (“real estate owned”) housing to the borrower. Housing in this state incurs a larger maintenance cost than usual, $v^{REO} > v^K$, designed to capture losses from foreclosure. With probability S^{REO} per period, REO housing is sold back to borrowers as owner-occupied housing. The existing stock of REO housing is denoted by K_t^{REO} , and the value of a unit of REO-owned housing is denoted p_t^{REO} .

Deposit Technology. Deposits in the model take the form of risk-free one-period loans issued from the depositor to the intermediary, where the price of these loans is denoted q_t^f , implying the interest rate $1/q_t^f$. Intermediaries must satisfy a leverage constraint (defined below in (20)) stating that their promised deposit repayments must be collateralized by their existing loan portfolio.

3.5 Borrower’s Problem

Given this model setup, the individual borrower’s problem aggregates to that of a representative borrower. The endogenous state variables are the promised payment A_t^B , the face value of principal M_t^B , and the stock of borrower-owned housing K_t^B . The representative borrower’s control variables are nondurable consumption C_t^B , housing service consumption H_t^B , the amount of housing K_t^* and new loans M_t^* taken on by refinancers, the refinancing fraction $Z_{R,t}$, and the mortgage default rate $1 - Z_{N,t}$.

The borrower maximizes (2) subject to the budget constraint:

$$\begin{aligned}
C_t^B = & \underbrace{(1 - \tau)Y_t^B}_{\text{disp. income}} + \underbrace{Z_{R,t} \left(Z_{N,t}M_t^* - \delta Z_{A,t}M_t^B \right)}_{\text{net new borrowing}} - \underbrace{(1 - \delta)Z_{A,t}M_t^B}_{\text{principal payment}} - \underbrace{(1 - \tau)Z_{A,t}A_t^B}_{\text{interest payment}} \\
& - \underbrace{p_t \left[Z_{R,t}Z_{N,t}K_t^* + (v^K - Z_{R,t})Z_{K,t}K_t^B \right]}_{\text{owned housing}} - \underbrace{\rho_t (H_t^B - K_t^B)}_{\text{rental housing}} \\
& - \underbrace{(\Psi(Z_{R,t}) - \bar{\Psi}_t)Z_{N,t}M_t^*}_{\text{net transaction costs}} - \underbrace{T_t^B}_{\text{lump sum taxes}}
\end{aligned} \tag{6}$$

the loan-to-value constraint

$$M_t^* \leq \phi^K p_t K_t^* \tag{7}$$

and the laws of motion

$$M_{t+1}^B = \pi_{t+1}^{-1} \zeta_{p,t+1} \left[Z_{R,t} Z_{N,t} M_t^* + \delta(1 - Z_{R,t}) Z_{A,t} M_t^B \right] \quad (8)$$

$$A_{t+1}^B = \pi_{t+1}^{-1} \zeta_{p,t+1} \left[Z_{R,t} Z_{N,t} r_t^* M_t^* + \delta(1 - Z_{R,t}) Z_{A,t} A_t^B \right] \quad (9)$$

$$K_{t+1}^B = Z_{R,t} Z_{N,t} K_t^* + (1 - Z_{R,t}) Z_{K,t} K_t^B \quad (10)$$

where π_t is the inflation rate, r_t^* is the interest rate on new mortgages, τ is the income tax rate, which also applies to the mortgage interest deductibility, ρ_t is the rental rate for housing services, $\bar{\Psi}_t$ is a subsidy that rebates transaction costs back to borrowers, and T_t^B are taxes raised on borrowers to pay for intermediary bailouts (defined below in (24)).

3.6 Intermediary's Problem

The intermediation sector consists of intermediary households (bankers), mortgage lenders (banks), and REO firms. The bankers are the owners, the equity holders, of both the banks and the REO firms. Each period, the bankers receive income Y_t^I , the aggregate dividend D_t^I from banks, and the aggregate dividend D_t^{REO} from REO firms. The latter two are defined in equations (23) and (25) below. Bankers choose consumption C_t^I to maximize (2) subject to the budget constraint:

$$C_t^I \leq (1 - \tau) Y_t^I + D_t^I + D_t^{REO} - v^K p_t H_t^I - T_t^I, \quad (11)$$

where T_t^I are taxes raised on intermediary households to pay for bank bailouts (defined in (24) below). Intermediary households consume their fixed endowment of housing services each period, $H_t^I = \bar{K}^I$.

Banks and REO firms maximize shareholder value. Banks lend to borrowers, issue deposits, and trade in the secondary market for mortgage debt. They are subject to idiosyncratic profit shocks and have limited liability, i.e., they optimally decide whether to default at the beginning of each period. When a bank defaults, it is seized by the government, which guarantees its deposits. The equity of the defaulting bank is wiped out, and bankers set up a new bank in place of the bankrupt one.

REO firms buy foreclosed houses from banks, rent these REO houses to borrowers, and sell REO housing in the regular housing market after maintenance.

Bank Portfolio Choice. Each bank chooses a portfolio of mortgage loans and how many deposits to issue. Although each mortgage with a different interest rate has a different secondary market price, we show in the appendix that any portfolio of loans can be replicated using only two instruments: an interest-only (IO) strip, and a principal-only (PO) strip. In equilibrium, beginning-of-period holdings of the IO and PO strips will correspond to the total promised interest payments and principal balances that are the state variables of the borrower's problem, and will therefore be denoted A_t^I and M_t^I , respectively. Denote new lending by banks in terms of face value by L_t^* . Then the end-of-period *supply* of PO and IO strips is given by:

$$\hat{M}_t^I = L_t^* + \delta(1 - Z_{R,t})Z_{A,t}M_t^I \quad (12)$$

$$\hat{A}_t^I = r_t^*L_t^* + \delta(1 - Z_{R,t})Z_{A,t}A_t^I. \quad (13)$$

Denote bank *demand* for PO and IO strips, and therefore the end-of-period holdings of these claims, by \tilde{M}_t^I and \tilde{A}_t^I , respectively. In equilibrium, we will have that $\hat{M}_t^I = \tilde{M}_t^I$ and $\hat{A}_t^I = \tilde{A}_t^I$.

The laws of motion for these variables depend on the level of indexation. Since they are nominal contracts, they also need to be adjusted for inflation:

$$M_{t+1}^I = \pi_{t+1}^{-1}\zeta_{p,t+1}\tilde{M}_t^I \quad (14)$$

$$A_{t+1}^I = \pi_{t+1}^{-1}\zeta_{p,t+1}\tilde{A}_t^I. \quad (15)$$

Banks can sell new loans to other banks in the secondary PO and IO market. The PO and IO strips trade at market prices q_t^M and q_t^A , respectively. The market value of the portfolio held by banks at the end of each period is therefore:

$$J_t^I = \underbrace{(1 - r_t^*q_t^A - q_t^M)L_t^*}_{\text{net new debt}} + \underbrace{q_t^A\tilde{A}_t^I}_{\text{IO strips}} + \underbrace{q_t^M\tilde{M}_t^I}_{\text{PO strips}} - \underbrace{q_t^f B_{t+1}^I}_{\text{new deposits}}. \quad (16)$$

To calculate the payoff of this portfolio in period $t + 1$, we first define the recovery rate of housing from foreclosed borrowers, per unit of face value outstanding, as:⁷

$$X_t = \frac{(1 - Z_{K,t})K_t^B(p_t^{REO} - v^{REO}p_t)}{M_t^B}. \quad (17)$$

⁷Note that X_t is taken as given by each individual bank. A bank does not internalize the effect of its mortgage debt issuance on the overall recovery rate.

After paying maintenance on the REO housing for one period, the banks sell the seized houses to the REO sector at prices p^{REO} .

Then the portfolio payoff is:

$$W_{t+1}^I = \underbrace{\left[X_{t+1} + Z_{A,t+1} \left((1 - \delta) + \delta Z_{R,t+1} \right) \right] M_{t+1}^I + Z_{A,t+1} A_{t+1}^I}_{\text{payments on existing debt}} + \underbrace{\delta(1 - Z_{R,t+1}) Z_{A,t+1} \left(q_{t+1}^A A_{t+1}^I + q_{t+1}^M M_{t+1}^I \right)}_{\text{sales of IO and PO strips}} - \underbrace{\pi_{t+1}^{-1} B_t^I}_{\text{deposit redemptions}}. \quad (18)$$

This is also the net worth of banks at the beginning of period $t + 1$.

Bank's Problem. Denote by \mathcal{S}_t^I all state variables exogenous to banks. At the beginning of each period, before making their optimal default decision, banks receive an idiosyncratic profit shock $\epsilon_t^I \sim F_\epsilon^I$, with $E(\epsilon_t^I) = 0$. The value of banks that do not default can be expressed recursively as:

$$V_{ND}^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I - \epsilon_t^I + E_t \left[\Lambda_{t,t+1}^I \max \left\{ V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I), 0 \right\} \right], \quad (19)$$

subject to the bank leverage constraint:

$$B_{t+1}^I \leq \phi^I \left(q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I \right), \quad (20)$$

the definitions of J_t^I and W_t^I in (16) and (18), respectively, and the transition laws for the aggregate supply of IO and PO strips in (12) – (15). The value of defaulting banks to shareholders is zero. The value of the newly started bank that replaces a bank liquidated by the government after defaulting, is given by:

$$V_R^I(\mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} - J_t^I + E_t \left[\Lambda_{t,t+1}^I \max \left\{ V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I), 0 \right\} \right], \quad (21)$$

subject to the same set of constraints as the non-defaulting bank.

Beginning-of-period net worth W_t^I and the idiosyncratic profit shock ϵ_t^I are irrelevant for the portfolio choice of non-defaulting and newly started banks, implying that all banks will choose identical portfolios at the end of the period. In the appendix, we show that we can define a value function after the default decision to characterize the portfolio problem

of all banks:⁸

$$V^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + E_t \left[\Lambda_{t,t+1}^I F_{\epsilon,t+1}^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right], \quad (22)$$

where

$$F_{\epsilon,t+1}^I \equiv F_{\epsilon}^I(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I))$$

is the probability of continuation, and $\epsilon_{t+1}^{I,-} = E[\epsilon_{t+1}^I | \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)]$ is the expectation of ϵ_{t+1}^I conditional on continuation. The objective in (22) is subject to the same set of constraints as (19).

Aggregation and Government Deposit Guarantee. By the law of large numbers, the fraction of defaulting banks each period is $1 - F_{\epsilon,t}^I$. The aggregate dividend paid by banks to their shareholders, the intermediary households, is:

$$\begin{aligned} D_t^I &= F_{\epsilon,t}^I \left(W_t^I - \epsilon_t^{I,-} - J_t^I \right) - \left(1 - F_{\epsilon,t}^I \right) J_t^I \\ &= F_{\epsilon,t}^I \left(W_t^I - \epsilon_t^{I,-} \right) - J_t^I. \end{aligned} \quad (23)$$

Bank shareholders bear the burden of replacing liquidated banks by an equal measure of new banks and seeding them with new capital equal to that of continuing banks (J_t^I).

The government bails out defaulted banks at a cost:

$$\text{bailout}_t = \left(1 - F_{\epsilon,t}^I \right) \left[\epsilon_t^{I,+} - W_t^I + \eta \delta (1 - Z_{R,t}) Z_{A,t} \left(q_t^A A_t^I + q_t^M M_t^I \right) \right],$$

where $\epsilon_t^{I,+} = E[\epsilon_t^I | \epsilon_t^I > V^I(W_t^I, \mathcal{S}_t^I)]$ is the expectation of ϵ_t^I conditional on bankruptcy. Thus, the government absorbs the negative net worth of the defaulting banks. The last term are additional losses from bank bankruptcies, which are a fraction η of the mortgage assets and represent deadweight losses to the economy. To finance the bailout, the government levies lump-sum taxes on all households, in proportion to their population share:

$$T_t^j = \chi_j \text{bailout}_t, \quad \forall j \in \{B, I, D\}. \quad (24)$$

The government bailout is what makes deposits risk-free, what creates deposit insurance.

⁸The value of the newly started bank with zero net worth is simply the value in (22) evaluated at $W_t^I = 0$.

REO Firm's Problem. There is a continuum of competitive REO firms that are fully owned and operated by intermediary households (bankers). Each period, REO firms choose how many foreclosed properties to buy from banks, I_t^{REO} , to maximize the NPV of dividends paid to intermediary households. The aggregate dividend in period t paid by the REO sector to the bankers is:

$$D_t^{REO} = \underbrace{\left[\rho_t + (S^{REO} - \nu^{REO}) p_t \right] K_t^{REO}}_{\text{REO income}} - \underbrace{p_t^{REO} I_t^{REO}}_{\text{REO investment}}. \quad (25)$$

The law of motion of the REO housing stock is:

$$K_{t+1}^{REO} = (1 - S^{REO}) K_t^{REO} + I_t^{REO}.$$

3.7 Depositor's Problem

The depositors' problem can also be aggregated, so that the representative depositor chooses nondurable consumption C_t^D and deposits B_t^D to maximize (2) subject to the budget constraint:

$$C_t^D \leq \underbrace{(1 - \tau) Y_t^D}_{\text{disp. income}} - \underbrace{\left(q_t^f B_{t+1}^D - \pi_t^{-1} B_t^D \right)}_{\text{net deposit iss.}} - \underbrace{\nu^K p_t H_t^D}_{\text{own housing maint.}} - \underbrace{T_t^D}_{\text{lump sum taxes}}. \quad (26)$$

and a restriction that deposits must be positive: $B_t^D \geq 0$. Depositors consume their fixed endowment of housing services each period, $H_t^D = \bar{K}^D$.

3.8 Financial Recessions

At any given point in time, the economy is either in a "normal" state, or a "crisis" state, the latter corresponding to a severe financial recession. This state evolves according to a Markov Chain with transition matrix Π . The financial recession state is associated with a higher value of $\sigma_{\omega,t}$, implying more idiosyncratic uncertainty; and a lower value of ζ_t , implying a fall in aggregate house prices. This discrete state provides the only source of aggregate risk aside from the aggregate income shock $\varepsilon_{y,t}$.

4 Model Solution

4.1 Borrower Optimality

The optimality condition for new mortgage debt,

$$1 = \Omega_{M,t} + r_t^* \Omega_{A,t} + \lambda_t^{LTV},$$

equalizes the benefit of taking on additional debt — \$1 today — to the cost of carrying more debt in the future, both in terms of carrying more principal ($\Omega_{M,t}$) and higher interest payments ($\Omega_{A,t}$), plus the shadow cost of tightening the LTV constraint. The marginal continuation costs are defined recursively:

$$\begin{aligned}\Omega_{M,t} &= \mathbb{E}_t \left\{ \Lambda_{t+1}^B \pi_{t+1}^{-1} \zeta_{p,t+1} Z_{A,t+1} \left[(1 - \delta) + \delta Z_{R,t+1} + \delta (1 - Z_{R,t+1}) \Omega_{M,t+1} \right] \right\} \\ \Omega_{A,t} &= \mathbb{E}_t \left\{ \Lambda_{t+1}^B \pi_{t+1}^{-1} \zeta_{p,t+1} Z_{A,t+1} \left[(1 - \tau) + \delta (1 - Z_{R,t+1}) \Omega_{A,t+1} \right] \right\}\end{aligned}$$

where an extra unit of principal requires a payment of $(1 - \delta)$ in the case of non-default, plus payment of the face value of prepaid debt, plus the continuation cost of non-prepaid debt. An extra promised payment requires a tax-deductible payment on non-defaulted debt plus the continuation cost if the debt is not prepaid.

The optimality condition for housing services consumption sets the rental rate to be the marginal rate of substitution between housing services and nondurables:

$$\rho_t = u_{c,t}^{-1} u_{h,t}.$$

The borrower's optimality condition for new housing capital is:

$$p_t = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^B \left[\rho_{t+1} + Z_{K,t+1} p_{t+1} \left(1 - v^K - (1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K \right) \right] \right\}}{1 - \lambda_t^{LTV} \phi^K}.$$

The numerator represents the present value of holding an extra unit of housing next period: the rental service flow, plus the continuation value of the housing if the borrower chooses not to default, net of the maintenance cost. The continuation value needs to be adjusted by $(1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K$ because if the borrower does not choose to refinance, which occurs with probability $1 - Z_{R,t+1}$, then she does not use the unit of housing to collateralize a new loan, and therefore does not receive the collateral benefit.

The optimal refinancing rate is:

$$Z_{R,t} = \Gamma \left\{ \underbrace{(1 - \Omega_{M,t} - \bar{r}_t \Omega_{A,t}) \left(1 - \frac{\delta Z_{A,t} M_t}{Z_{N,t} M_t^*} \right)}_{\text{equity extraction incentive}} + \underbrace{\Omega_{A,t} (\bar{r}_t - r_t^*)}_{\text{interest rate incentive}} - \underbrace{p_t \mathcal{C}_t \left(\frac{Z_{N,t} K_t^* - Z_{K,t} K_t^B}{Z_{N,t} M_t^*} \right)}_{\text{collateral expense}} \right\} \quad (27)$$

where $\bar{r}_t = A_t^B / M_t^B$ is the average interest rate on existing debt. The “equity extraction incentive” term represents the net gain from obtaining additional debt at the *existing* interest rate, while “interest rate incentive” term represents the gain from moving from the existing to new interest rate. The stronger these incentives, the higher the refinancing rate. The “collateral expense” term arises because housing trades at a premium relative to the present value of its housing service flow due to its collateral value. Refinancing is less desirable when taking on new debt would increase the cost of that collateral.

The optimality condition for the default rate pins down the default threshold $\bar{\omega}_t$:

$$\begin{aligned} \zeta_\omega(\bar{\omega}_t) & \left[\underbrace{(\delta Z_{R,t} + (1 - \delta)) M_t + (1 - \tau) A_t}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t}) (\Omega_{M,t} M_t + \Omega_{A,t} A_t)}_{\text{continuation cost of debt}} \right] \\ & = \underbrace{(1 - \nu^K - (1 - Z_{R,t}) \lambda_t^{LTV} \phi^K)}_{\text{continuation value of housing}} p_t \bar{\omega}_t K_t^B \end{aligned}$$

This expression relates the benefit of defaulting on debt, which is eliminating both the current payment and continuation cost, after indexation, against the cost of losing a marginal unit of housing at the threshold idiosyncratic shock level $\bar{\omega}_t$, and the cost of not being able to use that lost unit of housing to finance new borrowing under a refinancing.

4.2 Intermediary Optimality

The optimality condition for new debt L^* is:

$$1 = q_t^M + r_t^* q_t^A,$$

which balances the cost of issuing new debt, \$1 today, against the value of the loan obtained, 1 unit of PO strip plus r_t^* units of the IO strip. The condition implies that the first

term in (16) is zero.

The optimality condition for deposits is:

$$q_t^f = \mathbb{E}_t \left[\Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \right] + \lambda_t^I$$

where λ_t^I is the multiplier on the intermediary's leverage constraint (20). The default option, represented by the $F_{\epsilon,t+1}^I$ term in the expectation, drives a wedge between the valuation of risk free debt by intermediary households, $\mathbb{E}_t \left[\Lambda_{t+1}^I \pi_{t+1}^{-1} \right]$, and that of banks.

The optimality conditions for IO and PO strip holdings pin down their prices:

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[Z_{A,t+1} \left(1 + \delta(1 - Z_{R,t+1}) q_{A,t+1}^A \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}$$

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{A,t+1} \left((1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}.$$

Both securities issue cash flows that are nominal (discounted by inflation) and indexed to house prices (discounted by $\zeta_{p,t+1}$). Both securities can also be used to collateralize deposits, leading to the collateral premia in the denominators. The IO strip's next-period payoff is equal to \$1 for loans that do not default, with a continuation value of q_{t+1}^A for loans that do not prepay or mature. The PO strip's next-period payoff is the recovery value for defaulting debt X_{t+1} plus the payoff from loans that do not default: the principal payment $1 - \delta$, plus the face value of prepaying debt, plus the continuation value q_{t+1}^M for loans that do not mature or prepay.

The optimality condition for REO housing is:

$$p_t^{REO} = \mathbb{E}_t \left\{ \Lambda_{t+1}^I \left[\rho_{t+1} - \nu^{REO} p_{t+1} + S^{REO} p_{t+1} + (1 - S^{REO}) p_{t+1}^{REO} \right] \right\}.$$

The right-hand side is the present discounted value of holding a unit of REO housing next period. This term is in turn made up of the rent charged to borrowers, the maintenance cost, and the value of the housing next period, both the portion sold back to the borrowers, and the portion kept in the REO state.

4.3 Depositor Optimality

The depositor's sole optimality condition for deposits, which are nominal contracts, ensures that the depositor's Euler equation is at an interior solution:

$$q_t^f = \mathbb{E}_t \left[\Lambda_{t+1}^D \pi_{t+1}^{-1} \right].$$

5 Calibration

This section describes the calibration procedure for key variables, and presents the full set of parameter values in Table 1. The model is calibrated at quarterly frequency.

Exogenous Shock Processes. Aggregate endowment shocks in (1) have quarterly persistence $\rho_y = .977$ and innovation volatility $\sigma_y = 0.81\%$. These are the observed persistence and innovation volatility of log real per capita labor income from 1991.Q1 until 2016.Q1.⁹ In the numerical solution, this AR process is discretized as a five-state Markov Chain, following the [Rouwenhorst \(1995\)](#) method. Long-run endowment growth $g = 0$. The average level of aggregate income (GDP) is normalized to 1. The income tax rate is $\tau = 0.147$, as given by the observed ratio of personal income tax revenue to personal income.

The idiosyncratic housing quality shock distribution $\Gamma_{\omega,t}$ is parameterized as a log-normal distribution, so that

$$Z_{N,t} = \int_{\bar{\omega}}^{\infty} dF(\omega) = 1 - \Phi \left(\frac{\log \bar{\omega}_t + \sigma_{\omega,t}^2/2}{\sigma_{\omega,t}} \right)$$

$$Z_{K,t} = \int_{\bar{\omega}}^{\infty} \omega dF(\omega) = 1 - \Phi \left(\frac{\log \bar{\omega}_t - \sigma_{\omega,t}^2/2}{\sigma_{\omega,t}} \right)$$

where Φ denotes the standard normal distribution function.

The discrete state follows a two-state Markov Chain, with state 0 indicating normal times, and state 1 indicating crisis. The probability of staying in the normal in the next quarter is 97.5% and the probability of staying in the crisis in the next quarter is 92.5%.

⁹Labor income is defined as compensation of employees (line 2) plus proprietor's income (line 9) plus personal current transfer receipts (line 16) minus contributions to government social insurance (line 25), as given by Table 2.1 of the Bureau of Economic Analysis' National Income and Product Accounts. Deflation is by the personal income deflator and by population. We remove a linear trend to take a deterministic growth component and then take logs.

Under these parameters, the economy spends 3/4 of the time in the low uncertainty state and 1/4 in the high uncertainty state. This matches the fraction of time between 1991.Q1 and 2016.Q4 that the U.S. economy was in the foreclosure crisis, and implies an average duration of the normal state of ten years. These transition probabilities are independent of the aggregate endowment state. The low uncertainty state has $\bar{\sigma}_{\omega,0} = 0.200$ and the high uncertainty state has $\bar{\sigma}_{\omega,1} = 0.270$. These numbers allow the model to match an average mortgage default rate of 0.65% in expansions and of 2.3% in financial recessions, which are periods defined by low endowment growth and high uncertainty. The unconditional mortgage default rate is 1.2%. In the data, the average mortgage delinquency rate is 1.05% per quarter; it is 0.7% in normal times and 2.3% during the foreclosure crisis.¹⁰

Demographics, Income, and Housing Shares. We split the population into mortgage borrowers, depositors, and intermediary households as follows. We use the 1998 Survey of Consumer Finances to define for every household a loan-to-value ratio. This ratio is zero for renters and for households who own their house free and clear. We define mortgage borrowers to be those households with LTV ratio of at least 30%.¹¹ Those households make up for 34.3% of households ($\chi_B = .343$). They earn 46.9% of labor income ($s_B = .469$). For parsimony, we set all housing shares equal to the corresponding income share. Since the aggregate housing stock \bar{K} is normalized to 1, $\bar{K}^B = .469$.

To split the remaining households into depositors and intermediary households (bankers), we set the share of labor income for bankers equal to 6.7%. To arrive at this number, we calculate the share of the financial sector (finance, insurance, and real estate) in overall stock market capitalization (16.4% in 1990-2017) and multiply that by the labor income share going to all equity holders in the SCF. We set the housing share again equal to the income share. The population share of bankers is set to 2%, consistent with the observed employment share in the FIRE sector. The depositors make up the remaining $\chi_D = 62.7\%$ of the population, and receive the remaining $s_D = 46.4\%$ of labor income and of the housing stock.

Prepayment Costs. For the prepayment cost distribution, we assume a mixture distribution, so that with probability 3/4, the borrower draws an infinite prepayment cost,

¹⁰Data are for all residential mortgage loans held by all U.S. banks, quarterly data from the New York Federal Reserve Bank from 1991.Q1 until 2016.Q4. The delinquency rate averages 2.28% per quarter between 2008.Q1 and 2013.Q4 (high uncertainty period, 23% of quarters) and 0.69% per quarter in the rest of the period.

¹¹Those households account for 88.2% of mortgage debt and 81.6% of mortgage payments.

Table 1: Parameter Values: Baseline Calibration (Quarterly)

Parameter	Name	Value	Target/Source
<i>Technology</i>			
Agg. income persistence	ρ_{TFP}	0.977	Real per capita labor income BEA
Agg. income st. dev.	σ_{TFP}	0.008	Real per capita labor income BEA
Profit shock st. dev.	σ_ϵ	0.075	FDIC bank failure rate
Transition: Normal \rightarrow Normal	Π_{00}	0.975	Avg. length = 10Y
Transition: Crisis \rightarrow Crisis	Π_{11}	0.925	25% of time in crisis state
<i>Demographics and Income</i>			
Fraction of borrowers	χ_B	0.343	SCF 1998 population share LTV > .30
Fraction of intermediaries	χ_I	0.020	Stock market cap. share of finance sector
Borr. inc. and housing share	s_B	0.470	SCF 1998 income share LTV > .30
Intermediary inc. and housing share	s_I	0.067	Employment share in finance
Tax rate	τ	0.147	Personal tax rate BEA
<i>Housing and Mortgages</i>			
Housing stock	\bar{K}	1	Normalization
Housing st. dev. (Normal)	$\bar{\sigma}_{\omega,0}$	0.200	Mortg. delinq. rate US banks, no crisis
Housing st. dev. (Crisis)	$\bar{\sigma}_{\omega,1}$	0.270	Mortg. delinq. rate US banks, crisis
Inflation rate	$\bar{\pi}$	1.006	2.29% CPI inflation
Mortgage duration	δ	0.996	Duration of 30-yr FRM
Prepayment cost mean	μ_κ	0.370	Greenwald (2016)
Prepayment cost scale	s_κ	0.152	Greenwald (2016)
LTV limit	ϕ^K	0.850	LTV at origination
Maint. cost (owner)	v^K	0.616%	BEA Fixed Asset Tables
<i>Intermediaries</i>			
Bank regulatory capital limit	ϕ^I	0.940	Financial sector leverage limit
Deadweight cost of bank failures	η	0.090	Bank receivership expense rate
Maint. cost (REO)	v^{REO}	0.024	REO discount: $p_{ss}^{REO}/p_{ss} = 0.725$
REO sale rate	S^{REO}	0.167	Length of foreclosure crisis
<i>Preferences</i>			
Borr. discount factor	β_B	0.950	Borrower value/income, SCF
Intermediary discount factor	β_I	0.950	Equal to β_B
Depositor discount factor	β_D	0.998	3% nominal short rate (annual)
Risk aversion	γ	5.000	Standard value
EIS	ψ	1.000	Standard value
Housing preference (Normal)	$\bar{\xi}_0$	0.220	Borrower hous. expend./income
Housing preference (Crisis)	$\bar{\xi}_1$	0.160	HP growth volatility

while with probability 1/4, the borrower draws from a logistic distribution, yielding

$$\rho_t = \frac{1}{4} \cdot \frac{1}{1 + \exp\left(\frac{\bar{\kappa}_t - \mu_\kappa}{\sigma_\kappa}\right)}$$

The calibration of the parameters follows [Greenwald \(2016\)](#), who fits an analogue of (27).¹² The parameter σ_κ , determining the sensitivity of prepayment to equity extraction and interest rate incentives, is set to that paper's estimate (0.152), while the parameter μ_κ is set to match the average quarterly prepayment rate of 3.76% found in that exercise.

Mortgages. We set $\delta = .99565$ to match the fraction of principal US households amortize on mortgages.¹³ The maximum loan-to-value ratio at mortgage origination is $\phi^B = 0.85$, consistent with average standard mortgage underwriting norms.¹⁴ Inflation is set equal to the observed 0.57% per quarter (2.29% per year) for the 1991.Q1 - 2016.Q4 sample.

Banks. We set the maximum leverage that banks may take on at $\phi^I = 0.940$, following [Elenev, Landvoigt, and Van Nieuwerburgh \(2017\)](#), to capture the historical average leverage ratio of the leveraged financial sector. The idiosyncratic profit shock that hits banks has standard deviation of $\sigma_\epsilon = 7.50\%$ per quarter. This delivers a bank failure rate of 0.28% per quarter, consistent with historical bank failure rate data from the FDIC.¹⁵ We assume a deadweight loss from bank bankruptcies equal to $\eta = 9.00\%$ of bank assets. This number falls in the interquartile range [5.9%,15.9%] of bank receivership expenses as a ratio of bank assets in a FDIC study of bank failures from 1986 until 2007 ([Bennett and Unal, 2015](#)). Deadweight losses from bank failures amount to 0.06% of GDP in equilibrium.

¹²See [Greenwald \(2016\)](#), Section 4.2. The parameters are fit to minimize the forecast error $LTV_t = Z_{R,t}LTV_t^* + (1 - Z_{R,t})\delta G_t^{-1}LTV_{t-1}$, where LTV_t is the ratio of total mortgage debt to housing wealth, LTV_t^* is LTV at origination, and G_t is growth in house values.

¹³The average duration of a 30-year fixed-rate mortgage is typically thought of as about 7 years. This low duration is mostly the result of early prepayments. The parameter δ captures amortization absent refinancing. Put differently, households are paying off a much smaller fraction of their mortgage principal than 1/7th each year in the absence of prepayment.

¹⁴The average LTV of purchase mortgages originated by Fannie and Freddie was in the 80-85% range during our sample period. However, that does not include second mortgages and home equity lines of credit. Our limit is a combined loan-to-value limit (CLTV). It also does not capture the lower down payments on non-conforming loans that became increasingly prevalent after 2000. [Keys, Piskorski, Seru, and Vig \(2012\)](#) document CLTVs on non-conforming loans that rose from 85% to 95% between 2000 and 2007.

¹⁵Based on the FDIC database of all bank failure and assistance transaction from 1991-2016, we calculate the asset-weighted average annual failure rate to be 1.65%.

Housing Maintenance and REOs. We set the regular housing maintenance cost equal to $\nu^K = 0.616\%$ per quarter or 2.46% per year. This is the average over the 1991-2016 period of the ratio of current-cost depreciation of privately-owned residential fixed assets to the current-cost net stock of privately-owned residential fixed assets at the end of the previous year (source: BEA Fixed Asset Tables 5.1 and 5.4).

We calibrate the maintenance cost in the REO state to $\nu^{REO} = 2.40\%$ per quarter. It delivers REO housing prices that are 24.0% below regular housing prices on average. This is close to the observed fire-sale discounts reported by Fannie Mae and Freddie Mac during the foreclosure crisis. We assume that $S^{REO} = 0.167$ so that $1/6$ th of the REO stock is sold back to the borrower households each quarter. It takes eight quarters for 75% of the REO stock to roll off. This generates REO crises that take some time to resolve, as they did in the data.

Preferences. All agents have the same risk aversion coefficient of $\gamma_j = 5$ and inter-temporal elasticity of substitution coefficient $\psi = 1$. These are standard values in the literature. We choose the value of the housing preference parameter in normal times $\bar{\xi}_0 = 0.220$ to match a ratio of housing expenditure to income for borrowers of 18% , a common estimate in the housing literature.¹⁶ The model produces a ratio of 17.5% . To induce an additional house price drop, we set $\bar{\xi}_1 = 0.16$ in the crisis states. This additional variation yields a volatility of quarterly log national house price growth of 1.41% , compared to 1.56% in the data (source: Case Shiller).

For the time discount factors, we set $\beta^B = \beta^I = 0.950$ to target the ratio of housing wealth to quarterly income for borrowers of 9.09 , close to the same ratio for “borrowers” as defined above in the 1998 SCF (8.67). Finally, we set the discount rate of depositors $\beta^D = 0.998$ to match the observed nominal short rate of 3.0% per year or 0.70% per quarter. With these parameters, the model generates average borrower mortgage debt to housing wealth (LTV) of 64.1% , close to the corresponding value 61.6% for the “borrower” population in the 1998 SCF.

¹⁶Piazzesi, Schneider, and Tuzel (2007) obtain estimates between 18 and 20 percent based on national income account data (NIPA) and consumption micro data (CEX). Davis and Ortalo-Magné (2011) obtain a ratio of 18% after netting out 6% for utilities from the median value of 24% across MSAs using data on rents.

6 Results on Mortgage Indexation

The main exercise is to compare the economy with regular mortgages to hypothetical economies with varying degrees and forms of mortgage indexation. Specifically, we solve models with: (i) no indexation corresponding to $\iota_p = \iota_\omega = 0$, which is the benchmark; (ii) only aggregate indexation, such that $\iota_p = 1$ and $\iota_\omega = 0$; (iii) only local indexation, such that $\iota_p = 0$ and $\iota_\omega = 0.25$ (iv) aggregate plus local indexation, which we parameterize as $\iota_p = 1$ and $\iota_\omega = 0.25$. We conduct a long simulation for each of the four models. Table 2 shows averages of key prices and quantities computed from the simulated time series.

These stylized experiments are designed to showcase the different properties of aggregate and local indexation. While the typical SAM proposal does not distinguish between the source of house price movements, any indexation scheme can be decomposed into these two types. Moreover, we will show that these forms of indexation yield sharply different economic implications, which should be considered when designing a mortgage product. For the aggregate indexation experiment, we choose the extreme case of full insurance ($\iota_p = 1$) to generate clear qualitative results. For the local indexation experiment, we choose partial (25%) indexation. This limited insurance, perhaps against MSA-level variation in house prices, is designed to avoid moral hazard problems from indexing to the value of an individual property, as well as asymmetric information problems from assets whose cash flows are tied to hyper-local price indexes, as analyzed in [Hartman-Glaser and Hebert \(2017\)](#).

6.1 Benchmark Model

Unconditional Moments. Before turning to the indexation results, it is useful to briefly discuss the benchmark model. On the borrower side, the model generates average mortgage debt to annual GDP of 68.5%, matching the observed value of 69%. It generates an aggregate LTV ratio among mortgage borrowers of 64.1%. The average mortgage default rate of 0.95% per quarter matches the data, and the loss-given-default rate of 38.69% comes close to the data. The implied loss rate is 0.39% per quarter. The refinancing rate of 3.84% per quarter matches implied average rate at which mortgages are replaced excluding rate refinances. The maximum LTV constraint, which only applies at origination at caps the LTV at 85% always binds in our simulations, consistent with the overwhelming majority of borrowers taking out loans up to the limit.

On the intermediary side, we match the leverage ratio of the levered financial sector,

Table 2: Results: Quantities and Prices

	No Index	Aggregate	Local (25%)	Both
Borrower				
1. Housing Capital	0.456	0.457	0.465	0.465
2. Refi rate	3.84%	3.82%	3.79%	3.82%
3. Default rate	0.95%	0.91%	0.37%	0.38%
4. Household leverage	64.45%	64.43%	66.19%	66.06%
5. Fraction LTV binds at orig	1.000	1.000	1.000	1.000
6. Mortgage debt to GDP	263.17%	258.87%	289.06%	285.70%
7. Loss-given-default rate	38.69%	38.16%	37.62%	37.70%
8. Loss Rate	0.39%	0.38%	0.13%	0.14%
Intermediary				
9. Mkt fin leverage	93.78%	93.65%	93.96%	93.87%
10. Fraction leverage constr binds	99.42%	88.06%	99.63%	92.23%
11. REO maint	0.35%	0.33%	0.13%	0.14%
12. REO return	5.26%	8.41%	5.11%	7.32%
13. Bank dividend	0.010	0.014	0.011	0.013
14. REO dividend	0.005	0.005	0.002	0.002
15. Bank equity capital	0.192	0.195	0.206	0.207
16. Bank equity ratio	7.30%	7.53%	7.13%	7.25%
17. Bank default rate	0.28%	0.89%	0.15%	0.47%
18. DWL from bank defaults	0.06%	0.18%	0.04%	0.11%
Saver				
19. Deposits	2.483	2.435	2.733	2.694
Prices				
20. House Price	8.958	8.789	9.406	9.303
21. REO house price	6.781	6.702	7.077	6.957
22. Risk-free rate	0.70%	0.65%	0.73%	0.66%
23. Mortgage Rate	1.45%	1.48%	1.21%	1.21%
24. Credit spread	0.76%	0.83%	0.47%	0.55%
25. Mortgage Expec. Excess Ret	0.35%	0.46%	0.34%	0.41%

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with full indexation of mortgage payments to aggregate house prices (second column), a model with partial indexation to relative local prices (third column), and a model with both aggregate and partial local indexation (fourth column). All flow variables are quarterly.

which is 93.78% in the model. Banks' regulatory capital constraints bind in 99.42% of the periods. Bank equity capital represents about 4.9% of annual GDP (19.7% of quarterly

GDP) and 7.30% of bank assets in the model. Bank deposits (that go towards financing mortgage debt) represent just over 64.7% of annual GDP (258.9%/4). Bank dividends are 1.1% of GDP. The model generates a substantial amount of financial fragility. One measure thereof is the bank default rate. In the benchmark, it is 0.28% per quarter or 1.4% per year. Deadweight losses from bank bankruptcies are 0.06% of GDP on average.

The REO firms represent the other part of the intermediary sector. They spend 0.35% of GDP on housing maintenance on average, and pay 0.6% of GDP in dividends to their owners. REO firms earn very high returns from investing in foreclosed properties and selling them back to the borrowers: the return on equity is 5.5% per quarter (equal to the return on assets since the REO firms have no leverage).¹⁷

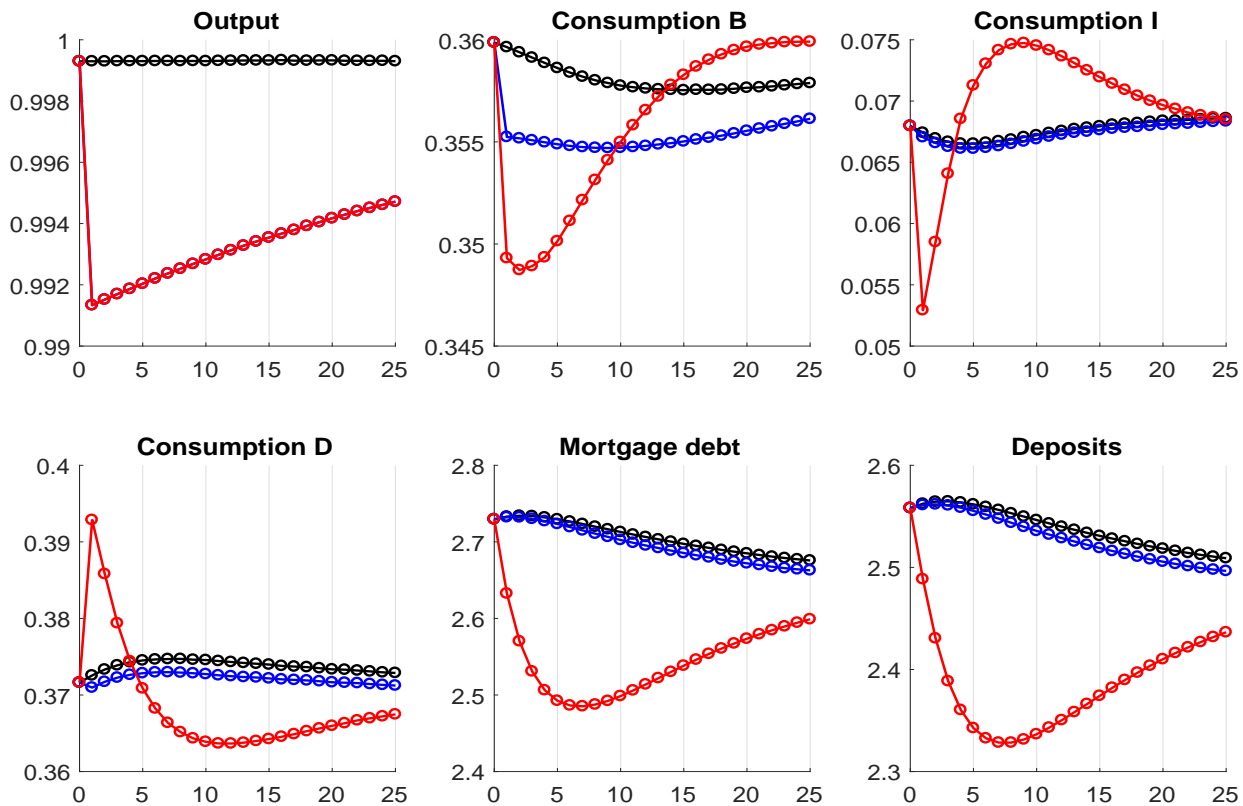
The model somewhat overstates housing wealth, which represents about 233.6% of annual GDP in the model and 153% in the data. This is an artifact of giving all agents the same housing to income ratio in the model, while the “borrower” type holds relatively more housing in the data than the other groups. At equilibrium, only borrower holdings of housing are relevant, so the quantitative effect of exaggerating total housing wealth is minimal. The mortgage rate exceeds the short rate by 78bps per quarter, which is close to the average spread between the 30-year fixed-rate mortgage rate and the 3-month T-bill rate of 89bps per quarter for 1991–2016. The model’s expected excess return, or risk premium, earned by banks on mortgages is 35bps per quarter.

Financial Crises. To understand risk-sharing patterns in the benchmark economy, it is instructive to study how the economy behaves in a financial recession and a non-financial recession. We a non-financial recession event as a one standard deviation drop in aggregate income while the economy is in the normal (non-crisis) state. In a financial recession, the economy experiences the same fall in income, but also transitions from the normal state into the crisis state, leading to an increase in house value uncertainty ($\bar{\sigma}_{\omega,0} \rightarrow \bar{\sigma}_{\omega,1}$) and a decrease in housing utility ($\bar{\xi}_0 \rightarrow \bar{\xi}_1$). We simulate many such recessions in order to average over the endogenous state variables (wealth distribution). Figures 1 and 2 plot the impulse-response functions, with financial recessions indicated by red circles, non-financial recessions in blue, and the black line indicating the average shock realization. By construction, the blue and red lines coincide in the top left panel of Figure 1.

A financial crisis results in a significant increase in mortgage defaults as well as bank failures. Bank equity falls, forcing banks to delever in the wake of the losses they suf-

¹⁷This return on equity in the model mimics the high returns earned by single-family rental firms like Blackstone’s Invitation Homes over the past five years.

Figure 1: Financial vs. Non-financial Recessions: Benchmark Model (part 1)



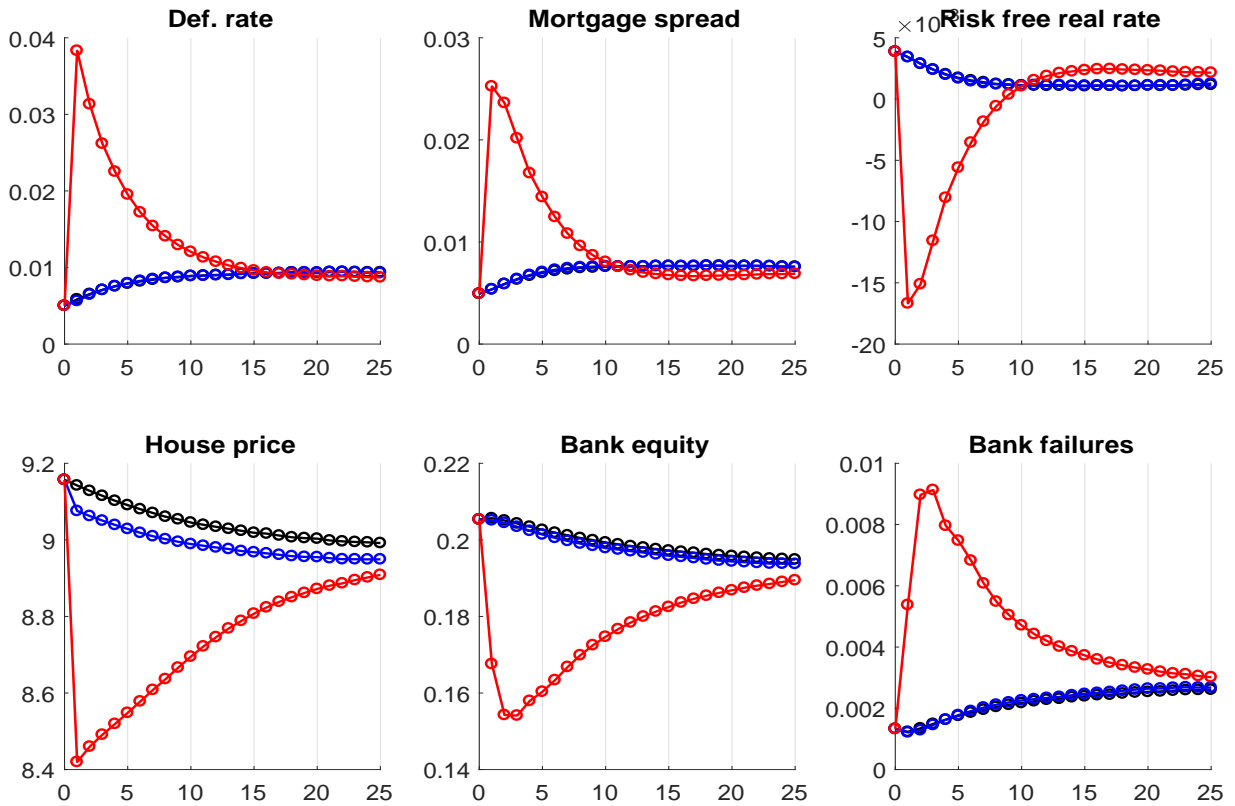
Blue line: non-financial recession, **Red line:** financial recession, **Black line:** no shocks.

fer. Banks shrink substantially, both in terms of their mortgage assets and their deposit liabilities. In order to induce depositor households to reduce deposits and increase consumption, the real interest rate falls sharply. Intermediary consumption falls heavily, as the owners of the intermediary sector absorb losses from mortgage default, since fixed payments on existing loans do not adjust for newly increased default risk. Borrower consumption also falls as borrowers cut back on new mortgage borrowing, and must help pay for the bank bailouts by paying higher taxes. After the shock, the economy gradually recovers as high mortgage spreads (and expected returns on mortgages) eventually replenish the bank equity.

6.2 Aggregate Indexation

The first experiment we consider is one where all mortgage payments are indexed to aggregate house prices. The conjecture in the literature is that this should reduce mortgage defaults and generally improve borrower's ability to smooth consumption. Surprisingly,

Figure 2: Financial vs. Non-financial Recessions: Benchmark Model (part 2)



Blue line: non-financial recession, **Red line:** financial recession, **Black line:** no shocks.

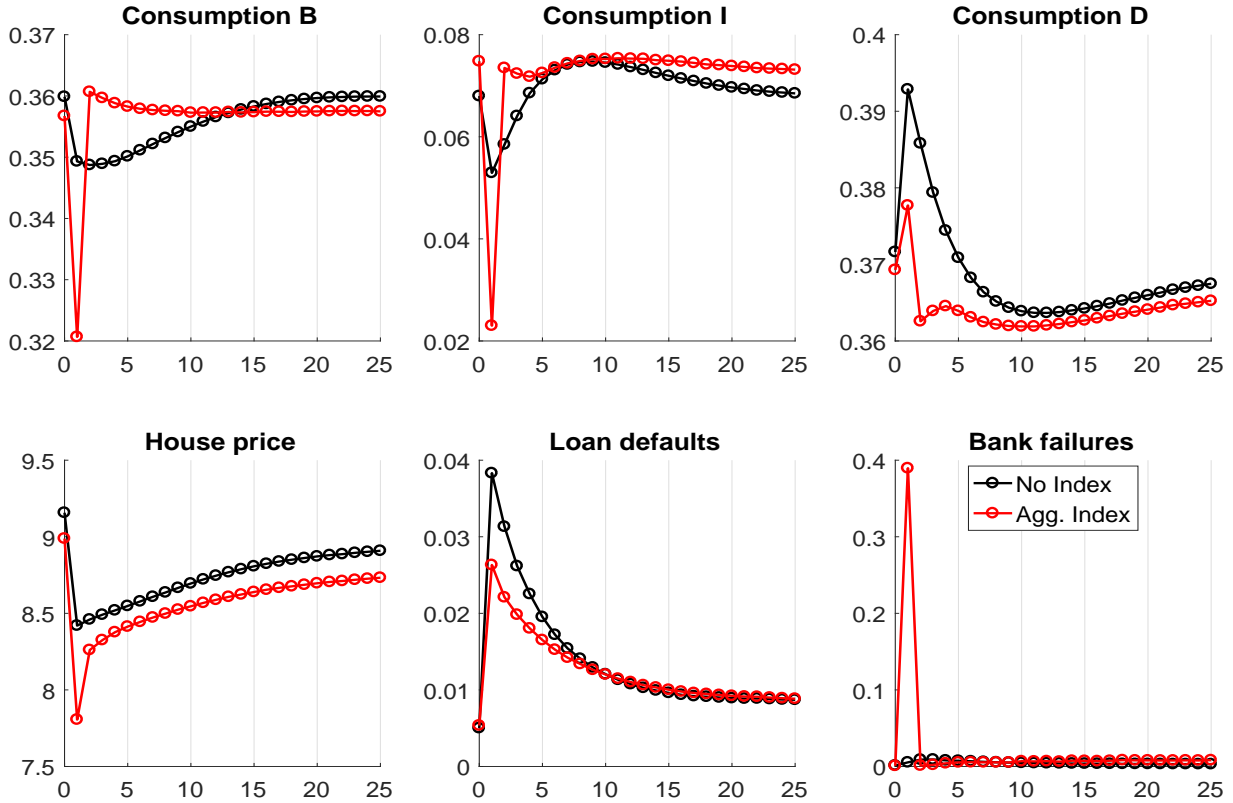
we find that this conjecture does not hold up in general equilibrium. To the contrary, Table 2 shows that by adding to financial fragility, aggregate indexation destabilizes borrower consumption while leaving mortgage default rates unchanged.

To understand this, we can turn to Figure 3, which compares financial recessions in the benchmark model to financial recessions in the model with aggregate indexation. Under aggregate indexation, banks find themselves exposed to increased risk through their loan portfolio. Although banks optimally choose to slightly decrease leverage and increase their capital buffer compared to the benchmark model, bank place their equity at much greater risk. Facing a trade-off between preserving charter value and taking advantage of limited liability, banks lean more toward their option to declare bankruptcy and saddle the government with the losses, explaining this behavior.

The combination of increased risk and the absence of precautionary capital means that the share of defaults upon entering a financial recession is three times larger in the aggregate indexation economy relative to the no indexation benchmark. This spike in bank

failures necessitates a wave of government bailouts, placing a large tax burden of nearly 6% of GDP on the population. An increase in tax payments to fund bailouts squeezes the borrower budget constraint, causing consumption to fall. It also depresses borrower housing demand, leading to a dramatically larger drop in house prices under aggregate indexation.

Figure 3: Financial Recessions: Benchmark vs. Aggregate Indexation Model



Black line: benchmark financial recession, **Red line:** aggregate index. financial recession.

Aggregate indexation provides a modest reduction in mortgage default in the financial recession. Although this indexation protects borrowers from the large fall in national house prices, it is unable to stave off the increase in defaults due to higher idiosyncratic dispersion $\sigma_{\omega,t}$. This occurs because aggregate indexation is indiscriminately targeted, providing equal relief to the hardest-hit and relatively unaffected regions/households alike, with limited effects on the number of foreclosures.

Next, Table 3 compares welfare and consumption outcomes across the different indexation regimes. The increased financial fragility results in incredibly volatile intermediary wealth (W^I growth volatility goes up 1448.2%) and intermediary consumption (C^I

growth volatility goes up 479.8%), as well as a larger drop in that consumption in a financial crisis. Borrower consumption growth volatility increases by 397.1%, albeit from a much lower base. Depositor consumption growth volatility decreases slightly. These results point to a deterioration in risk sharing between borrowers and intermediaries in the economy with aggregate indexation, measured by the volatility of the log marginal utility ratios between this pairs of agents in row 39 of Table 3. This ratios increases by 171.5%, respectively, indicating that markets have become more “incomplete.”

Table 3: Results: Welfare and Consumption

	No Index	Aggregate	Local (25%)	Both
Welfare				
26. Value function, B	0.376	-0.49%	+0.25%	-0.19%
27. Value function, D	0.374	-0.03%	-0.16%	-0.18%
28. Value function, I	0.068	+4.84%	-2.90%	+1.10%
29. Value function, Bank	0.202	+2.93%	+4.73%	+6.61%
30. Total housing maint	0.058	-2.10%	+1.40%	+0.33%
Consumption and Risk-sharing				
31. Consumption, B	0.359	-0.3%	+0.4%	+0.2%
32. Consumption, D	0.372	-0.8%	+0.0%	-0.7%
33. Consumption, I	0.069	+6.1%	-4.1%	+0.5%
34. Consumption gr vol, B	0.42%	+397.1%	+33.3%	+247.2%
35. Consumption gr vol, D	1.07%	-9.9%	-32.7%	-2.1%
36. Consumption gr vol, I	4.00%	+479.8%	-60.2%	+355.7%
37. WI gr vol	3.48%	+1448.2%	+8.5%	+699.0%
38. log (MU B / MU D) vol	0.025	-0.6%	-8.6%	-20.3%
39. log (MU B / MU I) vol	0.057	+171.5%	-48.9%	+125.4%

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with full indexation of mortgage payments to aggregate house prices (second column), a model with partial indexation to relative local prices (third column), and a model with both aggregate and partial local indexation (fourth column). All flow variables are quarterly. In rows 26-39, we calculate percentage differences between the models in columns 2-4 and the benchmark model.

To assess the gains from this policy, we aggregate agents’ value functions to obtain measures of welfare.¹⁸ Borrowers are made worse off (row 26), both because their con-

¹⁸There are many ways of computing aggregate welfare in incomplete markets economies with heterogeneous agents. The measure we present calculates welfare per capita for each agent type, multiplies it by the population share of each type, and sums across types. An alternative calculation one could do is to ask how much each agent would be willing to pay to avoid a switch to the aggregate indexation economy. One can then add up the resulting amounts. A negative sum would indicate that the transition is a Pareto

sumption has become more volatile (row 34) and because their consumption is lower (row 31) for reasons explained above. Borrowers have lower house prices and face higher mortgage rates. Depositors' welfare and risk exposure are roughly unchanged (rows 27 and 32). Their mean consumption is slightly lower mostly because they earn lower interest rates on their savings and accumulate less wealth as a result (row 19 in Table 2). However, their consumption also becomes less volatile (row 35), causing a neutral net effect on their overall welfare.¹⁹

Finally, and perhaps surprisingly, intermediary households are made better off. Intermediary consumption levels increase because of the higher risk premia they earn on mortgage assets from the banks they own, and because they earn higher returns on REOs from the REO firms they own. This benefit to intermediaries is purely due to this effect on the level of average consumption, as the volatility of intermediary consumption is massively increased under aggregate indexation, due to a deterioration in risk sharing. All told, we obtain the interesting distributional result that insuring borrower exposure to aggregate house price risk leads bankers to gain at the expense of the borrowers and the savers.

6.3 Adding Local Indexation

In sharp contrast to the results above, partially indexing mortgage debt to relative local house prices, as in the third and fourth columns of Table 2, causes the mortgage default rate to drop precipitously. This can be seen in Figure 4, which compares crises in the benchmark model to crises in a model with partial local indexation ($\iota_\omega = 0.25$). Facing less default risk, banks lower mortgage interest rates, pushing up house values. These higher values support increased household borrowing, raising the average stock of mortgage debt, in turn financed with a larger deposit base.

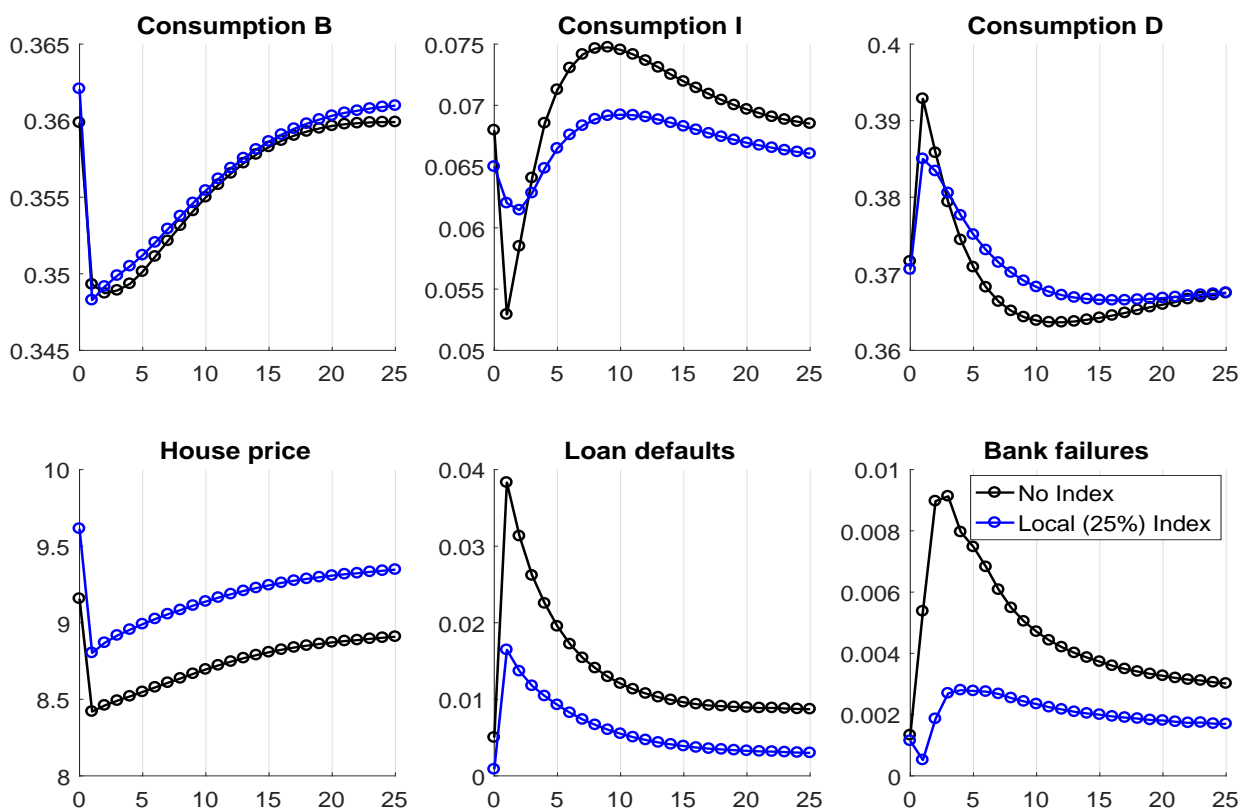
While partial local indexation does not prevent the aggregate drop in house prices, it is highly successful at reducing loan defaults, sending debt relief to the households in greater need. While banks react to this reduced risk by holding as little capital as allowed, the required minimum is sufficient to ensure a large decrease in risk overall. As a result, this economy does not suffer from financial fragility; bank failure rates fall to nearly zero as mortgage default risk dissipates, and bank wealth becomes dramatically less volatile.

improvement after transfers.

¹⁹Depositor consumption becomes less volatile because it experiences a smaller spike in financial recessions, see also Figure 3. This smaller spike is due to higher taxes that need to be raised to cover losses from bank bailouts.

The risk-free interest rate rises slightly due to the increase in the deposit base. At the same time, lower mortgage risk is reflected also in lower mortgage risk premia and mortgage spreads. Overall, the banking system is both safer and larger under this contract, but is compensated less on a per-loan basis.

Figure 4: Financial Recessions: Benchmark vs. Local Indexation Model



Black line: benchmark financial recession, **Blue line:** local index. financial recession.

The welfare effects from local indexation are the reverse of those from aggregate indexation. Borrowers and depositors gain while intermediaries lose. Risk sharing in the economy improves dramatically, as the volatility of marginal utility ratios between groups falls, especially between borrowers and intermediaries (rows 39, 40). Savers and intermediaries also see large reductions in consumption growth volatility, while borrowers experience increased volatility — albeit from a low level — due to larger housing and mortgage positions. The smaller changes in intermediary and depositor consumption during crises (top row of Figure 4) underscore this point. Savers earn higher interest rates under this system, while borrowers pay lower rates on their mortgages, helping to boost the consumption of each group. In contrast, intermediary households' mean consump-

tion falls by 4.1% as dividends from REO firms and banks decline.

In sum, even partial indexation to idiosyncratic house value shocks is highly effective at reducing the risk of foreclosures and financial fragility. More intermediation ensues, which makes both borrowers and savers richer. However, banking becomes less profitable.

6.4 Robustness: Liquidity Defaults

A potential concern with our approach is that many mortgage defaults are triggered — at least in part — by household liquidity shocks, while our model only considers strategic default. Appendix A.3 studies a model of liquidity defaults and shows that it gives rise to a similar threshold rule for default that depends on the borrower’s loan-to-value ratio, generating default dynamics similar to those found in our setting. The reason is that households who cannot make their payments after a liquidity shock can sell their properties rather than default if they are not underwater. Generating substantially different results would require a large fraction of *above-water* liquidity defaults, an assumption that is not supported by the data. This suggests that our findings are robust to the source of borrower defaults.

7 Conclusion

Redesigning the mortgage market through product innovation may allow an economy to avoid a severe foreclosure crisis like the one that hit the U.S. economy in 2008-2010. We study the welfare implication of indexing mortgage payments to aggregate or local house prices in a model with incomplete risk-sharing. A key finding is that indexation of mortgage payments to aggregate house prices may increase financial fragility. Inflicting large losses on highly-levered mortgage lenders in bad states of the world can cause systemic risk (high bank failure rates), costly tax-payer financed bailouts, meaningful house price declines, and higher risk premia on mortgages, all of which ultimately hurt the borrowers the indexation was trying to help. Moreover, aggregate indexation redistributes wealth from borrowers and depositors towards bankers, since a more fragile banking business also is a more profitable banking business.

In sharp contrast, indexation of idiosyncratic house price risk is highly effective at reducing mortgage defaults and financial fragility. It increases welfare for borrowers and

depositors, but reduces intermediary welfare as mortgage banking becomes safer but less profitable.

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A Appendix

A.1 Derivation of Bank FOCs

First, starting from the value function in (19), we can define a value function net of the idiosyncratic profit shock

$$V^I(W_t^I, \mathcal{S}_t^I) = V_{ND}^I(W_t^I, \mathcal{S}_t^I) + \epsilon_t^I$$

such that we can equivalently write the optimization problem of the non-defaulting bank after the default decision as

$$V^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + \mathbb{E}_t \left[\Lambda_{t,t+1}^I \max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I, 0 \right\} \right], \quad (28)$$

subject to the same set of constraints as the original problem. We can now take the expectation with respect to ϵ_t^I of the term in the expectation operator

$$\begin{aligned} & \mathbb{E}_\epsilon \left[\max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I, 0 \right\} \right] \\ &= \text{Prob}_\epsilon \left(\epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \mathbb{E}_\epsilon \left[V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I \mid \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right] \\ &= F_\epsilon^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right), \end{aligned} \quad (29)$$

with $\epsilon_{t+1}^{I,-} = \mathbb{E}_\epsilon \left[\epsilon_{t+1}^I \mid \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right]$ as in the main text. Inserting (29) into (28) gives the value function in (22) in the main text.

To derive the first-order conditions for the bank problem, we formulate the Lagrangian

$$\begin{aligned} \mathcal{L}^I(W_t^I, \mathcal{S}_t^I) &= \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I, \lambda_t^I} W_t^I - J_t^I + \mathbb{E}_t \left[\Lambda_{t,t+1}^I F_\epsilon^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right] \\ &\quad + \lambda_t^I \left(\phi^I \left(q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - B_{t+1}^I \right) \right), \end{aligned} \quad (30)$$

and further conjecture that

$$V^I(W_t^I, \mathcal{S}_t^I) = W_t^I + \mathcal{C}(\mathcal{S}_t^I), \quad (31)$$

where $\mathcal{C}(\mathcal{S}_t^I)$ is a function of the aggregate state variables but not bank net worth.

Before differentiating (30) to obtain first-order conditions, note that the derivative of the term in the expectation operator with respect to future wealth, after substituting in

this guess, is

$$\begin{aligned}
& \frac{\partial}{\partial W_{t+1}^I} F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \\
&= \frac{\partial}{\partial W_{t+1}^I} \left[F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) - \int_{-\infty}^{W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I)} \epsilon f_\epsilon^I(\epsilon) d\epsilon \right] \\
&= F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right).
\end{aligned}$$

Using this result, and differentiating with respect to L_t^* , \tilde{M}_t^I , \tilde{A}_t^I , B_{t+1}^I , and λ_t^I respectively, gives the first-order conditions

$$1 = q_t^M + r_t^* q_t^A, \quad (32)$$

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{A,t+1} \left((1-\delta) + \delta Z_{R,t+1} + \delta(1-Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{(1 - \phi^M \lambda_t^I)}, \quad (33)$$

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[Z_{A,t+1} \left(1 + \delta(1-Z_{R,t+1}) q_{A,t+1}^A \right) \right] \right\}}{(1 - \phi^A \lambda_t^I)}, \quad (34)$$

$$q_t^f = \mathbb{E}_t \left[\Lambda_{t+1}^I F_{\epsilon,t+1}^I \pi_{t+1}^{-1} \right] + \lambda_t^I, \quad (35)$$

and the usual complementary slackness condition for λ_t^I . Recalling the definition of J_t^I as

$$J_t^I = (1 - r_t^* q_t^A - q_t^M) L_t^* + q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - q_t^f B_{t+1}^I,$$

we note that the term in front of L_t^* is zero due to FOC (32), and we can substitute out prices q_t^M , q_t^A , and q_t^f from FOCs (33)-(35), both in J_t^I and in the constraint term in (30).

Further inserting our guess from (31) on the left-hand side of (30), and canceling and collecting terms, we get

$$\mathcal{C}(\mathcal{S}_t^I) = \mathbb{E}_t \left[\Lambda_{t,t+1}^I F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left(\mathcal{C}(\mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right], \quad (36)$$

which confirms the conjecture. $\mathcal{C}(\mathcal{S}_t^I)$ is the recursively defined value of the bankruptcy option to the bank. Note that without the option to default, one gets

$$\epsilon_{t+1}^{I,-} = \mathbb{E}_\epsilon \left[\epsilon_{t+1}^I \right] = 0.$$

Then the equation in (36) implies that $\mathcal{C}(\mathcal{S}_t^I) = 0$ and thus $V^I(W_t^I, \mathcal{S}_t^I) = W_t^I$. However, if the bank has the option to default, its value generally exceeds its financial wealth W_t^I by the bankruptcy option value $\mathcal{C}(\mathcal{S}_t^I)$.

A.2 Aggregation of Intermediary Problem

Before aggregating across loans, we must treat the distribution over $m_t(r)$, the start-of-period balance of a loan with interest rate r , as a state variable. In addition, the intermediary can freely choose her end-of-period holdings of these loans $\tilde{m}_t(r)$ by trading in the secondary market at price $q^m(r)$. In this case, the intermediary's problem is to choose nondurable consumption C_t^I , new debt issuance L_t^* , new deposits B_{t+1}^I , new REO investment I_t^{REO} , and end-of-period loan holdings $\tilde{m}_t(r)$ to maximize (2) subject to the budget constraint

$$\begin{aligned}
C_t^I = & \underbrace{(1 - \tau)Y_t^I}_{\text{disp. income}} + \underbrace{\int \left[X_t + Z_{A,t} \left(r + (1 - \delta) + \delta Z_{R,t} \right) \right] m_t(r) dr}_{\text{payments on existing debt}} - \underbrace{(1 - q_t^m(r_t^*)) L_t^*}_{\text{net new debt}} \\
& + \underbrace{q_t^f B_{t+1}^I - \pi_t^{-1} B_t^I}_{\text{net deposits}} - \underbrace{\int q_t^m(r) \left[\tilde{m}_t(r) - \delta(1 - Z_{R,t}) Z_{A,t} m_t(r) \right] dr}_{\text{secondary market trades}} \\
& + \underbrace{\left[\rho_t + (S^{REO} - \nu^{REO}) p_t \right] K_t^{REO}}_{\text{REO income}} - \underbrace{p_t^{REO} \left[I_t^{REO} - X_t A_t^I \right]}_{\text{REO investment}}
\end{aligned} \tag{37}$$

and the leverage constraint

$$q_t^f B_t^* \leq \phi^M \int q_t^m(r) \tilde{m}_t(r) dr + \phi^{REO} p_t^{REO} \tilde{K}_t^{REO}$$

with the laws of motion

$$\begin{aligned}
m_{t+1}(r) &= \pi_{t+1}^{-1} \zeta_{p,t+1} \tilde{m}_t(r) \\
K_{t+1}^{REO} &= (1 - S^{REO}) K_t^{REO} + (1 - Z_{K,t}) K_t^B
\end{aligned}$$

and where the recovery rate X_t is defined as before. From the optimality condition for end-of-period holdings for loans with a given interest rate $\tilde{m}_t(r)$, we obtain

$$q_t^m(r) = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I \pi_{t+1}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{A,t+1} \left(r + (1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) q_{t+1}^m(r) \right) \right] \right\}}{1 - \lambda_t^I \phi^M}$$

where λ_t^I is the multiplier on the intermediary's leverage constraint. To obtain aggregation, we can split $q_t(r)$ into an interest-only strip with value q_t^M and a principal-only strip with value q_t^A , so that

$$q_t^m(r) = r q_t^A + q_t^M.$$

Substituting into the equilibrium condition for $q_t^m(r)$ verifies the conjecture and yields

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I Y_{t+1}^M Z_{A,t+1} \left[1 + \delta(1 - Z_{R,t+1}) q_{t+1}^A \right] \right\}}{1 - \lambda_t^I \phi^M}$$

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I Y_{t+1}^M \left[X_{t+1} + Z_{A,t+1} \left((1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{1 - \lambda_t^I \phi^M}.$$

Importantly, due to our assumption on the prepayment behavior of borrowers (ensuring a constant $Z_{R,t}$ across the r distribution), the prices q_t^A and q_t^M are independent of r . Substituting into the budget constraint, and applying the identities

$$M_t^I = \int m_t(r) dr$$

$$A_t^I = \int r m_t(r) dr$$

now yields the aggregated budget constraint (11) and leverage constraint (20).

A.3 Liquidity Defaults

This section considers the case where defaults are driven by liquidity concerns (the need to stop making mortgage payments) rather than the strategic motive of the baseline model. Assume that each period, fraction θ_t of borrowers are hit by a liquidity or turnover shock, so that they cannot make their mortgage payments this period. After being hit with the shock, borrowers have the choice of whether to sell the house or to default. Since the proceeds from a sale are:

$$\omega_{i,t} p_t K_t^B - \zeta_\omega(\omega_{i,t}) \cdot \delta \zeta_{p,t} M_t^B,$$

while the proceeds from a default are zero, the threshold house quality shock at which the borrower defaults rather than sells is defined by

$$\bar{\omega}_t p_t K_t^B - \zeta_\omega(\bar{\omega}_t) \cdot \delta \zeta_{p,t} M_t^B = 0.$$

Substituting for ζ_ω and some additional algebra yields

$$\bar{\omega}_t = \frac{(1 - \iota_\omega) \cdot \delta \zeta_{p,t} M_t^B}{p_t K_t^B - \iota_\omega \cdot \delta \zeta_{p,t} M_t^B}.$$

Given this threshold, the mortgage default rate is $\theta_t \Gamma_\omega(\bar{\omega}_t)$, and our other key default ratios are given by

$$\begin{aligned} Z_{N,t} &= 1 - \theta_t \Gamma_\omega(\bar{\omega}_t) \\ Z_{K,t} &= 1 - \theta_t \left(1 - \int_{\bar{\omega}_t} \omega d\Gamma_{\omega,t} \right) \\ Z_{A,t} &= \iota_\omega Z_{K,t} + (1 - \iota_\omega) Z_{N,t}. \end{aligned}$$

This shows that the model with liquidity default generates the same implications as the model with strategic default, modulo the θ_t parameter. That θ_t could be endogenized to reflect the liquidity needs of consumers, or changed with economic conditions to reflect the hazard rate of falling into unemployment.