Missed Sales and the Pricing of Ancillary Goods*

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August 8, 2017

Abstract: Firms often sell a basic good as well as ancillary ones. Hold-up concerns have led to ancillary good regulations such as transparency and price caps. The hold-up narrative, however, runs counter to evidence in many retail settings where ancillary good prices are set below cost (e.g. free shipping, or limited card surcharging in countries where the “no-surcharge rule” was lifted). We argue that the key to unifying these conflicting narratives is that the seller may absorb partly or fully the ancillary good’s cost so as not to miss sales on the basic good. A supplier with market power on the ancillary good market then takes advantage of cost absorption and jacks up its wholesale price. Hold-ups occur only when consumers are initially uninformed or naïve about the drip price and shopping costs are high. The price of the basic good then acts as a signal of the drip price, since a high markup on the basic good makes the firm more wary of missed sales. Regardless of whether consumers are informed, uninformed-but-rational, or naïve, mandating price transparency and banning loss-making on the ancillary good leads to (i) an efficient consumption of the ancillary good, and (ii) a reduction of its wholesale price, generating strict welfare gains.

Keywords: add-ons, drip pricing, missed sales, give-aways, hold-ups.

JEL numbers: D83, L10, L41.

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*An earlier paper entitled “Shrouded Transaction Costs” forked into the present paper and an eponymous paper written in collaboration with Hélène Bourguignon. The present paper develops the general theory. The companion paper focuses on an open card payment system, makes specific assumptions (in particular it rules out repeat purchases, which play a prominent role in this paper), considers the case of “price coherence” and compares cash discounts and card surcharges. The authors are grateful to Alessandro Bonatti, Hélène Bourguignon, Matthias Lang, Yassine Lefouili, Bill Rogerson, Jean-Charles Rochet, Florian Schuett, Andrei Shleifer, Yossi Spiegel, Julian Wright, four anonymous referees, and seminar or conference participants at the TNIT Meeting (Microsoft), Paris School of Economics, the Tenth IDEI-TSE-IAS Conference on “The Economics of Intellectual Property, Software and the Internet,” the Norwegian School of Economics, the Second Berlin Workshop on the Economics of Platforms, La Poste, the 15th CSIO/IDEI Conference on Industrial Organization, and Tilburg University. Jean Tirole gratefully acknowledges financial support from the ERC grants FP7/2007-2013 No. 249429 (Cognition and Decision-Making: Laws, Norms and Contracts) and No.669217 (MARKLIM).

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1 Introduction

Firms often sell a basic good (physical product, hotel room, flight ticket) as well as ancillary ones/ add ons (shipping, assembly, in-room dining, baggage allowances, card payment), whose consumption can be avoided only at an inconvenience cost. The concern of consumer-advocacy groups and regulators about potential hold-ups on ancillary goods has led to policies such as mandated price transparency (whereby firms are required to jointly post all prices and fees),\(^1\) or caps on additional fees (e.g., the obligation to sell the ancillary good at no more its marginal cost).\(^2\)

The hold-up narrative, however, runs counter to evidence in many retail settings involving ancillary good pricing. One much discussed example comes from the payment industry. Regulators around the world recently lifted the so-called “no-surcharge rule” imposed by card platforms, thus allowing merchants to price discriminate according to the payment method.\(^3\) These reforms were designed to constrain the fee-setting power of card platforms, as merchants could now pass high merchant fees through to consumers. Yet, with notable exceptions that we will later discuss, card surcharging is infrequent or limited in magnitude in most industries:\(^4\) Rather than imposing surcharges, merchants typically opt to implicitly subsidize consumer card usage by absorbing the fees imposed by the card platform. Relatedly, there is little evidence that platform fees went down in reaction to the possibility of card surcharging.

Other instances of subsidization of the ancillary good include free or below-cost (regular or express) shipping or delivery by retailers (online and brick-and-mortar), complimentary baggage allowances, free warranties, packaging or gift wrapping, clothing adjustments, return and refund policies, or free broadcasting of sporting events to patrons of bars and restaurants. Rather than a source of hold-up, ancillary goods may be *give-aways*.

We argue that the key to explaining these observations and to unifying these conflicting narratives is the seller’s concern about *missed sales*, whereby some consumers may forgo purchasing the basic good when faced with an additional fee on the ancillary one. Concerns about missed sales are empirically important in one-sided (shipping) and two-sided (card surcharges) markets. For example, Forrester (2011) reports that online retailers find free shipping expensive (from 5% to 20% of their revenue) and yet use it not to miss sales. This policy is a response to consumers’ behavior: when asked to think about the last time they did not complete the trans-

\(^1\)In the European Union, price transparency is mandated by the *Unfair Commercial Practices* directive 2005/29/EC, in the UK by the *Consumer Protection from Unfair Trading Regulations* 2008, and in the US by the FTC Act 1914 and the several (industry-specific) guides and rules issued thereafter by the Federal Trade Commission.

\(^2\)In the context of payments means, the UK and Denmark (among other countries) have constrained merchants to surcharge card transactions by no more than what they pay for a card transaction. See, for instance, the UK *Consumer Rights (Payment Surcharges) Regulations* (2012).

\(^3\)Australia pioneered this regulation in 2003. A key provision of a long-standing litigation brought by a class of US retailers in 2005 was to force Visa to allow merchants to surcharge; since January 2013, merchants in the United States are permitted to impose a surcharge on consumers when they use a credit card, except in states with laws prohibiting surcharging, such as California, New York and Massachusetts (among others).

\(^4\)Studies conducted in Sweden, the UK, the Netherlands and Australia all find that between 70% and 95% of merchants do not surcharge. Surcharging occurs in specific industries such as travel (where surcharges can reach 20 times the merchant fee). See, for example, Bolt et al (2010) and Hayashi (2012).
action after putting items in the shopping basket, they listed “shipping costs were too high” as the primary reason (44%). In the case of payments, survey data from the Netherlands indicate that 5% of consumers give up the purchase when faced with card surcharges (see Bolt et al 2010). This paper introduces the concept of missed sales in the add-on pricing literature, develops a unified framework that explains the hold-up and give-away narratives, and provides a simple and robust regulatory response.

Section 2 introduces the framework. A firm sets prices for both a basic and an ancillary good. To provide a first analysis of the missed-sales concern, the baseline, perfect-price-information model posits that all consumers are informed about both prices, and therefore cannot be held up. After observing prices, consumers consider the possibility of buying the good. Doing so entails a non-pecuniary shopping cost (which accounts for the time and effort to go to the shop, inspect/try the good, compare shipping and pickup alternatives, manage cash holdings, evaluate do-it-yourself options, coordinate with fellow travelers or hosts...). Consumers then learn the inconvenience cost of bypassing the ancillary good (snail shipping, picking up the item themselves, paying by cash, self-assembly, taking fewer luggage,...), if they do not know it already. Finally, they decide whether to buy the basic good, and possibly the ancillary good as well.

The perfect-price-information model is interesting for three reasons. First, it is a fair description of markets with a high fraction of repeat customers (Amazon, Walmart, the local grocery store). Second, one of the standard regulatory interventions - mandating price transparency - precisely aims at transforming a situation of imperfect information about the ancillary good price into a perfect information environment. Third, the analysis of missed sales in this model is a building block that will be used verbatim in the analyses of uninformed-but-rational and naïve customers.

Section 3 first assumes that the ancillary good is self-supplied, or, equivalently, supplied by a competitive upstream industry. It shows that the firm passes the cost of the ancillary good through to consumers if the (endogenous) markup on the basic good is small, but absorbs partly or fully any ancillary good cost increase if this markup is high. Moreover, firms in markets characterized by lower shoppings costs engage in more cost absorption (i.e., subsidize more the consumption of the ancillary good). Intuitively, the firm does not want to run the risk of losing sales of the basic good, and prefers to sell the ancillary good (even at a loss) provided its marginal cost is not too high. The consumption of the ancillary good is thus always (weakly) subsidized, and under a simple condition, it is even given away. This is consistent with the practice of many online retailers of offering complimentary (express) shipping (such as Amazon), and not charging for card payments even when surcharging is allowed. Relatedly, Forrester (2011) reports that retailers with generous margins on their products offer free

5Ernst and Young’s third annual online retailing report (1999) states that over 60% of shoppers have abandoned an order when shipping costs were added. Blake et al (2017) find similar evidence using experimental data generated by StubHub.com.

6In Australia, the 2010 Consumer Payments Use Study reports that around half of consumers avoid paying card surcharges by either using a different payment method (such as cash), or giving up the purchase. See Bagnall et al (2011).
(express) shipping more often than others.

One might believe that low ancillary good prices are good for consumers. They are not. The resulting high price for the basic good (following a see-saw pattern, the two prices co-vary negatively) hurts those consumers who do not buy the ancillary good. The ancillary good’s price is inefficiently low, leading to over-consumption. As our analysis reveals, a regulation that sets a floor for the ancillary good price equal to its marginal cost (i.e., a ban on loss-making sales) is welfare-maximizing. This regulation pushes the firm to lower its basic good price, and induces efficient consumption of the ancillary good, therefore eliminating externalities on non-participants (which are those consumers who buy only the basic good).

Next, we extend the baseline model to assume that the ancillary good is sold to the firm by a supplier with market power (as is often the case of shipping, broadcasting and card payments). In conformity with recent developments in these industries, we show that the supplier often has an incentive to jack up the wholesale price of the ancillary good so as to benefit from the firm’s cost absorption strategy. Accordingly, the wholesale price is marked up substantially when the surplus from the basic good is high (in which case the potential profit loss from missed sales is large). As in the case of a competitive upstream market, welfare can be increased by banning loss-making sales on the ancillary good. This regulation de facto commits the firm not to absorb wholesale price increases and reduces the market power of the ancillary good supplier. Thus a no-loss-making requirement effectively responds to two distortions: skewed price allocation by the firm in favor of the ancillary good (i.e., regulation rebalances the firm’s price structure) and a wholesale price set by the supplier substantially above cost (i.e., regulation makes the supplier more accountable for its wholesale price increases).

Section 4 considers uninformed-but-rational consumers; such consumers learn the ancillary good’s price only at the checkout point (which is more likely to occur in industries characterized by infrequent shopping, or when consumers are inattentive), but they correctly foresee whether they will be held up or not. The ancillary good’s price can now be called a “drip price,” as consumers learn prices sequentially. The new insight gleaned from the introduction of uninformed consumers is that the price of the basic good acts as a signal of the price for the ancillary good: A high price for the basic good makes the firm more wary of missed sales and thus reassures uninformed consumers about the ancillary good price.

We show that, with uninformed-but-rational consumers, give-aways (ancillary-good subsidization) obtain exactly when they do under perfect information, and so arise when the shopping cost is small or the ancillary-good production cost is large. By contrast, when there is pass-through under perfect information, that is, when the shopping cost is high or the ancillary-good cost is small, consumers are held up (i.e., the price of the ancillary good exceeds its marginal cost). This accords with empirical evidence revealing that card surcharging entails hold-ups only in industries characterized by one-time or infrequent shopping (such as gas stations, travel agencies, airport shops, car rentals, etc), where consumers are likely to have imperfect infor-

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7Drip pricing is the practice whereby a headline price is advertised at the beginning of the purchase process, following which additional fees, only avoidable at a cost to consumers, are then incrementally disclosed or “dripped.”
mation about the pricing practices adopted by the merchant. Conversely, as reported by Forrester (2011), give-aways (e.g., in the form of free shipping) are more widespread for retailers with repeat-customer bases. Regulation that combines price transparency (preventing hold-ups) with a ban on loss-making sales on the ancillary good (preventing give-aways) is strictly welfare-enhancing regardless of the levels of shopping and ancillary-good costs.

Section 5 analyzes pricing and regulation in the presence of naïve consumers, who optimistically do not foresee that they will need, or will have to pay for, an ancillary good. The analysis is exactly the same as for uninformed-but-rational consumers, except for the consumers’ choice of whether to consider buying, a decision that is based on inflated benefits from doing so. Like in the literature on shrouded attributes (Gabaix and Laibson 2006), the firm may not want to unveil the existence of a drip price, and transparency (which “opens the eyes” of consumers to drip prices) must now be mandated. Perhaps surprisingly, cost absorption by the firm may occur even when consumers are naïve, as missed sales at the checkout point constrain the firm’s ability to pass ancillary good costs through to consumers.

Bringing together the analyses of Sections 2 through 5, we then conclude that, whether consumers are informed, uninformed-but-rational, or naïve (reality involves a mix of the three), jointly imposing mandated price transparency and banning loss-making sales on the ancillary good is welfare-enhancing.

Section 6 considers the case in which the ancillary good is supplied by a two-sided platform, such as a card payment network. Like in the case of a vertical structure (in which the ancillary good supplier had no interaction with the consumer), missed sales concerns imply that the platform can set a high merchant fee without it being passed through to consumers (which is consistent with empirical evidence documenting a modest incidence of card surcharging where it is allowed). The new feature specific to a two-sided market is that this high merchant fee in turn induces the platform to offer very favorable conditions to consumers (e.g., in the form of rewards and cash-backs), who then are heavily incentivized to pay by card. Under weak conditions, there is over-consumption of card payments. The payment card literature (see Rysman-Wright 2014 for a comprehensive survey) has obtained overconsumption results (e.g., Rochet-Tirole 2002, Wright 2012, Vickers 2005’s must-take card argument, and Edelman-Wright 2015’s broader analysis) in environments in which the drip price is constrained by the platform to be equal to zero (“no surcharge rule”, also called “price coherence”, “parity” or “most-favored nation” clause). Here no such constraint is imposed. The overconsumption is entirely driven by missed sales concerns, which are absent in that literature.

Section 7 discusses the robustness of the results, and Section 8 concludes.

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8See Reserve Bank of Australia (2011).
Related Literature

This paper first relates to the theory of complementary goods pricing. Consider two goods with general demand functions \( D_i(p_i, p_j) \) exhibiting complementarity \((\frac{\partial D_i}{\partial p_j} < 0)\) and marginal costs \( c_i \). The first-order condition for the maximization of total profit \( \sum_i (p_i - c_i) D_i(p_i, p_j) = MR_i + (p_i - c_i) \frac{\partial D_i}{\partial p_i} = 0 \), where \( MR_i \) is the marginal revenue on product \( i \). The extent to which the firm is willing to sacrifice profit on good \( i \) to boost that on good \( j \) thus depends on the markup on good \( j \), and conversely. In our model, other factors affect the relative markups across goods. First, goods are fundamentally asymmetric in two ways: the consumption of the ancillary good requires that of the basic good, and there exist bypass opportunities (inversely measured by the opportunity cost of doing without the ancillary good). Moreover, the sequentiality of price discovery (in the case of drip pricing) and the strategic behavior of the ancillary-good supplier are central features to our analysis, but ignored in the pricing theory of complementary goods. Relative to this abstract theory, the context considered here allows us to obtain new results, better match the evidence on add-on pricing, and derive new public policy implications.

Through its emphasis on missed sales, our paper differs from theories of aftermarket monopolization (e.g. Chen-Ross 1993, Farrell-Shapiro 1989, Klein 1996 and Shapiro 1995). This literature is motivated by antitrust decisions in the photocopier and car dealer markets, in which the seller in the primary market forecloses the complementary market for cartridges or repairs. Because consumers are locked-in by the time they consider buying the ancillary service, a high price on it cannot lead to missed sales on the primary good. That literature emphasizes under-consumption associated with hold-ups on the ancillary service, the question of consumer sophistication and the possibility that the ancillary service is used as a metering device for price discrimination purposes. The “see-saw” relationship between the basic and ancillary good’s prices (an increase in the ancillary good price leads to a reduction in the basic good price) is present in our paper as in the aftermarket literature, or for that matter in the related literatures on pricing in network industries, or add-ons.

The possibility of missed sales is ignored in the add-on models developed by Lal and Matutes (1994), Verboven (1999) and Ellison (2005). In these papers, sellers may conceal the prices of add-ons as a way to extract more rents from consumers. Verboven (1999) constructs an equilibrium where firms engage is loss-leading prices: They advertise the basic good at a low price and charge high prices for an add-on which most consumers find too costly to forgo buying. Lal and Matutes (1994) shows that, when consumers are homogeneous, an irrelevance result obtains: The bundle of goods each consumer purchases and the total amount each consumer pays are exactly the same with add-on pricing as they are when all prices are advertised. Assuming consumers are heterogeneous in their marginal utility of income, Ellison (2005) shows that add-on pricing works as a collusive device for firms, who can sustain higher profits if committing to hide add-on prices is possible. By introducing missed sales, our work offers an explanation to the low (even negative) markups on the ancillary good practiced in many industries, and generates markedly different regulatory prescriptions.

The hold-up literature gained new impulse with the seminal paper of Gabaix and Laibson
(2006) on shrouded attributes.\textsuperscript{10} That paper, like the aftermarket and add-on literatures, assumes away missed sales, and puts a particular emphasis on the role of consumer information and sophistication. Consumers can bypass the add-on at a cost (like in our model), but this requires to be aware of the existence of an add-on. Naïves are held up and therefore profitable. They end up subsidizing the purchases of the sophisticates. Gabaix and Laibson show that opacity is more likely when the number of naïve customers increases and that it may not disappear with product market competition. Firms cannot attract business by educating myopic consumers and offering transparent pricing: once informed, a naïve is better off shopping from a firm that shrouds the add-on price because this gives her a subsidy from the remaining naïves. Heidhues et al (2016, 2017) show how consumer naïveté distorts the choice of products. Armstrong and Vickers (2012) offer a comprehensive discussion of the redistributive effects of the co-existence of naïve and informed consumers.

A related literature interprets naïveté as overoptimism, which firms take advantage of. DellaVigna and Malmendier (2006) show that overoptimism can account for otherwise surprising patterns of pricing of gym visits. Sandroni and Squintani (2007) develop a model of insurance where agents are overoptimistic about their accident probability.\textsuperscript{11}

Our theoretical recommendations certainly overstate the power of transparency. While transparency has been shown to be effective in certain environments, it also confronts several difficulties. Consumers must have enough literacy to understand. Second, firms may neutralize the effects of disclosure by highlighting other information that distracts consumers. Third, regulation must be all-encompassing so as to prevent firms from offering alternative products that are not covered by the transparency requirement. We refer to Campbell (2016) for an extensive discussion of consumer protection in financial markets.\textsuperscript{12}

This paper only touches on two-sided platforms. Our companion paper (Bourguignon et al 2017) focuses on specific platforms, those associated with card payments. Its contributions relative to this paper are related to the specificities of the payment card market. In particular, it considers the case of “price coherence” (where card surcharges, i.e., drip pricing, are ruled out by the payment system). It also focuses on an open system that maximizes the volume of card transactions, while we consider a for-profit monopoly platform that maximizes profit. Finally, it studies the merchants’ incentive to price discriminate according to the payment method (see also Wright 2003). It compares cash discounts and card surcharges,\textsuperscript{13} demonstrating a fundamental asymmetry between the two.

\textsuperscript{10}We will not review the follow-up literature, if only because that paper has well over a thousand Google Scholar citations.

\textsuperscript{11}See Spiegler (2011) for a detailed discussion of this literature.

\textsuperscript{12}On the effects of transparency, see also Kosfeld and Schüwer (2011), Agarwal et al (2014) and Grubb (2015).

\textsuperscript{13}Discounts are particularly interesting in the card payment application, as payment platforms have traditionally allowed cash discounts while prohibiting surcharges. We do not study discounts in this paper.
2 Baseline model and preliminaries

2.1 Description

There is one firm and a unit-mass continuum of consumers with unit demands and homogeneous valuation $v > 0$ for a basic good.\(^\text{14}\) Actions unfold in three stages. At stage 1, the firm sets the basic price $p$ for the basic good with marginal cost $c \geq 0$. It also sets the price $\tau$, facing the marginal cost $\gamma \geq 0$, for an ancillary good that complements the basic one. Examples of ancillary goods include card payments, (express) shipping, assembly, packaging, warranties, baggage allowances, gift wrapping, clothing adjustments/repairs, sports broadcasting in bars, maintenance, help with operating the product, or complementary benefits.

In the baseline model, consumers jointly observe, at stage 2, the basic price $p$ and the ancillary good price $\tau$ (perhaps because they are repeat customers, or because regulation mandates price transparency). By contrast, we will later assume that consumers initially observe only the basic good price, and either have rational expectations (Section 4) or are naïve (Section 5) about the firm’s incentives regarding drip pricing.

After observing the price pair $(p, \tau)$, consumers decide whether to incur the shopping cost $s \geq 0$, which enables them to complete the purchase (if they decide to do so). This may capture transportation costs (for getting to the store) or the time filling online forms, customization costs (for choosing the right model and specification), coordination costs (with family or friends when planning a trip), or other costs associated with executing the purchase (selecting among different shipping and pickup alternatives, managing cash holdings, learning about do-it-yourself options, etc). For brevity, we say that consumers who pay the shopping cost $s$ “consider purchasing the good.”

Having incurred the shopping cost, consumers learn (stage 3) their inconvenience cost $b$ of not purchasing the ancillary good,\(^\text{15}\) and then decide what to purchase (if anything). This cost is independently drawn across consumers from a cumulative distribution $G$ with support on $\mathbb{R}_+$. The density of $G$ is $g$, its hazard rate is $\lambda_G \equiv g/1-G$ and its reverse hazard rate is $r_G \equiv G$. As is standard in pricing models, we assume that the functions $b - \lambda_G^{-1}(b)$ and $b + r_G^{-1}(b)$ are continuous and increasing in $b$.

The case where the shopping cost is nil ($s = 0$) allows us to capture situations where all information to be learnt about the value of the ancillary good is already known by consumers (prior to the consideration stage).\(^\text{16}\) Consumers then de facto face a single decision; namely, which products to purchase, given the preference pair $(v, b)$ and the price pair $(p, \tau)$.

Figure 1 summarizes the timing. Our solution concept is subgame perfect equilibrium.

\(^{14}\)We discuss the consequences of heterogeneity on consumers’ valuations in Section 7, and show in the Supplementary Material that the results are robust to this possibility.

\(^{15}\)This timing assumption may reflect some ex-post uncertainty only realized at the checkout point (e.g., about cash needs), or limited/delayed cognition (e.g., consumers only reckon the value of the ancillary service when they have to decide whether to purchase it or not).

\(^{16}\)Letting $s = 0$ de facto implies that consumers observe $b$ at stage 0. In this case, no consumer would give up buying the basic good at the checkout point, and $v$ can be interpreted as its valuation net of shopping costs.
2.2 Consumer behavior and firm profit

Once at the checkout point, facing the inconvenience cost $b$, the consumer makes two interdependent decisions. First, when purchasing the basic good, she buys the ancillary good if and only if $\tau \leq b$. Moreover, she buys the basic good if and only if:

$$v \geq p + \min\{\tau, b\}.$$ 

If the inequality above is violated, the consumer forgoes buying the basic good at the checkout point, in which case a missed sale occurs. Because some consumers will face high inconvenience costs ($b > \tau$), no missed sales occur at the price pair $(p, \tau)$ if and only if

$$v - p \geq \tau.$$ 

We refer to condition (1) as the no-missed-sale condition. In particular, if this condition is violated, no consumer will buy the ancillary good (regardless of $b$), which is equivalent to the firm not offering it.

Note that the shopping cost $s$ is sunk at the checkout point, and therefore does not influence the consumer’s purchasing decision. By contrast, the decision to pay the shopping cost $s$ is taken ex ante. Accordingly, consumers optimally consider purchasing the good if and only if $s$ is no greater than the anticipated surplus from reaching the checkout point:

$$s \leq \mathbb{E}\left[\max\{v - p - b, v - p - \tau, 0\}\right],$$

which we refer to as the consideration constraint.

To describe profit, note that, if the consideration constraint (2) evaluated at $(p, \tau)$ is violated, no consumer considers purchasing the good, and there is no profit: $\Pi (p, \tau) = 0$. Otherwise, the firm’s profit equals

$$\Pi (p, \tau) = 1_{\{v - p \geq \tau\}} [p - c + (1 - G(\tau))(\tau - \gamma)] + 1_{\{v - p < \tau\}} G(v - p) (p - c),$$
where \(1_{\{v - p \geq \tau\}}\) is the indicator function that equals 1 if the inequality \(v - p \geq \tau\) is verified, and 0 otherwise. Recall that, if the no-missed-sale condition (1) is satisfied, all consumers buy the basic good, but only a fraction \((1 - G(\tau))\) buy the ancillary good. The profit is then given by the first term in the right-hand side of (3). In turn, if the no-missed-sale condition (1) is violated, no consumer buys the ancillary good. In this case, only a fraction \(G(v - p)\) of consumers buys the basic good, and missed sales occur. Profits are then given by the second term in the right-hand side of (3).

### 2.3 Transaction costs and missed sales

When the no-missed-sale condition (1) is satisfied (i.e., \(v - p - \tau \geq 0\)), the transaction cost function

\[
T(\tau) \equiv s + \mathbb{E}[\min\{\tau, b\}]
\]

describes the expected cost that consumers face if they consider purchasing the good. In this case, the consideration constraint (2) is equivalent to

\[
v - p \geq T(\tau).
\]

Intuitively, consumers consider purchasing the good if and only if the anticipated transaction cost (generated by the shopping cost combined with the ancillary good price or the nuisance cost of avoiding this extra fee) does not exceed the gross surplus \(v - p\).

By contrast, when the no-missed-sale condition (1) is violated (i.e., \(v - p - \tau < 0\)), the consideration constraint (2) is equivalent to

\[
v - p \geq T(v - p).
\]

Intuitively, from the consumers’ perspective, the possibility of not purchasing the basic good at the checkout point is equivalent to facing a price for the ancillary good that is equal to the gross surplus \(v - p\). Accordingly, consumers consider purchasing the good if the transaction cost (evaluated at \(v - p\)) is not greater than the gross surplus \(v - p\).

The consumer’s decision to consider purchasing the good and the occurrence of missed sales naturally depend on the level of the ancillary good price \(\tau\). In this respect, it is convenient to define the critical price \(\hat{\tau}\) that solves the fixed-point equation

\[
T(\tau) = \tau.
\]

Because the function \(T\) is increasing and concave, the critical price \(\hat{\tau}\) is unique and \(T(\tau) \leq \tau\) if and only if \(\tau \geq \hat{\tau}\) (see Figure 2 for an illustration). Note also that the critical price \(\hat{\tau}\) is an increasing function of \(s\), and that \(\hat{\tau} = 0\) when \(s = 0\).

To rule out uninteresting cases, we impose the following condition on the primitives of the economy:
Assumption 1 $v - c > \hat{\tau}$.

This assumption says that the firm is able to reap positive profits even if it does not offer the ancillary good (or, equivalently, if the no-missed-sale condition (1) is violated).\(^{17}\)

3 The rationale for give-aways

This section explores the role of missed sales in determining equilibrium pricing strategies.

3.1 Firm behavior

The decision of a firm to sell the ancillary good depends on how its profit compares in case the no-missed-sale condition (1) is satisfied or violated. In the former case, the firm’s profit is:

$$
\Pi^*(\gamma, c) \equiv \max_{p, \tau} \{p - c + (1 - G(\tau))(\tau - \gamma)\} \quad \text{subject to} \quad v - p \geq \max\{\tau, T(\tau)\},
$$

where the inequality constraint captures the no-missed-sale condition (1) and the consideration constraint (2). When missed sales occur, the ancillary good price does not affect the firm’s profit (as long as $v - p < \tau$), and the firm’s profit is

$$
\Pi^{ms}(c) \equiv \max_p G(v - p)(p - c) \quad \text{subject to} \quad v - p \geq T(v - p),
$$

where the inequality constraint describes the consideration constraint (2) when missed sales occur. Comparing the solutions of these two problems, and their associated profit levels, the next proposition characterizes the optimal firm behavior.

**Proposition 1 (optimal firm behavior)** In the unique equilibrium, the ancillary good is offered by the firm if and only if the bundle is viable: $\gamma \leq v - c$. In this case, the price profile $(p^*, \tau^*)$ is such that:

\(^{17}\)Obtaining a positive profit while not offering the ancillary good is possible if and only if the consideration constraint is slack at $p = c$. This requires that $v - c > T(v - c) \iff v - c > \hat{\tau}$, as in Assumption 1.
(i) **Complete pass-through:** If $\gamma < \hat{\gamma}$, then $\tau^* = \gamma$ and $p^* = v - T(\gamma)$. The no-missed-sale condition is slack and the consideration constraint binds.

(ii) **Zero pass-through:** If $\hat{\gamma} \leq \gamma \leq \hat{\gamma}$, where $\hat{\gamma} \equiv \hat{\tau} + r^{-1}_G(\hat{\tau})$, then $\tau^* = \hat{\tau}$ and $p^* = v - \hat{\tau}$. Both the no-missed-sale condition and the consideration constraint bind.

(iii) **Incomplete pass-through:** If $\gamma > \hat{\gamma}$, then $\tau^*$ uniquely solves

$$\tau - \gamma = -\frac{1}{r_G(\tau)},$$

and $p^* = v - \tau^*$. The no-missed-sale condition binds, and, despite ex-ante symmetric information, consumers enjoy a rent.

The identity “profit = social surplus – consumer net surplus” implies that the choice of ancillary price obeys the familiar trade-off between efficiency and consumer rent extraction. Efficiency considerations first suggest that the ancillary good will not be put up for sale if the full package generates negative gains from trade (that is, $v - c < \gamma$). In this case, the gain from avoiding missed sales outweighs the loss from selling the ancillary good below cost. Conversely, as revealed by Proposition 1, it is always profitable for the firm to offer the ancillary good provided the “bundle” (basic + ancillary good) is viable (i.e., $v - c \geq \gamma$).

The ideal case for the firm is when it can conciliate efficiency and rent extraction. Efficient consumption of the ancillary good requires marginal cost pricing ($\tau = \gamma$); but efficiency also requires that the basic good be consumed, and therefore the absence of missed sales ($v - p \geq \tau$). In turn, rent extraction is maximal when the consideration constraint binds: $v - p = T(\tau)$. These two goals are compatible if and only if $T(\gamma) \geq \gamma$, or, equivalently, if and only if the cost of producing the ancillary good is small enough ($\gamma \leq \hat{\tau}$). In this case, the firm just passes the cost of the ancillary good through to consumers and fully extracts the consumer surplus through its basic price. This “sell-out contract” or “perfect two-part tariff” allows the firm to reach the highest possible profit, as in Bernheim and Whinston (1986, 1998).

When the cost $\gamma$ exceeds the critical threshold $\hat{\tau}$, pricing reflects two opposite concerns. On the one hand, the firm would like to raise the ancillary good price $\tau$ to match any increase in $\gamma$ (thus avoiding overconsumption of the ancillary good). But doing so implies either missed sales (and the concomitant loss of mark-up on the basic good) if the consideration constraint is binding, or lowering the basic price so as to keep satisfying the no-missed-sale condition with the unwanted consequence of leaving a positive rent to the consumers (making the consideration constraint slack). Over a range starting at the threshold $\hat{\tau}$, the efficiency loss is second order, while those losses associated with missed sales and rent extraction are first order. Resolving this trade-off then entails full cost absorption: the ancillary good price $\tau$ remains equal to the critical level $\hat{\tau}$.

As the cost keeps increasing ($\gamma > \hat{\gamma}$), the overconsumption creates too high an inefficiency loss, and the firm raises the ancillary good price. However, the basic price and the ancillary good price need to covary negatively one-for-one, as the no-missed-sale condition binds: $v = \gamma$. ...
Figure 3: The equilibrium ancillary good price (left-side panel) and basic price (right-side panel) as a function of the marginal cost $\gamma$ of the ancillary good (assuming $v - c > \hat{\gamma}$).

$p + \tau$. Because a unit increase in the basic good price affects all consumers, whereas the demand for the ancillary good is elastic, it is then optimal to absorb some of the cost increase (partial pass-through), as captured by the first-order condition (6). Accordingly, consumers enjoy a positive rent (in expectation) when considering purchasing the good, despite the fact that, at the time when prices are set, information is symmetric between consumers and the firm.

The characterization from Proposition 1 is summarized in Figure 3. The complete proof of this proposition (as well as of all others) appears in the appendix.

The result above accords with the practice of many online retailers (such as Amazon) of offering ancillary services (such as shipping) below cost to consumers. Only concerns about missed sales can explain this behavior. The next corollary investigates the effect of shopping costs on the equilibrium price of the ancillary good.

Corollary 1 (shopping costs) Holding the marginal cost of the ancillary good fixed at $\gamma = v - c$, its equilibrium price $\tau^*$ increases with the shopping cost $s$, and strictly so whenever $\tau^* = \hat{\tau}$.

According to Corollary 1, firms in markets characterized by lower shopping costs engage in more cost absorption. The next corollary reveals that our analysis is consistent with the common practice among online retailers (for whom shopping costs are arguably low) of offering ancillary goods at no cost (Amazon France, for instance, offers regular shipping for free, or, if the order contains books, for 1 cent, as recent regulation banned free shipping).

Corollary 2 (free shipping) Let $\gamma = v - c$ and suppose the shopping cost is zero ($s = 0$). Then, in equilibrium, the no-missed-sale condition always binds, and the ancillary good is sold strictly below cost by the firm. Moreover, if $r_G(0) < \infty$, the ancillary good is provided for free ($\tau^* = 0$) whenever its marginal cost is not too high: $\gamma \leq \hat{\gamma} = r_G^{-1}(0)$.

When there are relatively many consumers with negligible - or even negative - inconvenience costs (as implied by the requirement that $r_G(0) < \infty$), the ancillary good is provided

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18 It can also be shown that the equilibrium ancillary good price $\tau^*$ increases as the distribution of the inconvenience cost $b$ increases (in the sense of reverse hazard rate dominance).

19 This may be reasonable in some applications: some consumers prefer to pay by cash rather than by card, or prefer snail rather than express shipping (to avoid having to wait at home for delivery), or have a parent who enjoys
for free (i.e., $\tau^* = 0$) when the shopping cost is zero. This occurs because raising the ancillary good price above zero would bring little extra revenues from sales of the ancillary good, but generate substantial revenue losses from the reduction in the basic good price (which in needed to satisfy the no-missed-sale condition).  

**Remark 1 (free shipping)** In practice, free shipping might occur even when the conditions of Corollary 2 are violated. Suppose that the optimal ancillary good price obtained in Proposition 1 is low but positive; yet, the firm may nonetheless set it equal to zero for reasons outside the model: for example it may not want to “nickel and dime” the consumer; or it may attempt to make the low price more salient and memorable by setting it equal to zero. Alternatively, (express) shipping may be offered for free if consumers join loyalty-enhancing programs, such as Amazon Prime.  

### 3.2 Vertical chain

The ancillary good often is not directly produced by the firm, but provided by a third party with market power. Examples include payment cards, shipping, broadcast, internet access, etc. In this case, the ancillary good supplier (for short, “supplier”) sets its wholesale price anticipating the firm’s pricing decisions and the consumers’ reaction to the ancillary good price. We accordingly add to the baseline model of Section 2 a stage 0 where the supplier chooses the wholesale price $\gamma$ (which corresponds to the marginal cost perceived by the firm) facing a marginal cost of production equal to $\delta \geq 0$. The analysis thus far corresponds to the case where $\gamma \equiv \delta$, as it occurs in environments where the ancillary good is self-supplied by the firm, or provided in a perfectly competitive upstream market.

Let us denote by $\Xi(\gamma; \delta)$ the profit obtained by the supplier when the wholesale price is $\gamma \leq v - c$ (as otherwise the ancillary good is not sold, and profit is zero). Applying the characterization of Proposition 1, we obtain that

$$
\Xi(\gamma; \delta) = \begin{cases} 
(1 - G(\gamma))(\gamma - \delta) & \text{if } \gamma < \hat{\tau} \\
(1 - G(\hat{\tau}))(\gamma - \delta) & \text{if } \gamma \in [\hat{\tau}, \hat{\gamma}] \\
(1 - G(\tau^*(\gamma)))(\gamma - \delta) & \text{if } \gamma > \hat{\gamma},
\end{cases}
$$

assembling the furniture herself. It may also be a matter of indifference ($b = 0$), as when a baggage allowance is not needed. For other applications, though, assuming the reverse hazard rate $r_G$ is bounded is not warranted (it is hard to imagine that an online customer might prefer no shipping at all, i.e. to pick up the good at the warehouse).

One can argue that in some contexts $s$ and $b$ are likely to be negatively correlated: a high shopping cost raises the consumer’s incentive to search for alternatives for the ancillary good prior to the consideration stage. For example, it may be easier to get cash at an ATM before starting the shopping process than once in the store. In that case, an increase in the shopping cost has a direct positive effect and an indirect negative effect on the ancillary good price (due to consumers’ precautionary behavior). Under weak conditions, our policy implications are robust to this possibility (the reasoning is similar to the case of an elastic discussed in the Supplementary Material).

Einav et al (2015) find on eBay data that moving from a small shipping charge to free shipping increases the expected auction price substantially (while they confirm the earlier finding in the literature that consumers under-weight strictly positive shipping fees relative to the basic price, assigning a relative weight of only 82%).

Although entailing an up-front payment by consumers, Amazon Prime is responsible for a substantial part of the net shipping losses incurred by Amazon, which reached 7.2 billion dollars according to the company’s 2016 SEC fillings. See www.geekwire.com/2017/true-cost-convenience-amazons-annual-shipping-losses-top-7b-first-time.
where, for $\gamma > \hat{\gamma}$, $\tau^*(\gamma)$ solves the first-order condition (6). Over the range $\gamma < \hat{\gamma}$, the firm sets $\tau^* = \gamma$ (i.e., the pass-through rate is one), and the supplier obtains the standard monopoly profit. When $\gamma \in [\hat{\tau}, \hat{\gamma}]$, the equilibrium pass-through is zero (as $\tau^*$ is constant at $\hat{\tau}$), and the supplier’s profit is linearly increasing in $\gamma$. Finally, for sufficiently high wholesale prices, satisfying $\gamma > \hat{\gamma}$, the pass-through is again positive, but incomplete.

To analyze the supplier’s problem, let us consider first the complete pass-through region ($\gamma < \hat{\tau}$). Whenever a local maximum exists in this region, it has to equal the monopoly price $\tau^m(\delta)$, which is the unique value of $\gamma$ that satisfies the standard pricing formula:

$$
\gamma - \delta = \frac{1}{\lambda_G(\gamma)}.
$$

Note that $\tau^m(\delta)$ is nothing but the hold-up price that would be charged by the firm if it faces marginal cost $\delta$ and does not fear missing sales on the basic good (as in the aftermarket or shrouded attributes literatures, in which the basic good has already been sold when the ancillary good is offered for sale). Because $\tau^m(\delta)$ is increasing in $\delta$, it follows that $\tau^m(\delta) < \hat{\tau}$ if and only if $\delta < \hat{\delta} \equiv \hat{\tau} - \lambda_G^{-1}(\hat{\tau})$. In this case, conditional on the firm selling the ancillary good, the supplier’s profit has two peaks: one at the monopolist price $\tau^m(\delta)$, and the other in the giveaway region. Moreover, the second peak satisfies $\gamma > \hat{\gamma}$, as the supplier’s demand is inelastic for $\gamma \in [\hat{\tau}, \hat{\gamma}]$. By contrast, when $\delta \geq \hat{\delta}$, the supplier’s profit has a single peak satisfying $\gamma \geq \hat{\gamma}$.

These two possibilities are illustrated in Figure 4.

The optimal wholesale price $\gamma^*$ then solves:

$$
\max_{\gamma} \Xi(\gamma; \delta) \quad \text{subject to} \quad \gamma \leq v - c,
$$

where the inequality constraint in (8) guarantees that the firm is willing to sell the ancillary good. We refer to it as the firm’s acceptance constraint. The higher the gross surplus $v - c$,
the smaller the firm’s resistance to selling the ancillary good (i.e., the slacker is the acceptance constraint). To guarantee that the supplier is willing to operate, we assume that $v - c \geq \delta$.

The wholesale price set by the supplier depends crucially on the firm’s resistance to selling the ancillary good (or, equivalently, on how tight the acceptance constraint is). To see how, consider first the case where the supplier’s profit has two peaks (i.e., $\delta < \bar{\delta}$). If the firm’s resistance is low, the supplier sets the wholesale price in the cost-absorption region (i.e., $\gamma > \bar{\tau}$), exploiting the fact that missed sales concerns prevent the firm from passing high wholesale prices through to consumers. This is captured by the condition $v - c > \bar{\gamma}(\delta)$, where $\bar{\gamma}(\delta)$ is the wholesale price in the cost-absorption region that produces the same profit to the supplier as the monopolist price $\tau^m(\delta)$ (see Figure 4). In case the supplier’s profit has a single peak (i.e., $\delta \geq \bar{\delta}$), the wholesale price is always in the firm’s cost-absorption region (and it will lie in the zero-pass-through region only if the acceptance constraint binds). The next proposition summarizes this discussion.

**Proposition 2 (equilibrium wholesale price)** Suppose that a monopolistic supplier sets a wholesale price $\gamma$ for the ancillary good, facing a marginal cost $\delta \leq v - c$. Then there exists a threshold $\bar{\gamma}(\delta) \geq \bar{\tau}$ such that, in the unique equilibrium:

(i) If $v - c \leq \bar{\gamma}(\delta)$, the wholesale price is the monopoly price, $\gamma^* = \tau^m(\delta)$, and the ancillary good is sold at cost: $\tau^* = \gamma^*$.

(ii) Otherwise, the wholesale price is in the firm’s cost-absorption region, $\gamma^* \in (\bar{\tau}, v - c]$, and the retail price of the ancillary good is below cost: $\tau^* < \gamma^*$. The firm implicitly subsidizes the consumption of the ancillary good.

Moreover, the threshold $\bar{\gamma}(\delta)$ is weakly decreasing, with $\bar{\gamma}(\delta) = \bar{\tau}$ whenever $\delta \geq \bar{\delta}$.\(^\text{24}\)

The next corollary applies Proposition 2 to investigate the effect of shopping costs on the equilibrium wholesale price.

**Corollary 3 (shopping costs)** The equilibrium wholesale price $\gamma^*$ weakly decreases with the shopping cost $s$.

As the shopping cost decreases, the firm engages in more cost absorption (for a given wholesale price), which, in turn, induces the supplier to further raise the wholesale price.

### 3.3 Give-aways: Welfare and regulation

In this subsection, we analyze the welfare properties of equilibria and discuss regulation. Our social welfare measure accounts for the surplus jointly obtained by consumers and the firm, but does not account for that obtained by the platform providing the ancillary good (if any).\(^\text{25}\) This is of course inconsequential if the ancillary good is internally provided by the firm or if the

\(^{24}\)Note that, if $\delta \geq \bar{\delta}$, then $\bar{\gamma}(\delta) = \bar{\tau}$, which implies that $v - c > \bar{\gamma}(\delta)$ by Assumption 1.

\(^{25}\)For instance, the supplier might be a foreign company. More generally, the weight on the supplier’s profit may depend on how much of the profit is a rent distributed to investors, employed into wasteful advertising,
Figure 5: As a function of the marginal cost of production \( \delta \), the full curve depicts the ancillary good price when the upstream market is monopolistic, the dashed curve depicts the ancillary good price when the upstream market is competitive (in which case \( \gamma \equiv \delta \)), and the dotted curve depicts the wholesale price when the upstream market is monopolistic. It is assumed that \( \bar{\tau} < v - c < \bar{\gamma} \), so that the partial pass-through region of Figure 3 does not exist.

upstream market is competitive (in which case \( \gamma \) coincides with marginal cost of production \( \delta \)). To avoid repetitions, we abuse language and refer to either case as that of a competitive upstream market.

To describe the welfare measure, let the price profile \((p, \tau)\) satisfy the no-missed-sale condition (1) and the consideration constraint (2). The aggregate social welfare when the firm pays \( \gamma \) for the ancillary good is then

\[
W(\gamma, \tau) \equiv v - s - c - \int_{0}^{\tau} bg(b)db - (1 - G(\tau))\gamma.
\]

Note that the penultimate term captures the inconvenience costs faced by consumers who do not buy the ancillary good (i.e., those for which \( b \leq \tau \)), whereas the last term accounts for the firm’s procurement cost.

When the upstream market is competitive (in which case \( \gamma \equiv \delta \)), social welfare is solely a function of the equilibrium ancillary good price \( \tau \), and it is given by \( W(\delta, \tau) \). By contrast, when the upstream market is monopolistic, welfare depends on the equilibrium wholesale and retail price of the ancillary good (\( \gamma \) and \( \tau \), respectively). It should come as no surprise that the social welfare function \( W(\gamma, \tau) \) decreases with \( \gamma \).

To isolate the inefficiencies that are intrinsic to ancillary good pricing (as opposed to those that arise if the supplier has market power), we shall proceed in two steps. We will first hold constant the wholesale price \( \gamma \), and derive the ancillary good price \( \tau \) that maximizes social welfare \( W(\gamma, \tau) \). This enables us to identify the key distortion produced by ancillary good pricing or a reward for investment that benefits consumers. See Tirole (2011) for a discussion, in the context of the card payment application, of whether the payment system profits should be included into the measure of social welfare. The welfare comparisons to follow are robust to social welfare specifications of the form \( CS + \Pi + \alpha \Xi \), where \( CS \) is consumer surplus and \( \Pi \) is the firm’s profit, provided the weight \( \alpha \) on the supplier’s profit \( \Xi \) is small.
pricing in the presence of missed-sale concerns.

**Corollary 4 (over-consumption of the ancillary good)** Holding constant the wholesale price $\gamma$, pricing the ancillary good at cost, $\tau^w = \gamma$, maximizes welfare. As a result, the ancillary good is over-consumed in equilibrium (i.e., $\tau^* < \gamma^*$) provided:

(i) either the upstream market is competitive and the marginal cost of the ancillary good lies in the firm’s cost absorption region: $\delta > \bar{\tau}$;

(ii) or the supplier is a monopolist and firm resistance is low: $v - c > \tilde{\gamma}(\delta)$.

If neither condition holds, the ancillary good is efficiently consumed (as marginal-cost pricing prevails).

It follows from Corollary 4, and the fact that $W(\gamma, \tau)$ decreases with $\gamma$, that social welfare in equilibrium is always (weakly) lower than $\bar{W} \equiv W(\delta, \delta)$, which obtains when the ancillary good is purchased at cost by the firm ($\gamma = \delta$), and sold at its wholesale price to consumers ($\tau = \gamma$). Departures from efficiency entail violating either (or both) condition(s).

The price-balancing distortion identified in Corollary 4, whereby the ancillary good price is inefficiently low relative to the firm’s cost of provision $\gamma$, occurs irrespective of whether the upstream market is competitive or monopolistic. In addition to this distortion, a second inefficiency arises when the supplier has market power. The occurrence of both distortions is illustrated in Figure 5. When the upstream market is monopolistic and the firm resistance is low ($v - c > \tilde{\gamma}(\delta)$), not only the wholesale price is above the marginal cost of production $\delta$ (market-power distortion), but the incidence of prices is inefficient (as the ancillary good is over-consumed relative to the firm’s cost of provision $\gamma$). By contrast, if the firm resistance is high ($v - c \leq \tilde{\gamma}(\delta)$), only the market-power distortion is present. Finally, if the upstream market is competitive, the price-balancing distortion arises if and only if the marginal cost of the ancillary good is sufficiently high (i.e., $\delta > \bar{\tau}$).

One natural way to mitigate both distortions is to prevent the firm from engaging in cost-absorption, i.e. to mandate it to sell the ancillary good at no less than $\gamma$. The next proposition assesses the welfare impact of this regulation, both when the upstream supplier is competitive or monopolistic. Recall that $\Pi^{ms}(c)$ is the firm’s profit when the ancillary good is not offered, as defined in equation (5). Below, we use the superscript $r$ (for “regulation”) to denote equilibrium outcomes under the regulation that bans loss-making sales on the ancillary good.

**Proposition 3 (ban on loss-making sales)** Consider a regulation that imposes a floor on the ancillary good price set by the firm according to $\tau \geq \gamma$, and assume that $\delta \leq v - c - \Pi^{ms}(c)$.

(i) If the upstream market is competitive, this regulation binds provided $\delta > \bar{\tau}$, in which case welfare increases to its maximal level $\bar{W}$. Relative to laissez-faire, the ancillary good price increases to its efficient level, and the basic price decreases. The firm is strictly worse off and consumers are strictly better off.
(ii) If the upstream market is monopolistic, this regulation increases welfare, strictly so when \( v - c > \gamma^*(\delta) \). Relative to laissez-faire, there is a decrease in the wholesale price of the ancillary good, which is then priced at cost by the firm:

\[
\gamma^* > \gamma^r \quad \text{and} \quad \tau^* - \gamma^* < \tau^r - \gamma^r = 0.
\]

The supplier is strictly worse off, while consumers and the firm are weakly better off provided \( \gamma^* \leq \hat{\gamma} \).

To understand Proposition 3, consider the firm’s problem under regulation. First, note that, whenever the ancillary good is offered by the firm, the floor constraint \( \tau \geq \gamma \) always binds at the optimum (this follows from the quasi-concavity of the objective and the fact that the firm always prices the ancillary good at or below cost under laissez-faire). As a consequence, if the no-missed-sale constraint (11) is satisfied, the choice of the basic price \( p \) solves

\[
\max_p \{ p - c \} \quad \text{subject to} \quad v - p \geq \max\{ \gamma, T(\gamma) \}.
\]

The firm’s profit when the ancillary good is offered is then equal to

\[
\Pi^r(\gamma, c) = v - \max\{ \gamma, T(\gamma) \} - c,
\]

which is greater than \( \Pi^{ms}(c) \) if and only if \( v - c - \Pi^{ms}(c) \geq \gamma \). This reveals that the firm’s resistance is higher under regulation, as the firm is willing to carry the ancillary good for a smaller range of \( \gamma \)'s. To guarantee that regulation does not shut down the market for the ancillary good, we assume that \( \delta \leq v - c - \Pi^{ms}(c) \). This weak assumption is satisfied whenever the cost of production of the ancillary good is a small fraction of the welfare gain generated by it (which is arguably the case with card payments and shipping, for example).

**Rebalancing effect.** When the upstream market is competitive (\( \gamma = \delta \)), regulation makes the firm weakly worse off (strictly so if \( \delta > \hat{\tau} \)), as banning loss-making sales constrains its pricing choices. Consumer surplus however increases, as regulation eliminates the price-balancing distortion, thus reducing the basic price charged by the firm. By correcting *externalities on non-participants* (those consumers who only buy the basic good), the ban on loss-making sales raises welfare to its maximal level \( \bar{W} \).

**Accountability effect.** When the upstream market is monopolistic, banning loss-making sales is akin to enabling the firm to commit to a high pass-through rate, which, in turn, makes the supplier internalize the equilibrium effects of high wholesale prices. To understand how, note

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26When \( \gamma^* > \hat{\gamma} \), regulation increases welfare, but might hurt either consumers or the firm. The proof in the appendix derives conditions for either case.

27To understand why, note that \( \Pi^r(\gamma, c) = \Pi^{ms}(c) \) only if \( \gamma > \hat{\tau} \), in which case \( \Pi^r(\gamma, c) = v - \gamma - c \).

28Putting things differently, this assumption requires that a firm that internally produces the ancillary good would be better off by fully appropriating the gains from trade, while offering the ancillary good for free, rather than not offering it and missing sales.
that, under regulation, the supplier chooses a wholesale price $\gamma$ to

$$
\max_{\gamma} \{ (1 - G(\gamma)) (\gamma - \delta) \} \quad \text{subject to} \quad \gamma \leq v - c - \Pi^{ns}(c).
$$

Figure 6 illustrates the supplier’s profit under regulation and laissez-faire. Note that, in the range where $\gamma > \hat{\tau}$, the supplier’s profit is uniformly smaller under regulation than laissez-faire. This change is due to the fact that the ancillary good price is now constrained to reflect the wholesale price faced by the firm, which leads to a smaller consumption of the ancillary good. Moreover, because the pass-through on $\gamma$ is complete under regulation, the marginal profit obtained by the supplier from increasing $\gamma$ goes down. Together with the fact that the firm acceptance constraint is tighter under regulation, this implies that the wholesale price decreases relative to laissez-faire.

By requiring the ancillary good price to be no less than $\gamma$, regulation also eliminates the price-balancing distortion (as $\gamma^r - \tau^r = 0$). In conjunction with the reduction of the wholesale price, this implies that floor regulation strictly increases welfare (which is not maximal, however, as the wholesale price remains above the marginal cost of production due to supplier market power).

**Discussion.** When the supplier is a monopolist and $\gamma^* \leq \hat{\tau}$, both consumers and the firm benefit from regulation. This stands in contrast to the case of a competitive upstream market, where the firm always loses from a ban on loss-making sales. The accountability effect explains this difference (as the reduction in the wholesale price set by the supplier compensates the firm’s loss from facing the no-loss-making constraint on the retail price of the ancillary good).

**Remark 2 (market shutdown)** When the marginal cost of production of the ancillary good is sufficiently high (i.e., $\delta > v - c - \Pi^{ns}(c)$), which we argued is unlikely, banning loss-making sales makes the firm stop selling it, which is detrimental to welfare. Whether the upstream market is competitive or monopolistic, the firm is worse off relative to laissez-faire, while consumers are better off. This is immediate when the consideration constraint binds under laissez-faire in equilibrium (as consumer surplus is zero). When the no-missed-sales condition binds (in which case the net utility from buying the ancillary good is nil), consumers are also strictly better off, as the basic price decreases as a result of regulation.
4 From give-aways to hold-ups

Hold-ups at the checkout point, whereby the firm sells the ancillary good at a positive markup, are as ubiquitous as give-aways, whereby the firm subsidizes the consumption of the ancillary good. At first sight, this observation might appear at odds with the analysis of the previous section, which predicts that ancillary good prices are always at or below the marginal cost faced by the firm. The key to this discrepancy is the assumption that consumers enjoy perfect information about the ancillary good price. While reasonable in industries characterized by frequent shopping, this assumption is likely to fail in markets characterized by one-time (or sporadic) shopping, or where consumers are inattentive or face costs to absorb price information.

In order to better understand the occurrence of hold-ups, as opposed to give-aways, we now depart from the baseline model of Section 2 in assuming that consumers have imperfect information about the price of the ancillary good. The basic price, by contrast, is observed by consumers before the checkout point (for instance, through advertising or price posting at the beginning of the purchase process). Apart from this modification, the analysis below coincides with that of Section 2. The ancillary good price \( \tau \) should now be called a “drip price,” as consumers learn prices sequentially.

Because of imperfect information, consumers consider purchasing the good if the consideration constraint (2) is satisfied at the anticipated drip price \( t \), as opposed to the actual drip price \( \tau \). In this case, the firm’s profit at the price pair \( (p, \tau) \) is \( \Pi(p, \tau) \), as described in equation (3). By contrast, if the consideration constraint is violated at \( t \), the profit is zero.

To properly account for imperfect information, we look for a Perfect Bayesian equilibrium (which will turn out to be unique). An equilibrium is a triple \( (p^*, \tau^*, t^*(\cdot)) \), where the price-signaling function \( t^*(p) \) describes the drip price anticipated by consumers following \( p \). Equilibrium behavior requires that for all \( p \)

\[
t^*(p) \in \arg \max_{\tau} \Pi(p, \tau),
\]

that is, consumers correctly anticipate the drip price practiced by the firm following any basic good price announcement. Because the firm will never choose a basic price that leads to a violation of the consideration constraint (2) evaluated at \( \tau = t^*(p) \), optimality requires that

\[
p^* \in \arg \max_p \Pi(p, t^*(p)).
\]

The equilibrium drip price is then \( \tau^* = t^*(p^*) \).

4.1 Price signaling and firm behavior

Imperfect information about the drip price may generate a commitment problem for the firm, which cannot promise consumers that they will not be held up at the checkout point. To see why, recall that, under perfect information, the firm’s optimal policy in the complete pass-through region (i.e., \( \gamma < \hat{\tau} \)) is to price the ancillary good at marginal cost, \( \tau = \gamma \), and fully
extract rents from consumers by means of the basic price $p = v - T(\gamma)$ (as illustrated in Figure 3). This is no longer possible under imperfect information: The no-missed sale condition is slack at any such price pair, giving the firm a strict incentive to raise the drip price above $\gamma$ so as to extract more rents from consumers who have already incurred the shopping cost $s$.

By contrast, in the cost-absorption region (i.e., $\gamma \geq \hat{\tau}$), the commitment problem does not arise, as the no-missed sale condition binds at the optimum under perfect information. So, for $\gamma \geq \hat{\tau}$, the analysis of Section 3 holds unchanged, and Proposition 1 applies.

The next result, illustrated by Figure 7, describes how equilibrium prices under imperfect information vary with the marginal cost $\gamma$ of the ancillary good. Recall that $\tau_m(\gamma)$ is the monopoly price for marginal cost $\gamma$ and that $\lambda^{-1}_G(\hat{\tau})$ (in which case $\tau_m(\gamma) \geq \hat{\tau}$ if and only if $\gamma \geq \hat{\delta}$).

Proposition 4 (Optimal firm behavior under imperfect information) In the unique equilibrium:

(i) **Hold-up:** If $\gamma \leq \hat{\delta}$, the ancillary good is sold at the monopoly price, $\tau^* = \tau_m(\gamma)$, and the consideration constraint binds: $p^* = v - T(\tau_m(\gamma))$.

(ii) **Constrained hold-up:** If $\gamma \in (\hat{\delta}, \hat{\tau})$, the ancillary good is sold above marginal cost, at the critical price $\tau^* = \hat{\tau}$, and the consideration constraint binds: $p^* = v - \hat{\tau}$.

(iii) **Give-away (zero pass-through):** If $\gamma \in (\hat{\tau}, \hat{\delta})$, the ancillary good is sold below marginal cost, at the critical price $\tau^* = \hat{\tau}$, and the consideration constraint binds: $p^* = v - \hat{\tau}$.

(iv) **Give-away (incomplete pass-through):** If $\gamma \in (\hat{\delta}, v - c)$, the ancillary good is sold below marginal cost, at the price $\tau^*$ satisfying (6), and $p^* = v - \tau^*$. Despite ex-ante symmetric information, consumers enjoy a rent.

(v) **Missed sales:** If $\gamma > v - c$, the firm gives up selling the ancillary good, missed sales occur, and the basic price solves (5).

As illustrated by Figure 7, the drip price under imperfect information lies strictly above its perfect information level whenever $\gamma < \hat{\tau}$ (which corresponds exactly to the complete pass-through region in Figure 3). To understand this equilibrium outcome, let us consider first the

$^{29}$Namely, $\tau_m(\gamma)$ is the unique value of $\tau$ that solves $\tau - \gamma = \lambda^{-1}_G(\tau)$.
(unconstrained) hold-up case, where \( \gamma \leq \delta \). Under this condition, consumers anticipate the drip price to be at the monopoly level, \( t^*(p) = \tau^m(\gamma) \), following the announcement of any basic price \( p \) at which the consideration constraint is satisfied: \( p \leq v - T(\tau^m(\gamma)) \).\(^{30}\) At any basic price above this level, no consumer considers purchasing the good. Therefore, the firm optimally sets \( p^* = v - T(\tau^m(\gamma)) \).

Now consider the constrained hold-up case, where \( \gamma \in (\delta, \tilde{\tau}) \). In this case, if the basic price is large, the monopolistic drip price \( \tau^m(\gamma) \) would violate the no-missed-sale condition. Foreseeing that the firm wants to avoid missing sales, consumers (correctly) anticipate the drip price to be \( t^*(p) = \min\{\tau^m(\gamma), v - p\} \) following the announcement of any basic price \( p \) such that \( p \leq v - \tilde{\tau} \). For \( p \) above this level, no consumer considers purchasing the good.\(^{31}\) Therefore, the firm optimally sets \( p^* = v - \tilde{\tau} \), which is the “closest” it can get to the perfect-information optimum (where committing to a drip price is possible). In this case, the ancillary good is priced above marginal cost and the pass-through on \( \gamma \) is zero.

In the imperfect-information setting considered here, the basic price perfectly signals the drip price charged at the checkout point; namely, a higher basic price is associated with a weakly lower drip price. This accords with casual empiricism according to which consumers expect larger hold-ups at the checkout point from firms who advertise lower basic prices (as typically occurs with air-flight booking websites).

Proposition 4 offers valuable guidance in determining how variations on the shopping cost \( s \) affect the occurrence of hold-ups or give-aways in equilibrium.

**Corollary 5 (shopping costs)** Holding the marginal cost fixed at \( \gamma \leq v - c \), the equilibrium drip price \( \tau^* \) is a weakly increasing function of the shopping cost \( s \), and strictly so whenever \( \tau^* = \tilde{\tau} \). Hold-ups occur when \( s \) is large, whereas give-aways occur when \( s \) is small. For \( s \) large enough, such that \( \tilde{\tau} \) approaches \( v - c \), only hold-ups occur, whereas, when \( s = 0 \), only give-aways occur in equilibrium.

### 4.2 Hold-ups in a vertical chain

As in subsection 3.2, we will now analyze the behavior of an upstream supplier with market power, who anticipates that consumers have imperfect information about the drip price set by the firm. As in subsection 3.2, we now interpret \( \gamma \) as the wholesale price imposed by the supplier, who faces a marginal cost of production \( \delta \geq 0 \).

For convenience, the supplier’s profit is denoted by

\[
\tilde{\Xi}(\gamma; \delta) \equiv (1 - G(\tau^*(\gamma))) \left(\gamma - \delta\right),
\]

where we abuse notation by letting \( \tau^*(\gamma) \) describe the equilibrium drip price characterized in

\(^{30}\)Note that, as \( \tau^m(\gamma) < \tilde{\tau} \), the no-missed sale condition is slack at \((p, \tau^m(\gamma))\). The firm has however no incentive to raise the drip price above \( \tau^m(\gamma) \), which, by construction, maximizes the firm’s profit from the ancillary good.

\(^{31}\)The reason is that, for \( p > v - \tilde{\tau} \), the no-missed sale condition is slack at any price pair \((p, \tau)\) that satisfies the consideration constraint. Foreseeing they would be held up, consumers then abandon the purchase.
Proposition 4. The optimal wholesale price $\gamma^*$ then solves

$$\max_{\gamma} \tilde{\Xi}(\gamma; \delta) \quad \text{subject to} \quad \gamma \leq v - c,$$  

(9)

where, as before, the inequality constraint guarantees that the firm is willing to sell the ancillary good.

To analyze program (9), it is convenient to define the double marginalization price $\tau^{mm}(\delta)$ that maximizes $(1 - G(\tau^m(\gamma)))(\gamma - \delta)$ in $\gamma$.32 Accordingly, $\tau^{mm}(\delta)$ is the unique value of $\gamma$ that satisfies the first-order condition

$$\gamma - \delta = \frac{1}{\lambda_G(\tau^m(\gamma))} \left[ \frac{\lambda_G^2(\tau^m(\gamma))}{\lambda_G'(\tau^m(\gamma))} + 1 \right].$$  

(10)

Intuitively, the double marginalization price $\tau^{mm}(\delta)$ corresponds to the wholesale price that prevails in a vertical structure where both the upstream supplier and the downstream firm mark up their perceived marginal costs by a monopolistic margin. This is reflected in equation (10) by the last term in brackets, which captures the pass-through rate on $\gamma$ of the monopoly price $\tau^m(\gamma)$ practiced by the firm. It is convenient to define $\hat{\delta}$ as the unique solution to $\tau^{mm}(\delta) = \delta$, and note that $\tau^{mm}(\delta) < \delta$ if and only if $\delta < \hat{\delta}$.33 The next proposition characterizes the equilibrium wholesale price $\gamma^*$.

**Proposition 5 (equilibrium wholesale price under imperfect information)** Suppose that a monopolistic supplier sets a wholesale price $\gamma$ for the ancillary good, facing a marginal cost $\delta$. Then there exists a threshold $\bar{\gamma}(\delta) \geq \hat{\tau}$ such that, in the unique equilibrium under imperfect information:

(i) **Hold-up**: If $v - c \leq \bar{\gamma}(\delta)$, the wholesale price is $\gamma^* = \tau^{mm}(\delta)$, and the drip price is at its hold-up level: $\tau^* = \tau^m(\gamma^*)$.

(ii) **Give-away**: Otherwise, the wholesale price is in the firm’s cost-absorption region, $\gamma^* \in (\hat{\tau}, v - c)$, and the drip price is below cost: $\tau^* < \gamma^*$.

Moreover, the threshold $\bar{\gamma}(\delta)$ is weakly decreasing and continuous, with $\bar{\gamma}(\delta) = \hat{\tau}$ whenever $\delta \geq \hat{\delta}$.

The logic behind Proposition 5 is analogous to that of Proposition 2. For simplicity, consider the case where the marginal cost of production is sufficiently small (i.e., $\delta < \hat{\delta}$). Conditional on the firm selling the ancillary good, the supplier’s profit has two peaks: one at the double marginalization price $\tau^{mm}(\delta)$, and the other in the give-away region (satisfying $\gamma \geq \hat{\gamma}$). As before, the existence of the second peak reflects the fact that the no-missed-sale condition hinders the firm’s ability to pass the wholesale price through to consumers. The supplier finds it worthwhile to set the wholesale price in the hold-up region $[0, \hat{\tau}]$ if the firm’s resistance is high (i.e., $v - c \leq \bar{\gamma}(\delta)$). If it is low, the supplier sets it in the give-away region $[\hat{\tau}, v - c)$, as a large enough profit can be obtained from the firm’s cost absorption strategy.

---

32 We assume that the supplier’s profit $\tilde{\Xi}(\gamma; \delta)$ is quasi-concave in $\gamma$ over the range $\gamma \leq \hat{\delta}$. A sufficient condition for this is that the function $b - \lambda_G(b)\lambda_G^2(b) - 2\lambda_G^3(b)$ be strictly increasing, which is satisfied by most distributions of interest (e.g., uniform, exponential, power, Gumbel, and Pareto whenever its mean exists).

33 Note that $\hat{\delta}$ can be negative, which occurs if and only if $\tau^{mm}(0) > \hat{\tau}$. Moreover, $\hat{\delta} < \hat{\delta} < \hat{\tau}$. 

24
4.3 Hold-ups: welfare and regulation

In this subsection, we analyze the welfare properties of hold-up equilibria, and discuss how optimal regulation differs in the case of hold-ups and give-aways. We adopt the same welfare measure as in subsection 3.4; namely, the sum of consumers’ surplus and firm’s profit. As before, we distinguish between the equilibrium distortions originated from the supplier’s market power from those that pertain to the firm’s drip pricing behavior. The next corollary compares hold-up equilibria to the welfare benchmark from Corollary 4 (which establishes that, for a given wholesale price γ, welfare is maximal when the drip price is at marginal cost).

**Corollary 6** (under-consumption of the ancillary good) The ancillary good is under-consumed in equilibrium (i.e., τ^* > γ^*) provided:

(i) either the upstream market is competitive and the marginal cost of the ancillary good lies in the firm’s hold-up region: δ ≤ τ ;

(ii) or the supplier is a monopolist and firm resistance is high: v - c < ̅(δ).

If neither condition holds, the ancillary good is over-consumed in equilibrium, as implied by Corollary 4.

Two regulatory interventions are often proposed to mitigate the inefficiencies produced by hold-ups. One is simply to mandate price transparency, whereby firms are required to jointly announce the basic price and the drip price (so that no consumer considers purchasing the good without complete price information). The outcome of such regulation coincides with the equilibrium under perfect information and laissez-faire characterized in Propositions 1 and 2 (that apply to when the upstream market is competitive or monopolistic, respectively).

Another common (and more information intensive) intervention is to impose a cap on the drip price set by the firm. The next proposition reveals that price transparency is outcome-equivalent to cap regulation if the cap on the drip price is set at the firm’s marginal cost (which, in a vertical chain, is the wholesale price practiced by the supplier).

**Proposition 6** (equivalence between cap regulation and mandated transparency) Consider the regulation that prohibits the firm from marking up the ancillary good: τ ≤ γ. Regardless of whether the upstream market is competitive or monopolistic, the (unique) equilibrium under imperfect information and cap regulation is outcome-equivalent to the (unique) equilibrium under mandated transparency (i.e., laissez-faire under perfect information).

To understand Proposition 6, note that, absent regulation, the firm’s pricing behavior under perfect and imperfect information only differ in that, in the latter case, the ancillary good is marked up when γ < ̅ (whereas, in the former case, the ancillary good is sold at cost). If the drip price cap is set precisely at cost, the firm then chooses τ = γ (as implied by quasi-concavity), resulting in the same equilibrium outcome under perfect information. Intuitively, banning positive markups on the ancillary good gives the firm the same commitment power vis-à-vis consumers as perfect price information.
What is the effect of cap regulation, or, equivalently, mandated transparency, on welfare, producer and consumer surplus? When the upstream market is competitive ($\gamma \equiv \delta$), welfare strictly increases to its maximal level, as the firm is able to commit to set the drip price at cost (which is welfare-optimal, as implied by Corollary 4). Consumers are equally well-off under regulation and laissez-faire, as the consideration constraint binds in both cases.

When the upstream market is monopolistic, the comparison is less straightforward, as, in view of a lower drip price, the supplier has an incentive to change its wholesale price. If the wholesale price increases as a result of regulation, the net effect on welfare hinges on a horse race between the gains from a more efficient drip price and the losses from a higher supplier’s margin. While a complete characterization is elusive, we are able to identify a large class of environments where welfare increases as a result of cap or transparency regulation. This is the subject of the next definition.

**Definition 1 (Weakly decreasing pass-through)** The distribution $G$ satisfies the weakly decreasing pass-through property if $\frac{d}{d\delta}m(\delta)$ is weakly decreasing in $\delta$.33  

The weakly decreasing pass-through property is satisfied by most distributions of interest.34 The next proposition contains the main result of this subsection.

**Proposition 7 (Welfare: cap regulation and mandated transparency)** Consider a regulation that either mandates price transparency or caps the drip price at marginal cost.

(i) If the upstream market is competitive, this regulation binds provided $\delta < \hat{\tau}$, in which case welfare increases to its maximal level $\bar{W}$. Relative to laissez-faire, the drip price decreases to its efficient level, and the basic price increases. The firm is strictly better off and consumers are equally well-off.

(ii) If the upstream market is monopolistic, under the weakly decreasing pass-through property, this regulation weakly increases welfare, and strictly so if $\nu - c < \bar{\gamma}(\delta)$. Relative to laissez-faire under imperfect information, there is a decrease in the wholesale price of the ancillary good, which is then priced at cost by the firm: 

$$\gamma^* \geq \gamma^r \quad \text{and} \quad \tau^* - \gamma^* > \tau^r - \gamma^r = 0.$$ 

The firm is strictly better off, consumers are equally well-off, and the supplier is strictly better off.

The weakly decreasing pass-through property guarantees that the double marginalization price $\tau^{mm}(\delta)$ is greater than the monopoly price $\tau^m(\delta)$ at any $\delta \geq 0$. Under this assumption, the wholesale price in a hold-up equilibrium under laissez-faire is always greater than the wholesale price under cap regulation or mandated transparency. Coupled with the decrease in the firm’s markup on the ancillary good, this implies that social welfare unambiguously increases.

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33This is equivalent to $\lambda_{\gamma^*}^{-1}(\cdot)$ being weakly concave over the range $[\tau^m(0), +\infty)$. For more on incidence and pass-through, see Weyl and Fabinger (2013).

34Examples include the uniform, exponential, power, logistic, and Pareto whenever its mean exists. One notable exception is the Gumbel distribution.
Consumers are equally well-off under regulation and laissez-faire, as the consideration constraint binds in both cases. The firm is strictly better off, as it faces a lower wholesale price and benefits from the increase in commitment power vis-à-vis consumers (which allows it to credibly increase the basic price and reduce the drip price). The supplier is also strictly better off, as the consumption of the ancillary good is uniformly greater when double marginalization is avoided. The next remark explores a specific distribution satisfying the weakly decreasing pass-through property.

**Remark 3 (constant pass-through)** The pass-through rate is constant if and only if the distribution $G$ is Generalized Pareto.\(^\text{36}\) In this case, whenever $v - c \leq \bar{\gamma}(\delta)$, the equilibrium wholesale price is the same under cap regulation, mandated transparency or laissez-faire under imperfect information: $\gamma^* = \gamma^\tau$.

**Remark 4 (ban on loss-making sales under imperfect information)** When consumers have imperfect information about the drip price and the upstream market is monopolistic, the ban on loss-making sales studied in Proposition 3 might have unintended consequences if used on a stand-alone basis. To see why, suppose $v - c > \bar{\gamma}(\delta)$, in which case a give-away equilibrium occurs under laissez-faire. If $\delta < \hat{\delta}$ and $\bar{\beta}(\hat{\tau}; \delta) < \bar{\beta}(\pi^{\text{mm}}(\delta); \delta)$, banning loss-making sales produces hold-ups, as the supplier’s optimal wholesale price moves to $\pi^{\text{mm}}(\delta)$. This policy then replaces one inefficiency (over-consumption of the ancillary good) for another (under-consumption). However, if coupled with mandated transparency, banning loss-making sales has unambiguous welfare-enhancing effects, as described in Proposition 3.

5 Naïve consumers

The analysis of Section 4, in particular Proposition 7, implies that (under weak conditions) price transparency is in the interest of the firm, as it recreates its commitment power vis-à-vis consumers and induces the supplier to reduce the wholesale price. Yet, firms often fail to communicate drip prices before the checkout point, leading regulators to mandate price transparency. A natural question of course is why firms engage in this seemingly sub-optimal behavior.

One prominent explanation is that some consumers are naïve, and neglect the possibility of drip prices at the checkout point. In this world, the consumers’ decision to consider purchasing the good is taken under the (incorrect) assumption that $\tau = 0$, in which case the consideration constraint is satisfied provided $p \leq v - s$. Disclosing the drip price works as an “eye-opener” to naïve consumers, who, following disclosure, make store visiting decisions with correct expectations about the transaction costs to be incurred later in the purchasing process. Accordingly, the firm is better off by not disclosing the drip price, as doing so would tighten the consumers’

\(^{36}\) The distribution $G$ is Generalized Pareto with support starting at 0 when

$$
G(b) = \begin{cases} 
1 - (1 - \frac{k}{B} b)^{\frac{1}{k}} & \text{if } k \neq 0 \\
1 - \exp\left(-\frac{b}{\theta}\right) & \text{if } k = 0,
\end{cases}
$$

and $B > 0$. When $k = 1$, $G$ is uniform over $[0, B]$; when $k = 0$, $G$ is exponential; when $k < 0$, $G$ is a shifted Pareto; and when $k > 0$, $G$ is power.
consideration constraint and reduce profit.\footnote{Another prominent reason for why drip prices may not be disclosed before the checkout point is the existence of substantial menu costs for the firm, which needs to post next to the basic price, or more realistically as a general policy displayed prominently in the store, all possible additional fees (e.g., card surcharges for various card associations and proprietary systems). Furthermore, inattentive consumers may overlook the drip price announcement, or be overloaded by its information. In this case, consumer cognitive limitations render drip prices \textit{de facto} non-transparent. Another issue is that price publicity may not come from the firm. For example, the manufacturer of the basic good may run a national advertising campaign, where it is infeasible to disclose the policies of all retailers carrying the product.}

Perhaps surprisingly, cost absorption by the firm may occur even when consumers are naïve. To describe the firm’s pricing behavior, let $\hat{a}^n \equiv s - \lambda_G^{-1}(s)$ and $\hat{a}^n \equiv s + r_G^{-1}(s)$. Note that $\hat{a}^n < s < \hat{a}^n$.

**Proposition 8 (pricing with naïve consumers)** The price pair $(p^n, \tau^n)$ that maximize the firm’s profit when consumers are naïve is such that:

(i) **Hold-up**: If $\gamma \leq \hat{a}^n$, the ancillary good is sold at the monopoly price, $\tau^n = \tau^m(\gamma)$, and the consideration constraint binds: $p^n = v - s$.

(ii) **Constrained hold-up**: If $\gamma \in (\hat{a}^n, s]$, the ancillary good is sold at the shopping cost (above marginal cost), $\tau^n = s$, and the consideration constraint binds: $p^n = v - s$.

(iii) **Give-away (zero pass-through)**: If $\gamma \in (s, \hat{a}^n]$, the ancillary good is sold at the shopping cost (below marginal cost), $\tau^n = s$, and the consideration constraint binds: $p^n = v - s$.

(iv) **Give-away (incomplete pass-through)**: If $\gamma \in (\hat{a}^n, v - c]$, the ancillary good is sold below marginal cost, at the price $\tau^n$ satisfying (6), and $p^n = v - \tau^n$.

(v) **Missed sales**: If $\gamma > v - c$, the firm gives up selling the ancillary good, missed sales occur, and the basic price solves (5).

Consumer naïveté benefits the firm, as it relaxes the consideration constraint. The no-missed-sale condition is however unaffected, as consumers end up learning the drip price at the checkout point. Similarly to Proposition 4, the latter condition binds provided the marginal cost $\gamma$ is sufficiently high, leading to cost absorption.

The supplier’s behavior when consumers are naïve can be derived analogously to subsection 4.3, and we shall omit the formal statement for brevity. As under either perfect or imperfect information, if the firm’s resistance is low, the supplier jacks up the wholesale price so as to explore the firm’s cost absorption policy (leading to a give-away equilibrium). If implemented in isolation, banning loss-making sales is effective in avoiding give-aways, but may lead to hold-ups (similarly to Remark 4). In turn, mandated price transparency alone (which “opens the eyes” of consumers to drip prices) is effective in avoiding hold-ups, but does not prevent the inefficiencies associated with give-aways. In line with our previous findings, \textit{jointly} imposing mandated price transparency and banning loss-making sales on the ancillary good improves welfare.
6 Drip pricing in two-sided markets: Card surcharging

The analysis so far considered a “vertical” or one-sided setting, where the supplier is not able to incentivize consumers to buy the ancillary good. In this section, we will show how the results above can be straightforwardly adapted to a two-sided market, where a platform (say, a payment card network) interacts with both the seller and the buyer. We will focus primarily on the card payment application, indicating how the predictions of our model square with recent policy experiences that made card surcharging legal, and discussing regulation.\textsuperscript{38}

The supplier is now a platform with market power. Its fee structure is \((\gamma, f)\), where \(f\) is the consumer fee and \(\gamma\) is the merchant fee (i.e., the wholesale price of the ancillary good). Similarly to the previous sections, the platform fee structure \((\gamma, f)\) is chosen at stage 0, before the merchant (i.e., the firm) takes its pricing and acceptance decisions. To better fit the card payment application, we further assume that, if the consumer pays by card, the merchant obtains a non-pecuniary benefit \(\beta \geq 0\) (capturing, for instance, the convenience of electronic payments for security and accounting purposes).\textsuperscript{39}

Before tackling the platform pricing problem, it is convenient to define the effective consumer fee \(\tau^e = \tau + f\), which adds to the nominal consumer fee \(f\) the card surcharge (i.e., drip price) \(\tau\). Consumers enjoy a positive surplus from paying by card if and only if

\[
v - p \geq \tau + f = \tau^e, \tag{11}
\]

which is the two-sided version of the no-missed-sale condition (1). Accordingly, whenever (11) is satisfied, consumers consider purchasing the good if and only if

\[
v - p \geq T(\tau + f) = T(\tau^e), \tag{12}
\]

which is the two-sided version of the consideration constraint (2).

6.1 Why so little card surcharging?

Over the last decade, several countries lifted the no-surcharge rule imposed by card networks, which prohibited merchants from surcharging card transactions. These reforms were conceived as a way to constrain the fee-setting power of card networks, as merchants could now pass high card fees through to consumers. Yet, card surcharging is rarely observed in most industries, especially those characterized by repeat purchases (where the perfect-information assumption fits best).\textsuperscript{40}

These facts are consistent with what is predicted by our model. Let us consider first the

\textsuperscript{38}In the context of card payments, the Rysman and Wright (2014) review article observes (without building a model) that card surcharges may play a similar role to that of add-on (or drip) prices.

\textsuperscript{39}We could have introduced a convenience benefit \(\beta\) earlier without any change in the analysis. We do so for the two-sided market treatment so as to explicitly connect our results with the debate on the regulation of interchange fees in card payment systems.

\textsuperscript{40}See, for example, Bolt et al (2010).
perfect-information environment of Section 3, which describes the outcome produced by regulation mandating price transparency (whereby merchants are obliged to announce card surcharges before the checkout point). To understand the merchant’s pricing behavior, we proceed as in subsection 3.1, and start with the case where the no-missed sale condition is satisfied. The merchant’s optimal price profile $(p, \tau)$ then solves

$$\max_{p, \tau} \{p - c + (1 - G(\tau + f))(\tau - \gamma + \beta)\} \quad \text{subject to} \quad v - p \geq \max\{\tau + f, T(\tau + f)\}.$$ 

Letting $\gamma^e \equiv \gamma + f - \beta$ be the effective aggregate fee, we can rewrite this problem as

$$\max_{p, \tau^e} \{p - c + (1 - G(\tau^e))(\tau^e - \gamma^c)\} \quad \text{subject to} \quad v - p \geq \max\{\tau^e, T(\tau^e)\},$$

which is isomorphic to program (4) after replacing $\tau$ and $\gamma$ by their effective counterparts. Moreover, the merchant’s as well as the platform’s profits are unaffected by the platform’s pricing choices in the missed-sales case (where no consumer pays by card), and so the analysis is again unchanged. These observations establish the next corollary, which follows directly from Proposition 1.

**Corollary 7** *(neutrality of platform fees)* Let the platform fee structure be $(\gamma, f)$. Then, in the unique equilibrium, the merchant accepts card payments if and only if $\gamma^e \leq v - c$. In this case, the basic price $p^*$ and the card surcharge $\tau^*$ are as in Proposition 1, after replacing $\gamma$ by $\gamma^e$.

The merchant’s optimal card surcharge depends on the effective aggregate fee $\gamma^e$, but not the structure of fees (i.e., how $\gamma + f$ is decomposed into the merchant fee $\gamma$ and the consumer fee $f$). Moreover, concerns about missed sales limit the pass-through associated with card fees: Merchants engage in cost absorption (implicitly subsidizing consumers for card usage) if and only if the effective aggregate fee is sufficiently high: $\gamma^e > \hat{\tau}$, or, equivalently, $\gamma + f > \beta + \hat{\tau}$. This effect is more pronounced when shopping costs are small, in which case the card surcharge may be zero even when the card fee substantially exceeds the merchant benefit $\beta$ from card transactions. The next corollary describes such an equilibrium, as well as those where card payments are explicitly subsidized by the merchant.

**Corollary 8** *(absence of card surcharging despite high merchant fees)* Let $\gamma^e \leq v - c$ and suppose the shopping cost is zero ($s = 0$) and $r_G(0) < \infty$. Then, if $f \geq 0$ and $\gamma^e \leq \hat{\gamma} = r_G^{-1}(0)$, no card surcharging occurs: $\tau^* = -f^* \leq 0$.

In the (perhaps focal) equilibrium where consumers pay nothing for a card payment ($f^* = 0$), the card surcharge is nil, notwithstanding a merchant fee that exceeds the firm’s benefit $\beta$ from a card payment. This result accords with substantial empirical evidence that documents the low incidence of card surcharging when it is allowed by regulation.

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41This observation is reminiscent of the “neutrality” results obtained by Rochet and Tirole (2002), Vickers (2005), and, in greater generality, Gans and King (2003).
The merchant’s cost absorption policy induces the card platform to jack up the aggregate fee. To see why, note that, in light of Corollary 7, the platform’s problem can be written as

$$\max_{\gamma^e} \left\{ (1 - G(\tau^e(\gamma^e))) \left( \gamma^e - (\delta - \beta) \right) \right\} \quad \text{subject to} \quad \gamma^e \leq v - c.$$ 

This program is isomorphic to program (8), after replacing the wholesale price $\gamma$ by the effective aggregate fee $\gamma^e$, and marking down the marginal cost of a card payment to $\delta - \beta$.$^{42}$ Mutatis mutandis, Proposition 2 then characterizes the prices practiced by the merchant and the fees chosen by the platform in equilibrium. In light of Corollary 7, it should come as no surprise that there exist multiple equilibria that differ in the fee structure $(\gamma, f)$. However, all equilibria are outcome equivalent, as they lead to the same effective aggregate fee. In particular, whenever merchant resistance is low $v - c > \bar{\gamma}(\delta - \beta)$, the platform sets its fees such that the merchant (sometimes fully) absorbs the cost of card payments. This result accords with international experiences suggesting that lifting the no-surcharge rule did not result in either low platform fees, or more efficient card usage by consumers.

6.2 Regulation: Price transparency cum ban on loss-making

The results of section 4, concerning the case where consumers have imperfect information about the price price of the ancillary good, could be equally translated to the card payment application. For brevity, rather than re-stating formal results, we shall just notice that Propositions 4 and 5 remain true after replacing $\tau$ and $\gamma$ by their effective counterparts, $\tau^e$ and $\gamma^e$, and marking down the platform’s marginal cost from $\delta$ to $\delta - \beta$. As under perfect information, the effective fees $\tau^e$ and $\gamma^e$ are unique across all equilibria, whereas the (nominal) fee structure $(\gamma, f)$ is indeterminate. Hold-up equilibria occur when the merchant resistance is high, whereas give-away equilibria occur when the merchant resistance is low.$^{43}$

As predicted by our theory, the card surcharge exceeds the “net” merchant fee $\gamma - \beta$ only if consumers have imperfect information about the card surcharge practiced by the merchant. This accords with empirical evidence revealing that card surcharging entails hold-ups mostly in industries characterized by one-time or infrequent shopping (such as gas stations, airport shops, etc).

As implied by Corollary 7, mandated transparency (i.e., perfect information) is an effective regulation for avoiding hold-up equilibria, and raises welfare under weak conditions (see Proposition 7). This regulation however does not address the inefficiencies produced by give-away equilibria, which can occur under either perfect or imperfect information. Our analysis therefore suggests that competition authorities should simultaneously impose mandated transparency and ban loss-making payments. The former avoids hold-ups, while the latter avoids

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$^{42}$Analogously to Section 3, we assume that $v - c \geq \delta - \beta$, which guarantees that the platform is willing to operate.

$^{43}$Interestingly, there exists a hold-up equilibrium where card surcharging does not occur ($\tau^* = 0$)! In this equilibrium, the platform completely internalizes the merchant surcharging incentives, adjusting its fee structure in a way that the hold-up outcome of Proposition 5 is implemented with no card surcharges. This is achieved by setting $f^* = \tau^m(\tau^{mm}(\delta))$ and $\gamma^* = \beta - f^* - \tau^{mm}(\delta) < \beta$. 

31
gives-aways, leading to efficient card usage and unambiguous welfare gains. The next corollary summarizes this discussion.

**Corollary 9 (price transparency cum ban on loss-making card payments)** Under mandated transparency, the card surcharge is always weakly below the marginal cost perceived by the merchant (i.e., no hold-ups occur). When merchant resistance is low (i.e., \( v - c > \bar{\gamma}(\delta - \beta) \)) there is excessive card usage, as \( \tau^* < \gamma^* - \beta \). In this case, banning loss-making card payments increases welfare, reduces the equilibrium aggregate fee, and induces efficient card usage.

The condition for efficiency identified in Corollary 9 (namely, that \( \gamma^w - \tau^w = \beta \)) is a reinstatement of the “tourist or avoided cost test” developed in Rochet and Tirole (2011) in the context of card payment regulation. According to this test, the card surcharge should equal the “net” merchant fee incurred by the merchant: \( \tau = \gamma - \beta \). This condition guarantees that the cardholder internalizes the merchant’s welfare when choosing the payment method, and is thus the incarnation of the Pigouvian precept in the payment context. Yet, under mandated transparency, whenever merchant resistance is low, the consumer choice of payment method imposes a negative externality on the merchant, as \( \bar{\gamma}^* - \tau^* > \beta \) (i.e., the tourist test is violated in equilibrium). The ban on loss-making card payments from Corollary 9 restores efficiency, as it obliges the merchant to pass its fee through to consumers, which, in turn, induces the platform to reduce aggregate fees, and generates an unambiguous welfare improvement. For its simplicity, banning loss-making card payments is an attractive alternative to regulation that targets merchant fees (e.g., through caps on the interchange fee, as, for example, practiced in the European Union and the US).

### 7 Robustness

**Pre-purchasing fees.** As our analysis reveals, the no-missed-sale condition often hinders the firm’s ability to pass the ancillary good’s price through to consumers. One natural way to relax this condition is to offer a “pre-purchasing plan,” at a fee \( r \) (to be paid before shopping costs are incurred), which entitles consumers to a discount on the basic good whenever a purchase is realized. One example is the practice (common among train companies) of offering frequent-customer packages, which reduce the fares (say, by 20%) of any trips booked by consumers during a certain period of time (say, a year). Similar pre-purchasing plans are offered by retailers (especially on clothing) that entitle consumers to discounts on the posted prices of basic goods. Levying the pre-purchasing fee \( r \) enables the firm to lower \( p \), which relaxes the no-missed-sale condition and gives room to increasing the price of ancillary goods.

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44 We assume here that \( v - c - (\delta - \beta) \geq \Pi^{ms}(c) \), which clearly holds as the marginal cost of a card payment is close to zero.

45 The tourist test was adopted by the European Commission as its benchmark for the regulation of the merchant fees charged by Visa and MasterCard members.

46 The interchange fee is the fee paid by the merchant’s bank (the acquirer) to the cardholder’s bank (the issuer). The interchange fee is set by the platform (e.g., Visa or MasterCard). For extensive overviews of the economics of interchange fees, see Evans and Schmalensee (2005) and Evans (2011).
Although occasionally observed, pre-purchasing plans are more the exception than the rule, and may not be feasible for several reasons. First, selling a pre-purchasing plan may not be feasible if the consumer has a choice as to the timing of her learning $b$. To understand why, consider a consumer who bought a pre-purchasing plan allowing her to buy the basic good at a discounted price $\hat{p}$. In case $v \geq \hat{p} + r + \tau$, there is no change with the previous analysis (as $\hat{p} + r$ is the de facto price of the basic good), and the pre-purchasing fee $r$ is irrelevant. Offering a pre-purchasing plan relaxes the no-missed-sale condition when $v < \hat{p} + r + \tau$ but $v - \hat{p} \geq \tau > 0$, as the fee $r$ is sunk when purchasing decisions are made. In this case, however, the consumer has an incentive to move forward in time her learning of $b$ so as to refrain from paying the pre-purchasing fee when her ex-post surplus is negative (which occurs when the realized $b$ is higher than $\tau$). Early learning thus precludes the “cross subsidies” (across realizations of $b$) that are needed to relax the no-missed-sale condition.

Second, it might be hard to sell a pre-purchasing plan to consumers who face uncertainty regarding their demand for the basic good (e.g., only sure-to-be frequent travelers would buy a pre-purchasing plan for train tickets). If consumers are risk averse, substantial discounts on the basic price might then be needed to make the pre-purchasing plan an attractive deal. Such discounts might be more detrimental to firm’s profit than having the no-missed-sale condition binding.

Third, a pre-purchasing fee request may raise a suspicion about the quality of the basic good, quality that will later be ascertained at a cost included in the shopping cost $s$; so adverse selection considerations may also go against offering such plans.

**Heterogeneity in consumer information.** To simplify the exposition, we opted to consider separately the cases where consumers (i) are informed about the drip price before the checkout point (Section 3), (ii) are rational-but-uninformed about the drip price (Section 4), or (iii) are naïve, therefore anticipating it to be zero (Section 5). Allowing for heterogeneity in consumer information (for instance, assuming that some consumers are informed while others are rational-but-uninformed) would enable us to “continuously” move from the characterizations of Propositions 1 and 2 to those of Propositions 4 and 5. Importantly, the positive and normative implications of our analysis are robust to different assumptions about consumer information, and remain true if we introduce heterogeneity in this dimension.

**Downward sloping demand for the basic good.** To isolate the effects of missed sales and price transparency on the firm’s pricing policies, we assumed an inelastic demand for the basic good. When this demand is downward sloping (as when consumers are heterogeneous on their valuations for the basic good), ancillary good prices may also be used for price discrimination (as in Chen-Ross 1993). In this case, on top of the effects documented above, the firm has an incentive to raise the drip price so as to extract rents from consumers with high valuations for the basic good (for whom the consideration constraint or the no-missed-sales condition are slack). We discuss this possibility in the Supplementary Material, showing that our positive insights and regulatory implications remain valid in this richer setting.

**Information acquisition by consumers.** In Section 4, we have assumed that consumers do
not know the drip price $\tau$ before taking the purchasing decision; and that there is no way of learning this price independently prior to incurring the shopping cost. Note that the value of learning the drip price before incurring the shopping cost is nil. A rational-but-uninformed consumer correctly predicts the equilibrium drip price, and therefore does no gain from learning it even if the information is freely available. A naïve consumer by definition does not anticipate further costs and therefore acquiring information about this cost is not on her radar.

**Incomplete information about the basic good’s price.** We have assumed that the consumer knows the basic good’s price; say, she is alerted to it by a display banner ad, or she passes by a shop’s window offering the basic good. Alternatively, she may find out from search. In the latter case, we can append a prior stage in which the consumer searches for the good. This does not change our analysis.

### 8 Concluding remarks

Our analysis reconciles two opposite narratives on hold-ups and give-aways, delivers clear predictions on which obtains, and identifies information-light regulations that are robust to the information and sophistication of consumers as well as to the industry structure (supplier market power, one- vs. two-sided markets). Below, we summarize the positive and normative implications of our model.

**Positive Implications: Hold-ups or give-aways?**

1) Give-aways are more likely when repeat customers are numerous, the marginal cost of the ancillary good is high, and shopping costs are low. Hold-ups are more likely when one-time shoppers are numerous, the marginal cost of the ancillary good is low, and shopping costs are high.

2) If shopping costs are negligible, the ancillary good may be even provided for free (e.g., no card surcharging, free shipping).

3) Prices exhibit a see-saw pattern, as basic and ancillary good prices co-vary negatively. In markets characterized by infrequent shopping (imperfect information), a high price for the basic good then signals a low drip price at the checkout point.

4) If the ancillary good is supplied by a third party with market power, the supplier often has an incentive to jack up its wholesale price, so as to benefit from the firm’s cost absorption strategy. Give-aways (resp., hold-ups) are more likely when the gross surplus from the basic good is high (resp., low).

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47 For a general treatment of this question, see Armstrong, Vickers and Zhou (2009).
48 But like in the entire literature, some assumption must be made that ensures that the Diamond paradox does not obtain and the consumer is indeed willing to search. We are agnostic as to the reason for why the Diamond paradox does not prevail in practice; it might be due to the presence of negative search costs for a fraction of consumers, or a mixture of informed and uninformed consumers; in slight extensions of our model, consumers might also have a surplus due to either the multi-unit or the multi-product (as in Rhodes 2015) nature of their purchase.
5) Ancillary good prices increase, whereas wholesale prices (in case the ancillary good supplier has market power) decrease with shopping costs. As shopping costs decrease, give-away equilibria become more likely (irrespective of market structure in the upstream market).

6) When the market is two-sided (e.g., card payments), the merchant internalizes the fees charged to consumers, and only aggregate fees affect card usage (but not the fee structure). If shopping costs are negligible, card payments may be explicitly subsidized (i.e., the merchant would offer a card discount).

The predictions above are consistent with a wealth of evidence. The Reserve Bank of Australia (2011) reports that above-cost card surcharging more often occurs in industries characterized by one-time or infrequent shopping (such as gas stations, travel agencies, airport shops, car rentals, etc), whereas firms exhibiting a large fraction of repeat customers are more likely to subsidize card usage.\textsuperscript{49} Similarly, Forrester (2011) reports that give-aways (e.g., in the form of free shipping) are more widespread among retailers with large repeat-customer bases, and in markets where the profit margin on the basic good is higher. More broadly, the ubiquitous presence of give-aways in online markets (where shopping costs are arguably low), as well as the widespread concerns about the high wholesale prices of ancillary goods (for card payments,\textsuperscript{50} shipping,\textsuperscript{51} and broadcasting\textsuperscript{52}) are consistent with the predictions of our model.

**Normative Implications: Regulation**

Our policy conclusions challenge received wisdom concerning regulation:

7) If implemented in isolation, price transparency eliminates hold-ups, but does not prevent give-aways, which result in over-consumption of the ancillary good and an inefficiently high price for the basic good (externality on non-participants).

8) If implemented in isolation, a ban on loss-making sales may generate hold-ups, replacing, relative to laissez-faire, one inefficiency (over-consumption of the ancillary good, due to give-aways) with another (under-consumption, due to hold-ups).

9) Regardless of whether consumers are informed, uninformed-but-rational, or naïve, regulation jointly imposing price transparency and banning loss-making sales is welfare increasing (indeed, it is welfare-maximizing if the ancillary good is competitively provided). This regulation leads to (i) an efficient consumption of the ancillary good, and (ii) a decrease in its wholesale price (if the ancillary good is provided by a third-party with market power), generating strict welfare gains.

10) In the case of card payments, if surcharging caps are to be used, the cap should be set at the merchant fee minus the merchant’s convenience benefit from card payments: \( \tau \leq \gamma - \beta \). Recent cost-based regulations are more lenient, as they require that \( \tau \geq \gamma \) or that \( \tau \leq \gamma + X \), with \( X > 0 \).

\textsuperscript{49}See also Choice (2014).


Appendix: Proofs

The appendix collects all proofs omitted in the text.

Proof of Proposition 1. The decision of a firm to sell the ancillary good depends on how its profit compares in case the no-missed-sale condition (1) is satisfied or violated. In the former case, the firm solves

\[ P^0 : \max_{p, \tau} \{ p - c + (1 - G(\tau))(\tau - \gamma) \} \quad \text{subject to} \quad v - p \geq \max\{\tau, T(\tau)\}, \]

which value function is \( \Pi^*(\gamma, c) \). In the latter case, the firm solves

\[ P^1 : \max_p G(v - p)(p - c) \quad \text{subject to} \quad v - p \geq T(v - p), \]

which value function is \( \Pi^{ms}(c) \). The next two lemmas obtain the solutions to program \( P^0 \) and program \( P^1 \).

Lemma 1 (pricing with no missed sales) The price pair \( (p^0(\gamma), \tau^0(\gamma)) \) that solves problem \( P^0 \) is such that:

(i) **Complete pass-through:** If \( \gamma < \hat{\tau} \), then \( \tau^0(\gamma) = \gamma \) and \( p^0(\gamma) = v - T(\gamma) \).

(ii) **Zero pass-through:** If \( \hat{\tau} \leq \gamma \leq \hat{\gamma} \), then \( \tau^0(\gamma) = \hat{\tau} \) and \( p^0(\gamma) = v - \hat{\tau} \).

(iii) **Incomplete pass-through:** If \( \gamma > \hat{\gamma} \), then \( \tau^0(\gamma) \) solves (6) and \( p^0(\gamma) = v - \tau^0(\gamma) \).

Proof of Lemma 1. It is clearly suboptimal for the firm to choose a price pair \( (p, \tau) \) such that both the consideration constraint (2) and the no-missed-sale condition (1) are slack. So consider first the case where the consideration constraint (2) binds and the no-missed-sale condition (1) is slack. The firm’s problem is then equivalent to

\[ \max_{\tau} \quad v - T(\tau) - c + (1 - G(\tau))(\tau - \gamma) \quad \text{subject to} \quad \tau \leq \hat{\tau}, \]

where the inequality constraint follows from the fact that \( v - p = T(\tau) \geq \tau \) if and only if \( \tau \leq \hat{\tau} \). The derivative of the objective above is

\[ -g(\tau)(\tau - \gamma), \]

which crosses zero only once and from above at \( \tau = \gamma \) (implying that the objective function is quasi-concave). As a result, at the optimum, \( \tau = \gamma \) if \( \gamma \leq \hat{\tau} \) and \( \tau = \hat{\tau} \) if \( \gamma > \hat{\tau} \).

Now consider the case where the consideration constraint (2) is slack and the no-missed-sale condition (1) is slack. The firm’s problem is then equivalent to

\[ \max_{\tau} \quad v - \tau - c + (1 - G(\tau))(\tau - \gamma) \quad \text{subject to} \quad \tau \geq \hat{\tau}, \]
where the inequality constraint follows from the fact that \( v - p = \tau \geq T(\tau) \) if and only if \( \tau \geq \hat{\tau} \). The derivative of the objective above is

\[-G(\tau) - g(\tau)(\tau - \gamma).\]

This expression is positive if and only if \( \gamma \geq \tau + r_{G}^{-1}(\tau) \). Because \( \tau + r_{G}^{-1}(\tau) \) is increasing in \( \tau \), we conclude that the objective function is quasi-concave. Note that the solution to (6), or, equivalently, \( \gamma = \tau + r_{G}^{-1}(\tau) \), is greater than \( \hat{\tau} \) if and only if \( \gamma \geq \hat{\gamma} \equiv \hat{\tau} + r_{G}^{-1}(\hat{\tau}) \). As a result, at the optimum, \( \tau = \hat{\tau} \) if \( \gamma \leq \hat{\gamma} \) and \( \tau \) solves (6) if \( \gamma > \hat{\gamma} \).

Note that the constraints in the two problems above are mutually exclusive, except for \( \tau = \hat{\tau} \) (in which case the objectives coincide). As a result, at the optimum, \( \tau^0(\gamma) = \gamma \) if \( \gamma \leq \hat{\tau} \), \( \tau^0(\gamma) = \hat{\tau} \) if \( \gamma \in (\hat{\tau}, \hat{\tau} + r_{G}^{-1}(\hat{\tau})) \) and \( \tau^0(\gamma) \) solves (6) if \( \gamma \geq \hat{\tau} + r_{G}^{-1}(\hat{\tau}) \). Moreover, \( p^0(\gamma) = v - T(\tau^0(\gamma)) \) if \( \gamma \leq \hat{\tau} \) and \( p^0(\gamma) = v - \tau^0(\gamma) \) if \( \gamma > \hat{\tau} \), concluding the proof.

To describe the solution to \( P^1 \), let us define \( p^{ms}(c) \) as the unique \( p \) that satisfies

\[ p - c = \frac{1}{r_{G}(v - p)}. \tag{13} \]

It is straightforward to see that \( p^{ms}(c) \) is increasing in \( c \) with \( (p^{ms})'(c) \in (0, 1) \). It is also convenient to denote by \( \hat{c} \) the unique solution in \( c \) to \( p^{ms}(c) = v - \hat{\tau} \), and note that \( \hat{c} < v - \hat{\tau} \). The next lemma describes how the solution to \( P^1 \) varies with the marginal cost \( c \) of the basic good.

**Lemma 2** *(pricing with missed sales)* The basic price \( p^1(c) \) that solves \( P^1 \) is such that \( p^1(c) = p^{ms}(c) \) if \( c \leq \hat{c} \) and \( p^1(c) = v - \hat{\tau} \) if \( c > \hat{c} \).

**Proof of Lemma 2.** Differentiating the objective in \( P^1 \) leads to

\[-g(v - p)(p - c) + G(v - p).\]

The expression above is positive if and only if \( p - r_{G}^{-1}(v - p) \leq c \). Because \( p - r_{G}^{-1}(v - p) \) is increasing in \( p \), the inequality is satisfied if and only if \( p \leq p^{ms}(c) \). This implies that the objective function is quasi-concave. As a result, at the optimum, \( p^1(c) = p^{ms}(c) \) if \( p^{ms}(c) \leq v - \hat{\tau} \) and \( p^1(c) = v - \hat{\tau} \) if \( p^{ms}(c) > v - \hat{\tau} \). Using the definition of \( \hat{c} \), this is equivalent to \( p^1(c) = p^{ms}(c) \) if \( c \leq \hat{c} \) and \( p^1(c) = v - \hat{\tau} \) if \( c > \hat{c} \), as in the statement of the lemma.

Having studied the firm pricing decisions with and without missed sales, we can now characterize the firm’s optimal decision of whether to offer the ancillary good. First, recall that, at the optimum of problem \( P^0 \), the no-missed-sale condition is slack and the drip price is at marginal cost \( (\tau^0(\gamma) = \gamma) \) whenever \( \gamma \leq \hat{\tau} \). As a result, for \( \gamma \leq \hat{\tau} \), the optimal firm’s profit when offering the ancillary good reaches its upper bound

\[ \Pi^*(\gamma, c) = v - T(\gamma) - c > \Pi^{ms}(c). \]
Next, suppose that $\gamma > \hat{\tau}$. From Lemma 1, the no-missed-sale condition binds (i.e., $v - p = \tau$), and so, at the optimum of problem $\mathcal{P}^s$, the firm’s profit can be written as

$$\Pi^s(\gamma, c) = \max_{p: \tau} \left\{ p - c + (1 - G(v - p))(v - p - \gamma) \right\} \text{ subject to } v - p \geq \hat{\tau},$$

where the latter inequality above is equivalent to the consideration constraint being satisfied (which follows from the definition of $\hat{\tau}$, as the consideration constraint requires that $v - p = \tau \geq T(\tau)$). Because $v - c > \hat{\tau}$ (by Assumption 1), direct inspection reveals that

$$\Pi^s(v - c, c) = \Pi^{ms}(c).$$

Accordingly, the maximal firm’s profit with missed sales equals the maximal firm’s profit without missed sales when the ancillary good has marginal cost equal to the gross surplus of the basic good: $\gamma = v - c$. Because the optimal firm’s profit with no missed sales, $\Pi^s(\gamma, c)$, is decreasing in $\gamma$, while the optimal profit with missed sales, $\Pi^{ms}(c)$, is invariant to $\gamma$, we obtain the result.

**Proof of Corollary 1.** The result follows from Proposition 1 and the fact that $\hat{\tau}$ is increasing in $s$. ■

**Proof of Corollary 2.** When $s = 0$, $\hat{\tau} = 0$. The result then follows from Proposition 1 and the fact that $r_G^{-1}(0) > 0$ when the reverse hazard rate is bounded. ■

**Proof of Proposition 2.** Consider first the case where $\delta < \hat{\delta}$, which implies that the supplier’s profit has a local maximum at $\tau^m(\delta) < \hat{\tau}$. Let us define the threshold $\bar{\gamma}(\delta)$ as

$$\bar{\gamma}(\delta) \equiv \inf \left\{ \gamma \in [\hat{\tau}, +\infty) : (1 - G(\tau^0(\gamma)))(\gamma - \delta) \geq (1 - G(\tau^m(\delta)))(\tau^m(\delta) - \delta) \right\},$$

where $\tau^0(\gamma)$ is described in the proof of Proposition 1. If the set above is empty, we identify $\bar{\gamma}(\delta) \equiv +\infty$ (notice that the set above in non-empty for $\delta$ close to $\hat{\delta}$). Together with the envelope theorem, total differentiation reveals that, whenever $\bar{\gamma}(\delta) < +\infty$, the threshold $\bar{\gamma}(\delta)$ is decreasing in $\delta$.

Whenever $v - c \leq \bar{\gamma}(\delta)$, the supplier is better off by setting the wholesale price to equal the monopoly price, $\gamma^* = \tau^m(\delta)$, which is entirely passed through to consumers in equilibrium: $\tau^* = \tau^m(\delta)$. If, however, $v - c > \bar{\gamma}(\delta)$ and $\bar{\gamma}(\delta) \leq \hat{\gamma}$, the supplier optimally sets $\gamma^* = v - c$. Finally, if $v - c > \bar{\gamma}(\delta)$ and $\bar{\gamma}(\delta) > \hat{\gamma}$, the supplier’s optimal wholesale price satisfies $\gamma^* \geq \hat{\gamma}$. If the supplier’s profit function $\Xi(\gamma; \delta)$ is decreasing to the right of $\hat{\gamma}$, reflecting the fact that the pass-through on $\gamma$ is again positive, then $\gamma^* = \hat{\gamma}$. In turn, $\gamma^* > \hat{\gamma}$ if the supplier’s profit function $\Xi(\gamma; \delta)$ is increasing to the right of $\hat{\gamma}$.

Consider now the case where $\delta \geq \hat{\delta}$, and notice that $\bar{\gamma}(\delta) = \hat{\tau}$. Optimality requires the supplier to set $\gamma^* \in (\hat{\tau}, v - c]$. This reflects the fact that its profit function is monotonically increasing over the range $\gamma < \hat{\tau}$.

The statement in the proposition summarizes the observations above. ■
Proof of Corollary 3. Note that (i) \( \hat{\tau} \) and \( \hat{\gamma} \) are increasing in \( s \), and (ii) \( \Xi(\gamma; \delta) \) is invariant is \( s \) whenever \( \gamma \leq \hat{\tau} \) or \( \gamma \geq \hat{\gamma} \). The result then follows from Proposition 2.

Proof of Corollary 4. Holding \( \gamma \) fixed, social welfare is a function of \( \tau \) with derivative

\[
(-\tau + \gamma)g(\tau).
\]

As a result, social welfare is an inverse-U-shaped function of \( \tau \) with unique maximum at \( \tau = \gamma \). That over-consumption occurs when the upstream market is competitive and \( \delta > \hat{\tau} \) follows directly from Proposition 1. That over-consumption occurs when the upstream market is monopolistic and \( v - c > \bar{\gamma}(\delta) \) follows directly from Proposition 2.

Proof of Proposition 3. Consider first the case where the upstream market is competitive, and let \( \delta > \hat{\tau} \). Because \( \tau^r = \delta \) and the no-missed sale condition binds, it follows that the basic price under regulation \( p^r = v - \delta < v - \tau^r = p^* \). Welfare strictly increases, as implied by Corollary 4. That the firm is strictly worse off than implies that consumers are strictly better off.

Consider now the case where the upstream market is monopolistic. As established in the text following Proposition 3, the supplier’s problem is to

\[
\max_{\gamma} \left\{ (1 - G(\gamma)) (\gamma - \delta) \right\} \quad \text{subject to} \quad \gamma \leq v - c - \Pi^{ms}(c).
\]

This implies that the supplier’s optimal wholesale price under regulation is

\[
\gamma^r = \min \left\{ v - c - \Pi^{ms}(c), \tau^m(\delta) \right\}.
\]

Notice that

\[
\Xi(\gamma; \delta) \geq \hat{\Xi}(\gamma; \delta),
\]

with strict inequality whenever \( \gamma > \hat{\tau} \). Moreover, whenever \( \gamma > \hat{\gamma} \), we have that \( \frac{d\tau^r}{d\gamma}(\gamma) \in (0, 1) \). Therefore,

\[
\frac{\partial \Xi}{\partial \gamma}(\gamma; \delta) \geq \frac{\hat{\Xi}}{\hat{\gamma}}(\gamma; \delta)
\]

with strict inequality whenever \( \gamma > \hat{\gamma} \) (reflecting the fact that, under regulation, the firm pass-through is one). This implies that

\[
\tau^m(\delta) \leq \hat{\tau}(\delta) \equiv \arg \max_{\gamma} \Xi(\gamma; \delta),
\]

where \( \hat{\tau}(\delta) \) is the unconstrained global maximizer of \( \Xi(\gamma; \delta) \). Together with the fact that the firm acceptance constraint is tighter under regulation (as \( \Pi^{ms}(c) > 0 \)), this establishes that \( \gamma^r < \gamma^* \) whenever the regulation binds. This occurs if and only if \( v - c > \bar{\gamma}(\delta) \), as implied by Proposition 2. Moreover, the firm’s markup is now zero, as \( \gamma^r = \tau^r \). By Corollary 4, these two effects guarantee that social welfare strictly increases.

To analyze the effects of regulation on consumer and producer surplus, we consider a num-
ber of cases.

**Case 1:** If $\delta < \hat{\delta}$ and $v - c \leq \hat{\gamma}(\delta)$, regulation is not binding, and the equilibrium outcome is the same under laissez-faire.

**Case 2:** Let $\delta < \hat{\delta}$ and assume $\gamma^* \in (\hat{\tau}, \hat{\gamma}]$. This occurs if $v - c \leq (\hat{\tau}, \hat{\gamma}]$ or $\Xi(\gamma; \delta)$ is decreasing over the range $\gamma > \hat{\gamma}$. Under regulation, the equilibrium wholesale price is $\gamma^r = \gamma^m(\delta)$, which is less than $\gamma^* \in (\hat{\tau}, \hat{\gamma}]$. In turn, the retail price of the ancillary good under regulation is $\tau^r = \gamma^r = \gamma^m(\delta)$, which is also less than its counterpart under laissez-faire ($\tau^* = \hat{\tau}$).

Because the consumers’ consideration constraint binds under regulation and laissez-faire, consumers are equally well-off in this case. Because welfare strictly increases, we conclude that the firm’s profit strictly increases.

**Case 3:** Let $\delta < \hat{\delta}$ and assume $\gamma^* > \hat{\gamma}$. This occurs if $v - c > \hat{\gamma}$ and $\Xi(\gamma; \delta)$ is increasing over some range to the right of $\hat{\gamma}$. Under regulation, the equilibrium wholesale price is $\gamma^r = \gamma^m(\delta)$, which is less than $\gamma^* > \hat{\gamma}$. In turn, the retail price of the ancillary good under regulation is $\tau^r = \gamma^r = \gamma^m(\delta)$, which is also less than its counterpart under laissez-faire (as $\tau^* > \hat{\tau}$).

Because the consumers’ consideration constraint binds under regulation, but is slack under laissez-faire, consumers are strictly worse off in this case. Because welfare strictly increases, we conclude that the firm’s profit strictly increases.

**Case 4:** Now let $\delta \geq \hat{\delta}$ and $\gamma^* \in (\hat{\tau}, \hat{\gamma}]$. Under laissez-faire, the firm’s profit is

$$\Pi^*(\gamma^*, c) = G(\hat{\tau})(v - c - \hat{\tau}).$$

Under regulation, the equilibrium wholesale price is $\gamma^r = \min \{v - c - \Pi^{\text{ms}}(c), \gamma^m(\delta)\}$, and the firm’s profit is

$$v - \min \{v - c - \Pi^{\text{ms}}(c), \gamma^m(\delta)\} - c \geq \Pi^{\text{ms}}(c) = \Pi^*(\gamma^*, c),$$

with strict inequality if $v - c - \Pi^{\text{ms}}(c) > \gamma^m(\delta)$. This implies that the firm’s profit weakly increases under regulation in this case. Because the consumers’ consideration constraint binds under laissez-faire, but is slack under regulation, consumers are strictly better off in this case as well.

**Case 5:** Now let $\delta \geq \hat{\delta}$ and $\gamma^* > \hat{\gamma}$. Notice that the consumers’ consideration constraint is slack under both laissez-faire and regulation. Because the no-missed-sale condition binds, in both cases, consumer surplus is simply $\tau - T(\tau)$. Under regulation, the ancillary good (retail) price is $\tau^r = \gamma^r = \min \{v - c - \Pi^{\text{ms}}(c), \gamma^m(\delta)\}$, while under laissez-faire the ancillary good (retail) price $\tau^*$ solves the first-order condition

$$\tau + \gamma^{-1}_G(\tau) - \delta = \lambda^{-1}_G(\tau) \frac{d}{d\tau} [\tau + \gamma^{-1}_G(\tau)]$$

whenever $\gamma^* < v - c$, and equals $(\tau^0)(v - c)$ otherwise. Because $\tau - T(\tau)$ is increasing in the ancillary good price, consumers are worse off under regulation provided $\tau^r < \tau^*$, in which
case, the firm is better off.

We obtain that \(\tau_r < \tau^*\) provided \(v - c - \Pi^{ms}(c) \leq \hat{\gamma}\). The same is true when \(\hat{\gamma} < \tau^m(\delta) < v - c - \Pi^{ms}(c)\) and \(\gamma^* = v - c\) provided \(\tau^m(\delta) < (\tau^0) (v - c)\). Finally, consider the case where \(\gamma^* < v - c\) and \(\hat{\gamma} < \tau^m(\delta) < v - c - \Pi^{ms}(c)\). Here, because the supplier profit is quasi-concave, we obtain that \(\tau_r < \tau^*\) if and only if

\[
\left[ r^{-1} G^{-1}(\tau) - \lambda^{-1} G^{-1}(\tau) \frac{d}{d\tau} (r^{-1} G^{-1}(\tau)) \right]_{\tau = \tau^m(\delta)} < 0.
\]

If however \(\tau_r > \tau^*\), consumers are better off under regulation.

The statement in the proposition follows from the five cases considered above.

**Proof of Proposition 4.** Proceeding by backward induction, we start with the continuation game that follows the announcement of the basic price \(p\). Consider first a putative equilibrium, described by the drip price \(t^*(p)\), where all consumers consider purchasing the good, and the ancillary good is traded (which requires that both the no-missed-sale condition (1) and the consideration constraint (2) are satisfied at \(t^*(p)\)). The drip price in this continuation equilibrium has to satisfy

\[
t^*(p) = \min \left\{ v - p, T^{-1}(v - p), \tau^m(\gamma) \right\},
\]

as otherwise the firm could increase the actual drip price \(\tau\), while satisfying (1) and (2), and strictly increase profits.

To conclude that the putative equilibrium \(t^*(p)\) is indeed an equilibrium, we have to rule out two deviations that the firm may engage in. The first consists in choosing a drip price that holds up consumers (who would forego considering purchasing the good had they observed the firm’s deviation), while satisfying the no-missed-sale condition. The maximum profit from the hold-up deviation is

\[
\hat{\Pi}^h(p) \equiv \max_{\tau} \left\{ p - c + (1 - G(\tau))(\tau - \gamma) \right\} \quad \text{subject to} \quad v - p \geq \tau.
\]

This deviation is relevant when the firm gains by “surprising” consumers with a drip price that, while worth paying at the checkout point, would scare them off at the consideration stage.

The second deviation consists in choosing an out-of-equilibrium drip price such that no consumer buys the ancillary good (which amounts to violating the no-missed-sale condition). The profit from the missed-sale deviation is

\[
\hat{\Pi}^{ms}(p) \equiv G(v - p) (p - c).
\]

This deviation is relevant when the firm gains by “reneging” on the promise to sell the ancillary good, therefore incurring missed sales.

Accordingly, following the announcement of the basic price \(p\), there exists a continuation
equilibrium where the ancillary good is traded if and only if
\[ \hat{\Pi}^s(p) \equiv p - c + (1 - G(t^s(p)))(t^s(p) - \gamma) \geq \max \left\{ \hat{\Pi}^{ms}(p), \hat{\Pi}^h(p) \right\}. \]

The next lemma characterizes when such continuation equilibria exist, as well as the entire equilibrium set.

**Lemma 3 (Continuation equilibria)** Consider the continuation game following the announcement of the basic price \( p \).

(i) There exists an equilibrium where the ancillary good is traded if and only if
\[ \gamma \leq \max\{v - c, \delta\} \]
and \( v - p \geq \min \{\hat{\tau}, T(\tau^m(\gamma))\} \). In this case, the drip price is \( t^s(p) = \min\{v - p, \tau^m(\gamma)\} \).

(ii) There exists equilibria where consumers consider purchasing the good, but the ancillary good is not offered, if and only if \( v - c \leq \gamma \) and \( v - p \geq \hat{\tau} \).

(iii) There exists equilibria where consumers do not consider purchasing the good if and only if \( v - p \leq \hat{\tau} \).

**Proof of Lemma 3.** The best hold-up deviation strictly increases profit if and only if the consideration constraint binds and the no-missed-sale condition is slack at the putative equilibrium \( t^s(p) \). This does not occur if and only if
\[ v - p \geq \min \{\hat{\tau}, T(\tau^m(\gamma))\}, \tag{14} \]
in which case \( \hat{\Pi}^s(p) = \hat{\Pi}^h(p) \). Under this condition, the missed-sale deviation is never profitable provided the monopolist price \( \tau^m(\gamma) \) is less than the critical price \( \hat{\tau} \) (or, equivalently, \( \gamma \leq \delta \)), as selling the ancillary good at the monopolist (above cost) enables to firm to sell more of the basic good. In turn, when \( \tau^m(\gamma) \) is strictly greater than the critical price \( \hat{\tau} \) (or, equivalently, \( \gamma > \delta \)), selling the ancillary good (possibly below cost) at the drip price \( \tau = v - p \) is worthwhile if and only if \( \gamma \leq v - c \) (as inspecting \( \hat{\Pi}^{ms}(p) \) and \( \hat{\Pi}^s(p) \) reveals). The first claim in Lemma 3 summarizes these observations.

There exist equilibria such that consumers consider purchasing the good, but do not expect to purchase the ancillary good. Such equilibria exist following the announcement of the basic price \( p \) if and only if \( v - p \geq \hat{\tau} \) (which guarantees the consideration constraint is satisfied) and \( v - c \leq \gamma \), which implies that not selling the ancillary good is indeed optimal for the firm: \( \hat{\Pi}^{ms}(p) \geq \hat{\Pi}^s(p) \).

Finally, there exist equilibria where no consumer considers purchasing the good if and only if \( v - p \leq \hat{\tau} \) (in which case the consideration constraint is violated under the belief that the ancillary good is not offered). \( \blacksquare \)

Having characterized the set of continuation equilibria following the announcement of \( p \), we are now in a position to study the firm’s optimal choice of basic price. Whenever multi-

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53Such equilibria are however in weakly dominated strategies whenever there exists an equilibrium where the ancillary good is traded.
ple continuation equilibria exist, we shall select the (always unique) one that is not in weakly dominated strategies (and use the word “equilibrium” to refer to the unique Perfect Bayesian equilibrium under this refinement).

Let us consider first the (unconstrained) hold-up case, where \( \gamma \leq \hat{\delta} \), or, equivalently, \( \tau^m(\gamma) \leq \hat{\tau} \). Under this condition, consumers anticipate the drip price to be at the monopoly level, \( t^*(p) = \tau^m(\gamma) \), following the announcement of any basic price \( p \) at which the consideration constraint is satisfied: \( p \leq v - T(\tau^m(\gamma)) \). At any \( p \) above this level, the no-missed sale condition is slack whenever the consideration constraint is satisfied, in which case there exists no continuation equilibrium where the ancillary good is sold. The unique continuation equilibrium is such that no consumers consider purchasing the good. The optimal basic price is then \( p^* = v - T(\tau^m(\gamma)) \), as claimed.

Now consider the case where \( \gamma \in (\hat{\delta}, \hat{\tau}] \), which implies that \( \tau^m(\gamma) > \hat{\tau} \). Under perfect information, as revealed by Proposition 1, the firm’s optimal policy is to price the ancillary good at marginal cost, \( \tau = \gamma \), and fully extract rents from consumers by means of the basic price \( p = v - T(\gamma) \). This is no longer possible under imperfect information, as a basic price as high as \( p = v - T(\gamma) \) would attract no consumers to the checkout point (by fear of hold-ups). Because the firm’s problem under perfect information is quasi-concave, the firm’s optimum (under imperfect information) is to choose the highest basic price that makes consumers (correctly) believe that they will not regret considering purchasing the good. This is achieved at \( p^* = v - \hat{\tau} \), which induces the drip price \( \tau^* = \hat{\tau} \) in the continuation game that follows.

For \( \gamma > \hat{\tau} \), the firm’s optimum perfect information (as characterized by Lemma 1 and Proposition 1) can be implemented as an equilibrium under imperfect information. This implies that the optimal basic price coincides with the one practiced under perfect information, as claimed in the proposition.

Proof of Corollary 5. The result follows from Proposition 4 and the fact that \( \hat{\tau} \) is increasing in \( s \).

Proof of Proposition 5. Consider the problem of choosing \( \gamma \) to maximize \( (1 - G(\tau^m(\gamma)))(\gamma - \delta) \). The first-order condition of this problem is given by (10). By the definition of the monopoly price, we then have that

\[
\tau^m(\tau^{mm}(\delta)) = \tau^{mm}(\delta) + \frac{1}{\lambda_G(\tau^m(\tau^{mm}(\delta)))}.
\]

Plugging (10) into the equation above leads to

\[
\tau^m(\tau^{mm}(\delta)) = \delta + \frac{1}{\lambda_G(\tau^m(\tau^{mm}(\delta)))} \left[ \frac{\lambda'_G(\tau^m(\tau^{mm}(\delta)))}{\lambda_G^2(\tau^m(\tau^{mm}(\delta)))} + 2 \right].
\]

The equation above has a unique solution provided \( b - \lambda'_G(b)\lambda_G^3(b) - 2\lambda_G^{-1}(b) \) is increasing, as assumed in the text (see footnote 32). Because \( \tau^m(\gamma) \) is increasing in \( \gamma \), this guarantees that the function \( (1 - G(\tau^m(\gamma)))(\gamma - \delta) \) is quasi-concave.

43
The supplier’s profit function, \( \hat{\Xi}(\gamma; \delta) \), as a function of the wholesale price \( \gamma \), is then

\[
\hat{\Xi}(\gamma; \delta) = \begin{cases} 
(1 - G(\tau^m(\gamma)))(\gamma - \delta) & \text{if } \gamma \leq \hat{\delta} \\
(1 - G(\hat{\tau}))(\gamma - \delta) & \text{if } \gamma \in (\hat{\delta}, \hat{\gamma}) \\
(1 - G(\tau^*(\gamma)))(\gamma - \delta) & \text{if } \gamma \geq \hat{\gamma},
\end{cases}
\]

where \( \tau^*(\gamma) \) is as in Proposition 4. Notice that \( \hat{\Xi}(\gamma; \delta) \) is affine when \( \gamma \in (\hat{\delta}, \hat{\gamma}) \). Consider first the case where \( \delta < \hat{\delta} \), which implies that the supplier’s profit has a local maximum at \( \tau^m(\delta) < \hat{\tau} \).

Let us define the threshold \( \bar{\gamma}(\delta) \) as

\[
\bar{\gamma}(\delta) = \inf \{ \gamma \in [\hat{\tau}, +\infty) : (1 - G(\tau^*(\gamma)))(\gamma - \delta) \geq (1 - G(\tau^m(\tau^m(\delta)))(\tau^m(\delta) - \delta) \},
\]

where \( \tau^*(\gamma) \) is as in Proposition 4. If the set above is empty, we identify \( \bar{\gamma}(\delta) \equiv +\infty \) (notice that the set above in non-empty for \( \delta \) close to \( \hat{\delta} \)). Together with the envelope theorem, total differentiation reveals that, whenever \( \bar{\gamma}(\delta) < +\infty \), the threshold \( \bar{\gamma}(\delta) \) is decreasing in \( \delta \).

Consider first the case where \( \delta < \hat{\delta} \). Whenever \( v - c \leq \bar{\gamma}(\delta) \), the supplier is better off by setting the wholesale price to equal the double monopoly price, \( \gamma^* = \tau^m(\delta) < \hat{\tau} \), and the drip price in equilibrium is \( \tau^* = \tau^m(\tau^m(\delta)) \). If, however, \( v - c > \bar{\gamma}(\delta) \), then \( \gamma^* \in (\hat{\tau}, v - c) \) and the drip price is such that \( \tau^* < \gamma^* \).

Consider now the case where \( \delta \geq \hat{\delta} \), and notice that \( \bar{\gamma}(\delta) = \hat{\gamma} \). Because \( v - c > \hat{\gamma} \) (by Assumption 1), optimality requires the supplier to set \( \gamma^* \in (\hat{\tau}, v - c) \). This reflects the fact that its profit function is monotonically increasing over the range \( \gamma < \hat{\tau} \).

The statement in the proposition summarizes the observations above. ■

**Proof of Corollary 6.** That under-consumption occurs when the upstream market is competitive and \( \delta < \hat{\tau} \) follows directly from Corollary 4 and Proposition 4. That under-consumption occurs when the upstream market is monopolistic and \( v - c < \bar{\gamma}(\delta) \) follows directly from Corollary 4 and Proposition 5. ■

**Proof of Proposition 6.** By arguments analogous to those in the proof of Proposition 4, we obtain the following result.

**Lemma 4 (Continuation equilibria)** Consider the continuation game following the announcement of the basic price \( p \) under cap regulation.

(i) There exists an equilibrium where the ancillary good is traded if and only if \( \gamma \leq v - c \) and \( v - p \geq \hat{\tau} \). In this case, the drip price is \( t^*(p) = \gamma \).

(ii) There exists equilibria where consumers consider purchasing the good, but the ancillary good is not offered, if and only if \( v - c \leq \gamma \) and \( v - p \geq \hat{\tau} \).

(iii) There exists equilibria where consumers do not consider purchasing the good if and only if \( v - p \leq \hat{\tau} \).

Such equilibria are however in weakly dominated strategies whenever there exists an equilibrium where the ancillary good is traded.
Now notice that the perfect-information firm’s optimum characterized in Proposition 1 is implementable as an equilibrium under imperfect information and cap regulation. Because the perfect-information profit is an upper bound to the firm’s profit under imperfect information, the result follows.

\[\text{\textbf{Proof of Proposition 7.}}\] Consider first the case where the upstream market is competitive, and let \( \delta < \hat{\tau} \). Because \( \tau^r = \delta \) and the consideration constraint binds, it follows that \( p^r = v - T(\delta) > v - T(\tau^m(\delta)) \). Because the consideration binds when under either regulation or laissez-faire, consumers are equally well-off. Welfare strictly increases, as implied by Corollary 4. That consumers are equally well-off than implies that the firm is strictly better off.

Consider now the case where the upstream market is monopolistic. We first establish the following lemma.

\[\text{\textbf{Lemma 5}}\] Under the weakly decreasing pass-through property, \( \tau^m(\delta) \leq \tau^{mm}(\delta) \).

\[\text{\textbf{Proof of Lemma 5.}}\] To simplify notation, let \( a(b) \equiv \lambda^{-1}_G(b) \). Note that

\[
\frac{d\tau^m}{d\delta}(\delta) = \left[ \frac{\lambda_G^2(\tau^m(\delta))}{\lambda_G^2(\tau^m(\delta)) + 1} \right]^{-1} = [-a'(\tau^m(\delta)) + 1]^{-1}.
\]

Because \( \tau^m(\delta) \) increases with \( \delta \), the weakly decreasing pass-through property is then equivalent to \( a'(\cdot) \) being weakly decreasing over the range \([\tau^m(0), +\infty)\). This is obviously equivalent to \( a(\cdot) \) being weakly concave over this range.

To simplify notation, let \( x \equiv \tau^m(\delta), z \equiv \tau^{mm}(\delta) \) and \( y \equiv \tau^m(\tau^{mm}(\delta)). \) These quantities simultaneously solve the following first-order conditions:

\[
x = \delta + a(x), \quad z = \delta + a(y)(1 - a'(y)), \quad y = z + a(y).
\]

Note that \( y > x \), as implied by the assumption that \( 1 - a'(b) > 0 \) for all \( b \geq 0 \). Because \( a(\cdot) \) is weakly concave, it then follows that

\[
a(x) - a(y) \leq (x - y)a'(y).
\]

Employing the first-order conditions, we obtain that

\[
x - z = a(x) - a(y) + a(y)a'(y) \\
\leq (x - y)a'(y) + a(y)a'(y) \\
= a'(y)(x - y + a(y)) \\
= a'(y)(x - z)
\]

where the inequality from the first to the second line employs (15).

We intend to show that \( z \geq x \). To obtain a contradiction, let us suppose that \( x > z \). Consider first the case where \( a'(y) > 0 \). The inequality (16) then implies that \( a'(y) \geq 1 \), which contradicts...
that $1 - a'(b) > 0$ for all $b \geq 0$. Now consider the case where $a'(y) < 0$. The inequality (16) then implies that $x \leq z$, contradicting $x > z$. We then conclude that $z \geq x$, as claimed.

Now suppose $v - c < \bar{\gamma}(\delta)$. Notice that, by definition, $\bar{\gamma}(\delta) < \gamma(\delta)$. As result, $v - c < \bar{\gamma}(\delta)$ implies $v - c < \gamma(\delta)$. Under either cap regulation or mandated transparency, the equilibrium wholesale price is then $\gamma^* = \tau(m(\delta))$, whereas under laissez-faire and imperfect information, it is $\gamma^* = \tau_{mm}(\delta)$. Lemma 5 then implies that regulation reduces the equilibrium wholesale price. Because the drip price is at cost under regulation, it follows that social welfare strictly increases. Because the consideration binds when under either regulation or laissez-faire, consumers are equally well-off. That consumers are equally well-off implies that the firm is strictly better off. That the supplier is strictly better off follows from the fact that $\Xi(\gamma; \delta)$ is strictly greater than $\bar{\Xi}(\gamma; \delta)$ over the range $\gamma < \hat{\gamma}$.

**Proof of Proposition 8.** The arguments are identical to those of Propositions 1 and 4 after modifying the consideration constraint to be $v - p \leq s$.

**Proof of Corollary 7.** The result follows from the arguments in the text and Proposition 1.

**Proof of Corollary 8.** When $s = 0$, $\hat{\gamma} = 0$. The result then follows from Corollary 2.

**Proof of Corollary 9.** The result follows from the arguments in the text, Corollaries 4 and 7, and Proposition 3.
References


