Abstract

We study the impact of fiscal policy shocks on bond risk premia. Government spending level shocks generate positive covariance between marginal utility and inflation (term structure level effect) making nominal bonds a poor hedge against consumption risk leading to positive inflation risk premia. Volatility shocks to spending have strong slope effect (steepening) on the yield curve, producing positive nominal term premia. For level and volatility shocks to capital income tax, term structure level effects dominate, delivering negative risk premia. Fluctuations in term premia are entirely driven by volatility shocks. Lastly, fiscal shocks are amplified at the zero lower bound.

JEL classification: G12, E62.

Keywords: Term structure, Bond Risk Premia, Uncertainty, Fiscal Policy.

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1 Introduction

Fiscal policy shocks and fiscal volatility shocks have first order effects on economic activity. Government spending and taxation can impact corporate investment-borrowing choices, household consumption-saving behavior, and economic aggregates such as inflation. The study of fiscal policy commands a large area of literature in economics. The majority of papers focuses on optimal taxation or government spending and its impact on the output multiplier or consumption. Similarly, uncertainty about government spending and tax rates can alter the decision-making process faced by economic agents and firms. Bloom (2009) finds productivity uncertainty shocks produce large fluctuations in aggregate output and employment. More recently, Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) show that unexpected increase in the return on capital tax rate uncertainty has strong negative impact on output.

The link between fiscal policy and policy uncertainty with the term structure of interest rates, on the other hand, is less well established. Dai and Philippon (2005) provide empirical evidence of fiscal deficits driving nominal yield curve dynamics in a no-arbitrage affine macrofinance model, but the model does not accommodate endogenous inflation, which Piazzesi and Schneider (2007) document to be the main risk factor in generating bond risk premia. Furthermore, given that monetary policy was at the zero lower bound (ZLB) until recently and the high political uncertainty in the U.S., the impact of fiscal level and volatility shocks on bond risk premia has never been more relevant. In this paper, we estimate a dynamic stochastic general equilibrium (DSGE) model to investigate the effects of fiscal policy and policy uncertainty on the term structure of interest rates and bond risk premia. We focus

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2 Following the literature (see e.g., Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015)) we interpret the unexpected changes in the time-varying volatility of the fiscal instrument (e.g. government expenditure) innovations as a representation of unexpected variations in uncertainty about fiscal policy. We also use the term “uncertainty” as shorthand for what would more precisely be referred to as “risk”. See also Bachmann, Bai, Lee, and Zhang (2015) where the authors quantify the welfare costs of fiscal uncertainty in a neo-classical stochastic growth model.

3 For the purpose of exposition, “bond risk premia” and “nominal term premia” are used interchangeably to denote a combination of “inflation risk premia” (term structure level effect) and “real term premia” (slope effect).
on two specific aspects of fiscal policy: government spending and the tax rate on the return of capital.

Through the lens of the estimated model, we document four main findings in this paper. First, level shocks to government spending generate positive inflation risk premium as inflation is high precisely when consumption declines. This term structure level effect is the opposite for level and volatility shocks to the return on capital tax rate: inflation decreases in bad times producing negative inflation risk premium. Second, volatility shocks to government spending are observed to have substantial slope effect on the term structure. Increased volatility to government spending steepens the yield curve, producing positive term premium. Third, fiscal volatility shocks are the primary factors in generating term premia fluctuations. Fourth, when the nominal short rate is at zero, consumption, inflation, and long-term interest rate reactions are more pronounced following level and volatility shocks to fiscal policy, implying considerable bond risk premia.

In reduced form empirical analysis, excess return predictive regressions are performed for nominal bonds across maturities employing estimated fiscal level and volatility shocks as explanatory variables as well as controlling for bond supply. We document government spending level and volatility shocks predict positive future excess returns, while capital tax level and volatility shocks weakly predict negative excess returns. Furthermore, the government spending volatility shock dominates the other fiscal shocks in terms of return predictability in the regression specification when all four fiscal shocks are included. Model implied predictive regressions using simulated data are able to replicate these findings, further validating the performance of the estimated model.

The theoretical analysis is conducted in a general equilibrium model with production. Ricardian equivalence in the model is disrupted by introducing distortionary taxation for return on capital. The representative agent has Epstein and Zin (1989) recursive preferences. The production sector is in line with the standard New-Keynesian stochastic growth model. The production function is Cobb-Douglas employing transitory TFP shocks and permanent

\footnote{The intermediate-good firms adjust prices according to the Calvo (1983) process, under which only a fraction of the firms are allowed to maximize present value of their expected profits by choosing the optimal price each period. This mechanism induces monetary policy non-neutrality with respect to the real economy allowing us to make comparisons between fiscal policy and monetary policy impacts.}
labor productivity shocks. The monetary authority sets the nominal short-term interest rate using a Taylor rule with contemporaneous feedbacks from inflation and output growth plus a shock which represents any unexpected deviations of the nominal short rate. The fiscal authority chooses the amount of current period lump-sum taxes to collect. Government revenue is a combination of the lump-sum transfer and tax on the return of capital such that the government budget constraint is satisfied. Government spending is exogenous and shocks to government spending exhibits stochastic volatility following an autoregressive process.

There are eight economic shocks driving the dynamics of the theoretical model: transitory and permanent productivity shocks, volatility shocks to transitory productivity, monetary policy shocks, as well as level and volatility shocks to government spending and the tax rate of return on capital. Since the impact of both productivity shocks and monetary policy shocks have been examined in the equilibrium term structure literature, our analysis is centered on the four fiscal shocks.

A positive level shock to government spending drives up demand of output, and it also crowds out consumption of the agents. The wealth effect of lower consumption increases the labor supply and depresses real wage. The precautionary savings motive also drives investment higher. Increase in return on capital generates a spike in inflation immediately after the positive level shock is realized, producing positive average inflation risk premium. On the other hand, a positive shock to government spending volatility lowers government debt and inflation in our benchmark model. Increase in spending volatility makes capital investment more attractive over debt for consumption smoothing because government spending is expected to be high, implying higher future taxes. The oversupply of capital causes the return on capital to decline, while increase in labor supply puts downward pressure on real wage. This leads to lower inflation as marginal cost of production decreases, generating negative average inflation risk premium due to government spending volatility shocks.

That said, government spending volatility shocks have differential impact on short-maturity and long-maturity bonds. With higher uncertainty, the decline in real wage and return on capital are transitory, and the increase in investment and saving are short-lived. This makes short-maturity government bonds especially valuable as a consumption hedge.

For example, see Rudebusch and Swanson (2012), Kung (2015), and Hsu, Li, and Palomino (2015).
relative to long-maturity Treasuries. Short-dated bonds become more expensive compared to the long-dated bonds causing long-term bonds to be risky when marginal utility is high. As a result, the spending volatility shock steepens the yield curve and generates positive term premium. From the impulse response functions of the model, we find the positive term premium dominates the negative inflation risk premium such that the nominal term premium is positive on average following a positive second moment shock to spending.

We solve the model using perturbation methods (see Schmitt-Grohe and Uribe (2004)). We compute a third-order approximate solution of the model around its non-stochastic steady state using the pruning algorithm suggested by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) (AFVRR hereafter). Importantly, AFVRR provide closed-form solutions for first and second moments of the pruned DSGE model. This allows us to estimate our model using means, variances and contemporaneous covariances of macro and financial series through generalized method of moments (GMM). Last but not least, the impulse response functions in an economy approximated to third order depend on the values of the state variables. Motivated by the situation in the United States following the financial crisis, we analyze the propagation of fiscal level and volatility shocks when the model economy is at the zero lower bound (ZLB). We find that, when the nominal short rate is held at zero for prolonged periods of time after the initial fiscal shocks are realized, the impulse responses of output, investment and inflation are greatly amplified relative to normal times. The effects are especially exaggerated for the government spending volatility shock and the return on capital tax rate level shock. Each of which produces a decline in output of about 10% and a drop in inflation of more than 30%.

This paper belongs to a growing literature examining the relation between government policies, economic activity, and asset prices. The joint modeling of the yield curve and macroeconomic variables has received much attention since Ang and Piazzesi (2003), where the authors connect latent term structure factors to inflation and the output gap. More recently, many term structure studies incorporate monetary policy elements in their models using the fact that the nominal short rate is the monetary policy instrument. However,

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A first-order approximation of the model and bond price (i.e., a log-linearization) eliminates the term premium entirely and a second-order approximation to the solution of the model and bond price produces a term premium that is nonzero but constant.
these models are generally silent on the effects of fiscal policy on the term structure despite evidence suggesting that it has nontrivial effects on interest rates. The primary contribution of this paper is establishing the link between fiscal policy and risk premia on nominal bonds, namely the term premium and the inflation risk premium. The model shows loose fiscal policy and high government spending cause investors to demand higher returns in exchange for holding Treasury securities.

This paper is most closely related to the literature on term structure and bond risk premia in equilibrium. [Campbell (1986)] specifies an endowment economy in which utility maximizing agents trade bonds of different maturities. When the exogenous consumption growth process is negatively autocorrelated, term premia on long-term bonds are positive, generating upward sloping yield curves because they are bad hedges against consumption risk compared to short-term bonds. More recently, [Piazzesi and Schneider (2007)] using [Epstein and Zin (1989)] preferences, show that inflation is the driver that generates a positive term premium on nominal long-term bonds. Negative covariance between consumption growth and inflation translates into high inflation when consumption growth is low and marginal utility to consume is high. [Wachter (2006)] generates upward sloping nominal and real yield curves employing habit formation. In her model, bonds are bad hedges for consumption as agents wish to preserve previous level of consumption as current consumption declines. [Campbell, Pflueger, and Viceira (2015)] study the effect of monetary policy rule and uncertainty on bond risk premium. They find that intensified monetary policy focus on inflation increases bond risks while a shifting policy focus to stabilize output does the opposite.

[Rudebusch and Swanson (2008)] and [Rudebusch and Swanson (2012)] examine bond risk premia in general equilibrium where utility-maximizing agents supply labor to profit-maximizing firms to produce consumption goods. The best-fit model in the latter paper is successful in matching the basic empirical properties of the term structure using only transitory productivity shocks. [Palomino (2010)] studies optimal monetary policy and bond risk premia in general equilibrium. More specifically, he shows that the welfare-maximizing monetary policy affects inflation risk premia depending on the credibility of the monetary authority in the economy as well as the representative agent’s preference. [Kung (2015)] builds a equilibrium model with stochastic endogenous growth to explain the impact of monetary policy shocks on bond risk premium. [Hsu, Li, and Palomino (2015)] examine risk premia
on real bonds in general equilibrium. Calibrated to TIPS data, they find that productivity growth shocks alone generate negative term premium on real bonds, but the presence of wage rigidities makes term premium positive.

This paper is also related to the literature on the interaction between fiscal policy and asset pricing. Croce, Kung, Nguyen, and Schmid (2012) study the effects of fiscal policies in a production-based general equilibrium model in which taxation affects corporate decisions. They find that tax distortions have negative effects on the cost of equity and investment. Our interest is different. We analyze the impact of government spending level and uncertainty shocks on the term structure of interest rates. Our interest in fiscal volatility shocks is motivated by Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) who uncover evidence of time-varying volatility in tax and government spending processes for the United States and, using both a VAR and a New Keynesian model. They document that these fiscal volatility shocks can have a sizable adverse effect on economic activity. To the best of our knowledge, our paper is the first attempt to evaluate the dynamic consequences on the term premium of unexpected changes to fiscal volatility shocks.

The rest of the paper is structured as follows. The next section documents the estimation of the fiscal shocks as well as their impact on bond risk premium using reduced form regression analysis. Section 3 introduces the model. Section 4 discusses the data used for GMM estimation, presents our solution method and estimation approach. Section 5 presents detailed analysis of the model and associated term structure. Section 6 studies the implications of fiscal shocks at the ZLB on the model. Section 7 concludes. Detailed derivations are deferred to the Appendix.

2 Empirical Analysis

In this section we estimate fiscal rules with time-varying volatility using data on taxes and government spending. The estimated rules will discipline our quantitative experiments by assuming that past fiscal behavior is a guide to assessing current behavior. We then present our regression results using bond yields and predicted bond returns as dependent variables to explore their dependence on fiscal shocks.
2.1 Fiscal Policy Uncertainty

Our two policy instruments, i.e. government spending as a share of output and tax rates on capital income, evolve as follows:

\[ x_{t+1} = (1 - \phi_x)\theta_x + \phi_x x_t + e^{\sigma_x t+1} \epsilon_{x,t+1} \quad (1) \]

\[ \sigma_{x,t+1} = (1 - \phi_{\sigma_x})\theta_{\sigma_x} + \phi_{\sigma_x} \sigma_{x,t} + \sigma_{\epsilon_{x,t+1}} \quad (2) \]

for \( x \in \{g, \tau^k\} \) where \( g \) is government spending as a share of output, and \( \tau^k \) is the tax rate on capital income. Each policy instrument features stochastic volatility since the log of the standard deviation of the innovation, \( \sigma_{x,t} \), is random. The parameter \( \theta_{\sigma_x} \) determines the average standard deviation of a fiscal shock to the policy instrument \( x \), \( \frac{\sigma_{\sigma_x}}{\sqrt{1-(\phi_{\sigma_x})^2}} \) is the unconditional standard deviation of the fiscal volatility shock to instrument \( x \), and \( \phi_{\sigma_x} \) controls the shock’s persistence. Following Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015), we estimate Eqs. (1) and (2) for each fiscal instrument separately, and we set the means in equation (1) to each instrument’s average value (see Table 3 Panel A). We estimate the rest of the parameters following a Bayesian approach by combining the likelihood function with uninformative priors and sampling from the posterior with a Markov Chain Monte Carlo. Table 3 Panel B reports the posterior median for the parameters along with 95 percent probability intervals. Both tax rates and government spending as a share of output are persistent. E.g., the half-life of government spending is around \(-\log(2)/\log(0.98) = 34\) quarters. Deviations from average volatility last also for some time. The \( \epsilon_{x,t,s} \)s have an average standard deviation of \( 100 \times \exp(-4.84) = 0.79 \) and \( 100 \times \exp(-6.03) = 0.24 \) percentage point for tax and government spending, respectively. These results are in line with Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) (see in particular their Table 1).

Figure 1 allows us to build an analytic narrative of fiscal volatility shocks. Panels 1(a) and 1(b) display the 95 percent posterior probability intervals of the smoothed fiscal volatility

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7Specifically, for government spending we adopt a beta distribution for \( \phi_{\sigma_g} \) and \( \phi_g \) with mean 0.8 and 0.85 respectively, a uniform distribution between \(-11 \) and \(-3 \) for \( \theta_{\sigma_g} \), and an inverse gamma for \( \sigma_{\sigma_g} \) with mean 0.1. Correspondingly, for capital tax we use a beta distribution for \( \phi_{\sigma_{\tau^k}} \) and \( \phi_{\tau^k} \) with mean 0.85 and 0.8 respectively, a uniform distribution between \(-8 \) and \(-3 \) for \( \theta_{\sigma_{\tau^k}} \), and an inverse gamma for \( \sigma_{\sigma_{\tau^k}} \) with mean 0.2.
shock to government spending, $100 \exp(\sigma_{g,t})$, and capital tax rates, $100 \exp(\sigma_{k,t})$, over the sample. Next, we focus on government spending volatility and refer the interest reader to Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) for a similar analysis of the fiscal volatility shock to capital income tax rates. Our smoothed estimate of the government spending volatility was high in 1974-1975. These were indeed times of unusual fiscal policy uncertainty: for example, in a talk given at Stanford University on May 13, 1975, George P. Shultz (Secretary of the Treasury from June 12, 1972 to May 8, 1974) stated that “This is an age of ambiguity ... And the result is that people are experiencing a great sense of unease and uncertainty.” Volatility was climbing again in the early 80s. These years were difficult ones for fiscal policy, with numerous proposals being floated to address the large fiscal deficits created during the early years of the Reagan administration. The 1985 Economic Report of the President made deficit reduction one of the President’s priorities, with an emphasis on expenditure control. This event is reflected in a “moderation” of our volatility series. Our fiscal volatility then raises in the period from 2001:II to 2002:I. These quarters witnessed the 9/11 terrorist attacks (with their potentially vast fiscal implications) and the 2001–2002 recession.

2.2 Bond Yields, Bond Returns and Fiscal Policy: Basic Tests

Tables 1 and 2 shows regressions of yield spreads and future returns on our fiscal instruments. Throughout we use the filtered series of volatilities to remove any look-ahead bias present in the smoothed estimates. Also, we employ a one-sided filter to remove a decadal trend in the level of fiscal series, and we use the business cycle component of government spending and capital tax rates as regressors. Appendix B discusses in details this transformation, and provides additional robustness and interpretations. Finally, observations are quarterly.

The results of the yield regression are in Table 1. The first row in Panels A and B provide a benchmark: the government debt supply – as proxied by the maturity-weighted debt to GDP, see Greenwood and Vayanos (2014) – is an important determinant of the slope. The

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9More precisely, we use the median of the filtered volatility series obtained from our Bayesian estimation.
second specification in Panels A and B shows that the level and uncertainty of government spending improve substantially the fit of the regression with the $R^2$ increasing from 10% to 39%. The third specification shows that capital tax rates do not appear to play an important role for the slope of the term structure after controlling for government spending.

We next turn to the results on returns. Table 2 shows regressions of future returns on our fiscal level and volatility series:

\[ r_{x_{t+k,k}} = \beta_0 + \beta_1 g_t + \beta_2 \sigma_{g,t} + \beta_3 \tau_t^k + \beta_4 \sigma_{\tau,t} + \text{Controls + } u_{t+k} \]

where $r_{x_{t+k,k}}$ is the future $k$-year return of the $\tau$-year bond in excess of the $k$-year yield, and $\sigma_{x,t}$ is our fiscal volatility series, and $x \in \{g, \tau^k\}$. We perform this regression for one-year returns for all bonds in our sample, and for three- and five-year returns for the long-term bond. We report $t$-statistics using Newey and West (1987) standard errors and allowing for 6 quarters of lags. Allowing for more lags does not seem to affect the results.\(^{10}\)

We again start with a benchmark in Panel A: government debt supply is a strong predictor for future returns\(^{11}\). Panel B shows that the government spending level and uncertainty series more than double the adjusted $R^2$ for 1-year holding period returns on bond with maturity ranging from 2- to 10-years. We add the capital tax rate level and volatility series in Panel C. We observe that the $R^2$ are almost identical to those in Panel B. Similarly, the magnitude and significance of government spending volatility, and to a lesser extend government spending level, are hardly affected by the inclusion of capital tax rates. Importantly, both government spending and capital tax seem to convey independent information about future bond excess returns after controlling for government debt supply. Across all panels the bond supply is the main driver for 5-year long-term bond returns consistent with the view that supply captures a lower-frequency component of expected returns. Our fiscal level and uncertainty

\(^{10}\)Cochrane (2008) suggests using a parametric alternative to the non-parametric Newey-West. Bauer and Hamilton (2017) suggest using a bootstrap procedure to address small-sample distortions in bond returns predictive regressions. Although we use the simple Newey-West approach, our model will shed further light on the plausibility of our empirical results.

\(^{11}\)Our results largely replicates those in Greenwood and Vayanos (2014) despite our use of quarterly data from 1970-Q1 to 2007-Q4 (Greenwood and Vayanos (2014) uses monthly observations for the longer 1952-2007 sample period). The main difference lies in $R^2$: This is because Greenwood and Vayanos (2014) forecast bond returns, whereas we forecast bond excess returns.
instruments instead seem to capture a complementary, higher-frequency (mainly business cycle) component of risk premia. The additional robustness checks in Appendix B confirm the picture drawn by these basic regressions: fiscal policy, and in particular government spending, is an important determinant of bond risk premia. The discussion that follows will shed light on the exact mechanism through the lens of our model.

[Insert Table 1 and 2 about here.]

3 The Benchmark Model

We implement a New-Keynesian\textsuperscript{12} model with government spending and distortionary tax on the return of capital for the analysis. The monetary authority implements the Taylor rule and sets the nominal short rate as a function of inflation and output growth. On the production side, firms maximize profits under staggered price setting. The model also features nominal wage rigidities. We leave the description of the optimal investment decision and staggered wage setting for the appendix.

3.1 The Household Problem

The representative agent has the ability to save current income in order to smooth future consumption by purchasing government bonds. With Epstein and Zin (1989), the representative agent maximizes lifetime utility by solving the following:

\[ \max \quad V(C_t, N_t) = \left\{ (1 - \beta) \left( \frac{C_t^{1-\psi}}{1-\psi} - \lambda_t \frac{N_t^{1+\omega}}{1+\omega} \right) + \beta E_t \left[ V_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{1-\psi}}, \]

s.t. \[ P_tC_t + P_tInvt + Q^{(1)}_tB_t(t+1) + P_tTax_t \]
= \[ P_tW_tN_t + (1 - \tau^k_t)P_tR^k_tK_{t-1} + B_{t-1}(t) + P_t\Psi_t. \]

where \( \beta \) denotes the time discount factor, \( \psi \) is the inverse of the intertemporal elasticity of substitution (IES), the Epstein-Zin parameter \( \gamma \) is related to the coefficient of relative

\textsuperscript{12}For a detailed exposition on the New-Keynesian framework, see Clarida, Gali, and Gertler (1999).
risk aversion, and $\omega$ is the inverse of the Frisch elasticity of labor supply. $\lambda_t$ is the time varying parameter as a function of the permanent technology shock ($A_{1-t}^{1-\psi}$) in order to achieve balanced path in the wage demand equation.

$C_t$ and $N_t$ are real consumption and labor, respectively. $Inv_t$ denotes investment in real terms. $P_t$ is the price level in the economy. $B_t(t+1)$ is the amount of nominal bonds outstanding at the end of period $t$ and due in period $t+1$. $W_t$ refers to real labor income, which is the same across households in the economy. $Tax_t$ is real lump-sum tax collected by the fiscal authority to keep the real debt process from exploding, and $\Psi_t$ is dividend income coming from the firms. $K_t$ is capital and $R_t^k$ is the return on capital.

$V_t$ is the value function of the dynamic programming problem for the representative agent, and $V_{t+1}$ is the “continuation utility” of the value function. The budget constraint states that the agent has periodic after-tax income from labor, capital, and dividends as well as bonds maturing at time $t$. The agent then decides how much to consume after taxes, how much to invest, and how much to pay for newly issued bonds at time $t$ at price $Q_t^{(1)}$.

The nominal pricing kernel written in terms of return on consumption and return on labor income with distortionary taxes is

$$M_{t,t+1}^s = \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\psi} \right]^{\frac{1-\gamma}{1-\gamma}} \frac{P_t}{P_{t+1}} \left[ (1 - share_t)R_{t+1}^c + share_t R_{t+1}^l \right]^{\psi-\gamma},$$

where

$$R_{t+1}^c = \frac{(1 + P_{t+1}^c)C_{t+1}}{P_t^c C_t} \text{ and } R_{t+1}^l = \frac{(1 + P_{t+1}^l)LI_{t+1}}{P_t^l LI_t}.$$ 

$P^c$ and $P^l$ are prices of the consumption and labor claims, and $LI$ is labor income.

### 3.2 The Firm’s Problem

There is a dispersion of firms, denoted by $j$, with identical production technology in the economy. With nominal price stickiness and monopolistic competition, each firm is faced
with the following optimization problem:

$$
\max_{P_t^*(j)} E_t \left[ \sum_{s=0}^{\infty} \alpha^s M_{t,s}^g \{ P_t^*(j)Y_{t+s}(j) - P_{t+s} \left[ W_{t+s}N_{t+s}(j) + R_{t+s}^k K_{t+s}(j) \right] \} \right]
$$

s.t. 

$$
Y_{t+s}(j) = Z_{t+s}K_{t+s-1}(j)^\kappa (A_tN_t(j))^{1-\kappa}
$$

$$
P_{t+s}(j) = \left( \frac{P_t^*(j)}{P_{t+s}} \right)^{-\eta} Y_{t+s}
$$

$$
P_t = \left[ \int_0^1 P_t(j)^{1-\eta} dj \right]^{\frac{1}{1-\eta}} = [(1-\alpha)P_t^{1-\eta} + \alpha P_{t-1}^{1-\eta}]^{\frac{1}{1-\eta}}.
$$

Using Calvo (1983) pricing, a firm can choose to optimally adjust price to $P_t^*(j)$ with probability $(1 - \alpha)$ each period independent of the time elapsed between adjustments. The objective function of the firm is simply profit maximization: revenue minus labor cost and rent on capital. The within-period profits are discounted by the nominal pricing kernel and the probability that the firm has not been allowed to adjust its price optimally up to that period. Each period, with probability $\alpha$, the firm is stuck with the price from the previous period. The cash-flow stream is discounted by the nominal stochastic discount factor between times $t$ and $t+s$, $M_{t,s}^g$. $P_t^*(j)Y_{t+s}(j)$ is total sales for firm $j$ at time $t+s$. $W_{t+s}N_{t+s}(j)$ and $R_{t+s}^k K_{t+s}(j)$ are the real labor cost of and the real rental cost of capital, respectively. Notice real wage and real return on capital are determined in equilibrium with the households and are common across all firms.

There are three constraints faced by the firm in optimizing its profit. Equation (3) is the production function of firm $j$, where $Z_t$ is the transitory productivity shock, the parameter $\kappa$ is the capital share of input in the Cobb-Douglas production function, and $A_t$ is the permanent productivity shock driving growth in the economy. Equation (4) is the demand equation for firm $j$’s output as a function of the optimal price it sets at time $t$. Lastly, equation (5) is the price aggregator as a weighted average of the optimal price at time $t$ and the sticky price from time $t - 1$.

$P_t^*(j)$ is the optimal price the firm $j$ charges for one unit of the consumption good set at time $t$. $\alpha$ is the probability in each period $t+s$ that the firm is not allowed to adjust its price optimal so it has to keep charging $P_t^*(j)$. If a firm is not allowed to adjust its price optimally, then it charges $P_t^*(j)$ at time $t+s$, as the price is not indexed. All variables
indexed by $j$ is firm-specific. For example, $Y_{t+s}(j)$ means output of firm $j$ at time $t+s$ given the last time firm $j$ was able to set its optimal price was at time $t$. Without the index $j$, the variable is common across all firms, such as the price level $P_{t+s}$ and the productivity shock $Z_{t+s}$. Finally, $\eta$ determines the markup charged by the firm when it sets $P^*_t(j)$ due to monopolistic competition.

$Z_t$ is the economy-wide productivity shock on output. Log productivity follows an exogenous AR(1) process such that

$$z_{t+1} = \log(Z_t) = \phi z_t + \sigma z_{t+1} \epsilon_{z,t+1},$$

with $\epsilon_{z,t} \sim \text{i.i.d. } \mathcal{N}(0,1)$. The log growth rate of the permanent productivity shock evolves according to an AR(1) process with mean growth rate $g_a$:

$$\Delta a_t = (1 - \phi a) a_t + \phi a \Delta a_{t-1} + \sigma_a \epsilon_{a,t} ,$$

with $\epsilon_{a,t} \sim \text{i.i.d. } \mathcal{N}(0,1)$. Note that we allow for stochastic volatility in technology since uncertainty in transitory productivity has been shown to have a sizable impact on bond prices (see, e.g., Andreasen, 2012, and Kung, 2015), and we want our analysis of fiscal policy implications for term premia to be robust to this alternative channel.\footnote{Justiniano and Primiceri (2008) show that time-varying volatility in permanent productivity accounts for about 20 percent of the variance of GDP growth and real wages but they did not explore its implications for asset prices. Segal (2016) provides evidence for productivity volatility of different sectors as an important determinant of equity prices.}

The firm’s optimal price setting behavior has to satisfy the following equation in the presence of nominal price rigidities such that it can only adjust its price optimally each period with probability $\alpha$.

$$F_t = \frac{\nu (1 - \kappa - (1 - \kappa) R^*_t Z_t W_t (1 - \kappa) J_t)}{A_t^{1 - \kappa}},$$

where $\nu = \frac{n}{\eta-1}$ is the frictionless markup and $\Pi^*$ is the inflation target of the central bank.
$F_t$ and $J_t$ are recursively defined as

\begin{align}
F_t &= 1 + \alpha E_t \left[ M_{t,t+1}^{\text{nom}} \left( \frac{Y_{t+1}}{Y_t} \right) \Pi_t \right] \left[ M_{t,t+1}^{\text{nom}} \left( \frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1} J_{t+1} \right] \\
J_t &= 1 + \alpha E_t \left[ M_{t,t+1}^{\text{nom}} \left( \frac{Z_t}{Z_{t+1}} \right) \left( \frac{A_t}{A_{t+1}} \right) \left( \frac{R_{t+1}^K}{R_t^K} \right)^\kappa \left( \frac{W_{t+1}}{W_t} \right) \left( \frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1} J_{t+1} \right]
\end{align}

### 3.3 The Monetary Authority

Disengaging monetary policy neutrality by augmenting the model with the New-Keynesian framework, we assess the implications of fiscal policy on bond risk premia in the presence of an effective monetary authority. The Taylor rule used by the monetary authority to set the nominal short rate, $R_t^{(1)}$, in the model is:

\[
\frac{R_t^{(1)}}{R} = \left( \frac{R_t^{(1)}}{R} \right)^{\rho_r} \left( \frac{\Pi_t}{\Pi^*} \right)^{(1-\rho_{\pi})\rho_x} \left( \frac{Y_t/A_t}{Y_{t-1}/A_{t-1}} \right)^{(1-\rho_{\pi})\rho_x} \epsilon^u_t,
\]

where $R$ is the steady state nominal rate, $\Pi_t = \left( \frac{P_t}{P_{t-1}} \right)$ is inflation, $\Pi^*$ is the long-run inflation target, $Y$ is the steady state output, and $u_t$ is the monetary policy shock. The parameter $\rho_r$ is the autoregressive coefficient used for interest rate smoothing. The monetary rule is said to satisfy the Taylor principle when $\rho_{\pi} > 1$. Finally, the monetary policy shock follows an autoregressive process of order one

\[
u_t = \phi_u u_{t-1} + \sigma_u \epsilon_t^u,
\]

with $\epsilon_t^u \sim \text{iid } \mathcal{N}(0,1)$.

### 3.4 Equilibrium

The competitive equilibrium is characterized by the set of market clearing conditions: composite labor, capital stock, bonds, and final goods. Furthermore, given prices and wages of other households, each optimizing household chooses the optimal allocation to solve his/her
utility maximization problem. Finally, given wages and prices of other firms, each firm chooses the optimal production input to solve its profit maximization problem. In equilibrium, $N^d_t = N_t$. In this economy, total output has to equal to total private consumption and private investment plus total government spending:

$$Y_t = C_t + Inv_t + Gov_t. \tag{9}$$

In the model, because the market is complete and there is a representative marginal pricer, there exists an unique pricing kernel which allows us to price all assets in the economy, including long- and short-term bonds.

### 3.5 The Government’s Budget Constraint

The government’s flow budget constraint balances resources with uses:

$$P_t Tax_t + Q_t^{(1)} B_t(t + 1) = B_{t-1}(t) + P_t Gov_t,$$

where $Gov_t$ is consumption by the government or government spending. $Gov_t$ is not productive in the model economy. Furthermore,

$$Tax_t = \tau_t + \tau^k_t P_t^k u_t K_{t-1},$$

such that $\tau_t$ is the lump-sum tax described below. Government spending as a fraction of output, $g_t = \frac{Gov_t}{Y_t}$, and the capital tax rate, $\tau^k_t$, follow two independent AR(1) with stochastic volatility, c.f. Section 2.1, Eqs. (1)-(2).

The lump-sum tax is meant to be collected to keep the borrowing path of the government from exploding. Following standard procedure in the literature, we specify the lump-sum tax as a function of real debt and government spending.

$$\tau_t = \rho_b D_{t-1}(t) + \rho_g Gov_t,$$

where $D$ denotes real debt such that $D_{t-1}(t) = \frac{B_{t-1}(t)}{P_t}$. The simple fiscal rule is widely
used in the literature on the macroeconomic impact of fiscal policy shocks, see Gali, Valles, and Lopez-Salido (2007) and Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) for two recent examples. In the previous working version of the paper, we model long-term bonds directly using a geometrically declining series to proxy for the maturity structure of government debt similar to Cochrane (2001). We find that the modeling of long-term bonds using a geometric series did not alter the term structure implications we focus on here. For simplicity, we abstract away from that setup to obtain a simpler government budget constraint and fiscal rule.\footnote{The maturity structure of government debt is an interesting question to itself. There is no clear consensus in the literature on how it should be modeled. However, this is a question beyond the scope of our current paper.}

4 Inference and the Observable Variables

To estimate the parameters of our model we rely on the generalized method of moments (GMM) using first and second unconditional moments of macroeconomic and financial data. This section provides a detailed description of the estimation method and discusses the data used to evaluate the unconditional moments.

4.1 Data and Moments for GMM

The time unit is defined to be one quarter. We estimate the model using the following quarterly time series: (i) log output growth, $\Delta y_t$ (henceforth, $\Delta$ denotes the temporal difference operator); (ii) log investment growth, $\Delta \text{inv}_t$; (iii) log consumption growth, $\Delta c_t$; (iv) inflation, $\pi_t$; (v) the 1-quarter nominal interest rate, $r_t$; (vi) the 10-year nominal interest rate, $y_t^{(40)}$; (vii) the slope of the term structure, $y_t^{(40)} - r_t$. The sample spans 1970.Q1 to 2014.Q2.\footnote{The starting date follows Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) and it is dictated by the start of our fiscal series. We have also repeated our estimation exercise with moments computed from a sample period that exclude the financial crisis, from 1970.Q1 to 2007.Q4, and find that the results remain qualitatively the same.} Appendix A gives detailed variable definitions and sources.
To estimate model parameters we use the mean, the variance and the contemporaneous covariances in the data as moments. Provided the model’s solution is stable, Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) derive closed-form solutions for first and second unconditional moments of the (non linear) state-space of the DSGE. This is important since it allows us to compute in a reasonable amount of time the unconditional moments for our DSGE model solved up to third-order. Appendix D provides additional details.

4.2 Inducing Stationarity and Solution Method

The exogenous productivity process \( A_t \) displays a stochastic trend. This random trend is inherited by the endogenous variables of the model. We focus our attention on equilibrium fluctuations around this stochastic trend. To this end, we perform a stationarity-inducing transformation of the endogenous variables by dividing them by their trend component. Appendix C.6.1 describes this transformation and presents the complete set of equilibrium conditions in stationary form.

To analyze the role of fiscal shocks and the implications for time-varying risk premia, we solve the benchmark DSGE model using perturbation methods (see Schmitt-Grohe and Uribe (2004)). Given our interest in analyzing time-varying risk premia, we employ a third-order Taylor approximation of the policy functions that characterize the equilibrium dynamics of the model (see propositions 3 and 4 in Andreasen (2012) for how stochastic volatility affects any type of risk premia in a wide class of DSGE models). See Appendix D for more details.

To fit the term structure to data, we compute the yield curve implied by the model using the fact that bond prices beyond the policy rate, \( r_t = \log R_t^{(1)} \), do not affect allocations and prices. Taking advantage of this property, we follow Andreasen and Zabczyk (2015) and first solve the model without bond prices exceeding one period, and then we recursively compute

\[ \text{\footnotesize 16} \]

We have also repeated our procedure adding to the first and second moments used in the baseline estimation the first and fifth autocovariances to capture the persistence in the data. Our point estimates do not significantly change and the conclusion from model-implied moments remain qualitatively the same. Results are available upon request.

\[ \text{\footnotesize 17} \]

Our model has a relatively large number of state variables and eight shocks. Because of this high dimensionality, discretization and projection methods are computationally infeasible.
all remaining bond prices based on

\[ Q_{t}^{(k)} = E_t \left[ M_{t,t+1}^{S} Q_{t+1}^{(k-1)} \right], \]

where \( M_{t,t+1}^{S} = M_{t,t+1}^{1/H_{t+1}} \) denotes the nominal stochastic discount factor, and \( M_{t,t+1} \) denotes the real stochastic discount factor. We let \( k = 2, \ldots, 40 \) quarters. The nominal yield curve with continuous compounding is then given by \( y_{t}^{(k)} = -\frac{1}{k} \log Q_{t}^{(k)} \). We also compute the real term structure based on

\[ Q_{t,\text{real}}^{(k)} = E_t \left[ M_{t,t+1}^{\text{real}} Q_{t+1,\text{real}}^{(k-1)} \right]. \]

Finally, we define the 10-year nominal term premium to be the difference between the 10-year interest rate and the yield-to-maturity on the corresponding bond under risk-neutrality. The latter is computed by discounting payments by \( r_{t} \) instead of the stochastic discount factor.

5 Estimation Results

5.1 Parameter Estimates

Given the large scope of the model, we fix a small number of parameters to values commonly used in the literature, see Table 3 Panel A. In particular the rate of depreciation on capital is 0.02 as employed by Kaltenbrunner and Lochstoer (2010). This value implies a steady-state investment-output ratio of 21 percent. The capital share of intermediate output, \( \kappa \), is 0.33. The following parameter values are standard in New-Keynesian models. The price rigidity parameter, \( \alpha \), is 0.66. This means every period, two thirds of the firms in the economy are not able to adjust their prices to the optimal level. The higher the \( \alpha \), the stickier the nominal prices are. We also set the wage rigidity parameter, \( \theta \), to 0.66. The price markup parameter resulting from monopolistic competition, \( \eta \), and the wage markup parameter in union wage setting, \( \eta_{w} \), are both equal to 6. Hence, steady-state price and wage markup are both equal to 20%. Consistent with previous studies, our calibrated parameters imply a steady-state capital-output ratio, \( \frac{Y}{K} \), of about 2. We also set the monetary policy rule coefficient on inflation, \( \rho_{\pi} \), to the typical value of 1.5 used in the literature. We set the
government spending–output ratio, $\theta_g$, to 20.2%, and the mean of the tax rate, $\theta_k$, to 40%, according to the data. Finally, we calibrate the parameters for transitory productivity to values commonly adopted in the literature, see, e.g., Andreasen (2012) and Kung (2015).

As discussed in Section 2.1, we estimate the processes for capital tax rate and for government spending outside of the model, see Table 3 Panel B. This procedure has the benefit of ensuring that the latent fiscal (tax and government spending) volatility factors maintain their intended economic interpretation.

Table 3 Panel C reports the estimates of the structural parameters in our model.

The estimation assigns a relatively high value of 0.995 to $\beta$. This value is needed in order to obtain a sufficiently low mean value for the one-period nominal interest rate. The parameter $\gamma$ is estimated to be 181. Since the representative agent in the model can earn labor income as a mean to smooth consumption, his/her attitude toward risk is different than those who do not supply labor. Following Swanson (2012), we adjust the risk aversion parameter by taking into account the labor margin using the closed-form formula $\psi + \gamma - \psi_1$ with $\nu = \frac{n}{\eta-1}$. The representative saver’s true coefficient of relative risk aversion is therefore $\approx 111$. This may seem like a high value; however, other term structure studies using Epstein-Zin preferences also typically estimate a high coefficient of relative risk aversion: Piazzesi and Schneider (2007) estimate a value of 57, van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012) a value of about 66, and Rudebusch and Swanson (2012) a value near 110.

The only parameter which deserves attention is $\sigma_{\sigma_z}$. We set the volatility of volatility to 0.03 in line with Andreasen (2012). We do so for two reasons. First, this value implies an unconditional standard deviation in $\sigma_{z,t}$ of 0.19, which is the same as one would obtain from fitting a GARCH model on log productivity. Second, our chosen value for the vol-of-vol parameter lies on the higher hand of those used in the literature, and makes our results for fiscal policy conservative. Indeed, lower values for $\sigma_{\sigma_z}$ would only increase the relative contribution of fiscal volatility shocks relative to uncertainty in productivity.

Alternatively we could have used macro and financial variables (bond yields) to estimate the full fledged model with time-varying volatility in fiscal rules. However, bond yields may potentially compromising the interpretation of the volatility in government spending and capital tax rate. Our approach disciplines the stochastic volatility to fit the observed government spending and capital tax rate data only, instead.

Andreasen and Jorgensen (2016) propose a slightly modified utility kernel for Epstein-Zin preferences to address the puzzlingly high relative risk-aversion in DSGE models. We leave the analysis of such a utility kernel in our setting to future research.
The estimation procedure picks a low value for the IES, \(1/\psi \approx 0.53\). This value is consistent with estimates in the micro literature (e.g., Vissing-Jorgensen, 2002) and it has also been adopted by Rudebusch and Swanson (2012) in a general equilibrium context similar to ours. The low value for the IES helps to make consumption less volatile and real interest rates more volatile, both of which improve the fit to the macro moments in the data. The higher interest rate volatility also increases bond price volatility and improves the model’s fit with respect to the finance moments. Our estimates of the Frisch elasticity is in line with the literature. The response of the monetary policy authority to output growth, \(\rho_x\), is similar to that used in influential studies such as Judd and Rudebusch (1998), Taylor (1999) and Clarida, Galí, and Gertler (2000). The shock persistence and variance for permanent productivity, \(\phi_z\) and \(\sigma_z\), are broadly in line with, e.g., the estimates in Justiniano, Primiceri, and Tambalotti (2011). Finally, our estimates imply a substantial degree of adjustment costs in investment, in line with previous studies (e.g. Del Negro, Schorfheide, Smets, and Wouters (2007) and Smets and Wouters (2007)).

5.2 Model’s Fit

Given our GMM estimates, how well does the model fit the data? We address this question by comparing a set of statistics implied by the model to those measured in the data. Throughout the section we benchmark the model-implied term premium to the measure provided by Adrian, Crump, and Moench (2013).

Table 4 reports the model-implied as well as the corresponding empirical moments for two sets of variables: (1) the first set comprises the seven variables used in estimation; (2) the second set is composed of additional macro (wages and hours) and financial (3-, 5-, 7-year yields, and the 5- and 10-year term premium) variables whose moments are not directly targeted in the estimation. The table reports the median and the 90 percent probability intervals that account for parameter uncertainty for the standard deviation, autocorrelation, and contemporaneous correlation with output.\(^{21}\) Although in the estimation

\[^{21}\text{We draw the structural parameters from a Normal distribution with a variance-covariance matrix obtained from our second step GMM estimation procedure. The parameters governing the processes for the fiscal instruments are obtained from the posterior distribution reported in Table 3 Panel B. For each param-}\]
we target growth rates of output, consumption, and investment, Panel A displays hp-filtered moments for these macro variables (as well as for wages and hours) to make our analysis comparable to other studies on fiscal policy (see, e.g., Table 5 in Fernandez-Villaverde et al., 2015).

[Insert Table 4 about here.]

Our benchmark model matches the mean and standard deviation of yields over the whole maturity profile, as well as the slope for the nominal term structure (all values fall within the 90% confidence interval). In particular, the model is able to produce a sizable slope of 1.2% and to generate a volatile 10-year rate. With respect to term premium, the model is overall quite successful in reproducing a sizable mean 5-year term premium of about 0.9%, to be compared to 1.3% in the data. The model is also able to account for 0.61/0.86 ≈ 71% of the term premium unconditional standard deviation.

Furthermore, the model can simultaneously match key business cycle moments for real variables. In particular, the model matches fairly well the volatility of output, consumption, investment, and inflation. The series of hours, which is not targeted in estimation, also displays a model-implied volatility quite in line with the data. Finally, although not reported, the model matches the mean of growth rates in output and consumption, and it slightly under-predicts that of investment growth, with a (median) value of 2.8% against 3.5% in the data.

It is worth highlighting the substantial time variation of the nominal short-rate, slope and term premium within the model generated by stochastic volatility of fiscal instruments rather than higher variance in the shocks to fiscal instruments themselves. In untabulated results, we consider the benchmark model without stochastic volatilities in fiscal policies. In particular we set the unconditional variance in shocks to government spending and capital tax rate $\sigma_{x,t+1} = \sigma_x = \theta_{\sigma_x} + \sigma_{\sigma_x}$, with $x \in \{g, \tau^k\}$. Doing so ensures that the unconditional variance in fiscal instruments is comparable to the specification of our benchmark model with stochastic volatility. The experiment showed that a model without fiscal uncertainty

eter draw, we generate an artificial long sample (5000 quarters) of the observable variables after discarding 1000 initial observations. Hence we do not account for small sample uncertainty.
is not able to quantitatively match the variability in the short-rate and the slope for the nominal term structure (the model-implied 90% confidence intervals do not include the data values). Also, a model without time-varying uncertainty produces much lower term premium volatility. Overall, time-varying volatility in fiscal shocks seem to be an important driver for variation in the U.S. yield curve and term premia.

Turning to the persistence of quantities and prices, Table 4 reports the first-order autocorrelation coefficient while Figure 2 displays the entire autocovariance function of the data (black line) and the model (blue line), along with the 90 percent intervals that account for parameter uncertainty. Again, the figure includes all the observable quantities used to estimate the model, as well as additional macro (wages and hours) and financial (3-, 5-, 7-year yields, and term premia) variables whose moments are not directly targeted in the estimation. Overall, the model captures the decaying autocorrelation structure of real and financial variables reasonably well. The success is particularly impressive for the long-term rates (maturities ≥ 5 years) and the term premium, for which the data auto-correlations are always within the model-implied confidence bands. The model does a satisfactory job for output, consumption, and investment, but it generates slightly too much persistence in inflation and in the nominal short-term interest rate.

We conclude this section by discussing a few more quantitative implications of the model that will support the interpretation of our model-implied term structure. First, our model is able to match the empirical correlation of consumption growth and inflation. In our dataset, these two series are negative correlated at −0.14 over the 1970:Q1–2014:Q2 sample period; this negative correlation doubles and is equal to −0.30 over the period 1970:Q1–2007:Q4, which excludes the financial crisis. Consistently with the data, our model implies a negative correlation of −0.29. We will return to this negative correlation in our discussion of government spending level shocks and inflation risk premium, see Section 5.3.
To further discipline the model, we investigate its implications for the real term structure. Table 5 displays the means, volatilities, and first autocorrelations of real bond yields of different maturities and the ten-year minus two-year yield spread from the model. We compare these statistics with the real term structure obtained by splicing together yields data from Chernov and Mueller (2012) and from Gurkaynak, Sack, and Wright (2010)\textsuperscript{25}. The volatility of real yields for all maturities is in line with the data, although the average level of the real yield curve in our model is slightly higher than in the data. More importantly, the model-implied average slope and its standard deviation are close to the data, and particularly so for the period that does not comprise the financial crisis. Both in the data and in our model, the average slope of the real yield curve is positive. Similarly, Campbell, Shiller, and Viceira (2009) report that the real yield on long-term US TIPS has always been positive (see also discussion in Beeler and Campbell, 2012). An upward-sloping real yield curve implies that long-maturity real bonds have lower payoffs than short-maturity ones when expected consumption growth is low. We will return to this fact in our interpretation of government spending volatility shocks and the term premium.

5.3 Impulse Responses

A large literature in financial economics finds that bond risk premia are substantial and vary significantly over time (see Campbell and Shiller (1991) and Cochrane and Piazzesi (2005)); however, the economic forces that can justify such large and variable term premium

\textsuperscript{25}The data from Chernov and Mueller (2012) spans 1971:Q3 to 2002:Q4. We merge this data with those from Gurkaynak, Sack, and Wright (2010). Throughout, we remove data for 2003 due to a high illiquidity premium. For the same liquidity reason, we also consider a shorter sample that excludes the financial crisis. The relative (il)liquidity of TIPS from their inception until 2003, when the Treasury reaffirmed its commitment to the TIPS program, and in the aftermath of the Lehman bankruptcy in late 2008, which resulted in its considerable TIPS inventory being released into the market, have been discussed in Sack and Elsasser (2004) and Campbell, Shiller, and Viceira (2009) among others.
are less clear. In this section, we shed some light on this issue by examining the model’s impulse responses to shocks.

To understand the role of shocks for the term premium, Figure 3 shows the impulse responses of the stochastic discount factor (SDF, henceforth), inflation, long-term bond yield, and term premium to a positive one-standard-deviation shock to government spending level (column 1) and volatility (column 2), and to capital tax rate level (column 3) and volatility (column 4); Figure 4 shows the impulse responses to shocks in transitory productivity and its time-varying volatility (columns 1 and 2, respectively), to permanent productivity (column 3) and monetary policy shocks (column 4).

[Insert Figures 3 and 4 about here.]

Figures 3 and 4 show that fiscal shocks together with innovations in transitory productivity represent the main drivers of bond risk premia. On the other hand term premium fluctuations induced by permanent productivity and monetary shocks are minimal. Comparing the last row in Figure 3 with that in Figure 4 we see that fluctuations in term premium due to government spending volatility shocks are larger than those generated by volatility in productivity. Government spending level shocks too stand out as a source of term premium as important as level shocks in transitory productivity. Both government spending level and volatility shocks demand a positive, and quite persistent term premium.

Next, we investigate the behavior of inflation risk premium induced by government spending shocks. To this end, we look at the response of the SDF and inflation. A key and novel result conveyed by Figure 3 is that the relationship between consumption and inflation depends critically on the nature of the underlying fiscal shocks: government spending level shocks imply a negative correlation between consumption growth and inflation, while government spending uncertainty shocks imply exactly the opposite relation. Therefore, in our model, an increase in government spending level implies that inflation is high exactly when agents wish to consume more; but high inflation makes payoffs on nominal bonds low in real terms, and the positive covariance between marginal utility of consumption and inflation generates positive inflation risk premia. On the other hand, following a positive government
spending uncertainty shock, consumption growth and inflation move in the same direction, which in turn delivers an average negative inflation risk premia.

Figure 3 further shows that both the level and volatility shocks to government spending have positive impact on the nominal term premium for long-term bonds. This is straightforward to rationalize for level shocks since inflation risk premium is positive. The fact that government spending volatility shocks command a positive nominal term premium in the second column despite the negative inflation risk premium suggests that long-term nominal bonds are riskier relative to short-term nominal bonds. In other words, when the marginal utility is high (spike in the SDF), long-term bond price appreciates less than the price of short-term bonds. The overall implication of the government spending volatility shock on the nominal term structure is that it has a negative level effect but a positive slope effect. The steepening of the nominal yield curve due to a positive spending volatility shock is confirmed in Figure 3(b) by observing the large decline in the 1-quarter nominal yield.

Turning to capital tax shocks in the third and fourth columns of Figure 3, inflation risk premia are negative on average and nominal term premia fall in response to both level and uncertainty shocks to tax rate. When the marginal utility to consume is high following tax shocks, inflation declines thus making nominal bonds an effective hedge against real consumption risk, resulting in a negative inflation risk premium. The third row of Figure 3 shows long-term nominal yields drop significantly exactly when the stochastic discount factor spikes, resulting in further decline in the 5-year nominal term premium. In sum, both level and volatility shocks to the return on capital tax rate have negative level effects on the nominal term structure.

Our discussion here based on the impulse responses of the model can be validated in the regression analysis in Table 2 in several dimensions. First, government spending level and volatility shocks command positive term premium, in line with positive coefficient estimates $\beta_1$ and $\beta_2$ from the predictive regressions (see Panel B and C). Second, return on capital tax rate level and volatility shocks command negative term premium with large error bands, consistent with coefficient estimates $\beta_3$ and $\beta_4$, which are mostly negative or statistically insignificant (Panel C). Third, government spending volatility shocks dominate level shocks in driving term premium variation. This is similarly reflected in the comparison of statistical
significance between $\beta_1$ and $\beta_2$ in Panel C of Table 2.

We conclude this section by quantifying the contribution of each shock to the variability of macroeconomic and financial variables.\(^{26}\)

Table 6 Panel B shows that, consistent with the results in Table 1, uncertainty in government spending is the single most important source of variation in the slope of the term structure. Government spending uncertainty is also as important as volatility in productivity to generate movements in term premium. On the other hand, level shocks in transitory productivity generate negligible variability in term premium, a result which contrasts with Rudebusch and Swanson (2008). All shocks are important drivers of nominal yields movements, except for permanent productivity, whose effects are puny, and monetary shocks, whose effects dissipate quickly along the term structure of interest rates. Turning to the real side of the economy, Table 6 Panel A shows that transitory productivity level shocks are a key determinant of consumption and output volatilities. However, spending and capital tax (level and volatility) shocks generate sizable effects on investment, hours and inflation.

To summarize, we find that stochastic volatility in government spending shocks can generate sizable variation in the term premium without distorting the ability of the model to match key macroeconomic moments.

### 5.4 Model Implied Return Predictability

We compare the predictability of bond excess returns in the data to that obtained from simulations of our fiscal model\(^{27}\) in Table 7. Panel A of Table 7 shows that, similar to the

\(^{26}\)The task of measuring the contribution of each of the eight shocks in our model to aggregate fluctuations is complicated because, with a third-order approximation to the policy function and its associated nonlinear terms, we cannot neatly divide total variance among the shocks as we would do in the linear case. We follow Fernández-Villaverde, Guerón-Quintana, Rubio-Ramírez, and Uríbe (2011) and set the realizations of seven of the shocks to zero and measure the volatility of the economy with the remaining shock.

\(^{27}\)In the data, we use the maturity-weighted debt to GDP ratio and the filtered volatility from our Bayesian procedure to proxy for the supply of debt and fiscal uncertainty. In the model-implied regressions we use
data, the government debt level is an important predictor of bond excess returns. Panel B shows that the loadings on the level and volatility of government spending are positive and statistically significant in the model regressions. Panel C, shows that adding capital tax rates leaves unaffected the conclusion on the level and volatility of government spending. Moreover the level of capital tax rate enters almost always with a negative coefficient, albeit the point estimate is insignificant. All these implications from our model are in line with the data, see Table 2.

Before concluding we remark that Table 7 shows population results from a long simulation of the model where parameters are fixed at their point estimates; so our model-implied predictive regressions do not account for parameter uncertainty and small sample uncertainty. Accounting for these two sources of uncertainty would close the gap between the estimated coefficients on government spending level and volatility in the data (Table 2) and those implied by the model. In fact, in untabulated results, we show that the 90% confidence interval from finite sample simulation always include the point estimate in the data for the level of government spending, and gets closer to that for uncertainty. Similarly, finite sample simulations deliver a 90% interval for \( R^2 \) in Panel C equal to \([11\%, 24\%]\) for one-year holding period returns (across maturities), which encompasses the \( R^2 \approx 21\% \) measured in the data.

### 5.5 Economic Intuition of Inflation Risk Premium from Government Spending

The decomposition of nominal bond yields consists of real yields, expected inflation, and inflation risk premium. In closed form:

\[
i_t^{(n)} = r_t^{(n)} + \frac{1}{n} \left\{ E_t[\pi_{t,t+n}] + \text{cov}_t(m_{t,t+n}, \pi_{t,t+n}) - \frac{1}{2} \text{var}_t(\pi_{t,t+n}) \right\},
\]

where the conditional covariance of the marginal rate of consumption substitution between times \( t \) and \( t+n \) with inflation during the same period gives us the compensation for inflation risk for holding \( n \)-period to maturity nominal bonds. To derive some intuition on inflation instead the real maturing debt \( D_{t-1}(t) \) and the true volatility process for fiscal instruments. Also, we apply a (one-sided) hp-filter to the level of the fiscal variables within the model as we did in the data.
risk premium in the current model, we study this covariance term by examining the impact of fiscal shocks on $m_{t,t+1}$ and on $\pi_{t,t+1}$.

The real stochastic discount factor can be written in logs such that

$$m_{t-1,t} = \frac{1 - \gamma}{1 - \psi} \left[ \log(\beta) - \psi \left( c_t^o - c_{t-1}^o \right) \right] + \frac{\psi - \gamma}{1 - \psi} \log(R_{t}^{dl}),$$

where $R_{t}^{dl}$ is the return on the wealth (consumption and labor income) portfolio of the representative saver. Because the representative household is Ricardian with respect to government spending, positive level shocks to government spending increase saving while crowding out consumption. The resulting high marginal utility state generates higher $m_{t-1,t}$ because consumption growth ($c_t - c_{t-1}$) is low.

To decipher the impact of government spending shocks on inflation, we loglinearize the Phillips curve in Equation (6) after detrending the growth variables to get,

$$\frac{\alpha}{1 - \alpha} \pi_t + f_t = \log(\nu \kappa^{-\kappa}(1 - \kappa)^{-(1-\kappa)}) + \kappa r^K_t + (1 - \kappa) \tilde{w}_t + j_t - z_t, \quad (10)$$

where the tilde above a variable indicates stationarity. Therefore, $\tilde{w}_t = \log \left( \frac{W_t}{A_t} \right)$. The first term can be obtained by assuming the steady state log inflation, $\pi$, is zero. The interpretation of this equation is that inflation is not only functions of the contemporaneous marginal cost to the firm ($r^K_t$ and $\tilde{w}_t$), but also expected inflation and expected marginal cost, according to Equations (7) and (8), during the period before the optimal price can be set again.

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28For the ease of exposition, the remainder of this section contains lower case variables denoting the log-version of their upper case counterparts.
Rearrange Eq. (10), we have\textsuperscript{29}

\[
\frac{\alpha}{1 - \alpha} \pi_t = \log(\nu \kappa - \kappa) + \kappa r^K_t + (1 - \kappa)w_t - z_t + j_t - f_t \\
\approx \log(\nu \kappa - \kappa) + \kappa r^K_t + (1 - \kappa)w_t - z_t \\
\text{contemporaneous marginal cost} +\text{const} \left\{ \mathbb{E}_t \left[ \begin{array}{c}
\pi_{t+1} \\
-\Delta z_{t+1} + \kappa \Delta r^K_{t+1} + (1 - \kappa) \Delta w_{t+1}
\end{array} \right] \\
+ j_{t+1} - f_{t+1} \right\} + \frac{1}{2} \left[ \text{var}_t(\Delta z_{t+1}) + \kappa^2 \text{var}_t(\Delta r^K_{t+1}) + (1 - \kappa)^2 \text{var}_t(\Delta w_{t+1}) \\
+ (1 - 2\eta) \text{var}_t(\pi_{t+1}) + \text{var}_t(j_{t+1}) + \text{var}_t(f_{t+1}) \right].
\]  

(11)

Recall the last equality is an approximation after dropping the remaining covariance terms. There are a number of takeaways from this derivation. First, higher expected inflation raises current inflation. Second, higher expected marginal cost also raises current inflation. Third, stochastic volatility, which increases conditional variance of the endogenous variables, generally increases current inflation with the except of inflation variance since \( \eta \) is much greater than 1.

Figure 5-Panel (a) shows the impulse responses of endogenous variables to spending shocks and it allows us to inspect further the mechanism. Following a positive government spending level shock, output rises according to the market clearing condition, Equation (9). Firms intend to produce more in order to meet the demand by increasing labor and capital input. On the supply side, labor supply is high deriving from the negative wealth effect of the households due to lower consumption, but capital supply is low stemming from the desire of the households to invest in Treasury bonds over capital because they are safer. The result is a drop in real wage, but a strong increase in the return on capital, hiking the marginal cost

\textsuperscript{29}Given the linearized functional forms of \( f_t \) and \( j_t \) in Appendix C.5, we can simplify the loglinear Phillips curve in Eq. (10). First notice \( \text{const}_f = \text{const}_j = \text{const} \) since steady state \( \Upsilon = \Phi \) by assuming \( \pi = 0 \). Second, we ignore the covariance terms in the decomposition of the variance terms within \( f_t \) and \( j_t \) to keep the intuition simple. Furthermore, many of these covariance terms will cancel out in calculating \( j_t - f_t \).
for the firm. The increase in contemporaneous and expected marginal cost drive up inflation according to the loglinearized Phillips curve. Recall the same positive government spending level shock pushes up marginal utility by lowering consumption growth, thus the covariance generated by the government spending level shock between $m_{t,t+1}$ and $\pi_{t+1}$, $\text{cov}_t(m_{t,t+1}, \pi_{t+1})$, is positive implying positive inflation risk premium.

Similar to the level shock, a positive government spending volatility shock also raises the marginal utility of consumption. Under the lognormal framework, the second moment shock works through the expectation channel in the following way:

$$E_t[G_{t+1}] = E_t[e^{g_{t+1}}] = e^{E_t[g_{t+1}] + \frac{1}{2} \text{var}_t(g_{t+1})}.$$ 

Uncertainty about government spending affects the expectation of future government spending, amplifying household’s precautionary savings motive making current consumption fall. Unlike the level shock, however, because the volatility shock increases the expected return on capital causing marginal Q to rise through the investment equation, the savers prefer investment in capital as opposed to Treasury bonds. Firms, on the other hand, also anticipate the increase in expected demand and coordinate by shifting production from today to tomorrow. By decreasing labor and capital inputs today, current marginal cost goes down resulting in a decline in inflation. The fall in inflation is further reinforced by the increase of the conditional variance of inflation in Equation (11) stemming from the government spending volatility shock. Because $(1 - 2\eta) < 0$, higher inflation uncertainty translates into lower current inflation according to the loglinear Phillips curve. On average, the second moment shock to government spending generates low inflation in high marginal marginal state of the world making $\text{cov}_t(m_{t,t+1}, \pi_{t+1})$ negative.

5.6 Term Premium and Government Spending Volatility Shocks

The bottom row of Figure 3 shows that fiscal policy shocks have significant impact on the nominal term premium. The variation is especially pronounced for volatility shocks in the second column. After the realization of a positive one standard deviation government spending volatility shock, the 5-year term premium increases by about 25 bps, on aver-
Recall that term premium stems from the relative riskiness of long-maturity bonds vs. short maturity bonds. Intuitively, the term premium is positive (negative) when the return for long-maturity bonds is lower (higher) than the return for short-maturity bonds in high marginal utility states. Translating into yields, this implies long-term yields increase (decrease) more (less) compared with short-term yields, thus creating a yield curve steepening effect.

Figure 5–Panel (b) presents government spending volatility shock impulse responses for the real economy and the nominal short rate. Notice the 1-quarter nominal rate drops significantly relative to the decline in the 5-year nominal rate in the second column of Figure 3, implying short-dated bonds have greater price increase in bad times making long-dated bonds risky. To get some intuition on what is driving the relative change in bond prices, assume a positive government spending volatility shock is realized at the beginning of time $t$ so the SDF is elevated ($M_{t-1,t}^S \uparrow$). We compare the price of a one-period to maturity bond to the price of a $n$-period to maturity bond under CRRA utility:

$$P_{t}^{(1)} \uparrow = e^{-r_{t}^{(1)}} \downarrow = \mathbb{E}_{t} \left[ M_{t,t+1}^{S} \right] = \mathbb{E}_{t} \left[ e^{-\gamma \Delta c_{t+1} - \pi_{t+1}} \right],$$

$$P_{t}^{(n)} \uparrow = e^{-r_{t}^{(n)}} \downarrow = \mathbb{E}_{t} \left[ M_{t,t+1}^{S} M_{t+1,t+2}^{S} \cdots M_{t+n-2,t+n-1}^{S} P_{t+n-1}^{(1)} \right] = \mathbb{E}_{t} \left[ e^{-\gamma \Delta c_{t+n} - \pi_{t+n}} \right],$$

where the length of the arrows denotes magnitude. For the price of the one-period to maturity bond to increase more in comparison to the $n$-period to maturity bond, it has to be the case that the one-period expected consumption growth declines more than the $n$-period expected consumption growth (assuming inflation differential is trivial for now). Figure 5–panel (b) shows that the positive government spending volatility shock causes a temporary decrease in real wage and increase in saving (real debt) in the short-run. However, in the long-run, wage rebounds and debt level falls persistently. The implications of these impulse responses are consistent with a large drop in short-term expected consumption growth and a less dramatic decline in long-term expected consumption growth, which steepens the yield curve and raises term premium.

[Insert Figures 5 about here.]
5.7 Inspecting the Mechanism due to Capital Tax Rate Shocks

The third and fourth columns of Figure 3 document that level and volatility shocks to the return on capital tax rate induce substantial negative nominal term premia. To decipher the mechanism, we examine the impulse response functions to the real economy of the these shocks in panels (a) and (b) in Figure 6. In panel (a), a positive level shock to the tax rate lowers output and investment as the marginal return on capital decreases. As a result, marginal cost declines causing inflation to be low when consumption is also low. Moreover, debt issuance drops as the tax revenue increases driving up (down) bond prices (yields), especially at the short-end of the maturity curve. The negative level effect generated by the positive capital tax level shock results in negative inflation risk premium. It is interesting to note that the 1-quarter nominal rate in panel (a) of Figure 6 experiences a much more significant drop relative to the 5-year rate in the third column of Figure 3. The steepening of the yield curve implies a positive term premium, and yet the overall nominal term premium is negative following the tax rate level shock. Therefore, we conclude that the negative term structure level effect dominates the positive slope effect in this case.

[Insert Figures 6 about here.]

Opposite to the government spending volatility shock, the term premium driven by tax rate volatility shock is negative. A positive one standard deviation shock to the return on capital tax rate volatility leads to a 30 bps fall in the 5-year term premium in the fourth column of Figure 3. This is reflected in panel (b) of Figures 6. Following a positive one standard deviation volatility shock to the return on capital tax rate, households cut investment immediately because tax rate is expected to be high tomorrow. At the same time, real wage gets a temporary bump up while savings start to decline. Over the long horizon, investment recovers, and wage falls as aggregate demand stays below its steady state. In contrast to panel (a), the decrease in investment is more attenuated for tax rate volatility shocks compared to level shocks, and marginal cost actually increases slightly due to higher wage. However, in the long-run, as wage declines, expected marginal cost also lessens to produce lower inflation. This is a pure term structure level effect as the positive tax rate volatility shock induces a parallel shift downward of the yield curve. The 1-quarter
short rate decreases by roughly the same magnitude in panel (b) of Figure 3 as the 5-year nominal rate in the last column of Table 3.

Furthermore, the impulse response functions to a volatility shock of the capital return tax rate are broadly in line with the empirical findings documented by Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015). Notably our model replicates both the decrease in inflation (see Figure 3, column 4) and nominal interest rate documented in Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015), a fact that was challenging to obtain in their baseline model economy. Intuitively, faced with higher tax uncertainty, households want to save more. At the same time households invest less because of the increased probability of higher tax rate on capital income. The increased uncertainty surrounding capital tax raises the demand for bonds leading to decline in yields across maturities.

5.8 The Importance of Fiscal Shocks for the Term Premium

Ex ante, productivity, fiscal and monetary policy shocks could all be very important drivers of the term premium. Table 6 has already highlighted that, in fact, fiscal volatility shocks turn out to be a key driver of variation in term premium within our model. To further substantiate our claim that fiscal shocks represent a key determinant of the term premium, we feed our model with the filtered shocks from the estimated government spending and capital tax rate dynamics, see Eqs. (1) and (2). Figure 7(a) compares the model’s prediction for the term premium to the empirical measure of term premium obtained in Adrian, Crump, and Moench (2013). The left Panel presents the term premium obtained when we feed into our model shocks to government and capital rate level only; the right Panel presents the premium when we feed our model with both fiscal level and volatility shocks.

[Insert Figure 7 about here.]

We estimate the parameters in Eqs. (1) and (2) following a Bayesian approach. The particle filter delivers draws for the shocks. We feed each draw into the model; then, we compute the median and 95 percent probability intervals for the model-implied premium.
The figure shows how the fiscal level shocks make the model able to track the average term premium whereas the volatility shocks helps in capturing the variability of the term premium. Also, the interaction of shocks to level and volatility captures the trending down in the late 90s. Finally, our model captures the increase in term premium around the financial crisis. To our eyes, (the fiscal shocks in) the model provides a tantalizing account of the cyclical and longer-term fluctuations in the term premium.

We also quantify the relative contribution of real and inflation risk premia to the overall nominal compensation. Figure 7(b) shows the result. The Figure superimposes the model-implied nominal and real term premium, as well as their difference, the inflation risk premium. The left panel shows that fiscal level shocks generate both a sizable level effect via real term premium and, more importantly, substantial variability through movements in inflation risk premium. Looking at the right chart, we observe that adding fiscal volatility shocks leads to remarkable fluctuations in real term premia. In all, the compensation investors require for bearing real interest rate risk – the risk that real short rates don’t evolve as they expected – represent a force behind movements in nominal term premia as important as inflation risk premium according to our model. This finding bodes well with the reduced form results in Abrahams, Adrian, Crump, Moench, and Yu (2016).

6 Fiscal Shocks at the ZLB

In this section we study the propagation of fiscal shocks when the economy is already at the zero lower bound (ZLB) such that the nominal interest rate is zero. In the aftermath of the 2008 financial crisis, the Federal Reserve Bank aggressively lowered the Fed funds rate to close to zero as a response in order to stimulate economic activity. This led to the longest episode of zero interest in modern U.S. history until interest rate liftoff in late 2015. Over the last decade, the ZLB interest rate economy has been of intense interest to macroeconomists and financial economists alike. The study of fiscal policy at the zero lower bound is especially relevant as the central bank loses its main policy instrument which is the nominal short rate.

The expansionary impact of the fiscal policy response in the absence of monetary policy coordination has been the subject of great debate. In a well cited paper, Christiano,"
Eichenbaum, and Rebelo (2011) find that the government spending multiplier, or the dollar-to-dollar increase in GDP per dollar spent, is much larger when the nominal interest rate is at the ZLB. In our analysis here, we document that this amplifying effect of the ZLB on the real economy not only holds for government spending level shocks, but the effect is even more pronounced for government spending volatility shocks as well as capital income tax shocks. Implications of the impulse responses due to fiscal shocks at the ZLB lead to intensification of bond risk premia when the nominal short rate is at zero for a prolonged period.

To implement the ZLB analysis in our model, we follow Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) and Sims (2017) by treating the lower bound as an interest rate peg at zero. More precisely, employing news shocks in the Taylor rule of a standard DSGE model, Sims (2017) demonstrates how to solve for the news shocks that allow the interest rate to be held at a constant over various horizons. This technique lets the model produce conditional IRFs at the ZLB. Appendix E provides additional details regarding the implementation. In our analysis of the impact of fiscal level and volatility shocks at the ZLB, we perform two experiments. Assuming the interest rate is already at zero, we perturb the economy with fiscal shocks while forcing the interest to stay at the ZLB for 4 and 8 quarters. In this setting, there is no uncertainty about when departure from the zero interest rate is going to take place, which greatly reduces the complexity of the ZLB analysis. The 4-quarter peg is motivated by the evidence in Swanson and Williams (2014) that until the Fed put an explicit date for the ZLB into its communications (through fall 2011), professional forecasters expected the ZLB to bind for four quarters. The second scenario of 8 quarters instead strikes a compromise between the ex ante views of professional forecasters and the actual realization of events (which turned out to be roughly 28 quarters).

Figure 8 presents the impulse responses following a one standard deviation positive government spending level shock, panel (a), and a one standard deviation positive spending volatility shock, panel (b), conditional on the nominal short rate stays at zero for 4 quarters (long dashed line, ZLB 4Q) and 8 quarters (short-long dashes, ZLB 8Q) after the initial shock. The subplots also overlay the unconditional responses from the benchmark economy (solid line) for comparison purposes. Figure 8 panel (a) shows that following a positive level shock to government spending, output increases by more than 2% under ZLB 8Q relative
to the benchmark rise of 1%, consistent with \textcite{Christiano2011}. The increases in investment and inflation are even more pronounced: 4% and 3% respectively under ZLB 8Q compared with less than 1% for both under the benchmark. The drastic increase in output under ZLB 8Q leads to an immediate rise in real wage whereas the benchmark response shows a decline in wage. Higher wage combined with higher return on capital cause marginal cost to increase by more than 1%. On the other hand, the significant rise in investment when the lower bound is binding makes saving less attractive for consumption smoothing, thus the increase in debt is less relative to the benchmark case. Moreover, the ZLB has term structure implications for the spending level shock. The spike in the SDF following a positive spending level shock are similar across the three scenarios so we do not illustrate the impulses here\textsuperscript{31}. The fact that the spending shock generates much higher inflation when the lower bound is binding implies the covariance between the SDF and inflation is also higher at the ZLB resulting in greater inflation risk premium. Panel (a) further shows that with the nominal short rate held at zero for multiple periods after the initial shock, the long-term yield rises but not as much as the benchmark case. Whereas the level spending shock has a flattening effect on the yield curve in the benchmark case, the same shock steepens the yield curve slightly at the ZLB.

![Insert Figure 8 about here.](image)

Figure 8 panel (b) displays the impulse responses following a positive government spending volatility shock. The first striking result is that the impact of the volatility shock is greatly exacerbated when the ZLB is binding. The declines in output, investment, wages, inflation and marginal cost are orders of magnitude larger for ZLB 8Q than the benchmark. For example, with the short rate held at zero for 8 quarters following the initial shock, output drops by nearly 10% during that window, consistent with the finding of \textcite{Nakata2017}. Following an uncertainty shock at the zero lower bound, government debt is in high demand as investment falls and precautionary savings motive kicks into high gear. The 30% decline in inflation under ZLB 8Q implies a lower average inflation risk premium stemming from the spending volatility shock. Finally, the IRFs for short rate and the 5-year rate demonstrate

\textsuperscript{31}This is the case for all four fiscal shocks we examine at the ZLB. Therefore, the SDF IRFs are omitted in Figures 8 and 9.
the steepening effect of the uncertainty shock on the yield curve for ZLB 8Q, resulting in higher nominal term premium.

Next, we examine the consequences of capital return tax rate shocks at the ZLB. Figure 9 presents the IRFs for capital tax level and volatility shocks, in panels (a) and (b), respectively. Two takeaways are worth pointing out. First, the ZLB amplifies the impact of return to capital tax rate shocks on output, investment, wages, inflation and marginal cost by comparing ZLB 8Q and the benchmark in both panels. Second, unlike the government spending shocks in Figure 8, the positive level shock to the capital income tax rate is much more significant than the positive volatility shock in depressing the economy. When the nominal short rate is held at zero for prolonged period of time, a positive one standard deviation shock to the tax rate level causes output to decline by more than 10%, investment by more than 30% and inflation by almost 40% in panel (a). The corresponding declines in panel (b) are 2%, 8% and 4%. On the term structure, Figure 9 exhibits low inflation when the SDF is high for both level and volatility shocks to the capital income tax rate, suggesting inflation risk premium is more negative at the ZLB due to these shocks. Furthermore, with the nominal short rate held at zero after the shocks are realized, the 5-year yields dip in both panels (a) and (b). The flattening of the nominal yield curve following capital tax rate level and volatility shocks generates negative term premium when the ZLB is binding.

Together, Figures 8 and 9 establish the amplified impact of fiscal policy shocks on the real economy at the ZLB. Consistent with the findings of Christiano, Eichenbaum, and Rebelo (2011), a positive government spending level shock creates a substantial economic boom shifting savings from government debt to investment, which drives up inflation and inflation risk premium. On the other hand, a positive government spending volatility shock generates a severe economic downturn, pushing the demand curve for debt significantly outward when the zero interest rate is binding, in line with Nakata (2017). Since ZLB only binds temporarily and the shock is transitory, this causes long-term yields to rise as the short rate is fixed. The steepening yield curve in turn produces positive nominal term premium. Lastly, positive shocks to the level and volatility of capital income tax rate also result in
economic depressions at the ZLB, but the nominal yield curves tend to flatten following those shocks, making nominal term premia negative.

7 Conclusion

We document that our DSGE model featuring fiscal policy and policy uncertainty is successful in matching both macroeconomic and financial moments in the data. Importantly for our purpose, the model is quite successful in reproducing the average 5-year term premium, as well as its dynamic properties as captured by the autocorrelation function. Stochastic volatility in government spending allows to capture up to 70% of the overall term premium variability, whereas a model with no stochastic volatility would account for at most 13% of the term premium volatility.

We also show that the relationship between consumption and inflation depends critically on the nature of the underlying fiscal shocks: government spending level shocks imply a negative correlation between consumption and inflation, while government spending uncertainty shocks imply exactly the opposite relationship. Since the empirical relation between consumption and inflation was large and negative in the 1970s and early 1980s, but much smaller in the 1990s and 2000s (see Piazzesi and Schneider (2007) and Benigno (2007)), our finding suggests that the relative importance of transitory technology and government level shocks may have been larger in the 1970s and early 1980s than over the rest of the sample where monetary and government spending uncertainty shocks may have become dominant.

Finally, our analysis at the zero lower bound (ZLB) of the nominal interest rate reveals the following three points. First, effects of fiscal shocks on macroeconomic variables are amplified when the ZLB is binding. Second, this amplification is particularly sharp for government spending volatility shocks and capital income tax rate level shocks. Third, bond risk premia implications due to fiscal shocks remain substantial at the ZLB.

In all, we view our estimated DSGE model as an important step forward to understand what state variables drive variation over time in bond risk premia. Our finding speak to the key role played by shocks to the level and the uncertainty about fiscal policy.
References


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The figure displays the 95 percent posterior probability intervals of the smoothed fiscal volatility shock to policy instruments, $100\exp(\sigma_{x,t})$, over the sample. The panel shows by how many percentage points a one-standard-deviation innovation to the fiscal shock would have moved the government spending (Panel A) and the capital income tax rate (Panel B) at different moments.
Autocorrelation function of the observable variables in the baseline model and the data. The black line is the data. The blue line is the model’s median and the dashed lines are the model’s 5th and 95th percentiles. The sample period for the data runs from 1970.Q1 to 2014.Q2.
This figure plots the impulse responses of the stochastic discount factor, inflation, long-term bond yields and the term premium to positive one standard deviation shocks to government spending level ($g_t$), government spending volatility ($\sigma_{g,t}$), capital income tax level ($\tau^k$) and capital income tax volatility ($\sigma_{\tau^k,t}$).
Figure 4: Impulse Responses to Structural Shocks

This figure plots the impulse responses of stochastic discount factor, inflation, long-term bond yields and the term premium to positive one standard deviation shocks to transitory productivity level ($z_t$) and volatility ($\sigma^2_t$), to permanent productivity ($\Delta a_t$) and to monetary policy ($u_t$).
Figure 5: IRFs to Government Spending Shock

(a) Government spending level \((g_t)\).

(b) Government spending volatility \((\sigma_{g,t})\).

This figure plots the impulse responses to a one standard deviation shock to government spending level \((g_t)\) and volatility \((\sigma_{g,t})\).
Figure 6: IRFs to Capital Income Tax Shock

This figure plots the impulse responses to a one standard deviation shock to capital income tax level ($\tau^k$) and volatility ($\sigma_{\tau^k,t}$).
Panel A plots the model-implied term premium against the actual term premium for the period from 1970.Q1 to 2014.Q4. The solid blue line is the median, while the dashed lines are the 5th and 95th percentiles. The correlation between the data and the model-implied term premium is 0.50 in the left panel and 0.54 in the right panel. Panel B plots the model-implied nominal and real term premium as well as the inflation risk premium. The green line is the difference between the median nominal and median real term premium.
This figure plots the impulse responses to a one standard deviation shock to government spending level \((g_t)\) and volatility \((\sigma_{g,t})\). The solid lines show the responses under the benchmark case (zero lower bound not binding), the dashed lines under a four period peg, and the dashed-dotted lines under an eight period peg.

(a) Government spending level \((g_t)\).

(b) Government spending volatility \((\sigma_{g,t})\).
This figure plots the impulse responses to a one standard deviation shock to capital income tax level ($\tau^k$) and volatility ($\sigma_{\tau^k}$). The solid lines show the responses under the benchmark case (zero lower bound not binding), the dashed lines under a four period peg, and the dashed-dotted lines under an eight period peg.
Tables

Table 1: **Bond yields and Fiscal Policy:** Quarterly time-series regressions. The dependent variable is the slope of the yield curve as measured by the difference between the 5-year and the 1-year rates, $y^{(20)}_t - y^{(4)}_t$ (Panel A), or the 10-year and the 1-year rates, $y^{(40)}_t - y^{(4)}_t$ (Panel B). The independent variable are the maturity-weighted-debt-to-GDP ratio (MWD/GDP, see Greenwood-Vayanos, 2014), the level of government spending ($g_t$) and capital tax rate ($\tau_k$), and the filtered volatilities of government spending ($\sigma_{g,t}$) and capital income tax ($\sigma_{\tau_k,t}$) series. The $t$ -statistics, reported in parentheses, is based on Newey-West standard errors, with 12 lags. The coefficient on MWD/GDP is multiplied by 100.

**Panel A:** $y^{(20)}_t - y^{(4)}_t = \beta_0 + \beta_1 g_t + \beta_2 \sigma_{g,t} + \beta_3 \tau_k + \beta_4 \sigma_{\tau_k,t} + c \text{ MWD/GDP}_t + \epsilon_{t+k}$

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**Panel B:** $y^{(40)}_t - y^{(4)}_t = \beta_0 + \beta_1 g_t + \beta_2 \sigma_{g,t} + \beta_3 \tau_k + \beta_4 \sigma_{\tau_k,t} + c \text{ MWD/GDP}_t + \epsilon_{t+k}$

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Table 2: Bond returns and Fiscal Policy: Quarterly time-series regression. The dependent variable is the one-year, three-year, or five-year excess return of the $\tau$-year bond. The independent variable are the maturity-weighted-debt-to-GDP ratio (MWD/GDP, see Greenwood-Vayanos, 2014), the level of government spending ($g_t$) and capital tax rate ($\tau^k$), and the filtered volatilities of government spending ($\sigma_{g,t}$) and capital income tax ($\sigma_{\tau^k,t}$) series. The $t$-statistics, reported in parentheses, is based on Newey-West standard errors with 6 lags. The coefficient on MWD/GDP is multiplied by 100.

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<td>Govt Sp Vol.</td>
</tr>
<tr>
<td>$t$-Statistics</td>
</tr>
<tr>
<td>Capital Tax Lev.</td>
</tr>
<tr>
<td>$t$-Statistics</td>
</tr>
<tr>
<td>Capital Tax Vol.</td>
</tr>
<tr>
<td>$t$-Statistics</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
</tbody>
</table>
Table 3: Calibrated and Estimated Parameters: This table reports the parameter values for the baseline model.

<table>
<thead>
<tr>
<th>Panel A: Calibrated Parameters</th>
<th>Panel B: Separately Estimated Parameters</th>
<th>Panel C: Estimated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coefficient Description</strong></td>
<td><strong>Value</strong></td>
<td><strong>Coefficient Description</strong></td>
</tr>
<tr>
<td>$\delta$</td>
<td>capital depreciation</td>
<td>0.02</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>capital share of production</td>
<td>0.33</td>
</tr>
<tr>
<td>$\omega$</td>
<td>share of firms with rigid prices</td>
<td>0.65</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>steady-state of government spending</td>
<td>-0.85</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>volatility of government spending volatility</td>
<td>0.31</td>
</tr>
<tr>
<td>$\eta$</td>
<td>markup parameter</td>
<td>6.00</td>
</tr>
<tr>
<td>$\mu$</td>
<td>interest-rate smoothing coefficient</td>
<td>0.35</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Taylor rule coefficient on inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>autocorrelation of capital tax level</td>
<td>0.98</td>
</tr>
<tr>
<td>$\theta_{\tau}$</td>
<td>steady-state government spending level</td>
<td>0.20</td>
</tr>
<tr>
<td>$\phi_{\tau}$</td>
<td>autocorrelation of capital tax volatility</td>
<td>0.78</td>
</tr>
<tr>
<td>$\phi_{\sigma}$</td>
<td>steady-state of capital tax volatility</td>
<td>-1.84</td>
</tr>
<tr>
<td>$\phi_{\epsilon}$</td>
<td>volatility of capital tax volatility</td>
<td>0.54</td>
</tr>
<tr>
<td>$\phi_{\omega}$</td>
<td>autocorrelation of transitory productivity level</td>
<td>0.99</td>
</tr>
<tr>
<td>$\phi_{\eta}$</td>
<td>volatility of transitory productivity volatility</td>
<td>0.03</td>
</tr>
<tr>
<td>$\phi_{\theta}$</td>
<td>volatility of transitory productivity volatility</td>
<td>4.82</td>
</tr>
<tr>
<td>$\phi_{\sigma}$</td>
<td>volatility of transitory productivity volatility</td>
<td>0.03</td>
</tr>
<tr>
<td>$\phi_{\mu}$</td>
<td>volatility of monetary policy shock</td>
<td>0.004</td>
</tr>
<tr>
<td>$\phi_{\tau}$</td>
<td>volatility of permanent productivity shock</td>
<td>0.004</td>
</tr>
<tr>
<td>$\phi_{\omega}$</td>
<td>volatility of permanent productivity shock</td>
<td>0.004</td>
</tr>
</tbody>
</table>
Table 4: **Empirical and Model-Based Unconditional Moments**: This table reports the mean, standard deviations and correlations for observable variables in the baseline model. The sample period for the data is 1970.Q1 to 2014.Q2. All data, except nominal interest rates, term premium and inflation, are in logs, HP-filtered, and multiplied by 100 to express them in percentage deviation from trend. In Panel B, interest rates and the term premia are expressed at an annual rate. The slope is proxied by the spread between the ten-year and one-quarter rates.

### Panel A: Macro Moments

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SD</td>
<td>AR(1)</td>
</tr>
<tr>
<td>Output</td>
<td>1.49</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>[1.44; 1.58]</td>
<td>[0.71; 0.72]</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.23</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>[1.16; 1.39]</td>
<td>[0.70; 0.70]</td>
</tr>
<tr>
<td>Investment</td>
<td>7.01</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>[5.69; 9.21]</td>
<td>[0.70; 0.71]</td>
</tr>
<tr>
<td>Wages</td>
<td>1.16</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>[1.14; 1.21]</td>
<td>[0.90; 0.90]</td>
</tr>
<tr>
<td>Hours</td>
<td>1.49</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>[1.37; 1.67]</td>
<td>[0.72; 0.73]</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.69</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>[0.61; 0.80]</td>
<td>[0.92; 0.93]</td>
</tr>
</tbody>
</table>

### Panel B: Finance Moments

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Nominal Rate 1Q</td>
<td>5.62</td>
<td>4.09</td>
</tr>
<tr>
<td></td>
<td>[5.00; 6.59]</td>
<td>[3.60; 4.80]</td>
</tr>
<tr>
<td>Nominal Rate 3Y</td>
<td>6.39</td>
<td>3.22</td>
</tr>
<tr>
<td></td>
<td>[5.98; 7.04]</td>
<td>[2.80; 3.88]</td>
</tr>
<tr>
<td>Nominal Rate 5Y</td>
<td>6.53</td>
<td>2.85</td>
</tr>
<tr>
<td></td>
<td>[6.14; 7.09]</td>
<td>[2.44; 3.52]</td>
</tr>
<tr>
<td>Nominal Rate 7Y</td>
<td>6.65</td>
<td>2.54</td>
</tr>
<tr>
<td></td>
<td>[6.31; 7.06]</td>
<td>[2.20; 3.16]</td>
</tr>
<tr>
<td>Nominal Rate 10Y</td>
<td>6.84</td>
<td>2.18</td>
</tr>
<tr>
<td></td>
<td>[6.35; 7.12]</td>
<td>[1.88; 2.80]</td>
</tr>
<tr>
<td>Slope</td>
<td>1.23</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td>[0.37; 1.63]</td>
<td>[2.00; 2.36]</td>
</tr>
<tr>
<td>Term Premium 5Y</td>
<td>0.86</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>[0.38; 1.10]</td>
<td>[0.32; 1.01]</td>
</tr>
<tr>
<td>Term Premium 10Y</td>
<td>1.16</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>[0.30; 1.56]</td>
<td>[0.40; 1.28]</td>
</tr>
</tbody>
</table>
Table 5: **Real term structure of interest rates**: This table presents the mean, standard deviation, and first autocorrelation of the two-year (RY8), three-year (RY12), five-year (RY20), seven-year (RY28), and ten-year (RY40) real yields, and the 10-year and two-year spread from the model and the data. Interest rates are expressed at an annual rate.

<table>
<thead>
<tr>
<th></th>
<th>Slope</th>
<th>RY8</th>
<th>RY12</th>
<th>RY20</th>
<th>RY28</th>
<th>RY40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>mean:</td>
<td>0.48</td>
<td>3.42</td>
<td>3.69</td>
<td>3.78</td>
<td>3.83</td>
</tr>
<tr>
<td></td>
<td>std:</td>
<td>0.75</td>
<td>1.48</td>
<td>1.22</td>
<td>1.07</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>AC1:</td>
<td>0.91</td>
<td>0.95</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Data:1971:3 - 2007:4

<table>
<thead>
<tr>
<th></th>
<th>Slope</th>
<th>RY8</th>
<th>RY12</th>
<th>RY20</th>
<th>RY28</th>
<th>RY40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data:</td>
<td>mean:</td>
<td>0.47</td>
<td>2.33</td>
<td>2.41</td>
<td>2.56</td>
<td>2.67</td>
</tr>
<tr>
<td></td>
<td>std:</td>
<td>0.75</td>
<td>1.51</td>
<td>1.36</td>
<td>1.17</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>AC1:</td>
<td>0.76</td>
<td>0.89</td>
<td>0.91</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Data:1971:3 - 2014:2

<table>
<thead>
<tr>
<th></th>
<th>Slope</th>
<th>RY8</th>
<th>RY12</th>
<th>RY20</th>
<th>RY28</th>
<th>RY40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean:</td>
<td>0.62</td>
<td>1.87</td>
<td>1.96</td>
<td>2.19</td>
<td>2.33</td>
</tr>
<tr>
<td></td>
<td>std:</td>
<td>0.84</td>
<td>1.80</td>
<td>1.67</td>
<td>1.43</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td>AC1:</td>
<td>0.76</td>
<td>0.89</td>
<td>0.91</td>
<td>0.93</td>
<td>0.94</td>
</tr>
</tbody>
</table>
Table 6: **Variance Decomposition - The Effect of Structural Shocks**: This table reports the variance decomposition for the different structural shocks in the baseline model. $A$ and $Z$ stand for permanent and transitory productivity, respectively. $G$ stands for government spending. In Panel B, the one-quarter, three-year, five-year, seven-year, ten-year nominal yields, the slope (ten-year and one-quarter spread), and the term premia are expressed at an annual rate.

### Panel A: Macro Moments

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Wages</th>
<th>Hours</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Shocks</td>
<td>1.49</td>
<td>1.23</td>
<td>7.01</td>
<td>1.10</td>
<td>1.49</td>
<td>0.69</td>
</tr>
<tr>
<td>Only A</td>
<td>0.51</td>
<td>0.48</td>
<td>0.69</td>
<td>0.31</td>
<td>0.24</td>
<td>0.04</td>
</tr>
<tr>
<td>Only Monetary</td>
<td>0.15</td>
<td>0.10</td>
<td>0.38</td>
<td>0.04</td>
<td>0.22</td>
<td>0.03</td>
</tr>
<tr>
<td>Only Z Level</td>
<td>0.96</td>
<td>0.80</td>
<td>1.75</td>
<td>1.01</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>Only Z Uncertainty</td>
<td>0.10</td>
<td>0.04</td>
<td>0.50</td>
<td>0.03</td>
<td>0.15</td>
<td>0.29</td>
</tr>
<tr>
<td>Only G Level</td>
<td>0.45</td>
<td>0.30</td>
<td>0.34</td>
<td>0.19</td>
<td>0.68</td>
<td>0.07</td>
</tr>
<tr>
<td>Only G Uncertainty</td>
<td>0.12</td>
<td>0.15</td>
<td>1.41</td>
<td>0.13</td>
<td>0.16</td>
<td>0.21</td>
</tr>
<tr>
<td>Only Tax Level</td>
<td>0.44</td>
<td>0.20</td>
<td>3.60</td>
<td>0.17</td>
<td>0.65</td>
<td>0.21</td>
</tr>
<tr>
<td>Only Tax Uncertainty</td>
<td>0.12</td>
<td>0.23</td>
<td>2.01</td>
<td>0.11</td>
<td>0.17</td>
<td>0.08</td>
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</tbody>
</table>

### Panel B: Finance Moments

<table>
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<tr>
<th></th>
<th>1Q</th>
<th>3Y</th>
<th>5Y</th>
<th>7Y</th>
<th>10Y</th>
<th>10Y-1Q</th>
<th>5Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Shocks</td>
<td>4.09</td>
<td>3.22</td>
<td>2.85</td>
<td>2.54</td>
<td>2.18</td>
<td>2.17</td>
<td>0.61</td>
<td>0.72</td>
</tr>
<tr>
<td>Only A</td>
<td>0.20</td>
<td>0.18</td>
<td>0.16</td>
<td>0.15</td>
<td>0.13</td>
<td>0.14</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Only Monetary</td>
<td>0.53</td>
<td>0.06</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>0.51</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Only Z Level</td>
<td>1.12</td>
<td>0.88</td>
<td>0.84</td>
<td>0.80</td>
<td>0.75</td>
<td>0.51</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Only Z Uncertainty</td>
<td>1.75</td>
<td>1.49</td>
<td>1.37</td>
<td>1.26</td>
<td>1.10</td>
<td>0.65</td>
<td>0.29</td>
<td>0.44</td>
</tr>
<tr>
<td>Only G Level</td>
<td>0.42</td>
<td>0.32</td>
<td>0.30</td>
<td>0.28</td>
<td>0.26</td>
<td>0.20</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Only G Uncertainty</td>
<td>1.22</td>
<td>0.69</td>
<td>0.47</td>
<td>0.33</td>
<td>0.21</td>
<td>1.05</td>
<td>0.34</td>
<td>0.37</td>
</tr>
<tr>
<td>Only Tax Level</td>
<td>1.23</td>
<td>0.97</td>
<td>0.84</td>
<td>0.73</td>
<td>0.60</td>
<td>0.66</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>Only Tax Uncertainty</td>
<td>0.45</td>
<td>0.48</td>
<td>0.46</td>
<td>0.42</td>
<td>0.36</td>
<td>0.11</td>
<td>0.37</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Table 7: Bond returns and Fiscal Policy: Model-implied regression. The dependent variable is the one-year, three-year, or five-year excess return of the $\tau$-year bond. The independent variable are the maturing debt level $D_{t-1}(t)$, the level of government spending and capital tax rate, and the filtered volatilities of government spending and capital tax rate series. The $t$-statistics, reported in parentheses, is based on Newey-West standard errors with 6 lags.

**Panel A:** $r_{x}^{(\tau)}_{t+k,k} = \beta_0 + c \frac{MWD}{GDP}_t + \epsilon_{t+k}$

<table>
<thead>
<tr>
<th>$\frac{MWD}{GDP}$</th>
<th>1-yr, 2-yr bond</th>
<th>1-yr, 3-yr bond</th>
<th>1-yr, 4-yr bond</th>
<th>1-yr, 5-yr bond</th>
<th>1-yr, 10-yr bond</th>
<th>3-yr, 10-yr bond</th>
<th>5-yr, 10-yr bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.97</td>
<td>3.56</td>
<td>4.87</td>
<td>5.97</td>
<td>9.06</td>
<td>9.23</td>
<td>5.40</td>
<td></td>
</tr>
<tr>
<td>(6.95)</td>
<td>(7.00)</td>
<td>(6.98)</td>
<td>(6.94)</td>
<td>(6.68)</td>
<td>(5.33)</td>
<td>(2.97)</td>
<td></td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Panel B:** $r_{x}^{(\tau)}_{t+k,k} = \beta_0 + \beta_1 g_t + \beta_2 \sigma_{g,t} + c \frac{MWD}{GDP}_t + \epsilon_{t+k}$

<table>
<thead>
<tr>
<th>$\frac{MWD}{GDP}$</th>
<th>1-yr, 2-yr bond</th>
<th>1-yr, 3-yr bond</th>
<th>1-yr, 4-yr bond</th>
<th>1-yr, 5-yr bond</th>
<th>1-yr, 10-yr bond</th>
<th>3-yr, 10-yr bond</th>
<th>5-yr, 10-yr bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.96</td>
<td>3.56</td>
<td>4.88</td>
<td>5.97</td>
<td>9.02</td>
<td>9.31</td>
<td>5.65</td>
<td></td>
</tr>
<tr>
<td>(7.01)</td>
<td>(7.07)</td>
<td>(7.05)</td>
<td>(7.01)</td>
<td>(6.73)</td>
<td>(5.68)</td>
<td>(3.32)</td>
<td></td>
</tr>
<tr>
<td>Govnt Sp Lev.</td>
<td>0.33</td>
<td>0.50</td>
<td>0.65</td>
<td>0.80</td>
<td>1.63</td>
<td>2.33</td>
<td>1.23</td>
</tr>
<tr>
<td>(2.55)</td>
<td>(2.16)</td>
<td>(2.06)</td>
<td>(2.07)</td>
<td>(2.62)</td>
<td>(2.53)</td>
<td>(1.35)</td>
<td></td>
</tr>
<tr>
<td>Govnt Sp Vol.</td>
<td>0.13</td>
<td>0.22</td>
<td>0.29</td>
<td>0.36</td>
<td>0.63</td>
<td>1.30</td>
<td>1.25</td>
</tr>
<tr>
<td>(6.05)</td>
<td>(5.88)</td>
<td>(5.84)</td>
<td>(5.84)</td>
<td>(5.94)</td>
<td>(5.75)</td>
<td>(5.76)</td>
<td></td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
<td>0.06</td>
</tr>
</tbody>
</table>

**Panel C:** $r_{x}^{(\tau)}_{t+k,k} = \beta_0 + \beta_1 g_t + \beta_2 \sigma_{g,t} + \beta_3 \tau_k^t + \beta_4 \sigma_{\tau,k,t} + c \frac{MWD}{GDP}_t + \epsilon_{t+k}$

<table>
<thead>
<tr>
<th>$\frac{MWD}{GDP}$</th>
<th>1-yr, 2-yr bond</th>
<th>1-yr, 3-yr bond</th>
<th>1-yr, 4-yr bond</th>
<th>1-yr, 5-yr bond</th>
<th>1-yr, 10-yr bond</th>
<th>3-yr, 10-yr bond</th>
<th>5-yr, 10-yr bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.92</td>
<td>3.47</td>
<td>4.75</td>
<td>5.81</td>
<td>8.81</td>
<td>8.84</td>
<td>5.04</td>
<td></td>
</tr>
<tr>
<td>(7.18)</td>
<td>(7.22)</td>
<td>(7.20)</td>
<td>(7.17)</td>
<td>(6.94)</td>
<td>(5.75)</td>
<td>(3.06)</td>
<td></td>
</tr>
<tr>
<td>Govnt Sp Lev.</td>
<td>0.34</td>
<td>0.51</td>
<td>0.67</td>
<td>0.83</td>
<td>1.68</td>
<td>2.41</td>
<td>1.32</td>
</tr>
<tr>
<td>(2.67)</td>
<td>(2.28)</td>
<td>(2.18)</td>
<td>(2.20)</td>
<td>(2.75)</td>
<td>(2.74)</td>
<td>(1.50)</td>
<td></td>
</tr>
<tr>
<td>Govnt Sp Vol.</td>
<td>0.13</td>
<td>0.22</td>
<td>0.29</td>
<td>0.36</td>
<td>0.63</td>
<td>1.30</td>
<td>1.25</td>
</tr>
<tr>
<td>(6.10)</td>
<td>(5.92)</td>
<td>(5.87)</td>
<td>(5.86)</td>
<td>(5.97)</td>
<td>(5.78)</td>
<td>(5.82)</td>
<td></td>
</tr>
<tr>
<td>Capital Tax Lev.</td>
<td>-0.01</td>
<td>-0.04</td>
<td>-0.06</td>
<td>-0.06</td>
<td>0.02</td>
<td>-0.36</td>
<td>-0.96</td>
</tr>
<tr>
<td>(0.11)</td>
<td>(-0.26)</td>
<td>(-0.27)</td>
<td>(-0.24)</td>
<td>(0.05)</td>
<td>(-0.54)</td>
<td>(-1.34)</td>
<td></td>
</tr>
<tr>
<td>Capital Tax Vol.</td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.04</td>
<td>-0.05</td>
<td>-0.08</td>
<td>-0.12</td>
<td>-0.10</td>
</tr>
<tr>
<td>(2.58)</td>
<td>(-2.57)</td>
<td>(-2.58)</td>
<td>(-2.59)</td>
<td>(-2.65)</td>
<td>(-3.36)</td>
<td>(-3.59)</td>
<td></td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.09</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.11</td>
<td>0.09</td>
<td></td>
</tr>
</tbody>
</table>
A Data

We follow Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) and construct the macroeconomic observable variables used in the estimation as:

1. Output is real GDP (GDPC1).
2. Consumption is real personal consumption expenditures (PCECC96).
3. Investment is real gross private domestic investment (GPDIC96).
5. Real Per Capita GDP = (1) / (4).
6. Real Per Capita Consumption = (2) / (4).
7. Real Per Capita Investment = (3) / (4).
8. Inflation is GDP deflator (GDPDEF).
9. Hourly real wage is compensation per hour in the business sector (HCOMPBS) divided by the GDP deflator (GDPDEF).
10. Hours per capita are measured by hours of all persons in the business sector (HOABS).

Data for the period 1970:Q1–2014:Q2 are taken from the St. Louis Fed’s FRED database (mnemonics are in parentheses).

Government spending and capital tax rates data are from Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015). In particular, the tax data are constructed from national income and product accounts (NIPA) as in Leeper, Plante, and Traum (2010) (see also Appendix B in Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) for details). Government spending is government consumption and gross investment, both from NIPA.

With regard to the financial variables, the Treasury yield data are from Gurkaynak, Sack, and Wright (2007) (data are available for download on the website http://www.federalreserve.gov/Pubs/feds/2006/200628/feds200628.xls) and the series for the 10-year Term premia is from Adrian, Crump, and Moench (2013) (data available at https://www.newyorkfed.org/research/data_indicators/term_premia.html). We thank the authors for making these data available for download.


B Predictive regressions: Robustness

We perform a number of robustness tests. Table 8 shows that our results for the (level and volatility of) government spending and capital tax rate series remain significant after controlling for the one-year yield (Gertler and Karadi (2015) suggest to take the one-year government bond rate as the relevant monetary policy indicator, rather than the federal funds rate), and for trend inflation (see Kozicki and Tinsley (2001) show that highly persistent expected inflation dynamics determines the level of interest rates in the long run and across maturities; see also Cieslak and Povala (2015)).

Table 8: Quarterly time-series regression for bond returns. The dependent variable is the one-year, three-year, or five-year return of the τ-year bond. The independent variable is the filtered government spending volatility series. The regressions control for the MWD/GDP (see Greenwood-Vayanos, 2014), the maturity-weighted-debt-to-GDP ratio and for the one-year yield (Panel A) and an inflation trend (Panel B). The t-statistics, reported in parentheses, is based on Newey-West standard errors with 6 lags. The coefficient on MWD/GDP is multiplied by 100.

<table>
<thead>
<tr>
<th></th>
<th>1-yr, 2-yr bond</th>
<th>3-yr bond</th>
<th>1-yr, 4-yr bond</th>
<th>1-yr, 5-yr bond</th>
<th>1-yr, 10-yr bond</th>
<th>3-yr, 10-yr bond</th>
<th>5-yr, 10-yr bond</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MWD/GDP</strong></td>
<td>0.72</td>
<td>0.69</td>
<td>0.83</td>
<td>0.90</td>
<td>0.80</td>
<td>2.35</td>
<td>2.45</td>
</tr>
<tr>
<td><strong>Govt Sp Lev.</strong></td>
<td>(3.46)</td>
<td>(3.39)</td>
<td>(3.41)</td>
<td>(3.47)</td>
<td>(3.77)</td>
<td>(5.68)</td>
<td>(6.26)</td>
</tr>
<tr>
<td><strong>Govt Sp Vol.</strong></td>
<td>0.30</td>
<td>0.45</td>
<td>0.54</td>
<td>0.54</td>
<td>0.62</td>
<td>1.09</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>Capital Tax Lev.</strong></td>
<td>-0.13</td>
<td>-0.32</td>
<td>-0.51</td>
<td>-0.71</td>
<td>-0.60</td>
<td>-0.45</td>
<td>-0.55</td>
</tr>
<tr>
<td><strong>Capital Tax Vol.</strong></td>
<td>-1.97</td>
<td>-2.90</td>
<td>-4.80</td>
<td>-1.65</td>
<td>-2.09</td>
<td>-1.76</td>
<td>-1.54</td>
</tr>
<tr>
<td><strong>Infl. Trend</strong></td>
<td>0.01</td>
<td>-0.11</td>
<td>-0.27</td>
<td>-0.44</td>
<td>-1.19</td>
<td>0.01</td>
<td>0.16</td>
</tr>
<tr>
<td><strong>R^2</strong></td>
<td>0.23</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.48</td>
<td>0.54</td>
</tr>
</tbody>
</table>
Tables 1, 2, and 9 use the business cycle component of government spending and capital tax rates. The components are obtained using the one-sided filter of Ortu, Tamoni, and Tebaldi (2013). The filter amounts to remove from the $g_t$ series an 8-year (equally weighted) moving average based on past observation. Table 9 shows an alternative interpretation of this result. We focus on government spending for ease of exposition. Panel A in Table 9 shows regressions of bond returns on the business cycle component of government spending and its volatility, after controlling for the maturity-weighted debt to GDP, and a time trend. Panel B shows the results when we replace the business cycle component of government spending with the raw series. Despite the time trend being strongly statistically significant in Panel B, the two panels depict the same picture, with $R^2$ that are almost identical for maturities 3- to 10-years.

Table 9: Quarterly time-series regression for bond returns. The dependent variable is the one-year, three-year, or five-year return of the τ-year bond. The independent variable are the government spending level and the filtered government spending volatility series. The regressions control for the MWD/GDP (see Greenwood-Vayanos, 2014), the maturity-weighted-debt-to-GDP ratio and for a time trend. The $t$-statistics, reported in parentheses, is based on Newey-West standard errors with 6 lags. The coefficients on MWD/GDP and the time trend are multiplied by 100.

Panel A: $r^{(τ)}_{τ+k} = \beta_0 + \beta_1 g^{BC}_t + \beta_2 \sigma_{t+k} + c \text{ MWD/GDP}_t + d \text{ Time trend} + \epsilon_{t+k}$

<table>
<thead>
<tr>
<th></th>
<th>1-yr, 2-yr bond</th>
<th>1-yr, 3-yr bond</th>
<th>1-yr, 4-yr bond</th>
<th>1-yr, 5-yr bond</th>
<th>1-yr, 10-yr bond</th>
<th>3-yr, 10-yr bond</th>
<th>5-yr, 10-yr bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWD/GDP</td>
<td>0.65</td>
<td>1.11</td>
<td>1.49</td>
<td>1.84</td>
<td>3.37</td>
<td>2.67</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>(2.68)</td>
<td>(2.53)</td>
<td>(2.54)</td>
<td>(2.60)</td>
<td>(2.90)</td>
<td>(3.04)</td>
<td>(3.12)</td>
</tr>
<tr>
<td>Govt Sp Lev.</td>
<td>0.72</td>
<td>1.25</td>
<td>1.67</td>
<td>2.01</td>
<td>3.32</td>
<td>2.00</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>(3.06)</td>
<td>(2.82)</td>
<td>(2.66)</td>
<td>(2.53)</td>
<td>(2.12)</td>
<td>(2.82)</td>
<td>(2.83)</td>
</tr>
<tr>
<td>Govt Sp Vol.</td>
<td>0.36</td>
<td>0.60</td>
<td>0.80</td>
<td>0.97</td>
<td>1.88</td>
<td>0.99</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(3.30)</td>
<td>(3.20)</td>
<td>(3.18)</td>
<td>(3.20)</td>
<td>(3.24)</td>
<td>(2.54)</td>
<td>(1.60)</td>
</tr>
<tr>
<td>Time Trend</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(-.41)</td>
<td>(-.21)</td>
<td>(-.08)</td>
<td>(0.04)</td>
<td>(0.39)</td>
<td>(1.03)</td>
<td>(2.05)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.22</td>
<td>0.19</td>
<td>0.18</td>
<td>0.18</td>
<td>0.17</td>
<td>0.48</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Panel B: $r^{(τ)}_{τ+k} = \beta_0 + \beta_1 g_t + \beta_2 \sigma_{t} + c \text{ MWD/GDP}_t + d \text{ Time trend} + \epsilon_{t+k}$

<table>
<thead>
<tr>
<th></th>
<th>1-yr, 2-yr bond</th>
<th>1-yr, 3-yr bond</th>
<th>1-yr, 4-yr bond</th>
<th>1-yr, 5-yr bond</th>
<th>1-yr, 10-yr bond</th>
<th>3-yr, 10-yr bond</th>
<th>5-yr, 10-yr bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWD/GDP</td>
<td>0.55</td>
<td>0.94</td>
<td>1.27</td>
<td>1.58</td>
<td>2.93</td>
<td>1.77</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>(2.12)</td>
<td>(2.01)</td>
<td>(2.03)</td>
<td>(2.09)</td>
<td>(2.45)</td>
<td>(2.65)</td>
<td>(3.26)</td>
</tr>
<tr>
<td>Govt Sp Lev.</td>
<td>0.45</td>
<td>0.81</td>
<td>1.12</td>
<td>1.37</td>
<td>2.35</td>
<td>1.10</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(1.71)</td>
<td>(1.79)</td>
<td>(1.85)</td>
<td>(1.87)</td>
<td>(1.80)</td>
<td>(1.45)</td>
<td>(1.05)</td>
</tr>
<tr>
<td>Govt Sp Vol.</td>
<td>0.35</td>
<td>0.58</td>
<td>0.77</td>
<td>0.94</td>
<td>1.82</td>
<td>0.97</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(2.77)</td>
<td>(2.68)</td>
<td>(2.68)</td>
<td>(2.71)</td>
<td>(2.85)</td>
<td>(2.13)</td>
<td>(1.30)</td>
</tr>
<tr>
<td>Time Trend</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>0.06</td>
<td>0.11</td>
<td>0.07</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(1.52)</td>
<td>(1.66)</td>
<td>(1.80)</td>
<td>(1.91)</td>
<td>(2.27)</td>
<td>(2.12)</td>
<td>(0.78)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.16</td>
<td>0.15</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.40</td>
<td>0.39</td>
</tr>
</tbody>
</table>
The understand this result it is useful to think of the time trend in Panel B as a filtering device on its own, trying to remove the decadal trend in the raw government spending series. This is easily seen in Figure 10 where we show the government spending series, its business cycle component, and the series \( g_t + 0.03 \times \text{Time Trend} \). The correlation between the two filtered series is about 85%. Indeed, conclusions would be unchanged had we included a time trend as a regressor in Tables 1, 2, and 9, and used the raw series of government spending.

**Figure 10: Government Spending and the Business Cycle**

The figure displays the government spending series (solid, blue line), the business cycle component obtained using the decomposition of Ortu et al. (2013) (red, dashed line), and the detrended government spending series implied by the regressions in Panel B, Table 8 (green line with circles).
C Solving the Benchmark Model

C.1 Households with Epstein-Zin Preference

The savers’ optimization problem is:

$$\max \ V(C_t, N_t) = \left\{ (1 - \beta)U(C_t, N_t)^{1-\psi} + \beta E_t \left[ V_{t+1}^{\gamma} \right] \right\}^{\frac{1}{1-\gamma}}$$

s.t. \( E_t \left[ \sum_{s=0}^{\infty} M_{t,t+s}^s P_{t+s} N_{t+s} \right] \leq E_t \left[ \sum_{s=0}^{\infty} M_{t,t+s}^s (W_{t+s} P_{t+s} N_{t+s} - P_{t+s} T_{t+s} + P_{t+s} \Psi_{t+s}) \right] \),

where

$$C_t = \left[ \int_0^1 C_t(j) \frac{s-\psi}{\gamma} dj \right]^{\frac{1}{\gamma-1}}$$

and

$$U(C_t, N_t) = \left[ \frac{C_t^{1-\psi}}{1-\psi} - A_t^{1-\psi} N_t^{1+\omega} \right]^{\frac{1}{1-\psi}}.$$ 

The first order conditions are:

$$\frac{\partial V_t}{\partial C_t} = \frac{1}{1-\psi} [V_t^{1-\psi}]^{\frac{1}{1-\psi}-1} (1 - \beta) C_t^{\psi} - \lambda M_{t,t}^s P_t = 0 \quad (12)$$

$$\frac{\partial V_t}{\partial N_t} = \frac{1}{1-\psi} [V_t^{1-\psi}]^{\frac{1}{1-\psi}-1} (1 - \beta) (-A_t^{1-\psi} N_t^{1+\omega}) + \lambda M_{t,t}^s W_t P_t = 0 \quad (13)$$

$$\frac{\partial V_t}{\partial C_{t+1}} = \frac{1}{1-\psi} [V_{t+1}^{1-\psi}]^{\frac{1}{1-\psi}-1} \beta \left( \frac{1 - \psi}{1 - \gamma} \right) E_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}-1} (1 - \gamma) V_{t+1}^{\gamma} \frac{\partial V_{t+1}}{\partial C_{t+1}} - \lambda M_{t,t+1}^s P_{t+1} = 0. \quad (14)$$

Furthermore,

$$\frac{\partial V_{t+1}}{\partial C_{t+1}} = \frac{1}{1-\psi} [V_{t+1}^{1-\psi}]^{\frac{1}{1-\psi}-1} (1 - \beta) C_{t+1}^{\psi}. \quad (15)$$

Combining (12) and (13), I have the household’s intratemporal consumption and labor supply optimality condition:

$$\frac{\lambda(1 - \psi)}{V_t^{\psi}(1 - \beta)} = \frac{C_t^{1-\psi}}{P_t} = \frac{A_t^{1-\psi} N_t^{1+\omega}}{W_t P_t} \Rightarrow W_t = A_t^{1-\psi} C_t^{\psi} N_t^{\gamma}. $$

Finally, combining (12), (14) and (15), I obtain the intertemporal consumption optimality condition:

$$\frac{\lambda(1 - \psi)}{V_t^{\psi}(1 - \beta)} = \frac{C_t^{1-\psi}}{P_t} = \beta \left( \frac{C_{t+1}^{1-\psi}}{P_{t+1}} \right) \left( \frac{V_{t+1}^{\psi-\gamma}}{M_{t,t+1}^s} \right) E_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}-\gamma}. \quad (16)$$

To get the nominal pricing kernel, I solve for \( M_{t,t+1}^s \),

$$M_{t,t+1}^s = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\psi} \left( \frac{P_{t+1}}{P_t} \right)^{-1} \left[ \frac{V_{t+1}}{E_t[V_{t+1}^{1-\gamma}]} \right]^{\psi-\gamma}. \quad (16)$$
C.2 Wage Rigidities and Optimal Wage Setting

Optimal price setting in the presence of wage stickiness is done through the following optimization problem. There is a continuum of optimizing households in the economy, indexed by \(k\). Each period, only a fraction, \(1 - \theta\), of the optimizing households has the ability to adjust wage demand optimally. The objective function is:

\[
\max_{W^*_t(k)} E_t \left[ \sum_{s=0}^{\infty} \theta^s M_{t,t+s}^s \left( I^w_{t,t+s} W^s_*(k) N_{t+s}(k) - P_{t+s} MRS_{t+s}(k) N_{t+s}(k) \right) \right] 
\]

s.t. \( N_{t+s}(k) = \left( \frac{W^s_*(k)}{W^s_t} \right)^{-\eta_w} N^d_{t+s} \)

\[
W^s_t = \left[ \int_0^1 W_t(k)^{1-\eta_w} dk \right]^{1-\eta_w} 
= \left( 1 - \theta \right) W^s_{t-1}^{1-\eta_w} + \theta(I^w_{t-1}, W^{s}_{t-1})^{1-\eta_w} \left[ \int_0^1 W_t(k)^{1-\eta_w} dk \right]^{1-\eta_w},
\]

where \(W^s_*(\cdot)\) is the optimal nominal wage chosen at time \(t\) and \(I^w_{t+s}\) is the wage index in the case when \(W^s_*(\cdot)\) is not adjusted optimally in following periods. \(\eta_w\) is the wage markup parameter. \(MRS_{t+s}(k)\) is the marginal rate of substitution between consumption and labor dis-utility. \(W^s_t\) is the prevailing nominal market-clearing wage at time \(t\), and \(N^d_{t+s}\) is the aggregate labor demand. The Calvo (1983) style staggered wage setting is standard in the macroeconomic literature.

The optimal wage demand equation is:

\[
\left[ \frac{1}{1 - \theta} \left( W_t^{1-\eta_w} - \theta \left( I^w_{t-1}, W_{t-1}^{1-\eta_w} \frac{\Pi_{t-1}}{\Pi_t} \right) \right) \right]^{1-\eta_w} H_t = \nu_w A_t^{1-\psi} C_t^{\psi} N_t^{\omega} G_t,
\]

where

\[
H_t = 1 + \theta \Pi_t \left[ M_{t,t+1}^{nom} I^w_{t+1} - \eta_w \left( N^d_{t+1} \frac{W_{t+1}}{W_t} \right) \Pi_{t+1}^{\eta_w} H_{t+1} \right]
\]

\[
G_t = 1 + \theta \Pi_{t+1}^{1+\eta_w} \left[ M_{t+1,t}^{nom} I^w_{t+1} - \eta_w \left( A_{t+1} \frac{C_{t+1}}{C_t} \right)^{1-\psi} \left( \frac{N_{t+1}}{N_t} \right)^\psi \left( \frac{N^d_{t+1}}{N^d_t} \right)^\omega \right] \times \Pi_{t+1}^{1+\eta_w} \left( \frac{W_{t+1}}{W_t} \right)^{\eta_w} G_{t+1}.
\]

In the above formulation, \(W_t\) is real wage, \(\Pi_t\) is inflation, and \(\nu_w = \frac{\eta_w}{\eta_{w-1}}\) is the wage markup. The equilibrium condition states that the optimal real wage is equal to the marginal cost of providing an extra unit of labor \((A_t^{1-\psi} C_t^{\psi} N_t^{\omega})\) multiplied by a time-varying markup \(\left( \nu_w \frac{\Pi_t}{\Pi_{t-1}} \right)\) stemming from the monopolistic behavior of the agents in the labor market.
C.3 The Investment Decision

The households rent out capital to the firms in exchange for earning the return on capital, \( R_k \). The capital accumulation equation is standard with convex quadratic adjustment cost, \( \Phi \):

\[
K_t = (1 - \delta)K_{t-1} + \Phi \left( \frac{Inv_t}{K_{t-1}} \right) K_{t-1},
\]

where \( \delta \) is the rate of capital depreciation.

The representative agent’s optimal investment strategy has to satisfy the following equation:

\[
Q^{inv}_t = \mathbb{E}_t \left[ M_{t,t+1} \left( (1 - \tau_k) R_k^{t+1} + Q^{inv}_{t+1} \left( 1 - \delta \right) + \Phi \left( \frac{Inv_{t+1}}{K_t} \right) \right) \right],
\]

where \( Q^{inv}_t \) is the shadow price of investment, and \( \Phi' \) is the first derivative of the quadratic adjustment cost function.

Similar to the standard investment first order condition from \( Q \)-theory, we derive here the intertemporal relationship of investment’s \( Q \) as a function of the return on capital, the rate of depreciation, and the marginal rate of investment adjustment cost.

C.4 Monopolistic Producers and Price Rigidities

There is a dispersion of firms, denoted by \( j \), with identical production technology in the economy. With nominal price stickiness and monopolistic competition, each firm is faced with the following optimization problem:

\[
\max_{P^*} \quad \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \alpha^s M^{k}_{t+s} \left( P^*_t (\Pi^*)^s Y_{t+s} - W_{t+s} (P_t N_{t+s}) \right) \right]
\]

s.t. \( Y_{t+s} = Z_{t+s} N_{t+s} \) \( \quad (17) \)

\[
Y_{t+s} = \left( \frac{P^*_t (\Pi^*)^s}{P_{t+s}} \right)^{-\theta} Y_{t+s} \quad \quad (18)
\]

\[
P_t = \left[ \int_0^{1} P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} = \left[ (1 - \alpha) P_t^{1-\theta} + \alpha (P_{t-1} (\Pi^*)^{1-\theta}) \right]^{\frac{1}{1-\theta}}. \quad \quad (19)
\]

Using Calvo (1983) pricing, a firm can choose to optimally adjust price to \( P^*_t \) with probability \((1 - \alpha)\) each period independent of the time elapsed between adjustments. Furthermore, \( t + s | t \) denotes the value in period \( t + s \) given that the firm last adjusted price in period \( t \). \( \Pi^* \) is the natural level of inflation that firms use to adjust their prices to from period to period if they cannot optimally set the price, and \( Z_t \) is the productivity shock on output. Log productivity is an exogenous AR(1) process such that

\[
z_{t+1} = \ln(Z_{t+1}) = \phi z_t + \sigma_z \epsilon_{z,t+1},
\]
The first order condition for firm $j$ is:

$$
E_t \left[ \sum_{s=0}^{\infty} \alpha^s M^s_{t,t+s} Y_{t+s|t(j)} \left( P^*_t (\Pi^*)^s - \nu P_{t+s} \frac{W_{t+s|t(j)}}{Z_{t+s}} \right) \right] = 0,
$$

where $\nu = \frac{\theta}{\bar{\nu}}$ is the frictionless markup in the absence of price adjustment constraint. Utilizing (19) and the fact that $W_{t+s|t(j)} = W_{t+s}$, (21) can be rewritten as:

$$
\left( \frac{P^*_t}{P_t} \right) F_t = \nu W_t Z_t J_t
$$
or after manipulating (20):

$$
\left[ \frac{1}{1 - \alpha} \left( 1 - \alpha \left( \frac{\Pi^*}{\Pi_t} \right)^{1-\theta} \right) \right]^{\frac{1}{\theta}} F_t = \nu W_t Z_t J_t.
$$

$F_t$ can be recursively expressed as:

$$
F_t = 1 + E_t \left[ \sum_{s=1}^{\infty} (\alpha \Pi^*)^s M^s_{t,t+1} M^s_{t,t+s} \frac{Y_{t+s}}{Y_{t+1}} \left( \frac{Y_{t+1}}{Y_t} \right) \left( \frac{P^*_t}{P_{t+1}} \right)^{-\theta} \left( \frac{P_{t+1} (\Pi^*)^{s-1}}{P_{t+s}} \right)^{-\theta} \right]
$$

Similarly, $J_t$ has the following recursive formulation:

$$
J_t = 1 + \alpha \Pi^* E_t \left[ M^s_{t,t+1} \frac{Y_{t+1}}{Y_t} \left( \frac{\Pi^*}{\Pi_{t+1}} \right)^{-\theta} F_{t+1} \right].
$$

**C.5 Loglinearized Phillips Curve**

To linearize $F_t$ and $J_t$, we apply Taylor series expansion to the expectation terms in the following steps for Equation (17). First, define $\Upsilon_t = \log \mathbb{E}_t \left[ e^{m_{t,t+1+\Delta \tilde{y}_{t+1} + \Delta \tilde{a}_{t+1} + (\gamma-1) \pi_{t+1} + f_{t+1}}} \right]$. Then,

$$
F_t = 1 + \alpha \mathbb{E}_t \left[ M^\text{nom}_{t,t+1} \frac{Y_{t+1}}{Y_t} \Pi^\text{nom}_{t+1} F_{t+1} \right]
$$

$$
F e^{f_t} = 1 + \alpha \Upsilon_t e^{\log \mathbb{E}_t \left[ e^{m_{t,t+1+\Delta \tilde{y}_{t+1} + \Delta \tilde{a}_{t+1} + (\gamma-1) \pi_{t+1} + f_{t+1}}} \right]}
$$

$$
f + f_t = \log(1 + \alpha \Upsilon_t e^{\Upsilon_t})
$$

$$
= \log(1 + \alpha \Upsilon_t e^{\Upsilon_t}) + \frac{\alpha \Upsilon_t e^{\Upsilon_t}}{1 + \alpha \Upsilon_t e^{\Upsilon_t}} \Upsilon_t - \Upsilon_t.
$$
Notice a variable without a time subscript implies the non-stochastic steady state of the variable. In steady state, 
\( f = \log(1 + \alpha \Upsilon e^\Upsilon) \), so

\[
f_t = \text{const}_f \Upsilon_t - \text{const}_f \Upsilon
\]

\[
= \text{const}_f \log \mathbb{E}_t \left[ e^{m_{t+1} + \Delta \hat{y}_{t+1} + \Delta a_{t+1} + (\eta - 1)\pi_{t+1} + f_{t+1}} \right] - \text{const}_f \Upsilon
\]

\[
= \text{const}_f \left\{ \mathbb{E}_t \left[ m_{t+1} + \Delta \hat{y}_{t+1} + \Delta a_{t+1} + (\eta - 1)\pi_{t+1} + f_{t+1} \right]
+ \frac{1}{2} \text{var}_t \left( m_{t+1} + \Delta \hat{y}_{t+1} + \Delta a_{t+1} + (\eta - 1)\pi_{t+1} + f_{t+1} \right) \right\}
- \text{const}_f \Upsilon,
\]

in which the last equality relies on the lognormality assumption.

For \( J_t \), define \( \Phi_t = \log \mathbb{E}_t \left[ e^{m_{t+1} - \Delta z_{t+1} + \kappa \Delta r^K_{t+1} + (1 - \kappa) \Delta \hat{w}_{t+1} + \Delta \hat{y}_{t+1} + \Delta a_{t+1} + \eta \pi_{t+1} + j_{t+1}} \right] \), then the same procedure as above gives us the loglinearized Equation (8):

\[
j_t = \text{const}_j \Phi_t - \text{const}_j \Phi
\]

\[
= \text{const}_j \log \mathbb{E}_t \left[ e^{m_{t+1} - \Delta z_{t+1} + \kappa \Delta r^K_{t+1} + (1 - \kappa) \Delta \hat{w}_{t+1} + \Delta \hat{y}_{t+1} + \Delta a_{t+1} + \eta \pi_{t+1} + j_{t+1}} \right] - \text{const}_j \Phi
\]

\[
= \text{const}_j \left\{ \mathbb{E}_t \left[ m_{t+1} - \Delta z_{t+1} + \kappa \Delta r^K_{t+1} + (1 - \kappa) \Delta \hat{w}_{t+1} + \Delta \hat{y}_{t+1} + \Delta a_{t+1} + \eta \pi_{t+1} + j_{t+1} \right]
+ \frac{1}{2} \text{var}_t \left( m_{t+1} - \Delta z_{t+1} + \kappa \Delta r^K_{t+1} + (1 - \kappa) \Delta \hat{w}_{t+1} + \Delta \hat{y}_{t+1} + \Delta a_{t+1} + \eta \pi_{t+1} + j_{t+1} \right) \right\}
- \text{const}_j \Phi,
\]

where \( \text{const}_j = \frac{\alpha \Phi e^\Phi}{1 + \alpha \Phi e^\Phi} \).

C.6 The System of Equations for the Model with Growth

The full model presented in this section has thirty-one endogenous variables:
\( \{M, R^d, R^c, R^l, share, PC, P^d, C, LI, N, W, Tax, \tau, H, G, I^w, D, K, Inv, Y, \Phi, \Phi', R^l, K, Q, P_{\text{real}}, \Pi, F, J, M_{\text{nom}}, R^{(1)} \} \).

I have a system of thirty-three equations resulting from equilibrium conditions, first order conditions and policy rules:
Pricing kernel,

\[ M_{t-1,t} = \left[ \beta \left( \frac{C_t}{C_{t-1}} \right)^{-\psi} \right]^{\frac{1-\phi}{\psi-\gamma}} \left[ R^c_t \right]^{\frac{\psi-\gamma}{1-\psi}} \] (23)

\[ R^c_t = (1 - \text{share}_{t-1}) R^c_{t-1} + \text{share}_{t-1} R^l_t \] (24)

\[ \text{share}_t = \frac{1}{1 - \frac{(1+\omega)P_tC_t}{(1-\psi)P^c_{t-1}LI_t}} \] (25)

\[ R^c_t = \left( 1 + P^c_t \right) C_t \left/ \left( P^c_{t-1} C_{t-1} \right) \right. \] (26)

\[ 1 = \mathbb{E}_t [M_{t,t+1} R^c_{t+1}] \] (27)

\[ R^l_t = \left( 1 + P^l_t \right) LI_t \left/ P^l_{t-1} LI_{t-1} \right. \] (28)

\[ 1 = \mathbb{E}_t [M_{t,t+1} R^l_{t+1}] \] (29)

Labor income,

\[ LI_t = W_t N_t \] (30)

Fiscal rule,

\[ \text{Tax}_t = \tau_t + \tau^h_t R^h_t K_{t-1} \] (31)

\[ \tau_t = \rho_b D_{t-1}(t) + \rho_g G_{Gov_t} \] (32)

Wage setting of the saver,

\[ \left[ \frac{1}{1 - \theta} \left\{ W_t^{1-\eta_w} - \theta \left( I^w_{t-1,t} W_{t-1} \right)^{1-\eta_w} \right\} \right]^{\frac{1-\eta_w}{1-\psi}} H_t = \nu_w A_t^{1-\psi} C_t^\omega N_t^{\psi} G_t \] (33)

\[ H_t = 1 + \theta \mathbb{E}_t \left[ M_{t,t+1} I^w_{t,t+1} - \eta_w \left( N^d_{t+1} W^d_{t+1} \right)^{\eta_w} H_t + 1 \right] \] (34)

\[ G_t = 1 + \theta \mathbb{E}_t \left[ M_{t,t+1} I^w_{t,t+1} - \eta_w \left( A_t W^d_{t+1} \right)^{1-\psi} \left( C_t^{\psi+1} \right) \psi \left( N^d_{t+1} \right)^{\omega} \left( N^d_{t+1} \right)^{\omega} H_t^{1-\eta_w} \left( W_{t+1} \right)^{\eta_w} G_{t+1} \right] \] (35)

Wage indexing,

\[ I^w_{t-1,t} = e^{\eta_w} \] (36)

Production function,

\[ Y_t = Z_t K^\kappa_{t-1} (A_t N_t)^{1-\kappa} \] (37)
Capital accumulation,
\[ K_t = ((1 - \delta) + \Phi_t)K_{t-1} \] (38)

Capital adjustment cost,
\[ \Phi_t = b1 + \frac{b2}{(1 - 1/\zeta)} \left( \frac{Inv_t}{K_{t-1}} \right)^{1 - 1/\zeta} \] (39)
\[ \Phi'_t = b2 \left( \frac{Inv_t}{K_{t-1}} \right)^{-1/\zeta} \] (40)

Return on investment,
\[ R_I = E_t[\Pi_{t+1}] \] (41)
\[ R_I^tQ_{t-1} = (1 - \tau_k^t)R^K_t + Q_t \left( 1 - \delta + \Phi_t - \Phi'_t \right) \] (42)
\[ 1 = Q_t\Phi'_t \] (43)

Market clearing condition,
\[ Y_t = C_t + Inv_t + Gov_t \] (44)

Government budget constraint,
\[ D_{t-1}(t) = Tax_t - Gov_t + P^{real}_tD_t(t + 1) \] (45)

Capital labor ratio,
\[ W_t = \frac{(1 - \kappa)K_{t-1}}{N_t} \] (46)

Optimal price setting,
\[ \left[ \frac{1}{1 - \alpha} \left( 1 - \alpha \left( \frac{1}{\Pi_t} \right)^{(1-\eta)} \right) \right]^{1-\eta} F_t = \frac{\nu \kappa^{\kappa - 1}(1 - \kappa)^{-\eta}R^K_tW^{(1-\eta)}_{t+1}J_t}{Z_tA_{t-1}^{1-\kappa}} \] (47)
\[ F_t = 1 + \alpha E_t \left[ M_{t,t+1} \left( \frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1}^{\eta}J_{t+1} \right] \] (48)
\[ J_t = 1 + \alpha E_t \left[ M_{t,t+1} \left( \frac{Z_t}{Z_{t+1}} \right) \left( \frac{A_t}{A_{t+1}} \right)^{1-\kappa} \left( \frac{R^K_t}{R^K_{t+1}} \right)^{\kappa} \left( \frac{W_{t+1}}{W_t} \right)^{(1-\kappa)} \left( \frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1}^{(1+\eta)}J_{t+1} \right] \] (49)

Nominal pricing kernel,
\[ M_{t-1,t}^n = \frac{M_{t-1,t}}{\Pi_t} \] (50)
Euler equation,

\[
\frac{1}{R_t^{(1)}} = E_t[M_{t,t+1}^{\text{nom}}]
\]  

(51)

Real bond price,

\[
P_{t}^{\text{real}} = E_t[M_{t,t+1}]
\]  

(52)

Taylor rule,

\[
\frac{R_t^{(1)}}{R} = \left( \frac{R_{t-1}^{(1)}}{R} \right)^{\rho_r} \left( \frac{\Pi_t}{\Pi^\ast_t} \right)^{(1-\rho_r)\rho_r} \left( \frac{Y_t/A_t}{Y_{t-1}/A_{t-1}} \right)^{(1-\rho_r)\rho_r} \epsilon^{u_t},
\]

(53)

where \(g_t, u_t\) and \(z_t\) are exogenous shocks to government spending, monetary policy and productivity, respectively:

\[
\begin{align*}
g_{t+1} &= (1 - \phi_g)g_t + \phi_g g_t + e^{g_{t+1}}y_{t+1}^{1}\epsilon_{g,t+1} \\
\sigma_{g,t+1} &= (1 - \phi_{\sigma}^{g})\sigma_{g,t} + \phi_{\sigma}^{g} \sigma_{g,t} + \sigma_{\sigma g,t+1}^{g} \\
\tau_{t+1} &= (1 - \phi_{\tau})\tau + \phi_{\tau} \tau_t + e^{\tau_{t+1}^{k}}\epsilon_{\tau,t+1} \\
\sigma_{\tau,t+1} &= (1 - \phi_{\sigma}^{\tau})\sigma_{\tau,t} + \phi_{\sigma}^{\tau} \sigma_{\tau,t} + \sigma_{\sigma \tau,t+1}^{\tau} \\
z_{t+1} &= \phi_z z_t + e^{z_{t+1}}\epsilon_{z,t+1} \\
\sigma_{z,t+1} &= (1 - \phi_{\sigma}^{z})\sigma_{z,t} + \phi_{\sigma}^{z} \sigma_{z,t} + \sigma_{\sigma z,t+1}^{z} \\
\Delta_{a,t+1} &= (1 - \phi_a)a_t + \phi_a \Delta a_t + \sigma_a \epsilon_{a,t+1} \\
u_{t+1} &= \phi_u u_t + \sigma_u \epsilon_{u,t+1}.
\end{align*}
\]

C.6.1 The Stationary Model

To make the model stationary, output, consumption, investment, capital stock, real wage, real debt, government revenue, and government spending need to be detrended by the permanent component of productivity, \(A_t\).
Pricing kernel,

\[ M_{t-1,t} = \left[ \beta \left( \frac{C_t A_t}{C_{t-1} A_{t-1}} \right)^{-\psi} \right]^{\frac{1}{1-\gamma}} \quad [R^c_t]^{\frac{\psi}{1-\gamma}} \]  

\[ \Rightarrow M_{t-1,t} = \left[ \beta \left( \frac{\widetilde{C}_t e^{\Delta a_t}}{\widetilde{C}_{t-1}} \right)^{-\psi} \right]^{\frac{1}{1-\gamma}} \quad [R^c_t]^{\frac{\psi}{1-\gamma}} \]  

\[ R^c_t = (1 - share_{t-1}) R^c_{t-1} + share_{t-1} R^l_t \]  

\[ share_t = \frac{1}{\left( 1 - \frac{(1+\omega)P^c_t C_t}{(1-\psi)P^c_{t-1} L_{t-1}} \right)} \quad \Rightarrow share_t = \frac{1}{\left( 1 - \frac{(1+\omega)P^c_t \widetilde{C}_t}{(1-\psi)P^c_{t-1} \widetilde{L}_{t-1}} \right)} \]  

\[ R^c_t = \frac{1 + P^c_t \widetilde{C}_t}{P^c_{t-1} \widetilde{C}_{t-1}} A_t \quad \Rightarrow R^c_t = \frac{(1 + P^c_t \widetilde{C}_t)}{P^c_{t-1} \widetilde{C}_{t-1}} e^{\Delta a_t} \]  

\[ 1 = E_t[M_{t,t+1} R^c_{t+1}] \]  

\[ R^l_t = \frac{(1 + P^l_t \widetilde{L}_t)}{P^l_{t-1} \widetilde{L}_{t-1}} A_t \quad \Rightarrow R^l_t = \frac{(1 + P^l_t \widetilde{L}_t)}{P^l_{t-1} \widetilde{L}_{t-1}} e^{\Delta a_t} \]  

\[ 1 = E_t[M_{t,t+1} R^l_{t+1}] \]  

Labor income,

\[ \frac{LI_t}{A_t} = \frac{W_t}{A_t} N_t \quad \Rightarrow \widetilde{LI}_t = \widetilde{W}_t \widetilde{N}_t \]  

Fiscal rule,

\[ \frac{T_{ax_t}}{A_t} = \frac{\tau_t}{A_t} + \tau^k_t R^k_t \frac{K_{t-1}}{A_{t-1}} A_{t-1} \quad \Rightarrow \bar{T}_{ax_t} = \bar{\tau}_t + \tau^k_t R^k_t \bar{K}_{t-1} e^{-\Delta a_t} \]  

\[ \frac{\tau_t}{A_t} = \rho_b \frac{D_{t-1}(t) A_{t-1}}{A_t} + \rho_g \frac{Gov_t}{A_t} \quad \Rightarrow \bar{\tau}_t = \rho_b \bar{D}_{t-1}(t) e^{-\Delta a_t} + \rho_g \bar{Gov}_t \]
Wage setting of the saver,

\[
\begin{align*}
H_t &= \nu_w A_t^{1-\psi} C_t^\psi N_t^{\omega} G_t \\
\implies H_t &= 1 + \theta E_t \left[ M_t^{\text{nom}} I_{t+1}^{w} - \eta_w \left( \frac{N_t^{d+1}}{N_t^d} \right) \left( \Pi_{t+1} \frac{W_{t+1}}{W_t} \frac{A_{t+1}}{A_t} \right) \right]^{\eta_w} H_{t+1} \\
\implies H_t &= 1 + \theta E_t \left[ M_t^{\text{nom}} I_{t+1}^{w} - \eta_w \left( \frac{N_t^{d+1}}{N_t^d} \right) \left( \Pi_{t+1} \frac{W_{t+1}}{W_t} \frac{A_{t+1}}{A_t} e^{\Delta A_{t+1}} \right) \right]^{\eta_w} H_{t+1} \\
G_t &= 1 + \theta E_t \left[ M_t^{\text{nom}} I_{t+1}^{w} - \eta_w \left( \frac{C_i A_{t+1}}{C_i A_t} \right) \left( \frac{N_t^{d+1}}{N_t^d} \right) \left( \Pi_{t+1} \frac{A_{t+1}}{A_t} \right) \left( \frac{W_{t+1}}{W_t} \frac{A_{t+1}}{A_t} \right) \right]^{\eta_w} G_{t+1}
\end{align*}
\]

Wage indexing,

\[
I_{t-1,t}^w = e^{\eta_w}
\]

Production function,

\[
\frac{Y_t}{A_t} = Z_t \left( \frac{K_{t-1}}{A_{t-1}} \right)^\kappa N_t^{1-\kappa} \left( \frac{A_{t-1}}{A_t} \right)^\kappa \implies \tilde{Y}_t = Z_t \left( \frac{K_{t-1} e^{\Delta A_t}}{A_{t-1}} \right)^\kappa N_t^{1-\kappa}
\]

Capital accumulation,

\[
\frac{K_t}{A_t} = \left( (1 - \delta) + \Phi_t \right) \frac{K_{t-1}}{A_{t-1}} \left( \frac{A_{t-1}}{A_t} \right) \implies \tilde{K}_t = \left( (1 - \delta) + \Phi_t \right) \frac{K_{t-1} e^{\Delta A_t}}{A_{t-1}}
\]

Capital adjustment cost,

\[
\begin{align*}
\Phi_t &= b_1 + \frac{b_2}{(1 - 1/\zeta)} \left( \frac{I_{t-1}^w}{K_{t-1}} \frac{A_t}{A_{t-1}} \right)^{-1/\zeta} \implies \Phi_t = b_1 + \frac{b_2}{(1 - 1/\zeta)} \left( \frac{I_{t-1}^w e^{\Delta A_t}}{K_{t-1}} \right)^{-1/\zeta} \\
\Phi_t' &= b_2 \left( \frac{I_{t-1}^w}{K_{t-1}} \frac{A_t}{A_{t-1}} \right)^{-1/\zeta} \implies \Phi_t' = b_2 \left( \frac{I_{t-1}^w e^{\Delta A_t}}{K_{t-1}} \right)^{-1/\zeta}
\end{align*}
\]
Return on investment,

\[ 1 = E_t[M_{t+1}R^f_{t+1}] \quad (76) \]
\[ R^f_t Q_{t-1} = (1 - x^k_t)R^K_t + Q_t \left( 1 - \delta + \Phi - \Phi'_F \frac{I_{inv}^t}{K_{t-1}^{\text{real}}} A_t \right) \quad (77) \]
\[ \Rightarrow R^f_t Q_{t-1} = (1 - x^k_t)R^K_t + Q_t \left( 1 - \delta + \Phi - \Phi'_F \frac{I_{inv}^t}{K_{t-1}^{\text{real}}} e^{\Delta a_t} \right) \quad (78) \]
\[ 1 = Q_t \Phi'_F \quad (79) \]

Market clearing condition,

\[ \frac{Y_t}{A_t} = \frac{C_t + I_{inv}^t + Gov_t}{A_t} \Rightarrow \bar{Y}_t = \bar{C}_t + \bar{I}_{inv} + \bar{Gov}_t \quad (80) \]

Government budget constraint,

\[ \frac{D_{t-1}(t) A_{t-1}}{A_t} = T^ax_t + \frac{Gov_t}{A_t} + p^{\text{real}}_t D_t(t+1) \Rightarrow \bar{D}_{t-1}(t)e^{-\Delta a_t} = \bar{T}^ax_t - \bar{Gov}_t + p^{\text{real}}_t \bar{D}_t(t+1) \quad (81) \]

Capital labor ratio,

\[ \frac{W_t}{A_t} = \frac{(1 - \kappa)}{\kappa} \frac{R^K_t K_{t-1}}{A_t N_t} \Rightarrow \bar{W}_t e^{\Delta a_t} = \frac{(1 - \kappa)}{\kappa} \frac{R^K_t K_{t-1}^{\text{real}}}{N_t} \quad (82) \]

Optimal price setting,

\[ \left[ \frac{1}{1 - \alpha} \left( \frac{1}{\Pi_t} \right)^{(1-\eta)} \right] \left[ \frac{1}{1 - \alpha} \left( \frac{1}{\Pi_t} \right)^{(1-\eta)} \right] F_t = \frac{\nu \kappa - \kappa(1 - \kappa)^{-1(1-\kappa)} R^K_t \left( \frac{W_t}{A_t} \right)^{(1-\kappa)}}{Z_t} \quad (83) \]
\[ \Rightarrow \left[ \frac{1}{1 - \alpha} \left( \frac{1}{\Pi_t} \right)^{(1-\eta)} \right] \left[ \frac{1}{1 - \alpha} \left( \frac{1}{\Pi_t} \right)^{(1-\eta)} \right] F_t = \frac{\nu \kappa - \kappa(1 - \kappa)^{-1(1-\kappa)} R^K_t \left( \frac{\bar{W}_t}{\bar{I}_t} \right)^{(1-\kappa)}}{Z_t} \quad (84) \]
\[ F_t = 1 + \alpha E_t \left[ M_{t+1}^{\text{nom}} \left( \frac{Y_{t+1}}{A_t} \right) \frac{A_{t+1}}{A_t} \right] \Pi_{t+1}^{\eta_t} F_{t+1} \quad (85) \]
\[ \Rightarrow F_t = 1 + \alpha E_t \left[ M_{t+1}^{\text{nom}} \left( \frac{Y_{t+1}}{\bar{Y}_t} \right)^{\Delta a_{t+1}} \right] \Pi_{t+1}^{\eta_t} F_{t+1} \quad (86) \]
\[ J_t = 1 + \alpha E_t \left[ M_{t+1}^{\text{nom}} \left( \frac{Z_t}{Z_{t+1}} \right) \left( \frac{A_t}{A_{t+1}} \right)^{1-\kappa} \left( \frac{R^{K}_{t+1}}{R^K_t} \right)^{\kappa} \left( \frac{W_{t+1}}{W_t} \right) \left( \frac{Y_{t+1}}{Y_t} \right) \right] \Pi_{t+1}^{(1+\eta_t)} J_{t+1} \quad (87) \]
\[ \Rightarrow J_t = 1 + \alpha E_t \left[ M_{t+1}^{\text{nom}} \left( \frac{Z_t}{Z_{t+1}} \right) \left( \frac{R^{K}_{t+1}}{R^K_t} \right)^{\kappa} \left( \frac{\bar{W}_{t+1}}{\bar{W}_t} \right) \right] \Pi_{t+1}^{(1+\eta_t)} J_{t+1} \quad (88) \]
Nominal pricing kernel,

\[ M_{t-1,t}^{nom} = \frac{M_{t-1,t}}{\Pi_t} \]  

(89)

Euler equation,

\[ \frac{1}{R_t^{(1)}} = E_t[M_{t,t+1}^{nom}] \]  

(90)

Real bond price,

\[ P_{t}^{real} = E_t[M_{t,t+1}] \]  

(91)

Taylor rule,

\[ \frac{R_t^{(1)}}{R} = \left( \frac{R_{t-1}^{(1)}}{R} \right)^{\rho_r} \left( \frac{\Pi_t}{\Pi^*} \right)^{(1-\rho_r)\rho_x} \left( \frac{\tilde{Y}_t}{Y_{t-1}} \right)^{(1-\rho_r)\rho_x} e^{\epsilon_u}, \]  

(92)

where \( g_t, u_t \) and \( z_t \) are exogenous shocks to government spending, monetary policy and productivity, respectively:

\[ g_{t+1} = (1 - \phi_g)g_t + \phi_g g_{t+1} + \epsilon^{\sigma_{g,t+1}} \epsilon_{g,t+1} \]
\[ \sigma_{g,t+1} = (1 - \phi_{\sigma_g})\sigma_g + \phi_{\sigma_g} \sigma_{g,t+1} + \sigma^2 \epsilon_{g,t+1} \]
\[ \tau_{t+1} = (1 - \phi_{\tau})\tau + \phi_{\tau} \tau_{t+1} + \epsilon^{\sigma_{\tau,t+1}} \epsilon_{\tau,t+1} \]
\[ \sigma_{\tau,t+1} = (1 - \phi_{\sigma_{\tau}})\sigma_{\tau} + \phi_{\sigma_{\tau}} \sigma_{\tau,t+1} + \sigma^2 \epsilon_{\tau,t+1} \]
\[ z_{t+1} = \phi_z z_t + \epsilon^{\sigma_{z,t+1}} \epsilon_{z,t+1} \]
\[ \sigma_{z,t+1} = (1 - \phi_{\sigma_z})\sigma_z + \phi_{\sigma_z} \sigma_{z,t+1} + \sigma^2 \epsilon_{z,t+1} \]
\[ \Delta a_{t+1} = (1 - \phi_a)g_a + \phi_a \Delta a_t + \sigma_a \epsilon_{a,t+1} \]
\[ u_{t+1} = \phi_u u_t + \sigma_u \epsilon_{u,t+1}, \]  

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The steady state system of the model with growth

The steady state of the pricing kernel block, with the exception of share, can be determined right away by noting \( M = \beta e^{-\psi g} \): Pricing kernel,

\[
M = \beta e^{-\psi g}, \quad (93)
\]
\[
R^{cl} = \frac{1}{\beta e^{-\psi g}}, \quad (94)
\]
\[
R^c = \frac{1}{\beta e^{-\psi g}}, \quad (95)
\]
\[
P^c = \frac{\beta e^{(1-\psi)g}}{1 - \beta e^{(1-\psi)g}}, \quad (96)
\]
\[
R^l = \frac{1}{\beta e^{-\psi g}}, \quad (97)
\]
\[
P^l = \frac{\beta e^{(1-\psi)g}}{1 - \beta e^{(1-\psi)g}}, \quad (98)
\]

In steady state, capital cancel out in the capital accumulation equation such that

\[
\Phi = e^{\theta} + \delta - 1. \quad (99)
\]

Given \( \Phi = \delta \), the investment-capital ratio and \( \Phi' \) can be found using the adjustment cost functions

\[
\overline{I}_{nv} = \left[ (e^{\theta} + \delta - 1 - b1) (1 - 1/\zeta) \right]^{\zeta/(\zeta-1)} e^{-\theta} \bar{K}. \quad (100)
\]
\[
\Phi' = b2 \left[ (e^{\theta} + \delta - 1 - b1) (1 - 1/\zeta) \right]^{-1/(\zeta-1)}. \quad (101)
\]

Return on investment is also \( \frac{1}{\beta e^{-\psi g}} \) which allows us to find the rental cost of capital,

\[
R^l = \frac{1}{\beta e^{-\psi g}}, \quad (102)
\]
\[
Q = \frac{1}{b2} \left[ (e^{\theta} + \delta - 1 - b1) (1 - 1/\zeta) \right]^{1/(\zeta-1)}. \quad (103)
\]
\[
R^K = \frac{1}{1 - \tau} \left[ \frac{1}{\beta e^{-\psi g}} - e^{\theta} + (e^{\theta} + \delta - 1 - b1) (1 - 1/\zeta) \right] Q. \quad (104)
\]
To solve for the steady state inflation, we notice the following:

\[ M^{\text{nom}} = \frac{\beta e^{-\psi g_a}}{\Pi} \]  
\[ R^{(1)} = \frac{\Pi}{\beta e^{-\psi g_a}} \]  
\[ p^{\text{real}} = \beta e^{-\psi g_a}. \]  

From the Taylor rule,

\[ \Pi = \left( R \frac{\beta e^{-\psi g_a}}{\Pi^{1-\nu \psi}} \right)^{\frac{1}{1-\nu \psi}}. \]  

With steady state inflation given, equilibrium wage offer is:

\[ F = \frac{1}{1 - \frac{\alpha \beta e^{(1-\psi)g_a} \Pi^{\eta - 1}}{1 - \frac{1}{\alpha \beta e^{(1-\psi)g_a} \Pi^\eta}}}, \]  
\[ J = \frac{1}{1 - \frac{1}{\alpha \beta e^{(1-\psi)g_a} \Pi^\eta}} \]  
\[ \tilde{W} = \left\{ \left[ \frac{1}{1 - \alpha} \left( 1 - \alpha \left( \frac{1}{\Pi} \right)^{(1-\eta) \Psi} \right) \right]^{(1-\eta) \Psi} \kappa^\kappa \frac{(1-\kappa)(1-\kappa) F}{\nu J} R^{K-K} \right\}^{\frac{1}{1-\kappa}}. \]  

With steady state inflation given, equilibrium wage demand is:

\[ H = \frac{1}{1 - \frac{\theta \beta e^{-\psi g_a} \Pi^{\eta \omega - 1}}{1 - \frac{1}{\theta \beta e^{(1-\psi)g_a} \Pi^\eta \omega}}}, \]  
\[ G = \frac{1}{1 - \frac{1}{\theta \beta e^{(1-\psi)g_a} \Pi^\eta \omega}} \]  
\[ \tilde{W} = \tilde{C} \psi N^\omega \nu \omega \frac{G}{H} \left[ \frac{1}{1 - \theta} \left\{ 1 - \theta \left( \frac{1}{\Pi} \right)^{(1-\eta) \Psi} \right\} \right]^{\frac{1}{1-\kappa}}. \]  

Capital labor ratio delivers capital in terms of labor input,

\[ \tilde{K} = \frac{\kappa}{1-\kappa} \frac{\tilde{W} e^{g_a}}{R^{K-N}}. \]  

Combining the production function and the market clearing condition, we can solve for steady state labor
by writing consumption, investment, and capital in terms of labor:

\[
(\bar{K}e^{-g_a})^\kappa N^{1-\kappa} = \frac{\bar{C} + \bar{I}_n}{1 - \theta_g} \tag{116}
\]

\[
\left(\frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K}\right)^\kappa N = \frac{1}{1 - \theta_g} \left\{ \left(\frac{\bar{W}}{N^\omega \Psi}\right)^{1/\psi} + \left[ (e^{g_a} + \delta - 1 - b_1) \frac{(1 - 1/\zeta)}{b_2} \right]^{\zeta/(\zeta - 1)} \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right\} N \tag{117}
\]

\[
\left(\frac{\bar{W}}{N^\omega \Psi}\right)^{1/\psi} = \left\{ (1 - \theta_g) \left[ \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right]^\kappa - \left[ (e^{g_a} + \delta - 1 - b_1) \frac{(1 - 1/\zeta)}{b_2} \right]^{\zeta/(\zeta - 1)} \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right\} N \tag{118}
\]

\[
\frac{\bar{W}}{N^\omega \Psi} = \left\{ (1 - \theta_g) \left[ \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right]^\kappa - \left[ (e^{g_a} + \delta - 1 - b_1) \frac{(1 - 1/\zeta)}{b_2} \right]^{\zeta/(\zeta - 1)} \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right\}^\psi N^{\psi} \tag{119}
\]

\[
N^{\psi + \omega} = \frac{\bar{W}}{\Psi} \left\{ (1 - \theta_g) \left[ \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right]^\kappa - \left[ (e^{g_a} + \delta - 1 - b_1) \frac{(1 - 1/\zeta)}{b_2} \right]^{\zeta/(\zeta - 1)} \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right\}^{-\psi} \tag{120}
\]

\[
N = \left\{ \frac{\bar{W}}{\Psi} \left[ (1 - \theta_g) \left[ \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right]^\kappa - \left[ (e^{g_a} + \delta - 1 - b_1) \frac{(1 - 1/\zeta)}{b_2} \right]^{\zeta/(\zeta - 1)} \frac{\kappa}{1 - \kappa} \frac{\bar{W}}{R^K} \right] \right\}^{\frac{1}{\psi + \omega}} \tag{121}
\]

where the second equality uses the fact that \(\bar{W} = \bar{C}^\psi N^\omega \Psi\). Labor is now written in terms of parameters and known variables. Steady state capital can be calculated using the capital labor ratio. Steady state investment can be found using the adjustment cost function relating investment to capital.

Production function delivers the steady state output,

\[
\bar{Y} = (\bar{K}e^{-g_a})^\kappa N^{1-\kappa}. \tag{122}
\]

Market clearing condition pins down the steady state aggregate consumption,

\[
\bar{C} = (1 - \theta_g) \bar{Y} - \bar{I}_n. \tag{123}
\]

Steady state real debt can be calculated from the fiscal rule and the government budget constraint:

\[
\bar{D}e^{-g_a} = \bar{D}_m - \theta_g \bar{Y} + \beta e^{-\psi g_a} \bar{D} \tag{124}
\]

\[
\bar{D}e^{-g_a} = \rho_0 \bar{D}e^{-g_a} + \rho_0 \theta_g \bar{Y} + (1 - \mu)^{\mu} K^\mu \bar{K} e^{-g_a} - \theta_g \bar{Y} + \beta e^{-\psi g_a} \bar{D} \tag{125}
\]

\[
(e^{-g_a} - \rho_0 e^{-g_a} - \beta e^{-\psi g_a}) \bar{D} = (\rho_0 - 1) \theta_g \bar{Y} + \theta_g R^K \bar{K} e^{-g_a} \tag{126}
\]

\[
\bar{D} = \frac{(1 - \rho_0) \theta_g \bar{Y} - \theta_g R^K \bar{K} e^{-g_a}}{(\beta e^{-\psi g_a} + \rho_0 e^{-g_a} - e^{-g_a})}. \tag{127}
\]

Steady state lump-sum transfer is:

\[
\bar{\tau} = \left[ \rho_0 \left( \frac{1 - \rho_0}{(\beta e^{(1-\psi)g_a} + \rho_0 - 1)} + \rho_0 \theta_g \bar{Y} \right) \right]. \tag{128}
\]
Tax revenues are:

\[ \tilde{\text{Tax}} = \tilde{\tau} + \theta_k R^k e^{-g_u} \]  

(129)

(130)

Finally, the following steady states are trivial:

\[ \tilde{LI} = \tilde{WN} \]  

(131)

\[ \text{share} = \frac{1}{1 - \frac{1+\omega}{1-\psi} \frac{P_c}{P_L I}} \]  

(132)
D Solution and Estimation

To estimate model parameters we use the mean, the variance and the contemporaneous covariances in the data as moments. Hence, we let

\[
q_t = \begin{bmatrix}
data_t \\
de \text{diag} \begin{bmatrix} data_t, data_t' \end{bmatrix} \\
de \text{vech} \begin{bmatrix} data_t, data_t' \end{bmatrix}
\end{bmatrix}.
\]

Letting \( \theta \) contain the structural parameters, our GMM estimator is given by

\[
\theta_{GMM} = \arg\min_{\theta \in \Theta} \left( \frac{1}{T} \sum_{t=1}^{T} q_t - E[q_t(\theta)] \right) W \left( \frac{1}{T} \sum_{t=1}^{T} q_t - E[q_t(\theta)] \right)'
\]

Here, \( W \) is a positive definite weighting matrix and \( E[q_t(\theta)] \) contains the model-implied moments computed as described in the following subsection. We use the conventional two-step implementation of GMM by letting \( W_T = \text{diag} \left( \hat{S}^{-1} \right) \) in a preliminary first step to obtain \( \hat{\theta}^{\text{step 1}} \) where \( \hat{S} \) denotes the long-run variance of \( \frac{1}{T} \sum_{t=1}^{T} q_t \) when re-centered around its sample mean. Our final estimates \( \hat{\theta}^{\text{step 2}} \) are obtained using the optimal weighting matrix \( W_T = \text{diag} \left( \hat{S}^{-1}_{\hat{\theta}^{\text{step 1}}} \right) \), where \( \hat{S}^{\text{step } 1} \) denotes the long-run variance of our moments re-centered around \( E \left[ q_t \left( \hat{\theta}^{\text{step 1}} \right) \right] \). The long-run variances in both steps are estimated by the Newey-West estimator using 10 lags, but our results are robust to using more lags.

Given our interest in analyzing time-varying risk premia, we employ a third-order Taylor approximation of the policy functions that characterize the equilibrium dynamics of the model. However, higher-order terms may generate explosive sample paths thus precluding any estimation method that, like GMM, relies on finite moments from stationary and ergodic probability distribution (see e.g. Sims, Kim, Kim, and Schaumburg (2008) for a discussion of this issue within the context of second-order approximations). To ensure stable sample paths (and existence of finite unconditional moments) we adopt the pruned state-space system for non-linear DSGE models suggested by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017). Intuitively, pruning means we are going to omit terms of higher-order effects than the considered approximation order (third-order, in our case) when the system is iterated forward in time.\(^{32}\) Provided the linearized solution is stable, Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) derive closed-form solutions for first and second unconditional moments of the pruned state-space of the DSGE. This is important since it allows us to compute in a reasonable amount of time the unconditional moments for our DSGE model solved up to third-order.\(^{33}\)

\(^{32}\)For details on the pruning method, see Sims, Kim, Kim, and Schaumburg (2008) for second-order and Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017) for higher-order approximations to the solutions of DSGE models.

\(^{33}\)Although we solve the model by a third-order perturbation, we verified that our model moments are similar when we use a higher-order approximation and no pruning. In particular we checked that our results do not change when we use a fifth order solution to our DSGE model. To obtain a fifth order solution we use the tensor approach proposed by Levintal (2017). The corresponding results are available upon request.
E  Conditional IRF at the Zero Lower Bound

To implement an interest rate peg in the model we follow [Sims (2017)]. In particular we augment the Taylor rule with “news” shocks as follows:

\[ i_t = i^* + \phi_\pi (\pi_t - \pi^*) + \epsilon_{t}^{(0)} + \sum_{j=1}^{H-1} \epsilon_{t-j}^{(1)} \]

where \( \epsilon_{t-j}^{(j)} \), for \( j > 0 \) are news shocks, i.e. shocks known to agents in advance of them actually impacting the policy rule.

We can impose an interest rate peg as follows. First, solve the model as described in Section 4.2 but where we replaced the Taylor rule with one augmented with news shocks. Second, we simulate a long path of \( T - 1 \) observation so that all state variables are at their ergodic mean in absence of shocks (EMAS). Starting at the EMAS, we compute the IRFs of the economy to, e.g., a government spending shock (at this stage we still have \( \epsilon_{t-j}^{(j)} = 0 \)). We then solve for the value of the news shocks \( \epsilon_{t-j}^{(j)} \), for \( j = 0, \ldots, H - 1 \), which keeps the nominal rate pegged for a desired length of time, i.e. \( i_{T+s} = i_{T-1} \equiv i_{\text{EMAS}} \) for \( s = 0, \ldots, H - 1 \). Effectively, we can think about the effects of a shock under an interest rate peg as being something like the sum of the direct effect of the shock, plus the effects of current and anticipated monetary policy shocks so as to keep the nominal the interest rate unresponsive to a shock for the current and subsequent \( H - 1 \) periods, for a total of \( H \) periods. We refer to \( H \) as the peg period. In Section 6 we discuss two policy scenarios: a \( H = 4 \) period interest rate peg, and a \( H = 8 \) period interest rate peg.

Few final remarks are in order. First, an important advantage of this approach is that one can still solve the model with perturbations above first order. Second, it is important to write the innovations in the policy rule as anticipated shocks since this guarantees that, at the time of another shock, agents will anticipate that the interest rate will be unresponsive for \( H \) total periods. Third, the algorithm so far described is similar to the one used in [Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015)]. Whereas [Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015)] use a combination of innovations to preference and productivity shocks to force the economy to the ZLB, we instead solve for the news shocks which keep the interest rate unresponsive.

\[ \text{For expository purposes, we consider a simplified Taylor rule with no interest rate smoothing and no reaction to output growth.} \]
\[ \text{To augment the Taylor rule with news shocks is reasonably straightforward in Dynare. To do so, one simply needs to create some auxiliary state variables. E.g. suppose one wants a four period peg, } H = 4. \text{ Then one would introduce four new state variables.} \]

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