Granular Comparative Advantage*

Cecile Gaubert
cecile.gaubert@berkeley.edu

Oleg Itskhoki
itskhoki@princeton.edu

July 4, 2016

preliminary and incomplete

download the most up-to-date version from:
http://www.princeton.edu/~itskhoki/papers/GranularCA.pdf

Abstract

Firms play a pivotal role in international trade, shaping the comparative advantage of countries. We propose a 'granular' multi-sector model of trade, which combines fundamental comparative advantage across sectors with granular comparative advantage due to outstanding productivity of individual firms. We develop a SMM-based estimation procedure, which takes full account of the general equilibrium of the model, and jointly estimate the fundamental and the granular forces using French micro-level data with information on firm domestic and export sales across manufacturing industries. The estimated granular model captures the salient features of micro-level heterogeneity across firms and industries, without relying on variation in model parameters across sectors. The estimated model implies that thirty percent of trade flows is explained by granular forces, and that sectors with the highest export shares are more likely to be of granular origin than sectors with average export shares. Extending the model to allow for firm-level productivity dynamics explains the majority of the change in the country's comparative advantage over time. We further show that empirically measurable proxies of granularity have a substantial predictive ability for trade flows in the estimated model, even after controlling for fundamental comparative advantage of the sectors. Lastly, we show that the welfare gains from liberalizing a sector are shaped by the extent to which this sector is granular.

*We thank Ulrich Müller, Chris Sims, and seminar participants at Berkeley, Princeton, Michigan, Brown, Dartmouth, Duke and NOITS and ERWIT conferences for insightful comments, Dmitry Mukhin and Mark Razhev for excellent research assistance, and Davin Chor for sharing his data. Cecile Gaubert gratefully acknowledges financial support from the Clausen Center at UC Berkeley.

1 Introduction

Firms play a pivotal role in international trade. Much of exports is done by a small number of very large firms that enjoy substantial market shares in their markets across destination countries. One may thus conjecture that countries export not only the goods they are fundamentally good at producing, but also the goods that their individual firms happen to master to produce in an idiosyncratic way. As a consequence, if some of these individual firms disappeared, the country's comparative advantage would be dramatically altered. This paper contrasts such *granular* comparative advantage with the *fundamental* comparative advantage of a country, which we define as a sector-level characteristic stemming from technological differences or factor endowment differences across countries.

By doing so, we revisit the fundamental questions in international trade: What goods do countries trade? What is the source of countries' comparative advantage? The answers to these questions are interesting in themselves, even if the source of comparative advantage were not consequential for trade flows and welfare gains from trade. Furthermore, the knowledge of the specific source of comparative advantage is instrumental for our ability to predict changes in trade flows in response to a variety of shocks, such as reductions in trade costs and productivity changes across countries.

Casual empiricism suggests that individual firms play a central role in shaping country-level trade patterns: for example, the fate of Nokia in Finland or Intel in Costa Rica have shaped aggregate export patterns in these countries. In this paper we are concerned with the role of individual firms in determining the comparative advantage of a country. To what extent the exports of a country are due to its *fundamental* comparative advantage, immune to the fate of individual firms, versus *granular* comparative advantage embodied in individual firms and entrepreneurs? In more formal terms, to what extent the country's comparative advantage is shaped by the high-mean sectoral productivity versus outstanding productivity draw(s) from an otherwise unremarkable mean-productivity distribution? Furthermore, can one identify which specific export sectors are 'granular'?

The dominant theoretical framework in international trade has shifted towards modeling individual firms' exporting decisions, with a focus on firm heterogeneity and selection of the fittest firms into exporting. At the same time, these models typically maintain the assumption that sectors are comprised of a continuum of firms. Under this continuum assumption, the law of large numbers (LLN) applies. The productivity of any individual firm plays no role in shaping sectoral trade in such a model. Sectoral trade is instead fully determined by the fundamental sectoral productivity. In other words, there is no granularity left upon integration across heterogenous firms within sectors. We refer to such models as *continuous models*, which are equivalent in the aggregate with neoclassical Ricardian models. In contrast, a *granular model* dispenses with the assumption of a continuum of draws, and acknowledges that a finite number of firms operate in each sector. Hence, the realized sectoral productivity (which shapes comparative advantage) operates away from the LLN limit. Furthermore, it can be in part driven by the specific productivity draws of individual firms, acknowledging the role of these firms in shaping sectoral outcomes (see also Eaton, Kortum, and Sotelo 2012, for a further discussion).[1]

[1] On top of the discreteness of the number of firms, the fact that the distribution of firm productivities is fat-tailed plays a very important role here. Because of this fat-tailedness, the properties of the average sectoral productivity for a sector of N

In this paper, we aim to analyze the properties of a quantified granular model of international trade. We conduct our analysis by systematically benchmarking the properties of the granular model to those of its continuous counterpart. To that end, we start our analysis with a standard continuous model based on a multi-sector extension of Melitz (2003) with sectoral comparative advantage. Across sectors, this model embeds the Ricardian forces of Dornbusch, Fischer, and Samuelson (1977) in the sense that absent within-sector heterogeneity of firms, this model would converge to the Dornbusch, Fischer, and Samuelson (1977) model, with full specialization of countries across sectors according to their comparative advantage, and only inter-sectoral trade. Within sectors, we maintain the firm heterogeneity assumption of the Melitz (2003) model, but relax the assumption of the continuum of firms within sector, hence relaxing the LLN assumption. Firm productivities are drawn from a fat-tailed Pareto distribution consistent with the empirically observed Zipf's law in firm sales distribution. The finite number of draws from a fat-tailed productivity distribution results in granular effects: individual firms' outcome can shape sectoral productivity. Our model is a multi-sector extension of the Eaton, Kortum, and Sotelo (2012) granular model of trade, which allows us to nest simultaneously fundamental and granular comparative advantage in a unified framework.

We develop a simulated method of moments (SMM)-based estimation procedure for the granular model, consistent with the full general equilibrium of the model. We estimate the model using French firm-level data, which has information on both domestic and export sales of French firms across 117 4-digit manufacturing industries. We show that the estimated model reproduces the empirical cross-sectional distributions, in particular the cross-sectoral variation in the number of firms, in the fatness of tails of the sales distributions, in the market shares of the largest firm, and in the sectoral export intensity, without relying on heterogeneity in parameters across sectors. In contrast, the continuous model would be unable to account for many of these features of the data without bringing in parametric cross-sectional heterogeneity. We argue therefore that the granular model, by means of relaxing the convenient (yet counterfactual) LLN assumption, provides a significantly better fit to the data.

In the data, despite the fact that a median manufacturing sector features around 270 firms, the market share of the largest firm is on average around 20% due to the fat-tailed productivity distribution. As a result, our estimated granular model, which mimics these empirical patterns, acts very differently from the continuous LLN model. In the model, as in the data, the single largest firm in a sector tends to have a mass-point effect on the cumulative sectoral sales and exports. The estimated model exhibits strong granular forces, which play a first order role in shaping sectoral comparative advantage and the resulting trade flows.

Our SMM procedure estimates jointly the sector-level and granular forces in the model, combining the data on sectoral trade flows and the within-sector distribution of firm sales. Intuitively, sectors in which the largest firm stands out in terms of sales relative to the median firm, exhibit signs of granularity. Therefore, we use the cross-sectoral correlation between the relative size of the largest firm (in the domestic market) and sectoral exports in the model, along with other moments, in order to

firms behaves very differently from what the central limit theorem would predict. That is, the realization of the largest draws among N shapes in part sectoral outcomes, even for relatively large N (See for example Gabaix 2011). We come back to this point below.

discipline the relative roles of fundamental and granular comparative advantage.

Using the estimated model, we show that fundamental comparative advantage, which acts at the sectoral level, accounts for 70% and the granular residual for 30% of the variation in the realized export shares across sectors. Therefore, a substantial share of international trade is a granular phenomenon. Furthermore, we show that sectors with the extreme export shares are more likely to be of the 'granular' origin, as opposed to sectors with average export shares. We next show that measurable proxies of granularity, such as the market share of the top local firms in their domestic market, have substantial predictive ability for sectoral exports, even after controlling for the fundamental comparative advantage of the sector. We also develop a statistical filtering technique that identifies the specific likely granular sectors in the data.

We then extend our model to allow for industry dynamics driven by exogenous firm productivity process. We discipline the firm-level productivity dynamics with the evolution of firm market shares observed in the data, in order to study the implications of the dynamic granular model for the evolution of comparative advantage over time. We show that the dynamic granular model is consistent simultaneously with the hyper-specialization of countries in a few industries and the relatively fast mean reversion in the comparative advantage of the countries, documented by Hanson, Lind, and Muendler (2015). In particular, a granular model with empirically-disciplined firm-level dynamics, and featuring no dynamics in fundamental comparative advantage, accounts for about 50% of mean reversion over time in country's realized comparative advantage documented by these authors. We then propose another way to quantify the importance of individual firms in shaping trade patterns. We consider a counterfactual in which a top firm in a sector fails for an exogenous reason and has to exit the industry. We show that this has a dramatic effect on the export standing of the sector, often switching it from a net exporter into a net importer.

Lastly, we investigate the extent to which accounting for the granularity of sectoral trade leads to different predictions for how sectors react to a trade liberalization episode, compared to the continuous model benchmark. We find that trade elasticities are shaped by the extent of granularity in each sector. Granularity predicts part of the welfare gains from liberalizing a set of sectors, beyond what is predicted by the sufficient statistics for welfare from Arkolakis, Costinot, and Rodríguez-Clare (2012). Finally, granularity shapes the dynamic gains from liberalizing a set of sectors, like it shapes the dynamics of comparative advantage.

Related literature Granularity has been mostly explored in macroeconomics for the purpose of studying aggregate fluctuations. However, the granular forces must be at least as prominent in international trade flows, which are shaped at the country-industry level, where granularity is particularly pronounced in the data.

- Granularity and aggregate fluctuations: Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Carvalho and Gabaix (2013), Carvalho and Grassi (2014)
- Granularity, trade and volatility: Di Giovanni and Levchenko (2012), Di Giovanni, Levchenko, and Méjean (2014)

- Granular trade model: Eaton, Kortum, and Sotelo (2012)
- Empirics of granularity and dynamics of comparative advantage: Freund and Pierola (2012), Chor (2010), Costinot, Donaldson, and Komunjer (2012), Hanson, Lind, and Muendler (2015)
- Oligopolistic competition in trade and international macro: Neary (2003, 2012, 2015), Atkeson and Burstein (2008), Edmond, Midrigan, and Xu (2015), Hottman, Redding, and Weinstein (2015), Amiti, Itskhoki, and Konings (2015)
- Recent review of granular implications for trade: Bernard, Jensen, Redding, and Schott (2015).

2 The DFS-Melitz Benchmark

In this section, we review the *continuous* model, which serves as a benchmark in our analysis of the *granular* model in Section 3. As a benchmark continuum economy we consider a two-country multi-sector extension of the Melitz (2003) model, with Ricardian comparative advantage across a unit continuum of sectors indexed by $z \in [0,1]$, as in Dornbusch, Fischer, and Samuelson (1977).[2] We refer to this benchmark economy as DFS-Melitz. More specifically, within each sector z we consider the Chaney (2008) version of the Melitz model without free entry, in which an exogenous mass of firms $M(z)$ are present and their productivities are drawn from a Pareto distribution with a sector-specific lower bound $\underline{\varphi}(z)$ and a shape parameter θ common across all sectors. We show below that in this model, the overall sectoral productivity is determined by:

$$T(z) = M(z)\underline{\varphi}(z)^{\theta}. \tag{1}$$

Intuitively, a sector is more productive either if there are more potential entrants, $M(z)$, or if the average productivity of a potential entrant, given by $\frac{\theta}{\theta-1}\underline{\varphi}(z)$, is high. The two countries differ in these sectoral productivity measures, $\{T(z)\}$ at home and $\{T^*(z)\}$ in foreign, which is the source of the Ricardian comparative advantage across sectors. Some of the specific assumptions and exposition we adopt here are slightly different from what is conventional in the Melitz literature, and this is meant to mimic the granular model of Section 3 and ease the comparison between the two frameworks.

Households in each country have Cobb-Douglas preferences over the consumption of sectoral goods $Q(z)$:

$$Q = \exp\left\{\int_0^1 \alpha_z \log Q(z)\mathrm{d}z\right\}, \tag{2}$$

where $\{\alpha_z\}$ are the preference parameters which determine the shares of household income spent on consumption in sector z. This is the only source of cross-sectoral heterogeneity that we introduce into

[2] This model extends Melitz (2003) in a multi-sector way, the same way Costinot, Donaldson, and Komunjer (2012) extend the Eaton and Kortum (2002) (EK) model. Our multi-sector extension of the Melitz model is different from Bernard, Redding, and Schott (2007) who also considered an extension of the Melitz model to a multi-sector environment, however one in which sectors differed in their capital-labor intensity, as in the classical Heckscher-Ohlin model. The other papers which considered a multi-sector DFS-Melitz environment, albeit under somewhat different formulation, are Okubo (2009) and Fan, Lai, and Qi (2015).

the model. The sectoral consumption bundles are CES aggregators of individual varieties ω:

$$Q(z) = \left[\int_{\omega \in \Omega(z)} q_z(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where $\Omega(z)$ is the set of varieties available for consumption in sector z at home, and $\sigma > 1$ is the elasticity of substitution across varieties within sectors, common to all sectors. The foreign demand structure is symmetric, with $\Omega^*(z)$ replacing $\Omega(z)$. The households supply labor inelastically, with L units at home and L^* units in foreign.

With this demand structure, the home consumers' expenditure on variety ω in sector z is given by:[3]

$$p_z(\omega)q_z(\omega) = s_z(\omega)\alpha_z Y, \qquad \text{where} \qquad s_z(\omega) \equiv \left(\frac{p_z(\omega)}{P(z)} \right)^{1-\sigma}, \tag{4}$$

where $p_z(\omega)$ is the price and $s_z(\omega)$ is the within-sector market share of the variety, and

$$P(z) = \left[\int_{\omega \in \Omega(z)} p_z(\omega)^{1-\sigma} \mathrm{d}\omega \right]^{1/(1-\sigma)} \tag{5}$$

is the sectoral price index. Lastly, Y is the aggregate household income at home which is comprised of labor income and dividends:

$$E = wL + \Pi, \qquad \text{where} \qquad \Pi = \int_0^1 \left[\int_{\omega \in M(z)} \pi_z(\omega) \mathrm{d}\omega \right] \mathrm{d}z, \tag{6}$$

where w is the wage rate at home and Π is the aggregate profits of all domestic firms. Hence, we have assumed that home households own home firms, and the firms pay out their full profits as dividends.

The foreign households are symmetric and differ only in the aggregate supply of labor L^*, the equilibrium wage rate w^*, and the dividend from the ownership of the foreign firms Π^*. We choose the foreign labor as numeraire and thus normalize the foreign wage $w^* = 1$, so that $w = w/w^*$ equals the home's relative wage. Finally, consumers' expenditure on the varieties in the foreign market parallels that in the home market given in (4).

Firms in sector z operate their individual varieties ω and choose whether to become active in the home and in the foreign markets. The entry cost in the home market is F units of home labor and it is F^* units of foreign labor in the foreign market respectively, independently of the origin of the firm. This symmetry of entry costs is an assumption we adopt following EKS, and it proves useful in our granular setup in Section 3.

There are $M(z)$ home firms in sector z, each endowed with a linear technology with individual productivity $\varphi = \varphi(\omega)$ drawn from a Pareto distribution with a cumulative distribution function

[3] This derives from the fact that under Cobb-Douglas consumers spend a constant share α_z of their income Y on purchasing the varieties in sector z (i.e., $P(z)Q(Z) = \alpha_z Y$), and within sector z the CES demand for variety ω is given by $q_z(\omega) = \big(p_z(\omega)/P(z)\big)^{-\sigma} Q(z)$.

$G_z(\varphi) = 1 - \left(\varphi/\underline{\varphi}(z)\right)^{-\theta}$, and we require $\theta > \sigma - 1$ for stability. In order to produce one unit of its variety, a firm with productivity φ requires $1/\varphi$ units of labor. Lastly, international shipments involve an iceberg melting cost $\tau > 1$, so that τ units of a good must be shipped for one unit to reach the foreign market. Therefore, a home firm with productivity φ has a marginal cost w/φ for domestic shipments and $\tau w/\varphi$ for international shipments.

Each firm is infinitesimal in the markets it serves. Therefore, upon entry, firms compete according to monopolistic competition in each market. They set a constant markup $\sigma/(\sigma-1)$ over their marginal costs. This implies that the firm's operating profit in each market equals $1/\sigma$ of its revenues, and the overall profit of the firm can be written as:

$$\pi_z(\omega) = \left[\left(\frac{\sigma}{\sigma-1}\frac{w/\varphi(\omega)}{P(z)}\right)^{1-\sigma}\frac{\alpha_z Y}{\sigma} - wF\right]^+ + \left[\left(\frac{\sigma}{\sigma-1}\frac{\tau w/\varphi(\omega)}{P^*(z)}\right)^{1-\sigma}\frac{\alpha_z Y^*}{\sigma} - w^* F^*\right]^+, \quad (7)$$

where we substituted the markup pricing rule over the marginal cost into the expression for revenues (4), and we use the notation $[x]^+ \equiv \max\{0, x\}$.[4]

Firms with sufficiently high productivities profitably enter the home and the foreign markets respectively, as is conventional in the Melitz model. We denote with $\varphi_h(z)$ and $\varphi_f(z)$ the productivity cutoffs for a domestic firm to enter the home and foreign markets respectively in sector z, and provide the closed-form expressions for these cutoffs in Appendix A.1. The foreign firms are symmetric, and we denote with $\pi_z^*(\omega)$ their profits, and with $\varphi_h^*(z)$ and $\varphi_f^*(z)$ their productivity cutoffs for entry into the home and foreign markets respectively.

Sectoral Equilibrium Using (5), (7) and the markup pricing rule, we can calculate the price index in sector z in the home market (see Appendix A.1):

$$P(z) = \frac{\sigma}{\sigma-1}w\left[\frac{\kappa}{\kappa-1}\frac{T(z)}{1-\Phi(z)}\right]^{-1/\theta}\left(\frac{\sigma w F}{\alpha_z Y}\right)^{(\kappa-1)/\theta}, \quad (8)$$

where we denote $\kappa \equiv \theta/(\sigma-1)$ and

$$\Phi(z) = \frac{(\tau w^*)^{-\theta}T^*(z)}{w^{-\theta}T(z) + (\tau w^*)^{-\theta}T^*(z)} = \frac{1}{1 + \left(\frac{\tau w^*}{w}\right)^{\theta}\frac{T(z)}{T^*(z)}}, \quad (9)$$

is the *foreign share*, that is the share of foreign firms in the sales in the domestic market in sector z, a conventional object in analysis of quantitative models of international trade following Eaton and Kortum (2002). All else equal, the foreign share increases with the fundamental productivity advantage $T^*(z)/T(z)$ of the foreign country, and decreases with the relative wage w^*/w and with the variable trade costs τ.[5] The sectoral price in (8) increases in the local wage rate and in the relative fixed cost of

[4]Specifically, a home firm sets $p_z(\omega) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi(\omega)}$ in the home market, which results in revenues $\left(p_z(\omega)/P(z)\right)^{1-\sigma}\alpha_z Y$, according to (4), and the operating profits equal fraction $1/\sigma$ of these revenues due to constant markup pricing. Net profits are operating profits net of the fixed entry cost. Symmetric characterization applies to profits in the foreign market, with the difference that the marginal cost of delivering a good abroad is augmented by iceberg trade cost τ.

[5]We note that the foreign share in (9) does not depend on the fixed costs since both domestic and foreign firms are assumed

entry $(wF)/(\alpha_z Y)$, and decreases in sectoral productivity. It also increases in the home share $[1-\Phi(z)]$, as is familiar from the Arkolakis, Costinot, and Rodríguez-Clare (2012) analysis.

The definition of the foreign share, and its symmetric counterpart in the foreign country $\Phi^*(z)$, makes it straightforward to calculate sectoral exports of home and foreign countries respectively:

$$X(z) = \alpha_z \Phi^*(z) Y^* \qquad \text{and} \qquad X^*(z) = \alpha_z \Phi(z) Y. \tag{10}$$

In the appendix we also characterize the allocation of aggregate labor supply to sector z, which in the home market satisfies:

$$wL(z) = \alpha_z Y \left(\frac{\sigma\kappa - 1}{\sigma\kappa}[1 - \Phi(z)] + \frac{\kappa - 1}{\sigma\kappa}\Phi(z) \right) + \alpha_z Y^* \frac{\sigma - 1}{\sigma}\Phi^*(z). \tag{11}$$

The last term is labor used in production of goods for foreign market, while the first two terms is labor used for production and entry costs in the home market.[6] This fully characterizes the sectoral equilibrium in sector z given aggregate variables (w, w^*, Y, Y^*) and exogenous parameters, including sector-level productivity parameters $T(z)$ and $T^*(z)$.

General Equilibrium requires balanced trade and labor market clearing in both countries, which (together with our choice of numeraire $w^* = 1$) allow us to solve for (w, w^*, Y, Y^*). These three conditions also imply countries' budget balances (6) by Walras Law. Using (10), we aggregate sectoral net exports:

$$NX(z) \equiv X(z) - X^*(z) = \alpha_z \big[Y^* \Phi^*(z) - Y \Phi(z) \big]$$

to arrive to the trade balance condition:

$$Y \int_0^1 \alpha_z \Phi(z) \mathrm{d}z = Y^* \int_0^1 \alpha_z \Phi^*(z) \mathrm{d}z. \tag{12}$$

Recall from (9), that $\Phi(z)$ and $\Phi^*(z)$ can be written as function of relative wages w/w^* and the exogenous parameters of the model. Therefore, (12) links relative wages with relative incomes, Y/Y^*. A higher relative income Y/Y^* implies a larger relative demand for foreign goods, and hence must be met by a higher relative foreign wage w^*/w to ensure trade balance, replicating the standard DFS logic.

Next, aggregating sectoral labor demand in (11) across z and equalizing it with inelastic labor supply L, we obtain aggregate labor market clearing, which we manipulate in Appendix A.1 to arrive at:

$$wL = \frac{\sigma\kappa - 1}{\sigma\kappa} Y \qquad \text{and} \qquad w^* L^* = \frac{\sigma\kappa - 1}{\sigma\kappa} Y^*. \tag{13}$$

to face the same fixed costs of entry into the home market. As a result, fixed costs in this framework have little effect on the key variables which characterize equilibrium, apart from the price indexes $P(z)$ and $P^*(z)$, which increase with the fixed cost of entry into the market, thereby reducing local welfare. In Appendix A.1 we also describe the alternative case in which the fixed cost differs between domestic producers and importers. This appendix also discusses the DFS limit of the model, which obtains as $\theta, \sigma \to \infty$ and $F \to 0$ holding $\kappa, \sigma F$ and $\big(T(z)/T^*(z)\big)^{1/\theta}$ constant, so that $\Phi(z)$ and $\Phi^*(z)$ become step functions from 0 to 1 at some cutoff sectors $\underline{z}$ and $\bar{z}$ respectively.

[6] Specifically, we show that $(\sigma - 1)/\sigma$ of revenues goes to cover variable production labor costs (in the country of production) and fraction $(\kappa - 1)/(\sigma\kappa)$ goes to cover entry labor costs (in the country of entry). Note that the first term in (11) can be decomposed as $(\sigma\kappa - 1)/(\sigma\kappa) = (\sigma - 1)/\sigma + (\kappa - 1)/(\sigma\kappa)$.

Therefore, total labor income is a constant share of GDP (total income), with the complementary share coming from firm profits. Taking the ratio of the two equations in (13) results in:

$$\frac{Y}{Y^*} = \frac{wL}{w^*L^*},$$

which together with (12) allows to solve for both relative wage w/w^* and relative incomes Y/Y^*, again as in the DFS model. A more productive country supplies more goods on the foreign market, and trade balance requires that this is offset by both higher wages and incomes in this country.[7]

With these simple characterizations for the aggregate variables, we briefly returns to the cross-sectional sectoral outcomes. Specifically, we characterize the number of entrants and the labor allocations across sectors. We denote the number of entrants with $N(z) = M(z)\big(\varphi_h(z)/\underline{\varphi}(z)\big)^{-\theta}$. Using (13) together with the solution for $\varphi_h(z)$ given in the appendix leads to express the number of entrants as:

$$N(z) = \frac{\kappa - 1}{\sigma\kappa}[1 - \Phi(z)]\frac{\alpha_z Y}{wF}. \tag{14}$$

Larger markets $\alpha_z Y$ and markets with smaller fixed costs wF have more domestic firm-entrants; so do the markets with less foreign competition (smaller foreign share $\Phi(z)$). The sectoral labor allocation (11) can be simplified using (13):

$$L(z) = L\left[\alpha_z + \frac{\sigma - 1}{\sigma}\frac{NX(z)}{wL}\right], \tag{15}$$

In the closed economy $NX(z) = 0$, and therefore each sector gets $\alpha_z L$ units of labor, due to the Cobb-Douglas sectoral preference aggregator. The allocation of labor in the open economy additionally depends on the net exports of the sector.

3 The Granular Model

We now develop the granular version of the model outlined in Section 2. This model is a multi-sector version of Eaton, Kortum, and Sotelo (2012; henceforth EKS), nesting together fundamental comparative advantage across sectors, a (pair of) sectors-level characteristic, and granular comparative advantage arising from the individual productivity draws of firms. Much of the modeling environment is the same as in Section 2, with the exception that the number of firms in each sector is now discrete and finite. Specifically, there are still a continuum of sectors, but the number of varieties within each sector is finite. The preference structure is the same, but instead of the sectoral aggregator (3), we now have:

$$Q(z) = \left[\sum\nolimits_{i=1}^{K(z)} q_{z,i}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{16}$$

[7] Indeed, we can combine the two conditions as

$$\frac{wL}{w^*L^*} = \frac{\int_0^1 \alpha_z \Phi^*(z)\mathrm{d}z}{\int_0^1 \alpha_z \Phi(z)\mathrm{d}z},$$

and a higher productivity at home increases the right-hand side of this equation, while a higher w/w^* increases the left-hand side and reduces the right-hand side.

where i indexes the product varieties, $K(z)$ is the total number of products offered in sector z in the home market. The demand $q_{z,i}$ and market share $s_{z,i}$ of individual product i are still given by (4), where the price index is now a discrete version of (5):

$$P(z) = \left[\sum\nolimits_{i=1}^{K(z)} p_{z,i}^{1-\sigma}\right]^{1/(1-\sigma)}, \tag{17}$$

with $p_{z,i}$ denoting the prices of individual products.

In addition, instead of a fixed pool of entrants, we adopt the EKS assumption that the number of potential entrants from the home country in sector z is a random variable distributed Poisson with a parameter $M(z)$.[8] The productivity of each entrant, as in the DFS-Melitz model, is independently drawn from a Pareto distribution with a sector-specific lower bound $\underline{\varphi}(z)$ and a common shape parameter θ. We denote this distribution $G_z(\varphi)$, as in Section 2. EKS show that under these circumstances, the combined productivity parameter $T(z) \equiv M(z)\underline{\varphi}(z)^\theta$ is still a sufficient statistic for the country-sector productivity advantage, but now determines it only in expectation, as under granularity the realized comparative advantage becomes a random variable. Formally, EKS show that the number of home entrants with productivity equal or above any given level $\varphi \geq \underline{\varphi}(z)$ is a Poisson random variable with parameter $T(z)\varphi^{-\theta}$, increasing in $T(z)$ and decreasing in φ.[9]

The production technology is as in the continuous model. A domestic firm with productivity φ uses a linear technology in labor ($y = \varphi\ell$) to produce output of its product. The product can be marketed domestically at no additional variable cost, or it can be exported at an iceberg trade cost $\tau > 1$. Therefore, the marginal cost for the firm of delivering a unit of its product locally is w/φ and it is $\tau w/\varphi$ internationally. As before, in order to enter the home (resp. foreign) market, the firms need to incur a fixed cost F (resp. F^*) in units of home (resp. foreign) labor. While firms are large within their industries, they are still infinitesimal at the level of the whole economy, since unlike EKS we have a continuum of sectors. We thus make the (internally consistent) assumption that firms take the wage rate w as given.

Granularity leads to two important differences in the economic environment for firms compared to the continuous benchmark. The first difference concerns the nature of competition and price setting, and the second difference concerns entry. We discuss each in turn below. In order to do so effectively, we adopt the following notational convention: we let $i \in \{1, 2, \ldots, \tilde{M}(z) + \tilde{M}^*(z)\}$ rank all potential entrants in the home market in the order of increasing marginal costs. Specifically, there are $\tilde{M}(z)$ potential domestic entrants with marginal cost of supplying home market $mc_{z,i} = w/\varphi_{z,i}$ and $\tilde{M}^*(z)$ potential foreign entrants with marginal cost of supplying home market $mc_{z,i} = \tau w^*/\varphi^*_{z,i}$. We assign firm-product indexes i in such a way that $mc_{z,i} \leq mc_{z,i+1}$ for $i \geq 1$. We refer to the firms with lower indexes i as more efficient firms, independently of what is the source of their lower marginal costs of

[8]Formally, the realized number of entrants $\tilde{M}(z)$ has the probability distribution function $\mathrm{P}\{\tilde{M}(z) = m\} = e^{-M(z)}M(z)^m/m!$ for $m = 0, 1, 2, \ldots$.

[9]To avoid the issue of separately specifying the probability distribution for $\varphi < \underline{\varphi}$, it is convenient, as in EKS, to work in the limit of $\underline{\varphi}(z) \to 0$ and $M(z) \to \infty$, which maintains constant the value of $T(z)$.

supplying the market.[10] We denote with $K(z) \leq \tilde{M}(z) + \tilde{M}^*(z)$ the total number of firms (home and foreign together) that choose to enter the home market. Lastly, we introduce an indicator variable $\iota_{z,i}$, which equals 1 if the firm (ranked) i in the home market of sector z is of domestic origin and 0 if it is foreign. The foreign market is characterized in a symmetric way.

Price setting Since the number of firms in any given market is finite, the competition between them is oligopolistic, rather than monopolistically competitive. Each firm can have a non trivial impact on the sectoral price index, and they internalize this impact when making their pricing decisions. As a result, firms charge variable markups $\mu_{z,i} \geq 1$ over their marginal costs:

$$p_{z,i} = \mu_{z,i} mc_{z,i}. \tag{18}$$

This contrasts with the constant-markup pricing under monopolistic competition in Section 2. We make the assumption that firms compete in prices.[11] That is, firms that chose to enter a given market set prices as an equilibrium outcome of a Bertrand-Nash game by maximizing market-specific profits and taking prices of their competitors as given.

Under these circumstances, firms adopt the following markup policy:[12]

$$\mu_{z,i} = \frac{\varepsilon_{z,i}}{\varepsilon_{z,i} - 1}, \qquad \text{where} \qquad \varepsilon_{z,i} \equiv \varepsilon(s_{z,i}) = \sigma(1 - s_{z,i}) + s_{z,i}, \qquad s_{z,i} = \left(\frac{p_{z,i}}{P(z)}\right)^{1-\sigma}. \tag{19}$$

The variable $\varepsilon_{z,i} \in [1, \sigma]$ can be thought of as the effective elasticity of residual demand for the product of the firm, which is no longer constant when firms are large. In particular, the effective elasticity $\varepsilon_{z,i}$ decreases in the market share $s_{z,i}$ of the firm, and with this the markup of the firm increases.

The model is block recursive. The pricing block is solved as follows. Conditional on knowing the number and the set of entrants $[z_i]_{i\in 1\ldots K(z)}$, equations (18) and (19), together with the definition of the price index (17), can be solved to get the market shares, markups and prices of the entrants in the home market. Conditional on the set of entrants, this solution exists and is unique, although closed-form expressions are not available. More efficient firms (with lower ranks i) charge higher markups, but lower prices, and hence command greater market shares. Lastly, we can write the resulting profits

[10] Note that index i is not a property of a firm, but rather a property of a firm-market pair. A firm is characterized by its origin and productivity draw φ, and a given firm in general has different indexes i in the two markets. Mapping φ into i is not essential for characterizing equilibrium in this model.

[11] Much of the earlier granularity literature (including Carvalho and Grassi 2014, Di Giovanni and Levchenko 2012) adopts, however, an *ad hoc* assumption that markups are constant. The quantitative pricing-to-market literature following Atkeson and Burstein (2008) studies oligopolistic competition with variable markups, but adopts competition in quantities, which is qualitatively similar but results in greater markup variability (see detailed discussion in Amiti, Itskhoki, and Konings 2015). We adopt oligopolistic competition in prices, following EKS, which we view as a more natural assumption, and under which the difference from the constant markup environment is less pronounced.

[12] The operating profit of the firm (gross of fixed costs) is given by $(p_{z,i} - mc_{z,i})q_{z,i}$, and demand is given by $q_{z,i} = \frac{p_{z,i}^{1-\sigma}}{\sum_{j=1}^{K(z)} p_{z,j}^{1-\sigma}} \alpha_z E$, where we substituted the expression for price index $P(z)$ into (4). Maximizing with respect to own price $p_{z,i}$, given the prices of the competitors $p_{z,j}$ for $j \neq i$, results in (19).

of the firms in the home market in the following way:

$$\Pi_{z,i} = \frac{s_{z,i}}{\varepsilon(s_{z,i})}\alpha_z Y - wF. \tag{20}$$

Operating profits are a fraction $\frac{\mu_{z,i}-1}{\mu_{z,i}} = \frac{1}{\varepsilon_{z,i}}$ of revenues, and revenues are the firm's share $s_{z,i}$ of the sectoral expenditure $\alpha_z Y$ in the market. In equilibrium, firms with higher market shares command higher profits.

Entry An equilibrium of the entry game is achieved when for a subset of firms equilibrium profits given by (20) are non-negative, while for any additional entrant the profits upon entry would be negative. With a discrete number of potential entrants, there exists a multiplicity of equilibria in the entry game. This arises because the entry of a number of small unproductive firms can lower the price index enough to crowd out entry of a larger, more productive firm - and vice-versa. To avoid this issue, we consider a sequential entry game in which more efficient firms (with lower ranks i) have the priority to decide whether to enter the market. Equilibrium in the sequential entry game is unique and has the following cutoff property. All firms enter, until for a given firm i it is not profitable to do so; if firm i does not enter, then all firms with higher ranks do not find it profitable to enter either. The equilibrium of the entry game is achieved when the profits of the last (Kth) firm are non-negative, but if another $(K+1)$th firm were to enter, its profits would be negative.[13]

General equilibrium Equilibrium in each sector, including entry, can be fully characterized given aggregate incomes and wage rates (Y, Y^*, w, w^*). We maintain our normalization $w^* = 1$. The equilibrium values of (Y, Y^*, w, w^*) are determined from the definitions of aggregate income (analogous to (6)) and labor market clearing, which we can write for home as:

$$Y = wL + \int_0^1 \left[\sum\nolimits_{i=1}^{K(z)} \iota_{z,i}\Pi_{z,i} + \sum\nolimits_{i=1}^{K^*(z)} (1-\iota^*_{z,i})\Pi^*_{z,i}\right] \mathrm{d}z, \tag{21}$$

$$wL = \int_0^1 \left[\alpha_z Y \sum\nolimits_{i=1}^{K(z)} \iota_{z,i}\frac{s_{z,i}}{\mu_{z,i}} + \alpha_z Y^* \sum\nolimits_{i=1}^{K^*(z)} (1-\iota^*_{z,i})\frac{s^*_{z,i}}{\mu^*_{z,i}} + K(z)wF\right] \mathrm{d}z. \tag{22}$$

Aggregate home income is the sum of labor income and profits of the domestic firms in both the home and the foreign markets in all sectors. Home labor income is the sum of wages paid to production workers, both for home and foreign markets, and wages paid to workers engaged in entry activities in the home market. Note that there are $K^*(z)$ entrants into the foreign market of sector z and $\iota^*_{z,i}$ is the indicator for whether the ith entrant is a foreign firm. Further, $\alpha_z Y s_{z,i}$ equals firm sales in the home

[13] In order to characterize the equilibrium in the entry game, it is useful to introduce notation Π_i^K for the profit of firm ranked i in the home market of sector z (indicator omitted for brevity) when a total of K firms have entered. From (20), Π_i^K is an increasing function of market share s_i^K, defined in a similar way. We have the following properties: $s_i^K > s_{i+1}^K$ and $s_i^K < s_i^{K+1}$, and similarly for profits Π_i^K. Then entry is complete when $\Pi_K^K \geq 0$ and $\Pi_{K+1}^{K+1} < 0$, and this is satisfied for a unique value of K. Alternatively, it is convenient to define the minimum market share consistent with non-negative profits: $\underline{s}_z$ solves $\underline{s}_z/\varepsilon(\underline{s}_z) = wF/(\alpha_z E)$. Then entry is complete when $s_K^K \geq \underline{s}_z$ and $s_{K+1}^{K+1} < \underline{s}_z$. Note that if entry were not sequential, then it is possible to have equilibria in which inefficient firms enter and crowd out more efficient firms from entry.

market, and therefore $\alpha_z Y s_{z,i}/\mu_{z,i}$ equals firm operating costs for home market production, which are entirely spent on labor in the country of production. A similar logic applies in the foreign market.

In addition to (21)–(22), there are two symmetric equations for foreign, with one of these equations redundant due to Walras' Law. Substituting in the expressions for firm profits (20) into (21) and its foreign counterpart, it is easy to see that this aggregate equilibrium system is linear in (Y, Y^*, w, w^*), conditional on parameters of the model and (endogenous) equilibrium values of $\{K(z), K^*(z)\}_z$ and $\{s_{z,i}, s^*_{z,i}\}_{z,i}$, since both firm profits and markups are determined by the firm's market share. In Section 4.2 we describe a simple computational iterative procedure, which simultaneously solves for the sectoral and general equilibrium of the model. Lastly, we note that as the expected number of firm draws $M(z)$ and $M^*(z)$ increase, and the entry fixed cost F and F^* decrease, the granular model converges to the continuous limit of Section 2.[14]

Some analytical properties While a complete closed-form characterization of equilibrium is infeasible, there exists a useful analytical property in this formulation of the granular model, which was first pointed out in EKS. Given the structure of productivity draws (Poisson-Pareto) and market entry (i.e., a common destination-specific fixed costs), the distribution of market shares conditional on entry in a given market is the same for home and foreign firms. This implies that the respective distributions of markups and profits are also the same. The only ex-ante difference between home and foreign firms is the expected number of entrants. We expect more home firms in markets with greater home comparative advantage, or if home has lower relative wages. We also expect more home firms in the home market, due to the trade cost advantage. At the same time, conditional on observing a foreign and a home firm, they have the same expected market share, markup and profits. Formally, one can show that the expected share of foreign entrants in the home market of sector z, both in terms of firm count and market sales, is given by $\Phi(z)$, as defined in (9). In particular, the foreign share at home is given by:

$$\mathbb{E}\left\{\frac{X^*(z)}{\alpha_z Y}\right\} = \Phi(z) \ = \frac{1}{1 + \left(\frac{\tau w^*}{w}\right)^{\theta} \frac{T(z)}{T^*(z)}}, \tag{23}$$

and thus the granular model has the same sectoral trade shares as the continuous model, but in expectation only.[15] The actual foreign share is shaped in part by this mean level, driven by fundamental comparative advantage, and in part by the actual realization of firm-level productivities, whose average may depart from this mean, driven by outstanding draws. To highlight this distinction between trade driven by fundamental comparative advantage vs trade driven by a 'granular comparative advantage',

[14]In order to maintain the entry productivity cutoffs stable, $FT(z)$ and $F^*T^*(z)$ must stay constant as $T(z)$ and $T^*(z)$ increase together with the number of draws (see (A3)).

[15]Sketch of a proof: Recall that the number of home firms with productivity above φ is distributed Poisson with parameter $T(z)\varphi^{-\theta}$, and symmetrically for foreign firms. Since the sum of Poissons is a Poisson with parameters adding up, the total number of firms with marginal cost below c of delivering the good to the home market is Poisson with parameter $c^{\theta}[T(z)w^{-\theta} + T^*(z)(\tau w^*)^{-\theta}]$. Note that a foreign firm has a cost $\tau w^*/\varphi$ below c, if its productivity $\varphi > \tau w^*/c$. Additionally, the expected fraction of foreign firms with cost below c equals $T^*(z)(\tau w^*)^{-\theta}/[T(z)w^{-\theta} + T^*(z)(\tau w^*)^{-\theta}] = \Phi(z)$, and this is true independently of the value of c. This immediately implies that the foreign fraction of overall entrants is $\Phi(z)$, and the distributions of market shares (and hence markups and prices) within the groups of foreign and home entrants are the same.

we write the following accounting identity and decompose realized trade (here, foreign share at home) into its two components:

$$\frac{X^*(z)}{\alpha_z Y} = \mathbb{E}\left\{\frac{X^*(z)}{\alpha_z Y}\right\} + \left[\frac{X^*(z)}{\alpha_z Y} - \mathbb{E}\left\{\frac{X^*(z)}{\alpha_z Y}\right\}\right] = \Phi(z) \ + \Gamma(z). \tag{24}$$

In what follows, we call $\Gamma(z)$ the *granular residual*. It has mean 0, and is the part of trade patterns not explained by fundamental comparative advantage.

Using these results, we can calculate the aggregate foreign share in the aggregate home economy:

$$\Phi \equiv \frac{\int_0^1 X^*(z)\mathrm{d}z}{Y} = \int_0^1 \alpha_z\left[\Phi(z)+\Gamma(z)\right]\mathrm{d}z = \int_0^1 \alpha_z\Phi(z)\mathrm{d}z, \tag{25}$$

where we used that $\mathbb{E}_z\Gamma(z) = 0$ together with the assumption of the continuum of sectors. These ensure that granularity cancels out on average across sectors, so that the aggregate trade shares depend only on fundamental comparative advantage $T(z)/T^*(z)$, trade costs and the aggregate wage ratio w/w^*, as $\{\Phi(z)\}_{z\in(0,1)}$ do.

If we assume further that $\frac{T(z)}{T^*(z)}$ are drawn independently across sectors from a common distribution, then we can simplify expression (25) to $\Phi = \mathbb{E}_z\Phi(z)$. The expectation is taken across the draws of $\frac{T(z)}{T^*(z)}$.[16] In general, foreign share Φ is an increasing function of relative wage w/w^* and a decreasing function of τ. The foreign share Φ also decreases with neutral increases in relative home productivity (i.e., when all $\frac{T(z)}{T^*(z)}$ are multiplied by a constant greater than 1).

Using these properties, we can conveniently rewrite the general equilibrium conditions (21)–(22) as follows (see Appendix A.2):

$$wL = wFK + Y\frac{1-\Phi}{\mu} + Y^*\frac{\Phi^*}{\mu^*}, \tag{26}$$

$$Y = wL + (1-\Phi)\left[Y\frac{\mu-1}{\mu} - wFK\right] + \Phi^*\left[Y^*\frac{\mu^*-1}{\mu^*} - w^*F^*K^*\right], \tag{27}$$

where $K = \int_0^1 K(z)\mathrm{d}z$ is the aggregate number of firms in the home economy across all sectors and $\mu = \left[\int_0^1 \alpha_z\left(\sum_{i=1}^{K(z)} s_{z,i}\,\mu(s_{z,i})^{-1}\right)\mathrm{d}z\right]^{-1}$ is the average markup rate in the home economy. K^* and μ^* are defined symmetrically for the foreign economy, and two symmetric equations apply in the foreign economy. Note that $\left[\frac{\mu-1}{\mu}Y - wFK\right]$ in (27) are net profits in the domestic market, with fraction $(1-\Phi)$ of these profits accruing to domestic firms, and therefore the right-hand side of (27) characterizes the aggregate income (labor plus profits) equal to GDP Y of the domestic economy. Further, $Y(1-\Phi)/\mu$ equals the variable cost expenditure (on labor) of the domestic firms for production and sales in the domestic market, and therefore the right-hand side of (26) equals the total expenditure of all firms on home labor equal to labor income wL.

Equation (26) is the granular version of (13) in the continuous model. Combining it with (27) to

[16] We treat the Cobb-Douglas shares, $\{\alpha_z\}$ as deterministic parameters. A similar logic would hold if they were themselves stochastic variables, uncorrelated with $\{\frac{T(z)}{T^*(z)}\}_{z\in(0,1)}$.

solve out wL, we obtain the trade balance condition:

$$\Phi\left[Y - wFK\right] = \Phi^*\left[Y^* - w^*F^*K^*\right], \tag{28}$$

which is the granular version of (12) in the continuous model. Note that $[Y - wFK]$ are the sales in the home market net of the fixed cost of entry, and fraction Φ of these net sales constitutes foreign income from sales in the home market. The right-hand side of (28) is the symmetric income of home firms from sales abroad, ensuring trade balance. The equations above form the general equilibrium system and allow us to solve for $(Y, Y^*, w/w^*)$. With this we complete the description of the granular economy and turn to characterizing its quantitative properties.

4 Estimation of the Granular Model

In this section we develop an SMM (simulated method of moments) estimation procedure for the granular model and apply it to the French firm-level data. Importantly, the estimation procedure takes into account the general equilibrium restrictions imposed by the model. We then use the estimated model to study the quantitative properties of the granular mechanism, including model fit and the implied role of granularity for the country's comparative advantage. We close this section with counterfactuals, in particular those in which we study the role of the individual firms in shaping the comparative advantage of a country.

4.1 Data

We use a dataset of French firms (BRN), which reports information on the balance sheets of the firms declared for tax purposes. All firms with revenues over 730,000 euros are included. It reports in particular information on domestic and export sales, and 4-digit industry classification, at the firm level. We use 2005 as our reference year for calibration. We match this data with international trade data from Comtrade, to get the aggregate imports and exports of France in each industry. The industry classification used in the French data is the French NAF (based on European NACE classification), whereas the trade data uses ISIC rev3. We convert the French data into the ISIC rev3 classification (using the crosswalk between NACE and ISIC, available from UNstats). This leaves us with 117 4-digit manufacturing sectors.

4.2 Estimation

Estimation procedure We estimate a two-country model with France as home and the rest-of-the-world (ROW) as foreign. Home and foreign differ in labor endowments L and L^* and the country-sector productivities $\{T(z)\}$ and $\{T^*(z)\}$, which together determine the relative size of the home and foreign markets. We assume that $F^* = F$.[17] Given the Cobb-Douglas preference structure, all variables of in-

[17] Absent data on the number of foreign firms serving the domestic market, we cannot identify a fixed cost parameter separately for ROW.

terest in the model scale with the common level of productivity, so we can normalize $T^*(z) \equiv 1$ without loss of generality.[18] Furthermore, the model scales with L as long as we keep $\frac{L}{L^*}$ and $\frac{L}{F}$ constant. In other words, L simply determines the units of labor, and hence we normalize $L = 100$. The model, therefore, has four parameters $\{\sigma, \theta, \tau, F\}$ common across sectors and countries, in addition to sectoral Cobb-Douglas shares $\{\alpha_z\}_z$ common across countries, as well as Ricardian sectoral comparative advantage parameters $\{\frac{T(z)}{T^*(z)}\}_z$ and relative country labor endowment $\frac{L}{L^*}$.

We next parametrize the distribution of sector-level comparative advantage parameters $\frac{T(z)}{T^*(z)}$, to reduce the dimensionality of the problem. Hanson, Lind, and Muendler (2015) show evidence that in the cross section of countries, the distribution of measured comparative advantage is well-approximated by a log-normal distribution. We thus make the parameteric assumption that the distribution of the sector-level comparative advantage parameter is lognormal with parameters (μ_T, σ_T), i.e. $\log\left(\frac{T(z)}{T^*(z)}\right) \sim \mathcal{N}(\mu_T, \sigma_T^2)$.[19] Therefore, the parameter vector we need to estimate is $\Theta \equiv (\sigma, \theta, \tau, F, \mu_T, \sigma_T)$, along with Cobb-Douglas shares and relative labor endowment.

We proceed in two steps. In the first step, we read the Cobb-Douglas share from the data as:

$$\alpha_z = \frac{D(z) + M(z)}{\sum_{z'} D(z') + M(z')},$$

where $D(z)$ is domestic sales in sector z and $M(z)$ is total French imports in sector z. We also calibrate w/w^* using the ratio of the wage in France to the average wage of its trading partners weighted by trade volume. We find that French wages are 13% above the average wage of its trade partners. Therefore, we set $w/w^* = 1.13$. As we discuss below, this provides a general equilibrium restriction on other parameters.

In a second step, we follow a simulated method of moments (SMM) procedure to estimate the remaining six parameters (along with L/L^*), using French firm-level data across 117 manufacturing industries.

1. For a given (μ_T, σ_T), we get a vector of values for the relative sectoral productivity $\frac{T(z)}{T^*(z)}$ corresponding to 117 quantiles of the the lognormal distribution with parameters (μ_T, σ_T).

2. We use 4 replications of the French sectors,[20] where in each of the 4 replications the Cobb-Douglas shares are reshuffled randomly across the sectoral productivity draws to ensure that sectoral shares and productivities are not correlated.[21] The simulated economy has therefore $117 \times 4 = 468$ sectors.

[18]Note that if productivity in sector z of both countries doubles, the quantity in this sector doubles and the price halves without any effect on market shares within or across sectors.

[19]Note here that the comparative advantage distribution analyzed in Hanson, Lind, and Muendler (2015) as being well-approximated by a log-normal lumps together what we aim to distinguish here, i.e. the firm-level granular comparative advantage and the fundamental sector-level one. In our model, the distribution of the observed comparative advantage is an outcome of the combination of the distributions of fundamental and granular comparative advantages. It could therefore be that an alternative distribution for the fundamental comparative advantage provides an even better fit to the data. We will explore alternative distributions in further versions of the draft.

[20]The empirical Cobb-Douglas shares are divided by 4 to ensure that they sum to 1.

[21]Without such replication, the model is sensitive to the randomness in the match between sectoral shares and productivities, while with four replications the effect of this randomness is already negligible. In future drafts we will explore further the role of the fat tail in the joint distribution of sectoral productivities and Cobb-Douglas shares.

3. For a given θ and $T(z)$ in each sector z, we draw the productivities of potential entrants in a manner consistent with the structural assumptions of the model. We follow EKS in using rank-order statistics for the Pareto distribution to directly draw the productivity of the most productive firm, which follows an extreme value distribution, and each firm thereafter, with spacings following an exponential distribution. We draw enough "shadow firms" in each sector to ensure that the least productive ones never enter the market.[22]

4. We implement the following fixed point procedure:

   (i) given the calibrated value of $\frac{w}{w^*}$, take an initial guess for (Y, Y^*). This pins down the initial candidate general equilibrium vector.

   (ii) given general equilibrium variables, solve for sectoral equilibrium in each sector and each country, characterizing $\{K(z), K^*(z), \Phi(z), \Phi^*(z)\}_z$, $\{s_{z,i}, s^*_{z,i}\}_{z,i}$ and thus $(\Phi, \Phi^*, \mu, \mu^*)$, as described in Section 3.

   (iii) use the two general equilibrium equations (26) and (28) along with normalization $L = 100$ to solve for (Y, Y^*) given the values of $(\Phi, \Phi^*, \mu, \mu^*)$ calculated in (ii). Note that equations (26) and (28) are linear in (Y, Y^*) conditional on the outcomes of step (ii).

   In this step, we also recover the only value of L^* consistent with general equilibrium from the foreign counterpart to equations (26).

   (iv) update the values of (Y, Y^*) taking a half step between the initial vector from step (i) and the new vector from step (iii) and loop over until convergence.

   (v) upon convergence, compute the moments $\mathcal{M}(\Theta)$ and compare it with the moments from the data.

Given this mapping from parameters of the model to the moments, we choose Θ to minimize the distance between $\mathcal{M}(\Theta)$ and the empirical moments, using the unweighted quadratic objective.[23] We discuss the moment selection below. We estimate the model for two cases: (a) the constant markup case, in which we replace $\mu(s_{z,i})$ in (19) with $\bar{\mu} \equiv \sigma/(\sigma - 1)$, corresponding to the monopolistic competition markup; and (b) the variable markup case, in which we use (19), corresponding to the oligopolistic in prices (Bertrand) described in section 3.[24] The constant markup case is useful for reference, as the assumption is often made of a monopolistic competition setup even with a discrete number of firms.

Estimated parameters We start with the summary of the estimated parameters, reported in the top panel of Table 1 for both (a) the constant markup and (b) the variable markup versions of the model. Both specifications broadly agree on the parameter values, with a slight difference in the estimated

[22]Specifically, we use a set of 20,000 firm draws by sector for France and 80,000 for Foreign. For smaller sectors (in terms of Cobb-Douglas shares), we use 8,000 and 20,000 draws instead, to economize on the computing requirement. We check that these constraints do not bind around estimated parameter values, i.e. that not all shadow firms enter in any sector.

[23]We first find the value of the objective function on a sparse grid in the parameter space. Then, we start the local numerical optimization routine from the 15 points on the grid that have minimal value of the objective function. The convergence to the same parameter vector from these 15 points suggests that we are indeed finding a global minimum.

[24]In the future versions of the draft we will also consider the case of Cournot competition, which results in greater markup variation, and has been commonly used in quantitative models of pricing-to-market following Atkeson and Burstein (2008).

parameters of the fundamental comparative advantage (μ_T, σ_T). Both specifications imply a within-sector elasticity of substitution of about 5, consistent with conventional estimates in the literature at the 4-digit level (see Broda and Weinstein 2006). The two specifications also agree on the value of $\kappa = \theta/(\sigma - 1)$ of around 1, which determines the shape of the within-sector sales distribution, the so-called Zipf's law (see Gabaix 2009). The estimated iceberg trade costs are around 1.4, again broadly in line with the estimated in the literature (see Anderson and van Wincoop 2004).

Table 1: Estimated parameters

Parameter/variable	Model (a)	Model (b)
θ	4.339	3.751
σ	5.298	4.927
$\kappa = \theta/(\sigma - 1)$	1.009	0.955
τ	1.365	1.410
F	$0.460 \cdot 10^{-5}$	$0.430 \cdot 10^{-5}$
σ_T	1.180	1.284
μ_T	-0.581	-0.625
w/w^*	1.130	1.130
L^*/L	4.792	4.185
Y^*/Y	4.283	3.808
$1 - wL/Y$	0.170	0.132

Note: Model (a) refers to constant markup and Model (b) refers to variable markup. $1 - wL/Y$ is the profit share in GDP. (Standard errors to be computed by bootstrap.)

Lastly, the estimated mean μ_T of the relative productivity distribution $\log \frac{T(z)}{T^*(z)}$ is around -0.6. This distribution also exhibits substantial dispersion across sectors, with a standard deviation that is twice as high as the absolute value of the mean. This creates a lot of room for fundamental comparative advantage across sectors to drive trade flows, as we further discuss below. The negative mean of the relative productivity distribution implies that home is on average less productive than foreign. Recall however that $T(z) = M(z)\underline{\varphi}(z)^{\theta}$, so that $T(z)$ reflects both the average productivity and the number of draws. Since France is smaller than the rest of the world, it is natural to expect that ROW benefits from substantially more draws, which may offset a possible productivity advantage of France reflected in greater $\underline{\varphi}(z)$ across sectors. Therefore, we judge the relevance of this estimated parameter based on its implications for the equilibrium values of labor income and GDP for France and the ROW, which we discuss next.

The lower panel of Table 1 reports the general equilibrium variables consistent with the estimated models. In both cases, we have calibrated the wage in France to be 13% above the wage in the ROW, consistent with the data. The estimated constant-markup model implies that France is nearly 4.8 times smaller than the ROW in terms of labor endowment and 4.3 times smaller in terms of GDP. These factors in the variable-markup model are 4.2 and 3.8 respectively. In reality, France accounts for only 3.7% of world GDP and less than 1% of world population. Also France is about 5 times smaller that the European Union alone. This is however not the right interpretation to give to the values of these variables in the model. The model is a simplification in that it allows for only one foreign country with a single value of iceberg trade costs τ. One therefore needs to discount the countries in the ROW by their proximity to France as reflected in the trade costs. The countries that are close to France should be discounted less, while the countries that do not trade much with France should be discounted more heavily. Therefore, we view the estimated values of relative GDPs and labor endowments as plausible, and leave the fuller analysis of a multi-country world economy for future work. Lastly, we point out that the estimated model implies a profit share of GDP of about 15%, again in line with conventional values from NIPA. Interestingly, the estimation procedure does not target this moment directly or indirectly.

4.3 Model fit

Table 2 summarizes the moments used in the SMM estimation procedure, reporting the values of the moments in the French firm-level data, as well as in the two versions of the estimated model: (a) constant markups and (b) variable markups. We have chosen 14 moments to match in estimation, as we discuss in detail below, and overall both versions of the model have a very similar quality of fit.

Moments 1–3 in Table 2 characterize the distribution of the log number of firms across manufacturing sectors in France: average across sectors, standard deviation, and ratio of mean to the median in order to capture skewness in this distribution. In a typical (median) sector in France, there are about 270 firms with a substantial variation across sectors. Both versions of the model are able to match very closely the distribution of number of firms across sectors. We illustrate this in panel (a) of Figure 1, which contrasts the distribution of the log number of firms by sector in the constant-markup model, the variable-markup model, and in the data. We see from Figure 1a that the model distribution lies on top of the empirical distribution, with both distributions exhibiting a smooth bell-shape. The ability of the model to replicate closely the distribution of the number of firms across sectors is important for the analysis of granularity.

The next two moments (4 and 5) in Table 2 are the market share of the largest firm on the domestic market and the market share of the top three firms on the domestic market, measured relative to the sales of other French firms (that is, not taking into account imports in the total market size).[25] In the data, these statistics are equal to 20% and 36% on average across sectors, while both versions of the model somewhat over-predict these values (equal to 27% and 41% respectively in the constant-markup version of the model). Panel (b) of Figure 1 plots the distribution of the top-French-firm market share across sectors in the constant-markup model and in the data. We see that the fit of the model is rather decent, especially in the right tail of the distribution across sectors (i.e., for sectors with the largest firm capturing 20% or more of the market), while the model under-predicts the number of sectors with the largest firm accounting for less than 20% of sales. In addition, and quite intuitively, both the data and the model exhibit a negative relationship between the size of the largest firm and the number of firms in the market, with the respective elasticity equal to -0.09 in the data and -0.07 in the model.

The next two moments (6 and 7) characterize the distribution of the within-sector domestic sales

[25]Here and in what follows, we measure the size of French firms using the ratio of their domestic sales to the total domestic sales made by domestic firms. In other words, the individual firm size excludes exports, and the total market size excludes imports. We do so to get a proxy of the relative productivity of French firms that is not impacted by their export status.

Table 2: Moments used in SMM estimation

	Moment	Data	Model (a)	Model (b)
1	Log number of French firms, mean	5.601	5.569	5.603
2	Log number of French firms, std. dev.	1.445	1.351	1.262
3	Log number of French firms, mean/median	0.948	0.984	0.989
4	Domestic market share of the largest firm	0.198	0.266	0.265
5	Cum. dom. market share of three largest firms	0.358	0.413	0.426
6	Pareto shape of domestic sales, mean	0.897	1.005	0.962
7	Pareto shape of domestic sales, std. dev.	0.230	0.175	0.154
8	Exports relative to domestic sales, mean	0.631	0.678	0.684
9	Exports relative to domestic sales, std. dev.	0.690	0.581	0.574
10	Exports relative to domestic sales, mean/median	1.393	1.462	1.456
11	Fraction of sectors with exports > domestic sales	0.154	0.188	0.212
12	Aggregate Import share	0.320	0.468	0.476
13	Correlation top market share and export share	0.269	0.365	0.284
14	Correlation top-3 market share and export share	0.314	0.323	0.257

Note: Model (a) refers to constant markups and Model (b) refers to variable markups. See the description of the moments in the text.

across firms. Specifically, they report the cross-sector mean and standard deviation of the estimated Pareto shape parameters of the within-sector domestic sales distributions, estimated using a $\log(\text{rank}-0.5)$ regression for firms above the median size (see Gabaix and Ibragimov 2011). On average across sectors, the distributions of domestic sales exhibit Zipf's law, i.e. the estimated Pareto shape parameter is close to 1, however there is a substantial variation across sectors, with some sectors having shape parameters as high as 1.5 and others as low as 0.5. Panel (c) of Figure 1 illustrates the model fit of the whole cross-sectoral distribution of Pareto shapes. We observe that the model somewhat over-predicts the mean (distributions are less fat-tailed in the quantified model than what the data suggest) and under-predicts the variance of this distribution.

From the joint fit of moments (4), (5) and (6), we can see that the estimation procedure faces a trade-off between matching the market share of the top firm and the Pareto shape of the distribution, two moments that are particularly important to understand granularity. The estimated model over-fits the market shares of top firms, but also over-fits the Pareto shape of firm size distributions (i.e. under-fits how fat-tailed firm size distributions are). With smaller Pareto shapes, closer to the ones in the data, the distribution of firm sizes would be more fat-tailed in the quantified model, hence the top market shares would be even larger.[26]

Despite the lack of a perfect fit, panels (a)–(c) of Figure 1 illustrate that the model is capable of capturing the salient features of the cross-sector variation in the number of firms, top-firm market shares and the fatness of tails of the within-sector sales distributions. This is particularly striking given the tightness of the model parameterization, which features only six parameters common across sectors and countries. The only parameters that are allowed to vary across sectors are the Cobb-Douglas

[26] In a future version of the draft, we will explore a more conservative estimation where we put more weight on the top market share, so as to match the top market share more closely at the expense of the Pareto shape.

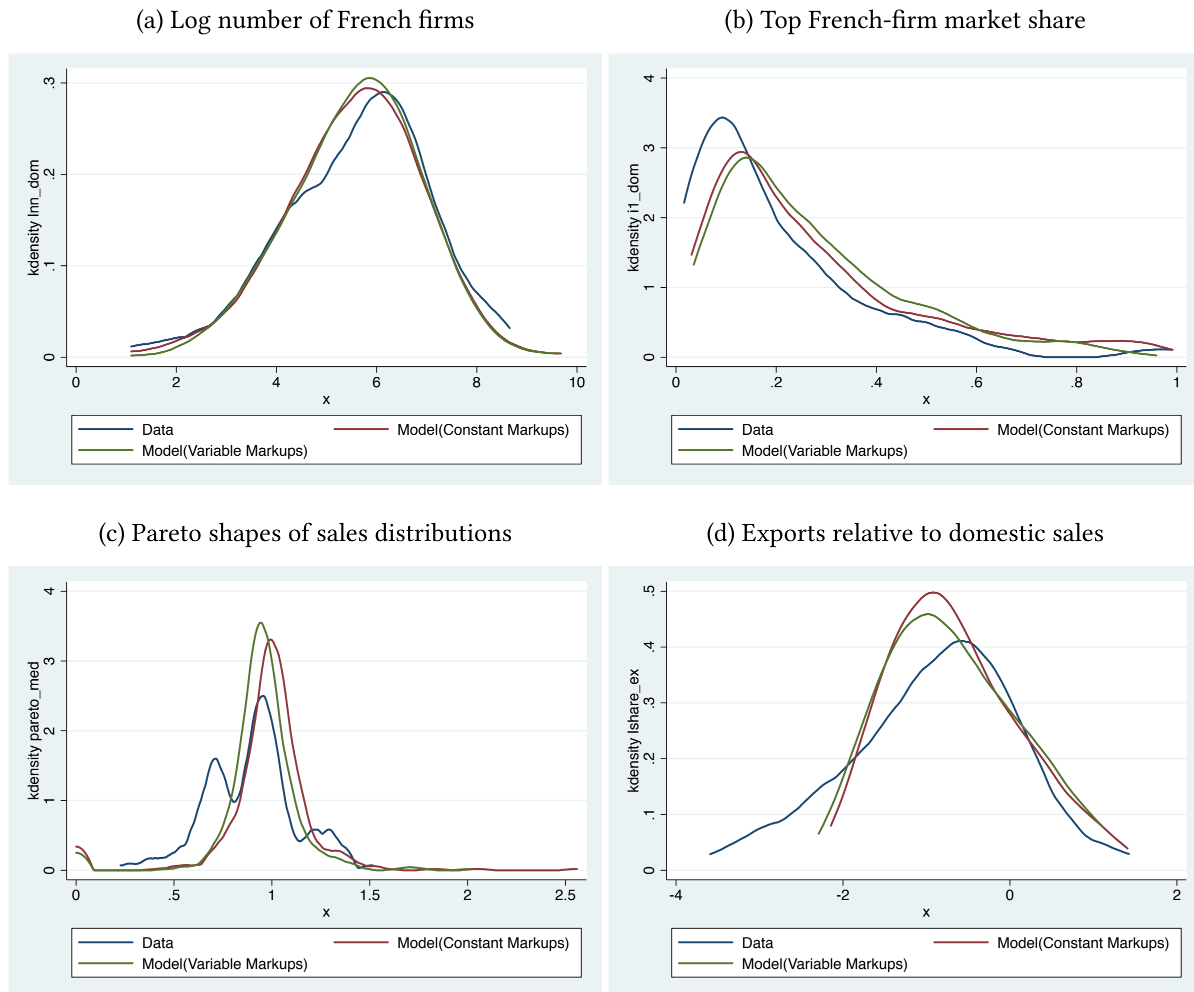


Figure 1: Distributions across sectors: model and data

Note: Top French firm market share is relative to other French firms. Pareto shapes are estimated using a log(rank−0.5) regression for within-sector sales of firms above the median size. Exports relative to domestic sales is the ratio of total sectoral exports to total sectoral domestic sales of the French firms.

sectoral shares. In Appendix Figure A2 we simulate the estimated model in which we shut down variation in the Cobb-Douglas shares keeping all other parameters constant. We see that the variation in Cobb-Douglas shares contributes very little to variation in the Pareto shapes and market shares of the top firm across sectors, and nearly all of the model-generated variation in these moments comes from the granularity of sectoral draws. A continuous model, therefore, would not be able to reproduce this variation without assuming rich heterogeneity in sectoral parameters (e.g., σ, θ, F or τ). For the variation in the number of firms across sectors, the heterogeneous Cobb-Douglas shares and the granularity contribute roughly equally.

The next 7 moments (8 through 15) in Table 2 characterize international trade. Moments 8–11 characterize the ratio of sectoral export sales relative to domestic sales, and its variation across sectors. Export sales in a typical French sector equal 63% of the domestic sales, with substantial variation across sectors, in particular with over 15% of sectors featuring export sales in excess of domestic sales. Panel

(d) of Figure 1 shows distribution of export-to-domestic sales ratio across sectors, in the data and in the estimated model. The model predicts accurately the mean of this distribution, yet it slightly underpredicts the variance and overpredicts the fraction of sectors in which this ratio exceeds one. At the same time, the model overpredicts the aggregate import share in French manufacturing (the ratio of imports to total domestic sales), which equals 32% in the data and 47% in the model.

In addition to these moments, we decompose the cross-sectoral variation in export sales into the extensive (number of exporters) and intensive (average exports per exporter) margins, and find that in the model the intensive margin explains about one-fourth of the cross-sectoral variation in exports, against two-thirds in the data. This notable discrepancy between the models and the data has been emphasized by Fernandes, Klenow, Meleshchuk, Pierola, and Rodríguez-Clare (2015). Importantly, however, the continuous model attributes nothing to the intensive margin (i.e., its contribution is nil, while the extensive margin drives all of the variation in export sales), and thus the granular model offers an important improvement. In Appendix C.2 we additionally discuss the decomposition of the variation in domestic sales into intensive and extensive margins.

The last two moments in Table 2 characterize the correlation between sectoral export intensity (i.e., the ratio of exports to domestic sales) and the relative size of the largest firm (resp. three largest firms) on the domestic market. These moments discipline the granularity force versus the fundamental comparative advantage: if sectoral export intensity is strongly correlated with the within-sector skewness of the domestic sales distribution, this is suggestive of the importance of the granular force. The models are broadly in line with the empirical correlations between these variables. Overall, we conclude that the model has a reasonable fit and is admissible for further quantitative explorations.

Identification We close this section with a brief discussion of identification in the estimation procedure. With 14 moments reported in Table 2, the model with six parameters is overidentified, and variation in any of the parameters tends to affect all moments simultaneously. Nonetheless, we can identify which moments are more directly linked to specific parameters, and thus what intuitively drives identification.

First, it is easy to see that the fixed cost of entry F largely determines the average number of firms that enter a sector. Second, the average Pareto shape parameter across sectors is pinned down, following the predictions of the theory, by $\kappa = \theta/(\sigma - 1)$. Third, given relative wages $\frac{w}{w^*}$, the aggregate import share Φ is decreasing in both μ_T and τ, as explained in Section 3, and thus this moment provides a joint restriction on these two parameters. Once these two parameters are known, we can calculate the import share in the foreign economy, Φ^*, and use it together with the trade balance condition (28) to discipline Y/Y^*. Indeed, a lower Φ^* relative to Φ requires a higher Y^*/Y in order to balance trade. A similar logic applies to the fraction of French sectors with export sales exceeding domestic sales, which would altogether be impossible if the foreign market were no greater than the home market. The mean and the variance of the export sales distribution (relative to domestic sales) also discipline the values of θ and τ, as θ is the elasticity of sectoral trade flows with respect to trade costs in the continuous limit of the model (as originally shown by Chaney 2008). Lastly, the value of σ_T determines the strength of the fundamental Ricardian comparative advantage: greater σ_T reduces the correlation between sectoral exports and the size of the largest home firm, with this correlation turning negative as σ_T increases further. Since this correlation is positive in the data, it puts a limit on the value of σ_T.

5 Quantifying Granular Trade

We use the estimated model to study the properties of trade flows, focusing in particular on the relative roles of the fundamental and granular components.[27]

5.1 Fundamental versus granular trade

We denote with $\Lambda^*(z) \equiv \frac{X(z)}{\alpha_z Y^*}$ the realized export share of home in sector z, that is the ratio of home exports to the total size of the foreign market. Recall that realized trade in the granular model is a random variable with the mean given by the expected share, $\mathbb{E}\Lambda^*(z) = \Phi^*(z)$. The expected share $\Phi^*(z)$ is an increasing function of fundamental comparative advantage $\frac{T(z)}{T^*(z)}$ (FCA for brief), as we discussed in Section 3. We decompose realized export shares into its fundamental and granular components (in parallel with (24)):

$$\Lambda^*(z) = \Phi^*(z) \ + \Gamma^*(z), \tag{29}$$

where $\Gamma^*(z)$ is the mean-zero *granular residual*, that measures the part of trade patterns not explained by fundamental comparative advantage. Note that *by definition* the granular contribution to aggregate trade is nil, as positive granular residuals in some sectors are offset by negative granular residuals in others.[28] Nonetheless, granularity contributes to the variation of exports across sectors, which is the focus of our analysis in this section. As the further properties of the granular component $\Gamma^*(z)$ are unavailable analytically, we proceed with a quantitative analysis using our estimated model.

First, to get the quantitative feel for the model, we simulate one realization of the economy with 468 sectors using the estimated values of the parameters (see Section 4.2), and plot the results in Figure 2. To facilitate this discussion, we need a small definitional digression:

- *Define* $\hat{A}(z) \equiv \mathcal{F}^{-1}(\Lambda^*(z))$, *where* $\mathcal{F}(\cdot)$ *is the function that maps* $\frac{T(z)}{T^*(z)}$ *into the expected trade share* $\Phi^*(z)$ *as a function of* $\frac{T(z)}{T^*(z)}$ *given the other parameters of the model* Θ. *That is:*

$$\Phi^*(z) = \mathcal{F}\left(\frac{T(z)}{T^*(z)};\Theta\right), \qquad \frac{T(z)}{T^*(z)} = \mathcal{F}^{-1}\big(\Phi^*(z);\Theta\big) \qquad \textit{and} \qquad \hat{A}(z) = \mathcal{F}^{-1}\big(\Lambda^*(z);\Theta\big).$$

In the continuous model, $\Lambda^*(z) = \Phi^*(z)$, and therefore $\hat{A}(z) = T(z)/T^*(z)$, or in words the observed trade flows allow to directly recover the fundamental comparative advantage. This property of a continuous model is commonly used for identification in the literature following Eaton and Kortum (2002). In the granular model, it is no longer the case that one can back out fundamental comparative advantage $T(z)/T^*(z)$ as $\hat{A}(z)$ from the observed trade shares $\Lambda^*(z)$. Furthermore, while realized trade shares

[27] We report the results based on the estimated variable markup model (referred to as Model (b) in Tables 1–2), yet most results are quantitatively similar for the estimated constant markup model (Model (a)).

[28] Formally, the total exports of a country equal $X = Y^* \int_0^1 \alpha_z \Lambda^*(z) \mathrm{d}z = Y^* \int_0^1 \alpha_z \Phi^*(z) \mathrm{d}z$ and $\int_0^1 \alpha_z \Gamma^*(z) \mathrm{d}z = 0$.

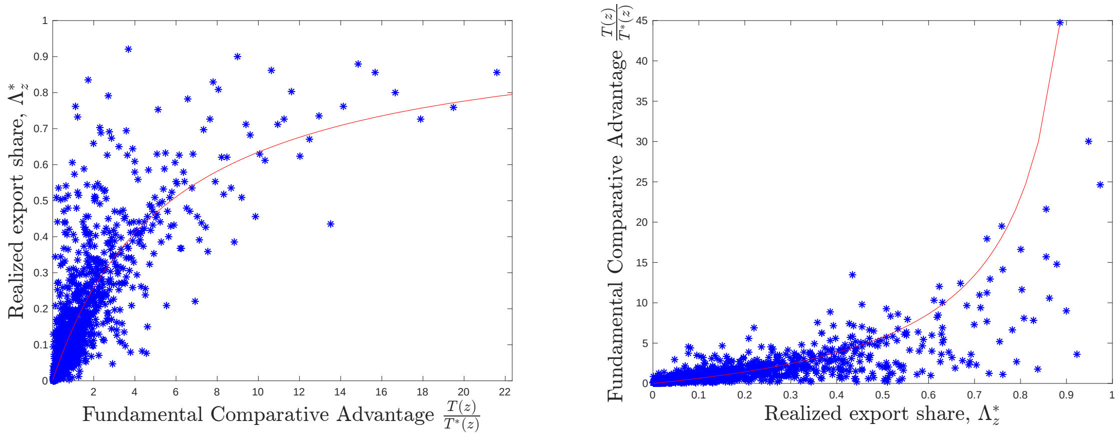


Figure 2: Comparative advantage and trade flows: one realization of the economy

Note: the left panel plots realized sectoral export shares $\Lambda^*(z) = \frac{X(z)}{\alpha_z Y^*}$ against the fundamental comparative advantage $\frac{T(z)}{T^*(z)}$, while the right panel reverses the axes and plots $T(z)/T^*(z)$ against $\Lambda^*(z) = \frac{X(z)}{\alpha_z Y^*}$. Each blue point corresponds to a sector z, and the figure shows one realization (draw) of the estimated economy with 468 sectors. The red line in both plots corresponds to the expected export share of the sector $\Phi^*(z) = \mathbb{E}\Lambda^*(z)$ as a function of $\frac{T(z)}{T^*(z)}$, which also equals the realized trade flows in the continuous limit of the model.

$\Lambda^*(z)$ are centered around $\Phi^*(z)$, the distribution of $T(z)/T^*(z)$ is not even centered around $\hat{A}(z)$, as we see below in Figure 2, where $\hat{A}(z) \equiv \mathcal{F}^{-1}(\Lambda^*(z))$ corresponds to the red curves in the two panels.

The left panel of Figure 2 plots realized sectoral trade shares $\Lambda^*(z)$ against the sector's fundamental comparative advantage $\frac{T(z)}{T^*(z)}$, which each blue point corresponding to an individual sector z. We see that, indeed, $\Lambda^*(z)$ is distributed around $\Phi^*(z)$, depicted in the figure with a red line, with a substantial variation in $\Lambda^*(z)/\Phi^*(z)$ reflecting the strength of the granular force in the model. In the continuous limit of the model, all blue points in the figure align on the red line with no granular effects left, corresponding to Figure A1.

It turns out to be interesting to swap the axis of the plot, as we do in the right panel of Figure 2. What we observe is that the sectors with the largest realizations of trade shares $\Lambda^*(z)$ are not necessarily the sectors with the largest fundamental comparative advantage. In other words, if we are trying to predict $\frac{T(z)}{T^*(z)}$ based on observed trade share $\Lambda^*(z)$, very large realizations of $\Lambda^*(z)$ seem to imply, based on this example, an extreme realization of the granular residual $\Gamma^*(z)$ rather than an extreme relations of $\frac{T(z)}{T^*(z)}$ and hence $\Phi^*(z)$.[29]

We further explore this intriguing finding in Figure 3, which describes the results across 300 simulations of the estimated economy, and in the $\{\Phi^*, \Lambda^*\}$ space rather than in the $\{\frac{T}{T^*}, \Lambda^*\}$ space. In the left panel of Figure 3, we plot the moments (mean and the percentiles) of the $\Lambda^*(z)$ distribution conditional on $\Phi^*(z)$, over 300 realizations of the quantified model. The figure confirms our observations for a single realization of the economy: for a given value of fundamental comparative advantage and associated expected trade share $\Phi^*(z)$, there exists a substantial variation in the realized trade share — accounted for by the granular residual $\Gamma^*(z)$. In other words, there is a wide distribution of

[29]We replicate the exact same analysis with a random draw for sectoral FCA $\frac{T(z)}{T^*(z)}$ rather than choosing them at fixed percentiles of the distribution as we do in estimation. This does not affect any results.

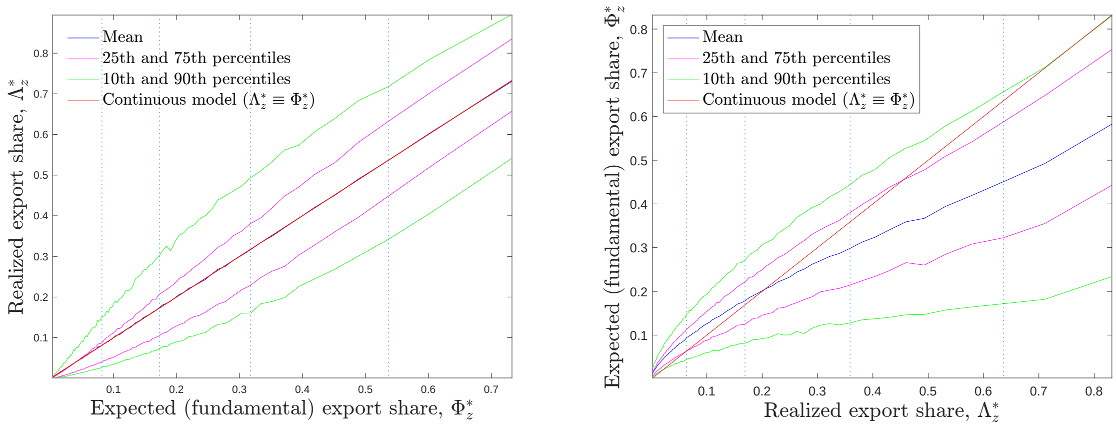


Figure 3: Comparative advantage and trade flows: distribution across realizations

Note: 300 simulations of the estimated economy. LEFT PANEL x-axis: $\Phi^*(z)$ (export intensity predicted by fundamental comparative advantage); y-axis: realized export share, $\Lambda^*(z) = \frac{X(z)}{\alpha_z Y^*}$. Blue line plots average realized export share for a sector across simulations as a function of $\Phi^*(z)$ of the sector; Pink (resp. Green) lines plot 25th and 75th percentiles across simulations (resp. 5th and 95th percentiles) of realized export shares for a given sector as a function of $\Phi^*(z)$ of the sector. RIGHT PANEL reverses the axes in the left panel. Blue line plots the average $\Phi^*(z)$ corresponding to a given observed value of export share $\Lambda^*(z) = \frac{X(z)}{\alpha_z Y^*}$ across simulations; Pink (resp. Green) lines plot 25th and 75th percentiles (resp. 5th and 95th percentiles) across simulations of $\Phi^*(z)$ corresponding to a given realized value of export shares. Red lines in both panels plot the 45 degree line. In the continuous limit, all realizations align on the red line. In both panels, the vertical black dotted lines corresponds to the 50th, 75th, 90th and 99th percentiles respectively of the distribution of the variable plotted on the x-axis.

possible export-intensity outcomes, for the same level of fundamental comparative advantage. Next, the right panel shows a more striking pattern, by plotting the distribution of the underlying fundamental comparative advantage $\frac{T(z)}{T^*(z)}$ — or rather of the export intensity predicted by this level of FCA, $\Phi^*(z) = \mathcal{F}\left(\frac{T(z)}{T^*(z)}; \Theta\right)$ — against the realized traded share $\Lambda^*(z)$.

The intriguing pattern in the right panel of Figure 3 is that the distribution of $\Phi^*(z)$ conditional on realized trade share $\Lambda^*(z)$ is not symmetric around $\Lambda^*(z)$, and the mean of this distribution (the blue line) diverges from $\Lambda^*(z)$ (the red line), as $\Lambda^*(z)$ increases. Equivalently, the distribution of fundamental comparative advantage $\frac{T(z)}{T^*(z)}$ conditional on realized trade share $\Lambda^*(z)$ is not symmetric around $\hat{A}(z)$, and the mean of this distribution diverges from $\hat{A}(z)$ as $\Lambda^*(z)$ increases.

That is, for large realized values of export intensity in a given sector, the actual fundamental comparative advantage level of the sector is systematically lower than what the continuous model would predict. Furthermore, this discrepancy increases with the export intensity of the sector. More specifically, the level of comparative advantage predicted by the continuous model $\hat{A}(z)$ exceeds the 75th percentile (the upper pink line) of the actual distribution of fundamental comparative advantage for very large realizations of $\Lambda^*(z)$, so that the bias from using a continuous model to infer fundamental comparative advantage can be substantial, especially for the most export-intensive sectors. This confirms the intuitive observation derived from the right panel of Figure 3, in which we plotted one realization of the granular economy. The granular residual $\Gamma^*(z)$ becomes particularly important to understand the export behavior of sectors with large realized export intensity $\Lambda^*(z)$.

Trade flows decomposition We next investigate quantitatively the extent to which trade flows are shaped by fundamental comparative advantage vs. granular comparative advantage. To that end, we propose a simple decomposition of the variation in export share $\Lambda^*(z)$ as well as in log exports $\log X(z)$ across sectors into the contributions of various components as follows:

$$\operatorname{var}\big(\Lambda^*(z)\big) = \operatorname{var}\big(\Phi^*(z)\big) + \operatorname{var}\big(\Gamma^*(z)\big) + 2\operatorname{cov}\big(\Phi^*(z), \Gamma^*(z)\big), \tag{30}$$

$$\operatorname{var}\big(\log X(z)\big) = \operatorname{var}\big(\log \alpha_z\big) + \operatorname{var}\big(\lambda^*(z)\big) + 2\operatorname{cov}\big(\alpha_z, \lambda^*(z)\big), \tag{31}$$

where $\lambda^*(z)$ is the log export share: $\lambda^*(z) \equiv \log \Lambda^*(z)$ and log exports is $\log X(z) = \log \alpha_z + \lambda^*(z) + \log Y^*$. The first decomposition captures the relative contributions of the fundamental and granular forces to the variation in trade shares across sectors. If we, however, are interested in the gross trade flows, the second decomposition additionally splits the variation in total exports across sectors into the size of the sector (captured by the Cobb-Douglas share) and the export share in the sector.[30]

Table 3: Decomposition of variation in trade flows

	Decomposition of $\operatorname{var}(\Lambda^*(z)) = 0.03$		
	$\Phi^*(z)$	$\Gamma^*(z)$	Covar
Variance-based	0.697	0.304	0.001
	Decomposition of $\operatorname{var}(\log X(z)) = 1.69$		
	$\log \alpha_z$	$\lambda^*(z)$	Covar
Variance-based	0.405	0.577	0.009

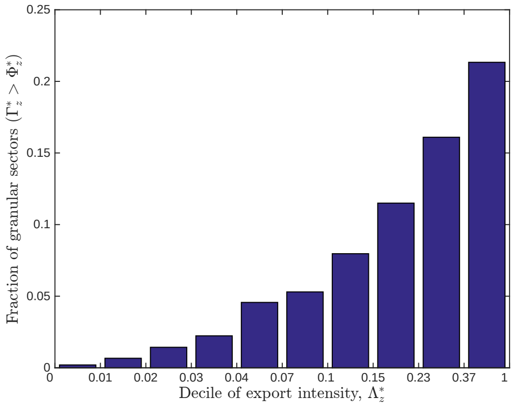
Table 3 reports the results from these decompositions. We observe that almost 60% of variation in gross export flows across sectors is due to the variation in trade shares $\lambda^*(z)$ and 40% is due to the size of the sectors. In turn, the variation in trade shares is 70% due to the fundamental comparative advantage and 30% due to granular residual.[31] We conclude that granularity plays an important role in shaping sectoral trade flows in our estimated model. It accounts for close to 20% of gross trade flows (30% of 60%) and 30% of the variation in export shares across sectors, which is a natural scale-free measure of comparative advantage.

[30] As an alternative to variance decompositions, we also considered regression-based decompositions, which conveniently feature no covariance terms. Given that the covariance terms are negligible in Table 3, regression-based decompositions give virtually the same results. For concreteness, consider the regression counterpart to variance decomposition (30):

$$\operatorname{var}\big(\Lambda^*(z)\big) = \operatorname{cov}\big(\Phi^*(z), \Lambda^*(z)\big) + \operatorname{cov}\big(\Lambda^*(z) - \Phi^*(z), \Lambda^*(z)\big),$$

and it can be conveniently obtained by regressing in turn $\phi^*(z)$ and $\lambda^*(z) - \phi^*(z)$ on a constant and $\lambda^*(z)$. The granular contribution in this case is measured as $\operatorname{cov}\big(\lambda^*(z) - \phi^*(z), \lambda^*(z)\big)/\operatorname{var}\big(\lambda^*(z)\big)$, as opposed to $\operatorname{var}\big(\lambda^*(z) - \phi^*(z)\big)/\operatorname{var}\big(\lambda^*(z)\big)$ in the variance decomposition (30). When the covariance term in (30) is close to zero, the two decompositions give similar results, as is the case in Table 3.

[31] The role of the granular component is somewhat larger, around 40%, in the estimated constant markup version of the model.



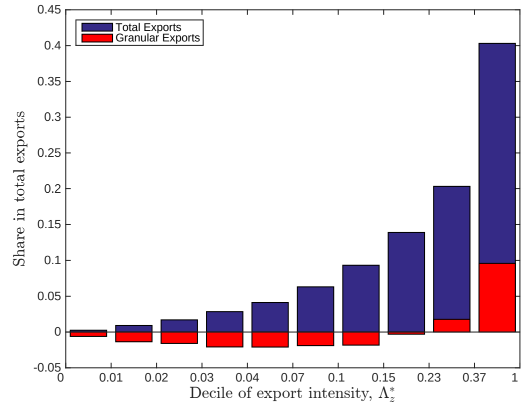


Figure 4: Granularity and export intensity

High export intensity and granularity If granularity explains 30% of variation in export shares on average across sectors, it is also important to note that the role of granularity is even more pronounced in high export-intensity sectors. We illustrate this point by reporting in Figure 4 how much granularity impacts trade flows differentially for high and low export intensity sectors. Specifically, we group sectors by decile of export intensity $\Lambda^*(z)$, and report the fraction of sectors in each decile for which export is largely driven by granularity. First, we call a sector *granular* if its exports are mostly accounted for by the granular component, as opposed to fundamental component. Formally, a sector is granular when $\Gamma^*(z) > \Phi^*(z)$, or equivalently $\Lambda^*(z) > 2\Phi^*(z)$. The left panel of Figure 4 exhibit a clear monotonic increasing pattern: granularity is particularly important to explain trade in high export-intensity sectors, with virtually no granular sectors in the bottom deciles and almost a quarter of granular sectors in the top decile.

The right panel of Figure 4 plots the contribution of sectors to total country exports by deciles of export intensity, as well as the separate contribution of the granular component.[32] As expected, the top export intensity decile accounts for over 40% of total exports. More surprisingly, the granular trade component is negative in all but the two largest deciles. Recall that the granular residual is defined to be mean zero, and therefore the sum of the red bars across deciles must sum to zero, however it is not zero within the deciles. The figure illustrates how the granularity of the large export sectors is offset by the negative granular components in the bottom eight deciles of sectors (i.e., these sectors export less than they would in a continuous model). Furthermore, within the top export decile, granular exports account for a quarter of total exports, which equals 10% of the total exports of the country.

To summarize, we have defined the granular contribution to be zero at the aggregate. Positive granular components in some sectors are offset by equally large negative granular components in other sectors. Granularity in the model explains about 30% of variation in trade across sectors (more trade in some sectors and less trade in others), though it does not create more trade on net. Granularity is

[32]Specifically, the figure plots $\left(\frac{Y^*}{X}\right)\int_{z\in D_k}\alpha_z\Lambda^*(z)\mathrm{d}z$ and $\left(\frac{Y^*}{X}\right)\int_{z\in D_k}\alpha_z\Gamma^*(z)\mathrm{d}z$, where $X = Y^*\int\alpha_z\Lambda^*(z)\mathrm{d}z$ is total exports and D_k is the kth decile of sectors sorted by export intensity $\Lambda^*(z)$.

important to understand the particularly large export intensity of certain sectors, i.e. the *export champions* of the country. Among the largest decile of exporting sectors, granular sectors play a particularly large role.

Properties of granular trade flows We now study the covariation properties of the granular component of trade flows with other variables in the estimated model. This exercise serves two purposes. First, it helps us understand the properties of the granular component of trade and in particular the determinants of granular nature of sectoral trade. Second, we want to find out empirical proxies for the granular component of trade flows using covariates that are in principle readily observable in the data, whereas fundamental comparative advantage and its counterpart $\Phi^*(z)$, used in the figures and decompositions above, are not. These proxies can guide a further empirical investigation of granular comparative advantage.

To this end, Table 4 reports regressions with the granular residual, $\Gamma^*(z) = \Lambda^*(z) - \Phi^*(z)$, as the dependent variable. The first two columns show that, in the estimated model, $\Gamma^*(z)$ is correlated with *neither* fundamental comparative advantage measured as $\Phi^*(z)$, *nor* with the size of the sector measured as α_z: both the coefficients and the R^2 in these regressions are virtually zero. Therefore, in the model, that granularity of trade is not systematically related with the size of the sector or the sector's fundamental comparative advantage.

Table 4: Properties of the granular residual

	(1)	(2)	(3)	(4)	(5)	(6)
Φ^*_z	−0.003					
$\log \alpha_z$		0.000				
$\log N_z$			−0.009		0.008	
$s_{z,(1)}$				0.281	0.302	0.296
$s^*_{z,(1)}$						−0.191
R^2	0.000	0.000	0.017	0.333	0.344	0.449

Note: The table reports coefficients and R^2 from the regressions of granular residual $\Gamma^*(z)$ on various proxies of granularity, where N_z stands for the number of home entrants in the home market, $s_{z,(1)}$ is the top market share of a home firm relative to other home firms in the home market, and $s^*_{z,(1)}$ is the top foreign market share among foreign firms in the foreign market.

The remaining columns show that the granular component of trade can be captured well by the size of the largest domestic firm as a our preferred measure of realized granularity in the sector's productivity draws. The size of the largest firm is one simple measure of the skewness in the realized productivity draws, and is a likely sign of a granular sector.[33] Column 4 of Table 4 shows that the size of the largest home firm (relative to other home firms) alone explains a third of the variation in $\Gamma^*(z)$ across sectors. In column 3, we also show that the number of firms in a sector (domestic entrants in the

[33]We have also experimented with two other proxies of granularity — concentration ratio of the three largest domestic firms (relative to other domestic firms) and the ratio of the top to median market shares of the domestic firms. Both of these variables have similar, albeit slightly lower, explanatory power for $\Gamma^*(z)$, but they do not improve the fit when combined together. Therefore, we only display results for our preferred proxy — the top home market share.

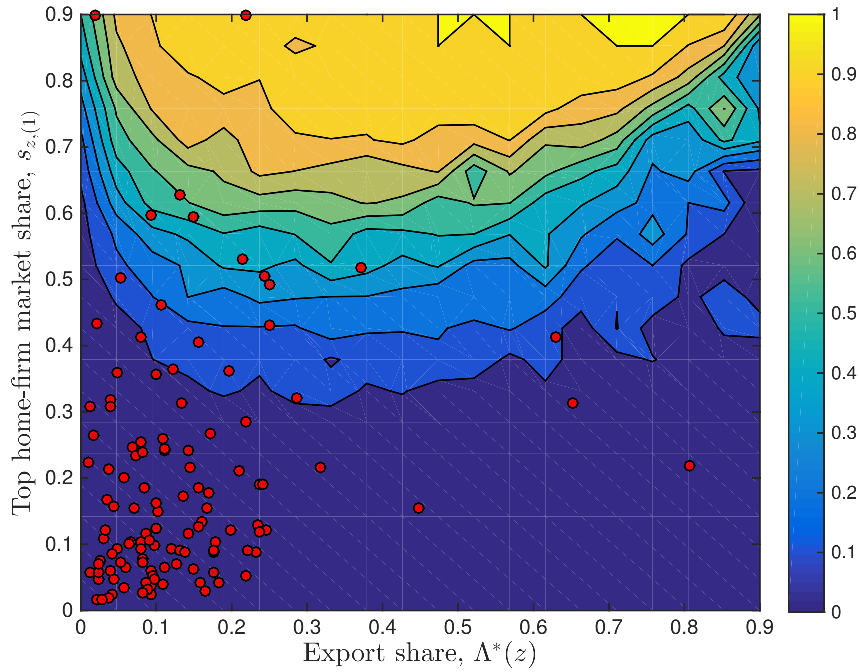


Figure 5: Probability $\Gamma^*(z) > \Phi^*(z)$ conditional on $(\Lambda^*(z), s_{z,(1)})$

domestic market) does not have much of an explanatory power for the granular component of trade. This underlines that granularity is not merely a reflection of small number of firms: in large sectors in terms of number of firms, the granular component can still be large, given the very fat-tailed distribution of productivity draws. Furthermore, adding the number of firms to the size of the largest firm does not improve the explanatory power of the regression (column 5).

As foreign granularity can be an equally important force in shaping trade flows, column 6 includes both the size of the largest home and foreign firms, as proxies for home and foreign granularity. These two variables can explain almost a half (45%) of the variation in the granular component $\Gamma^*(z)$ across sectors, up from a third when only the home granularity is proxied for in column 4. We conclude that a single easily observable variable — the relative size of the largest firm — acts as a powerful proxy for granularity, explaining nearly a half of the granular trade. The remaining portion of variation in granular trade is due to the other, less easily measurable, features of the sectoral productivity draws.

5.2 Identifying granular sectors

We now address the subject of identifying the specific granular sectors in the data. This is a challenging task, as neither $\Phi^*(z)$ nor $\Gamma^*(z)$ are observed, and the inference must be made using only the overall trade share $\Lambda^*(z)$ and the observable proxies for granularity, such as the relative size of the largest firm. We again define a sector to be granular if $\Gamma^*(z) > \Phi^*(z)$, that is more than a half of its exports have a granular origin. The estimated model offers a full description of the data generating process, and therefore we can answer the following question: in a given sector z, what is the likelihood of $\Gamma^*(z) > \Phi^*(z)$ (or in other words, $\Phi^*(z) < \Lambda^*(z)/2$) conditional on observing the realized trade share $\Lambda^*(z)$

and $r(z)$, a vector of other observed variables. In particular, we simulate the model to evaluate a joint distribution function for $(\Phi^*(z), \Lambda^*(z), r(z))$, and use it to evaluate the probability mass associated with $\Phi^*(z) < \Lambda^*(z)/2$, integrating across all possible values of the unobserved variable $\Phi^*(z)$.[34]

In Figure 5, we plot the resulting probability of granularity as a function of $(\Lambda^*(z), s_{z,(1)})$, where $s_{z,(1)}$ is the relative size of the largest home firm (as in Table 4 above). The blue colors indicate the region of low probability of granularity, while the green and yellow regions indicate the regions of high likelihood of granularity (high probability of $\Gamma^*(z) > \Phi^*(z)$). Since $\Gamma^*(z) > \Phi^*(z)$ is a rather unlikely event in the estimated model, concluding that the probability of this event is greater than a half requires a very large realization of the top home market share $s_{z,(1)}$ of above 50%. Conversely, if the top home market share is below 30%, the likelihood of a sector being granular (in the $\Gamma^*(z) > \Phi^*(z)$ sense) is below 10%.[35] Figure 5 additionally superimposes our French data as red circles. This way, for each of our 117 French sectors, we identify the probability of it being granular, in the $\Gamma^*(z) > \Phi^*(z)$ sense. Indeed, there are a few sectors in the French data that we can call as likely granular, i.e. in which the realized export shares exceed considerably those that would be predicted based on the sectors' fundamental productivity.

6 Dynamics of Comparative Advantage

In this section, we develop a dynamic extension for our granular model of trade in order to study the dynamics of comparative advantage. In a recent paper, Hanson, Lind, and Muendler (2015; HLM henceforth) emphasize two striking facts:[36]

(i) the hyper-specialization of exports, where a single top sector accounts on average across countries for 21% of a country's total exports and three top sectors account on average for 45% of total exports; and

(ii) the high turnover of comparative advantage, where a sector in the top-5% of exporting sectors has only a 41% chance of staying in the top-5% of exporting sectors two decades later.

A granular model is well suited to address these facts. Firm dynamics, which is straightforward to discipline empirically using data on the evolution of firm market shares, alters the granular comparative advantage over time, and therefore can explain, at least partially, the high turnover of exporting sectors, replicating simultaneously the hyper-specialization of countries at any given point in time. This is the issue we now address quantitatively.

[34] Formally, this probability is calculated as follows:

$$\mathbb{P}\{\Gamma^*(z) > \Phi^*(s)|\Lambda^*(z), r(z)\} = \frac{\int_{\Phi^* < \frac{1}{2}\Lambda^*(z)} h\big(\Phi^*, \Lambda^*(z), r(z)\big) \mathrm{d}\Phi^*}{\int_0^1 h\big(\Phi^*, \Lambda^*(z), r(z)\big) \mathrm{d}\Phi^*},$$

where $h(\Phi^*, \Lambda^*, r)$ is the joint density of $(\Phi^*(z), \Lambda^*(z), r(z))$ according to the estimated model, which can be factored into a density of $(\Lambda^*(z), r(z))$ conditional on $\Phi^*(z)$ and the partial density of $\Phi^*(z)$, resulting in the standard Bayesian formula.

[35] One can adopt alternative, less tight, cutoffs for granularity, for example $\Lambda^*(z) > 1.5 \cdot \Phi^*(z)$, in which case the probability schedule will shift upwards across the entire space $(\Lambda^*(z), s_{z,(1)})$ in Figure 5.

[36] Hanson, Lind, and Muendler (2015) use data which splits all products into 135 sectors.

Table 5: Firm dynamics and comparative advantage

Moment	Data	Model $\nu = 0.0092$	Model $\nu = 0.0120$
SR persistence $\text{std}(\Delta s_{z,i,t+1})$	0.0036	0.0025	0.0036
LR persistence $\text{corr}(s_{z,i,t+10}, s_{z,i,t})$	0.78	0.78	0.65
Top-1% of sectors trade share	21%	20%	20%
Top-3% of sectors trade share	45%	34%	34%
Turnover: stay in top-5% sectors after 20 years	41%-55%	68%	57%

Note: Empirical moments in the top panel from French data and in the bottom panel from HLM. Top-1% (3%) of sectors corresponds to 1 (3) sectors in the HLM data and to 3.5 (10) sectors in the simulation (with 468 sectors relative to 135 sectors in the data).

We introduce dynamics by assuming that the productivity of each firm evolves over time according to a random growth process. Consistent with our earlier assumptions, there is a given pool of firms in each sector drawn in the beginning of times from a Poisson distribution. These firms never leave and new firms never enter, yet the firms decide each period whether to be active or inactive. The decision to become active involves a per-period fixed cost, but no sunk costs, making this choice static. The productivity of every firm, whether active or inactive, evolves over time. In any given time period, firms play a static entry game, as described in Section 3, given the realized productivity distribution in that period. We choose the productivity process to replicate the same cross-sectional Pareto distribution of firm productivities as in the static model. Therefore, our dynamic economy is a sequence of static economies, with the only inter-temporal link through the persistent firm-level productivity process. We discipline this process using our French data on the evolution of firm market shares over time.

The specific firm productivity process we adopt is a geometric random walk with drift and a reflecting barrier at the lower bound for productivity draws. Formally, productivity of firm i in sector z evolves according to:

$$\varphi_{z,i,t} = \underline{\varphi}_z + |\mu + \varphi_{z,i,t-1} + \nu\varepsilon_{z,i,t} - \underline{\varphi}_z|, \tag{32}$$

where $\underline{\varphi}_z$ is the reflecting barrier, $\mu = -\theta\nu^2/2 < 0$ is the negative drift term, ν is the standard deviation of innovation, and $\varepsilon_{z,i,t}$ is iid standard normal. If the initial productivity draws $\varphi_{z,i,0}$ came from a Pareto distribution with the shape parameter θ and the lower bound $\underline{\varphi}_z$, the cross-sectional distribution of productivities $\varphi_{z,i,t}$ stays stable over time (see Gabaix 2009).

We normalize $\underline{\varphi}_z$ to a small number which we ensure never binds in the entry game,[37] and otherwise this parameter is inconsequential in the model. Given the restriction on the value of μ, this leaves us with only one parameter to estimate—the size of the productivity innovation ν. Importantly, this parameter does not affect any of the static moments used in the simulation, and therefore can be disciplined in a separable way from the rest of the model. Specifically, we choose to match two moments of the firm market share evolution—the short-run persistence of market share measured (inversely) by $\text{var}(\Delta s_{z,i,t+1})$ and the long-run persistence measured by $\text{corr}(s_{z,i,t+10}, s_{z,i,t})$, where one period correspond to a year. We choose the value $\nu = 0.0885$ to match these two moments, as we show in the

[37]Concretely, we use the minimum realized draw of the pool of shadow firms drawn initially.

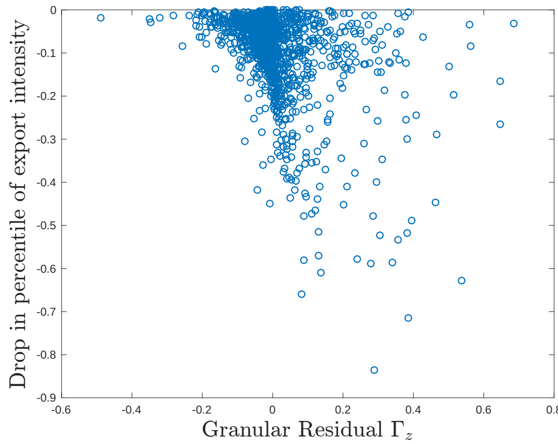


Figure 6: Trade effects of individual firm exit

top panel of Table 5. The model is broadly consistent with both the short-run and long-run persistence of market shares, and we choose to err on the conservative side and adopt the parameter value which results in somewhat greater persistence of market shares than in the data.

With this, we simulate the dynamic equilibrium path of the economy over a long period of time, and use the simulated firm data to calculate the HLM moments, as reported in the bottom panel of Table 5. The model matches the hyper-specialization of the top-1% of sectors and somewhat understates the hyper-specialization of the top-3% of sectors (34% versus 45%). The model also accounts for a large fraction of the turnover in comparative advantage sectors: over 20 years, the probability for a sector to remain among top-5% exporting sectors is 68%, against 41% in the data.[38] If we were to recalibrate the model to match the short-run (rather than long-run) persistence of market shares by setting $\nu = 0.012$ (last column of Table 5), we find that the probability of staying in top-5% of exporting sectors falls from 68% to 57%, very close to the empirical persistence of comparative advantage for developed countries. Therefore, a simple dynamic granular model accounts jointly for the hyper-specialization of industries and a substantial turnover in comparative advantage across industries over time. The turnover implied by the model is arguably a lower bound, since the model features no evolution over time in the fundamental comparative advantage $\frac{T(z)}{T^*(z)}$. Nonetheless, we conclude that a granular model offers an important quantitatively-disciplined source of dynamics for comparative advantage of countries.

Trade effects of individual firm exit Finally, we consider a counterfactual in which the largest firm in a randomly chosen sector fails for an exogenous reason and has to exit the industry. We study the impact of this shock for the aggregate export performance of the sector, which is absent in a model with a continuum of firms. In particular, in Figure 6, we plot the drop of the sector in the percentile of the export intensity distribution as a result of the exit of the largest firm, as a function of the sector's

[38]This moment in the HLM data is sensitive to the sample of countries, with developing countries having greater turnover. The corresponding probability for the developed countries is, in fact, 55%, closer to the number implied by our model calibrated to the French firm-level data.

granular residual $\Gamma^*(z) = \Lambda^*(z) - \Phi^*(z)$. We observe that in the non-granular sectors, the effects are small, with the relative standing of the sector in terms of export intensity barely changing. At the same time, a death of the sector leader in more granular sectors has much more dramatic consequences. On average, a death of a firm in a non-granular sector (in the bottom 7 deciles of the $\Gamma^*(z)$ distribution) results in a drop of the sector by 5 percentiles in the export intensity distribution of $\Lambda^*(z)$. In contrast, for the top decile of granular sectors, a death of a firm shifts the sector down on average by 18 percentiles in the import intensity distribution, however much greater drops are also typical. In the most extreme cases, a drop of over 80 percentiles — that is, from the top-10% of exporting sectors to the bottom-10% — merely due to the death of a single firm is possible.

Granularity, trade openness, and labor reallocation Idiosyncratic firm productivity shocks in a continuous model result in labor reallocation across firms, but not across sectors. Sectoral labor allocation in a continuous model depends on the sector's Cobb-Douglas share and is additionally increasing in the sector's net exports. Idiosyncratic firm productivity shocks wash out at the sectoral level, and hence affect neither net exports, nor labor shares across sectors.[39] Matters are different in a granular model, where idiosyncratic firm productivity shocks affect aggregate sectoral comparative advantage, as we discussed above. As a result, these shocks lead to labor reallocation not only within, but also across sectors — from sectors losing comparative advantage towards sectors gaining it. We quantify the amount of intersectoral labor reallocation in response to idiosyncratic firm productivity dynamics in the estimated dynamic model. We find that 20% of the total labor reallocation in response to idiosyncratic firm productivity shocks is across sectors. This between-sector labor mobility is entirely absent in the continuous counterpart of the model, emphasizing that granularity can be an important source of aggregate resource reallocation, especially in economies open to international trade.

7 Implications of GCA for the Impact of Trade Liberalization

We have established in section 5 that accounting for the granular nature of exports is important to understand the determinants of countries' comparative advantage. We turn next to studying whether accounting for this granular source of comparative advantage has implications for trade policy. Specifically, we ask first whether granular sectors respond differently to a trade liberalization episode. Second, we study the implications of granularity for evaluating the welfare gains from trade following such a trade liberalization episode.

7.1 Trade Elasticities

To evaluate how trade flows respond to a trade liberalization episode in the granular model, we take the estimated model of Section 5 and shock the economy with a reduction in trade costs of 10%. We

[39] Aggregate sectoral productivity shocks can change a sector's comparative advantage, and therefore its net exports and labor share. In the closed economy, even aggregate shocks do not affect sectoral labor allocations, given the Cobb-Douglas assumption, as sectoral labor income is proportional to sectoral expenditure share. While this exact relationship breaks down in the granular model, it gives a quantitatively-accurate benchmark: The sectoral labor shares are stable in the granular model in autarky, emphasizing the role of both granularity and trade openness for the results we now discuss.

then compute a measure of sectoral trade elasticity $\tilde{\theta}_z$ defined as:[40]

$$\tilde{\theta}_z = \frac{1}{2\Delta \log \tau} \Delta \log \left(\frac{\Lambda_z}{1-\Lambda_z} \frac{\Lambda_z^*}{1-\Lambda_z^*} \right) \tag{33}$$

This measure of trade elasticity plays a central role in the recent research in international trade. Arkolakis, Costinot, and Rodríguez-Clare (Arkolakis, Costinot, and Rodríguez-Clare; henceforth, ACR) show that for the class of models commonly used in the international trade literature this measure of trade elasticity form a sufficient statistics, along with the home shares $(1 - \Lambda_z)$, for evaluating the welfare gains from trade. In the continuous limit of our granular model, $\tilde{\theta}_z = \theta$ for all sectors z.

We evaluate this expression in the granular model, across 300 different simulations of the model. Figure 7 reports the results. Panel (a) plots the kernel density of the distribution of these sectoral trade elasticities. It also plots for reference the case of the continuous model, where these elasticities are equal to the parameter θ, constant across sectors, as well as the value of the aggregate trade elasticity, measured using aggregate trade flows:

$$\tilde{\theta}_{agg} = \frac{1}{2} \frac{1}{\Delta \log \tau} \Delta \log \left(\frac{\Lambda}{1-\Lambda} \frac{\Lambda^*}{1-\Lambda^*} \right), \tag{34}$$

where $\Lambda = X^*/Y$ and $\Lambda = X/Y^*$ are the home and foreign aggregate import shares.

Two facts emerge. First, trade elasticities are on average higher (at 4.24) than what the continuous limit of the model would predict (at 3.75). Second, they display a fair amount of variance across sectors, varying between 0.37 and 6.75, despite the facts that all sectors are governed by the same parameter θ. We investigate the determinants of this heterogeneity in realized trade elasticities. To that end, panel (b) plots the distribution of the sectoral trade elasticities conditional on the realized market share of the largest home firm in the sector (our preferred proxy for granularity from Section 5). Further, Table 6 reports the explanatory power of this trade elasticity using a few additional proxies of granularity, that is, the market share of the top foreign firm in the foreign market and the number of domestic and foreign firms. Granularity both at home and in foreign systematically lowers how much trade flows react to a reduction in trade costs. Beyond that, trade elasticity is higher the smaller the sector in terms of number of firms.

Table 6: Determinants of the Sectoral Trade Elasticity

	(1)	(2)	(3)
Top Firm MS on Domestic Market	-1.220	-1.163	-1.587
Top Firm MS on Foreign Market		-0.743	-1.007
log Number Dom. Firms			-0.172
R-squared	0.241	0.308	0.491

We conclude from this exercise that granularity (i) makes sectoral trade more sensitive, on average,

[40]Formula (33) is constructed by analogy with the continuous case, in which the same expression (given that in this case $\Lambda_z = \Phi_z$ and $\Lambda_z^* = \Phi_z^*$) allows to recover the structural parameter θ. The intuition behind the product of ratios in (33) is that such transformation cancels out the general equilibrium variables and thus allows to recover the (partial) elasticity of trade with respect to trade costs.

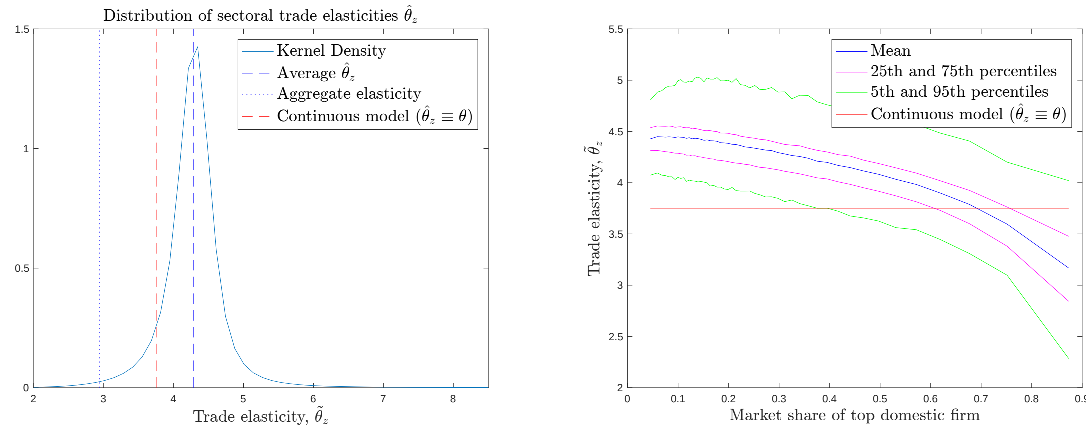


Figure 7: Distribution of Sectoral Trade Elasticity

to a reduction compared to the continuous limit of the modeland (ii) matters to explain cross-sectional variation in sectoral response to trade liberalization, in a way that can be well predicted using as a proxy for granularity the top market share of home and foreign firms, as well as the number of home firms in each sector.

7.2 Welfare Gains from Trade

We next investigate the extent to which the granular model has different implications for the welfare gains from trade, compared to its DFS-Melitz continuous limit. To that end, we follow the logic of ACR. They show that under a wide class of trade models, the measured trade elasticity $\tilde{\theta}_z$ together with the home shares $1 - \Lambda_z$ are sufficient statistics for the welfare gains from trade. That is, once these quantities are measured from the data, the welfare gains from trade can be computed using only these quantities, irrespective of the specific micro-structure of the model. Our granular model does not fall into this class of models, though its continuous limit does.[41] We therefore investigate quantitatively whether trade elasticities and home shares are still sufficient statistics for welfare gains from trade once we account for granularity in the world's exports, or whether the granular micro-structure plays a role in welfare gains beyond these statistics. We first briefly investigate this question at the aggregate level for completeness of the analysis, then focus on cross-sectoral heterogeneity.

Aggregate welfare gains Our first exercise is concerned with aggregate welfare gains. In our model, because of the assumption of a continuum of sectors, the LLN holds across sectors so that there is no granularity left in the aggregate upon integration across sectors. Granularity kicks in at the sectoral level, and our main focus is to investigate whether different sources of comparative advantage, fundamental or granular, have different implications for sector-level outcomes. Nevertheless, before moving

[41]Relaxing the continuum assumption of the DFS-Melitz model leads to two departures from the ACR framework. First, granularity gives rise to oligopolistic competition and variable markups, as analyzed in Edmond, Midrigan, and Xu (2015). Second, under finite number of draws, the realized productivity distribution is no longer Pareto, and therefore the macro restrictions of the ACR analysis do not hold in the granular model.

Table 7: Welfare gains from trade (%)

	Actual welfare gain	Multi-sector ACR	Single-sector ACR
Small Liberalization	5.97	5.85	5.76
Full Liberalization	20.66	19.76	21.11

to studying cross-sectoral heterogeneity in welfare gains from trade, we first briefly study aggregate questions. It is indeed not a priori clear whether the formulas commonly used for aggregate welfare gains following ACR hold here in the aggregate, since this granular model does not fall within the class of models considered by ACR. Our first step is therefore to check whether it is the case.

To that end, we first compute, within the granular model, the instantaneous welfare gains from a small trade liberalization as well as the welfare losses of going back to autarky, holding constant individual firm productivity draws before and after the trade liberalization. To implement the ACR formula, we evaluate the multi-sectoral ACR formula for welfare gains (which applies in the continuous limit):

$$\hat{W}_{ACR} = -\int \alpha_z \frac{\widehat{(1-\Lambda_z^*)}}{\tilde{\theta}_z} \mathrm{d}z, \tag{35}$$

where the hat notation denotes log changes.[42] We use the same estimator of $\tilde{\theta}_z$ described above in (33), which varies across sectors, for both liberalization episodes.[43] For completeness, we also report the outcome of the one-sector ACR formula, i.e. the one we would apply if the data were interpreted as coming from a continuous one-sector model:

$$\hat{W}_{ACR,agg} = -\frac{\widehat{1-\Lambda^*}}{\tilde{\theta}_{agg}}.$$

The results are reported in Table 7. We find that the ACR formula(s) perform remarkably well in the context of the static granular model, whether we use its multi-sector version or its one sector version. Both predict the actual gains from trade in the granular model with an error of less than 5%. These results echo the ones in Edmond, Midrigan, and Xu (2015) who study welfare gains from trade on Taiwanese data through the lens of a quantified model of Cournot competition, and find that the one-sector ACR formula provide an accurate approximation for aggregate welfare gains from trade.

Sectoral contribution to welfare gains We next move to the heart of our question, which is whether the contribution of each sector to a change in welfare following a trade liberalization is close to the one predicted by the continuous model, or whether a granular sector has different implications for welfare gains compared to a sector whose comparative advantage is driven by fundamental CA. That is, are the granular and the continuous models close sector-by-sector as well, or does the aggregate result mask substantial discrepancies at the sectoral level? To analyze this question, we first evaluate the

[42]In particular, in the case of a small reduction in trade costs, $\widehat{(1-\Lambda_z^*)} = \mathrm{d}\log(1-\Lambda_z^*)$ is the log change in home shares, measured in the simulated data before and after the liberalization. In the case of going back to autarky, $\widehat{(1-\Lambda_z^*)} = -\log(1-\Lambda_z^*)$.

[43]The sectoral trade elasticities are computed for a 10% reduction in trade costs, starting from the calibrated trade costs.

distribution of the sectoral contribution to welfare gains, defined as:

$$\hat{W}_z = \frac{\Pi_z}{\alpha_z Y}\Delta \log \Pi_z - \Delta \log P_z.$$

This expression is a way to evaluate the marginal contribution of a sector to the welfare gains from trade. In particular, we have that $\hat{W} = \int_0^1 \alpha_z \hat{W}_z \mathrm{d}z$, as shown in Appendix A.4.

Figure 8 reports the distribution across 300 simulations of the model of the sectoral contribution to welfare gains from trade of each sector, as a function of its fundamental comparative advantage. Panel (a) reports the results for a small reduction in trade costs, and Panel (b) plots the results for going back to autarky. The welfare contribution of individual sectors in the granular model can vary substantially from what their fundamental comparative advantage predicts, another sign that granularity shapes the comparative advantage of sectors. We next investigate the extent to which these potential welfare gains can be predicted by measures that are observable in the data before the liberalization episode. A natural candidate for this is the home share, $1 - \Lambda_z$. As shown in Appendix A.4, sectoral contribution to welfare can be expressed as a function of home shares and the model's parameter in the continuous model.

Table 8: Predicting outstanding welfare contribution

	(1)	(2)	(3)
Top Firm MS on Domestic Market	0.016	0.017	0.016
Top Firm MS on Foreign Market		-0.012	-0.013
log Number Dom. Firms			-0.001
R-squared	0.196	0.289	0.289

Figure 9 reports the distribution across 300 simulations of the model of the sectoral contribution to welfare gains from trade of each sector, as a function of its observed home share. Panel (a) reports the results for a small reduction in trade costs, and Panel (b) plots the results for going back to autarky. We see that home shares are a much tighter predictor of sectoral welfare gains, though sectors with high home shares have quite variant outcomes in terms of welfare gains. We see for example that for these high-home-share sectors, the distribution of welfare gains are very skewed: 5% of them have systematically a welfare contribution that is about twice what the mean sectoral contribution to welfare. Sectors that tend to bring in large welfare gains correspond to outliers compared to the continuous case. We report in Table 8 how these outstanding welfare contributions correlate with — and hence can be predicted by — the proxies for granularity we have already used above. To that end, we measure the difference between their sectoral welfare contribution $\hat{W}_z$ and the mean welfare contribution predicted by their fundamental comparative advantage (the red line in Figure 8). We then project this residual on proxies for granularity. Similarly to what we have seen for the trade elasticity patterns, the top market share of the domestic firm and the top market share of the foreign firm on their respective home markets are good predictors of these outstanding welfare contributions, though here the number of firms in the sectors brings in no additional predictive power. More precisely, an exceptionally large domestic firm in a given sector tends to bring in additional gains from trade compared to what the corresponding trade share would predict. Conversely, an exceptionally large foreign firm in a given

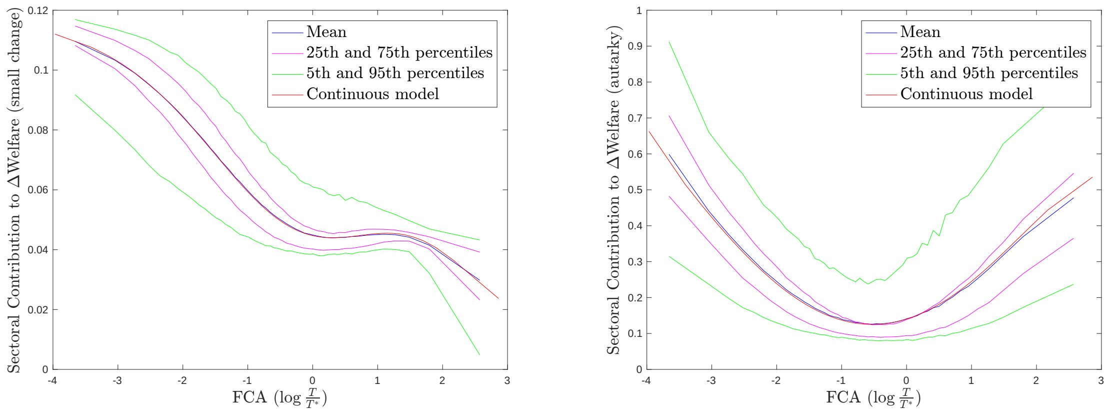


Figure 8: Sectoral Contribution to Welfare Gains from Trade

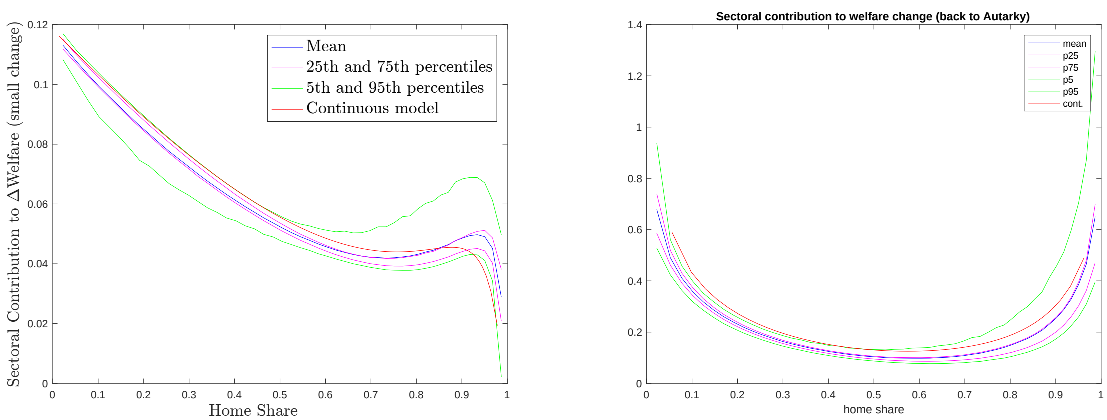


Figure 9: Sectoral Gains as a Function of Home Share, $1 - \Lambda_z$

sector tends to dampen systematically the gains from trade compared to what the corresponding home share would predict.

7.3 Sectoral contribution to welfare gains: dynamics

To illustrate quantitatively these systematic departures from the continuous benchmark, we run the following thought experiment. We study a liberalization episode in which only a subset of sectors see their trade costs change,[44] and ask how large are welfare gains depending on the set of sectors chosen to be liberalized, both in the short run and in the long run. For this exercise, we come back to modeling firm dynamics as we did in Section 6. Concretely, we measure the counterfactual gains from trade that would occur, over time, if one were to liberalize the top 2% of sectors chosen in terms of either

(i) their instantaneous contribution to welfare gains (computed myopically for year 1);

(ii) their home share $1 - \Lambda_z$ at the time of liberalization; and

[44] Formally, we bring back these sectors to autarky and report the negative of the corresponding losses.

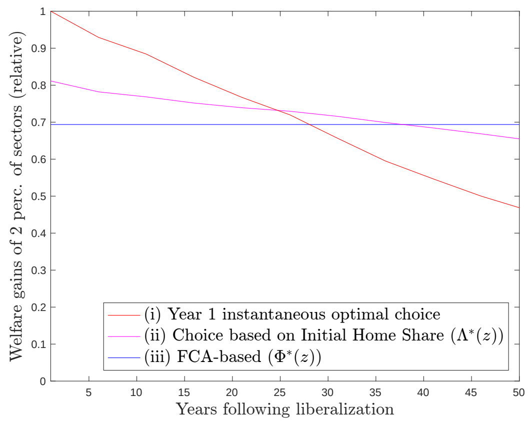


Figure 10: Dynamics Gains from Trade for a subset of sectors, relative to the optimal choice of sectors in year 1

(iii) their fundamental comparative advantage as captured by $1 - \Phi_z$.

In the continuous model, these three notions are equivalent, both in year 1 and over time. Therefore, the same measure of sectors would be selected according to either criteria. In the granular model, this is not the case anymore. First, (i) and (ii) differ because of the outlier sectors highlighted above. Second, (i) and (iii) differ because granular comparative advantage shapes 30% of trade patterns, so that fundamental comparative advantage alone is only a noisy predictor of the realized export patterns.

We let the economy evolve dynamically according to the process described in Section 6. Over time, individual firms' productivity evolve, and because of granularity, sectoral comparative advantage evolves in turn. Therefore, sectors that initially bring in large welfare gains because of granular effects tend to revert over time to bringing in the welfare gains that their fundamental comparative advantage predict.

Figure 10 plots the yearly contribution to welfare gains of the sectors selected according to selections rules (i), (ii) and (iii). Gains from trade are computed compared to a counterfactual with no change in trade costs, but with the same firm dynamics as in the liberalization episode. Welfare numbers are normalized by the maximal instantaneous welfare gains of 2% of sectors.

We see from Figure 10 that the optimal choice of sectors to liberalize depends on the time horizon considered. Nevertheless, the relative ranking of sectors is slow-moving, so that for example, as long as the time horizon considered in less than 25 years, the ranking of selection criteria is unambiguous: (i) is preferred over (ii), which is preferred over (iii). Only over the very long run are the yearly gains from trade higher for sectors chosen based on their fundamental comparative advantage. This suggests that accounting for the role played by large firms in bringing in welfare gains from liberalization matters both in the short and medium-to-long run.

8 Conclusion

Firms play a pivotal role in international trade: much of exports is done by a small number of very large firms that enjoy substantial market shares. This research develops a quantified general equilibrium model that is well-suited to account for this fact. It contrasts two sources of comparative advantage: a *fundamental* comparative advantage, which is a sector-level characteristic stemming from technological characteristics shared by all firms in a sector-country pair, and a *granular* comparative advantage, driven by goods that some individual firms happen to master to produce in an idiosyncratic way. To do so, we rely on an international trade model that features technology differences between sectors, and firm heterogeneity within sectors a la Melitz (2003). Different from the standard models in the literature, we relax the assumption of the continuum of firms within sector, allowing for individual firms to play a determinant role in shaping sectoral exports. The estimated model implies that thirty percent of trade flows is explained by granular forces, and that sectors with the highest export shares are more likely to be of granular origin than sectors with average export shares. Furthermore, extending the model to allow for firm-level productivity dynamics explains the majority of the change in the country's comparative advantage over time. Failure of a single large firm in a granular sector has dramatic effects on the relative export standing of the sector. Lastly, we show that the welfare gains from liberalizing a sector are shaped by the extent to which this sector is granular.

A Theory Appendix

A.1 Derivations and proofs for the continuous model of Section 2

Using (7) to write the zero profit conditions for entry into the home and foreign market, which define $\varphi_h(z)$ and $\varphi_f(z)$ respectively, we have:

$$\begin{cases} \pi_z(\omega) = wF\left[\left(\frac{\varphi(\omega)}{\varphi_h(z)}\right)^{\sigma-1} - 1\right]^+ + w^*F^*\left[\left(\frac{\varphi(\omega)}{\varphi_f(z)}\right)^{\sigma-1} - 1\right]^+, \\ \varphi_h(z) = \frac{\sigma}{\sigma-1}\frac{w}{P(z)}\left(\frac{\sigma wF}{\alpha_z Y}\right)^{1/(\sigma-1)}, \\ \varphi_f(z) = \frac{\sigma}{\sigma-1}\frac{\tau w}{P^*(z)}\left(\frac{\sigma w^*F^*}{\alpha_z Y^*}\right)^{1/(\sigma-1)}. \end{cases} \tag{A1}$$

Using (5) and the markup pricing rule

$$p_z(\omega) = \begin{bmatrix} \dfrac{\sigma}{\sigma-1}\dfrac{w}{\varphi_z(\omega)}, & \text{if variety } \omega \text{ is by home firm,} \\ \dfrac{\sigma}{\sigma-1}\dfrac{\tau w^*}{\varphi_z^*(\omega)}, & \text{otherwise,} \end{bmatrix} \tag{A2}$$

we can calculate the price index in sector z in the home market:

$$P(z) = \frac{\sigma w}{\sigma-1}\left(\frac{\kappa T(z)}{\kappa-1}\right)^{\frac{1}{1-\sigma}}\varphi_h(z)^{\kappa-1}\left[1 + \left(\frac{\tau w^*}{w}\right)^{1-\sigma}\frac{T^*(z)}{T(z)}\left(\frac{\varphi_h^*(z)}{\varphi_h(z)}\right)^{\sigma-1-\theta}\right]^{\frac{1}{1-\sigma}},$$

where we denote $\kappa \equiv \theta/(\sigma-1) > 1$. Combining this expression with the cutoff condition in (A1), we have:

$$\begin{cases} \varphi_h(z) = \left(\frac{\kappa T(z)}{\kappa-1}\right)^{\frac{1}{\theta}}\left[1 + \left(\frac{w}{\tau w^*}\right)^{\theta}\frac{T^*(z)}{T(z)}\right]^{\frac{1}{\theta}}\left(\frac{\sigma wF}{\alpha_z Y}\right)^{\frac{1}{\theta}}, \\ \varphi_h^*(z) = \frac{\tau w^*}{w}\varphi_h(z), \\ \varphi_f(z) = \tau\frac{P(z)}{P^*(z)}\left(\frac{w^*F^*/Y^*}{wF/Y}\right)^{\frac{1}{\sigma-1}}\varphi_h(z), \\ P(z) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi_h(z)}\left(\frac{\sigma wF}{\alpha_z Y}\right)^{\frac{1}{\sigma-1}} \end{cases} \tag{A3}$$

Symmetric equation hold for cutoffs and the price index in the foreign market. Therefore, we have fully solved for industry equilibrium in sector z given aggregate variables (w, w^*, Y, Y^*) and exogenous parameters, including Ricardian productivity $T(z)$ and $T^*(z)$.

We define $\Phi(z) = \int_{\omega\in M^*(z)} s_z(\omega)\mathrm{d}\omega$ and use equations (4), (A1) and (A3) to calculate this integral to obtain expression (9) in the text.[45]

[45] In a more general case in which the fixed cost of market entry depend both on the source-country j and the destination country i, e.g. F_{ji} paid in units of domestic labor (at cost w_jF_{ji}), the foreign share in country i is given by:

$$\frac{(w_j\tau)^{-\theta}(w_jF_{ji})^{1-\kappa}T_j(z)}{w_i^{-\theta}(w_iF_{ii})^{1-\kappa}T_i(z) + (w_j\tau)^{-\theta}(w_jF_{ji})^{1-\kappa}T_j(z)} = \frac{1}{1 + \left(\frac{w_j\tau}{w_i}\right)^{\theta}\frac{T_i(z)}{T_j(z)}\left(\frac{w_jF_{ji}}{w_iF_{ii}}\right)^{\kappa-1}},$$

Table A1: Revenue split in sector z in the home market (shares)

	Home expenditure on Home varieties $= [1-\Phi(z)]\alpha_z Y$		Home expenditure on Foreign varieties $= \Phi(z)\alpha_z Y$	
	Home Income	Foreign Income	Home Income	Foreign Income
Production (Labor)	$\frac{\sigma-1}{\sigma}$	–	–	$\frac{\sigma-1}{\sigma}$
Entry Cost (Labor)	$\frac{\kappa-1}{\sigma\kappa}$	–	$\frac{\kappa-1}{\sigma\kappa}$	–
Profits	$\frac{1}{\sigma\kappa}$	–	–	$\frac{1}{\sigma\kappa}$

Note: the total expenditure on sector z varieties at home equals $\alpha_z Y$, and the table reports the split of this expenditure into incomes of various factors in home at in foreign.

In Table A1 we provide income accounting in the domestic market for sector z, i.e. the split of the total sectoral expenditure $\alpha_z E$ into the incomes of various factors at home and in foreign. As just discussed, the share $\Phi(z)$ of the expenditure goes to foreign firms, while the complementary share $1-\Phi(z)$ goes to home firms. The revenues of the firms are split between production labor and operating profits, with the latter in turn split between net profits and fixed costs of entry (payed to labor in the country of entry). Given constant markup pricing and linear production, fraction $(\sigma-1)/\sigma$ of sales covers the operating costs (payment to production labor), while the remaining fraction $1/\sigma$ is operating profits. With Pareto productivity distribution and given that the marginal firm just breaks even paying the entry cost wF, we calculate that fraction $1/\kappa$ of operating profits goes to cover fixed costs and the remaining share $(\kappa-1)/\kappa$ is net profits of all firms in the sector, where recall that $\kappa \equiv \theta/(\sigma-1) > 1$. This applies to both domestic and foreign firms, however for foreign firms the operating costs and net profits are income of the foreign factors, while entry costs go to local labor. These results are summarized in Table A1 for the domestic market, and mirror-image results hold in the foreign market.

Next, we use Table A1 to calculate the aggregate demand for home labor (in value terms):[46]

$$wL = E\int_0^1 \alpha_z \left[\frac{\sigma\kappa-1}{\sigma\kappa}\big(1-\Phi(z)\big) + \frac{\kappa-1}{\sigma\kappa}\Phi(z)\right]\mathrm{d}z + E^*\int_0^1 \alpha_z \frac{\sigma-1}{\sigma}\Phi^*(z)\mathrm{d}z,$$

where the first integral is the aggregate demand for domestic labor used to supply goods in the domestic market (both in production and in entry), while the second integral is the aggregate demand for domestic labor used in production for exports. Since L units of labor are supplied inelastically, this equation can be also viewed as the labor market clearing condition. Manipulating the equation above

and hence also decreases in the relative fixed costs of entry for foreign firms, $(w_j F_{ji})/(w_i F_{ii})$.

[46]Since all workers are homogenous and command the same wage in the labor market, the total value spent on labor expenditure wL is sufficient, given the wage rate w, to recover the total demand for units of labor, L. In this model, it is more convenient to do the calculations in the value terms, however, one can arrive to the same aggregate labor demand by aggregating the physical demand for labor of individual firms.

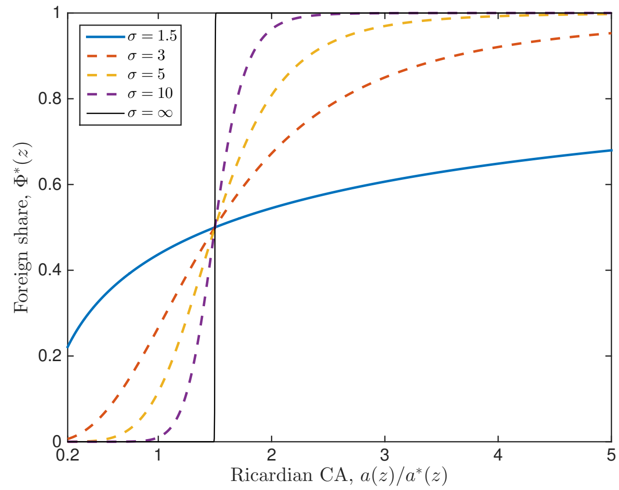


Figure A1: Foreign share across sectors in the DFS-Melitz model

Note: Symmetric countries with $\kappa = 1.25$, $\tau = 1.5$, $\alpha_z \equiv 1$, $M(z) \equiv 1$, $\underline{\varphi}(z)$ drawn from a uniform distribution on $[1, 5]$ and $\underline{\varphi}^*(z) \equiv 6 - \underline{\varphi}(z)$. The other parameters are inconsequential when countries are symmetric.

and using the trade balance condition (12), we can rewrite the labor market clearing condition as:

$$wL = \frac{\sigma\kappa - 1}{\sigma\kappa} E, \tag{A4}$$

and a symmetric condition holds for foreign. Condition (13) states that labor income in the model is a constant share of total income, with the complementary share coming from firm profits.

DFS Limit The continuous DFS-Melitz benchmark admits as a limiting case the classical DFS formulation when within-sector firm heterogeneity collapses. Specifically, the DFS model emerges as a limit of the DFS-Melitz model when $\theta, \sigma \to \infty$, $F \to 0$, while at the same time holding constant $\kappa = \theta/(\sigma - 1)$, σF and the following productivity parameters: $a(z) \equiv T(z)^{1/\theta}$ and $a^*(z) \equiv T^*(z)^{1/\theta}$. Note that our choice of notation $a(z)$ and $a^*(z)$ parallels the original notation in DFS, as in the limit of $\theta \to \infty$, $a(z)/a^*(z)$ indeed becomes the measure of relative sectoral productivity of the countries. In what follows we use $a(z)/a^*(z)$ as our measure of comparative advantage rather than $\frac{T(z)}{T^*(z)}$, as this makes easier the comparison across calibrations with different values of θ and σ. In the DFS limit, the foreign shares $\Phi(z)$ and $\Phi^*(z)$ in (9) become step functions, defined by two cutoffs $\underline{z}, \bar{z} \in [0, 1]$. Specifically, we rank all sectors $z \in [0, 1]$ such that $a(z)/a^*(z)$ is a monotonically increasing function of z, and define the cutoffs to satisfy:

$$\frac{a(\underline{z})}{a^*(\underline{z})} = \frac{w}{\tau w^*} \qquad \text{and} \qquad \frac{a(\bar{z})}{a^*(\bar{z})} = \frac{\tau w}{w^*}, \tag{A5}$$

which implies $\underline{z} < \bar{z}$. For sectors $z \in [0, \underline{z})$, foreign is the only supplier of the good on both domestic and foreign markets, goods $z \in (\underline{z}, \bar{z})$ are non-traded and produced in both countries, and for goods

$z \in (\bar{z}, 1]$ home is the only world supplier.[47]

More generally, we can study the behavior of the DFS-Melitz economy as we vary θ and σ holding constant κ and σF. We do this in Figure A1 by plotting the foreign share $\Phi^*(z)$ (of home firms in foreign market) across sectors against the measure of sectoral comparative advantage $a(z)/a^*(z)$ for various values of parameter σ and θ. Specifically, we consider $\sigma \in \{1.5, 3, 5, 10, \infty\}$ and corresponding values of θ holding constant $\kappa = 1.25$. Recall from (10) that $\Phi^*(z)$ is exports of home normalized by the foreign expenditure on good z, and hence can be viewed as the direct consequence of the home's comparative advantage. We see from Figure A1 that foreign share is indeed increasing in the comparative advantage of the sector. When σ is small, the foreign share is rather flat across sectors, and it becomes steeper approaching a step function as σ increases towards infinity.

A.2 Derivations and proofs for the granular model of Section 3

We can rewrite (21)–(22) and their foreign counterparts as:

$$wL = wFK + Y\frac{S}{\mu_H} + Y^*\frac{1-S^*}{\mu_H^*}, \tag{A6}$$

$$w^*L^* = w^*F^*K^* + Y^*\frac{S^*}{\mu_F^*} + Y\frac{1-S}{\mu_F}, \tag{A7}$$

$$Y = wL + \left[YS\frac{\mu_H - 1}{\mu_H} - wF\chi K\right] + \left[Y^*(1-S^*)\frac{\mu_H^* - 1}{\mu_H^*} - w^*F^*(1-\chi^*)K^*\right], \tag{A8}$$

$$Y^* = w^*L^* + \left[Y^*S^*\frac{\mu_F^* - 1}{\mu_F^*} - w^*F^*\chi^*K^*\right] + \left[Y(1-S)\frac{\mu_F - 1}{\mu_F} - wF(1-\chi)K\right], \tag{A9}$$

$$0 = \left[Y^*(1-S^*) - w^*F^*(1-\chi^*)K^*\right] - \left[Y(1-S) - wF(1-\chi)K\right], \tag{A10}$$

where (A10) is the sum of (A6) and (A8), which is also the sum of (A7) and (A9), and where we used the following notation:

$$K = \int_0^1 K_z \mathrm{d}z, \qquad K^* = \int_0^1 K_z^* \mathrm{d}z,$$

$$\chi = \frac{1}{K}\int_0^1 \left(\sum_{i=1}^{K_z} \iota_{z,i}\right)\mathrm{d}z, \qquad \chi^* = \frac{1}{K^*}\int_0^1 \left(\sum_{i=1}^{K_z^*} \iota_{z,i}^*\right)\mathrm{d}z,$$

$$S = \int_0^1 \alpha_z \left(\sum_{i=1}^{K_z} \iota_{z,i}s_{z,i}\right)\mathrm{d}z, \qquad S^* = \int_0^1 \alpha_z \left(\sum_{i=1}^{K_z^*} \iota_{z,i}^*s_{z,i}^*\right)\mathrm{d}z,$$

$$\mu_H = \left[\frac{1}{S}\int_0^1 \alpha_z \left(\sum_{i=1}^{K_z} \frac{\iota_{z,i}s_{z,i}}{\mu(s_{z,i})}\right)\mathrm{d}z\right]^{-1}, \qquad \mu_F^* = \left[\frac{1}{S^*}\int_0^1 \alpha_z \left(\sum_{i=1}^{K_z^*} \frac{\iota_{z,i}^*s_{z,i}^*}{\mu(s_{z,i}^*)}\right)\mathrm{d}z\right]^{-1},$$

$$\mu_H^* = \left[\frac{1}{1-S^*}\int_0^1 \alpha_z \left(\sum_{i=1}^{K_z^*} \frac{(1-\iota_{z,i}^*)s_{z,i}^*}{\mu(s_{z,i}^*)}\right)\mathrm{d}z\right]^{-1}, \qquad \mu_F = \left[\frac{1}{1-S}\int_0^1 \alpha_z \left(\sum_{i=1}^{K_z} \frac{(1-\iota_{z,i})s_{z,i}}{\mu(s_{z,i})}\right)\mathrm{d}z\right]^{-1}.$$

Lemma A1 *$\chi = S = 1 - \Phi$, $\chi^* = S^* = 1 - \Phi^*$, $\mu_H = \mu_F = \mu$ and $\mu_H^* = \mu_F^* = \mu^*$.*

This lemma follows from the property of EKS model described in footnote 15.

[47] The only remaining equilibrium condition in the limiting case is a version of (12), after substituting in the limiting versions of expression (13) $wL = Y$ and $w^*L^* = Y^*$, which reads $(wL)/(w^*L^*) = \int_{\bar{z}}^1 \alpha_z \mathrm{d}z / \int_0^{\underline{z}} \alpha_z \mathrm{d}z$, and together with (A5) allows to solve for equilibrium $(w/w^*, \underline{z}, \bar{z})$.

A.3 Trade elasticity

Continuous Model Home shares in the continuous model are:

$$\Lambda_z = \frac{w^{-\theta}T_z}{w^{-\theta}T_z + (\tau w^*)^{-\theta}T_z^*} \qquad \text{and} \qquad \Lambda_z^* = \frac{w^{*-\theta}T_z^*}{w^{*-\theta}T_z + (\tau w)^{-\theta}T_z}.$$

It follows that :

$$\frac{\Lambda_z}{1-\Lambda_z}\frac{\Lambda_z^*}{1-\Lambda_z^*} = \tau^{2\theta}.$$

And finally:

$$\theta = \frac{1}{2}\frac{1}{\Delta\log\tau}\Delta\log\left(\frac{\Lambda_z}{1-\Lambda_z}\frac{\Lambda_z^*}{1-\Lambda_z^*}\right). \tag{A11}$$

Granular Model We know that in the granular model, the first expressions above are valid for the *expected* trade shares, i.e.:

$$E(\Lambda_z) = \frac{w^{-\theta}T_z}{w^{-\theta}T_z + (\tau w^*)^{-\theta}T_z^*} \qquad \text{and} \qquad E(\Lambda_z^*) = \frac{w^{*-\theta}T_z^*}{w^{*-\theta}T_z + (\tau w)^{-\theta}T_z}.$$

Therefore, by a similar reasoning as above in the continuous model:

$$\theta = \frac{1}{2}\frac{1}{\Delta\log\tau}\Delta\log\left(\frac{E(\Lambda_z)}{1-E(\Lambda_z)}\frac{E(\Lambda_z^*)}{1-E(\Lambda_z^*)}\right). \tag{A12}$$

Of course, we do not observe $E(\Lambda_z)$ but instead one realization of it for each sector, Λ_z. We calculate Λ_z and Λ_z^* before and after the reduction in τ, and calculate $\hat{\theta}_z$ according to (33).

A.4 Sectoral Contribution to Welfare Gains from Trade

In this section of the appendix, we normalize $w = 1$ at Home and $\omega \equiv w^*/w$.

Continuous Model The total home firm profits in both the home and foreign markets in sector z are:

$$\Pi_z = \frac{1}{\sigma\kappa}\Phi_z\alpha_z Y + \frac{1}{\sigma\kappa}[1-\Phi_z^*]\alpha_z Y^*.$$

We aggregate:

$$\Pi = \frac{1}{\sigma\kappa}\int_0^1 \alpha_z\big(Y\Phi_z + Y^*[1-\Phi_z^*]\big)\mathrm{d}z = \frac{1}{\sigma\kappa}Y.$$

where the second equality uses the trade balance condition. Since profits are a share $1/(\sigma\kappa)$ of income, the residual share goes to labor income $wL = Y(\sigma\kappa-1)/(\sigma\kappa)$. Since trade is not balanced sector-by-sector, some sectors generate a greater or a smaller profit share for the home country in a trade equilibrium (in general, $\Pi_z \neq \alpha_z Y/(\sigma\kappa)$).

We can therefore write:

$$W = \frac{Y/L}{P} = \frac{wL + \Pi}{PL} = \frac{Y}{L} \cdot \frac{1 + \overbrace{\int_0^1 \alpha_z \left(\frac{\Pi_z}{\alpha_z Y} - \frac{1}{\sigma\kappa} \right) \mathrm{d}z}^{=0}}{\exp \int_0^1 \alpha_z \log P_z \mathrm{d}z}.$$

We therefore have the following decomposition of welfare changes:

$$\hat{W} = \int_0^1 \alpha_z \hat{W}_z \mathrm{d}z$$

where we use $\hat{Y} = \hat{w} = 0$ and we define sectoral gains as:

$$\hat{W}_z \equiv \frac{\mathrm{d}\Pi_z}{\alpha_z Y} - \mathrm{d}\log P_z \tag{A13}$$

We then show that:

$$\begin{aligned} \hat{W}_z &\equiv \frac{\mathrm{d}\Pi_z}{\alpha_z Y} - \mathrm{d}\log P_z \\ &= \frac{1}{\sigma\kappa}\Phi_z\hat{\Phi}_z - \frac{1}{\sigma\kappa}\Phi_z^*\hat{\Phi}_z^*\frac{Y^*}{Y} + \frac{1}{\sigma\kappa}[1-\Phi_z^*]\frac{Y^*}{Y}\hat{\omega} - \frac{1}{\theta}\hat{\Phi}_z \\ &= \frac{1}{\sigma\kappa}\Phi_z(1-\Phi_z)\theta(\hat{\tau}+\hat{\omega}) - \frac{1}{\sigma\kappa}\Phi_z^*(1-\Phi_z^*)\frac{Y^*}{Y}\theta(\hat{\tau}-\hat{\omega}) + \frac{1}{\sigma\kappa}[1-\Phi_z^*]\frac{Y^*}{Y}\hat{\omega} - \frac{1}{\theta}(1-\Phi_z)\theta(\hat{\tau}+\hat{\omega}), \end{aligned}$$

where $\hat{\omega} = \hat{w}^* - \hat{w} = \hat{w}^*$. The first three terms are from profits and they are proportional to the change in the net exports in the sector, so that we can rewrite:

$$\hat{W}_z = \frac{1}{\sigma\kappa}\frac{\mathrm{d}(X_z - X_z^*)}{\alpha_z Y} - \frac{1}{\theta}\hat{\Phi}_z.$$

So the welfare gains from the sector are a weighted sum of an increase in net exports (divided by $\sigma\kappa$ to convert to increase in profits) and a reduction in home share (divided by θ to convert to price index reduction).

After some further algebra, it is easy to show that Net exports are a function of Φ, Φ^*:

$$\frac{\mathrm{d}NX_z}{\alpha_z Y} = -\theta\hat{\tau}\big[\frac{Y^*}{Y}\Phi_z^*(1-\Phi_z^*) - \Phi_z(1-\Phi_z)\big]$$

and Foreign share are a function of Φ:

$$\mathrm{d}\log(1-\Phi_z) = -\Phi_z\theta(\hat{\tau}+\hat{\omega})$$

Granular Model We define the profits of domestic firms in sector z Π_z to be the ones made in both home and foreign markets:

$$\Pi_z = \sum_{i=1}^{K_z}(1-\iota_{z,i})\Pi_{z,i} + \sum_{i=1}^{K_z^*}\iota_{z,i}^*\Pi_{z,i}^*.$$

Welfare writes:

$$\log W = \log \frac{Y}{PL} = \log\left(w + \frac{\Pi}{L}\right) - \log P$$
$$= \log\left(w + \frac{1}{L}\int_0^1 \Pi_z \mathrm{d}z\right) - \int_0^1 \alpha_z \log P_z \mathrm{d}z$$

Given our normalization $w = 1$, we can calculate:

$$\hat{W} = \frac{1}{Y}\int_0^1 (\mathrm{d}\Pi_z)\mathrm{d}z - \int_0^1 \alpha_z \mathrm{d}\log P_z \mathrm{d}z.$$

We can define the contribution of sector z to welfare as:

$$\hat{W}_z = \frac{\Pi_z}{\alpha_z Y}\mathrm{d}\log\Pi_z - \mathrm{d}\log P_z$$

such that $\hat{W} = \int_0^1 \alpha_z \hat{W}_z \mathrm{d}z$.

B Estimation Appendix

Variable markup case is identical to the constant markup case except for the computation of markups and prices given general equilibrium variables (see Section 4.2). To make the procedure faster, we use the following approximation:

- We show that the number of firms in the variable markup case is larger than in the constant markup case (higher markups of large firms resulting in more entry by small firms), given the same parameters of the model. Therefore, we start with the set of firms that would enter in the constant markup equilibrium.
- For these firms, we compute the Atkeson-Burstein markup distribution and market share distribution, and the corresponding price index.

  For all less productive firms, we approximate their markup at the constant markup level and check whether with this markup and this price index they would enter or not.
- This is the equilibrium. It turns out to overpredict entry somewhat, but yields a good approximation overall. We compare the outcome (moments) under these procedure and under the full procedure, and the difference is negligible.

B.1 Identification

[TO BE COMPLETED]

B.2 Shutting down granularity

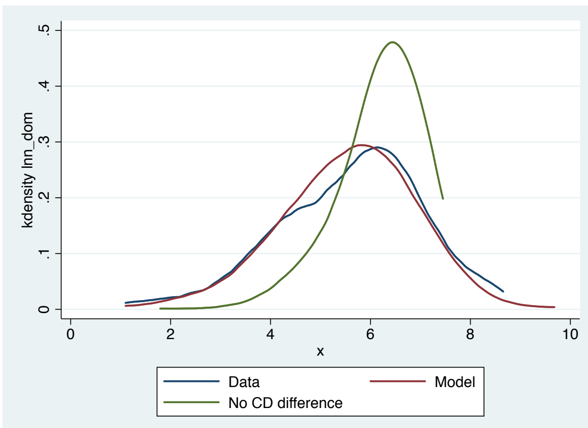
1. By increase θ.

2. By reducing σ.
3. By increase $T(z)$ and reducing F, keeping $FT(z)$ constant, to increase the median number of firms per sector from 270 to 2,700.
4. By replacing Pareto with log-normal with the same variance.

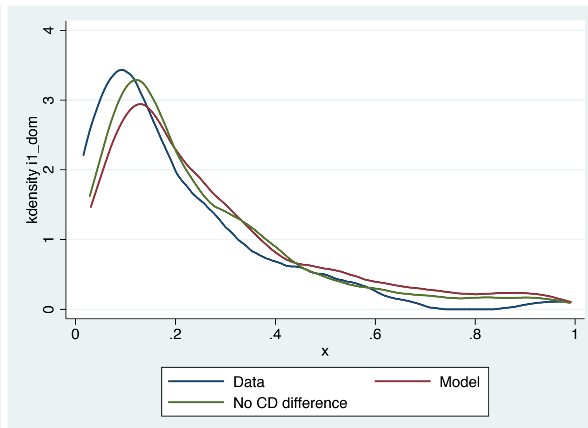
C Additional Results

C.1 Model without variation in the Cobb-Douglas shares

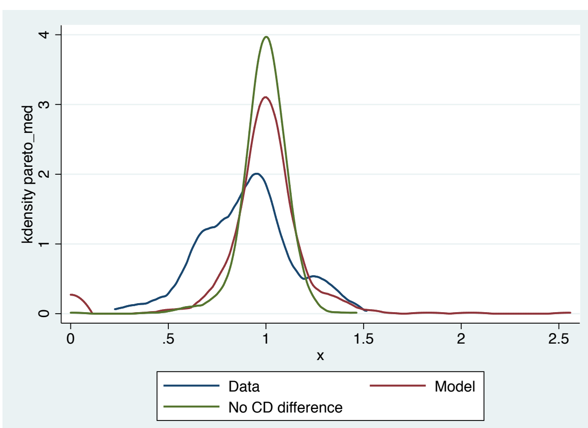
(a) Log number of French firms



(b) Top French-firm market share



(c) Pareto shapes of sales distributions



(d) Exports relative to domestic sales

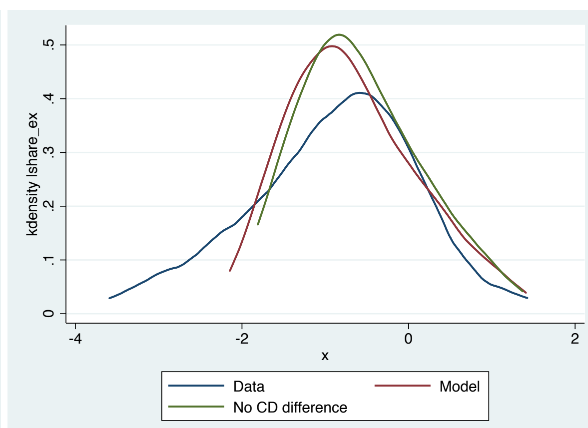
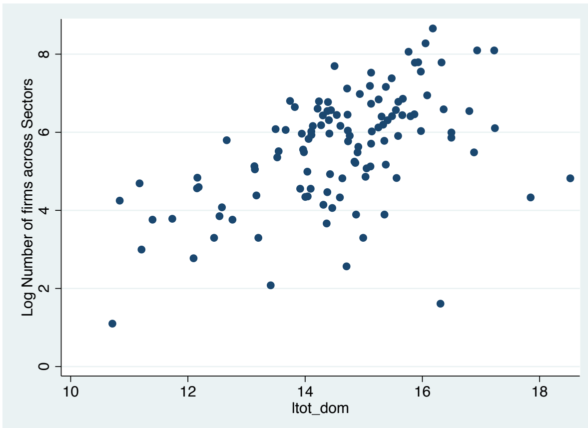


Figure A2: Distributions across sectors: model without variation in Cobb-Douglas shares

Note: the figure replicates figure 1 adding the results from a model (with constant markups) holding all parameters the same apart from the Cobb-Douglas shares which are made uniform across sectors (and equal to $1/468$ where 468 is the number of simulated sectors).

C.2 Extensive versus intensive margin

(a) Extensive margin in the data



(b) Extensive margin: model vs data

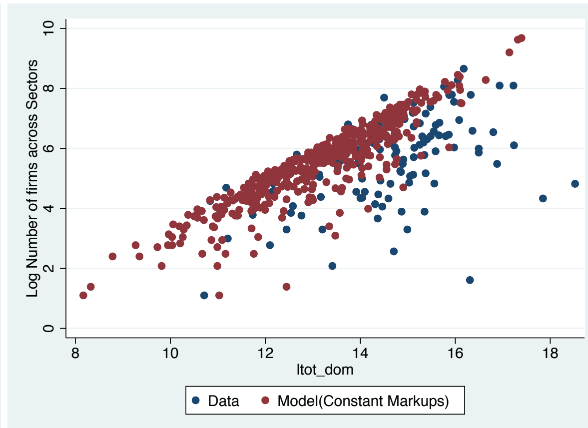


Figure A3: Extensive versus intensive margin of cross-sector variation in sales

Note: the figures plot the extensive margin—variation across sectors in the log number of firms against the total sectoral sales.

References

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012): "The Network Origins of Aggregate Fluctuations," *Econometrica*, 80(5), 1977–2016.

Amiti, M., O. Itskhoki, and J. Konings (2015): "International Shocks and Domestic Prices: How Large Are Strategic Complementarities?," http://www.princeton.edu/~itskhoki/papers/DomesticPrices.pdf.

Anderson, J. E., and E. van Wincoop (2004): "Trade Costs," *Journal of Economic Literature*, 42(3), 691–751.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?," *American Economic Review*, 102(1), 94–130.

Atkeson, A., and A. Burstein (2008): "Trade Costs, Pricing-to-Market, and International Relative Prices," *American Economic Review*, 98(5), 1998–2031.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2015): "Global Firms," .

Bernard, A. B., S. J. Redding, and P. K. Schott (2007): "Comparative Advantage and Heterogeneous Firms," *Review of Economic Studies*, 74(1), 31–66.

Broda, C., and D. Weinstein (2006): "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 121(2), 541–85.

Carvalho, V. M., and X. Gabaix (2013): "The Great Diversification and Its Undoing," *American Economic Review*, 103(5), 1697–1727.

Carvalho, V. M., and B. Grassi (2014): "Firm Dynamics and the Granular Hypothesis," http://www.crei.cat/~carvalho/research.html.

Chaney, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98(4), 1707–1721.

Chor, D. (2010): "Unpacking sources of comparative advantage: A quantitative approach," *Journal of International Economics*, 82(2), 152–167.

Costinot, A., D. Donaldson, and I. Komunjer (2012): "What Goods Do Countries Trade? A Quantitative Exploration of Ricardo's Ideas," *Review of Economic Studies*, 79(2), 581–608.

Di Giovanni, J., and A. A. Levchenko (2012): "Country Size, International Trade, and Aggregate Fluctuations in Granular Economies," *Journal of Political Economy*, 120(6), 1083–1132.

Di Giovanni, J., A. A. Levchenko, and I. Méjean (2014): "Firms, Destinations, and Aggregate Fluctuations," *Econometrica*, 82(4), 1303–1340.

Dornbusch, R., S. Fischer, and P. A. Samuelson (1977): "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, 67(5), 823–839.

Eaton, J., and S. Kortum (2002): "Technology, Geography, and Trade," *Econometrica*, 70(5), 1741–1779.

Eaton, J., S. S. Kortum, and S. Sotelo (2012): "International Trade: Linking Micro and Macro," NBER Working Papers No. 17864.

Edmond, C., V. Midrigan, and D. Y. Xu (2015): "Competition, markups, and the gains from international trade," *The American Economic Review*, 105(10), 3183–3221.

Fan, H., E. L.-C. Lai, and H. S. Qi (2015): "Heterogeneous Firms and Trade Liberalization: Theory and Evidence," http://ihome.ust.hk/~elai/.

Fernandes, A. M., P. J. Klenow, S. Meleshchuk, M. D. Pierola, and A. Rodríguez-Clare (2015): "The Intensive Margin in Trade: Moving Beyond Pareto," http://www.klenow.com/.

Freund, C., and M. D. Pierola (2012): "Export superstars," World Bank Policy Research Working Paper No. 6222.

Gabaix, X. (2009): "Power Laws in Economics and Finance," *Annual Review of Economics*, 1(1), 255–294.

——— (2011): "The Granular Origins of Aggregate Fluctuations," *Econometrica*, 79(3), 733–772.

Gabaix, X., and R. Ibragimov (2011): "Rank - 1 / 2: A Simple Way to Improve the OLS Estimation of Tail Exponents," *Journal of Business and Economic Statistics*, 29(1), 24–39.

Hanson, G. H., N. Lind, and M.-A. Muendler (2015): "The Dynamics of Comparative Advantage," unpublished.

Hottman, C., S. J. Redding, and D. E. Weinstein (2015): "Quantifying the Sources of Firm Heterogeneity," *Quarterly Journal of Economics*, forthcoming.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695–1725.

Neary, J. P. (2003): "Presidential Address: Globalization and Market Structure," *Journal of the European Economic Association*, 1(2-3), 245–271.

——— (2012): "Superstar Firms and Trade," Nottingham Lectures in International Economics.

Neary, P. (2015): "International Trade in General Oligopolistic Equilibrium," Economics series working papers, University of Oxford, Department of Economics.

Okubo, T. (2009): "Firm heterogeneity and Ricardian comparative advantage within and across sectors," *Economic Theory*, 38(3), 533–559.